Effects of boundary layer forcing on wing-tip vortices

Author: Samantha Shaw-Ward
Supervisor: Dr. David Birch

Thesis submitted for the degree of Doctor of Philosophy

Faculty of Engineering and Physical Sciences

September 2015
Statement of Originality

This is to certify that, to the best of my knowledge, the content of this thesis is original and my own work. I have not submitted it for any other degree or published work, and any sources I used have been acknowledged.

Samantha Shaw Ward
Abstract

The nature of turbulence within wing-tip vortices has been a topic of research for decades, yet accurate measurements of Reynolds stresses within the core are inherently difficult due to the bulk motion wandering caused by initial and boundary conditions in wind tunnels. As a result, characterization of a vortex as laminar or turbulent is inconclusive and highly contradicting. This research uses several experimental techniques to study the effects of broadband turbulence, introduced within the wing boundary layer, on the development of wing-tip vortices. Two rectangular wings with a NACA 0012 profile were fabricated for the use of this research. One wing had a smooth finish and the other rough, introduced by P80 grade sandpaper. Force balance measurements showed a small reduction in wing performance due to surface roughness for both 2D and 3D configurations, although stall characteristics remained relatively unchanged.

Seven-hole probes were purpose-built and used to assess the mean velocity profiles of the vortices five chord lengths downstream of the wing at multiple angles of attack. Above an incidence of \( \approx 4^\circ \), the vortices were nearly axisymmetric, and the wing roughness reduced both velocity gradients and peak velocity magnitudes within the vortex. Laser Doppler velocimetry was used to further assess the time-resolved vortex at an incidence of \( 5^\circ \). Evidence of wake shedding frequencies and wing shear layer instabilities at higher frequencies were seen in power spectra within the vortex. Unlike the introduction of freestream turbulence, wing surface roughness did not appear to increase wandering amplitude. A new method for removing the effects of vortex wandering is proposed with the use of carefully selected high-pass filters. The filtered data revealed that the Reynolds stress profiles of the vortex produced by the smooth and rough wing were similar in shape, with a peak occurring away from the vortex centre but inside of the core. Single hot-wire measurements in the 2-D wing wake revealed the potential origin of dominant length-scales observed in the vortex power spectra. At angles above \( 5^\circ \), the 2-D wing wake had both higher velocity deficits and higher levels of total wake kinetic energy for the rough wing as compared to the smooth wing.
Acknowledgments

I would like to thank Dr. David Birch for his patience and guidance throughout this entire research project. Without his encouragement this would not have been possible.

My gratitude also goes to Dr. Paul Hayden, Dr. Paul Nathan and Mr. Alan Wells for their help and assistance manufacturing equipment and software programming. Without their ceaseless efforts to ensure the smooth running of experiments and constant advancement of laboratory software and equipment, this research could not have been completed on time.

Last but not least, I would like to thank my family for their constant support throughout my academic career. My parents may not have always understood what I was doing, but they never lost their enthusiasm in my work. Jonathan, thank you for putting up with the highs and lows that come with this journey. Thank you all for never losing faith in me.
# Contents

List of figures vii

List of tables xvi

1 Introduction and Motivation 1
  1.1 Wake vortices 1
  1.2 Objectives 3
  1.3 Thesis layout 4

2 Vortex Flows 6
  2.1 Fundamental concepts 6
    2.1.1 Mean velocity profiles 7
    2.1.2 Consideration of turbulence 8
  2.2 Wing tip vortices 10
    2.2.1 Vortex Rollup 10
    2.2.2 Wandering 11
    2.2.3 Vortex Control 12
  2.3 Flows over Rough Walls 14
  2.4 Vortex Measurement Techniques 15
    2.4.1 Intrusive techniques 17
      2.4.1.1 Multi-hole Pressure Probes 17
      2.4.1.2 Hot wires 18
    2.4.2 Non-intrusive (optical) techniques 19
      2.4.2.1 Laser Doppler Anemometry 19
      2.4.2.2 Particle Image Velocimetry 20
3 Experimental Apparatus and Method

3.1 Wing profile selection, design and fabrication of wing

3.2 Wind tunnels, facilities and instrumentation

3.2.1 A-Tunnel
3.2.1.1 Seven hole probe fabrication and instrumentation
3.2.1.2 Uncertainty analysis
3.2.1.3 Novel 19 hole probe development
3.2.1.4 Hot wire setup

3.2.2 Aero-tunnel
3.2.2.1 Force balance
3.2.2.2 Uncertainty analysis
3.2.2.3 Laser Doppler anemometry experimental setup
3.2.2.4 Uncertainty analysis
3.2.2.5 Flow seeding

3.3 Novel multi-hole probe calibration scheme
3.3.1 Demonstration of need for new calibration scheme
3.3.2 Description of calibration procedure

3.4 Post-processing
3.4.1 Vortex centre
3.4.2 Velocity
3.4.3 Vorticity
3.4.4 Circulation
3.4.5 Lift and drag estimates
3.4.6 Power spectra
3.4.6.1 LDA
3.4.6.2 Hot-wire
3.4.7 LDA filter signal processing

4 Wing performance

4.1 Lift characteristics
4.2 Drag characteristics
E  Plots of raw LDA data 137

F  Paper presented at the AIAA 51st Aerospace Sciences Meeting, 2013 143

G  AIAA Journal paper, 2014 158

H  Journal of Fluid Mechanics paper, submitted August 2015 and currently in review 182
### List of Figures

3.1.1 (a) Surface map and (b) Roughness height distribution for the P80 grade sandpaper used ........................................... 24
3.2.1 A-Tunnel experimental setup schematic ................................. 25
3.2.2 Tip of a seven hole probe .................................................. 26
3.2.3 Numbering scheme of a 19 hole probe, alongside the tip geometry schematic .................................................... 28
3.2.4 Schematic of Aero tunnel using the 27 mm LDA lens ............... 31
3.2.5 Schematic of Aero tunnel using the 60 mm LDA lens ............... 31
3.3.1 Numbering scheme of a seven-hole probe, looking aft ............. 33
3.3.2 Seven hole probe co-ordinate system (adapted from Ericksen et al., 1995) ................................................................. 34
3.3.3 (a) Contours of normalized vorticity, $\zeta r_c/v_0$, and (b) Contours of normalized vorticity showing calibration sector overlay using the conventional calibration methods ..................................... 35
3.3.4 Contours of normalized vorticity, $\zeta r_c/v_0$, using novel calibration method. Data is identical to that in 3.3.3. ................... 37
3.4.1 Polar and Cartesian coordinate system definitions, looking up- stream, with the origin located at the vortex centre .................... 39
4.1.1 2-D lift data versus angle of attack .................................... 45
4.1.2 3-D lift data versus angle of attack .................................... 46
4.1.3 Variation of normalized tip vortex strength and 3D force balance lift data with $\alpha$ ................................................. 46
4.2.1 Drag vs lift coefficient ..................................................... 47
4.2.2 Variation of induced drag ($C_{D_i}$) and total drag ($C_D$) with $\alpha$ ... 48
5.0.1 Vortex parameters, in terms of $r_c/c$ and $v_0$, against $x/c$: ($\circ$), Smooth wing at $\alpha = 8^\circ$; ($\bullet$), Rough wing at $\alpha = 5^\circ$; ($\ast$), Devenport et al. (1996) at $\alpha = 5^\circ$. .......................................................... 50

5.1.1 Vortex size, in terms of $r_c/c$, against wing incidence: ($\circ$), Smooth wing; ($\bullet$), Rough wing; ($+$), Devenport et al. (1996). $Re_c = 1.1 \times 10^5$ for smooth and rough wings, and $Re_c = 5.3 \times 10^5$ for Devenport et al. (1996) data. .......................................................... 52

5.1.2 Vorticity contours for $\alpha = 5^\circ$. (a) smooth wing and (b) rough wing. Numerical values denote normalized vorticity $\zeta r_c/v_0$. .......................................................... 53

5.1.3 Tangential velocity contours for $\alpha = 5^\circ$: (a) smooth wing and (b) rough wing. Numerical values denote normalized velocity $v_\theta/U_\infty$. .......................................................... 54

5.2.1 Circulation profiles normalized by total circulation calculated from (3.4.7): ($\circ$), Smooth wing; ($\bullet$), Rough wing; (—), (2.1.1). .......................................................... 55

5.2.2 Normalized vorticity profiles: ($\circ$), Smooth wing; ($\bullet$), Rough wing; (—), (3.4.3). .......................................................... 55

5.2.3 Axial velocity profiles normalized by maximum tangential velocity: ($\circ$), Smooth wing; ($\bullet$), Rough wing; (—) and (—), (2.1.3). .......................................................... 56

5.2.4 Circumferentially averaged tangential velocity profiles normalized by maximum tangential velocity: ($\circ$), Smooth wing; ($\bullet$), Rough wing; (—), (2.1.5). .......................................................... 56

6.2.1 Reynolds stress as a function of non-dimensional position across the vortex - (a) smooth wing and (b) rough wing. (—), $\overline{u'^2}/v_0^2$; (—$\blacksquare$), $\overline{v'^2}/v_0^2$; (—), $\overline{w'^2}/v_0^2$. .......................................................... 59

6.2.2 Circumferentially averaged Reynolds stresses as a function of $\eta$: ($\circ$), $\overline{v'^2}/v_0^2$; (□), $\overline{v'^2}/v_0^2$; (△), $v'_\theta v'_\eta/v_0^2$. Open symbols, smooth wing; filled symbols, rough wing. .......................................................... 60

6.3.1 Smooth wing cross-flow velocity spectra normalized by standard deviation, $\alpha = 5$: (a) $v'$ spectra, $\Phi/v'^2$; (b) $w'$ spectra, $\Phi/w'^2$. Legend values indicate position $\eta$ across the vortex. .......................................................... 62

6.3.2 Rough wing cross-flow velocity spectra normalized by standard deviation, $\alpha = 5$: (a) $v'$ spectra, $\Phi/v'^2$; (b) $w'$ spectra, $\Phi/w'^2$. Legend values indicate position $\eta$ across the vortex. .......................................................... 63
6.3.3 Axial velocity spectra normalized by standard deviation, $\Phi/u'^2$ and $\alpha = 5$: (a) smooth wing; (b) rough wing. Legend values indicate position $\eta$ across the vortex. .................. 64

6.4.1 Circumferentially averaged $v_\theta$ vs. $\eta$: (◦), 12 Hz bandpass data; and (—), equation (2.1.5) .................. 65

6.4.2 (a) Isocontours of $v_\theta^2/v_0^2$ ($\times 10^{-4}$) and (b) Isocontours of $v_r^2/v_0^2$ ($\times 10^{-3}$) for the bandpass 12 Hz filter, (—) outline of the vortex core for reference. .................. 66

6.4.3 Circumferentially averaged $v_\theta$ vs. $\eta$ for the 330Hz bandpass filter: (◦), smooth wing; (bullet), rough wing; and (—), equation (2.1.5) .. 66

6.4.4 Smooth wing data for the bandpass 330 Hz filter (a) Isocontours of $v_\theta^2/v_0^2$ ($\times 10^{-4}$) and (b) Isocontours of $v_r^2/v_0^2$ ($\times 10^{-3}$), (—) outline of the vortex core for reference. .................. 67

6.4.5 Rough wing data for the bandpass 330 Hz filter (a) Isocontours of $v_\theta^2/v_0^2$ ($\times 10^{-4}$) and (b) Isocontours of $v_r^2/v_0^2$ ($\times 10^{-3}$), (—) outline of the vortex core for reference. .................. 68

6.5.1 Maximum tangential Reynolds stress for each highpass filter of frequency, $f_H$. Filled symbols denote rough wing data, and $f_H = 0$ Hz indicates unfiltered data. .................. 71

6.5.2 Circumferentially averaged high-pass filtered Reynolds stress profiles for (a) smooth wing - $f_H = 0.5$ Hz and (b) rough wing - $f_H = 1$ Hz; (◦), $v_\theta^2/v_0^2$; (□), $v_r^2/v_0^2$. .................. 72

7.1.1 Mean velocity profiles, filled symbols for rough wing data. (◦) $\alpha = 5^\circ$; (□) $\alpha = 8^\circ$; (△) $\alpha = 10^\circ$; (♦) Hah and Lakshminarayana (1982) .................. 74

7.1.2 2D drag data - open symbols from force balance measurements; filled symbols from wake data. (◦) rough wing; (□) smooth wing. 75

7.2.1 Turbulence intensity profiles, filled symbols for rough wing data. (◦) $\alpha = 5^\circ$; (□) $\alpha = 8^\circ$; (△) $\alpha = 10^\circ$; (♦) Hah and Lakshminarayana (1982) .................. 76
7.3.1 Spectra at the midpoint \((y/c = 0)\) of each wing wake. Successive spectra have been moved up a decade for clarity, and a \(-5/3\) slope has been shown for reference. ............................... 78

7.3.2 Hotwire data in the wake of the smooth wing at \(\alpha = 5^\circ\)
(a) Contours of power (logscale) from hotwire spectra and (b) Streamwise turbulence intensity. Frequencies \(f_1 = 11\) Hz, \(f_2 = 20\) Hz, \(f_3 = 51\) Hz, \(f_4 = 233\) Hz, and \(f_5 = 300\) Hz .............. 79

7.3.3 Hotwire data in the wake of the rough wing at \(\alpha = 5^\circ\) (a) Contours of power (logscale) from hotwire spectra and (b) Streamwise turbulence intensity. Frequencies \(f_1 = 11\) Hz, \(f_2 = 45\) Hz, \(f_3 = 150\) Hz, \(f_4 = 245\) Hz, and \(f_5 = 290\) Hz .............. 81

A.0.1 A schematic of the pitch automation traverse ................. 94

B.0.1 Numbering scheme of a seven-hole probe, looking aft. ........ 95

C.2.1 \(R^2\) of the surface fit, as a function of polynomial order. ......... 102

C.2.2 \(C_{P7}\) exhaustive calibration data (●) on the sixth order polynomial surface fit ......................................................... 102

C.2.3 Locations of D-Optimal sample sets for \(n = 28\) (○), \(n = 112\) (□), and \(n = 196\) (+) ............................................................. 103

C.2.4 Standard deviation as a percentage of dynamic pressure vs number of D-optimal points for each of the seven holes ............... 104

D.0.1 Velocity vector plot of seven hole probe data 5\(c\) behind the (a) Smooth wing and (b) Rough wing at \(\alpha = 2^\circ\) ................. 105

D.0.2 Velocity vector plot of seven hole probe data 5\(c\) behind the (a) Smooth wing and (b) Rough wing at \(\alpha = 4^\circ\) ...................... 106

D.0.3 Velocity vector plot of seven hole probe data 5\(c\) behind the (a) Smooth wing and (b) Rough wing at \(\alpha = 5^\circ\) ...................... 106

D.0.4 Velocity vector plot of seven hole probe data 5\(c\) behind the (a) Smooth wing and (b) Rough wing at \(\alpha = 6^\circ\) ...................... 107

D.0.5 Velocity vector plot of seven hole probe data 5\(c\) behind the (a) Smooth wing and (b) Rough wing at \(\alpha = 8^\circ\) ...................... 107
D.0.6 Velocity vector plot of seven hole probe data 5c behind the (a) Smooth wing and (b) Rough wing at α = 10°. . . . . . . . . . . . . 108
D.0.7 Velocity vector plot of seven hole probe data 5c behind the (a) Smooth wing and (b) Rough wing at α = 12°. . . . . . . . . . . . . 108
D.0.8 Velocity vector plot of seven hole probe data 5c behind the (a) Smooth wing and (b) Rough wing at α = 13°. . . . . . . . . . . . . 109
D.0.9 Velocity vector plot of seven hole probe data 5c behind the (a) Smooth wing and (b) Rough wing at α = 14°. . . . . . . . . . . . . 109
D.0.10 Axial velocity, \((u - U_\infty)/U_\infty\), profile of seven hole probe data at \(x = 5c\) and \(\alpha = 2°\); (○), Smooth wing; (●), Rough wing; (—) and (—−), (2.1.3). . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 110
D.0.11 Axial velocity, \((u - U_\infty)/U_\infty\), profile of seven hole probe data at \(x = 5c\) and \(\alpha = 4°\); (○), Smooth wing; (●), Rough wing; (—) and (—−), (2.1.3). . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 111
D.0.12 Axial velocity, \((u - U_\infty)/U_\infty\), profile of seven hole probe data at \(x = 5c\) and \(\alpha = 5°\); (○), Smooth wing; (●), Rough wing; (—) and (—−), (2.1.3). . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 112
D.0.13 Axial velocity, \((u - U_\infty)/U_\infty\), profile of seven hole probe data at \(x = 5c\) and \(\alpha = 6°\); (○), Smooth wing; (●), Rough wing; (—) and (—−), (2.1.3). . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 113
D.0.14 Axial velocity, \((u - U_\infty)/U_\infty\), profile of seven hole probe data at \(x = 5c\) and \(\alpha = 8°\); (○), Smooth wing; (●), Rough wing; (—) and (—−), (2.1.3). . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 114
D.0.15 Axial velocity, \((u - U_\infty)/U_\infty\), profile of seven hole probe data at \(x = 5c\) and \(\alpha = 10°\); (○), Smooth wing; (●), Rough wing; (—) and (—−), (2.1.3). . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 115
D.0.16 Axial velocity, \((u - U_\infty)/U_\infty\), profile of seven hole probe data at \(x = 5c\) and \(\alpha = 12°\); (○), Smooth wing; (●), Rough wing; (—) and (—−), (2.1.3). . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 116
D.0.17 Axial velocity, \((u - U_\infty)/U_\infty\), profile of seven hole probe data at \(x = 5c\) and \(\alpha = 13°\); (○), Smooth wing; (●), Rough wing; (—) and (—−), (2.1.3). . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 117
D.0.18 Axial velocity, \((u - U_\infty)/U_\infty\); profile of seven hole probe data at \(x = 5c\) and \(\alpha = 14^\circ\); (a), Smooth wing; (b), Rough wing; (—) and (—), (2.1.3). ................................. 118

D.0.19 Axial velocity contours obtained with the seven hole probe at \(x = 5c\) and \(\alpha = 2^\circ\); (a), Smooth wing; (b), Rough wing. Contour levels denote \((u - U_\infty)/U_\infty\) values. ................................. 118

D.0.20 Axial velocity contours obtained with the seven hole probe at \(x = 5c\) and \(\alpha = 4^\circ\); (a), Smooth wing; (b), Rough wing. Contour levels denote \((u - U_\infty)/U_\infty\) values. ................................. 119

D.0.21 Axial velocity contours obtained with the seven hole probe at \(x = 5c\) and \(\alpha = 5^\circ\); (a), Smooth wing; (b), Rough wing. Contour levels denote \((u - U_\infty)/U_\infty\) values. ................................. 119

D.0.22 Axial velocity contours obtained with the seven hole probe at \(x = 5c\) and \(\alpha = 6^\circ\); (a), Smooth wing; (b), Rough wing. Contour levels denote \((u - U_\infty)/U_\infty\) values. ................................. 120

D.0.23 Axial velocity contours obtained with the seven hole probe at \(x = 5c\) and \(\alpha = 8^\circ\); (a), Smooth wing; (b), Rough wing. Contour levels denote \((u - U_\infty)/U_\infty\) values. ................................. 120

D.0.24 Axial velocity contours obtained with the seven hole probe at \(x = 5c\) and \(\alpha = 10^\circ\); (a), Smooth wing; (b), Rough wing. Contour levels denote \((u - U_\infty)/U_\infty\) values. ................................. 121

D.0.25 Axial velocity contours obtained with the seven hole probe at \(x = 5c\) and \(\alpha = 12^\circ\); (a), Smooth wing; (b), Rough wing. Contour levels denote \((u - U_\infty)/U_\infty\) values. ................................. 121

D.0.26 Axial velocity contours obtained with the seven hole probe at \(x = 5c\) and \(\alpha = 13^\circ\); (a), Smooth wing; (b), Rough wing. Contour levels denote \((u - U_\infty)/U_\infty\) values. ................................. 122

D.0.27 Axial velocity contours obtained with the seven hole probe at \(x = 5c\) and \(\alpha = 14^\circ\); (a), Smooth wing; (b), Rough wing. Contour levels denote \((u - U_\infty)/U_\infty\) values. ................................. 122

D.0.28 Vorticity contours obtained with the seven hole probe at \(x = 5c\) and \(\alpha = 2^\circ\); (a), Smooth wing; (b), Rough wing. Contour levels denote \(\zeta r_c/v_0\) values. ................................. 123
| D.0.29 | Vorticity contours obtained with the seven hole probe at $x = 5c$ and $\alpha = 4^\circ$; (a), Smooth wing; (b), Rough wing. Contour levels denote $\zeta_r c / v_0$ values. | 123 |
| D.0.30 | Vorticity contours obtained with the seven hole probe at $x = 5c$ and $\alpha = 5^\circ$; (a), Smooth wing; (b), Rough wing. Contour levels denote $\zeta_r c / v_0$ values. | 124 |
| D.0.31 | Vorticity contours obtained with the seven hole probe at $x = 5c$ and $\alpha = 6^\circ$; (a), Smooth wing; (b), Rough wing. Contour levels denote $\zeta_r c / v_0$ values. | 124 |
| D.0.32 | Vorticity contours obtained with the seven hole probe at $x = 5c$ and $\alpha = 8^\circ$; (a), Smooth wing; (b), Rough wing. Contour levels denote $\zeta_r c / v_0$ values. | 125 |
| D.0.33 | Vorticity contours obtained with the seven hole probe at $x = 5c$ and $\alpha = 10^\circ$; (a), Smooth wing; (b), Rough wing. Contour levels denote $\zeta_r c / v_0$ values. | 125 |
| D.0.34 | Vorticity contours obtained with the seven hole probe at $x = 5c$ and $\alpha = 12^\circ$; (a), Smooth wing; (b), Rough wing. Contour levels denote $\zeta_r c / v_0$ values. | 126 |
| D.0.35 | Vorticity contours obtained with the seven hole probe at $x = 5c$ and $\alpha = 13^\circ$; (a), Smooth wing; (b), Rough wing. Contour levels denote $\zeta_r c / v_0$ values. | 126 |
| D.0.36 | Vorticity contours obtained with the seven hole probe at $x = 5c$ and $\alpha = 14^\circ$; (a), Smooth wing; (b), Rough wing. Contour levels denote $\zeta_r c / v_0$ values. | 127 |
| D.0.37 | Tangential velocity profile obtained with the seven hole probe at $x = 5c$ and $\alpha = 2^\circ$; ($\circ$), Smooth wing; ($\bullet$), Rough wing; ($-$) and ($-$), (2.1.5). | 128 |
| D.0.38 | Tangential velocity profile obtained with the seven hole probe at $x = 5c$ and $\alpha = 4^\circ$; ($\circ$), Smooth wing; ($\bullet$), Rough wing; ($-$) and ($-$), (2.1.5). | 129 |
| D.0.39 | Tangential velocity profile obtained with the seven hole probe at $x = 5c$ and $\alpha = 5^\circ$; ($\circ$), Smooth wing; ($\bullet$), Rough wing; ($-$) and ($-$), (2.1.5). | 130 |
D.0.40 Tangential velocity profile obtained with the seven hole probe at 
\( x = 5c \) and \( \alpha = 6^\circ \); (○), Smooth wing; (●), Rough wing; (—) and 
(−−), (2.1.5). ................................................................. 131

D.0.41 Tangential velocity profile obtained with the seven hole probe at 
\( x = 5c \) and \( \alpha = 8^\circ \); (○), Smooth wing; (●), Rough wing; (—) and 
(−−), (2.1.5). ................................................................. 132

D.0.42 Tangential velocity profile obtained with the seven hole probe at 
\( x = 5c \) and \( \alpha = 10^\circ \); (○), Smooth wing; (●), Rough wing; (—) and 
(−−), (2.1.5). ................................................................. 133

D.0.43 Tangential velocity profile obtained with the seven hole probe at 
\( x = 5c \) and \( \alpha = 12^\circ \); (○), Smooth wing; (●), Rough wing; (—) and 
(−−), (2.1.5). ................................................................. 134

D.0.44 Tangential velocity profile obtained with the seven hole probe at 
\( x = 5c \) and \( \alpha = 13^\circ \); (○), Smooth wing; (●), Rough wing; (—) and 
(−−), (2.1.5). ................................................................. 135

D.0.45 Tangential velocity profile obtained with the seven hole probe at 
\( x = 5c \) and \( \alpha = 14^\circ \); (○), Smooth wing; (●), Rough wing; (—) and 
(−−), (2.1.5). ................................................................. 136

E.0.1 Velocity vector plot of LDA data 5c behind the smooth wing at 
\( \alpha = 5^\circ \). ................................................................. 137

E.0.2 Contours of \((u - U_\infty)/u_0\). LDA data 5c behind the smooth wing 
at \( \alpha = 5^\circ \). ................................................................. 138

E.0.3 Axial velocity profile of LDA data 5c behind the smooth wing at 
\( \alpha = 5^\circ \), and (—), (2.1.3). .......................................... 138

E.0.4 Contours of normalised vorticity, \( \zeta r_c/v_0 \) of LDA data 5c behind 
the smooth wing at \( \alpha = 5^\circ \). ......................................... 139

E.0.5 Tangential velocity profile of LDA data 5c behind the smooth wing 
at \( \alpha = 5^\circ \), and (—), (2.1.5). ......................................... 139

E.0.6 Velocity vector plot of LDA data 5c behind the rough wing at 
\( \alpha = 5^\circ \). ................................................................. 140

E.0.7 Contours of \((u - U_\infty)/u_0\). LDA data 5c behind the rough wing at 
\( \alpha = 5^\circ \). ................................................................. 140

xiv
E.0.8 Axial velocity profile of LDA data 5c behind the rough wing at $\alpha = 5^\circ$, and $(-)$, (2.1.3). . . . . . . . . . . . . . . . . . . . . . . . . . . . . 141
E.0.9 Contours of normalised vorticity, $\zeta r_c/v_0$ of LDA data 5c behind the rough wing at $\alpha = 5^\circ$. . . . . . . . . . . . . . . . . . . . . . . . . . . . . 141
E.0.10 Tangential velocity profile of LDA data 5c behind the rough wing at $\alpha = 5^\circ$, and $(-)$, (2.1.5). . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 142
## List of Tables

<table>
<thead>
<tr>
<th>Number</th>
<th>Table Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.1</td>
<td>Table of experimental techniques used in vortex flows.</td>
<td>16</td>
</tr>
<tr>
<td>3.1</td>
<td>Error estimates for seven hole probe experiments (% of maximum value)</td>
<td>27</td>
</tr>
<tr>
<td>3.2</td>
<td>Range, precision and standard error (as % of full scale) of 6-component ATE-Ltd force and moment balance, supplied by ATE-Ltd</td>
<td>29</td>
</tr>
<tr>
<td>3.3</td>
<td>Filter specifications for LDA timetrace processing</td>
<td>43</td>
</tr>
<tr>
<td>5.1</td>
<td>Smooth wing vortex core parameters for all $\alpha$</td>
<td>51</td>
</tr>
<tr>
<td>5.2</td>
<td>Rough wing vortex core parameters for all $\alpha$</td>
<td>51</td>
</tr>
<tr>
<td>6.1</td>
<td>LDA vortex core data</td>
<td>59</td>
</tr>
</tbody>
</table>
Chapter 1

Introduction and Motivation

A vortex is a very complex phenomenon that researchers have spent decades studying. Despite this, there are still many things that are not yet understood about the formation and development of trailing vortices. The following section describes the motivation behind why vortices - specifically, wake vortices - were chosen as the subject of this research.

1.1 Wake vortices

It is well known that lifting wings produce long-lived vortices in their wake. Not limited to conventional aircraft wakes, tip vortices also affect flows around helicopter rotor blades, propellers, and turbine blades. The aviation industry is concerned about these vortices primarily because of their hazardous effects on flight safety. The long-lasting nature of the vortex in the airplane wake causes congestion on airport runways due to the space and time required for vortex dissipation to occur. Accepted practices require minimum distances between landing aircraft dependent on the weights of the leader and follower aircraft (Spalart, 1998). Larger aircraft require longer distances and, with the size and number of commercial aircraft flights continually increasing, there is a strong desire to reduce and minimize these lengths as much as possible. However, in order to assess these hazards, it is necessary to be able to predict the formation, motion, and persistence of these flows (Gerz et al., 2002).
The development of the vortex over the wing and in the wake is reasonably well understood (Widnall, 1975; Spalart, 1998). However, in contrast to a general understanding that these tip vortices form with the roll-up of the wing boundary layers, relatively little is known about the role of turbulence within the roll-up period. Much of the early fundamental research on vortical flows focused on finding analytical solutions to the velocity profiles of a vortex (Batchelor, 1964; Hoffmann and Joubert, 1963), but the close agreement between laminar and turbulent analytical solutions gives little insight to the state of the vortex (Birch, 2012). As a result, there are contradicting theories about if and how turbulence is contained or transported within a well-developed vortex.

A vortex is generally accepted to have a form analogous to boundary layers: an inner core dominated by viscosity, an outer core dominated by momentum effects, and an outer region characterized by initial conditions (Phillips, 1981). However, unlike boundary layers, there are no external forces or mechanisms to generate turbulent production within the vortex. As such, the definition of a vortex as turbulent is unclear and is generally accepted as having high levels of turbulent kinetic energy near the centre, with velocity spectra indicative of turbulence. Unfortunately, measurements of such quantities are difficult due to effects from initial and boundary conditions inherent in most experimental and numerical setups (Jammy et al., 2014).

The aviation industry’s concern with wing tip vortices is not a new one, and as such, decades of research have been devoted to methods of reducing the effects of the vortex on both aircraft performance and safety. The induced drag of an airplane is a consequence of the vortex formation, and methods currently in place to improve efficiency include changes in wing planform and additions of wing tip devices such as winglets and endplates, among others (Kroo, 2001). Yet these changes give relatively little improvement to the safety concerns of the long-lasting nature of vortices in the wake of an airplane. More emphasis now needs to be placed methods to reduce the strength of the vortex or force vortex breakdown to occur sooner than natural. It has been established that old vortices are subject to long-wave Crow (1970) instabilities, and perturbation analysis has shown that a vortex can become unstable if perturbed at particular frequencies, providing the axial velocity is large enough (Lessen and Paillet, 1974; Leibovich and Stewartson, 1974).
1983). It, then, becomes a question of whether it is possible to perturb these wing tip vortices in a way to induce these instabilities.

Recently, there has been an increase in researching how turbulence affects the flow in and around vortices. Most of this research has involved varying grid turbulence upstream of a wing, while only a few studies have taken a more direct approach by adding turbulence within the wing boundary layer, and the analysis of the effects on trailing vortices is limited. Increasing freestream turbulence intensity can increase the amplitude of vortex wandering and the rate of decay of tangential velocity downstream of the wing (Bailey et al., 2006; Bailey and Tavoularis, 2008), while adding roughness elements onto the wing reduces the pressure drop on the wing during vortex rollup and increases the turbulence intensity in the 2-D wing wake (Katz and Galdo, 1989; Zhang et al., 2004). Increasing surface roughness has more practical applications in the aviation industry; however, most of the research investigating wing surface roughness is focused on wing boundary layer transition characteristics and changes in the very near wing wake. While the changes in the early development of the vortex are promising, the effects of surface roughness on the tip vortex still remain unclear.

1.2 Objectives

The aim of this research is to determine the effects that increased wing surface roughness has on the development of a wing-tip vortex. The idea has been inspired by both the potential industrial applications and possible contributions to the fundamental research of vortex flows. Introducing fine-scale and broadband turbulence into the formation of the vortex can not only add to the understanding of how turbulence is rolled into the vortex but also how the turbulence is transported and decays within the vortex.

This study uses a baseline smooth wing to analyze and compare mean and time-dependent results measured in the wake of a rough wing. After carefully selecting the characteristic roughness height of the rough wing, four types of experimental data are used to assess the performance of the wing and to characterize the wing wake before and after vortex formation. Analytical solutions provide a method of validating the existence of a well-developed axisymmetric vortex behind both
wings. Velocity power spectra are used to identify dominate frequencies and structures within the 2-D wing wake and correlate them to structures in the vortex five chord lengths behind the trailing edge. Finally, a new method is proposed to remove effects of vortex wandering without apriori knowledge of the form of the Reynolds stress distributions.

1.3 Thesis layout

The second chapter presents a review of literature which outlines the current understanding of axisymmetric wing-tip vortices. The chapter begins with the analytical descriptions of a simple Lamb-Oseen vortex and continues with the accepted mean velocity profiles in laminar and turbulent line vortices. Topics prevalent in wing-tip vortex research are then discussed - including vortex roll-up, vortex wandering, and methods of vortex control. A summary of experimental measurement techniques, as applied to vortex flows, is then presented.

Chapter three describes the facilities and instrumentation developed and used during the course of this research. After detailing the selection process of the wing profile and surface roughness characteristics, a summary of the wind tunnels and experimental instrumentation is given. Problems with the initial calibration of the seven-hole probe (given for reference in appendix B) are identified, and a new calibration method is proposed as detailed in the corresponding paper, included in appendix G. The chapter concludes with the details of the post-processing methods and equations.

The first chapter of results, chapter four, summarizes the performance of the wings with an emphasis on the effects of increased surface roughness. Published data is used to validate the lift characteristics of the 2-D smooth wing, to which the 2-D rough wing data is also compared. Lift and drag characteristics of the 3-D wing are also presented as obtained from both force balance and wake scan data.

Wake survey data from the seven hole probe in chapter five gives the first look at the mean wing-tip vortex. Vortex core characteristics - such as peak vorticity, core radius, and core circulation - are presented for all wing incidences tested. Profiles of axial and tangential velocities, vorticity, and circulation within the vortex are given for two angles and validated against analytical solutions given in chapters
two and three, while the profiles from the remaining angles are given in appendix D.

Two-component LDA data is used to gain a better understanding of how small-scale fluctuations are distributed within the vortex in chapter six. Reynolds shear stresses are used as an initial measure of the magnitude and width of turbulence in the vortex behind both wings at the same wing incidence. Velocity power spectra indicate what frequencies dominate the flow and how the dominant length-scales change with the additional broadband turbulence in the wing boundary layer. Finally, a novel method of removing the apparent contribution to Reynolds stresses from vortex wandering is proposed, with further details of the method provided in appendix H.

The final chapter of results details the data for a single hot-wire placed in the 2-D wake of the smooth and rough wings. Mean velocity profiles and Reynolds stress measurements show the effects of the sandpaper on the 2-D wake width. The analysis then highlights similarities and differences between the dominate length-scales in the power spectra in the wake and the wing-tip vortex downstream.

Chapter eight concludes the research by summarizing the important findings of the research and highlighting the implications of the data. Further experiments which would help clarify some of the results are also suggested. The appendices included in this report contain data collected and methods used as a part of this research, but was not considered to be essential in the arguments presented. Appendix A is a description of an essential piece of equipment developed specifically for this project by the author, and B details the calibration of the seven hole probe initially used before deriving an improved method described in section 3.3. The paper published as a part of the probe work is reproduced in appendix G. The optimal calibration methods detailed in appendix C were developed in conjunction with the novel calibration method as a way to reduce wind tunnel time spent on calibration and presented in the conference paper given in appendix F. Lastly, the plots in appendices D and E contain data collected as a part of this research and are included as a means of giving a complete data set for any future research. The paper in appendix H is currently under review for publication and details the methods and theory behind the post-processing of the LDA data described in section 6.5.
Chapter 2

Vortex Flows

2.1 Fundamental concepts

A wing-tip vortex is only one type of vortex, one that is formed as a result of
the pressure difference between the top and bottom surfaces on an aircraft wing.
The simplest type of viscous vortex is the Lamb-Oseen vortex (Saffman, 1992),
which is a laminar, axisymmetric, two-dimensional ($v_r = u = 0$ where $v_r$ is radial
velocity and $u$ is axial velocity) vortex in incompressible flow. While the three-
dimensional wing-tip vortex does not easily lend itself to an analytical description,
these simplified conditions lead to an exact solution to the Navier-Stokes equations,

$$\frac{\Gamma}{\Gamma_0} = 1 - \exp \left( -\frac{r^2}{4\nu t} \right),$$  \hspace{1cm} (2.1.1)

where $r$ is the radius of the vortex, $\Gamma$ is the circulation ($\Gamma_0$ denotes total
circulation), $\nu$ is the kinematic viscosity, and $t$ is the relative age of the vortex.
The circulation is an indication of the strength of the vortex and is defined by

$$\Gamma = \oint_S \mathbf{v} \cdot d\mathbf{S},$$  \hspace{1cm} (2.1.2)

where $S$ is a closed path containing the vortex centre. Given a circular path around
an axisymmetric vortex, $d\mathbf{S}$ is a circular arc segment and the tangential velocity,
$v_\theta$, is constant. From (2.1.1), it can be seen that the circulation and tangential
velocity should have a Gaussian variation with radius. The vorticity of a vortex,
ζ, is an alternative way to describe the velocity distribution of a vortex, and can be obtained from spatial derivatives.

The overall structure of a vortex consists of a core and a surrounding region with a velocity profile akin to that of a boundary layer (Hoffmann and Joubert, 1963; Batchelor, 1964). The outer boundary of the vortex core is defined as the radial coordinate at which the tangential velocity reaches a maximum. Vortices are considered to be highly stabilizing, with a near solid-body rotation and dominating tangential viscous forces in the inner core (Hoffmann and Joubert, 1963). This is evidence that streamline curvature has a stabilizing effect in the core by introducing much larger strain rates than simple shear layers. If the core is considered to be made up of concentric circular streamlines, then any displaced fluid element to either a larger or smaller radial position would be forced to return to its original position by the mean pressure gradient (Bradshaw, 1973). The solid-body rotation of the core is thought to cause any turbulence present in the roll-up process to rapidly decay, a phenomenon known as relaminarization. In support of this theory, the work of Chow et. al (1997) and Bandyopadhyay et al. (1991) show initial high levels of turbulence which then appear to dissipate. The low levels of turbulence in the outer core are consistent with the characteristics of a laminar vortex.

### 2.1.1 Mean velocity profiles

Building on the Lamb-Oseen solution by assuming small non-zero axial velocity gradients, Batchelor (1964) determined that a decay in tangential velocity produces an axial pressure gradient, and therefore, an axial velocity component in a laminar vortex. Under these assumptions, the axial velocity profile for a trailing line vortex is

\[
\frac{U_\infty - u(r)}{u_0} = \exp(-\alpha \eta^2), \tag{2.1.3}
\]

where the similarity variable \( \eta = r/r_c \) (\( r_c \) is the core radius), the maximum axial velocity deficit at the vortex centre \( u_0 \) is proportional to downstream distance \( x \), and \( \alpha \) is defined by the transcendental \( \exp(\alpha) = 1 + 2\alpha \) giving \( \alpha \approx 1.25643 \). The freestream axial velocity, \( U_\infty \), is arbitrary and vanishes for a stationary vortex.
The axial velocity deficit may be either positive or negative, giving a jet-like or wave-like vortex. Batchelor (1964) also showed that an increase in axial velocity must result in radial inflow, and vice-versa. Consequently, the radial velocity profile is

\[ \frac{v_r(r)}{v_{r0}} = \frac{1}{r} \left( 1 - \exp\left(-k r^2\right) \right), \]  

(2.1.4)

where \( v_{r0} \) is the maximum radial velocity, and \( k \) is a scaling constant. Batchelor also derived a similarity solution for the tangential velocity in a laminar vortex,

\[ \frac{v_\theta(\eta)}{v_0} = \left(1 + \frac{1}{2\alpha}\right) \frac{1}{\eta} \left( 1 - \exp\left(-\alpha \eta^2\right) \right), \]  

(2.1.5)

where \( v_0 \) is the peak tangential velocity. Taken together, (2.1.3), (2.1.4), and (2.1.5) satisfy the Navier-Stokes and continuity equations. Both (2.1.5) and (2.1.1) also show that the circulation and tangential velocity profiles are self-similar.

The above Batchelor (1964) solutions are valid for low Reynolds number flows, and assume a laminar vortex. The laminar profile could explain the slow decay of a wing-tip vortex, and also appears to be supported by the free flight observations discussed by Ciffone (1974) and Iverson (1976). However, in contrast to those data sets which support these assumptions, there is evidence of high levels of turbulence within the vortex core from the studies of Green and Acosta (1991), Birch and Lee (2005), and Beninati and Marshall (2005), among others. As such, mean velocity profiles of high Reynolds number, turbulent vortex flows have been derived by Hoffmann and Joubert (1963) and Phillips (1981).

### 2.1.2 Consideration of turbulence

Hoffman and Joubert (1963) developed a solution to the Navier Stokes equations for the case of a turbulent vortex, dividing the vortex into two parts outside of the viscous core: a viscosity dominated near region, and a far region in which turbulent stresses dominate. Applying dimensional analysis and the conservation of momentum as well as inner and outer matching, they concluded that
\[
\frac{\Gamma(\eta)}{\Gamma_c} = A_0 \eta^2 \quad 0 \leq \eta \leq 0.4 \tag{2.1.6}
\]
\[
\frac{\Gamma(\eta)}{\Gamma_c} = 1 + A_1 \ln(\eta) \quad 0.5 \leq \eta \leq 1.4, \tag{2.1.7}
\]

where experiments showed that \(A_0 = 1.83\) and \(A_1 = 0.929\), and \(\Gamma_c\) is the core circulation. This model requires two important assumptions: that there exists an inner laminar region of solid body rotation such that \(\Gamma \propto rv_\theta\), and that the tangential inertial forces within and just outside of this region dominate over viscous forces.

Phillips (1981) took Hoffman and Joubert’s work a step further and divided a developed core into three regions: 1) the innermost region dominated by solid-body rotation characteristics, 2) the region in which the logarithmic circulation distribution applies and the boundary of the vortex core is contained, and 3) the outermost region containing distinct turns of the vortical wake shed from the lifting surface. Phillips applied known line-vortex boundary conditions to the Reynolds-averaged momentum equation, resulting in a circulation profile of the form

\[
\frac{\Gamma(\eta)}{\Gamma_c} = \sum_{i=1}^{n} B_i \eta^{2i} \quad 0 \leq \eta \leq 1.3, \tag{2.1.8}
\]

where \(B_i\) are constants and \(n \to \infty\). Through inner and outer matching, he was able to provide values for the first three constants: \(B_1 = 1.7720\), \(B_2 = -1.0467\), and \(B_3 = 0.2747\). The Hoffmann and Joubert (1963) and Phillips (1981) solutions agree well with each other, and the results of several studies (Birch et al., 2004; Green and Acosta, 1991; Ramaprian and Zheng, 1997, among others) support them. These models have also been shown to be robust to both unsteady vortices (Ramaprian and Zheng, 1997; Birch and Lee, 2005) and vortices in high-levels of free stream turbulence (Beninati and Marshall, 2005; Bailey and Tavoularis, 2008).

While there have been experimental evidence to support both the laminar and turbulent solutions, it is important to note that the laminar vortex model from Batchelor (2.1.5), and the turbulent vortex models from Hoffman and Joubert (2.1.6) and (2.1.7), and Phillips (2.1.8) agree well with each other over the range
0 \leq \eta \leq 1.3. Birch (2012) even provides evidence that a logarithmic region within the circulation profile does not necessarily indicate the existence of scale-independent turbulence inside a vortex. By defining tangential velocity in terms of circulation, Birch proves this is also the case for the core velocity profile. Clearly, this agreement adds to the difficulty of determining whether a vortex is laminar or turbulent by using mean velocity profiles alone.

2.2 Wing tip vortices

Experimental studies on quasi-two-dimensional, trailing vortices typically use vortex generators composed of at least one lifting surface to produce a trailing line vortex. Such vortex generators can consist of split-wings or multiple lifting surfaces which can be less susceptible to wind tunnel effects (Phillips and Graham, 1984), oscillating wings to simulate helicopter rotor blades (Birch and Lee, 2005), or fixed wings to study flow over and behind turbine blades and aircraft wings (Zhang et al., 2004; Devenport et al., 1996). Aircraft trailing vortices are formed as the vortex sheet rolls up on the wing tips, entrapping the wing boundary layers within them. While free-flight tests such as McCormick et al. (1968) are the best way to understand these vortices, they are not practical for investigations of the fundamental nature of wing-tip vortices due to the costs and small length-scales within the vortices. Wind tunnels provide a convenient environment for studying how the vortex forms on the wing and how the vortex is affected by small changes in the flow field and wing structure.

2.2.1 Vortex Rollup

Before the vortex conforms to the mean velocity profiles derived by Batchelor (1964), it is highly three dimensional and asymmetrical during and just after the rollup process (Hah and Lakshminarayana, 1982). The formation of the vortex occurs as the flow on the pressure side moves to the suction side of the wingtip, creating a vortical flow structure (Chow et al., 1997). This requires, then, that the boundary layer on the surface of the wing is entrained by the vortex roll-up process, injecting the turbulence and vorticity contained within it into the
vortex. Once the vortex achieves a nearly symmetric profile, the loading and physical characteristics of the wing have negligible influence on the development of the wake (Hah and Lakshminarayana, 1982), implying that any influence on the vortex wake must occur during the roll-up process. The formation process is also characterised by secondary structures (see Birch et al., 2003, 2004; Chigier and Corsiglia, 1971) which eventually merge into a single trailing vortex, resulting in high axial velocity gradients within the vortex core (Chow et al., 1997; Chigier and Corsiglia, 1971). Wing endcap geometry influences the development of secondary structures during the roll-up process, with round or body of revolution geometry producing fewer secondary structures (Sohn and Chang, 2012; Birch et al., 2004; Giuni, 2013). Anderson and Lawton (2003) also showed the shape of the endcap affects the size of the fully-formed vortex. The vortex is considered to be mostly formed only after about two chord lengths downstream of the wing trailing edge (Ramaprian and Zheng, 1997).

2.2.2 Wandering

Because mean velocity profiles do not easily distinguish between laminar and turbulent vortices, it becomes necessary to look at time-resolved data and higher-order velocity statistics within the vortex. However, initial and boundary conditions inherent in wind tunnel and numerical setups make the vortex prone to a stable motion known as wandering. Wandering is the tendency of vortices to have a low-frequency Gaussian-random motion, and it has a convolution effect on velocity profiles, making the dimensions of a time-averaged vortex appear larger than the instantaneous vortex. Of further consequence, wandering also pollutes the higher-order velocity statistics, making any uncorrected turbulence measurements within the core invalid.

One of the first attempts to provide a quantitative description of the effects of wandering on velocity profiles, Devenport et al. (1996) developed an analytical correction for wandering in laminar vortices, and Bailey et al. (2006) extended the correction for high-turbulence cases. Both of these methods require knowledge of and assumptions about the velocity distributions before beginning the experiment (Iungo and Skinner, 2009). However, care must be taken when applying these
methods, as the Gaussian convolution proposed by Devenport et al. (1996) will force any function toward the assumed Gaussian velocity profile when the standard deviation of the wandering amplitude is sufficiently large (Birch, 2012). Therefore, in flows with high amplitude vortex wandering, such as those with intense free stream turbulence, the original form of the vortex may be lost given the assumptions necessary for these corrections. Birch (2012) demonstrated this by applying the corrections to a nonphysical flow field, producing a velocity profile which closely approximated (2.1.5).

The random wandering motion of vortices is due to the initial and boundary conditions imposed by wind tunnels (Jammy et al., 2014), a conclusion that is consistent with the absence of this motion in free-flight tests (McCormick et al., 1968). The amplitude of wandering is very sensitive to free stream turbulence intensity (Bailey et al., 2006; van Jaarsveld et al., 2011), and the turbulence detected within the vortex is highly sensitive to any corrections imposed on the velocity field. Jammy et al. (2014) use direct numerical simulations (DNS) to show that second-order velocity statistics exhibit a stronger sensitivity to boundary conditions and wandering than first-order spatially-averaged velocities. The uncorrected circumferentially-averaged Reynolds stresses have significantly higher peak magnitudes and a near Gaussian distribution, which could be interpreted as a turbulent core. Once corrected, the turbulence levels drop significantly and a peak in Reynolds stresses appears within the core but away from the vortex centre, providing evidence of secondary structures and turbulent production within the core. This study also showed that the bulk-motion wandering should be considered as an “inactive” phenomenon, only negligibly contributing to turbulent production and dissipation.

2.2.3 Vortex Control

Vortex formation occurs around the wing tip, and any influence on the quasi-steady vortex must happen before or during rollup. Some studies have looked at the effect of wing tip shape (Souza and Faghani, 2001), used vortex generators (Heyes and Smith, 2005), or delta-wing attachments on the wing-tip (Lee and Su, 2012), among other methods (Carlin et al., 1989; Kroo, 2001), in an attempt to modify or control
the wing-tip vortices. One method proven to affect the behaviour of the vortex has been the addition of turbulence to the freestream. Bailey et al. (2006) and Bailey and Tavoularis (2008) performed tests on a wing-tip vortex investigating the influence of freestream turbulence on vortex development and decay. An increase in freestream turbulence was found to increase the amplitude of wandering, but had no effect on the dominant wavelength of wandering. The increased turbulence intensity also increased the rate of decay of the peak tangential velocity but had no effect on the radial location of the peak velocity, implying a loss of streamwise angular momentum within the vortex core. Beninati and Marshall (2005) found small-scale turbulence was injected into the vortex core from the boundary layer of the vortex generator blade during vortex formation. A related numerical study (Marshall and Beninati, 2005) found that strong initial turbulence intensity creates strong turbulence within the vortex core and the large-scale vortex core breaks up. Significantly, van Jaarsveld et al. (2011) found that the addition of upstream grid turbulence did not affect lift characteristics of the wing. Being able to increase the vortex decay rate without altering the performance of the wing would reduce the amount of time required between aircraft and allow airports to have increased traffic on runways.

Since creating upstream turbulence in the flight-path of an airplane is not a practical method of controlling vortex behaviour, an alternative method of injecting turbulence into a wingtip vortex that has not been extensively studied is that of changing the surface roughness on the wing. Katz and Galdo (1989) took surface pressure measurements and used flow visualisation techniques on a hydrofoil with roughness elements painted onto it. Their findings indicated that the physical dimension of the vortex was only slightly changed by surface roughness, and a reduction in the magnitude of the pressure drop associated with the vortex occurred on the wing. This change in pressure drop causes a reduction in the tip vortex strength, reducing further with increasing roughness. The surface roughness also appeared to either reduce or completely eliminate the presence of secondary structures during vortex roll-up. Zhang et al. (2004) showed that the velocity distributions in the near-field airfoil wake were more sensitive to an increase in surface roughness than to variations in freestream turbulence. They also demonstrated that an increase in surface roughness had a cascading effect
starting with increased the boundary layer thickness and trailing edge separation, resulting in wider wakes containing more momentum diffusion and vorticity. Wake surveys at one chord length behind the trailing edge also showed that peak velocity deficits increased with roughness. In addition to those already mentioned, most of the known experiments investigating effects of roughness elements on wings focus solely on the boundary layer transition on the wing and on the development of the very near wake (including those of Kerho and Bragg, 1997; Kim et al., 2006). While this data is useful to an extent, it is the far-field trailing vortex which is of greatest concern to the present day aviation industry. Without detailed wake surveys of the quasi-steady vortex, there is no way to determine if and how roughness elements on the wing change the turbulence structure of the wing-tip vortex.

2.3 Flows over Rough Walls

In the present application, the primary interest in boundary layers is their influence on drag and performance characteristics. In general, rough surfaces produce thicker boundary layers and consequently, more drag. However, this is not always the case. Some experiments have used rough surfaces to actually reduce skin friction and drag by up to 10% (Bechert et al., 1997), many of which use models inspired by wings and rough surfaces found in nature (Sirovich and Karlsson, 1997; Bechert et al., 2000). These are very specialised roughness profiles and an upper value of the equivalent sand roughness $k^+_s \approx 4$ is the limit for which drag is the same for a smooth wall (Jimenez, 2004). The goal of this study is not to necessarily reduce overall drag with surface roughness; it is to reduce the strength and life of the trailing vortex by increasing turbulence in the wing boundary layer without diminishing wing performance.

In order to be able to characterize the roughness of a surface and determine what degree of roughness is required for this research, a short discussion of rough wall flow is necessary. The part of the boundary layer closest to the wall is dominated by viscosity (below $y^+ \approx 5$), while the most active regions are the buffer and logarithmic regions. The term logarithmic refers to the distribution of mean streamwise velocity,
\[ u^+ = \frac{1}{\kappa} \ln y^+ + B \quad y^+ > 30, \quad y/\delta < 0.3, \quad (2.3.1) \]

where \( \delta \) is the thickness of the boundary layer, \( u^+ = u/u_\tau \), \( u_\tau \) is the friction velocity \( (u_\tau = \sqrt{\tau_w/\rho} \), where \( \tau_w \) and \( \rho \) are the tangential wall stress and fluid density), \( y^+ = y u_\tau/\nu \) \( (\nu \) is the kinematic viscosity), and the Karman constant \( \kappa \approx 0.4 \).

Equation (2.3.1) is only valid away from the wall \( (y^+ \gg 1) \), and \( B \) represents an offset that depends on the behaviour of the buffer and viscous layers. For rough walls, this offset is generally a function of the roughness height or an “effective” surface roughness, \( k_s \). A roughness length, \( k_0 = 0.33k_s \), can also be used (Jimenez, 2004). The buffer layer is the transition region between the viscous sublayer and the turbulence dominated region for which the above log-law holds (Pope, 2000). Most of the turbulent energy in moderate Reynolds number flows is generated in the buffer and log-law regions (Jimenez and Moin, 1991).

Jimenez (2004) discusses two important parameters in rough wall flows: the roughness Reynolds number \( (k_s^+ = k u_\tau/\nu \), where \( k \) is the height of the roughness elements) and the ratio of boundary layer thickness to the roughness height \( (\delta/k) \). The roughness Reynolds number measures the effect of surface roughness on the buffer layer, and \( \delta/k \) indicates whether a logarithmic region survives. Generally, if \( \delta/k \leq 50 \), the surface roughness has an effect extending across the boundary layer, and most of the dynamics present in smooth wall flows no longer exists. Hence, for a given flow with known Reynolds number, a roughness height can be estimated which would add turbulence into the boundary layer without destroying it - limiting the increase in profile drag of the wing while hopefully reducing the induced drag caused by the vortex.

### 2.4 Vortex Measurement Techniques

As mentioned previously, there have been a number of vortex studies over the decades using both numerical and experimental techniques. Free from experimental error, numerical methods provide results that are well resolved in space and time. However, for turbulence research, Reynolds-averaged and large eddy simulation methods assume that small and large scale turbulence
do not interact. They only model the behaviour of small-scale turbulence, so the solutions are only as good as the models. Vortex flows severely challenge these models, making these methods inadequate for the current research. Direct numerical simulations do not require any turbulence models, but the Reynolds numbers of interest require more computational power than is currently available. Experimental techniques can capture the full range of turbulence data within reasonable time frames, and a summary of the most commonly used techniques in vortex flows is presented in table 2.1.

Most of the techniques typically used in vortex flows are single-point techniques, including multi-hole pressure probes, hot-wire anemometers, and laser Doppler anemometry (LDA). A single wake scan can take several hours depending on the measurement area, but the hot-wire and LDA techniques offer very high temporal resolution (up to 100s kHz). In contrast, particle image velocimetry (PIV) is a field technique, capturing the entire measurement plane in each frame. The data rate for PIV experiments, however, is limited to the frame rate of the camera, typically less than 50 Hz. A description of the techniques is given in table 2.1 and a discussion of their application to vortex flows follows.

<table>
<thead>
<tr>
<th>Intrusive? (Y/N)</th>
<th>Multi-hole probes</th>
<th>Hot-wire</th>
<th>LDA</th>
<th>PIV</th>
</tr>
</thead>
<tbody>
<tr>
<td>Point/field?</td>
<td>Point</td>
<td>Point</td>
<td>Point</td>
<td>Field</td>
</tr>
<tr>
<td>Bandwidth</td>
<td>&lt; 1Hz</td>
<td>100s kHz</td>
<td>100s kHz</td>
<td>&lt; 50 Hz</td>
</tr>
<tr>
<td>Spatial Resolution</td>
<td>~ 3mm diam.</td>
<td>&lt; 3mm diam.</td>
<td>&lt; 2mm</td>
<td>&lt; 10µm²</td>
</tr>
<tr>
<td>Incompressible/Compressible</td>
<td>Both</td>
<td>Incompressible</td>
<td>Both</td>
<td>Both</td>
</tr>
<tr>
<td>Quantities Measured</td>
<td>3-comp. mean velocity</td>
<td>3-comp. velocity</td>
<td>2-comp. velocity</td>
<td>2-comp. velocity</td>
</tr>
</tbody>
</table>

Table 2.1: Table of experimental techniques used in vortex flows.
2.4.1 Intrusive techniques

2.4.1.1 Multi-hole Pressure Probes

Multi-hole pressure probes are used to obtain point-wise velocity and pressure information in flows with high angularity, such as those found in turbomachinery or wind turbine laboratory tests. There have been a variety of configurations and geometries developed for specific applications, but the basic principle they all have in common is the ability to determine the steady-state velocity magnitude and direction from measured pressure differences. Two types of multi-hole probes which are common in vortex and wing wake applications are the five- and seven-hole probes. The application of these probes is not new, but the development of accurate and efficient calibration methods has continued to be of interest in recent years (Wenger and Devenport, 1999; Sumner, 2002; Silva et al., 2003; McParlin et al., 2013; Shaw-Ward et al., 2014). Pressure probes of this kind have the benefit of a calibration matrix linearly dependent on freestream velocity magnitude, but due to their longer response time, only resolve average flow quantities. As discussed previously, vortex wandering causes a smoothing effect in time-average measurements and require the application of a correction to remove these effects. There has been some work using fast-response five-hole probes (Iungo and Skinner, 2009), but due to the increased size and higher cost, the more common type is those with a time-averaged response.

Seven-hole probes are more widely used in vortex flows than five-hole probes. The extra holes allow it to resolve flows with higher angularity and with more accuracy (Gallington, 1980; Zilliac, 1989). Recent developments into multi-hole probes with more than seven holes have also been undertaken in an effort to increase the range of high angle flows measurable (Ramakrishnan and Rediniotis, 2007; Wang et al., 2012) and probe sensitivity (Shaw-Ward et al., 2014). There have also been recent studies into the use of functions relating measured pressures directly to flow properties (Pisasale and Ahmed, 2002, 2004). Notably, Iungo and Skinner (2009) used a fast-response five hole probe to determine a method for correcting for vortex wandering. Unlike some other measurement techniques, multi-hole probes also introduce a possible interference with the flow. High levels of shear and the possible introduction of instabilities make vortices particularly
sensitive to intrusive measurement. Payne et al. (1989) used a seven hole probe to measure leading edge vortex flows on a delta wing, and examined the interference of the probe on vortex development. Using laser Doppler anemometry (LDA), they were able to show that for most cases, the probe had minimal interference, with the exception being in flows when the natural vortex breakdown occurred near or just after the probe position. One of the most widely cited studies using the seven hole probe was produced by Chow et al. (1997). Focusing on the near field of a wing tip vortex, they were able to provide a detailed description of mean velocity profiles within the vortex, supplementing the data with multi sensor hot wire measurements. Birch et al. (2003) used a seven hole probe in conjunction with particle image velocimetry (PIV) to document the near field behaviour of a tip vortex. In a subsequent study, Birch et al. (2004) showed how the Maskell (1972) and Kusunose (1997) methods can be used to calculate the induced drag generated by a tip vortex using the velocity profiles obtained with a seven hole probe. While these probes offer a relatively simple method of measuring mean velocity data, turbulence research requires the measurement of high frequency velocity fluctuations, so another method must be used.

2.4.1.2 Hot wires

Multi-sensor hot wires are probably one of the most widely used measurement techniques in turbulence research. Using three or four wires in conjunction, they are able to resolve high frequency velocity magnitude and direction, but with poor spatial resolution due to the size of the measurement volume. The high frequency is necessary for turbulence measurements, the characteristics of which in a vortex are a widely studied subject. One of the drawbacks in the practical use of hot wires is that they lose their calibration relatively quickly, which is especially consequential when a single detailed wake survey can take several hours. Multi-wire hot wires require a longer calibration procedure than multi-hole probes due to their non-linear dependence on velocity (Mathioudakis and Breugelmans, 1985; Lekakis et al., 1989), and they also introduce possible effects of probe interference. Both multi-hole probes and hot wires are point-wise techniques and cannot resolve vortex structures. As such, multi-wire techniques also require a correction for
vortex wandering in the post-processing of data.

As discussed earlier, the presence and degree of turbulence in the core of a vortex has not been confirmed to any degree of certainty. Some of the most frequently cited experiments using hot wires are those by Devenport et al. (1996) and Bailey and Tavoularis (2008), both offering a correction for vortex wandering based on their measurements; Phillips and Graham (1984); and Hah and Lakshminarayana (1982), who address the effectiveness of numerical turbulence models at predicting turbulence in the vortex core, to name a few. It is necessary to measure the turbulence statistics inside of a vortex to investigate not only how freestream and boundary layer turbulence effects the roll up process, but also how turbulence is convected in the vortex. A good understanding of the turbulence characteristics inside of a vortex could lead to more efficient methods of controlling vortex roll-up and decay.

2.4.2 Non-intrusive (optical) techniques

2.4.2.1 Laser Doppler Anemometry

Laser Doppler anemometry (LDA) is a non-intrusive technique used to measure velocity data in a vortex. LDA is a point-wise measurement technique that extracts single particle position data to then determine flow velocity characteristics. Able to resolve high-frequency velocity data, LDA is however prone to the same limitations as hot-wires with respect to vortex wandering. LDA requires seeding of the flow which, although theoretically non-intrusive, can have its own effect on vortex flows, depending on the size of the particles (Phillips and Graham, 1984). Birch and Martin (2013) also argued that seed trajectories within the vortex might not follow fluid pathlines accurately. Due to the solid-body rotation characteristics of the vortex core, it can be difficult to entrain particles inside of the core, resulting in low resolution velocity characteristics in this region (Giuni, 2013).

While not widely used in vortex measurements to date, LDA offers another method in which to measure time-resolved velocity data. Unlike hot wires and multi-hole probes, this technique does not require additional time to be devoted to probe calibration for each separate experiment, and there is minimal to no interference (depending on the position of the probe) to complex and unstable
flows, such as those which occur throughout vortex development. Obtaining turbulence characteristics and power spectra from LDA data is more complex due to the irregularity of the measurement time intervals, but several techniques have been developed, some of which have been reviewed by Benedict et al. (2000). Velte et al. (2014) found the primary limitations in the LDA data include a small dip as the spectra levels off at higher frequencies related to the "dead time" experienced during data collection and high noise levels due to the unevenly spaced data points. Despite the increased time required in post-processing, the benefits of no calibration procedure and minimal probe interference make LDA an ideal method for collecting time-resolved vortex velocity data.

2.4.2.2 Particle Image Velocimetry

Particle image velocimetry (PIV) is a non-intrusive field technique which measures velocity data in a vortex. PIV uses a laser sheet to take rapid successive instantaneous pictures of the flow, and after 2D cross-correlations over small sub-areas, velocity data can be extracted. PIV is able to resolve instantaneous vortex structures but only on inertial-range timescales. As an optical technique, it also requires well disbursed seeding to detect any flow structures.

PIV can be useful when considering vortex wandering. Rather than having to determine the position of the vortex centre from local time-mean velocity measurements (Bailey and Tavoularis, 2008), PIV produces a picture of the whole instantaneous vortex in each data plane collected. The cores can then be physically aligned in post-processing, without having to introduce any corrections into the data before extracting velocity statistics (Giuni, 2013). However, if a sufficient amount of seeding particles are not entrained within the vortex core, then the position of the centre is still an estimate to some degree. A few examples of experiments using PIV include those in the near-field (Birch et al., 2004; Souza and Faghani, 2001), and to enhance the smoke visualisation techniques used to investigate tip configurations on vortex formations (Sohn and Chang, 2012). While the advantages of the PIV technique include resolving structures in the data field and ease of correcting for wandering without the use of deconvolution methods, the low data rate cannot capture the high-frequency fluctuations characteristic of
turbulent flows.
Chapter 3

Experimental Apparatus and Method

3.1 Wing profile selection, design and fabrication of wing

To test the effects of surface roughness on wing-tip vortices, a pair of identical round-tipped, rectangular NACA0012 wings with a chord $c$ of 15.7 cm and an aspect ratio of 2.5 were constructed using a stereo-lithographic rapid prototyping machine, commonly known as a 3D printer. Previously published experiments using the symmetric NACA0012 profile (such as Devenport et al., 1996, among others) provide a range of data sets with which to compare the findings from this study. The freestream velocity $U$ was fixed at 10 m/s for all experiments, resulting in a chord Reynolds number $Re_c = Uc/\nu = 1.08 \times 10^5$.

Boundary layer forcing was accomplished by covering one of the wings with P80 grade sandpaper. The goal of this investigation was to increase the broadband turbulence within the boundary layer, without completely destroying the mechanics of the boundary layer. The sandpaper grade was chosen based on the roughness height $k$ and length scale $y^+$ discussed in Jimenez (2004). The desired height of the roughness elements was within the buffer layer ($y^+ > 5$), but not high enough to destroy the buffer-layer near-wall cycle ($k^+ \geq 50 - 100$). Given these two constraints, the simplest way to determine the necessary height is to
The characteristics of the boundary layer were not known \textit{a priori}, so an estimate of the tangential wall stress ($\tau$) had to be used. For the case of a turbulent boundary layer over a flat plate with zero pressure gradient,

$$\tau \approx 0.664 \rho U^2 \frac{2}{2Re^{0.5}},$$

(Schlichting and Gersten, 2001) where $Re = Ux/\nu$ is the streamwise Reynolds number and $x$ is the distance from the leading edge of the plate. For the present application, $x$ was taken to equal $c$, giving $\tau = 0.124$ Pa. This approximation has a few shortcomings: the wing surface is not flat, the boundary layer is not subject to a zero pressure gradient, and it is unlikely that the wing boundary layer is growing from scratch. However, (3.1.2) provides a rough estimate that should be accurate to within an order of magnitude. With a value for $\tau$, (3.1.1) was used to calculate $y^+ \geq 5$ for the roughness to enter the buffer layer, giving $y = 228 \mu m$ as a minimum.

The P80 grade sandpaper is generally accepted to have an average roughness height range of 190 to 265 $\mu m$, corresponding to $1.2 \leq k/c \leq 1.7(\times10^{-3})$, making it the first choice for these experiments. To verify the random and even distribution of the roughness height, the topology was digitized using an optoNCDT ILD2200-20 displacement transducer (with a measurement precision of 0.3 $\mu m$). An $x-y$ traverse was built for this purpose, and a surface area of $2 \times 2$ mm was scanned at 0.02 mm intervals. The minimum value of $k$ measured is defined as $k = 0$. The resulting surface map and roughness-height probability distribution function is included in figure 3.1.1, showing an average roughness height of 154 $\mu m$ and a maximum height of 433 $\mu m$.

### 3.2 Wind tunnels, facilities and instrumentation

All of the experiments carried out for this project were conducted in the EnFlo Laboratory at the University of Surrey.
Figure 3.1.1: (a) Surface map and (b) Roughness height distribution for the P80 grade sandpaper used

3.2.1 A-Tunnel

The A-Tunnel in the Enflo laboratory was used for all of the seven hole probe experiments as well as the hot wire experiments. This is an open return tunnel with a rectangular cross section of 0.6 m × 0.9 m and a working section 5 m long. The freestream velocity was maintained constant to within measurement precision by means of a closed-loop active control system. The control system used a Pitot probe mounted at the test section entrance connected to a Furness micromanometer with a full-scale range of 196 Pa to take velocity averages over 30 s intervals. The wind tunnel has a streamwise turbulence intensity level of less than 0.5%.

The wings were mounted vertically on the floor of the tunnel and were fitted in a purpose-built automated pitch positioning system with a 0.25° angular precision (see Appendix A for detailed schematic). The tunnel was also equipped with a five degree-of-freedom spherical-Cartesian traverse on which the probes were mounted. The x, y, z traverse was capable of a precision of ±5 µm, and the yaw and pitch angles had a precision of ±0.2°. The experimental setup and wind tunnel coordinate system are illustrated in figure 3.2.1.
3.2.1.1 Seven hole probe fabrication and instrumentation

Seven hole probes were purpose built for this research. They were constructed by assembling and soldering together lengths of 21-gauge stainless steel needle stock (having an outside diameter of 0.9 mm) and manually finishing with a tip cone angle of 60° (see figure 3.2.2). With an overall outside diameter of 2.9 mm, the seven hole probes were connected to a customised array of pressure transducers. Seven Honeywell 163PC01D75 temperature compensated, differential, amplified pressure transducers were assembled into a purpose built array. The array can be installed inside of the wind tunnel, allowing for quicker response due to the shorter connections between probe and transducer. This array can also have an additional eighth transducer attached, which was the configuration required when used with a novel eight hole probe. The transducers have a full-scale error of less than 0.25% and used wind tunnel static pressure as the reference. During wind tunnel use, the pressure transducers were calibrated in situ simultaneously against the micromanometer, by exposing the probe tip to known pressures. The transducers
were calibrated before and after measurements, and the data was discarded if the drift was larger than 1%. A full description of probe calibration is in section 3.3.

![Figure 3.2.2: Tip of a seven hole probe](image)

3.2.1.2 Uncertainty analysis

The seven hole probe is intended to capture mean pressure data in the vortex, and one of the primary sources of error is not capturing long enough periods of data for a converged mean value. Convergence tests were carried out to ensure that the standard error in the mean was less than 0.2%.

Detailed error analysis for the seven hole probe data was based on the capabilities of the pressure transducers, according to the manufacturer’s specification. The full-scale repeatability of the transducers is 0.25%, so random errors up to this amount were added to the raw pressure data of an existing vortex wake scan and the calibration data associated with it. Percentage errors were then calculated for \((u, v, w)\) values as well as how they propagate through to vortex properties such as core radius and circulation. For example, the error in estimating the vortex core radius was 1%, and the difference between maximum vorticity estimates was 4.7%. The remaining uncertainties are in table 3.1. It is important to note that
<table>
<thead>
<tr>
<th>Property</th>
<th>Mean Error</th>
<th>Maximum Error</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V$</td>
<td>2.6</td>
<td>12.8</td>
<td>2.0</td>
</tr>
<tr>
<td>$U$</td>
<td>1.8</td>
<td>6.1</td>
<td>1.2</td>
</tr>
<tr>
<td>$W$</td>
<td>2.6</td>
<td>12.1</td>
<td>2.0</td>
</tr>
<tr>
<td>$v_\theta$</td>
<td>6.8</td>
<td>156.2</td>
<td>16.4</td>
</tr>
<tr>
<td>$\Gamma$</td>
<td>92.4</td>
<td>176.8</td>
<td>54</td>
</tr>
<tr>
<td>$\zeta_x$</td>
<td>3.9</td>
<td>7.1</td>
<td>1.1</td>
</tr>
</tbody>
</table>

Table 3.1: Error estimates for seven hole probe experiments (% of maximum value)

most of the maximum errors quoted occurred well outside of the vortex core in regions prone to high levels of noise.

3.2.1.3 Novel 19 hole probe development

Concurrently, a novel 19 hole probe was designed and built for the purpose of extracting mean vorticity directly from pressure data. The numbering scheme for this probe is depicted in figure 3.2.3. An array of 35 low-cost, high gain Honeywell PCAFA6D differential pressure sensors was also designed and built with a net sensitivity of $0.04 Pa/V$ for use with the 19 hole probe. An analogue signal multiplexer was built to use in conjunction with this transducer array. The multiplexer allows for use of as many or as few channels as required at any given time, by cycling through five sets of seven signals. The signals from the transducers were then digitized using existing acquisition systems. This probe has been shown to provide slightly higher precision measurements of velocity than a seven hole probe as well as local measurements of velocity gradients (Shaw-Ward et al., 2014). The size of the 19 hole probe made it impractical for use in the bulk of the wake scans, with a tip diameter of $4 \pm 0.08$mm diameter. While successfully obtaining some velocity gradients, care must be taken in vorticity measurements near the vortex centre, due to the high degree of sensitivity in measurement error.
3.2.1.4 Hot wire setup

Single-wire experiments in the wing wake were carried out in the A-Tunnel. The hot wire was calibrated against the pitot-static probe used for the tunnel speed regulation. During experiments, the hot wire was calibrated every 60 minutes, and the data were discarded if the calibration drift was over 1%. The probes were driven by a Newcastle constant temperature anemometer. The hot wire was mounted on the \((x, y, z)\) tunnel traverse. The hot wire was used to measure across the 2D wing wake \(0.25b\) from the wing tip and \(0.3c\) behind the wing. The data rate was 5600 Hz and data was taken over a time period of 1 minute in blocks 4096 points long.

3.2.2 Aero-tunnel

All laser Doppler anemometry and force balance experiments were conducted in the Aero-tunnel at the University of Surrey. This is a closed return tunnel with a \(1.1 \times 1.4\) m rectangular working section, and a measured streamwise turbulence intensity of \(\sim 0.15\%\). The tunnel is equipped with a 3 degree of freedom traverse, allowing movement in the \(x – y – z\) planes, and an automated pitch traverse (see appendix A) was also installed in the floor of the tunnel. The freestream velocity was maintained constant to within measurement precision by means of a closed-loop active control system using a Pitot probe at the entrance of the test section.
3.2.2.1 Force balance

Wing performance was evaluated using the 6-component force and moment balance installed in the Aero-tunnel. The full-scale range, precision, and standard error of the Aerodynamic Test Equipment Ltd (ATE-Ltd) balance is detailed in table 3.2. A 6 channel signal conditioning unit mounted on the balance provides digital, high resolution signals. The present experiments only used two of the available components for lift and drag measurements. Due to the orientation of the wing after it was mounted on the balance, the component used for lift measurements was the component labeled “Side Force” by the manufacturer (and as in table 3.2). For 2-dimensional airfoil measurements, an end plate with a sharp leading edge was constructed and installed parallel to the tunnel floor, 3 mm from the wing tip.

<table>
<thead>
<tr>
<th>Component</th>
<th>Range</th>
<th>Precision</th>
<th>Standard Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lift (N)</td>
<td>-650 to +450</td>
<td>0.0255</td>
<td>0.0122</td>
</tr>
<tr>
<td>Pitch (Nm)</td>
<td>-40 to +80</td>
<td>0.00488</td>
<td>0.03</td>
</tr>
<tr>
<td>Drag (N)</td>
<td>0 to 200</td>
<td>0.00435</td>
<td>0.0296</td>
</tr>
<tr>
<td>Side Force (N)</td>
<td>±180</td>
<td>0.00978</td>
<td>0.075</td>
</tr>
<tr>
<td>Yaw (Nm)</td>
<td>±10</td>
<td>0.00081</td>
<td>0.068</td>
</tr>
<tr>
<td>Roll (Nm)</td>
<td>±70</td>
<td>0.00481</td>
<td>0.021</td>
</tr>
</tbody>
</table>

Table 3.2: Range, precision and standard error (as % of full scale) of 6-component ATE-Ltd force and moment balance, supplied by ATE-Ltd

3.2.2.2 Uncertainty analysis

Force balance measurements were required to assess the effect of sandpaper roughness on wing performance. However, accurate assessment of the performance requires a high level of sensitivity to detect small changes in lift and drag. McCroskey (1987) cites that an accuracy of 0.0005 in $C_D$ is desired to assess changes in model configuration, which is 0.0048 N for the current conditions. Based on the full-scale ranges of each component, the precision of the lift component is 0.0255 N, while the precision in drag measurements is 0.00978 N. The data
presented has been averaged over two minute periods to limit the effect of noise on the readings. The standard error in drag measurements was less than 0.002 and 0.014 in lift measurements over the full range of angles investigated. The precision of the angle in balance measurements is ±0.5°.

3.2.2.3 Laser Doppler anemometry experimental setup

The laser Doppler anemometry (LDA) technique was used to measure the time-resolved velocity field. The LDA system allows for non-intrusive measurement of the flow-field, and is a well established technique (Buchhave et al., 1979). The LDA system installed in the Aero-tunnel consists of a Coherent Innova 70C argon laser connected to a Dantec Dynamics 2-component Fibreflow backscatter system. Two probes were used for the data collection. The first, installed inside of the tunnel to measure cross-flow components of velocity \((v, w)\), was 27 mm in diameter, and was fitted with a lens requiring a 160 mm stand-off distance from the measurement volume. The measurement volume length for this probe is 1.57 mm with a diameter of 0.074 mm. The probe was installed to measure upstream flow and mounted parallel to the free stream on the tunnel 3-component traverse system, as shown in figure 3.2.4.

To capture the streamwise component of velocity \((u)\), a 60 mm diameter LDA probe was used. This probe has a 0.8 m stand-off distance, and was mounted outside of the tunnel walls, perpendicular to the free stream flow (see figure 3.2.5). The measurement volume of the 60 mm probe is 1.1 mm long and 0.062 mm in diameter. Both probes used the Dantec Dynamics F60 flow processor, outputting the data to the automated tunnel acquisition system.

For this set of experiments, the wing incidence was held constant at \(\alpha = 5^\circ\). The data plane was 5 chords lengths downstream of the wing trailing edge, and both the 800 mm and 160 mm focal length lenses were used. Data reporting \((v, w)\) was taken with the 160 mm lens at \(\Delta y = \Delta z = 1\) mm, while the \((u)\) data was taken with the 800 mm lens at \(\Delta y = \Delta z = 1.5\) mm.
Figure 3.2.4: Schematic of Aero tunnel using the 27 mm LDA lens

Figure 3.2.5: Schematic of Aero tunnel using the 60 mm LDA lens
3.2.2.4 Uncertainty analysis

As an initial test, the mean data from the LDA experiments were compared to the data from the seven hole probe to ensure consistency between setups. Based on free stream measurements, the systematic measurement error in LDA data on average is $\sim 1\%$. The effect of random errors in the timetraces on mean quantities is similar to that of seven hole probe measurements. The random sampling rate of an LDA timetrace can also present a velocity bias on the measured mean velocity - higher data rates are more likely for higher velocity fluctuations in the flow. Edwards and Jensen (1983) state that the bias in the mean velocity is proportional to the square of the turbulence intensity. As an example, the maximum bias on tangential velocity data for the rough wing case is $\sim 6\%$ of $v_0$.

Spectra generated from LDA data necessarily contain some amount of uncertainty, due to the random nature of data capture. Data rates were kept above a nominal value of 2000 Hz with real-time seeding control during experiments. LDA spectra require a re-construction of the timetraces at constant time intervals, before taking the Fourier transform of the autocorrelation function. The frequency at which this re-sampling is evaluated has one of the greatest effects on the spectra (Adrian and Yao, 1987; Nobach et al., 1998), and the mean data rate of the LDA sample has been shown to produce spectra closest to that produced by hot wire data. This particular “sample and hold” technique is straightforward in implementation, but is limited by effectively introducing a low pass filter frequency of the order $\frac{\dot{N}}{2\pi}$, where $\dot{N}$ is the mean data rate.

3.2.2.5 Flow seeding

Selection of seeding particle diameter is an important consideration in optical measurements of vortex flows. Accurate velocity measurement in vortex flows is very sensitive to seeding particle diameter. For LDA measurements, the flow was seeded with a TSI 9307 atomiser, using olive oil as the seeding fluid. Building on the work of Dring (1982) and Melling (1997) (among others), Birch and Martin (2013) present criteria for particle tracking errors to remain below 1% for each velocity component in vortex flows in terms of maximum Strouhal numbers. The atomiser creates a typical seeding diameter of $\sim 1 \mu m$, well within the maximum
values derived by Birch and Martin (2013). Seeding was controlled remotely via a real-time feedback loop to give a desired minimum sampling frequency during testing periods. The flow was seeded from a single position at the end of the working section, giving ample time for homogeneity of seeding particles throughout the flow before returning to the working section.

3.3 Novel multi-hole probe calibration scheme

3.3.1 Demonstration of need for new calibration scheme

Using established techniques, such as those described by Zilliac (1989) and Gallington (1980), the calibration of multi-hole probes is dependent upon non-dimensional pressure coefficients derived from pressure differences. These coefficients are used to determine flow speed and direction. For reference, the numbering scheme of the holes on a seven-hole probe is as shown in figure 3.3.1. The probe is divided into seven sectors, each corresponding to a different hole and calibration region. For a full description on current seven hole probe calibration procedures, see appendix B.

![Figure 3.3.1: Numbering scheme of a seven-hole probe, looking aft.](image)

Most conventional calibration methods include using mutually exclusive and non-overlapping calibration regions in cone ($\theta$) and roll ($\phi$) space, as defined in 3.3.2. These regions can be discontinuous, and the algorithm used to select the appropriate calibration space could fail if the stagnation point falls in between
holes such that the pressures recorded at more than one hole are within the range of experimental uncertainty. Calibration of multi-hole probes can also be very time consuming to obtain a dense enough matrix of calibration data which yields reasonably low errors when using conventional interpolation schemes. Previous studies have proven a calibration matrix density of five degrees in pitch and yaw to be sufficient for seven hole probes, resulting in a $29 \times 29$ matrix for pitch and yaw angles ranging from -70 to 70 degrees (Zilliac, 1993). There have been a number of papers published on various methods of interpolation and data reduction for multi-hole probe calibration (Silva et al., 2003; Zilliac, 1993; Ericksen et al., 1995; Wenger and Devenport, 1999, among others). In fact, one outcome of this research is a method of data reduction using optimal design of experiments and D-Optimality. This method was presented in a conference paper and is described in detail in appendix C (a copy of the paper is also provided for reference in appendix F).

During the first stage of data gathering for this research, problems stemming from the conventional methods of seven hole probe calibration were overwhelmingly evident. Using accepted methods of probe calibration, as detailed in appendix B, figure 3.3.3 shows wake survey data from a seven-hole probe behind a smooth NACA0012 wing with a rounded tip at an incidence of $10^\circ$ taken $5c$ downstream
Figure 3.3.3: (a) Contours of normalized vorticity, $\zeta r_c/v_0$, and (b) Contours of normalized vorticity showing calibration sector overlay using the conventional calibration methods

of the wing. The free-stream velocity was 10 m/s, and the probe was calibrated *in-situ* before the test began. The figure shows contours of normalised vorticity, with an overlay of the calibration sectors. Discontinuities such as these were not visible in raw pressure readings from the probe and are impossible in a vortex. Therefore, they must arise from the discontinuities in the calibration space (Shaw-Ward et al., 2014, and reproduced for convenience in G).

The conventional calibration of multi-hole probes requires the identification of pitch and yaw (or cone and roll) coefficients to determine flow angularity, reducing the number of independent variables from $n$ to two (where $n$ is the number of holes in the probe). For probes having more than five holes, this reduces the sensitivity of the probe as pressures have to be averaged at some point (see (B.0.1) for an example). Interpolation of flow velocities from calibration functions has also been shown to be a potential source of error (Shaw-Ward et al., 2014).
3.3.2 Description of calibration procedure

It was clear that, in order to proceed with this research, a new calibration procedure, which ideally eliminates the need for discontinuous functions, was necessary. Another motivation for this particular technique was to increase angular precision by keeping the number of independent variables at \( n \), which also, consequently, means that the same approach can be used for any probe of \( n \) holes, with arbitrary geometry. The local stagnation pressure can still be approximated as the maximum pressure \( P_{\text{max}} \) recorded, as in the conventional seven hole calibration. Without knowing the exact geometry or hole arrangement, the static pressure can be assumed as the minimum pressure \( P_{\text{min}} \) recorded not subject to separated flow. The pressure coefficients of an \( n \) hole probe can then be defined as

\[
C_{P_i} = \frac{P_{\text{max}} - P_i}{P_{\text{max}} - P_{\text{min}}} \tag{3.3.1}
\]

\[
C_0 = \frac{P_{\text{max}} - P_0}{P_{\text{max}} - P_{\text{min}}} \tag{3.3.2}
\]

\[
C_s = \frac{P_{\text{max}} - P_s}{P_{\text{max}} - P_{\text{min}}} \tag{3.3.3}
\]

where \( P_i \) is the pressure recorded at the \( i \)th hole (\( i = 1, 2, ..., n \) for \( P_i \neq P_{\text{max}} \)), and \( P_0 \) and \( P_s \) are the reference total and static pressures, respectively. Given a set of calibration data collected at many angles \((\alpha, \beta)\), the functions \( f_\alpha \) and \( f_\beta \) can be defined such that

\[
\alpha = f_\alpha (C_{P1}, C_{P2}, ..., C_{Pn}) \tag{3.3.4}
\]

\[
\beta = f_\beta (C_{P1}, C_{P2}, ..., C_{Pn}) \tag{3.3.5}
\]

where \( f_\alpha \) and \( f_\beta \) are empirical functions defined by calibration data collected in constant, uniform flow at a single velocity. The functions have the advantage of being continuous over the full range of the probe, eliminating the need to select between discrete functions. The arrangement and indexing of the holes
becomes arbitrary as well. Given experimental measurements of \((C_{P1}, C_{P2}, ..., C_{Pn})\), the flow angle can be interpolated from the calibration data set. The velocity magnitude can then be obtained using the conventional approach (see (B.0.11)). By removing the sectors in the calibration procedure and keeping the number of independent variables at \(n\), the vorticity field from figure 3.3.3 becomes axisymmetric as shown in figure 3.3.4.

![Figure 3.3.4](image-url)  

Figure 3.3.4: Contours of normalized vorticity, \(\zeta r_c/v_0\), using novel calibration method. Data is identical to that in 3.3.3.

For small \(n\), it is also possible to fit polynomials to the continuous functions \(f_\alpha\) and \(f_\beta\) of order \(k\) with \(n\) variables. Related work (McParlin et al., 2013, included in F) suggested that a polynomial on the order of at least \(k = 6\) is required. This results in 28 terms for a seven hole probe, but for example, if \(n = 19\) (as for the probe described in Shaw-Ward et al., 2014) then 177,100 terms are required. This technique can also be applied to the calibration of triple-wire probes. However, the velocity response of triple-wire probes is nonlinear, so calibration is required in pitch, yaw, and speed, making a fine calibration grid impractical.
3.4 Post-processing

3.4.1 Vortex centre

Finding the centre of a vortex is not trivial, and parameters such as tangential velocity can be severely affected by small changes in the estimated location of the centre. There are several ways to determine the coordinates of the centre (see Giuni, 2013). The method used in this work is that of finding the location of minimum cross-flow velocity. This method is independent of origin, and does not require any a priori assumptions about the flow. Using the coordinate system from figure 3.2.1, the vortex centre is determined to be at the point where \( v^2 + w^2 \) reaches its minimum value. To improve the spatial resolution of the data, a bicubic surface-fit was used to interpolate the location of the minimum velocity from high resolution resampling of the data.

3.4.2 Velocity

Describing the velocity field of a vortex is done most conveniently in polar coordinates, since a vortex is fundamentally axisymmetric. Data is collected with a wind tunnel fixed coordinate system, which is best done in a Cartesian coordinate system \((x, y, z)\). The Cartesian velocities obtained from the probe \((u, v, w)\) then need to be converted into the polar radial \((v_r)\) and tangential \((v_\theta)\) velocities. Assuming the vortex exists in a space causing the fluid to rotate anti-clockwise about the \(x\)-axis, the system has an origin defined as being coincident with the centre of the vortex (fig 3.4.1).

3.4.3 Vorticity

The axial vorticity \( \zeta_x(y, z) \) of a vortex is a measure of the strength of the vortex and is defined as

\[
\zeta_x(y, z) = \frac{\partial v}{\partial z} - \frac{\partial w}{\partial y},
\]

or, in polar coordinates,
Figure 3.4.1: Polar and Cartesian coordinate system definitions, looking upstream, with the origin located at the vortex centre.

\[
\zeta_x(r, \theta) = \frac{v_\theta}{r} + \frac{\partial v_\theta}{\partial r} - \frac{1}{r} \frac{\partial v_r}{\partial \theta}.
\]

This is approximated from wake scan data with first-order finite central differences. Higher order differences were not used due to the potential smoothing out of the data fields. Assuming an axisymmetric vortex, the analytical solution to the vorticity distribution can be obtained directly from the Lamb solution to the Navier Stokes equations,

\[
\zeta = \frac{v_0}{r_c} (1 + 2\alpha) \exp \left( -\alpha \eta^2 \right).
\]

Just as circulation and tangential velocity have a Gaussian distribution, the distribution of vorticity within a Batchelor vortex is also Gaussian. Hence, the distribution of vorticity can be used to assess the axisymmetry of the vortex. The maximum vorticity occurs at the vortex centre ($\eta = 0$) and has a theoretical dimensionless value of $\zeta x r_c / v_0 = 1 + 2\alpha \approx 3.5$. 

39
3.4.4 Circulation

The circulation of a vortex, as defined in 2.1.2, is the closed-path integral of the tangential component of velocity. The total circulation of the vortex is defined as

$$\Gamma_0 = \Gamma(r) \bigg|_{r \to \infty} \quad (3.4.4)$$

However, it is very difficult to compute this from experimental data. Computing line integrals from sparse data is not straightforward and prone to large errors, and the signal-to-noise ratio becomes erratic as $r$ becomes large. In practice, the circulation is generally computed using Green’s theorem,

$$\Gamma = \int_A \zeta_x(x, y, z) dA, \quad (3.4.5)$$

where $A$ is the area bounded by $S$ in (2.1.2). From (2.1.1), the core circulation (at $\eta = 1$), yields $\Gamma_c/\Gamma_0 = 0.71533$. This can be supported from experiments with numerical integration of (3.4.5), but determining $\Gamma_0$ is difficult given the amount of uncertainty spacial averaging introduces as well as the level of noise present in experimental data. As such, the total circulation was calculated by expressing (2.1.1) in terms of tangential velocity,

$$v_\theta = \frac{\Gamma_0}{2\pi r} \left(1 - \exp \left(-\frac{r^2}{4\nu t}\right) \right), \quad (3.4.6)$$

given $\Gamma(r) = 2\pi rv_\theta$, and time, $t$, is the relative age of the vortex defined by $r_c = \sqrt{4\alpha \nu t}$. Evaluating (3.4.6) for the maximum tangential velocity, a solution for $\Gamma_0$ presents itself as

$$\frac{\Gamma_0}{2\pi \sqrt{4\alpha \nu t}} = v_0 \left(1 + \frac{1}{2\alpha}\right). \quad (3.4.7)$$

3.4.5 Lift and drag estimates

Total lift and drag were recording using the force balance in the Aero tunnel, but approximations of these values can be made directly from vortex wake scans. The lift per unit length $L/b$ (where $b$ is the wing span) experienced by a wing in potential flow is
\[ \frac{L}{b} = \rho U_\infty \Gamma_0. \]  

(3.4.8)

In terms of lift coefficient, \( C_L \), lift can be expressed as

\[ L = \frac{1}{2} \rho U_\infty^2 b c C_L, \]  

(3.4.9)

where \( \rho \) is the air density, and the wing is assumed to be two-dimensional and rectangular. Therefore, combining 3.4.8 and 3.4.9 gives

\[ \Gamma_0 = \frac{1}{2} U_\infty c C_L, \]  

(3.4.10)

where \( \Gamma_0 \) is calculated from (3.4.7). Assuming incompressible and adiabatic flows, Kusunose (1997), (1998) showed that the induced drag, \( D_i \), on a three-dimensional wing model can be obtained from

\[ D_i = \int \int_A 1 \frac{1}{2} \rho_\infty (v^2 + w^2) dy dz, \]  

(3.4.11)

where \( A \) is a surface contained within the wake survey plane. Drag, similar to circulation, is a very difficult quantity to measure from vortex wake data alone, and is prone to high levels of uncertainty.

### 3.4.6 Power spectra

#### 3.4.6.1 LDA

The nature of data obtained from LDA experiments makes generating spectra from raw data impossible. LDA data is only recorded when a particle enters the measurement volume, which occurs at random intervals. To obtain velocity spectra, the data needs to be re-sampled at regular time intervals. The method used in the present work was reviewed in detail by Adrian and Yao (1987) and consists of a “sample and hold” technique. The LDA velocity data is re-sampled at the mean data rate, by assuming that the flow velocity is held constant between recorded data points. A fast Fourier transform is then applied to the re-sampled data set to obtain the velocity spectra. The output of the Fourier transform was then squared to obtain the power (\( \Phi \)). To remove some of the noise from the
spectra, the data was then averaged in logarithmically spaced sections, increasing in size with frequency. For LDA data sets taken in four blocks (longer blocks were not possible due to system memory limitations), the mean data rate was calculated from the full timetrace, but each block was re-sampled separately as described above. The Fourier transform for each block was computed and averaged together to produce the spectra for that data point.

3.4.6.2 Hot-wire

Single hot-wire power spectra were computed in a similar fashion to the LDA spectra. Power spectra were obtained from discrete Fourier transforms of the data. Hot-wire data was taken in multiple blocks, and the power spectra from each block was averaged to compute spectra for that data point.

3.4.7 LDA filter signal processing

Filters were applied to re-sampled LDA timetraces as a means of isolating certain spectral frequencies. A Cauer elliptic filter was used to implement highpass and bandpass filters to the data. Details of the stop and start frequencies for each filter used, as well as other important parameters, are shown in table 3.3. For all filters, the mean velocity was subtracted from the timetraces before applying the filter then added back in to the filtered time trace. The filtered timetraces were also truncated as necessary to remove any signal noise that was a product of the filter.
<table>
<thead>
<tr>
<th>Highpass filter</th>
<th>Stop freq (Hz)</th>
<th>Pass freq (Hz)</th>
<th>Attenuation (dB)</th>
<th>Ripple allowed (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 Hz</td>
<td>2</td>
<td>3</td>
<td>60</td>
<td>1</td>
</tr>
<tr>
<td>3 Hz</td>
<td>3</td>
<td>4</td>
<td>60</td>
<td>1</td>
</tr>
<tr>
<td>5 Hz</td>
<td>5</td>
<td>6</td>
<td>60</td>
<td>1</td>
</tr>
<tr>
<td>8 Hz</td>
<td>8</td>
<td>9</td>
<td>60</td>
<td>1</td>
</tr>
<tr>
<td>10 Hz</td>
<td>10</td>
<td>11</td>
<td>60</td>
<td>1</td>
</tr>
<tr>
<td>15 Hz</td>
<td>15</td>
<td>17</td>
<td>60</td>
<td>1</td>
</tr>
<tr>
<td>20 Hz</td>
<td>20</td>
<td>22</td>
<td>60</td>
<td>1</td>
</tr>
<tr>
<td>Bandpass filter</td>
<td>12 Hz</td>
<td>10/14</td>
<td>11/13</td>
<td>60</td>
</tr>
<tr>
<td></td>
<td>330 Hz</td>
<td>290/380</td>
<td>300/370</td>
<td>60</td>
</tr>
</tbody>
</table>

Table 3.3: Filter specifications for LDA timetrace processing
Chapter 4

Wing performance

Apart from determining how surface roughness affects the wingtip vortex, it is also important to evaluate how roughness affects wing performance. For applications on aircraft wings and helicopter rotor blades, an increase in drag for a given incidence results in a decrease in fuel efficiency - a consequence that could cause this concept to be discarded with increasing fuel prices. Other important performance characteristics include the lift curve and stall angle. A significant decrease in either of these would most likely cancel out any positive effects on the vortex. All force balance data were taken at a freestream velocity of 10 m/s, or \(Re_c \approx 1.2 \times 10^5\), in the Aero-tunnel. Estimates of total circulation and induced drag were also calculated from the wake scan data taken in the A-tunnel with the seven hole probe, using (3.4.10) and (3.4.11).

4.1 Lift characteristics

First, the performance of the wing needed to be validated with previously published data. While are there a number of wind tunnel studies on the performance of the NACA 0012 airfoil to choose from (McCroskey, 1987), the data from Jacobs and Sherman (1937) has been used based on \(Re_c\) agreement. Figure 4.1.1 shows the coefficient of lift for the 2-D airfoil as compared with the theoretical value for a thin airfoil of \(C_l = 2\pi\alpha\), and NACA 0012 data published by Jacobs and Sherman (1937). The decrease in lift curve slope between smooth and rough surfaces is
~ 20%, and the stall angle remained constant to within 0.5°. The 3-D wing lift characteristics are given in figure 4.1.2. The theoretical lift coefficient has been adjusted to include the efficiency factor, $e$, and an aspect ratio, $A_R = 5$, for the 3-D wing. The decrease in lift curve slope has been reduced to only ~ 5% with the addition of roughness, and the stall angle has increased by 1° from the 2-D configuration.

![Graph showing lift coefficient $C_L$ versus angle of attack $\alpha$.](image)

Figure 4.1.1: 2-D lift data versus angle of attack

Total circulation estimates from wake surveys is presented in figure 4.1.3. These measurements are prone to a large amount of uncertainty, and something as simple as the vortex not centered in the wake survey plane can result in low estimates of circulation. This is most likely the reason for the slight dip in circulation at $\alpha = 10^\circ$ and $13^\circ$ in the smooth wing data. However, the low circulation estimates at $\alpha = 10^\circ$ and $14^\circ$ in the rough wing data are too severe for a simple misalignment. These are considered to be bad points and not considered to affect the analysis.
Figure 4.1.2: 3-D lift data versus angle of attack

Figure 4.1.3: Variation of normalized tip vortex strength and 3D force balance lift data with $\alpha$
4.2 Drag characteristics

Not only are the lift characteristics important, but a large increase in drag for a given angle would make any change in vortex behaviour irrelevant. Figure 4.2.1 shows the drag characteristics for the 2-D and 3-D configurations of both the smooth and rough wings. For reference, the profile drag ($C_{D0}$) from Jacobs and Sherman (1937) has also been included. As seen in the lift-curve slope, the added surface roughness has a greater effect on the drag in the 2-D airfoil than on the 3-D wing. At low angles ($\leq 5 - 7^\circ$), the difference in lift-to-drag ratio is within experimental accuracy. This demonstrates that for a given amount of lift, or vortex strength, the surface roughness has little effect on the overall drag of the wing.

![Figure 4.2.1: Drag vs lift coefficient](image)

Using the methods of Kusunose (1997), (1998), wake survey data was used to estimate induced drag, $C_{Di}$ (figure 4.2.2). It would appear that the estimates of induced drag using (3.4.11) are greatly overestimated for higher $\alpha$. However, it is difficult to determine what the area, $A$, should be exactly in (3.4.11), and the data in figure 4.2.2 used the entire wake survey plane as $A$. As such, if part of the wing wake is contained within the wake survey plane, then any nonzero values
of \( v \) and \( w \) would not be contributing to the strength of the vortex, but would increase the estimate of induced drag. This would also be evident for any nonzero values of vorticity outside of the vortex, most likely stemming from any residual wing boundary layers not yet rolled up into the vortex. Evidence of this can be seen in figure 5.1.2.

![Figure 4.2.2: Variation of induced drag (\( C_{Di} \)) and total drag (\( C_D \)) with \( \alpha \)](image)

Figure 4.2.2: Variation of induced drag (\( C_{Di} \)) and total drag (\( C_D \)) with \( \alpha \)
Chapter 5

Mean vortex

The following chapter focuses on the characteristics of the mean vortex as compared to analytical solutions introduced in Chapters 2 and 3. The data was collected with a seven hole probe at a range of angles of attack at 5 chord lengths downstream of the wing trailing edge. A distance of $x = 5c$ was chosen based on previous studies by Devenport et al. (1996) which show that between $5 \leq x/c \leq 30$ there is no significant change in the core characteristics. A sample of these results is reproduced in figure 5.0.1 along with some data taken behind the smooth and rough wings between $3 \leq x/c \leq 10$. All wind tunnel runs were at 10 m/s, and the seven hole probe was calibrated in situ before data capture. The probe axis was aligned with the freestream by finding the angle at which the pressure recorded at hole 7 was at the maximum and setting $\theta = \phi = 0$ there. Wake surveys were taken at increments of $\Delta y = \Delta z = 1.5$ mm with around 1100 grid points for each data plane.

5.1 Vortex core characteristics

The aim of the seven-hole probe experiments was to establish the properties of the mean wing-tip vortex. To be able to show the influence of boundary layer forcing on a vortex, it is important to establish that a well developed, axisymmetric vortex occurs with and without any forcing applied. This is accomplished, in part, by showing that the vortex conforms to the analytical solutions derived by Batchelor...
Figure 5.0.1: Vortex parameters, in terms of $r_c/c$ and $v_0$, against $x/c$: (○), Smooth wing at $\alpha = 8^\circ$; (●), Rough wing at $\alpha = 5^\circ$; (∗), Devenport et al. (1996) at $\alpha = 5^\circ$.

(1964). Earlier arguments suggest that the simple conformation to the analytical solutions does not conclusively demonstrate the presence of a vortex. However, the analytical profiles together with comparisons to similar vortex data (see Devenport et al., 1996) are the only tools available. Core parameters, such as core radius and maximum vorticity are shown in tables 5.1 and 5.2. Setting $\eta = 1$ in (2.1.1), the theoretical value of $\Gamma_c/\Gamma_0 \approx 0.71533$, which agrees well with the angles above $4^\circ$.

Wake surveys below $\alpha \approx 5^\circ$ are prone to more error as most of the wake is not yet rolled into the vortex. For all $\alpha$, peak tangential velocity ($v_0$) is lower for the rough wing than the smooth wing. Axial velocity deficits are lower for the rough wing for most $\alpha$, excluding $\alpha = 8$, 12, and 13. The vortex size, $r_c/c$, is consistent with the results of Devenport et al. (1996) at $x/c = 10$ for $\alpha \geq 4$, as seen in figure 5.1.1. Vortex sizes below $\alpha = 4^\circ$ are much higher for both the smooth and rough wing, most likely because the data is taken closer to the wing and at a lower chord Reynolds number.

Figure 5.1.2 shows normalized vorticity contours of the tip vortex generated from a wing at $\alpha = 5^\circ$. These contours show that at $5c$ downstream of the wing, the vortex appears axisymmetric with no visible wake still being rolled into the vortex. Contours of tangential velocity, $v_\theta$, are shown in figure 5.1.3. The asymmetry in
<table>
<thead>
<tr>
<th>( \alpha )</th>
<th>( \zeta r_c/v_0 )</th>
<th>( r_c/c )</th>
<th>( \Gamma_c/\Gamma_0 )</th>
<th>( v_0/U_{\infty} )</th>
<th>( (U_{\infty} - u_0)/U_{\infty} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>4.56</td>
<td>0.037</td>
<td>0.70</td>
<td>0.11</td>
<td>0.08</td>
</tr>
<tr>
<td>4</td>
<td>3.74</td>
<td>0.038</td>
<td>0.69</td>
<td>0.26</td>
<td>0.22</td>
</tr>
<tr>
<td>5</td>
<td>3.45</td>
<td>0.035</td>
<td>0.69</td>
<td>0.33</td>
<td>0.37</td>
</tr>
<tr>
<td>6</td>
<td>3.55</td>
<td>0.037</td>
<td>0.70</td>
<td>0.39</td>
<td>0.39</td>
</tr>
<tr>
<td>8</td>
<td>3.64</td>
<td>0.040</td>
<td>0.70</td>
<td>0.49</td>
<td>0.30</td>
</tr>
<tr>
<td>10</td>
<td>3.31</td>
<td>0.040</td>
<td>0.70</td>
<td>0.51</td>
<td>0.35</td>
</tr>
<tr>
<td>12</td>
<td>3.53</td>
<td>0.045</td>
<td>0.70</td>
<td>0.63</td>
<td>0.27</td>
</tr>
<tr>
<td>13</td>
<td>3.66</td>
<td>0.044</td>
<td>0.70</td>
<td>0.61</td>
<td>0.28</td>
</tr>
<tr>
<td>14</td>
<td>3.72</td>
<td>0.047</td>
<td>0.70</td>
<td>0.64</td>
<td>0.21</td>
</tr>
</tbody>
</table>

Table 5.1: Smooth wing vortex core parameters for all \( \alpha \)

<table>
<thead>
<tr>
<th>( \alpha )</th>
<th>( \zeta r_c/v_0 )</th>
<th>( r_c/c )</th>
<th>( \Gamma_c/\Gamma_0 )</th>
<th>( v_0/U_{\infty} )</th>
<th>( (U_{\infty} - u_0)/U_{\infty} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>4.09</td>
<td>0.068</td>
<td>0.71</td>
<td>0.052</td>
<td>0.06</td>
</tr>
<tr>
<td>4</td>
<td>4.27</td>
<td>0.054</td>
<td>0.64</td>
<td>0.13</td>
<td>0.12</td>
</tr>
<tr>
<td>5</td>
<td>3.64</td>
<td>0.044</td>
<td>0.69</td>
<td>0.21</td>
<td>0.19</td>
</tr>
<tr>
<td>6</td>
<td>3.51</td>
<td>0.039</td>
<td>0.69</td>
<td>0.29</td>
<td>0.30</td>
</tr>
<tr>
<td>8</td>
<td>3.33</td>
<td>0.044</td>
<td>0.70</td>
<td>0.42</td>
<td>0.41</td>
</tr>
<tr>
<td>10</td>
<td>3.64</td>
<td>0.044</td>
<td>0.69</td>
<td>0.21</td>
<td>0.19</td>
</tr>
<tr>
<td>12</td>
<td>3.28</td>
<td>0.052</td>
<td>0.71</td>
<td>0.54</td>
<td>0.47</td>
</tr>
<tr>
<td>13</td>
<td>3.29</td>
<td>0.055</td>
<td>0.71</td>
<td>0.56</td>
<td>0.52</td>
</tr>
<tr>
<td>14</td>
<td>3.64</td>
<td>0.044</td>
<td>0.69</td>
<td>0.21</td>
<td>0.19</td>
</tr>
</tbody>
</table>

Table 5.2: Rough wing vortex core parameters for all \( \alpha \)
Figure 5.1.1: Vortex size, in terms of $r_c/c$, against wing incidence: (○), Smooth wing; (●), Rough wing; (+), Devenport et al. (1996). $Re_c = 1.1 \times 10^5$ for smooth and rough wings, and $Re_c = 5.3 \times 10^5$ for Devenport et al. (1996) data.
the tangential velocity around \( r_c \) is most likely due to the radial component of the vortex trajectory or a slight offset in the probe origin.

## 5.2 Vortex profiles

The Batchelor (1964) model for a laminar vortex offers analytical profiles for circulation, vorticity, axial velocity, and tangential velocity. Phillips (1981) and Hoffmann and Joubert (1963) also offer analytical profiles for a turbulent vortex, but Birch (2012) has shown these profiles exhibit remarkably close agreement to each other in the vortex core up to \( r/r_c \approx 1.3 \). Data from the smooth and rough wings at \( \alpha = 5^\circ \) and \( 10^\circ \) is compared with the Batchelor solutions as a function of \( r/r_c \) in figures 5.2.1, 5.2.2, 5.2.3, and 5.2.4, respectively. Figure 5.2.4 shows \( v_\theta \) data that has been circumferentially averaged about the vortex centre. All data agrees relatively well up to at least \( \eta \approx 1.3 \), at which point the validity of the model begins to break down. Axial velocity deficits show the core for both smooth and rough wings is wake-like. For clarity, only two of the nine angles at which data was collected are included in this section. They are representative of the general
trends in the data, and the corresponding plots of the remaining data are included in D.

On the whole, the addition of the sandpaper appears to reduce the strength of the vortex for most angles of attack. The vortex core radius becomes larger, and the velocity gradients within the vortex decrease. It is, perhaps, interesting to note that even a vortex produced from a very rough wing, which would presumably have higher levels of turbulence within the vortex, still follows the laminar profiles of Batchelor (1964) very well. Because of the well known long lasting nature of these vortices, it is not expected that any vortex instabilities or break down would occur at $5c$ downstream of the wing.
Figure 5.2.1: Circulation profiles normalized by total circulation calculated from (3.4.7): (○), Smooth wing; (●), Rough wing; (—), (2.1.1).

Figure 5.2.2: Normalized vorticity profiles: (○), Smooth wing; (●), Rough wing; (—), (3.4.3).
Figure 5.2.3: Axial velocity profiles normalized by maximum tangential velocity: (○), Smooth wing; (●), Rough wing; (—) and (—−), (2.1.3).

Figure 5.2.4: Circumferentially averaged tangential velocity profiles normalized by maximum tangential velocity: (○), Smooth wing; (●), Rough wing; (—), (2.1.5).
Chapter 6

Vortex turbulence characteristics

It has now been established that increasing surface roughness changes properties of the mean vortex for a given set of freestream conditions. The vortex size \( r_c \) increases, and velocity gradients and magnitudes are reduced within the core. However, a much more interesting question is how wing surface roughness changes the small-scale velocity fluctuations, or turbulence, within a vortex. The introduction of a completely rough surface (i.e. sandpaper) introduces broadband noise within the velocity spectra without isolating any particular scale or frequency. A wingtip vortex, by nature, is formed when the boundary layer on the trailing edge of a wing rolls up due to the pressure difference between the two sides of the wing. It would follow, then, that any turbulence (or lack thereof) present in the wing boundary layer would be contained within the wingtip vortex. However, the presence of turbulence within the vortex core has been notoriously difficult to measure. Even analytical solutions to the Navier-Stokes equation which include nonzero Reynolds stresses bear a remarkable resemblance to solutions which assume a laminar vortex (Birch, 2012). Measurement of inertial subranges in velocity spectra is difficult due to the phenomenon of vortex wandering (Devenport et al., 1996; Jamie et al., 2014; Phillips and Graham, 1984; Bailey and Tavoularis, 2008). It is largely accepted that wandering is a passive bulk movement and contributes minimally to any velocity fluctuations within the core. There have been a few attempts to remove the effects wandering from data (see Devenport et al., 1996; Bailey and Tavoularis, 2008), but Birch (2012) has shown that these
methods can actually force data to Gaussian distributions. There is evidence of two kinds of characteristic vortex cores. The first, an inactive “solid” core, is thought to be dominated by viscosity, dissipating any turbulence within it (Devenport et al., 1996; Phillips and Graham, 1984). The second, is a core containing high levels of turbulence, seen by Bandyopadhyay et al. (1991) and Beninati and Marshall (2005), among others. LDA data analysis in this chapter will attempt to characterize how the addition of broadband turbulence within the wing boundary layer affects the development of the wing-tip vortex once it has achieved an axisymmetric state.

6.1 Mean vortex characteristics

Laser Doppler velocimetry data provides not only a validation of the mean vortex properties observed in the seven hole probe data, but also a means of assessing time-resolved velocity fluctuations. Given the data was taken in two different tunnels with two different experimental setups, an initial check of consistency between mean vortex parameters was completed (see table 6.1). In agreement with data presented in chapter 5, the addition of surface roughness to the wing reduces the velocity magnitudes within the vortex and increases the size ($r_c$) of the vortex core. It is noted that while the trends are the same, the values are not exactly consistent. The reduction in velocity magnitude is not as extreme in the LDA data as in the seven hole probe data. The vortices measured with the LDA are smaller than those measured with the multi-hole probe, by a factor of 1% of the vortex core. This is within the level of uncertainty in the seven hole probe measurements, but could be partially attributed to the degree to which the vortex wanders. The Aero tunnel cross-section is 3 times larger than the cross section of the A tunnel, which would presume to reduce the amount the vortex interacts with the tunnel boundaries. The freestream turbulence is also higher in the A tunnel - a condition which has been shown to increase the amplitude of wandering (Bailey and Tavoularis, 2008). Contour plots and profiles of velocities, vorticity and circulation are also consistent with the data presented in chapter 5 and included for reference in appendix E.
### Table 6.1: LDA vortex core data

<table>
<thead>
<tr>
<th></th>
<th>Smooth wing</th>
<th>Rough wing</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\zeta r_c/v_0$</td>
<td>4.05</td>
<td>4.56</td>
</tr>
<tr>
<td>$r_c/c$</td>
<td>0.029</td>
<td>0.034</td>
</tr>
<tr>
<td>$\Gamma_c/\Gamma_0$</td>
<td>0.69</td>
<td>0.69</td>
</tr>
<tr>
<td>$v_0/U_\infty$</td>
<td>0.327</td>
<td>0.285</td>
</tr>
<tr>
<td>$(U_\infty - u_0)/U_\infty$</td>
<td>0.29</td>
<td>0.24</td>
</tr>
</tbody>
</table>

#### 6.2 Reynolds stresses

As a first measure of the distributions of Reynolds stresses in the vortex, the $(u, v, w)$ components are plotted along a line of constant $z$ through the vortex centre in figure 6.2.1. The similar values of $v'^2$ and $w'^2$ imply the vortex has reached a nearly axisymmetric state (Beninati and Marshall, 2005). The maxima of $v'^2$ and $w'^2$ occur very near the vortex centre ($y = 0$). The turbulent fluctuations in $u'^2$ are significantly smaller than the cross-flow components.

![Figure 6.2.1: Reynolds stress as a function of non-dimensional position across the vortex - (a) smooth wing and (b) rough wing. (—), $w'^2/v_0^2$; (—), $w'^2/v_0^2$; (—), $u'^2/v_0^2$](image-url)
Figure 6.2.2 shows polar Reynolds stresses for the smooth and rough wings. The maxima of $v_\theta^2$ and $v_r^2$ occur near the vortex centre and are nearly Gaussian in distribution. The scale of $v_\theta'v_r'$ is an order of magnitude smaller and within the range of experimental uncertainty. The high level of normal stresses within the vortex core implies a significant amount of turbulent fluctuations at the centre of the vortex; however, Jammy et al. (2014) has shown that this could be a result of the random low-amplitude wandering, not necessarily the presence of turbulent fluctuations in the core. The increase in magnitude of normal stresses for the rough wing case is expected, but it is the similar width of distributions which is interesting. This agreement indicates that the magnitude of wandering is not dependent on the energy scales in the wing shear layer.

![Figure 6.2.2: Circumferentially averaged Reynolds stresses as a function of $\eta$; (o), $v_\theta^2/v_0^2$; (□), $v_r^2/v_0^2$; (△), $v_\theta'v_r'/v_0^2$. Open symbols, smooth wing; filled symbols, rough wing.](image-url)
6.3 Velocity spectra

Velocity power spectra can indicate at which frequencies, or equivalent length scales, velocity fluctuations dominate in the flow. For flows with well developed isotropic turbulence, a -5/3 power law is visible somewhere within the spectrum. The author is not aware of any published results reporting a -5/3 slope exactly, but a few have seen slopes between -2.3 to -3 within the vortex core (see Beninati and Marshall, 2005; Devenport et al., 1996). There have been a few data sets which also report peaks within the velocity spectra. These vary in wavelength and have been attributed to several different mechanisms, ranging from probe interference to vortex shedding frequencies (see Bailey and Tavoularis, 2008; Yarusevych and Boutilier, 2011).

Three components of velocity power ($\Phi$) spectra for both the smooth and rough wings in the present study are given in figures 6.3.1, 6.3.2, and 6.3.3. The spectra have been normalized by the standard deviation, taken as the integral of the power spectra. Shown on a log-log scale, the data in these spectra were taken in a line at a constant $z$ value. The value of $z$ was chosen simply by taking the line of data from the wake scan closest to the vortex centre.

Immediately apparent is a sharp spike in the smooth wing spectra at 12 Hz for all velocity components. The spike is not only visible in the spectra near the vortex core, but it is present in all points of the wake scan. Given the sharpness and strength of this spike, the first concern was whether this was result of probe vibration during experiments. However, the spike is present in data recorded with two different LDA lenses (in two different setups), so it could not be a probe vibration. One possibility is that it is interference from equipment running parallel with the LDA system. This is unlikely given the data was taken on separate days, and there is not any recorded data to confirm this theory. Interference from laboratory lights or tunnel breathing is also inconsistent with the 12 Hz frequency. The only consistency between experiments is the wing itself, and it is unknown whether vibration of the wing occurred for any set of experiments. However, this is also unlikely given that the spike only occurs in the smooth wing data, and if the wing was vibrating, it should not be able to distinguish between a smooth or rough surface. Assuming, now, that this is part of the vortex, not some
external factor, one possible explanation of the spike is that it is a remnant of the vortex shedding frequency. The shedding frequency, \( f \), typically has a Strouhal number, \( St = fc/U_\infty \approx 0.2 \), although studies on wing wakes have found a range of \( 0.16 \leq St \leq 0.22 \) which satisfy this condition (see Huang and Lin, 1995; Huang and Lee, 2000, and references cited therein). In this case, with \( f = 12 \), \( St \approx 0.19 \), well within the acceptable range. The reason this spike is not visible in the rough wing data is unclear. The additional turbulence from the rough wing could promote enough mixing and decay to effectively dissipate these scales sooner.

![Figure 6.3.1: Smooth wing cross-flow velocity spectra normalized by standard deviation, \( \alpha = 5 \): (a) \( v' \) spectra, \( \Phi/v'^2 \); (b) \( w' \) spectra, \( \Phi/w'^2 \). Legend values indicate position \( \eta \) across the vortex.](image)

The second notable feature of the spectra is a smaller, slightly more broad, peak around \( \sim 330 \) Hz (\( \pm 30 \) Hz in some positions). This peak is not dependent on surface roughness, but it does appear to be somewhat dependent on radial position. The spatial dependence of this peak could be the remnant of a similar peak in the wake spectra (see Chapter 7). The spectral peak in the wake data is dominant on one side of the wake, a characteristic not unusual given that the boundary layers on either side of the wing are exposed to different pressure gradients. Given that the peak frequencies are so similar, this is compelling evidence that the turbulence
visible within a wing-tip vortex comes directly from wake itself. If this assumption is made, it could explain the spatial dependence of the peak within the vortex velocity spectra. Previously published results have revealed similar spectral peaks at high frequencies, and have attributed them to dominant frequencies in the external flow (Beninati and Marshall, 2005; Bandyopadhyay et al., 1991).

Well-developed steady-state turbulence, by definition, has a balance between production and dissipation. As a result, the spectra have a -5/3 slope in the dissipation range, a relationship originally derived by Kolmogorov (1941), and well documented since for boundary layers and grid turbulence (see Pope, 2000, and references cited therein). Given that, unlike boundary layers, there are no external forces within a vortex to aid production, a -5/3 slope in the spectra of the vortex is considered to be old turbulence from the initial condition. However, power spectra in vortex cores are rarely reported, and those that have been published have slopes between -2.3 and -3 (Devenport et al., 1996; Beninati and Marshall, 2005). For reference, lines with a -5/3 and -2.5 slope are shown in each spectral plot. Like the spectral peaks already discussed, the slope of the the dissipation range varies
with radial position. This variance is not so easily explained. However, the slope does seem to vary with the strength of the 330 Hz peak.

\[ f \text{(Hz)} \]

![Figure 6.3.3: Axial velocity spectra normalized by standard deviation, $\Phi/u'^2$ and $\alpha = 5$: (a) smooth wing; (b) rough wing. Legend values indicate position $\eta$ across the vortex.](image)

6.4 Bandpass filters

Using bandpass filters on the re-sampled timetraces for select frequencies can lead to a more complete picture of where those turbulent structures occur in the vortex. The bandpass filters focus on the two dominant wavelengths previously discussed at 12 Hz and 330 ± 30 Hz.

6.4.1 12 Hz filter

The 12 Hz filter is only presented for the smooth wing data, given the lack of this peak in the rough wing spectra. Figure 6.4.1 shows the mean tangential velocity after the filter to show that, while noisier, the mean velocity field remains Gaussian.

Reynolds normal stresses are presented in figure 6.4.2. If this wavelength was the result of electrical noise in the wind tunnel, the distribution of normal stresses
should not be sensitive to vortex location. However, contours of the Reynolds stresses across the wake reveal that this is not the case, and there is a clear concentration within the vortex core. This does not rule out the possibility of wing vibration during the experiments, but alludes to some vortex-centred phenomenon.

### 6.4.2 330 Hz filter

The bandpass filter for the 330 Hz frequency was applied to both the smooth and rough wing wake scans. The mean tangential velocity distribution remains Gaussian, given in figure 6.4.3 for both wings. It should also be noted that $v_0$ also remains unchanged to within a 5% relative error.

While both the smooth (figure 6.4.4) and the rough (figure 6.4.5) Reynolds stress contours both indicate a concentration in the vortex centre, there are also small pockets on the outer edges of the wake scan. These pockets are more pronounced in the smooth wing data, which is consistent with increased rates of dissipation in the rough wing data. The broadband turbulence introduced in
Figure 6.4.2: (a) Isocontours of $\frac{\langle v'_\theta \rangle^2}{v_0^2}$ ($\times 10^{-4}$) and (b) Isocontours of $\frac{\langle v'_r \rangle^2}{v_0^2}$ ($\times 10^{-3}$) for the bandpass 12 Hz filter, (---) outline of the vortex core for reference.

Figure 6.4.3: Circumferentially averaged $v_\theta$ vs. $\eta$ for the 330Hz bandpass filter: (○), smooth wing; (●), rough wing; and (—), equation (2.1.5)
the wing boundary layer would have increased mixing and dissipation, resulting in more evenly distributed turbulence. The concentration of this turbulence in the core in both cases could be from the roll-up of the wake and subsequent entrainment due to the solid body rotation of the vortex core. However, the persistence of the 330 Hz frequency in the rough wing vortex core suggests that dissipation in this region occurs at a slower rate, indicating that the increased turbulence in the wing boundary layer has less of an effect on the dissipation rate in the vortex core.

Figure 6.4.4: Smooth wing data for the bandpass 330 Hz filter (a) Isocontours of $\overline{v'^2}/v_0^2 (\times10^{-4})$ and (b) Isocontours of $\overline{v'^2}_r/v_0^2 (\times10^{-3})$, (---) outline of the vortex core for reference.

6.5 Wandering

6.5.1 Theory

It is clear that surface roughness on the wing affects not only the mean vortex but also the time dependent velocity measurements. One aspect of processing vortex velocity fields that has not been addressed is the low-amplitude, low-frequency, random, Gaussian motion of a vortex measured within a bounded volume. This
“wandering” motion is largely accepted to be a result of initial and boundary conditions imposed by wind tunnels (Jammy et al., 2014). Historically, this motion has been corrected for in post-processing experimental data with deconvolution methods (Devenport et al., 1996) or with two-point correlation of the real-time location of the vortex centre (Bailey and Tavoularis, 2008). However, Birch (2012) showed that using a correction such as the one proposed by Devenport et al. (1996) actually drives measured results toward the assumed solution. Numerical research by Jammy et al. (2014) shows that wandering can cause artificially high Reynolds stress values at the vortex centre, which all but disappear when correcting for the vortex location. The results of Beninati and Marshall (2005) suggest that wandering primarily contributes to the spectral energy at low frequencies. One of the primary goals of the current research was to develop an alternative way of “correcting” for the wandering motion. Being able to account for vortex wandering is inherently important to this research so accurate conclusions can be made about turbulence within the vortex core.

First, the extent of vortex wandering must be assessed before any corrections can be applied. For this, it is necessary to establish a set of conditions for which the Reynolds stress distributions can be considered representative of
an axisymmetric vortex that is neither incompletely developed nor dominated by wandering. The good agreement between the laminar (2.1.5) and turbulent (2.1.7) solutions for the mean velocity profiles derived by Batchelor (1964) and Hoffmann and Joubert (1963), respectively, is a good place to start. Demonstration of this agreement is in Shaw-Ward and Birch (2015) and is valid for $0.65 \lesssim \eta \lesssim 1.3$.

As a consequence, the momentum terms and the viscous terms in the Reynolds-averaged Navier-Stokes equations will approximately equal in this range. If the assumptions ($\partial/\partial \eta \gg \partial/\partial \zeta \gg \partial/\partial \theta$, where $\zeta = x/r_c$ is the non-dimensional axial coordinate) of Batchelor (1964) and Hoffmann and Joubert (1963) are assumed, then

\[
\frac{\overline{v_{\theta}^2}}{v_0^2} = \frac{\partial}{\partial \eta} \left( \eta \frac{\overline{v_r^2}}{v_0^2} \right)
\]

(6.5.1)

\[
\frac{v_{\theta} v_r}{v_0^2} = \frac{C_1}{\eta^2}
\]

(6.5.2)

\[
\frac{v_r v_x}{v_0^2} = \frac{C_2}{\eta}
\]

(6.5.3)

where $C_1$ and $C_2$ are arbitrary constants. For (6.5.1), (6.5.2), and (6.5.3) to hold as $\eta \to 0$, then $C_1 \approx C_2 \approx 0$, or $\overline{v_{\theta} v_r} \approx \overline{v_r v_x} \approx 0$ (Shaw-Ward and Birch, 2015).

This argument is supported by results found in Beninati and Marshall (2005) and Jammy et al. (2014), which have shown that Reynolds shear stresses are much smaller than Reynolds normal stresses in a turbulent vortex. The turbulence within a vortex can be considered equivalent to grid turbulence for the case of a well-developed and stable vortex, such that sufficient time (or $\zeta$) has elapsed for the vortex to become independent of initial conditions. Well-developed grid turbulence is increasingly isotropic, and only small remnants of the initial turbulence remain.

Assuming a passive vortex, in which initial turbulence is simply being advected through the vortex, this same condition of isotropy, $\overline{v_{\theta}^2} \approx \overline{v_r^2} \approx \overline{v_x^2}$, can be applied. This assumption supports $\overline{v_{\theta} v_r} \approx \overline{v_r v_x} \approx 0$, and following from (6.5.1)

\[
\overline{v_{\theta}^2} \approx \overline{v_r^2} = C_0,
\]

(6.5.4)
where $C_0$ is a constant dependent on the age of the vortex. Consequently, (6.5.4) fails if the vortex is anisotropic and exerts a significant effect upon the energy cascade (Shaw-Ward and Birch, 2015). It is also important to remember that this relationship fails if insufficient time has passed for the vortex to become independent of initial conditions. Evidence of the relationships (6.5.4) and $\overline{v'_\theta v'_r} \approx \overline{v'_r v'_x} \approx 0$ have been demonstrated by Shaw-Ward and Birch (2015) with data collected by Phillips and Graham (1984).

### 6.5.2 Highpass filters

Given the distribution of Reynolds stresses presented in figure 6.2.2, it is clear that the wing-tip vortices generated from the smooth and rough wings are dominated by wandering from the high levels of Reynolds stresses near the vortex centre. Assuming wandering is a low-frequency passive phenomenon, it would be reasonable to assume that the turbulent scales and wandering scales may be effectively filtered out of velocity timetraces. Without knowing, for certain, at which frequency or frequencies the wandering fluctuates, a series of high pass filters were applied with increasing frequency, $f_H$. For details on the stop/start frequencies for each filter, see Section 3.4.7. Figure 6.5.1 shows the maximum tangential Reynolds stress post-filtering for both the smooth and rough wings. Both data sets have a sharp decrease in peak $\overline{v'^2_\theta}$ for low $f_H$, then remain relatively constant as $f_H$ increases. If the peak Reynolds stresses in a well-developed vortex were still dominated by low frequency spectral energy from the wing shear layers (see fig 7.3.1), then they would remain almost constant for low frequency filters. The sharp decline in $\overline{v'^2_\theta}$ at low $f_H$ is consistent with high contributions to Reynolds stress from inactive wandering of the vortex.

The high-pass-filtered results of the Reynolds stress profiles are shown in figure 6.5.2, with $f_H = 0.5$ and 1 for the smooth and rough wings, respectively. Recall that the contribution of Reynolds stresses from the wandering is assumed negligible if $\overline{v'^2_i}$ is constant for some range near $\eta \approx 1$, see (6.5.4). A region satisfying (6.5.4) near $\eta = 1$ is clearly emerging from the smooth wing data in figure 6.5.2a, while a flattening of $\overline{v'^2_r}$ may be developing in the rough wing data, although not as clearly. The filtered stresses presented here are still only a spatial
Figure 6.5.1: Maximum tangential Reynolds stress for each highpass filter of frequency, $f_H$. Filled symbols denote rough wing data, and $f_H = 0$ Hz indicates unfiltered data.
average over an area determined by the probability of the vortex centre, so they should be interpreted loosely. However, applying high-pass filters to two cases with very different total kinetic energy and similar mean velocity profiles results in remarkably similar Reynolds stress profiles. This indicates that the large amount of turbulence introduced in the wing shear layers had minimal effects on the vortex development, calling into question whether there is a distinct difference between laminar and turbulence vortex Reynolds stress profiles.

Figure 6.5.2: Circumferentially averaged high-pass filtered Reynolds stress profiles for (a) smooth wing - $f_H = 0.5$ Hz and (b) rough wing - $f_H = 1$ Hz; (o), $v_{\theta}^2/v_0^2$; (□), $v_r^2/v_0^2$. 
Chapter 7

Wing wake profiles

This research has shown that velocity spectra in the vortex five chord lengths behind the wing have characteristic peaks at different frequencies depending on the wing roughness. At this point, it is unknown whether these structures are from the tip vortex or a remnant of the boundary layer roll-up process. There have been several studies on how the vortex forms over the wing tip (Birch and Lee, 2005; Ramaprian and Zheng, 1997; Chow et al., 1997; Hah and Lakshminarayana, 1982), but little characterization of the spectra within the roll-up process or how it relates to spectra within the vortex further downstream. Many studies in the wing wake have centred on characterizing the vortex shedding frequencies with the non-dimensional Strouhal and Rossby numbers. Regardless of airfoil profile or experimental conditions, these numbers seem to collapse to a small range \( 0.14 \lesssim St \lesssim 0.21 \) (Huang and Lee, 2000; Yarusevych and Boutilier, 2011). This shedding frequency and structures related to the development of shear layer instabilities have been the focus of several papers (Huang and Lin, 1995; Yarusevych et al., 2008; Yarusevych and Boutilier, 2011).

7.1 Mean velocity profiles

Data was taken in the wake of the smooth and rough wings at three angles of attack: \( \alpha = 5, 8, \) and \( 10^\circ \). Mean streamwise velocity data for all six cases is in figure 7.1.1 alongside data obtained by Hah and Lakshminarayana (1982) behind
a NACA 0012 wing at $\alpha = 6^\circ$. The Hah and Lakshminarayana (1982) wake data is not as wide, but this could be explained in part by a slightly upstream measuring position - $x/c = 0.28$ in the Hah and Lakshminarayana (1982) experiments, where $x$ is measured along a streamline curvature. The measurement position could also account for the greater velocity deficit at a similar wing incidence.

In general, wing roughness increases the velocity deficit and wake width for a given incidence, and an increase in incidence increases the velocity deficit. However, one exception is the data at $\alpha = 5^\circ$, for which the maximum velocity deficit in the wake is equal to within experimental accuracy. Section 5.1 shows that the addition of surface roughness decreases the velocity deficit in the wing-tip vortex by 7%, the opposite effect to that seen in the wing wake. Integration of the wake profile data is presented in figure 7.1.2 as a coefficient of drag and compared against the 2D force balance measurements discussed in chapter 4. Drag calculated from wake measurements, in general, over estimates the 2D drag on the wing. The higher drag at $\alpha = 5^\circ$ in the smooth wing data indicate that it is a bad point, which could also explain the similar wake profiles at $\alpha = 5^\circ$ in figure 7.1.1.

Figure 7.1.1: Mean velocity profiles, filled symbols for rough wing data. (○) $\alpha = 5^\circ$; (□) $\alpha = 8^\circ$; (△) $\alpha = 10^\circ$; (♦) Hah and Lakshminarayana (1982).
Figure 7.1.2: 2D drag data - open symbols from force balance measurements; filled symbols from wake data. (○) rough wing; (□) smooth wing.

7.2 Reynolds stress measurements

Apart from the similarities in mean velocity deficits at $\alpha = 5^\circ$, surface roughness increases the mean velocity deficits in the wing wake for a given wing incidence. Turbulence levels in the wake, however, tell a different story. Normal Reynolds stress profiles for all data sets are in figure 7.2.1. The single-sensor hot wire is sensitive to $(u'^2 + w'^2)^{1/2}$, but in the present case it may be argued that $u'^2 >> w'^2$, an assumption that is often made in boundary layer work. Not surprising, the turbulence levels in the rough wing wakes are higher than the smooth wing wakes for a given angle, indicating the sandpaper is introducing broadband turbulence into the wing wake. All of the rms profiles have two peaks, which Yarusevych and Boutilier (2011) found to develop from the cores of the shear layers on the upper and lower surfaces of the airfoil. However, what is interesting are the changes in peak turbulence intensity for the pressure and suction sides of the wing. For all cases, the pressure side of the wing is on the left ($y/c \leq 0$) of figure 7.2.1. Hah and Lakshminarayana (1982) found that the streamwise turbulence intensity was higher on the suction side than on the pressure side in the near wake region for
angles ranging from 3° to 9° at Re = 3.8 × 10^5. The rough wing data at α = 10° and
the smooth wing data at α = 8° and 10° agree with Hah and Lakshminarayana
(1982). The remaining cases disagree, although the rough wing peak levels at
α = 10° are less than 3% apart. It appears that as the wing incidence increases,
the maximum intensity switches from the pressure to the suction side, and the
increased surface roughness delays this switch.

The overall turbulence levels are highest in the 5° data and lowest for the 8°
data. The reason for this fluctuation in levels is unclear, but most likely is related
to the thickness and characteristics of the boundary layers (i.e. laminar, turbulent,
separation, etc.) on the wing. The wake is widest and about equal for both wings
at α = 5°. The dip seen in all of the data appears in the near and far wake regions
of the Hah and Lakshminarayana (1982) data as well as in the smooth wing data
of Zhang et al. (2004), which is a result of the merging boundary layers. Hah and
Lakshminarayana (1982) also point out that the width of this dip is related to the
boundary-layer thickness.

Figure 7.2.1: Turbulence intensity profiles, filled symbols for rough wing data.
(○) α = 5°; (□) α = 8°; (△) α = 10°; (♦) Hah and Lakshminarayana (1982).
7.3 Wake spectra

To further investigate any structures within the wing wake, velocity power spectra are shown in figure 7.3.1. All of the spectra are from the data point at $y/c = 0$, and each successive spectrum has been moved up one decade for clarity. Immediately apparent are two trends: 1) all six spectra have a peak around $\sim 300$ Hz, and 2) the $\alpha = 5^\circ$ spectra are uniquely showing spikes at lower frequencies. It is important to note that the anomalies observed in the $\alpha = 5^\circ$ setup earlier are not expected to affect the data presented in this section.

The higher frequency peak is the most persistent one in the data, even though the peak is only just discernible at $\alpha = 10^\circ$. The frequency at which the peak occurs increases slightly with angle of attack, ranging from about 290 - 400 Hz. This range is almost exactly the same as the peak seen in the wing-tip vortex, suggesting that the wing wake is the origin of those turbulence scales. For the $\alpha = 5^\circ$ and $8^\circ$ cases, the additional higher frequency peaks are the harmonics of the first frequency. A similar trend is seen in the data of Yarusevych et al. (2008) in which a fundamental frequency is identified for each Re and $\alpha$ combination around which other peaks develop. Yarusevych et al. (2008) relates their fundamental frequency to disturbances caused by a separated shear layer on the surface, and just as in the present data, the peaks are more prominent at $\alpha = 5^\circ$ than $\alpha = 10^\circ$. These higher frequency peaks are also perhaps related to the shear-layer instability waves discussed by Huang and Lin (1995). The data presented by Huang and Lin (1995) for high $\alpha$ shows an increase in the instability waves with an increase in $\alpha$, a trend that is consistent with the data in figure 7.3.1. Huang and Lin (1995) saw a close relationship between the behaviour of instability waves in the suction-side boundary layer and the vortex shedding frequencies.

The lower frequency peaks in the $\alpha = 5^\circ$ data are perhaps just as interesting given similar peaks found in the vortex spectra in Section 6.3. The first peak is at $f = 11$ Hz, which is, once again, remarkably close to the 12 Hz peak in the vortex data. The smooth wing $\alpha = 5^\circ$ has a second peak at $f = 20$ Hz, and the second peak of the rough wing data is at 45 Hz. The origin of these frequencies is unknown, but it would be reasonable to assume that differences of the 20Hz and 45 Hz peaks are related to the surface roughness. Zhang et al. (2004) tested
the effects of different surface roughness heights in compressible flows, and found a decrease in the shedding frequency with an increase in roughness - the opposite trend seen here. For vortex shedding frequencies \( f_s \), the Strouhal number tends to fall within a small range \((0.14 \lesssim 0.21)\), regardless of Reynolds number or body geometry. Given that

\[
St = \frac{f_s d}{U},
\]

(7.3.1)

where \( U = U_\infty \) and \( d \) is taken as the cross-stream length scale of the body \((d = 0.0137 \text{ m for } \alpha = 5^\circ)\), the shedding frequency should be \(100 \lesssim 150\), which is not visible for any case in figure 7.3.1. Roshko (1954) also introduced a “universal” Strouhal number \( (St^*) \) based on wake parameters alone, which remained equal to 0.164 for a variety of body geometries. The wake Strouhal number was not measured during data collection, because an estimate could be made from the data that was collected. A rough estimate of \( St^* \) using the wake width \( d = 0.25c \) and \( f_s = 51 \), gives \( St^* = 0.196 \), agreeing with results of Huang and Lin (1995).
Without more extensive data collection in the near wake, it is not possible to say conclusively if any of the spectral peaks in figure 7.3.1 are the shedding frequency.

![Figure 7.3.2: Hotwire data in the wake of the smooth wing at $\alpha = 5^\circ$ (a) Contours of power (logscale) from hotwire spectra and (b) Streamwise turbulence intensity. Frequencies $f_1 = 11 \text{ Hz}$, $f_2 = 20 \text{ Hz}$, $f_3 = 51 \text{ Hz}$, $f_4 = 233 \text{ Hz}$, and $f_5 = 300 \text{ Hz}$](image)

It may be useful to look at all of the available spectra from the wing wake for a better picture of how the energy is distributed. Figures 7.3.2 and 7.3.3 show logscale contours of power across the wake of the smooth and rough wings at $\alpha = 5^\circ$, respectively. The turbulence intensity from figure 7.3.1 for each case has been reproduced in (b) for reference. Clearly, there are a few more dominant
scales visible in the wider wake than at the mid-point spectra discussed already. What is interesting is the additional peak in the smooth wing data at 51 Hz ($f_3$). This could be the shedding frequency previously discussed, given the proximity to the rough wing peak at 45 Hz. The decrease in frequency with the increase in roughness would agree with the results of Zhang et al. (2004). It must be noted, however, that a frequency of 51 Hz is quite close to mains frequency of 50 Hz. The $f_5$ peaks from both the smooth and rough data were discussed earlier, but neither of the slightly lower $f_4$ peaks were. Mostly likely these peaks are related to the instabilities within the merging boundary layers as well.

Surface roughness has several effects on the turbulence characteristics, including some rather puzzling ones. The streamwise component of normal stress levels within the wake are increased due to the higher broadband turbulence from the rough surface. The rough surface also changes the location of maximum normal stress levels for a given angle of attack. A possible vortex shedding frequency has been identified from the wake data, and instability waves from the merging shear layers dominate the wake spectra. While there is still a marginal difference in the frequencies of structures in the wake and tip vortex, it is reasonable to conclude that at least some of the turbulence within the vortex originated in the wing wake.
Figure 7.3.3: Hotwire data in the wake of the rough wing at $\alpha = 5^\circ$ (a) Contours of power (logscale) from hotwire spectra and (b) Streamwise turbulence intensity. Frequencies $f_1 = 11$ Hz, $f_2 = 45$ Hz, $f_3 = 150$ Hz, $f_4 = 245$ Hz, and $f_5 = 290$ Hz.
Chapter 8
Conclusions and Recommendations

8.1 Concluding remarks

Wing-tip vortices have classically been identified as either laminar or turbulent based on the level of Reynolds stresses measured within the core. However, the bulk motion low frequency wandering of a vortex in wind tunnels often skews turbulence measurements. Measurements of wing-tip vortices generated by two wings with the same NACA 0012 profile but different degrees of surface roughness have been taken in an effort to show how initial turbulence within the wing shear layer effects the development of a wing-tip vortex. Broadband turbulence was introduced into the wing boundary layer on one of the rectangular wings with P80 grade sandpaper. Several different methods of measurement techniques were used to look at overall wing performance as well as time-resolved measurements within the wing wake and axisymmetric tip-vortex downstream.

Force balance measurements revealed, as expected, a small reduction in wing performance due to surface roughness for both 2-D and 3-D configurations. Stall characteristics remained relatively unchanged. The increase in drag on the 3-D wing due to surface roughness was within the range of experimental uncertainty.

Mean velocity measurements of the tip vortices 5 chord lengths downstream of the wing were taken with purpose-built seven hole probes. During the initial
stages of wake survey data collection, it was clear that the historical method of seven hole probe calibration resulted in non-physical discontinuities in the flow-field. As a result, a novel approach for both the collection and the interpretation of the calibration data was developed. The new techniques reduce the time required for probe data collection and eliminate the discontinuities in the calibration space. Using the new calibration techniques, this first wake survey data agreed well with published results and revealed the vortices at a wide range of angles of attack to be axisymmetric and consistent with analytical solutions given by Batchelor (1964). In general, the roughness increased the core radius of the vortex by $\sim 10\%$ and reduced peak tangential and axial velocities for a given $\alpha$. This is an important result for the real-life application of this research - the reduction in velocities is a direct indication that the strength of the vortex decreased, reducing the safety hazards caused by the vortices on airport runways. Data taken with the LDA system in a different tunnel agreed with these trends, although a comparison between the two sets of data seemed to indicate that the vortices generated in a larger tunnel were subject to smaller amplitudes of wandering.

Laser Doppler velocimetry measurements at $\alpha = 5^\circ$ were then used to assess the distribution of the turbulence within the vortex. Velocity power spectra from the LDA measurements in the tip vortex revealed several dominant wavelengths. Spectra for the smooth wing from all three velocity components (taken with two different LDA probes) showed a spike at 12 Hz throughout the whole wake survey plane. Several possible explanations for the spike were explored, but were ultimately inconclusive and further confused by the absence of the peak in the rough wing spectra. Two probable causes are that it is related to the wake shedding frequency or the vibration of the wing itself. A higher frequency peak of $\sim 330 \pm 30$ Hz is also visible in both the smooth and rough wing spectra and agrees remarkably well with peaks related to the wing shear layers seen in spectra taken in the 2-D wing wake with a single hotwire.

Reynolds stress distributions in both the smooth and rough wing-tip vortices were remarkably similar in shape and width. These profiles can be an indication of the magnitude of vortex wandering, and this similarity demonstrates that the wandering magnitude is not dependent on the structures or increase in energy in the wing shear layer. However, contributions to the Reynolds stresses from
wandering skew these profiles, so a new method for removing the effects of wandering in data post-processing was developed. Applying carefully selected high-pass filters to the re-sampled LDA time traces, peak values of $v_\theta^2$ decreased sharply then remained constant with increased filter frequency. The post-filtered Reynolds stress distributions showed the peak values of $v_\theta^2$ actually occur in the core away from the centre and an emerging region of $v_\theta^2 = v_r^2 = \text{constant}$ around $\eta \approx 1$. The magnitudes of the post-filtered stresses were considerably higher for the rough wing, consistent with higher levels within the wing shear layer. Most importantly, a large increase in broadband turbulence within the shear layer did not significantly change the distribution of shear stresses. This is the first experimental evidence supporting the theory that there are few fundamental differences between laminar and turbulent vortices.

Single hot-wire measurements were taken in the 2-D wing wake, in an effort to characterize the turbulence before being rolled up into the tip vortex. Mean axial velocity deficits increased in the wake with the addition of surface roughness, although the case of $\alpha = 5^\circ$ showed relatively little difference. Kinetic energy levels also increased with surface roughness, but the distribution of energy throughout the wake changed dramatically with wing incidence. Peak energy levels occurred on the pressure side of the wake for low $\alpha$ and gradually changed to the suction side as $\alpha$ increased. The same trend was seen for both wings, but the rough wing data had much higher pressure side peaks at low $\alpha$ and a delay in transition of that peak to the suction side. Wake velocity power spectra revealed possible vortex shedding modes at low frequencies and higher-frequency structures likely associated with shear layer instability waves. The remarkably close agreement with dominant frequencies in the LDA spectra suggest that the turbulence in the shear layer is passively rolled into the vortex.

8.2 Future recommendations

A number of investigations in this thesis could lead to further studies.

The first variation on this research would be to test different degrees of surface roughness. Would it be possible to produce a reduction in tip vortex strength with a lower surface roughness? This would, presumably, improve wing performance,
but what are the tradeoffs? Is there an ideal wing roughness for both performance and tip vortex strength? Force balance and wake surveys would be required for these tests.

This research showed that the increased surface roughness also increased the total drag measured by a force balance, but the smaller size of the wing prevented a precise measurement of that increase. It would be useful to understand what drag components contributed to the increase in total drag - was there just an increase in profile drag, or did a change in induced drag also occur? Detailed characterization of the wing boundary layers at different incidence angles could show any separation regions and how much the profile drag of the wing is affected by surface roughness. Separation regions could be detected with simple pressure tappings on the surface, but LDA or hot-wire data would be required for any detailed characterization of the boundary layer.

The results of this thesis indicate that the turbulence in the wing shear layers is passively rolled into the tip vortex. However, there are several aspects of the data which are unclear. The origin of the 12 Hz peak in the velocity spectra of the vortex is still a mystery. Wing vibration could be measured by using a displacement laser focused on the wing surface while in the wind tunnel. If this is a remnant of vortex shedding frequency, then detailed, time-resolved measurements of the tip vortex during roll-up and in the near wake would be necessary. This data could also reveal if any secondary structures occur on the wing tip and if they change with the increase in surface roughness. Time-resolved LDA or triple wire measurements of the near wake could also show more conclusively exactly how turbulent structures in the wing wake are transferred into the vortex.

To see how roughness affects the life and ultimate breakdown of a wing-tip vortex, measurements would need to be taken in the far wake, \( x/c > 40 \) or farther. Given the size of a conventional wind tunnel, this would most likely need to be completed in a water tunnel to allow sufficient time for vortex breakdown to occur. This would reveal whether the increased surface roughness actually results in a shorter lifespan of the vortex, or if it just decreases the strength of the vortex, as already shown in this thesis.


Appendix A

Pitch Traverse

The pitch positioning system was designed to be installed outside of the tunnel wall, accessible from within the tunnel through a small hole. The wing can be mounted into the system via an M12 bolt that had previously been built into the root of the wing. The pitch traverse is driven by a Schneider M-Drive 23 Plus motor attached to a worm and wheel setup (see figure A.0.1).

![Figure A.0.1: A schematic of the pitch automation traverse](image)

Figure A.0.1: A schematic of the pitch automation traverse
Appendix B

Current multi-hole probe calibration practice

The established calibration techniques of Zilliac (1989) and Gallington (1980) are dependent upon non-dimensional pressure coefficients derived from pressure differences. These coefficients are used to determine flow speed and direction. As discussed in 3.3, the numbering scheme of the holes on a seven-hole probe is shown again in figure B.0.1 for easy reference. The probe is divided into seven sectors, each corresponding to a different hole. These sectors allow the probe to be used beyond angles where separation occurs on the probe tip, by discarding data from those holes which are on the lee side of the probe.

Figure B.0.1: Numbering scheme of a seven-hole probe, looking aft.
In conventional use, the probe axis is aligned with the freestream flow. For low angularity flow, when the centre hole is subjected to the highest measured pressure, the directional pressure coefficients can be defined as

\[
C_{P_\beta} = \frac{\frac{1}{2}(P_5 + P_6) - \frac{1}{2}(P_3 + P_2)}{P_7 - \overline{P}} \tag{B.0.1}
\]

\[
C_{P_\alpha} = \frac{P_4 - P_1}{P_7 - \overline{P}} \tag{B.0.2}
\]

\[
C_{P_{total}} = \frac{P_7 - P_{total}}{P_7 - \overline{P}} \tag{B.0.3}
\]

\[
C_{P_{static}} = \frac{\overline{P} - P_{static}}{P_7 - \overline{P}}, \tag{B.0.4}
\]

where \(\overline{P}\) is the average of the pressure measured from the 6 radial holes, \(\alpha\) is the pitch angle, and \(\beta\) is the yaw angle (see fig 3.3.2). The numerators of the directional coefficients \((C_{P_\beta} \text{ and } C_{P_\alpha})\) are a measure of the pressure difference on either side of the probe tip, and the denominator is an approximation of the local dynamic pressure. For a probe aligned parallel to the oncoming flow, the pressure measured at the centre hole is an approximation of the total pressure (B.0.3), and similarly, the average of the radial holes is representative of a static pressure measurement (B.0.4). The freestream stagnation and static pressures are represented by \(P_{static}\) and \(P_{total}\).

At angles for which the central hole does not measure maximum pressure, alternative coefficients and procedures are used. At high coning angles, only the holes which are adjacent to the hole measuring maximum pressure are used. It is assumed that the flow only remains attached to these adjacent holes, making the measurements of those remaining invalid. For high angle flows, the flow is more easily described by using the cone angle, \(\theta\), and roll angle, \(\phi\), as shown in figure 3.3.2. Pressure coefficients sensitive to cone and roll angles can then be defined for each of the holes 1 through 6.
\[ C_{i\theta} = \frac{P_i - P_f}{P_i - \overline{P}} \]  
(B.0.5)

\[ C_{i\phi} = \frac{P_{cw} - P_{ccw}}{P_i - \overline{P}} \]  
(B.0.6)

\[ \overline{P} = \frac{P_{cw} - P_{ccw}}{2} \]  
(B.0.7)

where \( i \) refers to the hole index, \( P_{cw} \) and \( P_{ccw} \) are the pressures adjacent to the hole measuring maximum pressure in the clockwise and counter-clockwise directions, respectively, and \( \overline{P} \) is the redefined approximated local static pressure, according to (B.0.7). Together, equations (B.0.1), (B.0.2), (B.0.5) and (B.0.6) yield seven independent sets of non-dimensional pressure coefficients.

In standard calibration practice, these equations need to be interpolated directly or by curve-fitting to find the angle of the flow relative to the probe in normal use. Sumner (2002) gives a comparison of these techniques. Once the angle of the flow and the local static and total pressures have been determined, the velocity magnitude \( |V| \) and orthogonal velocity components \( u, v, \) and \( w \) can be obtained as

\[ |V| = \sqrt{\frac{2}{\rho} (P_i - \overline{P})(1 + C_{P_{static}} + C_{P_{total}})} \]  
(B.0.8)

\[ u = |V| \cos \beta \cos \alpha = |V| \cos \theta \]  
(B.0.9)

\[ v = |V| \cos \beta \sin \alpha = |V| \sin \theta \cos \phi \]  
(B.0.10)

\[ w = |V| \sin \beta = |V| \sin \theta \cos \phi. \]  
(B.0.11)
Appendix C

Optimal calibration methods

As discussed in appendix B, a large amount of flow facility time is required to obtain calibration data sets of sufficient density and quality to yield high confidence experimental results. As such, there is a desire to reduce the required number of calibration points without reducing experimental accuracy. Using the accepted techniques of optimal design of experiments and D-Optimality, the next step was to develop more efficient sampling of calibration data for directional velocity probes. What follows is a description of the D-Optimal calibration as a general principle and a demonstration of the method as applied to a seven hole probe. The application of this method to seven hole probe calibration and possible further applications of the method to multi-sensor hot-wire probes was demonstrated in McParlin et al. (2013).

C.1 D-Optimality

A system with $k$ inputs and a single output, $y$, can be expressed as

$$y = f(x_1, x_2, ..., x_k),$$  \hspace{1cm} (C.1.1)

where $x_1, x_2, ..., x_k$ are the inputs, and the function $f$ is unknown. The function $f$ can also be approximated as a Taylor polynomial of order $q$ with $k$ variables, giving
\[
y \approx c_0 + c_{11}x_1 + c_{12}x_1^2 + c_{13}x_1^3 + \\
+ c_{21}x_1x_2 + c_{22}x_1x_2^2 + ... + c_{kk}...x_k^q,
\]

(C.1.2)

where \(c_0, c_{11}, ..., c_{kk}...\) are constants unique to this system yet to be determined. The number of terms \(p\) in the Taylor polynomial will be

\[
p = \frac{(k + q)!}{k!q!}.
\]

(C.1.3)

To determine the values of the constants in (C.1.2), consider a series of \(n\) experiments which measure the inputs \(x_1, x_2, ..., x_k\) and the output \(y\) of the system in (C.1.2). The system can then be represented as

\[
y_1 = c_0 + c_{11}x_{11} + c_{21}x_{21} + c_{11}x_{11}^2 + c_{12}x_{11}x_{21} + ... + c_{kk}...x_{k1}^q + \epsilon_1 \\
y_2 = c_0 + c_{12}x_{12} + c_{22}x_{22} + c_{11}x_{12}^2 + c_{12}x_{12}x_{22} + ... + c_{kk}...x_{k2}^q + \epsilon_2 \\
: \\
y_n = c_0 + c_{1n}x_{1n} + c_{2n}x_{2n} + c_{11}x_{1n}^2 + c_{12}x_{1n}x_{2n} + ... + c_{kk}...x_{kn}^q + \epsilon_n.
\]

(C.1.4)

where \(x_{ij}\) is the value of the \(i\)th input during the \(j\)th experiment, \(y_j\) is the output for the \(j\)th experiment, and \(\epsilon_j\) is the random, experimental error associated with each experiment. The values of \(c\) remain unchanged between experiments and \(\epsilon\) contains both the experimental uncertainty and any residual error associated with the truncation of the polynomial.

To simplify, (C.1.4) can also be written in matrix form,

\[
y = Xc + \epsilon,
\]

(C.1.5)

where \(y = [y_1 \ y_2 \ ... \ y_n]'\), \(c = [c_0 \ c_1 \ c_{11} \ c_{12} ... c_{kk}]'\), and \(\epsilon = [\epsilon_1 \ \epsilon_2 \ ... \ \epsilon_n]'\). The matrix \(X\) contains all of the inputs, such that
The dimension of the input matrix $X$ is $(n \times p)$ and the array of constants $c$ is $(p \times 1)$, where $p$ is the number of terms in the polynomial, not the order of the polynomial. The order of the polynomial is usually chosen to be as small as possible while still providing a reasonable approximation of $f$.

For a given experiment, the values of $c$ are calculated to give the best possible fit between (C.1.5) and the set of experimental data $(X,y)$. To achieve this, (C.1.5) is optimized to give a minimum error value, $\epsilon$. Re-arranging (C.1.5), the function

$$\epsilon^2 = (y - Xc)^2$$

(C.1.7)

can be minimized by expanding and differentiating with respect to $c$. The value of $c$ for which the global minimum is achieved occurs when

$$c = (X'X)^{-1}X'y.$$  
(C.1.8)

The minimum number of independent experiments, $n$, required is $p$, given $X$ is the size $(n \times p)$. In practice, however, the number of experiments is typically chosen within a given cost or time. The inputs, $X$, are then defined to minimize the error ($\epsilon$) in the values of $c$ within a chosen tolerance. There are many different ways to accomplish this, but the method used in this research, D-Optimality, selects $X$ such that the values of $c$ are least sensitive to small changes or uncertainty in the locations of $X$.

Following from (C.1.8), the sensitivity of $c$ to small changes in $X$ is minimized when the determinant of $X'X$ is maximized. A number of iterative schemes exist to solve for the values of $X$, reviewed by St. John and Draper (1975) and Box and Wilson (1951). The algorithm adopted by Federov (1972) is used in the present work.
C.2 Application of D-Optimality to multi hole probes

The implementation of D-Optimality to the calibration of a seven hole probe requires an a priori knowledge of the form of the response surface for readings from each hole, according to (C.1.6) and (C.1.8). The form of the response surface will not change between probes of similar types, but the polynomial constants will change with every probe. Therefore, an exhaustive calibration matrix was carried out first, as described in appendix B.

The method applied to the response from each hole is the same, so the following will only detail the method used for a single hole as an example. The pressures recorded from hole 7 of a conventional 7-hole probe, \( C_{P7} \), are considered as the dependent variable, and the probe position in \((x_1 = \alpha, x_2 = \beta)\) as the two independent variables in the expression from (C.1.1). The data was initially fit to response surfaces of increasing polynomial order, and the quality of surface fit increased (i.e. \( \epsilon \) decreased) with increasing polynomial order, as shown in fig C.2.1. The \( R^2 \) of the surface fit increases by less than 0.1\% between \( 6 \leq p \leq 9 \). Given the increase in complexity and minimum number of \( n \) required for each increase in polynomial order, a value of \( p = 6 \) was chosen to be sufficient. Figure C.2.2 shows the calibration data points on the sixth order surface.

For a sixth-order polynomial in two independent variables, the number of coefficients in \( c \) is 28, giving the minimum number of sample points required as \( n = 28 \). The location of D-optimal sample points were then generated in multiples of 28, some of which are shown in figure C.2.3 for non-dimensionalized space. The points were generated with maximum residuals of 0.01\% of the full scale range.

The probe was then re-calibrated using each of the D-optimal sample set data points (where \( 28 \leq n \leq 196 \)), and a response surface model (RSM) was obtained for \( C_P \) for each of the seven holes. To determine the minimum number of points needed for calibration, statistical variance between the calibration data and RSM was obtained. Values of the standard deviation between RSM and calibration data is shown in figure C.2.4, and the regression coefficient \( R^2 \) was greater than 0.99 in all cases. Application of this method can be seen in McParlin et al. (2013) to a typical wing wake scan, resulting in the peak magnitudes of vorticity varying by
Figure C.2.1: $R^2$ of the surface fit, as a function of polynomial order.

Figure C.2.2: $C_{PT}$ exhaustive calibration data (●) on the sixth order polynomial surface fit
Figure C.2.3: Locations of D-Optimal sample sets for $n = 28$ (◦), $n = 112$ (□), and $n = 196$ (+)

no more than 2.3% and the peak magnitudes of tangential velocity by no more than 4.1%. This, then, reduces the time and cost required for calibration by an order of magnitude, relative to a typical calibration procedure.
Figure C.2.4: Standard deviation as a percentage of dynamic pressure vs number of D-optimal points for each of the seven holes
Appendix D

Plots of mean vortex data

Figure D.0.1: Velocity vector plot of seven hole probe data 5c behind the (a) Smooth wing and (b) Rough wing at $\alpha = 2^\circ$. 
Figure D.0.2: Velocity vector plot of seven hole probe data 5c behind the (a) Smooth wing and (b) Rough wing at $\alpha = 4^\circ$.

Figure D.0.3: Velocity vector plot of seven hole probe data 5c behind the (a) Smooth wing and (b) Rough wing at $\alpha = 5^\circ$. 
Figure D.0.4: Velocity vector plot of seven hole probe data 5c behind the (a) Smooth wing and (b) Rough wing at $\alpha = 6^\circ$.

Figure D.0.5: Velocity vector plot of seven hole probe data 5c behind the (a) Smooth wing and (b) Rough wing at $\alpha = 8^\circ$. 
Figure D.0.6: Velocity vector plot of seven hole probe data $5c$ behind the (a) Smooth wing and (b) Rough wing at $\alpha = 10^\circ$.

Figure D.0.7: Velocity vector plot of seven hole probe data $5c$ behind the (a) Smooth wing and (b) Rough wing at $\alpha = 12^\circ$. 

108
Figure D.0.8: Velocity vector plot of seven hole probe data 5c behind the (a) Smooth wing and (b) Rough wing at $\alpha = 13^\circ$.

Figure D.0.9: Velocity vector plot of seven hole probe data 5c behind the (a) Smooth wing and (b) Rough wing at $\alpha = 14^\circ$. 
Figure D.0.10: Axial velocity, \((u - U_\infty)/U_\infty\), profile of seven hole probe data at \(x = 5c\) and \(\alpha = 2^\circ\); (\(\circ\)), Smooth wing; (\(\bullet\)), Rough wing; (\(\_\_\_\_\) and (\(\_\_\_\_\)), (2.1.3).
Figure D.0.11: Axial velocity, $(u - U_\infty)/U_\infty$, profile of seven hole probe data at $x = 5c$ and $\alpha = 4^\circ$; (o), Smooth wing; (●), Rough wing; (—) and (—−), (2.1.3).
Figure D.0.12: Axial velocity, \((u - U_{\infty})/U_{\infty}\), profile of seven hole probe data at \(x = 5c\) and \(\alpha = 5^\circ\); (\(\circ\)), Smooth wing; (\(\bullet\)), Rough wing; (\(\_\_\_\) and (\(\_\_\)\(\_\)), (2.1.3).
Figure D.0.13: Axial velocity, $(u - U_\infty)/U_\infty$, profile of seven hole probe data at $x = 5c$ and $\alpha = 6^\circ$; ($\circ$), Smooth wing; ($\bullet$), Rough wing; (—) and (——), (2.1.3).
Figure D.0.14: Axial velocity, $(u - U_\infty)/U_\infty$, profile of seven hole probe data at $x = 5c$ and $\alpha = 8^\circ$; (○), Smooth wing; (●), Rough wing; (—) and (——), (2.1.3).
Figure D.0.15: Axial velocity, \( (u - U_\infty)/U_\infty \), profile of seven hole probe data at \( x = 5c \) and \( \alpha = 10^\circ \); (○), Smooth wing; (●), Rough wing; (—) and (---), (2.1.3).
Figure D.0.16: Axial velocity, \((u - U_\infty)/U_\infty\), profile of seven hole probe data at \(x = 5c\) and \(\alpha = 12^\circ\); (o), Smooth wing; (●), Rough wing; (—) and (——), (2.1.3).
Figure D.0.17: Axial velocity, \((u - U_\infty)/U_\infty\), profile of seven hole probe data at 
\(x = 5c\) and \(\alpha = 13^\circ\); (○), Smooth wing; (●), Rough wing; (—) and (—you), (2.1.3).
Figure D.0.18: Axial velocity, \((u - U_\infty)/U_\infty\), profile of seven hole probe data at \(x = 5c\) and \(\alpha = 14^\circ\); (a), Smooth wing; (b), Rough wing; (—) and (—), (2.1.3).

Figure D.0.19: Axial velocity contours obtained with the seven hole probe at \(x = 5c\) and \(\alpha = 2^\circ\); (a), Smooth wing; (b), Rough wing. Contour levels denote \((u - U_\infty)/U_\infty\) values.
Figure D.0.20: Axial velocity contours obtained with the seven hole probe at $x = 5c$ and $\alpha = 4^\circ$; (a), Smooth wing; (b), Rough wing. Contour levels denote $(u - U_\infty)/U_\infty$ values.

Figure D.0.21: Axial velocity contours obtained with the seven hole probe at $x = 5c$ and $\alpha = 5^\circ$; (a), Smooth wing; (b), Rough wing. Contour levels denote $(u - U_\infty)/U_\infty$ values.
Figure D.0.22: Axial velocity contours obtained with the seven hole probe at $x = 5c$ and $\alpha = 6^\circ$; (a), Smooth wing; (b), Rough wing. Contour levels denote $(u - U_\infty)/U_\infty$ values.

Figure D.0.23: Axial velocity contours obtained with the seven hole probe at $x = 5c$ and $\alpha = 8^\circ$; (a), Smooth wing; (b), Rough wing. Contour levels denote $(u - U_\infty)/U_\infty$ values.
Figure D.0.24: Axial velocity contours obtained with the seven hole probe at $x = 5c$ and $\alpha = 10^\circ$; (a), Smooth wing; (b), Rough wing. Contour levels denote $(u - U_\infty)/U_\infty$ values.

Figure D.0.25: Axial velocity contours obtained with the seven hole probe at $x = 5c$ and $\alpha = 12^\circ$; (a), Smooth wing; (b), Rough wing. Contour levels denote $(u - U_\infty)/U_\infty$ values.
Figure D.0.26: Axial velocity contours obtained with the seven hole probe at $x = 5c$ and $\alpha = 13^\circ$; (a), Smooth wing; (b), Rough wing. Contour levels denote $(u - U_\infty)/U_\infty$ values.

Figure D.0.27: Axial velocity contours obtained with the seven hole probe at $x = 5c$ and $\alpha = 14^\circ$; (a), Smooth wing; (b), Rough wing. Contour levels denote $(u - U_\infty)/U_\infty$ values.
Figure D.0.28: Vorticity contours obtained with the seven hole probe at $x = 5c$ and $\alpha = 2^\circ$; (a), Smooth wing; (b), Rough wing. Contour levels denote $\zeta r_c/v_0$ values.

Figure D.0.29: Vorticity contours obtained with the seven hole probe at $x = 5c$ and $\alpha = 4^\circ$; (a), Smooth wing; (b), Rough wing. Contour levels denote $\zeta r_c/v_0$ values.
Figure D.0.30: Vorticity contours obtained with the seven hole probe at $x = 5c$ and $\alpha = 5^\circ$; (a), Smooth wing; (b), Rough wing. Contour levels denote $\zeta_{rc}/v_0$ values.

Figure D.0.31: Vorticity contours obtained with the seven hole probe at $x = 5c$ and $\alpha = 6^\circ$; (a), Smooth wing; (b), Rough wing. Contour levels denote $\zeta_{rc}/v_0$ values.
Figure D.0.32: Vorticity contours obtained with the seven hole probe at $x = 5c$ and $\alpha = 8^\circ$; (a), Smooth wing; (b), Rough wing. Contour levels denote $\zeta r_c/v_0$ values.

Figure D.0.33: Vorticity contours obtained with the seven hole probe at $x = 5c$ and $\alpha = 10^\circ$; (a), Smooth wing; (b), Rough wing. Contour levels denote $\zeta r_c/v_0$ values.
Figure D.0.34: Vorticity contours obtained with the seven hole probe at $x = 5c$ and $\alpha = 12^\circ$; (a), Smooth wing; (b), Rough wing. Contour levels denote $\zeta_{rc}/v_0$ values.

Figure D.0.35: Vorticity contours obtained with the seven hole probe at $x = 5c$ and $\alpha = 13^\circ$; (a), Smooth wing; (b), Rough wing. Contour levels denote $\zeta_{rc}/v_0$ values.
Figure D.0.36: Vorticity contours obtained with the seven hole probe at \(x = 5c\) and \(\alpha = 14^\circ\); (a), Smooth wing; (b), Rough wing. Contour levels denote \(\zeta r_c/v_0\) values.
Figure D.0.37: Tangential velocity profile obtained with the seven hole probe at $x = 5c$ and $\alpha = 2^\circ$; (○), Smooth wing; (●), Rough wing; (—) and (—−), (2.1.5).
Figure D.0.38: Tangential velocity profile obtained with the seven hole probe at $x = 5c$ and $\alpha = 4^\circ$; (○), Smooth wing; (●), Rough wing; (—) and (——), (2.1.5).
Figure D.0.39: Tangential velocity profile obtained with the seven hole probe at $x = 5c$ and $\alpha = 5^\circ$; (○), Smooth wing; (●), Rough wing; (—) and (—–), (2.1.5).
Figure D.0.40: Tangential velocity profile obtained with the seven hole probe at $x = 5c$ and $\alpha = 6^\circ$; (o), Smooth wing; (●), Rough wing; (—) and (—−), (2.1.5).
Figure D.0.41: Tangential velocity profile obtained with the seven hole probe at $x = 5c$ and $\alpha = 8^\circ$; $(\circ)$, Smooth wing; $(\bullet)$, Rough wing; $(--)$ and $(---)$, (2.1.5).
Figure D.0.42: Tangential velocity profile obtained with the seven hole probe at $x = 5c$ and $\alpha = 10^\circ$; (o), Smooth wing; (●), Rough wing; (—) and (—−), (2.1.5).
Figure D.0.43: Tangential velocity profile obtained with the seven hole probe at $x = 5c$ and $\alpha = 12^\circ$; (○), Smooth wing; (●), Rough wing; (—) and (−−), (2.1.5).
Figure D.0.44: Tangential velocity profile obtained with the seven hole probe at $x = 5c$ and $\alpha = 13^\circ$; (o), Smooth wing; (●), Rough wing; (—) and (—–), (2.1.5).
Figure D.0.45: Tangential velocity profile obtained with the seven hole probe at $x = 5c$ and $\alpha = 14^\circ$; (○), Smooth wing; (●), Rough wing; (—) and (——), (2.1.5).
Appendix E

Plots of raw LDA data

Figure E.0.1: Velocity vector plot of LDA data 5c behind the smooth wing at $\alpha = 5^\circ$. 
Figure E.0.2: Contours of \((u - U_\infty)/u_0\). LDA data 5c behind the smooth wing at \(\alpha = 5^\circ\).

Figure E.0.3: Axial velocity profile of LDA data 5c behind the smooth wing at \(\alpha = 5^\circ\), and (---), (2.1.3).
Figure E.0.4: Contours of normalised vorticity, $\zeta r_c/v_0$ of LDA data $5c$ behind the smooth wing at $\alpha = 5^\circ$.

Figure E.0.5: Tangential velocity profile of LDA data $5c$ behind the smooth wing at $\alpha = 5^\circ$, and (—), (2.1.5).
Figure E.0.6: Velocity vector plot of LDA data 5c behind the rough wing at $\alpha = 5^\circ$.

Figure E.0.7: Contours of $(u - U_\infty)/u_0$. LDA data 5c behind the rough wing at $\alpha = 5^\circ$. 
Figure E.0.8: Axial velocity profile of LDA data 5c behind the rough wing at $\alpha = 5^\circ$, and (--), (2.1.3).

Figure E.0.9: Contours of normalised vorticity, $\zeta r_c/v_0$ of LDA data 5c behind the rough wing at $\alpha = 5^\circ$. 

141
Figure E.0.10: Tangential velocity profile of LDA data 5c behind the rough wing at $\alpha = 5^\circ$, and (—), (2.1.5).
Appendix F

Optimal calibration of directional velocity probes

Stephen C. McParlin\textsuperscript{1}, Samantha. S. Ward\textsuperscript{2} and David M. Birch.\textsuperscript{3}

University of Surrey, Guildford, Surrey, GU2 7XH, UK

A novel approach has been considered for the process of calibrating directional velocity probes for use in wind tunnels. The probe response was modeled as a polynomial surface, and a set of optimal sample points was determined using the process of D-optimality. This offers the potential of selecting the allowable calibration uncertainty \textit{a priori} (in order to minimize the required number of calibration points required), as well as of optimizing the locations (in parametric space) at which calibration data is collected. This procedure is demonstrated in the calibration and testing of a typical seven-hole pressure probe, and is shown to reduce the required number of calibration points by an order of magnitude (relative to an exhaustive calibration procedure) to achieve similar experimental uncertainty. The applicability of the procedure to the calibration of triple-sensor hot-wire probes is also demonstrated.

Nomenclature

\begin{itemize}
\item ANOVA = Analysis of Variances
\item DoE = Design of Experiments
\item $P_i$ = Measured pressure at point $i$
\item $C_p$ = Pressure coefficient
\item $C_\alpha$ = Differential pressure function with pitch at low coning angle
\item $C_\beta$ = Differential pressure function with yaw at low coning angle
\item $C_\theta$ = Differential pressure function with pitch at high coning angle
\item $C_\phi$ = Differential pressure function with yaw at high coning angle
\item $A$ = Array of polynomial coefficients
\item $n$ = Data points
\item $m$ = Number of polynomial dimensions
\item $p$ = Order of the polynomial
\item $x$ = Array of polynomial variables
\item $\varepsilon$ = Error term
\item RSM = Response Surface Model
\item $\alpha$ = Angle of incidence
\item $\beta$ = Angle of sideslip
\item $\phi$ = Euler angle in roll
\item $\theta$ = Euler angle in pitch
\item $\psi$ = Euler angle in yaw
\item $\sigma$ = Standard deviation
\item $U$ = Freestream velocity
\item $c$ = Wing chord
\item $\nu$ = Kinematic viscosity
\item $\zeta$ = Streamwise vorticity
\item $v_\theta$ = Tangential velocity
\end{itemize}

\textsuperscript{1}Associate Tutor, Division of Mechanical, Medical and Aerospace Engineering, Associate Fellow AIAA.
\textsuperscript{2}Doctoral Candidate, Division of Mechanical, Medical and Aerospace Engineering, student member AIAA.
\textsuperscript{3}Lecturer in Aerospace Engineering, Division of Mechanical, Medical and Aerospace Engineering, member AIAA.
I. Introduction

The calibration of multi-hole pressure probes has been an issue of some interest, owing to the large amount of flow facility time typically required to obtain calibration data sets of sufficient density and quality to yield high confidence experimental results. Though the behaviour of an ideal multi-hole pressure probe may be obtained analytically\(^1\), the probe response is highly sensitive to small errors in geometry, so an experimental calibration is almost always necessary. Though there has been some effort toward applying advanced numerical techniques and neural-network algorithms toward probe calibration\(^2\) in order to improve efficiency, the most commonly used calibration methods are least-squares or piecewise-polynomial functional approximations\(^3,4\), though direct linear interpolation schemes are also frequently encountered\(^5,6\). For both techniques, the quality of the resultant data increases with the number of calibration points. Significantly, Sumner\(^7\) showed that for flows of large angularity, the direct interpolation method resulted in lower uncertainties, while the functional approximation method was more accurate for flows of small angularity. Consequently, neither technique can be optimal for flows spanning both large and small angles.

In nearly all cases, either rectangular or logarithmic calibration grids are used, primarily as a matter of convenience; indeed, unstructured calibration grids render some calibration schemes impractical. However, the sensitivity of the sensors is not uniform throughout the calibration space, so a grid of arbitrary spacing is also necessarily suboptimal.

This paper presents a novel approach to the calibration of multi-hole pressure probes, derived from the theory of optimal design of experiments (DoE). A conventional, exhaustive calibration was carried out in order to obtain a set of high-confidence calibration data. A formal statistical analysis was then carried out to fit these data to functional response surfaces, eliminate insignificant terms in the functions, identify outliers and determine the bounds of validity for each of the functions. Well-established optimization techniques were then used to determine the locations of the data points required in order to obtain the set of response surface coefficients with minimum error.

The calibration of experimental measurement equipment is fundamental to the accuracy and repeatability of all quantitative observations. There are numerous factors involved, not least the time taken for measurements supporting calibration. This is particularly important for instruments such as hot-wires, which are subject to time and temperature-dependent drift and therefore require frequent recalibration. Formalized approaches to calibration (which can be automated) reduce the amount of time required and ensure that repeatability between calibrations can be assessed rapidly. This is important for both the formal and technical quality assurance associated with subsequent data measurements.

Introducing an optimal DoE-based approach assures repeatability between tests by ensuring that a standard set of calibration points are used that offer minimum probability of error while limiting the number of sample points to the minimum required to achieve continuous fits to the measurements of the desired level of accuracy. Representation of the measured variables as a continuum also allows rapid inversion of the calibration matrix for subsequent experimental observations.

II. Calibration of multi-hole pressure probes

A. Current practice

The function of a multi-hole pressure probe is to obtain spatially-resolved flow velocity components from pressure measurements. The static and stagnation pressures are approximated based on the pressures recorded, and with some empirical adjustment from calibration, are used to determine the velocity magnitude. The flow angularity is obtained from the pressure differences between the holes, though this is typically a discontinuous process as the flow can separate in the region of the probe tip.

Because seven-hole probes are among the most commonly used, these shall be considered in order to demonstrate the application of optimal DoE-based calibration methods. However, it will also be shown that the process is equally applicable to \(n\)-hole probes, and other directional velocity probes having a response which can be characterized by a polynomial function.

1. Approximation of local static and stagnation pressure

A seven-hole probe typically has a conical or ogive tip with one hole located centrally, and the remaining six arranged in close-packed configuration around the central hole (figure 1a). The holes are conventionally numbered clockwise from 1-7, with hole 1 being at the bottom position and the central hole numbered as 7 (figure 1b).
In normal use, the probe will be orientated parallel to the free stream direction. When aligned parallel to an undisturbed free-stream, then, the pressure coefficient measured at the central hole,

\[ C_{p7} = \frac{p_7 - P}{P_0 - \bar{P}} \]

(where \( p_7 \) is the pressure measured at hole 7, \( P \) is the local static pressure, and \( P_0 \) is the local total pressure) will be close to unity. Depending on the probe head geometry, the pressure coefficients at the peripheral holes should range between 0 and 1 (corresponding to the approximate local static and stagnation pressure, respectively).

From the geometry of the seven-hole probe tip, it is clear that a probe with a sufficiently 'sharp' tip will begin to approximate a Pitot-static tube. Indeed, for small flow angles, the peak pressure is recorded at hole 7, and provides an approximation of the local stagnation pressure. Similarly, the mean pressure from holes 1-6 under these conditions may be taken as a measure of local static pressure. However, as shown in figure 1, the peripheral holes are often oriented towards the free-stream direction, and thus the pressure measured by these will be above local static pressure. Since the difference between the actual and approximated static and stagnation pressures will be a repeatable function of the flow angle, it is common to apply an empirical correction obtained through calibration.

![Fig. 1. (a) Photograph of the tip of a typical miniature seven-hole probe; (b) radial arrangement of pressure holes on a seven-hole probe, looking aft.](image)

The principle behind the calibration of probes with different numbers of holes is formally similar to that for seven-hole probes; the only difference is that the coefficients are obtained as some other function of the pressures recorded at particular holes (see, for example, Treaster and Yocum\(^8\) and Wang \textit{et al.}\(^9\)).

2. Treatment at low cone angle

For a typical seven-hole probe, it is assumed that the flow is attached everywhere around the probe tip when the stagnation point is in the vicinity of the central hole (so that hole 7 records the highest pressure). Under these conditions, pressure coefficients sensitive to the pitch angle, \( \alpha \), and yaw angle, \( \beta \), may be defined such that,

\[ C_\alpha = \frac{p_7 - p_1}{\bar{P}} \]

\[ C_\beta = \frac{\frac{1}{2}(p_3 + p_6) - \frac{1}{2}(p_3 + p_2)}{p_7 - \bar{P}} \]

where \( \bar{P} \) is the mean pressure recorded at holes 1 through 6. In these expressions, the numerators represent a measure of the pressure difference on either side of the probe tip, while the denominator is an approximation of the local dynamic pressure. The calibration process for these functions therefore involves traversing the probe through \( \alpha \) and \( \beta \) for the space in which the maximum pressure is recorded by hole 7. However, the boundaries in \((\alpha, \beta)\) space...
at which the maximum pressure shifts from hole 7 is not known \emph{a priori} and may be a function of Reynolds number, with implications for the extent of the validity of any calibration. For the case when the maximum pressure is recorded at hole 7, then,

\begin{equation}
C_\alpha = f_1(\alpha, \beta)
\end{equation}

\begin{equation}
C_\beta = f_2(\alpha, \beta)
\end{equation}

3. Treatment at high coning angle

Where the maximum pressure measured is not at the central hole, alternative procedures are used and outputs derived\textsuperscript{11}. It is assumed that the flow is only attached over the holes adjacent to that measuring maximum pressure, and thus only the measured data from these holes is valid. The local static pressure is then approximated as the mean pressure at the holes closest to the stagnation point, such that

\begin{equation}
P \approx \frac{P_{cw} + P_{ccw}}{2},
\end{equation}

where $P_{cw}$ and $P_{ccw}$ are the pressures recorded at the holes located adjacent to the hole registering the maximum pressure, in the clockwise and counter-clockwise directions, respectively. It is also therefore more natural to describe flows of large angularity using the cone angle $\theta$ and roll angle $\phi$. Pressure coefficients sensitive to cone and roll angles then may be defined for each of holes 1 through 6 (where $i$ indicates the hole index), as

\begin{equation}
C_{i,\theta} = \frac{P_i - P_7}{P_1 - P},
\end{equation}

\begin{equation}
C_{i,\phi} = \frac{P_{cw} - P_{ccw}}{P_1 - P},
\end{equation}

where $P$ may be approximated from (6). Then, in flow of high angularity (when the maximum pressure is not recorded at the central hole),

\begin{equation}
C_{i,\theta} = f_{i,1}(\theta, \phi)
\end{equation}

\begin{equation}
C_{i,\phi} = f_{i,2}(\theta, \phi),
\end{equation}

Equations (2) and (3), together with (7) and (8), yield seven different sets of fitted representations in matrix form, which need to be inverted to return the angle of the flow relative to the probe when in normal use.

4. Summary

It is important to observe in the above that, for an $n$-hole pressure probe, the calibration process will necessarily require $n$ mutually independent calibration spaces $f$ to be constructed, and the selection of the calibration space used to reduce the $n$ pressures to three components of velocity will typically depend only upon which hole registers the highest pressure. These independent calibration spaces, then, correspond to non-overlapping regions in $(\theta, \phi)$ space over which each calibration function is valid.

Conventional calibration procedures, therefore, have a number of fundamental shortcomings. First, the calibration will yield multiple independent calibration regions which are necessarily discontinuous. Furthermore, the algorithm used for selecting the appropriate calibration function may fail if the stagnation point falls somewhere between holes, such that the pressures recorded at more than one hole are within the range of experimental uncertainty. Because the functions are not continuous, this may result in substantial error. Finally, there is no formal treatment included in the above to verify that the pressures recorded are indeed within a region of attached flow.
B. Additional areas for improvement

It should be noted that, although the relations obtained for $C_\alpha$, $C_\beta$, $C_{i,0}$ and $C_{i,\theta}$ are derived using the absolute measured pressures, the results will be identical if $P_1-P_7$ are substituted with incremental pressures relative to free-stream static pressure, or indeed, pressure coefficients, normalized by free-stream dynamic pressure. Using the latter of these approaches, it is possible to verify the validity of individual pressure measurements. If the flow is assumed to be adiabatic, then the stagnation pressure seen by the head of the probe will always be equal to or less than free-stream total pressure. This provides a maximum value of pressure coefficient beyond which measurements are suspect. Conversely, the minimum pressure coefficient experienced by any pressure hole under attached flow conditions can be determined by inspection of the data measured during the calibration process. Rather than determining the validity of pressure measurements on the leeward side of the probe head based on measurements at the windward side, then, it should be possible to determine the validity of an individual point based on whether it falls within prescribed bounds. The same criterion used to determine the validity of the pressure measured at the most leeward point can also be applied, at higher $\theta$, to determine the validity of the central point, and hence the limit of applicability of the probe.

As the logic defining the function of the probe breaks down at the boundaries of the calibration spaces of adjacent holes (where pressure measurements at the holes become equal), it is desirable to find an alternative approach in which these boundary regions overlap, so as to provide continuity and potentially increased accuracy. Indeed, once a necessary and sufficient condition for flow attachment around a particular hole has been established, all of the available valid measurements may be used to determine the flow angle. At the centre of the domain, then, all seven pressures should be valid, while at the boundaries, only the pressures at the points closest to the windward side will be valid. Between these extremes, the number of overlapping measurements will reduce progressively, giving a gradual reduction in available accuracy as the onset flow angle becomes more extreme. The current scheme assumes that the data for the three holes surrounding the most windward point remain attached up to the limit of coning angle for the probe.

III. Optimal Design of Experiments

A. Design of Experiments

The field of design of experiments seeks to produce formal processes and schemes for quantitatively determining the sensitivity of outputs to variations in associated inputs. In practice, this means designing efficient sampling schemes to identify which factors are significant in the generation of an output, and then characterizing the relationships between input factors and output responses. In classical design of experiments, experimental design schemes start with full-factorial schemes, in which sample points are chosen at the centre, vertices and face centres of hypercubes of the dimensions of the input factors, and then alternative schemes with progressively smaller sample sets are used. For a 2D hypercube, a full-factorial sample set to support a quadratic fit would consist of nine sample points arranged as a regular $3 \times 3$ array. For a 3D hypercube, the corresponding number of sample points in a full-factorial scheme would be 27, in a $3 \times 3 \times 3$ arrangement. The number of sample points in a quadratic full-factorial scheme varies as $3^n$, where $n$ is the number of factors involved.

The second element in design of experiments involves the analysis of sample data to formally determine the significance of individual or coupled factors. The analysis of variance (ANOVA) is a standard series of statistical tests used to evaluate the results of sampling, identify outliers and non-significant relationships and thus determine the relationship between input factors and output responses. The resulting relationships are then expressed in algebraic terms as a response surface model (RSM).

In optimal DoE, rather than follow a set sampling scheme (with the number of sample points being a consequence of the number of input factors), the objective is to optimize sampling locations given a fixed set of input variables and a fixed number of sample points. Among the advantages of this approach, the ability to make use of blocks of pre-existing sample points and cope with constraints on the boundaries of the sampling domain makes optimal DoE a great deal more flexible than classical schemes, while the rationale for the technique is to reduce the cost and time required to take samples.
B. D-Optimality

In the creation of an RSM for a function \( y \), it is assumed that the domain of the model is continuous and can be represented as a surface fit. For a simple polynomial fit such that \( x = (x_1, x_2, \ldots, x_m) \), we can express \( y \) as a polynomial with \( p + 1 \) terms in \( m \) variables, as

\[
y = A_0 + A_1 x_1^{B_{11}} + \cdots + x_m^{B_{pm}} + \epsilon,
\]

where \( \epsilon \) is a small random error term which is assumed to be normally distributed, having a variance \( \sigma^2 \). This polynomial can also be expressed as

\[
y = A x_p + \epsilon
\]

where

\[
A = (A_0, A_1, \ldots, A_p)^T
\]

is a vector of real coefficients which must be determined to fit the response surface model over the domain of interest.

Given \( n \) data points in \( m \) dimensions, \((X_{11}, X_{12}, \ldots, X_{1m})\) through \((X_{n1}, X_{n2}, \ldots, X_{nm})\), the data points can be arranged in the form of a matrix, as

\[
X = \begin{bmatrix}
(X_{11}^{B_{11}}, X_{12}^{B_{12}}, \ldots, X_{1m}^{B_{1m}}) & \cdots & (X_{n1}^{B_{11}}, X_{n2}^{B_{12}}, \ldots, X_{nm}^{B_{1m}}) \\
\vdots & \ddots & \vdots \\
(X_{11}^{B_{m1}}, X_{12}^{B_{m2}}, \ldots, X_{1m}^{B_{mm}}) & \cdots & (X_{n1}^{B_{m1}}, X_{n2}^{B_{m2}}, \ldots, X_{nm}^{B_{mm}})
\end{bmatrix}
\]

where \( p \) is again the order of the polynomial fit. For any such set of points, the values of the coefficients \( A^* \) yielding the best possible fit is then simply given by

\[
A^* = (X^TX)^{-1}X^TY.
\]

where

\[
Y = (y_1, y_2, \ldots, y_n)^T
\]

is the vector containing the exact functional values. In addition, the variance, \( \sigma \), may be obtained from the expression,

\[
A^* = (X^TX)^{-1}\sigma^2
\]

D-optimality has the objective of finding the specific \( n \) sample points which minimize this variance and hence providing an optimal estimate of \( A^* \) by maximizing the determinant of \( X^TX \). The sample size, \( n \), to be used for D-optimality is arbitrary, and provides a means by which to compromise between accuracy and the time or cost associated with the generation of samples.

The minimum value of \( n \) required is that necessary to produce a non-singular matrix, which is \( p \). In practice, the ratio of \( np \) is chosen so as to give the minimum variance for a given cost or time. Then, for a given \( m \) and a specified \( p \), \( n \) may be uniquely determined. For example, a quadratic description of a space with two and three independent variables, \( p = 6 \) and \( p = 10 \), respectively. The minimum sample set for a D-optimal experimental design is smaller than for a full-factorial representation of the same space, while offering much larger savings in sample points for larger numbers of independent factors. For a 7-dimensional space, 2187 sample points are required for a quadratic full-factorial scheme, while the minimum D-optimal sample set for a quadratic RSM would be \( n = 120 \). Typically, however, the actual value of \( np \) is determined by a compromise that gives an adequate level of fit to the sample data set, as determined by the regression coefficients and ANOVA analysis.
III. Application of D-Optimality to probe calibration

A. Determining the form of the response surface models.

1. Exhaustive calibration of seven-hole pressure probe

As seen in (14) and (15), the implementation of D-optimality to the calibration of a sensor would necessarily require an *a priori* knowledge of the form of the sensor response. A comprehensive, exhaustive calibration of a typical seven-hole pressure probe was therefore carried out.

A miniature seven-hole pressure probe (having a probe tip diameter of 3.0 mm) was specially constructed and used for this work. The probe was connected to an array of temperature-corrected and Honeywell 163PC01D75 differential amplified pressure sensors (having internal signal conditioning and a full-scale error of less than 0.25%), using the local static pressure as the reference. The transducers were calibrated both before and after each set of measurements, and data were discarded if the calibration drift was greater than 1%.

The probe was mounted in a high-precision, servomotor-controlled traverse with three translational and two rotational degrees of freedom, allowing *in-situ* directional calibration with an uncertainty in angular position of ±0.2°. Experiments were carried out in an open-return wind tunnel with a working section of 0.9 m × 0.6 m. All calibrations were performed with a free-stream velocity of 10 m/s, which was actively controlled and maintained constant to within the measurement precision.

The pressure transducer signals were digitized using a Data Translation DT9836 data acquisition system, and a total of $10^4$ samples were collected over a total time of 20 seconds per measurement location in order to obtain a statistically converged mean. For the purposes of obtaining a high-resolution (exhaustive) calibration field, a regular grid of 843 data points were collected in increments of 5° in pitch and yaw. The location of the origin of the angular displacements of the traverse was determined iteratively, and was defined as the position at which the the centre hole returned the maximum possible pressure.

The response of the probe could then be mapped by plotting the surfaces described by the pressure coefficients $C_{Pi}$ obtained at each hole over the range of $\alpha$ and $\beta$ tested. The results are shown in figure 2, together with the piecewise quadratic surfaces of best fit. While the results and surface fits are shown in the coordinate system in which they were measured, the effect of fitting these using cone and roll angle was also investigated. The surface fits to the existing data based on a polar representation appeared to be slightly worse in terms of regression coefficient, although it was not determined whether this was due to the data not being sampled as a regular array in these two variables.

The results show that the variation in the pressure measurements with angle is reasonably regular, and agrees well with the response typically expected. The surface fits to these data is also reasonably good, particularly in regions near the maxima. Though the agreement for $C_{Pi}$ is good throughout the range tested, the agreement worsens away from the maxima for the peripheral holes, suggesting that the fit could potentially be improved by eliminating some of the sample points furthest away from the maximum pressure measured for each hole (which, notably, is where the pressure measurements are most likely to be invalid).

2. Selection of sample points

The complexity of shape of the surfaces generated for the radial pressure ports suggested that it might be necessary to consider whether the use of a quadratic relationship was a limiting constraint on the process. The surfaces shown in figure 2 were generated using a piecewise, rather than continuous, surfacing function, thus increasing the complexity of the task. In addition, while progressively increasing the number of sample points in a set, it was necessary to ensure that the number of repeat points in the sample set was controlled directly.

A commercial D-optimal sample generation software package was used to assess the data, and the likely goodness of fit for polynomial response surface models up to sixth order was assessed. The predicted goodness of fit for the 7-hole probe increased with order of polynomial, with the sixth-order treatment being predicted to be marginally better than quartic or quintic treatments. For a sixth-order polynomial expression in two independent variables, the number of coefficients to be determined is 28. Hence the size of the non-singular sample set is 28 points. The software was then used to generate the required D-optimal sample points in multiples of 28. Some of these are shown in figure 3 for a non-dimensionalised space. Note that, in all cases, the number of data points in the D-optimal sample sets was at least an order of magnitude smaller than the size of the exhaustive calibration data set. The process was repeated using a 5-hole pressure probe, and the resulting surfaces also agreed reasonably well.
Figure 2: Comparison between piecewise quadratic functional approximations and calibration data.
B Generating D-Optimal sample data

The seven-hole pressure probe was then re-calibrated using each of the sets of D-optimal sampling points, with \( n \) ranging from 28 to 196 in increments of 28, and an RSM was obtained for \( C_p \) for each of the seven holes. Initial fits to each of the pressure responses for these points showed a standard deviation between the fitted surfaces and the sample points of typically 1 to 4\% of dynamic pressure. The regression coefficient \( R^2 \) was in all cases greater than 0.99, and typical values of the standard deviation are shown in Table 1. Figure 4 shows the percent variance (relative to the dynamic pressure) for each of the holes. For the larger sample sizes \( (n/p > ~3) \), the variance is independent of \( n \), suggesting that further increases in \( n \) would yield only marginal improvements in the accuracy of the fit. In all cases, the variance was less than 7\% of the dynamic pressure.

Next, the calibration coefficients obtained using the exhaustive calibration procedure were compared directly to those obtained using only those points prescribed by the D-optimality, with \( n = 196 \) \( (or, \ equivalently, \ n/p = 7) \). Figure 5 (a) maps the exhaustive calibration data against the values of the RSM obtained from the D-optimal calibration points, for the case of hole 7. Clearly, an unacceptable error is incurred for larger angles; a similarly poor agreement is obtained for the peripheral holes.

By applying a typical sensitivity analysis, it emerged that the sixth-order polynomial RSMs (and therefore the variances) were highly sensitive to the data points collected well outside of the range of sensitivity of a given hole. Those data points lying outside the range of sensitivity were identified, and typically corresponded to flow angles less than \(-60^\circ\). By removing those data points, the RSM was able to predict the exhaustive calibration data to within the typical experimental uncertainty, though the error does again increase with increasing flow angularity (figure 5 b).

Figure 5 also demonstrates that the optimal calibration process is compatible with legacy software systems requiring the construction of exhaustive look-up tables at prescribed points in \((\alpha, \beta)\) space; once the D-optimal data has been collected and the RSMs obtained, discrete look-up tables may be constructed from the continuous RSMs having any arbitrary number of data points, with no loss of accuracy (providing, of course, that the number of points in the look-up table is greater than \( n \)).

Figure 3 - D-Optimal sample sets for \( n = 28 \) (○), \( n = 112 \) (□) and \( n = 196 \) (+).
Table 1: Quantitative parameters for response surface representations of probe outputs

<table>
<thead>
<tr>
<th>$\sigma$</th>
<th>196-point D-Optimal set</th>
<th>Exhaustive set</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_{p2}$</td>
<td>2.38%</td>
<td>4.14%</td>
</tr>
<tr>
<td>$C_{p3}$</td>
<td>2.79%</td>
<td>3.91%</td>
</tr>
<tr>
<td>$C_{p7}$</td>
<td>2.64%</td>
<td>2.40%</td>
</tr>
</tbody>
</table>

Figure 4: Variation in the standard deviation between the RSM and the exhaustive calibration data with $n$, for each of the seven surfaces.

Figure 5: Reconstruction of the exhaustive calibration data for hole 7 using (a) all 196 D-optimal points, and (b) only those D-optimal points to which the hole was sensitive.
IV. Application to typical wind tunnel measurements

As a final test of the use of D-optimality in the calibration of a 7-hole pressure probe, a typical wing wake scan was carried out. Measurements were collected behind a NACA 0012 wing model, having a chord \( c = 157 \text{ mm} \), an aspect ratio of 2.5 and no twist. The tip of the wing was fixed with a body-of-revolution end cap in order to minimize the generation of secondary vortices (see, for example, Birch et al.\(^{12}\)). The wing was fixed at an angle of attack of 5°, and the measurement plane was located 5\( c \) downstream of the wing trailing edge. Measurements were collected at a free-stream velocity \( U = 10 \text{ m/s} \), corresponding to a chord Reynolds number \( \text{Re} = \frac{Uc}{\nu} = 1.05 \times 10^5 \), where \( \nu \) is the kinematic viscosity. Velocity measurements were made using the same probe, data acquisition system and sampling parameters. Data were collected at 64 \( \times \) 64 locations in the measurement plane, with a constant spatial resolution of \( \Delta z = \Delta y = 1.5 \text{ mm} \). The data were then reduced to three components of velocity using either the look-up table from the exhaustive calibration (using a second-order interpolation) or using the RSM model.

Figure 6 shows normalized isovorticity contours within the tip vortex roll-up region for the same data set reduced with the exhaustive and D-optimal calibrations, scaled against the vortex core radius \( r_c \) (defined by fitting the circumferentially-averaged tangential velocity to a Batchelor \( q \) vortex\(^{13}\)). The tip vortex region is characterized by a high degree of flow angularity, and is therefore a good test case for the procedure. The D-optimal calibration correctly captures the structure of the developing vortex, as well as the presence of a highly concentrated secondary structure to the right of the vortex core. There are some noticeable differences in the vorticity fields around the noise level (-0.5 < \( \zeta_c / U \) < 1.5, where \( \zeta \) is the streamwise vorticity). The peak normalized vorticity returned by the exhaustive and D-optimal calibrations were 30.41 and 29.70, respectively.

![Figure 6. Contours of \( \zeta_c / U \) obtained by reducing a data set using (a) the exhaustive calibration look-up table and (b) the D-Optimal RSM](http://mc.manuscriptcentral.com/aiaa-masm13)

Similarly, figure 7 compares the isocontours of normalized tangential velocity \( v / U \) from the same data set reduced using the exhaustive look-up table and the D-optimal RSM (the origin was taken as the location of minimum velocity). Again, the profiles show very good agreement. The asymmetry apparent in the tangential velocity field is due primarily to the incomplete roll-up of the vortex and continuing influence of the wing wake, and was captured by the D-optimal RSM. Some small differences in the isocontours are evident around the location of maximum tangential velocity (where the flow angularity is greatest), but the peak magnitudes vary by no more than 4.1%.

American Institute of Aeronautics and Astronautics
V. Application of D-optimal calibration procedure to triple-sensor hot-wire probes

The extension of the D-optimal calibration procedure to the use of multi-sensor hot-wire probes was also considered, as an extension of this work. Providing that the calibration spaces can be well-represented by polynomial surfaces, the D-optimal calibration procedure may be applied to any sensing system; indeed, if the RSM may be described as a sixth-order polynomial (that is, \( p = 28 \)), the locations at which calibration data is required will necessarily be the same as well.

![Figure 7: Contours of \( \frac{v}{U} \) obtained by reducing a data set using (a) the exhaustive calibration look-up table and (b) the D-Optimal RSM](image)

Exhaustive calibration data from a \( 120^\circ \times 45^\circ \), triple-sensor hot-wire probe was collected at a single free-stream velocity of \( U = 10 \) m/s over the angular range \((-45^\circ \leq \alpha \leq 45^\circ, -45^\circ \leq \beta \leq 45^\circ)\) with a uniform resolution of \( \Delta \alpha = \Delta \beta = 0.5^\circ \). As the purpose was to assess only the form of the RSM, \( U \) was not included as a dimension in the calibration space (there is evidence, however, that the velocity response of the hot-wire system is nearly quadratic\(^{14}\), so the nonlinear velocity response will not significantly affect the form of the RSM).

Figure 8 shows the calibration spaces for each of the three hot-wire sensors, together with the sixth-order polynomial surface of best-fit. As expected, there is a strong rotational symmetry, and a decrease in sensor sensitivity with increasing flow angularity. These results show that a sixth-order RSM fits the data well (\( \sigma \approx 2.5\% \) of voltage range), and as a consequence, the D-optimal calibration process may be applied to these sensors.

While the multi-sensor hot-wire probe may be calibrated at the same D-optimal points (in \( \alpha, \beta \) space) as the multi-hole pressure probe at each velocity, the resulting three-dimensional calibration space would necessarily be sub-optimal owing to the nonlinear velocity sensitivity. A formal optimization would require that the quadratic velocity response be included in the optimized parametric set, resulting in an RSM having \( p = 84 \).

VI. Conclusion

A formal approach to the optimization of the calibration of directional velocity probes has been developed by applying the theory of D-optimality. The multiple, independent calibrations required for a multi-hole pressure probe have been represented as response surfaces, and it has been shown that by selecting the appropriate points at which calibration data are collected, similar levels of measurement confidence may be obtained with an order of magnitude fewer calibration points. The applicability of the D-optimal calibration procedure to multi-sensor hot-wire probes has also been demonstrated.
Figure 8: Triple-sensor hot-wire calibration surfaces at $U = 10$ m/s.

Acknowledgments

This work has been funded by the U.K. Engineering and Physical Sciences Research Council under grant number EP/H030360/1. Mr. Jason Nunes is thanked for his contribution in the development and imaging of the seven-hole probe.

References


The calibration and use of $n$-hole velocity probes

Samantha Shaw-Ward*, Alex Titchmarsh, David M. Birch†

University of Surrey, Guildford, Surrey, UK GU2 7XH

A generalized calibration process is presented for multi-hole, pressure-based velocity probes which is independent of the number of holes and probe geometry, allowing the use of probes with large numbers of holes. The calibration algorithm is demonstrated at low speeds with a conventional seven-hole pressure probe and a novel nineteen-hole pressure probe. Because the calibration algorithm is independent of probe configuration, it is very tolerant of data corruption and imperfections in the probe tip geometry. The advantages of using probes with large numbers of holes is demonstrated in a conventional wing wake survey. The nineteen-hole probe offers a higher angular sensitivity than a conventional seven-hole probe, and can accurately measure velocity components even when an analytical calibration scheme is used. The probe can also provide local estimates of the diagonal components of the cross-flow velocity gradient tensor in highly vortical flows.

I. Introduction

Despite their comparative simplicity, multi-hole pressure probes continue to be used in the characterization of three-dimensional flows owing to their reliability, robustness and ease of manufacture. Furthermore, because they can provide local measurements of the three components of fluid velocity as well as of the local static and total pressure, they are of particular use in wake surveys (see [1, 2, 3, 4] and references cited therein), for which optical methods may present difficulties owing to the potential flow interference arising from particle injection [5] and particle momentum effects [6].

The design, calibration and use of five- and seven-hole probes is already well-developed (see, for example [7, 8, 9, 10, 11]). In general, the calibration process involves the identification of nondimensional pressure coefficients which are as sensitive as possible to the flow angularity but are insensitive to the velocity magnitude. These coefficients are then measured in steady flow over a range of incident flow angles during a calibration procedure; the range of angular sensitivity will depend on both the velocity magnitude and the probe tip geometry. Because the flow angularity has two degrees of freedom, at least two independent coefficients are required. Estimates of the local static and total pressure are also identified, and the errors between the estimates and actual

*Graduate Research Assistant, Department of Mechanical Engineering Sciences, Associate Member AIAA
†Lecturer, Department of Mechanical Engineering Sciences, Senior Member AIAA, d.birch@surrey.ac.uk
values (which are also a function of the flow angle) are similarly nondimensionalized and measured over a range of flow angles. The result of the calibration process is typically a set of four calibration functions mapping the nondimensionalized pitch angle, yaw angle, static pressure and total pressure to the pressures measured at the probe ports. These functions are generally either stored in the form of a look-up table (see [12]) or approximated as a polynomial expansion [9, 8]. A detailed comparison of these two calibration techniques is provided by Sumner [13].

Given any experimental measurement, then, the four coefficients are computed from the pressure readings, and the corresponding pitch angle, yaw angle, static pressure and total pressure are obtained either by interpolation or by functional approximation. The velocity magnitude may be determined from the static and total pressures, and the Cartesian velocity components may then be resolved. For probes with tips having well-defined geometries, it is also possible to obtain theoretical estimates of the four calibration functions based on either analytical or numerical solutions for the surface pressures; these techniques, however, are hindered by the high sensitivity of these idealized solutions to small manufacturing imperfections in the probe geometry, as well as by a loss of sensitivity in flows of very high vorticity [14].

More recently, a number of novel geometries and calibration algorithms have been proposed for probes having twelve and more holes, capable of resolving even reverse flow ([15, 16, 17]). The calibration technique proposed by Ramakrishnan & Rediniotis [15] is particularly attractive, as it is generalized and independent of both probe geometry and hole position; this method, however, still relies upon the identification and use of piecewise functions to represent the calibration surface. Calibration schemes such as that of Benay [18] are also of great value, as the procedure is generalized in the sense that it does not require the division of the probe measurement space into sectors, nor does it constrain the probe geometry.

The purpose of this work is to demonstrate the use of a continuous, generalized calibration scheme with probes having an arbitrary number of arbitrarily-arranged holes, and assess the robustness of the data reduction algorithm against some of those discussed above. In addition, the use of probes with large numbers of holes for the measurement of velocity components without calibration, as well as for the direct measurement of the local velocity gradients, is investigated.

II. Experimental setup

Experiments were carried out in an open-return wind tunnel with a working section of 0.9 m × 0.6 m. The free-stream velocity magnitude was set to $U_\infty = 10$ m/s for all of the measurements, and was maintained constant to within measurement precision by means of a closed-loop active control system. The control system sampled the average free-stream velocity averaged over 30-second intervals just upstream of the main measurement station, using a Pitot probe and a Furness micromanometer having a full-scale range of 196 Pa.

The directional velocity probes being tested were mounted in a five degree-of-freedom traverse
capable of translation in \( x, y \) and \( z \) (the streamwise, vertical and transverse axes, respectively) with a precision of \( \pm 5 \) \( \mu \)m, and rotation in cone angle \( \theta \) and roll \( \phi \) with a precision of \( \pm 0.2^\circ \) (where \( \phi \) is a rotation about the \( x \)-axis). Probes were calibrated \textit{in situ} over an angular range of \( -60^\circ \leq \alpha \leq 60^\circ \) and \( -60^\circ \leq \beta \leq 60^\circ \) (where \( \alpha \) and \( \beta \) are the pitch and yaw angles of the probe axis, respectively). The probe measurement volume was not held stationary through the calibration process; however, scans carried out within the envelope of probe movement showed the variation in the freestream velocity was less than the overall measurement uncertainty.

The probes were connected via lengths of silicone tubing to an array of low-cost Honeywell PCAFA6D differential pressure sensors, referenced to the wind tunnel static pressure and driven by Burr-Brown INA125 bridge signal amplifiers to a net sensitivity of \( \sim 0.04 \) Pa/V. The analogue signals were routed through DG408 analogue signal multiplexers, and digitized using a Data Translation DT9836 data acquisition system. In all cases, a total of \( 10^4 \) samples were collected over 20 s, in order to ensure statistical convergence of the mean pressures. The pressure transducers were calibrated simultaneously against a micromanometer, by exposing the probe tip to quiescent air at controlled pressures. Transducer calibration was carried out before and after each experiment, and data were rejected if the calibration coefficients varied by more than 1%. The experimental setup and wind tunnel coordinate system are illustrated in Figure 1.

Two probes were constructed and tested. The first was a conventional miniature seven-hole probe, having a diameter of \( 2.5\pm0.04 \) mm and an apex angle of \( 30^\circ \). The probe tip was precision-machined from solid brass; the holes were drilled to a nominal diameter of 0.5 mm, with a centre-to-centre spacing of \( 1.0\pm0.06 \) mm. The second probe had nineteen holes, with seven central holes.
in a closed-packed arrangement, surrounded by twelve peripheral holes arranged axisymetrically. The probe was constructed by assembling and soldering together lengths of 21-gauge stainless steel tubing, resulting in holes $0.51 \pm 0.04$ mm in diameter, with a centre-to-centre spacing of $0.81 \pm 0.04$ mm. The probe outer diameter was $4 \pm 0.08$ mm, and the probe tip was precision-machined after assembly to a hemispherical profile having a radius $R = 3.0 \pm 0.2$ mm. The configuration and hole index conventions for the seven- and nineteen-hole probes are shown in Figure 2.

Figure 2. Schematic of probe geometries, including hole numbering conventions used and photographs of probe tips. (a) Conventional seven-hole probe; (b) nineteen-hole probe.

The probes were used to collect wake survey data behind a finite wing model, as wing tip vortices offer a velocity field which is strongly three-dimensional, highly vortical and easily validated as tip vortices tend to closely approximate a Batchelor vortex \[19, 20\]. The wing used had a uniform NACA 0012 profile with no taper or twist, and was fitted with a matching NACA 0012 body-of-revolution end-cap to minimize the generation of secondary vortices (see \[21\]). The wing had a chord $c = 157$ mm and an aspect ratio of 2.5, resulting in a chord Reynolds number $Re_c = U_\infty c/\nu \sim 1.05 \times 10^5$ (where $\nu$ is the kinematic viscosity). The wing was set at an angle of attack of ranging from $5^\circ$ to $12^\circ$ relative to the tunnel axis, and in all cases the wake surveys were collected at $x/c = 5$ downstream of the trailing edge, with the probe axis aligned with the free-stream flow.

III. Calibration algorithms

A. Conventional seven-hole probe calibration technique

As discussed above, there exist a number of different conventional techniques for the calibration of seven hole probes, and the definitions of the nondimensional coefficients will vary. Here, the
sectorized normalization technique of Wenger & Devenport [12] is adopted.

The seven-hole probe calibration process requires the assumption that the flow remains attached only in the immediate vicinity of the hole registering the maximum pressure. For small flow angles, then, the central hole will register the largest pressure. In this case, the pressures are converted into nondimensional coefficients as

\[
C_\alpha = \frac{P_4 - P_1}{P_7 - \overline{P}}, \\
C_\beta = \frac{P_5 + P_6 - P_2 - P_3}{2(P_7 - \overline{P})},
\]  

(1)

where \(C_\alpha\) and \(C_\beta\) are coefficients sensitive to the pitch yaw angle, respectively; \(\overline{P}\) is defined here as

\[
\overline{P} = \frac{1}{6} \sum_{k=1}^{6} P_k,
\]  

(2)

and the subscripts indicate the hole indices, defined as shown in Figure 2. In order to obtain local measurements of the velocity magnitude, approximations of the local static and stagnation pressures are also required. The stagnation pressure is approximated as the pressure at the central hole, and the static pressure is approximated as the mean pressure at the six peripheral holes; the difference \(P_7 - \overline{P}\) therefore approximates the local dynamic pressure. The stagnation pressure coefficient \(C_0\) and static pressure coefficient \(C_s\) are then defined as

\[
C_0 = \frac{P_7 - P_0}{P_7 - \overline{P}}, \\
C_s = \frac{\overline{P} - P_s}{P_7 - \overline{P}},
\]  

(3)

where \(P_0\) and \(P_s\) are the true stagnation and static pressures measured in the free-stream flow (generally by an independent reference probe).

For flows of large angularity, the maximum pressure will be recorded at some hole \(i\) such that \(i \neq 7\), and it may be assumed that the flow is attached only in the immediate vicinity of hole \(i\). In this case, it becomes more convenient to express the flow angles in spherical coordinates; the different flow angles and velocity components are illustrated in Figure 3 for clarity. Then,

\[
C_{\theta i} = \frac{P_i - P_7}{P_i - \overline{P}} \quad i = 1, 2, ..., 6, \\
C_{\phi i} = \frac{P_{cw} - P_{ccw}}{P_i - \overline{P}},
\]  

(4)

where \(C_{\theta i}\) and \(C_{\phi i}\) are sets of coefficients sensitive to \(\theta\) and \(\phi\), respectively; \(P_{cw}\) and \(P_{ccw}\) are the pressures recorded at the holes located adjacent to the \(i\)th hole in the clockwise and counter-
clockwise directions, respectively, and \( \overline{P} \) (the approximation of the static pressure) must be redefined as \( \overline{P} = \left( P_{cw} + P_{ccw} \right)/2 \). The static and stagnation pressure coefficients may then be defined as

\[
C_0 = \frac{P_i - P_0}{P_i - \overline{P}}, \quad C_s = \frac{\overline{P} - P_s}{P_i - \overline{P}},
\]

where it has been recognized that for large flow angles, \( P_i \) (as the maximum pressure recorded on the probe tip) provides the best approximation of \( P_0 \). The functional dependence of the coefficients \( C_\alpha \) and \( C_\beta \) (or, equivalently, \( C_{\theta_i} \) and \( C_{\phi_i} \)), \( C_0 \) and \( C_s \) upon \( \alpha \) and \( \beta \) (or \( \theta \) and \( \phi \)) may then be determined by calibration. Seven sets of discrete (but presumably piecewise-continuous) calibration functions will result; the appropriate calibration functions are selected depending on which hole \( i \) registers the highest pressure.

![Graphical representation of velocities in pitch/yaw and spherical coordinate systems.](image)

When subjected to an unknown velocity, the hole registering the maximum pressure is identified and the appropriate coefficients are evaluated from either (1) or (4). The flow angularity is determined from the corresponding calibration function, together with the corresponding values of \( C_0 \) and \( C_s \). The individual velocity components may then be resolved, as

\[
U = |V| \cos(\alpha) \cos(\beta) = |V| \cos(\theta),
\]
\[
V = |V| \sin(\alpha) = |V| \sin(\theta) \sin(\phi),
\]
\[
W = |V| \cos(\alpha) \sin(\beta) = |V| \sin(\theta) \cos(\phi).
\]

The velocity magnitude \( |V| \) is obtained from (3) or (5) and the Bernoulli equation, as

\[
|V| = \left( \frac{2\Delta P}{\rho} (C_s - C_0 + 1) \right)^{1/2},
\]
where \( \Delta P \) is the difference between the approximations of the stagnation and total pressures (in this case, \( \Delta P = P_t - P \)), and \( \rho \) is the fluid density [10].

**B. Generalized, \( n \)-hole probe calibration algorithm**

Consider now a probe with a tip of arbitrary geometry, having \( n \) holes. As was the case for the conventional seven-hole probe, the local stagnation pressure may be approximated as the maximum pressure \( P_{\text{max}} \) recorded from the \( n \) holes. However, because the tip geometry and hole arrangement is arbitrary, no combination of holes can be identified \textit{a priori} from which to obtain an average measure of the local static pressure. Instead, the closest available measure of static pressure is the minimum pressure \( P_{\text{min}} \) recorded from the \( n \) holes.

Using \( P_{\text{min}} \) and \( P_{\text{max}} \) as defined above, pressure coefficients may then be defined, as

\[
\begin{align*}
C_j &= \frac{P_{\text{max}} - P_j}{P_{\text{max}} - P_{\text{min}}} \quad j = 1, 2, \ldots, n \\
C_0 &= \frac{P_{\text{max}} - P_0}{P_{\text{max}} - P_{\text{min}}} \\
C_s &= \frac{P_{\text{min}} - P_s}{P_{\text{max}} - P_{\text{min}}} 
\end{align*}
\]

where \( P_j \) is the pressure recorded at the \( j \)th hole, and \( P_0 \) and \( P_s \) are again the reference total and static pressures, respectively. Note that \( C_j = 0 \) identically for the hole registering the largest pressure. These definitions are based upon the same reasoning used to obtain (1) and (3): that the error in the approximations of local stagnation and static pressure will become velocity-independent when normalized against the approximation of local dynamic pressure.

The pressure coefficients defined above have the advantages of being continuous throughout the range of calibration, and of being independent of the hole arrangement and the probe tip geometry. However, they are consequently more susceptible to error arising from flow separation (and therefore loss of accuracy in flows of high angularity); furthermore, if either \( P_{\text{min}} \) or \( P_{\text{max}} \) is recorded in a region of separated flow, this approach will necessarily fail.

With the pressure coefficients defined as shown in (8), a set of calibration data may be collected by recording the values of these coefficients with the probe oriented at a range of angles in \( \alpha \) and \( \beta \) (or, equally, \( \theta \) and \( \phi \)) in constant, uniform flow at a single velocity. Assuming that all of the coefficients \( C_j \) are mutually independent, then \( \alpha, \beta, P_0 \) and \( P_s \) will each be continuously and uniquely defined within the \( n \)-dimensional parameter space, so that

\[
\begin{align*}
\alpha &= f_\alpha(C_1, C_2, \ldots, C_n) \\
\beta &= f_\beta(C_1, C_2, \ldots, C_n) \\
C_0 &= f_0(C_1, C_2, \ldots, C_n) \\
C_s &= f_s(C_1, C_2, \ldots, C_n) 
\end{align*}
\]
where \( f_\alpha, f_\beta, f_0 \) and \( f_s \) are empirical functions defined by the calibration data. If the probe is then subjected to an unknown flow, the coefficients \((C_1, C_2, ..., C_n)\) obtained in that flow will describe a unique location within the \( n \)-dimensional hypercube. The flow angularity, \( C_0 \) and \( C_s \) (and thereby \(|V|\)) may then be obtained by evaluating the functions \( f_\alpha, f_\beta, f_0 \) and \( f_s \) at that point. This may be accomplished in the same way as is done for five- and seven-hole probes, using either look-up tables or curve fitting. The Cartesian velocity components may then be resolved in the same way as in the conventional seven-hole probe calibration procedure using (6) with \( \Delta P = P_{\text{max}} - P_{\text{min}} \).

Alternatively, it is possible to approximate \( f_\alpha, f_\beta, f_0 \) and \( f_s \) as continuous functions over the entire domain by fitting to polynomials of order \( k \) having \( n \) variables. However, previous work [22] has suggested that a polynomial of at least \( k = 6 \) is required. Then, if \( n = 19 \) (for example), this results in 177,100 terms, and the inversion of the calibration polynomial matrix becomes computationally intractable.

For the purposes of this work, the coefficients were in all cases obtained from the calibration data using third-order interpolation (see, for example, [8, 9]). Formally, then, \( f_\alpha, f_\beta, f_0 \) and \( f_s \) were approximated as piecewise bicubics.

C. Extension of generalized calibration scheme to high-speed flows

The generalized \( n \)-hole probe calibration scheme described above requires that the fluid density remain constant; consequently, it is necessarily limited to flows of low Mach numbers. However, when multi-hole probes are used in high-speed flows, the directionality of the flow is obtained in much the same way as it is in low-speed flows.

Conventionally, the nondimensional coefficients \( C_\alpha \) and \( C_\beta \) (or \( C_\theta \) and \( C_\phi \)) are defined using the same pressure differences as in (1), except that the pressures are normalized against the upstream dynamic pressure (which needs to be determined separately, and may require iteration) [23]. Because the generalized calibration scheme described above operates on nondimensional coefficients sensitive only to flow angularity, it may equally be used to resolve the directionality of high-speed flows using an \( n \)-hole probe with an appropriate tip geometry.

D. Analytically-derived calibration function for the nineteen-hole probe

For the particular case of a probe with a hemispherical tip, the probe geometry is such that analytical relationships between the hole pressure and local flow velocity may be obtained [14, 24, 25]; in this way, the probe may be used without requiring empirical calibration. In all cases, however, the analytical calibration of probes requires some idealization of the probe tip geometry. Because the probe performance tends to be highly sensitive to the tip geometry, the small imperfections which are unavoidable in the manufacture of any probe contribute significantly to measurement error and generally preclude the use of analytically-derived calibration functions (especially at higher Reynolds numbers). On the other hand, for the case of probes having a large number of holes, the
impact of error arising from imperfections affecting only some small number of the holes will be reduced as a consequence of the high level of data redundancy. The use of analytically-derived calibration functions for the nineteen-hole probe was therefore investigated.

The flow around the probe tip is assumed to approximate potential flow around a sphere, so that the local surface pressure (normalized by the far-field dynamic pressure) varies linearly with the square of the cosine of the relative flow cone angle, such that

$$\frac{2}{\rho |V|^2} (P - P_s) = \frac{9}{4} \cos^2(\theta') - \frac{5}{4},$$  \hspace{1cm} (10)

where $P$ is the surface pressure at some point $p$ on the sphere, and $\theta'$ is the angle subtended between the incident velocity vector and the position vector of $p$ (relative to an origin at the centre of the sphere). If the cone and roll angles describing the position vector of $p$ on the surface of the probe tip are $\theta_p$ and $\phi_p$, respectively, then

$$\cos(\theta') = \sin(\theta) \cos(\phi) \sin(\theta_p) \cos(\phi_p) + \sin(\theta) \sin(\phi) \sin(\theta_p) \sin(\phi_p) + \cos(\theta) \cos(\theta_p).$$  \hspace{1cm} (11)

Substituting (11) into (10) will then yield a single equation relating the pressure at $p$ to the magnitude and direction of the velocity incident upon the sphere. Given a hemispherical-tip probe having $n$ pressure ports, the pressures $P_1, P_2, ..., P_n$ are known; equally, since the probe tip geometry is fixed, the locations of each hole $\theta_p = \theta_1, \theta_2, ..., \theta_n$ and $\phi_p = \phi_1, \phi_2, ..., \phi_n$ are also known. Then, (10) yields a system of $n$ equations in $\theta$, $\phi$ and $|V|$. If $n = 3$, the system may be solved exactly; however, for cases of $n \geq 4$, more robust estimates of $\theta$, $\phi$ and $|V|$ may be obtained by treating the system as an unconstrained optimization problem. In this case, (10) may be alternatively expressed as a set of $n$ equations,

$$\frac{2}{\rho |V|^2} (P_j - P_s) - \frac{9}{4} \cos^2(\theta'_j) + \frac{5}{4} = \epsilon_j \hspace{1cm} j = 1, 2, ..., n,$$  \hspace{1cm} (12)

where $\epsilon_j$ is a measure of the error at each hole. The total error $\epsilon_0$ may be defined such that

$$\epsilon^2_0 = \sum_{j=1}^{n} \epsilon^2_j.$$  \hspace{1cm} (13)

The system of equations given by (12) may then be solved, subject to the minimization of (13). For the purposes of the present work, a generic search function is used to determine $\theta$, $\phi$ and $|V|$ to within a resolution of at least 0.1%.

Because this data reduction procedure is sensitive to the probe tip geometry, and because the
probe tip geometry is likely to be subject to some manufacturing errors, the sensitivity of the probe response to errors in hole position has been assessed for the case of the 19-hole probe. A synthetic data set \( P_1, P_2, ..., P_{19} \) was generated using (10), assuming a uniform flow field having \( U = V = 0 \). A random error of up to \( \delta \) in hole position (including an error in local \( R \)) was then applied to the known hole locations, and the resultant velocity components were obtained by minimizing (13) and applying (6). Any measured cross-flow velocity magnitude \( V_{xy} = (V^2 + W^2)^{1/2} \) is therefore an artifact of the data reduction process and is indicative of the resultant error. This process was repeated until the mean error achieved statistical convergence. Figure 4 shows \( V_{xy}/U_\infty \) as a function of \( \delta/R \). The error increases almost linearly with the error in tip geometry, with \( V_{xy}/U_\infty \approx 40\delta/R \). These results suggest that the analytical calibration of the nineteen-hole probe was sufficiently robust that even large tolerances in the probe tip geometry will still result in an acceptable error magnitude.

![Figure 4](image_url)

**Figure 4.** Error in cross-flow velocity magnitude as a function of error in hole position. \( \circ \), computed values of \( V_{xy}/U_\infty \); \( \cdots \cdots \), \( 40\delta/R \)

### IV. Results

#### A. Validation of generalized calibration algorithm

In order to assess the effectiveness of the generalized calibration process, a single calibration data set was collected with the seven-hole probe, and the probe was then used to carry out a wake survey behind the wing model set at an angle of attack of \( 12^\circ \). Trailing vortex flows are fundamentally three-dimensional, and are characterized by both angularity and shear. These flows therefore provide a good test-case for the assessment of velocity probes.

Wake scan data were processed using both the conventional, sector-based seven-hole probe algorithm (1) - (5) and the new generalized algorithm (9). The normalized streamwise vorti-
ity $\zeta c/U_\infty$ was computed from the cross-flow velocity field using local bicubic fitting, and the resulting isovorticity contours are plotted in Figure 5. The maximum self-normalized vorticity $\zeta r_c/v_0$ (where $r_c$ is the core radius and $v_0$ is the peak tangential velocity) was 2.626 and 2.484 for the conventional and generalized calibration techniques, respectively. However, the conventional, sector-based calibration technique resolved a secondary structure which was not apparent when the generalized calibration technique was used (Figure 5 a).

![Figure 5](image)

**Figure 5.** Contours of normalized vorticity $\zeta c/U_\infty$ from seven-hole probe measurements behind the wing at $12^\circ$ incidence, using (a) conventional sector-based calibration technique, and (b) generalized calibration technique.

Since secondary structures are not typically expected to persist in wing wake surveys as far downstream as $x/c = 5$, the existence of a pronounced secondary vortex in the wake was investigated further. Figure 6 (a) shows isocontours of the pressure coefficient $C_P = 2P_i/\rho U_\infty^2$ from the central hole of the seven-hole probe. The contours are skewed toward the positive-$z$ axis, suggesting either a manufacturing defect in the probe tip or an initial misalignment between the probe axis and the tunnel axis. However, there are no localized disturbances in the pressure fields at the location of the secondary structure. The pressure fields from the six peripheral holes (not shown) likewise do not demonstrate any localized irregularities. Since concentrations of vorticity are normally associated with local pressure defects, these results appear to be contradictory.

Figure 6 (b) shows the isovorticity contours obtained using the conventional seven-hole probe calibration algorithm (as in Figure 5 b) together with the spatial regions in which the discrete calibration function for each hole $i$ was used. From this plot, it is apparent that the secondary structure occurs directly upon the interface of two calibration sectors. Since there is no evidence of the existence of a secondary structure in the direct pressure measurements, it may be concluded that the secondary structure was an artifact of the conventional calibration scheme. Since secondary structures within regions of high vorticity can be common in wake surveys [21], the use of discrete calibration functions may yield misleading results.
B. Validation of nineteen-hole probe using generalized calibration

Because probes with hemispherical tip geometries have characteristically low ranges of sensitivity, the response of the nineteen-hole probe to flows of high angularity was assessed directly and compared to that of the conical seven-hole probe. The probes were first calibrated, and then positioned at a series of prescribed angles \((\alpha, \beta)\) in steady flow at constant \(U_\infty\). The flow angles returned by the probes (using the generalized calibration and data reduction scheme) were then compared to the prescribed angles. The nineteen-hole probe was accurate to within a mean error of 0.5\(^\circ\) over the range \(-60^\circ \leq \alpha \leq 60^\circ\) and \(-60^\circ \leq \beta \leq 60^\circ\), compared to a mean error of 1.2\(^\circ\) for the seven-hole probe (Figure 7). The nineteen-hole probe also demonstrated a much higher level of accuracy at large angularity. Note that the calibration remained monotonic within this range of flow angles, and so did not appear to be affected by any flow separation on the probe tip.

The relative accuracy of the probes is quantitatively demonstrated in Figure 8 (a), which shows the mean error in flow angularity \(\Delta(\alpha, \beta)\) as a function of the prescribed flow cone angle \(\theta_0\), where

\[
\Delta(\alpha, \beta) = (\alpha - \alpha_0)^2 + (\beta - \beta_0)^2 \right)^{1/2}
\]

and \(\alpha_0\) and \(\beta_0\) are the prescribed pitch and yaw angles, respectively. The plot shows results from the seven-hole and nineteen-hole probes, both using the generalized calibration scheme within an angular range of \(-60^\circ \leq \alpha \leq 60^\circ\) and \(-60^\circ \leq \beta \leq 60^\circ\). Note that results are also shown for the analytically-calibrated nineteen-hole probe, though over a reduced range of \(-30^\circ \leq \alpha \leq 30^\circ\) and \(-30^\circ \leq \beta \leq 30^\circ\). Throughout the range of angles, the calibrated nineteen-hole probe provides an error of less than 0.75\(^\circ\), and provides typical improvement in accuracy of \(\sim 0.25^\circ\) over the seven hole probe. Figure 8 (b) shows the probability distributions of \(\Delta(\alpha, \beta)\) for the same data. The
calibrated nineteen-hole probe has both a narrower distribution and a substantially reduced tail relative to the seven-hole probe.

Both the seven-hole probe and the nineteen-hole probe were then used to obtain wake survey data behind the wing, set at a 5° angle of attack. The cross-flow velocity vectors, streamwise vorticity fields and streamwise velocity fields obtained with the two probe systems are compared in Figure 9. As expected, the results are almost indistinguishable. The nineteen hole probe does, however, appear to have slightly better resolved the velocity and vorticity at the centre of the vortex, likely as a consequence of its higher sensitivity to flow angularity.

The tip vortex formed downstream of a finite wing is expected to agree well with the ax-
Figure 9. (a) Cross-flow velocity vector fields, and contours of (b) $\zeta_c/U_\infty$ and (c) $U/U_\infty$ for the seven-hole probe (left) and nineteen hole probe (right).

isymmetric Batchelor [19] profile, through a wide range of experimental parameters [20]. Radial profiles of self-scaled circulation $\Gamma(r)/\Gamma_c$ (where $r$ is the radial coordinate relative to the vortex centre, and $\Gamma_c$ is the circulation at $r = r_c$) were computed from the vorticity fields measured with
both probe systems, and the results were compared to the self-similar Batchelor solution,

\[
\frac{\Gamma(r)}{\Gamma_c} = \frac{1 - \exp\left(-ar^2/r_c^2\right)}{1 - \exp(-a)},
\]

where \(a \approx 1.25643\) is Lamb’s constant. The circulation profiles obtained with both probe systems agree very well with (15) for \(0 \lesssim r/r_c \lesssim 1.5\).

![Figure 10. Core-normalized radial circulation profiles. – – –, Seven-hole probe; - - -, nineteen-hole probe; ——, (15).]

C. Data redundancy and robustness of generalized calibration scheme

In order for a pressure-based velocity probe to adequately resolve the velocity components in three-dimensional flow, at least four mutually independent pressure signals from the probe tip are required. For probes having \(n > 4\), then, a generalized calibration scheme (which is independent of the probe tip geometry) would enable the probe to function should one or more of the pressure signals be deemed unusable in post-processing.

To test the robustness of the calibration scheme described by (9), the pressure signals collected by the nineteen-hole probe during the wake survey shown in Figure 9 were re-processed using only data from some number \(k\) of randomly-selected holes (where \(k = 4, 5, ..., n - 1\)). A cross-flow velocity error field \(\epsilon(k)\) was defined, as

\[
\epsilon(k) = \left| \frac{(V_k^2 + W_k^2)^{1/2} - (V_n^2 + W_n^2)^{1/2}}{(V_n^2 + W_n^2)^{1/2}} \right|
\]

(16)

(where the subscripts \(k\) and \(n\) indicate the number of holes used to obtain the corresponding estimates of \(V\) and \(W\)). This estimate of error has the advantage of being a scalar quantity sensitive
to differences in both the direction and magnitude of the velocity vector. The mean error $\bar{\epsilon}$ was then computed as a spatial average over the cross-flow field (which had a maximum flow angularity of $\pm \sim 25^\circ$). This process was repeated, eliminating different randomly-selected holes, until $\bar{\epsilon}$ achieved statistical convergence. The variation of $\bar{\epsilon}$ with $k$ is plotted in Figure 11, which also shows the extrema obtained for individual combinations of holes removed. For $k > 12$, the error was always less than $\sim 1\%$. However, for $k \leq 8$, the mean error in the cross-flow velocity fields remained within $\sim 1\%$, while the maximum error was within $\sim 3\%$. Measurements of the velocity components are therefore possible using the nineteen-hole probe and the current calibration technique with as many as any eleven of the individual sensors inoperative.

![Figure 11. Variation in the mean cross-flow velocity error parameter $\bar{\epsilon}$. Error bars indicate range of values obtained.](image)

D. Assessment of the analytical calibration scheme with the nineteen-hole probe

In order to assess the validity of the analytical calibration scheme described in Section D, data was collected with the probe positioned at a range of prescribed angles $(\alpha, \beta)$ in a uniform free-stream flow. Although this technique derives from the assumption of inviscid flow and therefore low Reynolds numbers $Re_D = U_\infty D/\nu$ (where $D$ is the diameter of the probe tip), Pisasale & Ahmed [25] show that flows of angularity of less than $60^\circ$ may be resolved for $Re_D \lesssim 1600$. In the present work, $Re_D \sim 3300$, so care was taken in the validation and assessment of the range of sensitivity.

The response of the probe is plotted in Figure 12, which shows the prescribed pitch and yaw angles, together with the corresponding pitch and yaw angles obtained from the data reduction algorithm. For angles within $\pm \sim 15^\circ$, the error in flow angularity is within the range of measurement uncertainty. For flow angles up to $\pm \sim 30^\circ$, the error in pitch and yaw increases to as much as $2.5^\circ$. The error distributions within this range of flow angles are also shown quantitatively in
Figure 8, together with the calibrated seven-hole and nineteen-hole probe results for comparison. Surprisingly, at flow angles of $\theta < 10^\circ$, the analytically calibrated probe was more accurate than the experimentally calibrated one, though the mean error increases rapidly with increasing $\theta$ above $10^\circ$, and the distribution of error is broad.

![Figure 12. Demonstration of the angular range of the analytical calibration. $\circ$, prescribed angle; $\bullet$, measured angle.](image)

The nineteen-hole probe may therefore be expected to provide good accuracy, providing that measurements are made in flow fields having small angularity ($\lesssim \pm 15^\circ$ in pitch and yaw). Since the calibrated post-processing of the wake survey data from the wing at $5^\circ$ incidence (see Figure 9) showed regions with flow angularities both within and outside of this range, these data were used to assess the use of the analytically-calibrated nineteen-hole probe in a vortical flow field.

Figure 13 shows contours of $\zeta c/U_\infty$ and $U/U_\infty$ for the analytically-calibrated nineteen-hole probe; these are directly comparable to the data shown in Figures 9 (b) and (c). These results are almost indistinguishable from the results obtained using the calibrated probes; the contours are nearly circular, and the maximum and minimum values are within the range of experimental uncertainty.

E. Direct measurement of local velocity gradients using generalized calibration

Intrusive probes are occasionally used for the direct measurement of local velocity gradients, either using multiple hot-wire elements [26] or pressure taps [27]. Typically, these probes provide independent measures of velocity at several locations in space, separated by distances with length-scales of the order of those of the probe measurement volume. By assuming that the velocity is constant within the probe volume (which is equivalent to the assumption that $(R/U_\infty)dV_i/dx_j$ is
negligible), mean velocity gradients within the volume may be obtained. While estimates of local velocity gradients may always be obtained from wake survey data by computing the gradients of the velocity fields, these estimates will be subject to increased error owing to the sensitivity of the gradients to small errors in the measurement locations. Also, computing spatial gradients from a wake survey grid requires the assumption that $(\Delta x/U_\infty) dV_i/dx_j$ is negligible (where $\Delta x$ is the spatial resolution of the measurement grid). Consequently, for flows with high, local concentrations of vorticity (such as wing wakes), local measurements of the gradients are preferable.

Because the nineteen-hole probe is able to obtain velocity measurements always accurate to within $\sim 2\%$ with as many as ten arbitrarily selected pressure signals discarded (for flows of angularity of at least $\pm 25^\circ$; see Figure 11), it is possible to obtain multiple, independent local measures of velocity by separately processing data from subsets of the nineteen holes. As an extension, if the holes in the probe head are assigned to four overlapping quadrants (as shown in Figure 14), quasi-independent measurements of the velocity components will be available at the approximate spatial locations $(x, y \pm R/4, z \pm R/4)$, where $(x, y, z)$ is the nominal measurement point. Since both $V$ and $W$ will be independently available from two different known locations in $y$ and two different known locations in $z$ within the same cross-flow plane, it is possible to obtain local estimates of the cross-flow velocity gradients.

Figure 15 shows isocontours of the normalized velocity gradients $(c/U_\infty) dV/dy$ and $(c/U_\infty) dW/dz$ obtained from the single-point nineteen-hole probe measurements (left) and from conventional field estimates (right); these are the same data as presented in Figure 9. Significant differences between the two gradient estimates are observed. The local measurements have a vanishing value near the origin, and lobes of positive and negative values in each of the four quadrants (though the estimate of $dV/dy$ was corrupted by some bad vectors in the $z > 0$, $y < 0$
quadrant), while the field estimates have a local maximum near the origin.

These results may be compared to the gradients of an axisymmetric Batchelor vortex,

\[
\frac{dW}{dz} = -\frac{dV}{dy} = \frac{2V_0}{r_c^3} \left( 1 + \frac{1}{2a} \right) \frac{yz}{\eta^4} \left[ 1 - \frac{\eta^2}{\eta^2 + 1} \exp(-a\eta^2) \right],
\]

(17)

(where \( \eta = r/r_c \)) which has extreme values of \( dW/dz = \pm 0.5242V_0/r_c \) at \( z = \pm y = 0.8448r_c \), and vanishes along the \( y \) and \( z \) axes. For the data shown, (17) predicts local extrema of \( (c/U_\infty)dW/dz = \pm 5.29 \) at \( y/c = \pm z/c \sim 0.028 \). The large, nonzero values of the gradients obtained at the vortex centre by field estimates is therefore likely to be an artifact of the poor spatial resolution of the scan relative to the scale of the vortex core (for the data shown in Figure 15, \( r_c/\Delta x \sim 3 \)). The peak magnitudes of the gradients and the spatial locations of these peaks were similar for both the local measurements and the field estimates; these also agreed with those predicted by (17).

The velocity gradients \( dW/dy \) and \( dV/dz \) could not be obtained reliably from the test-case velocity field using this technique. The distribution of the gradients obtained were subject to a high degree of noise and distortion. This poor agreement is likely due to the magnitude of the gradient. For the case of a Batchelor vortex,

\[
\frac{dW}{dy} = \frac{2V_0}{r_c^3} \left( 1 + \frac{1}{2a} \right) \frac{y^2}{\eta^4} \left[ 1 - (\eta^2 + 1) \exp(-a\eta^2) \right] - \frac{r_c^2}{2y^2} \eta^2 \left[ 1 - \exp(-a\eta^2) \right],
\]

(18)

which has an extreme value of \( dW/dy = 1.7564V_0/r_c \) at the origin (note that \( dV/dz = -dW/dy \) when subjected to a 90° rotation). The peak absolute magnitude of \( (c/U_\infty)dW/dy \) expected was therefore \( \sim 18 \), corresponding to \( (R/U_\infty)dW/dy \sim 0.37 \), which is not negligible. This large gradient is likely to have resulted in significant error due to probe interference effects [28, 29], especially since the sampling holes have been clustered together (rather than being randomly distributed). However, the results presented in Figure 11 suggest that a probe of this design may be
V. Conclusions

The use of a miniature, nineteen-hole pressure probe and a generalized calibration algorithm in low-$Re$ wing wake surveys is demonstrated. The calibration algorithm is particularly useful, since it is independent of the probe geometry and the number of active pressure taps, and therefore tolerant of data corruption and imperfections in probe manufacture.

The nineteen-hole probe was tested in the vortex wake behind a wing, as this flow offers a well-characterized and strongly three-dimensional velocity field with high angularity and shear. The nineteen-hole probe was able to accurately return the three components of velocity in the vortex wake, and yielded vorticity fields which were more closely axisymmetric than those obtained with a conventional seven-hole probe. The large number and high concentration of holes in the nineteen-hole probe, together with an $n$-dimensional calibration function, results in velocity mea-
surements which are less susceptible to error resulting from high velocity gradients or calibration data interpolation.

The large number of holes also allows the more accurate use of the probe with an analytical calibration function for flows with angularity of less than \( \sim 15^\circ \), though this process necessarily requires a nominally hemispherical probe tip geometry. The sensitivity to error in probe tip geometry has been quantified, demonstrating that a mean error of as much as \( 0.1\Delta R \) in hole position will result in a measurement error of only \( \sim 3\% \).

Quasi-independent velocity estimates were obtained from different subsets of holes in the nineteen-hole probe tip, in order to obtain local estimates of the cross-flow velocity gradients in a vortex wake. The diagonal components of the gradient tensor were accurately reproduced, and agreed well with the distribution characteristic of axisymmetric vortex flows. By comparison, finite-difference field estimates of the vorticity exhibited a high degree of error near the vortex centre, owing to the high vorticity and low spatial resolution of the wake scan. The off-diagonal components of the gradient tensor could not be obtained using the nineteen-hole probe, as the error sensitivity was too high in the vortex flow field.

Acknowledgments

This work was supported in part by the UK Engineering and Physical Sciences Research Council under grant number EP/H030360/1. The authors are very grateful to Dr. Paul Nathan for his assistance in the fabrication and integration of the instrumentation systems used.

References


Appendix H

Journal of Fluid Mechanics paper, submitted August 2015 and currently in review
Is there really such a thing as a turbulent vortex?

SAMANTHA SHAW-WARD AND DAVID M. BIRCH
Department of Mechanical Engineering Sciences, University of Surrey, Guildford, Surrey GU2 7XH UK

(Received 18 August 2015)

It is difficult to ascertain from experimental data whether a typical trailing vortex is laminar or turbulent. Because the laminar and turbulent mean velocity profiles are nearly identical, determining the state of the vortex necessarily requires the consideration of higher-order velocity statistics. However, the effect of random vortex ‘wandering’ on the higher-order moments can dominate. A set of criteria are developed which indicate if the second-order velocity moments measured at fixed points within a well-developed axisymmetric vortex are mostly free of the effects of wandering. By re-examining a legacy data set in which these criteria happened to be met, evidence emerges that the vortex played a passive role in the transport of turbulence: that the vortex was effectively a superposition of freely-decaying turbulence upon a laminar vortex field. Experiments carried out using roughened vortex generators also suggested that fixed-point measurements of velocity moments could be corrected for wandering in post-processing using an adaptive filtering technique requiring no a priori assumptions about the velocity profile. Once corrected, the non-wandering criteria are satisfied and the vortex again exhibits the characteristics of freely-decaying turbulence superimposed upon a laminar velocity field. This result raises an important question about the nature of stable trailing vortex flows, and under what circumstances it would be formally correct to describe such flows as being turbulent.

Key Words: turbulent flows, vortex dynamics, vortex flows

1. Introduction

Despite their importance in both fundamental and engineering applications, vortices continue to attract relatively little attention compared to other canonical shear flows (such as boundary layers and shear layers). Like boundary layers, vortices are characterized by a laminar inner region dominated by viscosity (the inner core) surrounded by, first, an ‘outer core’ region in which the flow is dominated by momentum effects, and finally an outer ‘development’ region in which the flow is dominated by the initial conditions (in practice, the spiral wake of the wing or vortex generator). Indeed, Bradshaw (1969) demonstrated that a vortex is exactly analogous to a stratified boundary boundary layer, such that the centripetal terms replace the buoyancy terms in the momentum equation.

Like stratified boundary layers, vortices are understood to be highly stabilizing as the near solid-body rotation in the inner core causes any turbulence there to rapidly decay. Unlike boundary layers, though, there is no equivalent of wall shear or the near-wall cycle in vortex flows: there are no external forces applied along the axis, and no mechanism to drive turbulent production there. This ‘inactive core’ model is supported by a number of published results; see, for example, Devenport et al. (1996), Chow et al. (1997) and
Pathways do exist to transfer kinetic energy from the bulk rotation to smaller scales, although these are usually limited to the extreme cases of vortices with very large axial velocities (which may be subject to absolute instability; see Lessen & Paillet 1974; Jacquin & Pantano 2002) or very old vortices, which are subject to long-wave Crow (1970) instabilities. The robust self-similarity and persistence of trailing vortices is also consistent with predominantly viscous energy transport.

There are, however, other studies which have shown relatively high levels of turbulent kinetic energy near the centre of a vortex, and local time traces and velocity spectra which are consistent with turbulent flow (see, among others, Green & Acosta 1991; Sarpkaya 1992; Birch et al. 2003; Beninati & Marshall 2005; Birch & Lee 2005). These results also appear to be fairly insensitive to the magnitude and sense of the axial velocity profile (wake-like or jet-like), the Reynolds number $Re = \Gamma_0/\nu$ (where $\Gamma_0$ is the circulation and $\nu$ is the kinematic viscosity), and initial conditions.

Because of this seemingly contradictory evidence, there has been some debate about whether a vortex core is actually laminar or turbulent in the absence of absolute instability (at typical laboratory-scale $Re$), and by what mechanisms (if any) turbulence may be transported into the core. This issue is further complicated by a number of practical difficulties and limitations which cause vortex flows to be particularly resistant to investigation.

1.1. Mean velocity profiles

For the case of very low $Re$, Batchelor (1964) presented an extension of Lamb’s exact solution for the velocity profiles of a laminar vortex, yielding the tangential velocity profile

$$V_\theta(\eta) = \left( 1 + \frac{1}{2\alpha} \right) \frac{1}{\eta} \left( 1 - \exp\left( -\alpha\eta^2 \right) \right),$$

(1.1)

where $V_\theta$ is the tangential velocity (normalized against the peak tangential velocity $V_0$), $\eta$ is the radial coordinate (normalized against the core radius $r_c$, such that $V_\theta(1) = 1$), $\alpha \approx 1.25643$ is Lamb’s constant, and it has been assumed that $\partial/\partial r \gg \partial/\partial z \gg \partial/\partial \theta$.

The Batchelor solution also admits a Gaussian profile of normalized axial velocity,

$$W(\eta) = \frac{W_0}{V_0} \exp\left( -\alpha\beta\eta^2 \right),$$

(1.2)

where $\beta$ is an arbitrary scaling factor for the axial profile, $W_0$ is the dimensional peak axial velocity (which may be either jet-like or wake-like), and it should be noted that the axial velocity has been normalized against the peak tangential velocity.

Similarly, for the case of a vortex at very high $Re$, Hoffman & Joubert (1963) developed a turbulent solution based on scaling arguments alone, assuming once again that $\partial/\partial r \gg \partial/\partial z \gg \partial/\partial \theta$. This analysis resulted in a self-scaled, normalized tangential velocity profile of

$$V_\theta(\eta) = \frac{1}{\eta} \left( 1 + \log(\eta) \right);$$

(1.3)

like the log law for turbulent boundary layers, (1.3) is not valid for small $\eta$, as the effects of viscosity within this region are no longer insignificant (using the available data, the authors proposed a range of validity of $0.5 \leq \eta \leq 1.4$). The first major difficulty in determining whether a given vortex is laminar or turbulent is that, coincidentally, (1.3) agrees remarkably well with (1.1) over the range of at least $0.65 \leq \eta \leq 1.3$ (Figure 1).

An even better agreement is obtained between (1.1) and the piecewise series solution of Phillips (1981) for a turbulent vortex with a viscous core, over the range of $0 \leq \eta \leq 1.2$. 

Is there really such a thing as a turbulent vortex?

It is therefore impossible to distinguish a laminar vortex from a turbulent one given the mean velocity profiles alone: even those vortices exhibiting unusually high levels of turbulent kinetic energy, such as the near-field of wing-tip vortices formed during deep dynamic stall (Birch & Lee 2005) or in intense free-stream turbulence (Bailey & Tavoularis 2008), collapse with (1.1) to within typical experimental uncertainty (see Birch 2012).

1.2. Vortex wandering

Because the mean velocity profiles do not distinguish laminar vortices from turbulent ones, the higher order velocity statistics must necessarily be considered. However, vortices confined within a finite space (such as a wind tunnel or a bounded numerical domain) exhibit a stable, low-amplitude and apparently random modulation of trajectory in both space and time. The mechanisms behind this ‘wandering’ are not yet entirely understood, but the phenomenon has been shown to be inactive (such that it contributes only negligibly to production and dissipation) and strongly dependent upon the boundary conditions (Jammy et al. 2014). The amplitude of this wandering can be significant; instantaneous excursions of up to \( \sim 0.3r_c \) and \( \sim 2r_c \) have been recorded in the presence of very low and very high levels of background turbulence, respectively (Bailey & Tavoularis 2008). As a consequence, any velocity statistics obtained either as instantaneous spatial averages or as fixed-point temporal averages will be dominated by the effects of this passive bulk wandering.

Assuming that wandering is random and inactive, any spatial distributions of velocity statistics will be subjected to convolution with a Gaussian kernel when measured within a fixed frame of reference, and any finite distribution will appear Gaussian for sufficiently large wandering amplitude (Birch 2012). In experimental studies, single-point velocity statistics are often corrected for the effect of wandering (using, for example, the technique of Davenport et al. 1996), but the corrections are usually limited to applying some form of deconvolution to the profiles. This process is not straightforward: because (1.1) itself is based on a Gaussian profile, its form remains unchanged through the convolution. The standard deviation of the wandering, therefore, must be inferred from the statistics, and no information about the actual form of the original profiles is available. Figure 2 shows self-scaled profiles of the (uncorrected) tangential Reynolds normal stress from

![Figure 1. Comparison of vortex solutions. ——, (1.1); - - -, (1.3); – – –, Phillips (1981) solution, to fifth order.](image)
a number of different studies of vortex-like flows affected by wandering, collected under very different conditions (including peak axial velocities, Reynolds numbers and upstream turbulence levels; see table 1). The data have been arbitrarily re-scaled here to normalize against the large variations in wandering amplitudes and Reynolds stresses. The collapse of the data upon the Gaussian profile, independent of initial conditions and Re, is strongly indicative of the dominance of wandering in these flows.

Although the velocity statistics collected using two- and three-dimensional tomographic optical measurement techniques (such as particle image velocimetry) would not require correction, these techniques cannot yet be easily used for turbulence measurements owing to bandwidth limitations and flow seeding issues (Birch & Martin 2013). On the other hand, direct numerical simulation (DNS) can provide three-dimensional fields of three-component velocity data in the time domain. However, DNS requires significant computational resources, and therefore remains limited to Reynolds numbers much lower than those of typical engineering interest. Approaches which model the finer scales of turbulence (such as large-eddy simulations) may achieve higher Re, but will necessarily need to be calibrated based on experimental data which themselves are likely to have been contaminated by wandering. The savings in computational time achieved through sub-grid modelling are also limited, owing to the sensitivity of vortices to even small scale coherent structures (Melander & Hussain 1993; Goto 2008).

Finally, even at the very low Re of DNS solutions, vortex wandering has convention-
ally been considered to be a fundamental feature of the vortex development in the post-
processing of results, so no corrections are implemented. As a result, some seemingly
counter-intuitive results emerge in the velocity statistics, such as significant Reynolds
stresses at the vortex centre (Marshall & Beninati 2005; Duraisamy & Lele 2006). Again,
even at these low Re (and depending on the domain size and boundary conditions se-
lected), the wandering can cause the Reynolds stress profiles to appear artificially Gaus-
sian (Jammy et al. 2014).

Whether from experimental measurements or DNS solutions, then, great care must be
taken in interpreting the Reynolds stress distributions; like the mean velocity profiles,
distributions of these higher-order statistics might not reflect the structure of the tur-
bulence within the vortex, or provide any useful information about the mechanisms of
transport.

1.3. Reynolds stresses in a well-developed, axisymmetric vortex

Before developing any argument about the transport mechanisms within a turbulent
vortex based on the Reynolds stress distributions, it is first useful to establish a set of
necessary and sufficient conditions under which any fixed-point measurements may be
considered as representative of a true, axisymmetric vortex (and not one which is either
incompletely developed or dominated by inactive wandering).

Consider a stationary, axisymmetric vortex. As a consequence of the good agreement
between (1.1) and (1.3) over \( 0 < \eta \lesssim 1.3 \), the momentum terms and the viscous terms
in the Reynolds-averaged Navier-Stokes equations will be approximately equal in this
range, similar to the case of grid turbulence. The remaining Reynolds stress terms must
therefore vanish, yielding (without simplification)

\[
\begin{align*}
\frac{1}{\eta} \frac{\partial}{\partial \eta} \left( \eta \bar{v}_\theta^2 \right) + \frac{1}{\eta} \frac{\partial}{\partial \theta} \bar{v}_\theta \bar{v}_r - \frac{1}{\eta} \bar{v}_\theta^2 &= 0 \\
\frac{1}{\eta^2} \frac{\partial}{\partial \eta} \left( \eta^2 \bar{v}_\theta \bar{v}_r \right) + \frac{1}{\eta} \frac{\partial}{\partial \theta} \bar{v}_\theta \bar{v}_\theta &= 0 \\
\frac{1}{\eta} \frac{\partial}{\partial \eta} (\eta \bar{v}_r \bar{v}_z) &= 0,
\end{align*}
\]

where \( \zeta = z/r_c \) is the nondimensional axial coordinate; \( v_\theta, v_r \) and \( v_z \) are the tangential,
radial and axial defect velocities, respectively (all normalized by \( V_0 \)), and the overbars
indicate time-averages. For the trivial case of laminar flow (where all of the Reynolds
stresses vanish), these expressions are identically satisfied. Applying once again the as-
sumptions of Batchelor (1964) and Hoffman & Joubert (1963) (that \( \partial/\partial \eta \gg \partial/\partial \zeta \gg \partial/\partial \theta \)), (1.4), (1.5) and (1.6) reduce to

\[
\begin{align*}
\bar{v}_\theta^2 &= \frac{\partial}{\partial \eta} \left( \eta \bar{v}_\theta^2 \right) \\
\bar{v}_\theta \bar{v}_r &= \frac{C_1}{\eta^2} \\
\bar{v}_r \bar{v}_z &= \frac{C_2}{\eta},
\end{align*}
\]

where \( C_1 \) and \( C_2 \) are arbitrary constants invariant with \( \zeta \). If it is required that (1.7), (1.8)
and (1.9) hold as \( \eta \to 0 \) and that both \( \bar{v}_\theta \bar{v}_r \) and \( \bar{v}_r \bar{v}_z \) remain finite, then \( C_1 = C_2 = 0 \)
and these two shear stresses vanish. However, (1.1) was only assumed to approximate
(1.3) over a finite range of \( \eta > 0 \). Nevertheless, an exponential increase in shear stress
with decreasing \( \eta \) is inconsistent with decreasing energy toward the centre of the vortex;
furthermore, experimental and numerical results from a number of sources suggest that the Reynolds shear stresses are much smaller than the Reynolds normal stresses in a turbulent vortex (Beninati & Marshall 2005; Duraisamy & Lele 2006; Jammy et al. 2014). It may therefore be argued that \( C_1 \) and \( C_2 \) are vanishingly small, and so \( \overline{v_r v_r} \approx \overline{v_r v_\theta} \approx \overline{v_r v_z} \approx 0 \). Note that no inferences may yet be made about \( \overline{v_\theta v_\theta} \).

Finally, consider the case of a stable, well-developed stationary turbulent vortex, such that sufficient time (or, equivalently, \( \zeta \) for the case of a trailing vortex) has elapsed for the vortex to become entirely independent of the initial conditions. As with grid turbulence, then, it should be expected that only small-scale vestiges of the initial turbulence will remain, and that this turbulence will become increasingly isotropic as the integral scale separation increases. To some extent, because of the absence of energy input or an equivalent to wall shear to drive the turbulence cycle, the vortex can be considered well-developed and independent of initial conditions if and only if the turbulence is isotropic. Under these conditions, then, (1.8) and (1.9) necessarily require that all three Reynolds shear stresses vanish. Also, if the turbulence is isotropic, \( \overline{v^2_\theta} \approx \overline{v^2_r} \approx \overline{v^2_z} \). Then, following from (1.7),

\[
\overline{v^2_\theta} \approx \overline{v^2_r} \approx \overline{v^2_z} = C_0,
\]

where \( C_0 \) is a spatially-invariant constant dependent upon the ‘age’ of the vortex and initial turbulent kinetic energy. Furthermore,

\[
k = \frac{3}{2} C_0,\]

where \( k \) is the turbulent kinetic energy normalized against \( V_0 \). On the other hand, the vortex itself is strongly anisotropic, so if it is exerting a significant effect upon the energy cascade then (1.10) would be expected to fail. This relationship, then, represents a good indication of whether the velocity statistics are free of the effects of wandering, and if so, whether the decaying initial turbulence is simply being advected through a well-developed vortex in a passive, ‘Taylorian’ sense.

2. Turbulent kinetic energy budget

If the scales of the turbulence within a well-developed vortex are small relative to the scale of the vortex itself and the turbulence is approximately isotropic, the energy equation can provide additional insight into the transport mechanisms. Applying the Batchelor vortex assumptions (that \( \partial / \partial \eta \gg \partial / \partial \zeta \gg \partial / \partial \theta \) and that the pressure is invariant in time), the normalized turbulent kinetic energy transport equation in polar coordinates (see Moser & Moin 1985) reduces to

\[
\frac{2}{\tau} \frac{d k}{d \tau} = k_C' + k_P' + k_T' + \frac{1}{R_c} k'_\nu + \frac{1}{R_c} k'_\epsilon,
\]

where \( R_c = V_0 r_c / \nu \approx 0.7153 \) is the core Reynolds number, \( \tau = V_0 t / r_c \) is the normalized time, and \( k_C', k_P', k_T', k'_\nu \) and \( k'_\epsilon \) are the normalized convection, production, turbulent diffusion, viscous diffusion and dissipation terms, respectively, given by

\[
k_C' = -V_r \frac{\partial}{\partial \eta} \left( v_r^2 + v_\theta^2 + v_z^2 \right)\]

\[
k_P' = -2 \left( v_r \frac{\partial}{\partial \eta} v_r + \frac{1}{\eta} v_\eta v_r \right)\]

\[
k_T' = -\frac{1}{\eta} \frac{\partial}{\partial \eta} \left( v_r v_\theta + v_r v_\theta + v_r v_z \right)\]

\[
\frac{2}{\tau} \frac{d k}{d \tau} = k_C' + k_P' + k_T' + \frac{1}{R_c} k'_\nu + \frac{1}{R_c} k'_\epsilon,
\]
Is there really such a thing as a turbulent vortex?

\[ k' = 1 \frac{\partial}{\partial \eta} \left( \eta \frac{\partial}{\partial \eta} \left( \bar{v}_r^2 + \bar{v_\theta}^2 + \bar{v}_z^2 \right) \right) \]  
(2.5)

\[ k'_c = - \left( \left( \frac{\partial v_r}{\partial \eta} \right)^2 + \left( \frac{\partial v_\theta}{\partial \eta} \right)^2 + \left( \frac{\partial v_z}{\partial \eta} \right)^2 + \frac{1}{\eta^2} \left( \bar{v}_\theta^2 + \bar{v}_r^2 \right) \right), \]  
(2.6)

\[ V_r \] is the mean radial velocity, and all velocities and velocity moments have been normalized by \( V_0 \).

Considering first the convection term (2.2), the earlier analysis showed the Reynolds normal stresses to be invariant in \( \eta \) (over some range near \( \eta = 1 \)) if the turbulence is again assumed to be isotropic. Because the convective term is proportional to the radial gradients in the Reynolds normal stresses, \( k'_c \) vanishes under these assumptions. Likewise, \( k'_\nu \) vanishes identically.

Next, the production term may be simplified by substituting (1.10) into (2.3), yielding

\[ k'_P = -2\bar{v}_\theta \frac{1}{\eta} \frac{\partial}{\partial \eta} (\eta V_r). \]  
(2.7)

However, continuity requires (without simplification)

\[ \frac{\partial}{\eta} (\eta V_r) + \frac{\partial}{\eta} V_\theta + \frac{\partial V_z}{\partial \zeta} = 0. \]  
(2.8)

Substituting (2.8) into (2.7), assuming only that \( \partial/\partial \theta << \partial/\partial r \) (and making no assumption about the relative magnitude of the axial gradients),

\[ k'_P = 2\bar{v}_\theta \frac{\partial}{\partial \zeta} V_z. \]  
(2.9)

The production of turbulent kinetic energy is therefore driven by the acceleration of tangential (and radial) velocity fluctuations by vortex stretching; equally, a canonical, decaying turbulent trailing vortex will be characterized by weak, negative production. Either way, if the axial gradients are small relative to the radial ones, then \( k'_P \sim 0 \).

The turbulent transport term (2.4) varies with the radial gradients of the third-order velocity moments. Nothing further may be inferred here about these third-order moments, although it is possible to model the third-order moments in terms of the second-order ones and some calibration constants; see, for example, Weinstock (1989). However, in many turbulent flows, the turbulent transport term is negligible; experimental measurements in high-\( Re \) vortices have shown that the peak normalized triple moments are of order \( 10^{-6} \) (Bandyopadhyay et al. 1991), and two to three orders of magnitude smaller than the second-order moments. For a well-developed turbulent vortex, then, it may be assumed that \( k'_T \approx 0 \).

With the convection, production, transport and viscous diffusion terms shown either to vanish or be of negligible magnitude, the energy equation is dominated by dissipation. Combining (1.10), (1.11), (2.1) and (2.6), and again assuming that the turbulence is reasonably isotropic,

\[ -\frac{3}{2} Re \frac{dC_0}{d\tau} = \frac{1}{\eta^2} C_0 + \frac{3}{2} \left( \frac{\partial v_\theta}{\partial \eta} \right)^2. \]  
(2.10)

Because \( C_0 \) varies with \( \tau \) but not \( \eta \) (at least over the range of interest, \( 0.65 \lesssim \eta \lesssim 1.3 \)), the second term must necessarily dominate in (2.10), and must be independent of \( \eta \). This \( \eta \)-independence may also be used as a more sensitive indication than (1.10) of the suitability of the isotropy approximation. The energy equation then may then be
re-expressed as

\[ \frac{dk}{d\tau} \approx - \frac{3}{2Re} \left( \frac{\partial \theta}{\partial \eta} \right)^2, \]  

which is exactly analogous to the case of grid turbulence in a state of final decay. Therefore, the turbulent kinetic energy within a well-developed, axisymmetric vortex should be expected to decay according to the Kármán-Howarth equation,

\[ k \left( \frac{V_0}{W_\infty} \right)^2 \propto \left( \frac{\xi}{L} \right)^{-n} \]  

(2.12)

(where \( W_\infty \) here is a global Lagrangian convective velocity scale, \( \xi \) is the mean convective path length, \( L \) is a reference length scale dependent upon the initial conditions and \( n \) is some constant). In typical grid-turbulence studies, \( n \sim 1.3 \); as viscous dissipation begins to dominate in (2.1), \( n \to 5/2 \).

2.1. Consideration of available data sets

Because of the difficulties in obtaining high-confidence, high-\( Re \) turbulence statistics (independent of wandering) from an axisymmetric vortex either experimentally or numerically, it is extremely difficult to validate the above arguments. Thankfully, though, one experimental data set does exist in the literature which is almost ideally suited for examining the turbulence statistics within a vortex at reasonably high \( Re \).

Phillips & Graham (1984) produced a turbulent vortex using the two-bladed vortex generator (VG) of Poppleton (1971a), capable of controlling the mean axial velocity profile via mass flux through the hub. The vortex generator was installed in the exhaust plane of a very-low-noise, 30-inch (0.762 m) diameter blower, and operated at a Reynolds number of \( \sim 5.0 \times 10^4 \) based on the free-stream speed and the chord of the vortex generator. The vortex was bounded only by the laboratory floor and side wall, which were both more than 1.4 m (\( \sim 43r_c \)) from the jet axis. A series of low-interference perforated baffles were used to ensure that there were no mean axial pressure or velocity gradients in the undisturbed jet; the wandering of the vortex would therefore have been minimized. The upstream velocity \( W_\infty \) produced by the blower was highly uniform over the exit plane (having a maximum variation in mean axial velocity of \( \sim 0.2\% \)), and had a turbulence intensity of approximately 0.2\%. Measurements were taken at the locations \( z/b = 3, 5.2 \) and 7.3 from the trailing edge of the vortex generator (where \( b \) is the full span of the vortex generator), allowing ample time for the entrainment of the vortex generator blade wakes. Five cases were tested, with peak normalized axial velocities \( W_0/W_\infty = 1.26, 1.08, 1.00, 0.92 \) and 0.87. Note that scaling parameters and additional data from these experiments are available from Poppleton (1971b) and Vogel (1968).

The quality of the data from Phillips & Graham (1984) is demonstrated in figure 3. The self-scaled tangential velocity profiles \( V_\theta(\eta) \) show remarkably good collapse with (1.1) or (1.3) for all cases tested, over the fairly wide range \( 0 \leq \eta < 4 \) (figure 3 a). There were a small number of outlying points, though these were all from the strong jet case, and significant axial gradients (as would be introduced by a strong jet at the vortex axis) would be expected to interfere with the self-similarity, and promote instability. Additionally, since

\[ V_0 = \frac{\alpha \Gamma_0}{\pi r_c (1 + 2\alpha)} \]  

(2.13)
Is there really such a thing as a turbulent vortex?

in a Batchelor vortex, it follows from (1.1) that

$$\frac{V_0}{W_\infty} = \left( \frac{\Gamma_0}{W_\infty c} \right) K \frac{c}{r_c},$$

(2.14)

where $c$ is the vortex generator blade chord and

$$K = \frac{\alpha}{\pi(1 + 2\alpha)} \approx 0.1138.$$  

(2.15)

Consequently, for vortices of equal circulation, the normalized peak tangential velocity is linearly proportional to the reciprocal of the normalized core radius. Figure 3 (b) shows this relationship for the data of Phillips & Graham (1984). For all cases, the data lie upon the same line, indicating that the additional mass and/or momentum introduced by the vortex generator hub did not have a significant effect upon the vortex strength. Applying a least-squares fit to this data yields $\Gamma_0/W_\infty c \approx 0.66$, even for the strong jet case.

The Reynolds stresses reported by Phillips & Graham (1984) were then considered more carefully. The shear stresses were typically two orders of magnitude smaller than the normal stresses, so the assumption that $\overline{v_\theta v_r} \approx \overline{v_r v_z} \approx 0$ is justified. The normal stresses from the cases having the weakest jet and weakest wake ($W_0/W_\infty = 1.00$ and 0.92, respectively) are plotted in Figure 4 at the most downstream station, resulting in swirl numbers sufficiently low that no instabilities in the vortices should be expected (Lessen & Paillet 1974). In both cases, the isotropic condition that $\overline{v_\theta^2} \approx \overline{v_r^2} \approx \overline{v_z^2}$ is reasonably-well met (to within the experimental uncertainty reported by the authors) over the range $0.6 \lesssim \eta \lesssim 3$. The collapse of the normal Reynolds stresses to a constant value (in this case, $C_0 \sim 7 \times 10^{-3}$) over this same range of $\eta$ is also consistent with (1.10). Therefore, the assumptions made in developing (1.10) were met: namely, (a) that the Reynolds stress profiles recorded in the vortex were not significantly affected by vortex wandering, and (b) that the turbulence within the vortex had decayed sufficiently in the near-field that it was mostly independent of the initial condition. It is also apparent that this collapse to $\overline{v_\theta^2} \approx \overline{v_r^2} \approx \overline{v_z^2} \approx C_0$ is not the result of a nonzero noise floor, as the
Reynolds stresses begin to diverge and further decay for \( \eta \gtrsim 3 \). The same collapse was observed (although not as clearly, and over varying ranges of \( \eta \)) in the other cases.

### 2.2. Re-assessing dissipation rates

Figure 5 (a) shows the axial evolution of normalized turbulent kinetic energy interpolated from the data of Phillips & Graham (1984) (the authors measured the different components of the Reynolds stress tensor at slightly different locations), with data points evenly distributed in \( \eta \) over the range \( 0.7 \leq \eta \leq 1.1 \), corresponding roughly with the range over which (1.1) and (1.3) agree well. There is no clear trend apparent in the data; some of the cases (notably those having the strongest jet and wake) exhibit a power-law decay rate, though other cases (such as those having \( W_0/W_\infty = 1.08 \) and 1.00) appear to demonstrate an increase in \( k \) over \( 3 < z/b < 7.3 \), in violation of (2.12).

For the case of a vortex, though, it may be argued that using the downstream distance \( z/b \) (or \( \zeta \)) as a mean convective path length in (2.12) is not appropriate. If the scales of the turbulence are small relative to the scale of the vortex itself, and if \( V_0/W_\infty \) is not negligible, then the turbulence originating from the initial condition is advected downstream along a helical path over a distance which varies with \( \eta \) and which is significantly larger than \( z/b \) for \( \eta \sim 1 \). The straight-line axial distance \( z/b \), then, does not provide a length-scale proportional to the transit time.

Instead, a normalized helical path length \( s/b \) can be defined, such that

\[
\frac{s(\eta)}{b} = V_0(\eta) \frac{V_0 z}{W_\infty b},
\]

which necessarily requires the assumption that the radial transport of turbulence is negligible. Since the most dominant modes of radial transport are bulk advection (which will be small for vortices having \( d/dz \sim 0 \), as has been assumed here) and turbulent diffusion (which will be small if the turbulence length scales are much smaller than the length scale of the mean vortex, and has already been shown to be negligible), this assumption does not severely restrict the generality. Figure 5 (b) shows the same data as figure 5 (a), scaled against \( s/b \). This revised scaling collapses the data reasonably well (given the experimental uncertainty) onto a straight line; a small number of outlying points do
Is there really such a thing as a turbulent vortex?

remain, although these are mostly from the station closest to the vortex generator, where the vortex was likely still developing.

Because these scaled results agree well with (2.12) with \( n \approx 5/2 \) and the condition of (1.10) is satisfied, it may be concluded that in a well-developed, axisymmetric turbulent vortex, the turbulence within the vortex is dominated by vestigial structures originating from the initial condition (in this case, the boundary layer of the vortex generator), passively convected by the mean velocity as they undergo final decay. This result is consistent with (2.11), and was only observable because of the absence of vortex wandering.

2.3. Application to wandering vortices

The experiments of Phillips & Graham (1984), however, were fairly unconventional in that they were specifically designed in such a way that the vortex wandering was minimized. In typical experiments and simulations, the effects of inactive wandering dominate, and the analysis described in §1.3 and §2 is precluded. On the other hand, if a wandering vortex is (a) axisymmetric about its own trajectory, (b) well-developed and independent of the initial conditions, and (c) passively advecting turbulence from the initial condition, then (1.10) should be satisfied if a correction for wandering is implemented and is effective.

A series of experiments were conducted in order to assess this hypothesis, and in particular to re-examine the transport of Reynolds stresses within a typical, experimental-scale trailing vortex once the wandering has been corrected.

3. Experimental demonstration

3.1. Flow facility

Experiments were carried out using a recirculating wind-tunnel at the University of Surrey. The wind tunnel test-section measures 1.1 m × 1.4 m in the cross-flow plane, and produces flow with a free-stream turbulence intensity of less than 0.15%. The free-stream speed is actively controlled, and maintained steady to within less than 0.5%. The facility is also equipped with an actively regulated heat exchanger, so that the temperature within its closed volume could be held constant.
Direct force measurements were collected using the facility’s integrated bespoke six-component force and moment balance. The balance sensing plate is mounted flush with the test-section floor. In the configuration used for these measurements, the force sensors have a full-scale range of ±180 N and an uncertainty of less than 0.1%.

Two-dimensional wake scans were carried out using a single-component thermal anemometry probe having a welded sensing element 2 mm long and 5 µm in diameter. The probe was driven by an in-house constant-temperature anemometer, having a bandwidth of at least 10 kHz. Signals were acquired using a Data Translation DT-9836 data acquisition system, and no analogue signal conditioning was used. The probe was calibrated in situ by moving the probe into the undisturbed free-stream flow far from the model. The probe was calibrated against a Pitot-static tube connected to a Furness micromanometer (having a full-scale range of 196 Pa), and the calibration was repeated after each scan; data were rejected if the calibration constants varied by more than 0.5% over the scan.

Cross-flow velocity measurements were collected using a Dantec two-component laser-Doppler velocimetry (LDV) system in backscatter mode. The LDV probe, connected to the optics system via fibre-optic cable, has an outside diameter of 27 mm and a stand-off distance of 160 mm. Verification of individual velocity component statistics against non-intrusive measurements taken through a pane of optical glass flush with the tunnel side-wall (using a much larger LDV lens system) demonstrated that the probe had a negligible effect on the flow behaviour. The LDV probe measurement volume was ~1570 µm in length (along the streamwise direction) and ~74 µm in diameter (in the cross-flow plane). The LDV signals were pre-processed using a Dantec F60 burst spectrum analyzer (BSA). The flow was seeded with atomized oil using a TSI 9307 atomizer, resulting in droplets having a mean diameter of ~1 µm. Seeding particles were introduced into the return section of the tunnel, to allow for a homogeneous distribution of particles at the test section entrance. The seeding rate was actively controlled in real-time to yield an optimal sampling frequency during measurement (nominally 2 kHz), independent of the position of the probe. The overall uncertainty in velocity measurements using the LDA system is ~1%.

The LDA probe was mounted at the end of a sting ~800 mm long, fixed to a precision servo-traverse capable of displacements along the three Cartesian axes. The traverse has an absolute positional uncertainty of approximately ±10 µm in the cross-flow axes and ±100 µm in the streamwise direction.

3.2. Wing models

To produce a well-developed turbulent trailing vortex, short-span wing models were installed in the wind-tunnel test section. The wing models were rectangular, with constant NACA0012 sections and no taper or twist. The wings had a chord $c = 157$ mm, an aspect ratio of 2.5 to ensure rapid wake roll-up, and body-of-revolution end caps to minimize the generation of secondary vortices (see Birch et al. 2003). The wings were mounted on a cylindrical support 16 mm in diameter, such that the gap between the wing root and the wind tunnel floor was negligible (the wind tunnel boundary layer was less than 10 mm thick at the location of the wing). In cases where the two-dimensional behaviour of the wing section was considered, the wing was fixed such that the root was approximately 60 mm from the tunnel floor. A rectangular end-plate with a sharpened leading edge, measuring approximately 300 mm in the streamwise direction, was suspended just beneath the wing root so that the gap between the root and the end-plate was negligible. Beneath the end-plate, the support was protected from the free-stream flow by an airfoil-shaped shroud. The rounded end-caps of the wing models were removed, and a second end-plate was suspended over the wing from the wind tunnel ceiling.
Is there really such a thing as a turbulent vortex?

Two different wings were tested. The wings were fabricated from PVC using a precision additive manufacturing process, resulting in a geometric tolerance of less than ∼100 µm. One wing was then sanded and polished, to obtain as smooth a surface finish as possible. The other was covered with commercial P80 sandpaper, having a mean roughness height $h / c \sim 160$ µm (corresponding to a relative roughness scale $h / c \sim 0.1\%$). The sandpaper topology was scanned using a Micro-Epsilon ILD2200-20 laser displacement transducer (with a resolution of 0.3 µm) mounted in a precision $x-y$ table. The roughness was found to be isotropic, with a near-normal distribution (figure 6).

The trailing vortex produced by this particular wing geometry has already been extensively validated and documented; for additional details, see Shaw-Ward et al. (2015). By introducing additional turbulence into the vortex flow field, the effects of the boundary layer turbulence upon the vortex development could be promoted without significantly altering the integral characteristics of the vortex. Increasing the wall shear velocity on the wing surface introduces additional fine-scale turbulence to the vortex without increasing the background turbulence levels, which are known to significantly affect wandering amplitudes (Bailey & Tavoularis 2008; Jammy et al. 2014). Furthermore, vortex flows have been shown to both act as a spatial filter for turbulence structures and to organize background turbulence (Melander & Hussain 1993), and the interaction between the primary structure and any external secondary structures was outside of the scope of this work.

Experiments were all carried out at a Reynolds number of $1.1 \times 10^5$ based on the wing chord and free-stream velocity. To ensure a well-developed, axisymmetric vortex had formed within the test section, the wings were set at an incidence angle of $\alpha = 5^\circ$. As the roughness thickened the rough wing’s boundary layer, it developed slightly less lift than the smooth wing. The difference at $\alpha = 5^\circ$, however, was sufficiently small that no change in the wing incidence was required in order to ensure that the difference in the vortex strength remained within the limits of experimental uncertainty. A schematic of the experimental setup and coordinate system is included in figure 7.

3.3. Data post-processing

The vortex centre was defined as the point of minimum mean cross-flow velocity $(V_x^2 + V_y^2)^{1/2} = (U^2 + V^2)^{1/2}$ identified from the two-dimensional rectangular data grids using third-order interpolation, as this minimized the effect of measurement uncertainty. The
vortex centre was, in each case, defined as the origin of the polar coordinate system used in the subsequent analysis.

For the purposes of statistical analysis and filtering, the LDA data were converted to a regular time-base according to the technique of Adrian & Yao (1987), and re-sampled at the mean data rate using third-order interpolation. To assess the effects of the low frequency spectral contents, the fluctuating component of the resampled time-domain data was high-pass filtered (using a Cauer elliptic filter having an attenuation of 60 dB per decade), and then added back to the mean. Direct spectral decomposition of the LDV signals was precluded by the limited bandwidth of the LDV system.

The axial vorticity fields were computed from the cross-flow velocity components in Cartesian space. The spatial velocity gradients were obtained using third-order numerical differentiation over the measurement grid.

To obtain circumferentially averaged quantities, the measured field quantities were first spatially re-sampled using bicubic interpolation to increase data grid densities by a factor of 20. Radial profiles $g(\eta)$ were then obtained from the resampled fields $g(\eta, \theta)$ as spatial averages, such that

$$g(\eta) = \frac{1}{2\pi \Delta \eta} \int_0^{2\pi} \int_{\eta - \Delta \eta/2}^{\eta + \Delta \eta/2} g(\xi, \theta) \, d\xi d\theta,$$

with a radial discretization of $\Delta \eta \approx 0.1$. Circumferentially-averaged tangential velocity profiles were then shifted in $\eta$ to ensure that $V_\theta = 0$ at $\eta = 0$. The offset at $\eta = 0$ was a consequence of the uncertainty in the location of the vortex centre (see Giuni 2013), and was in all cases less than $\sim 15\%$ of $r_c$.

3.4. Model characterization

The lift characteristics of the wing model (and its constituent two-dimensional airfoil section) are illustrated in figure 8. The two-dimensional section exhibits a behaviour very similar to that of Lee & Gerontakos (2004), who tested a high aspect-ratio NACA 0012 model at the slightly higher chord Reynolds number of $Re \sim 1.4 \times 10^5$. As expected,
Is there really such a thing as a turbulent vortex?

Figure 8. Wing lift as a function of incidence. □, smooth wing; ◦, rough wing; open symbols, 2D configuration; filled symbols, 3D configuration; ⬤, Lee & Gerontakos (2004), at $Re \sim 1.4 \times 10^5$.

Figure 9. Two-dimensional wing wake profiles at $5^\circ$ incidence. (a), $WV_0/W_\infty$; (b), $\overline{w^2}/W_\infty^2$. ——, smooth wing; - - -, rough wing; – – –, (1.2).

The rough wing develops less lift than the smooth wing, but the difference is small; for $\alpha \lesssim 10^\circ$, the difference is almost negligible. This result agrees with the findings of Katz & Galdo (1989). Importantly, there is no significant difference in the stall characteristics, so the roughness has not altered the stall mechanism and is only affecting the wall shear velocity on the wing.

Figure 9 shows the profiles of streamwise velocity and Reynolds normal stress behind the wings in two-dimensional configuration at $5^\circ$ incidence. Both $W$ and $\overline{w^2}$ have been re-normalized here against the free-stream velocity, as $V_0$ in this case is undefined. These profiles were collected at $z/c = 0.3$ (or $0.3c$ downstream of the location of the trailing edge at zero incidence) using a single-sensor hot-wire probe. For clarity, the origin in these plots has been taken as the location of minimum axial velocity for each case; Note that the wing is lifting in the direction of $x/c > 0$.

The mean axial velocity profiles agree very well with (1.2), and the surface roughness
appears to have a very small effect on the wake strength. These results further suggest that the increase in drag on the rough wing is small, and due primarily to the increase in skin friction. A greater difference is observed between the rough and smooth wings in the approximation of the normal Reynolds stress (note that the single-sensor hot wire is sensitive to $(V^2 + W^2)^{1/2}$ and not $W$); a small amount of the energy is displaced toward the pressure side of the wing. However, the total normalized axial turbulent kinetic energy $E_w$, which may be defined as

$$E_w = \frac{1}{2L} \int_{-L/2}^{L/2} \bar{w}^2 \left( \frac{V_0}{W_\infty} \right)^2 dx,$$

(3.2)

(where $L$ is the wake width) increases by only 0.3% in the rough-wing case.

The wing boundary layer turbulence was also characterized by examining the spectral contents of the velocity in the near-wake of the 2D wing configuration. The power spectral density $\Phi$ of the normalized velocity signals was computed at $x/c = \pm 0.2$ for both the rough and smooth wing; these are shown in figure 10 as functions of the Strouhal number $St = f c / W_\infty$ (where $f$ is the frequency). Note that some environmental noise is evident at $St \sim 4$ with a harmonic at $St \sim 8$, and has been suppressed in post-processing. The power spectra are similar for both the smooth- and rough-wing case, and both are in general agreement with the results of Devenport et al. (1996). More illustrative, perhaps, is the spatial map of power spectral density, shown in figure 11. Here, some skewing of the isocontours toward the pressure side of the wing is indicative of additional energy arising from the mean acceleration, but otherwise the power is decaying from the wing trailing edge with a similar distribution for both the rough and smooth wings. Furthermore, the peak power occurs at the lowest frequencies, and decays with increasing $St$.

### 3.5. Vortex development

The normalized streamwise mean vorticity fields $\omega_z c / W_\infty$ of the tip vortices formed $5c$ downstream of the wing models are shown in Figure 12. The vortices in both cases are well-developed and nearly axisymmetric, with no evidence remaining of discrete wake spirals. The critical vortex parameters for each of the two cases are listed in Table 2. The differences in the size of the two vortices was fairly small, and the dimensional core
Is there really such a thing as a turbulent vortex?

Figure 11. Spatial distribution of normalized velocity power spectral density. Isocontours show levels of $\log_{10}(\Phi(St)/W_\infty)$. (a) Smooth wing; (b) rough wing.

Figure 12. Isocontours of normalized axial vorticity $\omega_z r_c/V_0$. (a) Smooth wing; (b) rough wing.

circulation $\Gamma_c$ remained constant to within less than 2%, which is within the limit of experimental uncertainty.

Figure 13 compares the circumferentially-averaged mean tangential velocity profiles with (1.1). As expected, the results collapse very well with (1.1) and (1.3) for $0 < \eta \lesssim 1.3$, which includes the range of interest.

The circumferentially averaged Reynolds stresses are shown in figure 14; note that the rough-wing profiles have been re-scaled by a factor of 0.5 for comparison, as the vortex formed behind the roughened wing exhibited about twice the turbulent kinetic energy. The cross-flow shear stress is negligibly small throughout, and falls within the range of
experimental uncertainty. The normal stress distributions are approximately Gaussian, subject to a small offset (the Reynolds normal stresses do not vanish for $\eta \to \infty$ as a result of the flow facility and electronic background noise). Values for $\eta < 0.25$ are not shown, as these have fairly low confidence owing to the uncertainty in the location of the vortex centre. It is, however, significant to note that the Reynolds stress profiles are nearly identical between the rough and smooth wing cases when self-scaled. If the normal distribution was due primarily to the wandering, the similarity of the width of the distributions between the two cases indicates that the magnitude of the wandering does not depend on the energy or scales within the wing shear layer.

3.6. Wandering correction
Because $dV_\theta/d\eta \sim 1$ in the vicinity of a vortex core, even small temporal fluctuations in the location of the vortex centre can result in very large contributions to the Reynolds stresses recorded at a fixed point. The results of Beninati & Marshall (2005) and others suggest that the spectral energy contribution from wandering is limited to fairly low frequencies. If the vortex is well-developed and if the Reynolds stresses are dominated by vestigial wing boundary layer turbulence having a relatively flat distribution for lower frequencies (see figure 10), it would be expected that the peak Reynolds stresses would remain relatively unchanged through a high-pass filter with a sufficiently low filter frequency $f_H$. Figure 15 shows the peak value of $\overline{v_\theta^2}$ obtained under a variety of nondimensional filter frequencies $St_H = f_H c/W_\infty$ for both the smooth and rough wing tip vortices at $z/c \sim 5$. In both cases, $\overline{v_\theta^2}$ (which may be interpreted as a general indicator of $k$)
Is there really such a thing as a turbulent vortex?

Figure 14. Radial distribution of Reynolds stresses. ◦, $v_r^2$; □, $v_\theta^2$; △, $10v_r v_\theta$; ---, Best-fit Gaussian distribution. Open symbols, smooth wing; filled symbols, rough wing (rescaled by a factor of 0.5 for ease of comparison).

Figure 15. Variation in peak $\overline{v_\theta^2}$ with high-pass filter frequency in tip vortex. ◦, smooth wing; □, rough wing.

...drops rapidly with increasing $St_H$ for low $St_H$, and then remains relatively constant with further increases in $St_H$. This is consistent with $k$ being dominated by the high-amplitude, inactive wandering and suggests that the high pass filter may be used to correct for the effects of the wandering upon the Reynolds stress distributions.

The filtering process had no observable effect upon the circumferentially-averaged mean tangential velocity profile. This was expected, as the error introduced into the mean velocity as a consequence of the wandering was estimated using the technique of Devenport et al. (1996), and found to be $\sim 0.15\%$.

Figure 16 shows the circumferentially-averaged profiles of the Reynolds stresses computed using the high-pass-filtered velocity signals. Dimensionless filter frequencies of $St_H = 0.031$ and 0.079 were selected for the smooth and rough wing cases, respectively, as these appeared to be the lowest frequencies at which the effects of wandering were...
mostly attenuated (see figure 15). In addition, if the Reynolds stresses are approximately constant over some range near $\eta \sim 1$ as required by (1.10), then $d\overline{v_\theta^2}/d\eta \approx d\overline{v_r^2}/d\eta \sim 0$ and the wandering will have a negligible effect on the higher-frequency spectral contents of the fixed-probe measurements within that region (providing that the length scale of the wandering is sufficiently small).

The application of the high-pass filter significantly reduces the peak Reynolds stress, and results in a region in which $\overline{v_\theta^2} \approx \overline{v_r^2} \approx C_0$, at least to within the experimental scatter. Further increasing $St_H$ had no significant effect upon the distributions.

The results presented in figure 16 are consistent with the emergence of a region of $\overline{v_\theta^2} = \overline{v_r^2} = C_0$ near $\eta = 1$, and the general form of the profiles away from the constant region broadly agree with the results of Phillips & Graham (1984) as shown in figure 4. For the case of the smooth wing, the filtering revealed a region of constant Reynolds stress at around $0.6 \lesssim \eta \lesssim 1$. For the rough-wing case, there is evidence of a flattening of the $\overline{v_r^2}$ profile at around the same relative magnitude as in the smooth wing (note that the vertical scale in figure 16b is scaled by 0.5 relative to the vertical scale in figure 16a), but this is not as clear. The condition of $\overline{v_\theta^2} \approx C_0$ does not appear to have been met in this case. Further increasing $St_H$ has no significant effect upon the distributions. The application of the low-pass filter therefore appears to have successfully attenuated the effects of wandering from the Reynolds stresses for the smooth-wing case, but the evidence in the rough-wing case is less compelling.

Finally, figure 17 shows the decay rate of the turbulent kinetic energy, premultiplied by $(s/b)^{5/2}$ and plotted against the Lagrangian path length given by (2.16). Note that the approximation $\overline{w^2} \approx (\overline{v_\theta^2} + \overline{v_r^2})/2$ was made here in order to estimate $k$, as the streamwise Reynolds normal stress was not available from the two-component LDV system. Data are shown over the range $0.7 < \eta < 1.1$, corresponding to the approximate range over which $\overline{v_\theta^2} \approx \overline{v_r^2} \approx C_0$ for the smooth-wing case and $\overline{v_r^2} \approx C_0$ for the rough-wing case in figure 16. This also matches the range of data from Phillips & Graham (1984) plotted in figure 5. The corrected smooth-wing data collapse well onto the line $(s/b)^{5/2}kV_0^3/W_0^3 = 2.5 \times 10^{-3}$, with reasonably good agreement for the rough-wing case as well. No such trend is apparent in the uncorrected data. Taken together, these results suggest that once
Is there really such a thing as a turbulent vortex?

4. Discussion

The results presented here have raised some interesting questions about the role played by turbulence in the development of stable laboratory-scale vortices and the distribution of Reynolds stresses in highly vortical flows. Conventional deconvolution correction techniques for the effects of vortex wandering cannot change the Gaussian form of the Reynolds stress distributions, but a Gaussian distribution of Reynolds stress does not satisfy the Reynolds-averaged Navier-Stokes equations for a well-developed vortex having good agreement between the mean velocity profile and one of the accepted turbulent vortex profiles. The correction of Reynolds stress profiles for the effect of wandering using these techniques may therefore result in inconsistencies.

Because wandering is a passive phenomenon, there should be sufficient scale separation between the wandering scales and the turbulence scales of interest that the dominating effects of the local mean shear upon the fixed-point Reynolds stress measurements may be filtered out. The filtered Reynolds stresses must still be interpreted with some caution: they will only represent a weighted spatial average over some area given by the probability distribution of the vortex trajectory.

Applying this filter to two typical trailing vortices having a similar mean velocity profile but different total turbulent kinetic energy revealed some interesting results. First, a relatively small increase in the peak streamwise Reynolds normal stress in the wing wake (and negligible change in the total turbulent kinetic energy) resulted in a twofold increase in the tangential Reynolds normal stresses recorded in the vortex developed downstream. This suggests that either the initial roll-up of the vortex provides a mechanism for the transfer of energy from the mean flow to the smaller scales, or that the spanwise flow around a finite wing contributes significantly to turbulence production.

Another important observation was that despite the doubling of the turbulent kinetic energy...
energy, the amplitude of the vortex wandering did not appear to be affected. One important distinction here was that the additional turbulence was introduced to the vortex through the wing boundary layers rather than by means of upstream turbulence grids. Since wandering amplitude is known to increase with increasing background turbulence levels (as has been extensively documented for the case of tip vortices formed in grid turbulence), this demonstrates that it is the background turbulence in the surrounding free-stream flow which interacts with the vortex to increase wandering amplitude rather than the turbulence rolled into the vortex itself.

Most importantly, the corrected velocity statistics showed that a small region of constant Reynolds normal stress was indeed emerging around the core radius (although this was less clear for the higher-turbulence case), satisfying the momentum equation for the case of a well-developed vortex in which the turbulence is isotropic and the turbulent kinetic energy transport is dominated by viscous dissipation. The introduction of additional turbulent kinetic energy into the vortex in the initial condition had a minimal effect on its development, and the filtered turbulent kinetic energy was shown in both cases to have a dissipation rate consistent with final decay. This provides additional support for a model in which the vortex is passively advecting the freely-decaying turbulence from the wing boundary layer in a process analogous to the free decay of grid turbulence.

Taken together with a mean velocity profile that agrees very well with the exact laminar solution, it becomes somewhat unclear whether or not it is appropriate to describe the vortex itself as being turbulent. Returning to the analogy of boundary layers, if a hypothetical, stable wall-bounded flow is dominated by viscous dissipation, has a Blasius mean velocity profile and contains only decaying turbulence from an arbitrary upstream initial condition, should this flow be described as a ‘turbulent boundary layer’? In the absence of any accepted convention on what conditions must be met by a stable, vortex-like flow in order for it to be considered turbulent, great care must be taken in the description of these flows. Depending on the context, it may be misleading to define a stable vortex as being turbulent only because a broad-band distribution of spectral energy is observed at some location within the flow.

5. Conclusions

An analytical treatment similar to that used to derive the Kármán-Howarth equation has been applied to the case of a turbulent vortex, recognizing that the mean velocity profiles for vortices closely agree for the cases of \( \Re \to 0 \) and \( \Re \to \infty \) in the vicinity of \( \eta \sim 1 \). A set of criteria were developed which could be used to ascertain if the Reynolds stress distributions were dominated by the inactive, random wandering of the vortex. Legacy data from a particular series of careful experiments were shown to satisfy these criteria, and these data could be used to demonstrate that the turbulence was undergoing classical final-stage decay if the Lagrangian path length was taken as the advective lengthscale. Conventionally, these characteristics would be masked by the random wandering of the vortex.

An experimental demonstration was carried out in which two trailing vortices having similar size and strength but different initial turbulence levels were produced. By introducing the additional turbulence through the wing boundary layer, the increased turbulence levels did not result in increased wandering amplitude. The effects of wandering were attenuated by high-pass filtering the velocity signals, and the resulting velocity statistics were consistent with isotropic turbulence dominated by viscous diffusion. These characteristics would not have been revealed by conventional wandering correction techniques based on Gaussian deconvolution.
Is there really such a thing as a turbulent vortex?

The two vortices characterized, having typical strength and turbulent kinetic energy as those used in laboratory wing-tip vortex studies, were therefore consistent with a model in which freely-decaying, isotropic turbulence is passively advected in a Lagrangian sense.

Acknowledgments

This work was supported in part by the U.K. Engineering and Physical Sciences Research Council (EPSRC) under grant number EP/H030360/1. The authors are grateful to the staff of the McGill University Shulich Library archives for making available the legacy data, and to Nick Hills for very useful discussions.

EPSRC data access compliance statement

The authors confirm that data underlying the findings are available without restriction. Details of the data and how to request access are available from the University of Surrey publications repository, at [address to be supplied].

REFERENCES


Poppleton, E. D. 1971a Effect of air injection into the core of a trailing vortex. J. Aircr. 8 (8), 672–673.


