Do multineutrons exist?

N. K. Timofeyuk

Physics Department, University of Surrey, Guildford, Surrey GU2 7XH, England, UK

(Dated: Received: February 8, 2008)

In a recently reported experiment [1] events were observed that exhibit the characteristics of a multineutron cluster liberated in the breakup of $^{14}$Be, most probably in the channel $^{10}$Be+$^4n$. The lifetime of order 100 ns or longer suggested by this measurement, would indicate that the tetraneutron is particle stable. The existence of a bound tetraneutron, if confirmed, could challenge our understanding of nuclear few-body systems and nucleon-nucleon (NN) interactions. In the present letter, I would like to make several observations in this context.

I

The tetraneutron-like events seen in [1] deserve attention because the breakup $^{14}$Be $\rightarrow$ $^{10}$Be+$^4n$ represents one of the best possible tools to search for a tetraneutron. In earlier experiments, the tetraneutron was searched for using heavy-ion transfer reactions such as $^7$Li($^{11}$B, $^{14}$O)$^4n$ [2], $^7$Li($^7$Li, $^{16}$O)$^4n$ [3] and double exchange reaction $^4$He($^1\bar{p}$, $^1\bar{p}$)$^4n$ [4, 5]. These reactions require considerable reconfiguring of the target nuclei and should be strongly suppressed. A negative outcome from these reactions could be easily anticipated. On contrary, the nucleus $^{14}$Be consists of a strongly bound core $^{10}$Be and four valence neutrons whose separation energy is only about 5 MeV. In the attractive field of $^{10}$Be these four neutrons could form a tetraneutron-like configuration which might be shaken off in the $^{14}$Be breakup.

II

No proper ab-initio four-body calculations of the tetraneutron with realistic two-body and three-body NN forces are known to the author. However, several calculations of the tetraneutron are available.

(i) In Ref. [6] the tetraneutron, studied in the translationally invariant $4\hbar\omega$ oscillator shell model, was found to be unbound by 18.5 MeV. However, the oscillator basis is not appropriate for the description of unbound systems.

(ii) No bound tetraneutron was found in Ref. [7] within the angular potential functions method with semirealistic NN interactions. No search for the four-body resonance state was carried out.

(iii) No bound tetraneutron was found in Ref. [8] within the stochastic variational method on a correlated Gaussian basis for a range of simple effective NN potentials. No search for the four-body resonance state was carried out.

(iv) A search for four-body resonances in the lowest order of the hyperspherical functions method (HSFM) gave a null result [9, 10].

(v) The energy behaviour of the eigenphases, studied in Ref. [11] within the HSFM using the $K_{max} = 6$ model space, led the authors to the conclusion that the tetraneutron may exist as a resonance in the four-body continuum at an energy between 1 and 3 MeV. However, a clear indication of the resonance has been seen with only one of the NN potentials used in the calculations, namely with the Volkov effective NN force V1 [12]. Volkov effective NN interactions reproduce the experimental binding energy of another four-body system, $^4$He, but they bind a dineutron. In the particular case of V1, the dineutron is bound by 0.547 MeV. Therefore, V1 cannot be used in calculations of multineutron systems. Another NN potential, used in Ref. [11], namely that of Reichstein and Tang (RT) [13], reproduces the n-p triplet and p-p singlet scattering lengths and does not bind a dineutron. With this potential the energy derivatives of the eigenphases monotonically decrease with increasing energy. This means that resonances in the four neutron system are absent, at least within the model space considered. Since no convergence of the eigenphases with an increase of the hyperangular momentum has been achieved in these calculations, such a conclusion does not look convincing.

To understand whether the RT potential can produce any resonance or bound state if the model space of the HSFM is increased, I have studied the tetraneutron within the extreme uncoupled adiabatic approximation of the HSFM [14] which provides a lower limit for the binding energy [15] within the model space considered. The model space in these calculations has been increased up to $K_{max} = 16$. In this limit it is sufficient to diagonalise the matrix of the hyperradial potentials $V_{K^+\gamma^+K^+\gamma^+}(\rho)$, which include both nuclear and centrifugal force, and to solve a Schrödinger-like equation with the lowest diagonalised potential $V_{diag}(\rho)$ [16]. The hyperradial potentials have been calculated using the technique developed in Ref. [16].

The RT potential has only central triplet and singlet even components $V_{TE}$ and $V_{SE}$ and the singlet and triplet
odd potentials are usually obtained from them using an arbitrary parameter $u$ so that $V_{SO} = (u - 1)V_{SE}$ and $V_{TO} = (u - 1)V_{TE}$. The present calculations have been performed with $u = 1$ which corresponds to no interaction in the odd partial waves. These calculations provide a lower limit for the binding energy because realistic values of $u$, which lie in the range of 0.93 to 0.98, induce repulsion between some of neutrons, which can only decrease the binding.

The diagonalised hyperradial potentials $V_{\text{diag}}(\rho)$, calculated for different model spaces $K_{\text{max}}$, are plotted in Fig.1a as a function of hyperradius $\rho$. They have almost converged and are purely repulsive, monotonically decreasing with the hyperradius, without any sign of local attractive pockets. Therefore, the RT potential can neither bind the tetraneutron nor produce any resonances. To get a bound tetraneutron, the value of $u$ should be increased to $u = 2.3$. Such a value corresponds to a unphysical attraction in odd partial waves and binds the isotopes $^4\text{H}$ and $^5\text{He}$ with respect to the $t + n$ and $^4\text{He} + n$ thresholds by about 6.5 MeV and 32 MeV respectively. In reality, these nuclei are unbound by 3 and 0.9 MeV.

For comparison, the calculations of the hyperradial potentials have been performed with Volkov NN potential V1. The diagonalised hyperradial potentials $V_{\text{diag}}(\rho)$, shown in Fig. 1b, reveal local attractive pockets for $K_{\text{max}} > 6$. These pockets become negative for $K_{\text{max}} > 12$, but they are too shallow to form a bound state. To get a bound tetraneutron, the Majorana parameter $m$ should be changed from its standard value of 0.6 to $m = -0.2$. Such a change provides $E(^4\text{n}) = -1.2$ MeV and does not influence the binding energy of $^4\text{He}$, however, it binds $^4\text{H}$ and $^5\text{He}$ with respect to the $t + n$ and $^4\text{He} + n$ thresholds by 7 MeV and 25 MeV respectively.

### III

The results of the theoretical calculations suggest that the tetraneutron must be unbound and most likely should not exist as a resonance, at least when only two-body central forces are considered. It is remarkable that even when an effective NN potential binds the dineutron (the case of V1), it still cannot bind two dineutrons, although a resonance in such a system should exist. In order for the two-body central force to be able to bind the tetraneutron, a huge unphysical attraction must be introduced in the triplet odd potential. Such an attraction would strongly overbind $^4\text{H}$, $^5\text{He}$ and other known $A > 4$ nuclei. The necessity for such a huge attraction lies in the relative number of nucleon pairs in even and odd states as determined by the Pauli principle. The NN force binds two nucleons only in the triplet even state. It is not sufficiently attractive in singlet even states, and is repulsive in the triplet and singlet odd states.

The probabilities $P$ to find a pair of nucleons in all these states are shown in Table 1 for the dominant components $[22]^{11}S$, $[31]^{33}P$ and $[4]^{11}S$ of $^4\text{n}$, $^4\text{H}$ and $^4\text{He}$ respectively, together with binding energies of $^4\text{H}$ and $^4\text{He}$. One can see that the binding energy of a nucleus is strongly correlated with the probabilities to find a pair of nucleons in even states, especially in the triplet even state where the $n - p$ bound state exists. In $^4\text{H}$, a decrease of the probabilities $P_{TE}$ and $P_{SE}$ by 30% in comparison with $^4\text{He}$ leads to a dramatic decrease of its binding energy by about 20 MeV.
TABLE I: Binding energies (in MeV) and the probabilities $P$ to find a pair of nucleon in the triplet even (TE), singlet even (SE), singlet odd (SO) and triplet odd (TO) states for different $A = 4$ isotopes and for $^6_a$, $^8_s$, $^{10}_o$.

<table>
<thead>
<tr>
<th></th>
<th>Binding energy</th>
<th>$P_{TE}$</th>
<th>$P_{SE}$</th>
<th>$P_{SO}$</th>
<th>$P_{TO}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^4_n$</td>
<td>-5.2</td>
<td>1/2</td>
<td>1/2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$^4_H$</td>
<td>-28.3</td>
<td>1/2</td>
<td>1/2</td>
<td>1/2</td>
<td>1/2</td>
</tr>
<tr>
<td>$^6_n$</td>
<td>-6/15</td>
<td>9/15</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$^8_n$</td>
<td>5/14</td>
<td>9/14</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$^{10}_n$</td>
<td>1/3</td>
<td></td>
<td></td>
<td></td>
<td>2/3</td>
</tr>
</tbody>
</table>

In the tetraneutron, triplet even components (the only ones which produce binding in the NN system) are absent and the probability to find two neutrons in the repulsive triplet odd state increases by 50% in comparison with $^4_H$. Therefore, it is not unreasonable to assume that moving from $^4_H$ to $^4_n$ one can lose a comparable amount of binding energy as going from $^4_{He}$ to $^4_H$. This would definitely make the tetraneutron unbound.

IV

The conclusion of particle instability of $^4_n$ made above has been drawn on the basis of only two-body NN forces. The presence of the 3N and, possibly, 4N forces could, in principle, modify this conclusion. The contribution of the 3N potential to the binding energies of neutron drops $^7_3$ n and $^8_3$ n formed in an external field has been calculated with Green’s function Monte Carlo Methods in Ref. [17]. It ranges from 1 to 5 MeV. A similar contribution from the 3N force could be expected in the case of $^4_n$. But such a contribution will not significantly lower the hyperradial potential from Fig. 1a to produce a potential well in which a bound state of four neutrons could be formed.

As for the 4N force, the results of the ab-initio calculations of the $3 \leq A \leq 8$ nuclei suggest that either the 4N contributions to the observed binding energies of these nuclei are smaller than 1%, or their effects are included in some parts of the 3N forces [17]. The latest models of the 3N force used to calculated the binding energies of these nuclei have not yet been tested in the description of polarization observables of the low-energy $Nd$ scattering where the contribution of the 3N force is important [18, 19]. Besides, the latest numerically accurate calculations employing a 3N force do not reproduce the proton analyzing power for the $p^6_{He}$ scattering [20]. The simultaneous fit of the $Nd$ and $p^3_{He}$ polarization observables and of binding energies of the lightest nuclei may leave room for a 4N force.

In order to get an idea of how strong the 4N force should be to bind the tetraneutron, the HSFM calculations of the present paper have been repeated with the RT potential using a 4N force simulated by the potential $V_{4N}(\rho) = W_0 e^{-\alpha \rho}$. The values $\alpha = 0.7, 1.2$ and 1.5 $fm^{-1}$ used in these calculations, were the same as the range of the phenomenological term of the 3N spin-orbit force introduced in Ref. [18]. The tetraneutron becomes bound if the corresponding values $W_0$ are equal to $-410, -1460$ and $-2530$ MeV respectively. These values are two orders of magnitude larger than the values $-1, -10, and -20$ MeV obtained for the 3N spin-orbit force [18]. For the Volkov potential V1, these strengths are similar: $W_0 = -320, -1565$ and $-2900$ MeV. If the same 4N force existed in $^4_{He}$, the binding energy of $^4_{He}$, 0.5 MeV, predicted with V1, would be $-88, -82$ and $-134$ MeV respectively instead of $-28.3$ MeV, which would be impossible not to notice.

V

The reported observation of the tetraneutron revives an old question “Do multineutrons exist?” Theoretical investigations of multineutrons have been carried out only for $^6_n$ and $^8_n$. The $6\hbar \omega$ oscillator shell model, with the same effective interactions that unbind the tetraneutron by 18.5 MeV, predicts that $^6_n$ is unbound only by 6.5 MeV [21]. However, with the same interaction, the hydrogen isotope $^6_{He}$ should be unbound with respect to the $t + n + n + n$ decay only by 1.3 MeV and thus should be less unbound than recently discovered isotope $^5_{He}$ [22]. It this was true, the $^6_{He}$ isotope would have been already observed. Other calculations of multineutrons, performed within the angular potential functions method [4], predict that $^6_n$ together with another multineutron $^4_n$ are unbound. No search for six- or eight-body resonances have been carried out in these works.

In the present letter, the hyperradial potentials $V_{\text{diag}}(\rho)$ have been calculated for $^6_n$ with the same NN potentials as for $^4_n$, for $K_{\text{max}}$ ranging from 4 to 12. The calculations with the RT potential and $u = 1$, shown at Fig. 1a, imply that $^6_n$ should be even more unbound than $^4_n$ and that it can not exist as a six-body resonance. The reason for this lies in further decrease of the number of the singlet even NN pairs, where attraction occurs, and increase of the number of repulsive triplet odd pairs (see Table 1). In the case of the NN potential V1, which binds the dineutron, the model space is not large enough to make definite conclusions about existence of $^6_n$. It is possible that further increase of the model space will create a local attraction pocket where a six-body resonance could be formed but it is unlikely that it will lead to a six neutron bound state. If repulsion in the triplet odd components was replaced by attraction in
such a way that $^4n$ is bound ($u = 2.3$ case for the RT potential and $m = -0.2$ case for V1) then $^6n$ would be bound by about 22 MeV or 50 MeV respectively. This results does not look surprising because there is more repulsion in $^6n$ than in $^4n$ and substitution of the repulsion by attraction should lead to more binding in $^6n$ than in $^4n$.

Finally, the hyperradial potentials for $^8n$ and $^{10}n$ have been calculated with the RT potential up to $K_{\text{max}}$ equal to 12 and 14 respectively. These potentials, shown in Fig.1a, do not exhibit any sign of either a bound or a resonant state which is consistent with further decrease of attraction and increase of repulsion in these systems demonstrated in Table 1.

Summarizing, due to the small probability for a pair of neutrons to be in the singlet even state, the two-body NN force cannot by itself bind four neutrons, even if it could bind a dineutron. Unrealistic modifications of the NN force would be needed to bind the tetranucleon. Therefore, it is unlikely that the events in the breakup of $^{14}\text{Be}$ were caused by a formation of a bound tetranucleon. A different explanation should be sought for this experiment and new experiments are needed to clarify this issue. As for experimental searches of other multineutron systems, the calculations presented here suggest that they might be unsuccessful.

The support from the EPSRC grant GR/M/82141 is acknowledged. I am grateful to Prof. R.C. Johnson and Prof. I.J. Thompson for useful comments concerning my paper.