On the existence of a bound tetraneutron

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Following recent work in which events which may correspond to a bound tetraneutron \((^4n)\) were observed, it is pointed out that from the theoretical perspective the two-body nucleon-nucleon force cannot by itself bind four neutrons, even if it can bind a dineutron. A very strong phenomenological four-nucleon (4N) force is needed in order to bind the tetraneutron. Such a 4N force, if it existed, would bind \(^4\text{He}\) by about 100 MeV. Alternative experiments such as \((^4\text{He},^3n)\) are proposed to search for the tetraneutron.

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In a recently reported experiment \([1,2]\) events were observed that exhibit the characteristics of a multineutron cluster liberated in the breakup of \(^{14}\text{Be}\), most probably in the channel \(^{10}\text{Be}+^4n\). The lifetime of order 100 ns or longer suggested by this measurement, would indicate that the tetraneutron is particle stable. If confirmed, the existence of a bound tetraneutron is of great importance as it would challenge our understanding of nuclear few-body systems and nucleon-nucleon (NN) interactions. Here I would like to make several observations in this context.

I

As discussed in Refs \([1,3]\) the breakup \(^{14}\text{Be} \rightarrow ^{10}\text{Be}+^4n\) represents an attractive channel to search for a tetraneutron. The nucleus \(^{10}\text{Be}\) is strongly bound and the four-neutron separation energy for \(^{14}\text{Be}\) is only about 5 MeV with respect to the \(^{10}\text{Be} \rightarrow ^{10}\text{Be}+^4n\) breakup. It is known that in an external potential well neutrons can form drops \([3,4]\). In the attractive field of \(^{10}\text{Be}\) the last four neutrons could form a tetraneutron which might be liberated in the breakup.

In earlier experiments, the tetraneutron was searched using heavy-ion transfer reactions such as \(^7\text{Li}(^{11}B,^{14}O)^{14}n\) \([5,6]\) and double exchange reaction \(^{4}\text{He}(\pi^-,\pi^+)^{14}n\) \([5,7]\). These reactions should be strongly suppressed by spectroscopic reasons and a negative outcome from them could be easily anticipated.

II

No proper \textit{ab-initio} four-body calculations of the tetraneutron with realistic two-body and three-body NN forces are known to the author. However, several theoretical calculations of the tetraneutron are available.

(i) In Ref. \([8]\) the tetraneutron, studied in the translationally invariant, symmetrized oscillator basis using the 4\(\hbar\omega\) model space, was found to be unbound by 18.5 MeV. However, the oscillator basis is not appropriate for the description of unbound systems.

(ii) No bound tetraneutron was found in Ref. \([9]\) within the angular potential functions method with semirealistic NN interactions. No search for the four-body resonance state was carried out there.

(iii) No bound tetraneutron was found in Ref. \([10]\) within the stochastic variational method on a correlated Gaussian basis for a range of simple effective NN potentials. No search for the four-body resonance state was carried out there.

(iv) A search of the four-body resonance in the lowest order of the hyperspherical functions method (HSFM) gave a null result \([11,12]\).

(v) The energy behaviour of the eigen phases, studied in Ref. \([13]\) within the HSFM using the \(K_{\text{max}}=6\) model space, led the authors to the conclusion that the tetraneutron may exist as a resonance in the four-body continuum at an energy of about 1 – 3 MeV. However, such a conclusion is not convincing because no convergence of the eigen phases with an increase of the hyperangular momentum has been achieved in these calculations. Besides, a clear indication of the resonance has been seen with only one of the NN potentials used in the calculations, namely with the Volkov effective NN force \(V_1\) \([15]\). Volkov effective NN interactions reproduce the experimental binding energy of another four-body system, \(^4\text{He}\). However, their singlet even and triplet even components are equal, and therefore with these potentials a singlet dineutron has exactly the same binding energy as the deuteron. In the particular case of \(V_1\), the dineutron is bound by 0.547 MeV. Therefore, \(V_1\) cannot be used in calculations of the multineutron systems. Another NN potential, used in Ref. \([13]\), namely that of Reichstein and Tang (RT) \([16]\), reproduces the n-p triplet and p-p singlet scattering lengths and does not bind a dineutron. With this potential the energy derivatives of the eigen phases monotonically decrease with increasing energy. This means that resonances in the four neutron system are absent, at least within the model space considered.

To understand whether the RT potential can produce any resonance or bound state if the model space of the HSFM is increased, I have calculated the hyperradial potentials for the tetraneutron up to \(K_{\text{max}}=16\) using the technique developed in Ref. \([17]\). The RT potential has only central even components \(V_{10}\) and \(V_{01}\) and the odd potentials are usually obtained from them using an arbitrary parameter \(u\) so that \(V_{00}=(u-1)V_{01}\) and \(V_{11}=(u-1)V_{10}\). The present calculations have been performed with \(u=1\) which corresponds to no-interaction in the odd partial waves. The diagonalised
hyperradial potentials \( V_{\text{diag}}(\rho) \), calculated for different model spaces \( K_{\text{max}} \), are plotted in Fig. 1 as a function of hyperradius \( \rho \). Such potentials can be used to obtain a quick approximate solution for the binding energy in the extreme adiabatic approximation of the HSFM \[13\]. One can see that the \( V_{\text{diag}}(\rho) \) have almost converged. They are purely repulsive, monotonically decreasing with the hyperradius, and do not show any sign of local attractive pockets. Therefore, the RT potential can neither bind the tetraneutron nor produce any resonances. To get a bound tetraneutron, the value of \( \rho \) should be increased up to \( \rho = 2.3 \). However, with this value, the isotopes \(^4\text{H}\) and \(^5\text{He}\) must become bound with respect to the \( t + n \) and \( ^4\text{He} + n \) thresholds by about 6.5 MeV and 32 MeV respectively. In reality, these nuclei are unbound by 3 and 0.9 MeV.

![Diagonalised hyperradial potentials](image)

**FIG. 1:** Diagonalised hyperradial potentials \( V_{\text{diag}}(\rho) \) of the tetraneutron calculated with \( K_{\text{max}} = 2, 4, ..., 16 \) with two different central NN potentials.

For comparison, the calculations of the hyperradial potentials have been performed with Volkov NN potential V1 as well. The diagonalised hyperradial potentials \( V_{\text{diag}}(\rho) \), shown in Fig. 2, reveal local attractive pockets for \( K_{\text{max}} > 6 \). These pockets become negative for \( K_{\text{max}} > 12 \), but they are too shallow to form a bound state with respect to the four-body decay. Although with \( K_{\text{max}} = 16 \) convergence has not yet been reached, the general trend seen in Fig. 2, suggests that it is unlikely, that a tetraneutron, bound with respect to the \(^2\text{n} + ^2\text{n}\) decay, may exist. To get a bound tetraneutron, the Majorana parameter \( m \) should be changed from its standard value of 0.6 to \( m = -0.2 \). Such a change provides \( \text{E}(^2\text{n}) = -1.2 \) MeV and does not influence the binding energy of \(^4\text{He}\), however, it binds \(^4\text{H}\) and \(^5\text{He}\) with respect to the \( t + n \) and \(^4\text{He} + n \) thresholds by 7 MeV 25 MeV respectively.

And finally, calculations with the realistic two-body isospin-conserving central NN interaction Argonne v4 \[19\] have been performed as well and the corresponding hyperradial potentials \( V_{\text{diag}}(\rho) \) are shown in Fig. 1. These purely repulsive potentials have almost converged and their behaviour excludes any possibility of either a bound or a resonance state.

**III**

The results of the theoretical calculations suggest that the tetraneutron must be unbound and most likely should not exist as a resonance, at least when only two-body central forces are considered. It is remarkable that even when an effective NN potential binds the dineutron (the case of V1), it still cannot bind two dineutrons, although a resonance in such a system should exist. In order for the two-body central force to be able to bind the tetraneutron, a huge unphysical attraction should be introduced in the triplet odd potential. Such an attraction would strongly overbind \(^4\text{H}\), \(^5\text{He}\) and other known \( A > 4 \) nuclei. The reason for necessity of huge attraction lies in the relative number of nucleon pairs in even and odd states allowed by the Pauli principle. For the tetraneutron with \( L = 0 \) and \( S = 0 \) the probabilities \( P \) to find a pair of nucleon in the singlet even (01) and triplet odd (11) states are equal, \( P_{01} = P_{11} = \frac{1}{2} \). For the tetraneutron with \( L = 1 \) and \( S = 1 \), these probabilities are \( P_{01} = \frac{1}{5} \) and \( P_{11} = \frac{4}{5} \). The NN force is not sufficiently attractive in the singlet even state, and is repulsive in the triplet odd state, which leads to the absence of the tetraneutron. For comparison, for the \( L = 0, S = 0 \) state of the closed shell nucleus \(^4\text{He} \) \( P_{10} = P_{01} = \frac{1}{2} \) and for the \( L = 1, S = 1 \) state of \(^4\text{H} \) \( P_{10} = P_{01} = P_{11} = \frac{1}{3} \). The binding energies of \(^4\text{He} \) and \(^4\text{H} \) are 28.3 MeV and 5.2 MeV respectively. It is clear that the binding of a nucleus is strongly correlated with the probabilities to find a pair of nucleons in even states, especially in the triplet even state (10) where the \( n + p \) bound state exists.

**IV**

The fact that, despite the theoretical predictions made with central two-body forces, six events \(^{14}\text{Be} \to ^{10}\text{Be} + ^4\text{n} \) have possibly been observed, means that either non-
central interactions and/or three-nucleon (3N) forces, and/or a strong four-nucleon (4N) force may bind the tetraneutron. If not, then a different interpretation of the six events from Refs 1, 2 should be looked for.

The first possibility should be rejected, because the calculations of Ref. 10 indeed included the spin-orbit and tensor forces. The second possibility does not look convincing either because the contribution of the 3N potential to the binding energies of the neutron drops $\tau_n$ and $\tau_8$, calculated with Green's function Monte Carlo Methods in Ref. 20, is about 1 to $5\text{ MeV}$ which is small compared to $4\text{He}$ and to other $A = 7$ and $A = 8$ nuclei. A similar contribution from the 3N force would be too small to bind the tetraneutron. Therefore, if what has been seen in the $^{14}\text{Be}$ experiment was indeed the tetraneutron, it could indicate at presence of a 4N force.

At present, the results of the ab-initio calculations of the $3 \leq A \leq 8$ nuclei suggest that a 4N force is not needed to fit the observed binding energies of light nuclei at the 1% level 24. Therefore, either the 4N contributions are smaller than 1% for these nuclei or parts of the 3N forces are masking up their effects 24. On the other hand, the latest models of the 3N force used to calculated the binding energies of these nuclei have not yet been tested in the description of polarization observables of the low-energy $Nd$ scattering where the contribution of the 3N force is very important 21, 22. Besides, the latest numerically accurate calculations employing a 3N force do not reproduce the proton analyzing power for the $p^{3}\text{He}$ scattering 23. The simultaneous fit of the $Nd$ and $p^{3}\text{He}$ polarization observables and of binding energies of the lightest nuclei may leave room for a 4N force.

In order to get an idea of what strength is needed for the 4N force, the HSFM calculations of this paper have been repeated with the RT potential using a 4N force simulated by the potential $V_{4N}(\rho) = W_0 e^{-\alpha \rho}$. The values $\alpha = 0.7, 1.2 \text{ and } 1.5 \text{ fm}^{-1}$, used in calculations, were the same as the range of the phenomenological term of the 3N spin-orbit force introduced in Ref. 21. The tetraneutron becomes bound if the corresponding values $W_0$ are equal to $-410, -1460$ and $-2530 \text{ MeV}$ respectively. These values are two orders of magnitude larger as compared to the values $-1, -10, \text{ and } -20 \text{ MeV}$ obtained for the 3N spin-orbit force 21. For the Volkov potential V1, these strengths are similar: $W_0 = -320, -1565$ and $-2900 \text{ MeV}$. If the same 4N force existed in $^{4}\text{He}$, the binding energy of $^{4}\text{He}$, calculated with V1, would be $-88, -82$ and $-134 \text{ MeV}$ respectively. Therefore, the 4N force, if it exists, should be strongly $T$-dependent. In principle, such a strong $T = 2$ 4N force could be noticeable in $A - Z \geq 4$ nuclei, especially near the neutron edge of stability, where the number of the $T = 2$ four-nucleon states increases. However, HSFM calculations of the neutron-rich helium isotopes $^{6,8}\text{He}$, performed with V1 21, do not indicate a need for a strong 4N force.

To get a real understanding of the role of the 4N force, ab-initio calculations of the tetraneutron with realistic 2N and 3N forces must be performed. The HSFM is the best method for such calculations because it is well suited to systems without bound subsystems, which is the case for the tetraneutron, and because it can also treat a four-body continuum.

Since the discovery of the tetraneutron would most certainly require a revision of modern realistic models of the three-nucleon interaction, it is extremely important to confirm or reject the results reported in Ref. 1. Therefore, independent searches for the tetraneutron in alternative reactions should continue. Among such reactions, $\alpha$ transfer reactions $A(8\text{He},4\text{n})A + \alpha$ deserve special attention. Using a light target and detecting a product nucleus $A + \alpha$, one can study, not only a hypothetical...
tetranucleon bound state but also the four-nucleon continuum as well.

The spectroscopic factor $S$ for the $(^8\text{He})_4^{14}\text{He} \otimes 4\ n$ overlap can be estimated using the translation invariant shell model [24]. If the probability amplitudes of the $|L = 0, S = 0\rangle$ and $|L = 1, S = 1\rangle$ configurations are $\alpha_0$ and $\alpha_1$ in $^8\text{He}$ and $\beta_0$ and $\beta_1$ in $^4\ n$ respectively, then $S = \left(\sqrt{2} \alpha_0 \beta_0 + \sqrt{2} \alpha_1 \beta_1\right)^2$.

The choice of the target $A$ must be determined by kinematic and spectroscopic considerations. It should be light enough for residual nucleus $A + \alpha$ to come out of the target and its wave function should overlap strongly with the $A + \alpha$ wave function. The obvious targets of this type are the deuteron, triton, $^3\text{He}$ and $^4\text{He}$. The list of other light targets with large spectroscopic factors $S_\alpha$ is given in Ref. [27].

As an example, in Figs. 3 and 4 the DWBA $\alpha$-transfer cross sections of the $d(^8\text{He},^4\ n)^6\text{Li}$ and $^{12}\text{C}(^8\text{He},^4\ n)^{16}\text{O}$ reactions are shown as a function of the four-body binding energy of the hypothetical tetraneutron. The energies of these reactions have been chosen based on availability of optical potentials in the entrance and exit channels, which have been taken from [20] and [27]. In addition, the energy choice for the $^{12}\text{C}$ target was also motivated by experimental measurements of the similar reaction $^{12}\text{C}(^6\text{Li},\alpha)^{16}\text{N}$ which revealed very large cross sections at small angles, about 25 mb/sr

The latest theoretical value $S_\alpha = 0.87$ [28] has been used for $^6\text{Li}$ and $S_\alpha$ for $^8\text{He}$ was taken to be $\frac{1}{2}$. According to Figs. 3 and 4, the $d(^8\text{He},^4\ n)^6\text{Li}$ reaction is better suited for the tetraneutron search because its cross sections are not strongly influenced by unknown tetraneutron binding energy. The cross sections and $^8\text{He}$ beam intensities make this experiment feasible in the very near future. In contrast, the cross sections of the $^{12}\text{C}(^8\text{He},^4\ n)^{16}\text{O}$ reaction decrease dramatically with increasing the binding of the tetraneutron and become small if the tetraneutron is bound.

Summarizing, due to the large probability for a pair of neutrons to be in the triplet odd state, the two-body nucleon-nucleon force cannot by itself bind four neutrons, even if it can bind a dineutron. A very strong phenomenological $T$-dependent four-nucleon force is needed in order to bind the tetraneutron. Alternative experiments such as $(^8\text{He},^4\ n)$ are proposed to search for the tetraneutron.

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