Coulomb Excitation of $^{206}$Hg at Relativistic Energies

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1 Abstract

The region of the nuclear chart surrounding the doubly-magic nucleus $^{208}$Pb provides a key area to constrain and develop contemporary nuclear structure models. One aspect of particular interest is the transition strength of the first excited $2^+$ state in even-even nuclei; this work describes the measurement of this value for the case of $^{206}$Hg, where the $Z=80$ line meets the $N=126$ shell closure.

The nuclei of interest were synthesized using relativistic-energy projectile fragmentation at the GSI facility in Germany. They were produced in the fragmentation of a primary $^{208}$Pb beam at an energy of 1 GeV per nucleon, and separated and identified using the Fragment Separator. The secondary beams with an energy of 140 MeV per nucleon were Coulomb excited on a secondary target of 400 mg/cm$^2$ gold. Gamma-rays were detected with the Advanced GAamma Tracking Array (AGATA). The precise scattering angle for Doppler-correction was determined with position information from the Lund-York-Cologne-CAlorimeter (LYCCA).

Using the sophisticated tracking algorithm native to AGATA in conjunction with pulse-shape analysis, a precise Doppler-correction is performed on the $\gamma$ spectra, and using a complex n-dimensional analysis, the B(E2) value for $^{206}$Hg is extracted relative to the known value also measured in $^{206}$Pb. A total of 409 million $^{206}$Hg particles were measured, and a cross-section of 50 mb was determined for the $2^+$ state at 1068 keV. The measurement of the B(E2) transition strength was found to be $1.109^{+0.031}_{-0.029}$ W.u. This result is compared to a number of theoretical calculations, including two Gogny forces, and a modified shell model parametrization and is found to be smaller than all calculated estimations, implying that the first excited $2^+$ state in $^{206}$Hg is uncollective in nature.
2 Acknowledgements

Those who contribute to a large project such as this one are almost too many to count, though as is the nature of modern physics experiments, the sum of their contributions is greater than any could achieve as an individual. Without the help of these professors, fellows, postdoctorates and PhD students none of this would have been possible, and I would like to extend my gratitude to everyone involved in the AGTAA-PreSPEC campaign at GSI.

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3 Introduction

Atoms are composed of neutrons, protons, and electrons, in a structure that was first described by the Rutherford model in the early 1900s. At the center of a cloud of electrons is the nucleus, which consists of neutrons and protons. The positive charge of the protons allows the atom to retain electrons, and a net charge of zero. This same charge however, results in a repulsive force between the protons, which must be countered and overcome if the nucleons are to remain together. This leads to the introduction of the strong force, which is exerted and felt by both neutrons and protons. In the resulting formation, these constituent particles are neither dormant nor stationary. The nucleus is a non-trivial configuration, and various details and intricacies of the structure, and the interactions of neutrons and protons inside it have far-reaching consequences: The line of stability, the formation and abundance of elements, and even the macroscopic scale of stars.

Understanding the nature of the strong force requires an examination of both the nucleus, and its constituent particles. The field itself contains a wide range of approaches to study an array of phenomena observed that already describe the nucleus as a complex system that is more than a simple, spherical collection of idle particles. The study of nuclear structure commonly resides in γ-ray spectroscopy, which studies the transition between energy states within the nucleus. The relative energy, ordering, spin and lifetimes of these states describes key aspects of the nucleus, such as the deformation. Furthermore, studies such as those performed on mirror nuclei can reveal the vital differences between protons and neutrons, by examining isotopes with matched, and swapped numbers of each.

This work narrows the large scope this endeavor to the region of $^{208}$Pb, a nucleus that resides on an intersection point of two magic numbers; N=126 and Z=82. This results in a very stable and spherical nucleus, which makes approximating the basic properties of this nucleus far simpler. On this basis, of approximating a core of potential, and
describing the properties of the outermost nucleons, theoretically calculations can begin to calculate the observable properties of nuclei in the area much more easily. The case that will be focused on is that of $^{206}$Hg, and the measurement of the transition strength. This value describes the presence of collectivity in the energy state. This value has been measured in other even-even nuclei along the line at $Z=80$, however the nucleus $^{206}$Hg has been yet unmeasured, and lies on the N=126 shell closure, making this measurement of significant interest.

The measurement described in this work utilises Coulomb excitation at relativistic energies. The requirements of such an experiment are high, and demand extreme precision in $\gamma$-ray spectroscopy, and careful attention to detail in calculations and corrections of the trajectory of the particle as it interacts with the target. Such requirements are only met through the use of new detectors and technologies, and thus this experiment serves to demonstrate the capabilities of these detectors. Utilising new techniques in deducing time-of-flight and particle positions, the Lund-York-Cologne-Calorimeter (LYCCA) is combined with the Advanced Gamma Tracking Array (AGATA), which allows the position of individual interactions and scatters of $\gamma$-rays to be measured within the crystal using electronic segmentation and pulse-shape analysis.
4 Theory

4.1 The Shell Model

To understand the nature of the motivations supporting the work described here, it is important to include some detail concerning our understanding of the nucleus, and how we interpret our existing observations into a depiction of structure. Our first impressions of the nucleus stem from systematic observations across the nuclear landscape, in particular along the line of stability; amongst the most prominent of these is the binding energy of the last nucleon, which changes significantly for particular numbers of protons and neutrons (counted separately). This attribute is reflected further still in the relative abundances of nuclei observed throughout the universe. The low neutron-capture cross-section of heavy nuclei is dramatically affected by such aspects, and thus the rate of synthesis of heavy elements is influenced to a vital degree by this feature of nuclear structure. Figure 1 shows the separation energy of the last two neutrons for even-even nuclei, these “magic numbers” are signs of increased stability, and exist independently for both neutrons and protons.

The excitation energy of first-excited states can also be utilised to reveal these magic numbers, with systematic measurements made for even-even nuclei, which decay by electromagnetic means, releasing γ-rays which can be detected and measured. The experimentally determined magic numbers are 2, 8, 20, 28, 50, 82 and 126. These magic numbers operate independently for protons and neutrons, and as such there are only a few observed nuclei that are considered ”doubly-magic”. In order to describe the effects of these numbers, we are led to use a shell model. Since the scale of the nucleus is of the order of $10^{-15}$ m (1 Fermi), nucleons are best described by quantum, rather than classical mechanics. As such, the aspects of a nuclear wavefunction are commonly described in terms of the principal quantum number $N$, the orbital angular momentum $l$, the projection of that angular momentum along the symmetry axis $m$, the intrinsic angular momentum, (often called spin), $s$, and finally the total angular momentum $j$ [3].
Figure 1: Known magic numbers are 2, 8, 20, 28, 50, 126 [1]. The binding energy drops significantly at each of these, more so than anticipated by the classical approach of Niels Bohr’s liquid-drop model [2]. The 2n separation energy is shown here to remove the odd-even staggering that arises from a pairing interaction between nucleons.

The particles within a nucleus are described as a collection of point-like bodies, each of which can move between discreet energy levels, and together forming a system with observable properties that can be approximated by the characteristics of the outermost nucleons. Those nucleons in core of the system tend towards a more stable and spherical function; a summation of contributions from this system is described by the mean field. This provides a significant simplification to deducing the electromagnetic transitions, which would otherwise involve resolving all possible combinations of momentum couplings and nucleon-nucleon interactions [1]. The mean field and residual interaction ($\hat{H}_0$ and $\hat{V}_r$ respectively), can be written as

$$\hat{H} = \hat{H}_0 + \hat{V}_r$$  \hspace{1cm} (1)
\[ \hat{H}_0 = \sum_i (\hat{T}_i + \hat{V}_i) \]  
\[ \hat{V}_r = \sum_{i<j} \hat{V}_{i,j} - \sum_i \hat{V}_i \]

where \( r \) is the radius of the field, and both \( i \) and \( j \) take values from 1 to the total number of nucleons \( A \). The resulting arrangement of shells in the nucleus from using this potential is shown in Figure 2. While this approach does reproduce the first magic numbers 2, 8 and 20, deviations can be observed soon after; the clear predictions of shell closures at 40, 58, 92 and 112 are incorrect. It is therefore necessary to make improvements to this simple depiction of the nucleus in order to adhere to experimental evidence. The mean-field potential itself is the first aspect to be examined; the Woods-Saxon potential is favoured here over the harmonic oscillator or infinite well. The main features of this potential provide a more realistic view of the nucleus by utilising a steep but tapered gradient, which approximates the distribution of nuclear density with respect to radius more reliably. This also produces an accurate description of the nuclear force itself, exerting a strong attractive force on those nucleons near the outside of the nucleus, but at the same time maintaining the short-ranged nature of that force [1].

\[ V(r) = -V_0 \left[ 1 + \exp \left( \frac{r - R_0}{\alpha} \right) \right] \]

Where \( R_0 = r_0 A^{1/3} \); a measure of the nuclear radius, with \( r_0 = 1.2 \) fm, and \( A \) is the number of nucleons. The value \( r_0 \) is determined from electron scattering measurements, which can be used to determine the nuclear charge radius for heavy nuclei [4]. Fully characterising the shape of this potential therefore requires measurements of the mean radius, and the ‘skin thickness’ [1]. While this potential is an improvement over the infinite well or harmonic oscillator, it still does not reproduce all of the known magic numbers. In order to accurately predict the higher magic numbers, it is necessary to account for other interactions between the nucleons within the nucleus, in particular the
spin–orbit coupling. This potential can be written as

\[
\frac{1}{\hbar^2} V_{so}(r) l \cdot s
\]  

(5)

where \(l\) and \(s\) are the angular momentum and spin operators for a nucleon. The introduction of this spin-orbit term therefore splits levels with \(l > 0\); and produces a more negative total potential when the spin of a nucleon is aligned with the angular momentum, and a more positive potential when it is anti-aligned. The result of this approach is the correct reproduction of the magic numbers, which are now better described as closed shells [3].

Figure 2: The energy levels calculated for cumulative nucleons. When the spin-orbit interaction is introduced, any level with \(l > 0\) splits the level into two. The known magic numbers are accurately reproduced [1].
The arrangement and configuration of sub-shells in this model are flexible, with respect to parameters such as those relating to the deformation of the nucleus. An example of subshell ordering is shown in Figure 3. Even in nuclei close to the closed shells at magic-numbers, it is observed that these configurations must be carefully examined with relation to the nuclei in the vicinity, by examining the energy and multipolarity of transitions. In cases where the parity ($\delta\pi$) changes between states, transitions are further restricted by the stipulation: $\delta\pi = (-1)^L$ for electric transitions, and $\delta\pi = (-1)^{L+1}$ for magnetic transitions. When parity does not change, even-electric and odd-magnetic transitions are favoured. While one transition mode is favoured, others that abide by the aforementioned conditions are not impossible. The ratio of these transitions is described by the mixing ratio. This can be applied to the single-hole nucleus $^{207}$Tl, and extrapolated further to $^{206}$Hg.

The ordering of energy shells in the vicinity of $^{206}$Hg is provided by a study of the single-particle states in $^{207}$Tl [6]. The level scheme of $^{207}$Tl is shown in Figure 4. By evaluating the angular distribution of observed $\gamma$-rays, the spin of each state can be determined. It is not uncommon to refer to energy state configurations in terms of the absence of nucleons from an otherwise complete subshell. The spin of the lowest state will describe the highest orbital, which will have one proton hole present, as the subshell is complete save for a single proton. The first excited state will describe the excitation of a proton from a lower-lying orbital to this hole, which in turn will leave a single hole and therefore, the spin of the state will entirely describe the spin of the next orbital down in the shell structure. The energy of the observed transition, and the prompt emission of the associated $\gamma$-ray and the angular distribution agree well with the stipulation that this would be an M1 transition, which describes to us a $d_{3/2}$ orbital situated below an $s_{1/2}$ orbital. A long-lived state lies above this, hindered as a spin-trap isomer. The lifetime of the state, as well as the energy provide good agreement with the expectation of a $h_{11/2}$ state [7]. The ordering of these three subshells allow us to fully construct a depiction of the low-lying states of nuclei in the surrounding area, including $^{206}$Hg.
Figure 3: The inclusion of a spin-orbit interaction, and adjustment of the shape of the potential allows the shell structure to agree well with experimental measurements of energy and spin [5]. Subshell ordering can be adjusted based on experimental data.

In the case of $^{206}$Hg, the low-lying energy structure is characterised by the presence of two proton-holes, situated beneath the closed shell at $Z = 82$. A level scheme of $^{206}$Hg can be seen in Figure 5. This provides the majority of freedom for excitation in the nucleus, while the neutron core remains unbroken until high energies (several MeV) are reached [9]. The ground state is described with the two proton holes beneath the $Z = 82$ shell closure situated in an $s_{1/2}$ orbital. The even number of nucleons with opposing spin-orientations in a shell creates the $0^+$ ground state spin; in $^{206}$Hg this fills the $d_{3/2}$ subshell. When examining the first excited state, there is good agreement between the observed energy and the expected energy of 1068 keV from the calculation for a $2^+$ state.
Figure 4: Partial energy level scheme for $^{207}$Tl [8]. On the right is shown the proton shell structure, and the promotion of protons from the $d_{3/2}$ and $h_{11/2}$ orbitals to the proton-hole in the $s_{1/2}$ orbital, to create the first and second excited states.

As per selection rules, a single proton state, promoted from the $d_{3/2}$ to the $s_{1/2}$ shell can produce a state such that the transition is $|J_i - J_f| \geq \lambda \geq |J_i + J_f|$, giving us an M1 transition. The observed transition lifetime is too short to be measured directly, which implies the transition itself is highly favoured, so this is considered to be a good fit.

Above this state there are a number of available options, however certain considerations make the spin of the second excited state clear: The lifetime of this state is 2.15 $\mu$s, and the energy is 2.1 MeV. Both of these measurements indicate the state is a 5- state, which would be a spin-trap isomer (discussed in section 2.4) with lifetime of the order 1 $\mu$s. This state is formed by the excitation of a proton from the $h_{11/2}$ orbital to the $s_{1/2}$, resulting in a total spin of 5-.

4.2 Collectivity and Transition Strengths

In the vicinity of closed shells, the shell model is remarkably successful at approximately describing the properties of nuclei. Key features and observations in describing these
nuclei include the energies of the first excited states, the ratio of those energies, and the strengths of the transitions. The ratio $R_{42} = E(4^+_{1})/E(2^+_{1})$ represents a vibrational state should the value be between 2 and 2.5, and a perfectly rotational state at its maximum value of 3.33 [1]. Rotational 'bands' can be characterised by a distinctly regular increase of energy between spin-consecutive states. These bands are observed in such deformed nuclei as $^{156}\text{Dy}$, where the odd and even parity states form two distinct bands of consistently increasing transition energy [10].

In addition to rotational and vibrational states, collective states can arise where a nucleus begins to deform from a spherical shape. A collective state is not a single particle state, but rather a state in which multiple nucleons are actively contributing to the total energy. An example of this is the first excited state in the doubly-magic nucleus $^{208}\text{Pb}$;
with two closed shells, this nucleus is particularly stable. The first excited state is a $3^-$
 at 2614 keV above the ground state, and decays via E3 $\gamma$ emission [11].

For nuclei situated near magic numbers, it is common for the interactions between
valence nucleons situated above closed shells, and nucleon 'holes', indicating particles
that are missing from shells, to be described by two-body matrix elements, which can
be adjusted empirically. As progression is made away from doubly-magic nucleus, other
parameters become necessary to describe the evolution of structure. Deformity can be
introduced to the Woods-Saxon potential ($V$) as

$$V(r) = -V_0 \left[ 1 + e^{\frac{r - R(r; \beta_2)}{\alpha}} \right]$$

(6)

In this case, $\beta_2$ describes the nature of the deformation, as well as the magnitude. The
subscripted 2 specifies this deformation more specifically as a quadrupole deformation;
more exotic shapes do exist than those formed from this addition. For negative values of
$\beta_2$, the nucleus takes of an oblate shape, and for positive values, a prolate shape. The
axis of a spheroid can be considered:

$$\beta_2 = \frac{4}{3} \sqrt{\frac{\pi}{5}} \frac{\delta R}{R_{av}}$$

(7)

Values of $\beta_2$ can take both positive and negative values with each end of this scale
concerning to oblate shapes and prolate shapes respectively (see Figure 6). While this
deformation parameter cannot be measured directly, in the case of a quadrupole-deformed
nucleus, the transition strength $B(E2)$ value can provide a significant limitation to the
range of this deformation, via the Quadrupole Moment $Q_0$. The transition strength
$B(E2)$ is a reduced matrix element used to describe the transition from an initial state
with spin $J_2$ to a final state with spin $J_1$, or $\langle J_1 | E2 | J_2 \rangle$, and can be written as:

$$B(E2; J_1 \rightarrow J_2) = \frac{5}{16\pi} e^2 Q_0^2 \langle J_1 K 2 0 | J_2 K \rangle^2$$

(8)
where $K$ is the projection of total angular momentum along the symmetry axis, and the angular momentum $J_1$ and $J_2$ can take values $\geq K$. When used in combination with a measurement of the nuclear spherical radius, the $\beta_2$ deformation parameter can be determined:

$$Q_0 = \frac{3}{\sqrt{5\pi}} Z R_0^2 \beta_2$$

(9)

This transition can be described in either direction, with the final state having higher or lower energy than the initial state. The reduced matrix element can be related to the transition probability of the state by the equation:

$$T_{fi}(\lambda L) = \frac{8\pi (L + 1)}{\hbar L ((2L + 1)!!)^2} \left( \frac{E_\gamma}{\hbar c} \right)^{2L+1} B(\lambda L; J_i \rightarrow J_f)$$

(10)

$L$ is the multipolarity of the transition from the initial state to the final state. The transition probability is related to the mean lifetime of the state as $T_{fi}(\lambda L) = 1/\tau$.

The type of this transition, denoted as electric or magnetic with $E$ and $M$ respectively, are determined by $2^L$, signified with the symbol $\lambda$. For any given nucleus, it is therefore
Figure 7: The energies of the first excited states in even-even nuclei around $^{208}\text{Pb}$ are well known, however measurements of the transition strengths are limited \cite{12} \cite{13} \cite{14}. It is possible to estimate the half-life of a state for each possible transitional multipolarity (see Appendix). This can be used as a guide when deducing the structure of a nucleus; E2 transitions are expected to be collectively enhanced in deformed nuclei. Weisskopf units are commonly used to describe the reduced matrix element in quantitative terms; for an E2 transition: $1WU = 5.94 \times 10^{-6} A^{4/3} e^2 b^2$. Transition strengths are generally expected to have values in the range of 10-50$WU$, however near closed shells, this estimation can be lower.

4.3 Coulomb Excitation

In the specific case of $^{206}\text{Hg}$, there exist a number of estimations of the reduced matrix element describing the transition strength of the decay of the first excited $2^+$ state. The energy of the $2^+$ state is well-known, and the low-lying energy structure of the nucleus fairly well-established, however to properly constrain calculations in the area, the lifetime of the states must also be known. From the estimations of the reduced matrix element, approximate lifetimes can be determined, which can in turn be themselves measured, allowing the calculations to be more precisely constrained.

Since all expectations suggest the decay is beyond the reach of typical (direct) lifetime measurements, a different approach must be employed: Coulomb excitation. This is the technique by which a known number of particles are excited via Coulomb interaction with...
a stationary target. The nucleus of interest then rapidly decays, emitting a $\gamma$-ray, which can be detected. The most probable mode is E2, and thus this method is widely used to study the low-lying states in even-even nuclei [1]. This time-limited window of detection allows us to deduce the transition rate with sufficient statistics. This method of study relies on the energy of the projectile being within a ‘safe’ range for only the Coulomb interaction to occur. Beginning with the expression for the Coulomb force:

$$ F = \frac{kq_1q_2}{r^2} $$ \hspace{1cm} (11)

Where $q_1$ and $q_2$ are the charges of the particles, $r$ is the distance between them, and $k$ is a constant of proportionality known as Coulomb’s constant. In the centre-of-mass frame, the Coulomb barrier between the two nuclei can be expressed as:

$$ V_{\text{Coulomb}}(\text{MeV}) = 1.44 \frac{A_1 + A_2}{A_2} \frac{Z_1Z_2e^2}{1.25(A_1^{1/3} + A_2^{1/3} + 5\text{fm})} $$ \hspace{1cm} (12)

As the distance of closest approach nears the nuclear radius, the two nuclei begin to interact with the nuclear force, at which point the Coulomb interaction can no longer accurately describes the cross section [1]. In order to minimise the probability for further interaction between the two nuclei, we can stipulate the ‘safe’ energy of a given projectile. For this we use an upper limit at the DeBroglie wavelength ($\lambda$ in Equation 13); as a general rule of thumb, the distance of closest approach to the nucleus must not be less than twice this wavelength:

$$ b(\theta) = a_0(1 - \frac{1}{\sin(\frac{\theta}{2})}) $$ \hspace{1cm} (13)

$$ \frac{b(\theta = 180)}{2\lambda} = \frac{Z_1Z_2e^2}{h\nu} =: \eta >> 1 $$ \hspace{1cm} (14)

Where $\eta$ is otherwise known as the Sommerfield parameter, which approximately describes the reaction rate between the two nuclei. In the case of $^{206}\text{Hg}$, this implies a maximum safe energy of 7.9 MeV per nucleon. The differential cross-section for excitation
of the nucleus from initial state $\ket{i}$ to a final state $\ket{f}$ is given by

$$\left(\frac{d\rho}{d\Omega}\right)_{sf} = \left(\frac{d\rho}{d\Omega}\right)_{Ruth} \dot{P}_{sf}$$

(15)

where $P_{sf}$ is the transition probability, $\rho$ is the density and $\Omega$ is the cross-section; the term denoted with the subscript $\text{Ruth}$ denotes the differential cross section for Rutherford scattering [15]. $\left(\frac{d\rho}{d\omega}\right)_{\text{Ruth}}$ is defined as

$$\left(\frac{d\rho}{d\omega}\right)_{\text{Ruth}} = \left(\frac{a_0}{2}\right)^2 \sin^{-4}\left(\frac{\theta}{2}\right)$$

(16)

where $a_0$ is the

There is a linear relation between this Coulomb excitation cross-section and the reduced matrix element (transition probability). Using this relation the reduced matrix element can therefore be derived by measuring the cross-section of a transition, either by directly evaluating the number of excitations per interacting nucleus in a system of known efficiency, or by measuring the cross-section relative to a previously measured transition. The latter method is utilised in this work, deducing the associated cross-section in $^{206}\text{Hg}$ by evaluating the ratio of the integral of Coulomb excitation events to that of the known transition in $^{206}\text{Pb}$. A level scheme for this nucleus is shown in Figure 8.

In this case of Coulomb excitation of $^{206}\text{Hg}$, a variation termed ‘Intermediate-energy’ Coulomb excitation is utilised, characterised by energies significantly above this ‘safe’ range. This technique is utilised when a nucleus of interest is far from stability, and cannot be produced without in-flight separation. This method of production, discussed in Section 2.4, relies on fragments having high velocity ($\beta = 0.25 - 0.65$).

The high velocity of the resulting fragment of interest forces complications on the typical measurement of Coulomb excitation, in particular a very large Doppler-shift of the $\gamma$-peak. In addition to this, the cross-section for Coulomb excitation drops at higher energies; an example is shown in figure 9.

Since the number of excitations that will occur is the product of this cross-section
Figure 8: A partial known level-scheme of $^{206}$Pb [9]. Transition energies are shown in units of keV.

and the target thickness, it can be favourable to use this method; the higher energies often required to produce the exotic nuclei facilitate the use of thicker targets, which counteracts the reduction in cross-section due to the higher energy.

$$N_x = \sigma_{i \rightarrow f} N_B N_T \epsilon$$  \hspace{1cm} (17)

Where $\sigma_{i \rightarrow f}$ is the Coulomb-excitation cross section from initial state $i$ to final state $f$, $N_T$ is the target density, $N_B$ is the number of beam particles, and $\epsilon$ is the efficiency of the measurement system. Particles that are Coulomb-excited in this way will scatter according to the distance of closest approach. For experiments utilising low-energy Coulomb excitation, the scattering angle of nuclei can be used to remove instances where
Figure 9: Cross-sections for Coulomb excitation of $^{40}\text{S}$ incident on a gold target, for a range of beam energy. In addition to excitation of the $2^+$ state is the giant dipole resonance (GDR) and giant quadrupole resonance (GQR) [16].

the projectile nucleus has interacted too closely with the target. For Coulomb excitation:

$$\theta_{lab} = \frac{2Z_1 Z_2 e^2}{m_1 c^2 \beta^2 \gamma b}$$ \hspace{1cm} (18)

Where $m_1$ is the rest mass of the projectile, and $b$ is the distance of closest approach between projectile and target [17]. In the case examined in this work, $^{206}\text{Hg}$ incident on $^{197}\text{Au}$ at an energy of 140 MeV has a grazing angle of 21.3 mrad.

**4.4 Relativistic Energy Fragmentation**

Fragmentation is the result of energetic interactions between nuclei with many constituent nucleons. This method is widely used in modern nuclear structure physics experiments as a method of producing exotic nuclei of interest [18]. When describing the collision of free nucleons, certain stipulations can be quickly made, but limitations of these must be
Figure 10: A depiction of fragmentation of a $^{238}$U projectile on a $^9$Be target particle [21].

also considered: The cross-section for scattering is inversely proportional to the energy of the incident particle, and momentum transferred is proportional to $1/a$, where $a$ is the range of the nuclear force, (which is independent of the energy of the particles) [19]. In cases where the momentum transferred is small in comparison to the momentum of the incident particle, the nucleons cannot be considered free particles; the mean separation of nucleons within the nucleus is smaller than that of the interacting distance [20]. When the energy of the projectile nucleus is roughly 50 MeV/µ or greater, the projectile and target nuclei interact at a relative velocity that significantly exceeds the Fermi velocity of their constituent nucleons: In this case, the Abrasion-Ablation model successfully and simply describes the fragmentation in two separate stages [19].

The first stage is inelastic abrasion; nucleons are abraded from the projectile and target nuclei into an intermediate zone of overlap, sometimes described as a hot ‘fireball’. The remaining nuclei of the projectile and target are called ‘prefragments’, and consist of nucleons that for this approximation are merely spectators, while those nucleons within the fireball itself are responsible for the majority of the interaction. The number of nucleons involved in this way is determined by the impact parameter, with a higher impact parameter producing more peripheral reaction, with fewer nucleons involved. In this model, the target and projectile can be interchanged, and the physics of this stage of the interaction is unaffected [22]. Some extended abrasion models exist that include
a frictional interaction of spectator nucleons, to compensate for low expectations of the excitation energy after the removal of one or two nucleons from the target [23]. Following this stage is the Ablation stage; the resulting prefragments are now able to evaporate and ablate one or several nucleons (including deuterons and alpha particles etc.) as a means of de-excitation, as well as the emission of γ-rays [22]. The period of this de-excitation can range from $10^{-16}$s to $10^{-21}$s, depending on the energy involved; this process occurs faster at higher energies. Accurate estimations of each of these occurrences can be produced through a statistical examination, with the neutron-proton ratio and excitation energy considered. One problem of the more simplistic ‘clean cut’ abrasion model described previously is that the transfer of angular momentum is not included, [23] however the abraded mass is expected to have a dependency on the angular-momentum distribution of the nuclei [24]. The excitation induced in the prefragment can be estimated by the sum of the energies of the holes in the single-particle scheme of the initial nucleus, which are left by abraded nucleons. This estimation is favourable for the abrasion of only a few nucleons, while more significant reactions can expect more reliance of the calculation of a spin-cutoff parameter to describe the resulting excited state of the final nucleus resulting from the fragmentation. In peripheral reactions, the angular momentum of the nucleus will be lower, with the root-mean-square of the spin-cutoff parameter increasing to a maximum at roughly half the total number of nucleons of the prefragment. For such central collisions however, the abrasion-ablation model is less applicable [25].

4.5 Isomers

An isomeric state is a quasi-long-lived energy state within a nucleus. The lifetime of a given state has a strong dependence on the energy and multipolarity of the decay. In both principle and in practice, this infers that there exist states with significantly longer lifetimes, arising as a direct result of the configuration of energy bands within a nucleus. (It should be noted that given the arbitrary nature of the term, there is no defined point at which a state is considered “isomeric”, however a common lower-limit for such a term
Figure 11: An example of a spin-trap isomer on the yrast line. A large change in angular momentum ($I$) causes a change in the energy required, extending the lifetime of the state. Yrast is a term used to describe states that are the lowest energy for a given value of spin [26].

is a half-life of the order of 1ns.) Figures demonstrating the relation between lifetime and energy of various electromagnetic transitions can be seen in the appendix.

There are a number of different causes for isomeric states. One type of isomer is the spin-trap isomer, which arise from situations where a single-particle state undergoes a large change in angular momentum between the initial and final state. An example of this is the $5^-\rightarrow$ state in $^{206}$Hg (see Figure 5): The low-lying states in this nucleus are universally single-particle states, however given the configuration of energy bands available near the closed shell at $Z = 82$, in the decay from the 5- second excited state to the $2^+$ first excited state, a proton must transition from $h_{11/2}^-\rightarrow d_{3/2}^+$, which amounts to a drastic change in angular momentum ($\lambda = 3$). The resulting half-life of this state is $2.15\mu$s.

Another form of the spin-trap isomer is K-trap isomer. This isomer arises from a change in orientation of the spin vector. The quantum number K refers to the projection of total spin along the symmetry axis of the nucleus; naturally this requires that the nucleus be deformed, and only occurs in prolate shapes, that are far from the spherical and oblate shapes found near closed shells [27]. Since logically, the multipolarity of a transition for a single shape should be such that $\lambda \geq \Delta K$, a transition that violates this
rule is not only intrinsically hindered, but indicates either a shape change, or a breaking of the symmetry [3]. The existence of these isomers indicates that this rule is only favoured, and not strictly enforced.

An example of this is $^{178}$Hf [28], in which an isomeric state has $K^\pi = 16^+$, which decays into a state $K^\pi = 8^-$ with a half-life of 31 years [29]. The decay of this state is strongly hindered, by a factor of nearly 100 for each degree of K-forbiddenness: $v = \Delta K - \lambda$. The spin vectors of particles contributing to the state are aligned, creating the high value of $K^\pi$. The M4 transition between the 16+ and 12- states has $\Delta K = 16$, which is therefore significantly hindered to a degree of $100^8$ [30].
5 Experimental Details

5.1 Experimental Requirements / Overview

To facilitate the Coulomb excitation of $^{206}$Hg, a beam must be produced with the correct energy to produce the reaction, as well as the purity to minimise the background present from other sources. This represents a challenge in and of itself, as this cannot be produced directly with ease. Instead, fragmentation of a heavier particle can be employed, and the resulting products purified by a fragment separator; with sufficient cross-sections and efficiency of the fragment separation, secondary beams can be produced with sufficient intensity for such a measurement. For this experiment, a primary beam of $^{208}$Pb is provided by the SIS heavy-ion synchrotron at GSI (Gesellschaft für Schwerionenforschung Helmholtz Centre for Heavy Ion Research), near Darmstadt, Germany. The primary beam begins upon being extracted by a Penning Ionization Gas (PIG) ion source; the initial energy of the ions is very low, and are first sent through a linear accelerator (UNILAC) in order to raise the velocity of the beam to meet the acceptance of the SIS18 synchrotron. The SIS18 consists of 24 1.8 T dipole magnets, in a circle with an approximate radius of 34 m, allowing the system to accelerate beams with magnetic rigidities up to 18 Tm [31]. The dipole magnets are accompanied by triplet and sextupole

Figure 12: The GSI facility; the experiment described here takes place at the location marked 1 [31].
magnets for continuous re-focusing of the beam. The beam is accelerated inside the SIS18 by means of two radio-frequency resonator cavities; an oscillating electromagnetic field accelerates and bunches the beam, while the dipole magnets are ramped up to match the velocity accordingly. In the case of a beam of $^{208}$Pb, the ions will reach the SIS18 with an energy of between 3 and 13 MeV/$\mu$, and after the strength of the dipoles is completely ramped up, the beam is sent to the Fragment Separator (FRS) with an energy of 1 GeV/$\mu$, for synthesis and separation of secondary fragments [32].

This resultant primary beam is incident upon a primary production target of $^9$Be with a thickness of 2.5 g/cm$^2$. The resulting fragmentation produces a ’cocktail’ beam of many different nuclei. A Niobium stripper is utilised to minimise the presence of charge-states in this beam, to facilitate optimised separation of the fragments in the FRS.

5.2 Fragment Separator

The FRagment Separator (FRS) is chiefly comprised for a series of four 30 degree dipole magnets. These are used to refine the beam, and eliminate unwanted contaminating fragments by means of consecutive A/Q cuts. The initial velocity spread of the beam is minimal; ions with a specific $B\rho$ are guided through the magnetic field of the dipoles, which will spread the fragments according to their charge. Momentum acceptance of the spectrometer is approximately 2%. Quadrupoles are also used both before and after each dipole to provide first-order focusing. $B\rho$ is otherwise known as the magnetic rigidity, and is used as a general measure of the FRS magnet setting - it describes a particular desired ratio of A/Q, and for a given particle can be written as

$$B\rho = \beta \gamma c u \frac{A}{Q}$$

where $\beta$ is the velocity of the fragment as a fraction of the speed of light (v/c), $\gamma$ is the Lorentz factor, and A and Q are the mass and charge. The $B\rho$ for a given fragment can be calculated from the strength of the magnetic fields (B), radius of path curvature ($\rho$)
and measured x-position at the focal-plane. For a set of two sets of two dipole magnets, where each pair have average magnetic field strengths of $B_1$ and $B_2$ respectively, the $B\rho$ can therefore be measured and calculated as:

$$B\rho = \rho_0 \frac{B_2 + B_3}{2} \left(1 + \frac{x_{s4} - M_1 x_{s2}}{D_1}\right)$$

(20)

where $D_1$ is the dispersion parameter, and $M_1$ is the magnification, which are fixed for a given set of magnets. The x-position of the particle between the two sets of dipole magnets is denoted by $x_{s2}$, and after the final dipole magnets by $x_{s4}$. It can be extrapolated from this equation that a measurement of x-position will can produce an estimate of the A/Q. Since the momentum acceptance of each dipole magnet must be sufficient enough to allow for the intrinsic momentum spread of the secondary beam, some unwanted fragments can also be transferred through an initial A/Q cut. This can also occur when a contaminating fragment exists in a non-stripped charge-state. For example, a beam of $^{200}$Pt has an A/Q of 2.564, but the primary beam of $^{208}$Pb with a single electron has an A/Q of 2.568, making it particularly difficult to separate.

![Figure 13: A general diagram of the FRS [31].](image)

In order to address this, energy-degraders are used: The energy loss of a charged particle is due to the coulomb interaction with the electrons of the matter, and is described by the Bethe-Bloch formula [3]. Consequently, the energy of the beam is reduced proportional to $Z^2/\nu^2$, and hence additional cuts can be made, according to $\Delta B\rho/B\rho$. A
simple block degrader is available at the S1 position, while at the S2 position, a wedge degrader is used, which can be angled to produce achromatic, and monochromatic beams, which favour discreet spatial distribution and discreet energy distribution respectively. Monochromatic settings are favoured for secondary beams where minimising the momentum spread is a concern, such as those used in the experiment described here [31]. A summary of beam settings, including degrader thicknesses and magnet strengths can be found in the Appendix.

Figure 14: Beam profiles for an achromatic setting (top) and monochromatic setting (bottom). Dipoles are denoted sequentially as $D_1$ to $D_4$, and the focal planes are $F_1$ to $F_4$. The dashed line denotes a 1% deviation in momentum. As can be seen at the $F_2$ and $F_4$ focal points, the monochromatic setting will create smaller momentum spreads for each fragment, while simultaneously producing a wider beam in $x$ [32].

5.2.1 Scintillators

Scintillator detectors are used to measure the time-of-flight of the secondary beam as it travels through the FRS. The scintillators work by utilising a scintillating material: When a charged particle passes through the material, an electron is excited (and subsequently de-excited), and emits a photon. This photon is received by a the photocathode of a photomultiplier tube (PMT), where electrons are emitted via the photoelectric effect. The electrons are then focused toward a series of dynodes, each of which will release
a number of secondary electrons for each incident electron. These secondary electrons are passed to the next dynode in the chain, which are held at increasing potentials to maintain this amplification of the initial signal through several orders of magnitude [1]. Finally the electrons are passed to an anode, and a voltage can be measured. Figure 15 shows a simple diagram of a scintillator detector.

In the scintillators used in the FRS for measuring the time-of-flight of the particles, each piece of scintillator material is equipped with two PMTs are positioned on either side (in the x-plane of the beam), which are referred to as Left and Right signals. These can be used in combination to eliminate the intrinsic bias that would otherwise arise from the speed of the light travelling through the material to the PMTs.

In the event that all left and right signals are present, then an average of the time differences between the two left signals (termed left-left) and the two right signals (termed right-right) can be utilised to produce a more accurate time-of-flight for the particle. The time-of-flight measurement is used in combination with position and energy measurements in order to produce accurate estimations of the A/Q and Z of the fragments. There are four scintillator detectors bound to the FRS-identification of particles: Two in the S2 position (called Sc21 and Sc22), between dipoles D$_2$ and D$_3$, and two in the S4 position.
Figure 16: By using equal delays between left and right signals, the correct time-of-flight can be extracted for a particle travelling between the two scintillators, without the need for estimations of arbitrary timing offsets.

(called Sc41 and Sc42), after the last dipole magnet D₄.

5.2.2 Time-Projection Chambers (TPC)

Time-Projection Chambers (TPCs) measure the position of the fragments in x and y. These detectors operate using a gas-filled chamber, to which an electric field is applied. When a charged particle passes through the gas, it is partially ionised, causing a current to flow between the anode at the bottom of the chamber, and the high-voltage cathode at the top. The position of a particle in Y can be determined using the electron drift time along the field of the cage:

\[ y = w_d t_d + y_{off} \]  

(21)

Where \( t_d \) is the anode signal, and \( w_d \) and \( y_{off} \) are calibration constants. The X position of the particle is determined by the time difference between left and right sides of the delay line.

\[ x = w(t_l - t_r) + x_{off} \]  

(22)

Where \( t_l \) and \( t_r \) are the left and right times, and \( x_{off} \) is a calibration constant. When used in combination with the time-of-flight measurement from the scintillators 21 and 41, the A/Q of a given fragment can be deduced, as the path-radius can now be determined,
thus providing a measurement of “effective $B\rho$” (see equations 18 and 22).

$$A/Q = B\rho/(\beta\gamma c)$$

(23)

5.2.3 Multi-Sampling Ionisation Chamber (MUSIC)

Multi-Sampling Ionisation Chambers (MUSICs) provide energy-deposition measurements of each fragment, which is proportional to the $Z^2/v^2$ of a particle. This allows the $Z$ of a given fragment to be calculated, regardless of the charge-state, which makes this measurement particularly useful for eliminating unwanted fragments that would have otherwise been removed by the A/Q cuts from the dipole magnets. As the name implies, a MUSIC detector consists of a chamber of ionised gas, which is held at a constant pressure (1 bar), in a homogeneous electric field. When the particle moves through the gas, electron-hole pairs are created, which causes a current to flow between the cathode at the base of the chamber, and the eight anode strips at the top of the chamber. The total length of the gas chamber is 420 mm, and each anode has an active length of 50 mm. The position-dependence of the interaction is removed by the presence of a Frisch grid - a plane of thin wires at an intermediate potential within the chamber, situated just beneath the anodes at the top of the chamber. The Frisch grid is transparent to the flow of electrons, and ensures a consistent response for a given energy deposition. The anode signal is then passed to a preamplifier, and voltage recorded. The MUSIC80 detectors in use here are expected to be capable of operating in rates of up to 200 kHz, however if the experiment described here the rates are significantly below this [33].

While any of the anodes can be used to provide a measurement of the energy deposition, it is more accurate to instead utilise all present signals to produce a measurement (improvements proportional to $\sqrt{N}$, where N is the number of measured signals). The $\Delta E$ is used in conjunction with the measured time-of-flight to produce an accurate representation of the $Z$ of a given particle. Once calculated, this value can be used in combination with the calculated $A/Q$ of a particle to identify and separate fragments
Figure 17: The internal layout of a MUSIC detector [33].

during the analysis.

5.3 LISE++ Simulations

Integral to the experiment itself are the simulations of the secondary beam. LISE++ is software that allows the optics of a fragment separator to be modeled quickly and easily, such that all relevant aspects of fragments can be estimated: Energy loss in degraders and detector materials, ion trajectories through dipole magnets, contaminant spread at focal positions, degrader angle for achromatic and monochromatic settings, momentum spread at target positions. The energy and rate of the primary beam is provided to a virtual model of the FRS, as well as the thickness of the production target. The magnet settings can be tuned for any fragment of interest; the population of each is estimated using cross-sections from an empirical database included in the software [34]. The layout of the FRS, along with precise measurement information between each piece of material in the beamline, is provided to the simulation. This information includes, for example, the window, and transitional material at the opening and exit of various detectors such as the MUSICs, where the active detector material is not the only material to interact with the beam. Since there are expectations of imperfections in the actual thicknesses of materials at key locations (S2 in particular), the effective thickness is measured at the start of the experiment. Similarly, the effective $B \rho$ must be determined by centering the
primary beam for a known energy. This technique is further detailed with the calibration of the FRS in section 4.3.4.

Using initial conditions (beam intensity, energy), the total thickness of the degraders used is adjusted such that the energy of the beam at the center of the secondary target is approximately identical for all secondary beam settings. The energy chosen for this experiment is 140 MeV/\(\mu\) at the center of the secondary target. This is amongst the most important stipulations of the simulation; if measurements of transition strengths are made at different energies, they cannot be accurately compared. A complete summary of the beam settings, including magnet settings, degrader angles and thicknesses, and expected beam energies at various stages can be found in the appendix.

The magnet settings are calculated to center the fragment of interest along the beam axis, however it is also possible to instead center the beam at the position of the slits, when separation of unwanted contaminants is an issue. The angle of the degrader can also be calculated for either achromatic, or monochromatic settings. In favour of uniform measurements of the transition strengths, a monochromatic degrader is favoured throughout this experiment.

The profile of the beam can be estimated at any stage, allowing the positions of the slits to be optimised to eliminate unwanted fragments. The population of charge states is also estimated, and accounting for these is of explicit importance in this type of high-energy experiment: Ordinary particle identification methods are already at their limit of utility, given the intrinsically high momentum-spread of the beam; the introduction of charge states can make accurate and reliable separation of fragments physically impossible without compromising beam intensity. An example of this is the removal of primary beam charge states in the \(^{200}\text{Pt}\) setting: The \(A/Q\) of \(^{200}\text{Pt}\) is 2.5641, and the \(A/Q\) of \(^{208}\text{Pb}^{81+}\) is 2.5679. The profile of the beam with and without a degrader at S1 is shown in Figure 18. The intensity of this primary beam charge state is sufficient enough to cause damage to the detectors at S2, which would severely hinder particle identification, so it must be removed before the beam reaches the scintillators. This is done by utilising a degrader.

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Figure 18: The x-position beam profile of the $^{200}$Pt setting at S2. The left image shows the beam without a degrader at S1, and the right image shows the beam with a 1 gram degrader at S1. The scale is logarithmic; without an S1 degrader, the beam is only a few millimeters from an intensity of several million particles per second, which is potentially enough to damage the detector material. With the degrader in place, contamination by the next charge state of $^{208}$Pb is favoured, which has a much more manageable intensity.

at S1, allowing two A/Q cuts to be made early. The total thickness of degrader material used remains unchanged; it can be distributed freely between multiple points without affecting the final energy of the beam. The profile of this secondary beam is shown with and without the S1 degrader in Figure 17.

The LISE++ simulations also provide the user with expectations of particle identification plots. If one or more structures can be identified in a particle identification plot, and that plot is correlated with the simulation, all other structures can be deduced by extrapolation. Identification of fragments is detailed in section 4.5, and makes use of isomeric tagging. In addition to using a thin target of 400 mg/cm$^2$ Au, a thick plastic stopper is used for each setting, for a short period of time. This stops the secondary beam completely in front of AGATA, allowing the delayed $\gamma$-ray emissions from isomeric states to be detected. Since for each detection, detector information about each particle remains, the profile and structure of the beam in each detector combination can be correlated: The simulation can be verified. Additionally, the isomeric ratio of each secondary beam can be calculated, which is involved in the calculation of the B(E2), later. The population of fragments for the settings of $^{206}$Hg and $^{206}$Pb at the S4 position can be seen in Figures 19 and 20.
5.4 Particle Tracking: LYCCA

The Lund-York-Cologne CAlorimeter (LYCCA) is a series of detectors designed to provide position and time-of-flight information in experiments studying the structure of exotic nuclei. The LYCCA-0 demonstrator, which is employed in this experiment, comprises of three scintillator detectors to provide time-of-flight measurements, an array of small double-sided silicon-strip detectors (DSSD) to provide energy loss and position measurements after the target, and an array of thick Caesium-Iodide detector modules are positioned at the end of the beamline, which give a final measurement of total energy for particle identification [35]. The time-of-flight measurement is crucial for an accurate Doppler-correction, as is the position information, so as to provide the outward angle of the fragment after interaction with the target. Position tracking of the fragments around the target can also provide a measure of scattering angle, which can be used to gauge the effective distance between interacting target and beam particles [36].

Time-of-flight detectors provided by LYCCA-0 consist of circular membranes of plastic scintillator material. Similar to the FRS scintillator detectors, photomultiplier tubes are
utilised to amplify the photon emissions caused by the fragments, allowing the time of the interaction with the material to be measured. By utilising multiple PMTs for a single interaction, the precision of the time measurement can be improved by taking an average of all of these measurements. The uncertainty in a single time measurement performed in this way is reduced: \( \sigma_{\text{total}} = \frac{\sigma_t}{\sqrt{N}} \), where \( \sigma_t \) is the uncertainty of each single measurement, and \( N \) is the number of simultaneous measurements of a given event by the PMTs [36].

The “Start” and “Target” detectors (positioned before and after the target, respectively) use a membrane of scintillator material that is approximately 25cm in diameter, and each of these detectors has a total of 32 PMTs. With all 32 signals present, the timing resolution can be as little as, and potentially lower than 30 ps [35] [36]. The “Target” detector is a smaller detector designed to fit more closely to the secondary target, and is positioned just outside the target chamber. A slightly different scintillator material is used in the Target-ToF detector, and only 12 PMTs are utilised.

The LYCCA-0 demonstrator utilises a large number of DSSDs to provide accurate position information around the secondary target, as well as energy measurements of the fragments, which can later be used for particle identification. This modular design favours flexibility, allowing the array to be arranged to meet the specific requirements of
experimental techniques such as intermediate-energy Coulomb excitation. Each DSSD module is 58 mm by 58 mm, and each side is divided into 32 strips of 0.3 mm thick silicon. The P side is divided vertically, providing segmentation relative to the X position, while the N side is divided horizontally. Due to insufficient available electronics, these strips are effectively doubled-up, reducing the overall position resolution of the detector system. By monitoring which P and N strips were hit, the position can be determined, accurate to the width of those strips. For 16 strips across a distance of 58 mm, the LYCCA DSSDs have a pixel size of 3.6 mm$^2$. Ionising fragments create electron-hole pairs in the silicon, which in turn causes a current to flow, which is proportional to the energy deposited. The energy lost by a beam fragment is proportional to $Z^2/v^2$. This measurement can be used in combination with the CsI detectors and the measured velocity of the particles in order to produce identification of particles after and around the secondary target position [35]. A single DSSD module is located just prior to the secondary target (58 mm) to provide position information as close as possible to the studied reaction. The “DSSD Wall” consists of 16 of these DSSD modules, and is positioned at the end of the beamline, roughly 3.6 m after the secondary target, such that the position resolution
of these detectors is sufficient to produce very precise scattering angles for the Doppler correction, and refinement of the data. The layout of the wall can be seen in Figure 23. The energy resolution (FWHM) of the DSSD detectors is roughly 1% $\Delta E/E$ for a particle with 0.1 GeV total energy [37].

CsI detectors form the last component of the LYCCA-0 demonstrator. Each DSSD Wall module is equipped with a small array of 9 CsI crystals placed 10 mm behind the silicon (see Figure 24). The beam is completely absorbed in this material, and the resulting scintillation light is measured using photodiodes that are well-matched to the scintillation emissions of the CsI. Each crystal is isolated with Extended Specular Reflectance (ESR) foil, and then again with Al foil, eliminating almost all cross-talk that could occur between crystals due to partial transparency of the ESR foil [37].

The energy resolution (FWHM) of each CsI crystal is $\Delta E/E<1\%$ at 10 GeV. The expected energy of the secondary beams utilised in this work are universally above this value (up to 40 GeV), and thus the resolving power this this detector as a whole is potentially reduced. The total energy of the fragments is directly proportional to their mass.
5.5 Gamma-Ray Spectroscopy: AGATA

Paramount to the accuracy of a Doppler correction is the angle $\theta$ between the scattered particle, and the emitted $\gamma$-ray. At the energy utilised in this experiment, the velocity of the particle at the point of interaction with the secondary target is approximately 0.5$c$, and the resulting Doppler shift of the peak is several hundred keV. At higher velocities such as this, the angular dependence becomes significant, and without precise measurement will result in a broadening of the peak. Figure 27 shows the broadening at different angles due to uncertainty in the measured angle between the emitting particle and the $\gamma$-ray. The precision of the Doppler correction must be such that the total counts in the peak must be condensed into a small enough width for it to be visible above the random fluctuations of the surrounding background. Since the energy of the beam is high, this background is expected to also be high, and so a high precision is called for, more than is provided by a typical high-purity Germanium detector.

The Advanced GAmma Tracking Array (AGATA) is a collection of high-purity germanium detectors combined with dedicated digital electronics to facilitate ambitious new experiments in the study of nuclear structure. AGATA is constructed of clusters, each with two or three electrically-segmented, close-end coaxial n-type high-purity germanium crystals. The crystals are 9 cm in length, and roughly 8 cm in diameter (varying on the three different crystal geometries), with an energy resolution of 2 keV at 1332 keV [38].
Compton-suppressors are commonly employed to eliminate events in which a detected γ-ray partially scatters out of the detecting material, however in the case of AGATA, these are simply replaced with yet more germanium, and the detector is analysed collectively, with each interaction deduced in terms of position, energy and time, to be reconstructed with a tracking algorithm. The utilisation of γ reconstruction using such tracking algorithms has the potential to provide dramatic improvements to signal-to-background ratios in experiments where Coulomb excitation is used [39]. A completed AGATA will consist of 120 to 180 crystals, depending on the geometry of choice [39] (Figure 25).

For intermediate-energy Coulomb-excitation the velocity of the emitting nucleus is near 0.5c, resulting in a significant Doppler-shift. Since this shift has an angular dependence, both the location and the energy of the γ-ray must be known with precision in order to accurately calculate the γ-ray energy in the emitting frame of reference. A typical high-purity germanium crystal lacks the sufficient angular resolution to perform this task, as the shifted γ-rays will be spread by the angular dependence over a range of several hundred keV.

To address this problem, pulse-shape analysis (PSA) can be employed. Each AGATA
Figure 26: Left: The 36-fold segmentation of an AGATA crystal [38]. Right: A larger potential configuration utilising 180 crystals, which is currently the favoured final geometry of AGATA. The different colours indicate the different crystal types, which have slightly varying hexagonal dimensions [39][40].

crystal is electronically segmented into 36, thus providing 37 signals, when including the core signal, which is split and amplified with both high and low gain settings. By examining the amplitude and shape of the signals present in each segment, (as well as the transients of those signals), the distance of each measurement from the interaction point within the crystal can be deduced. These shapes can be calculated with accuracy for the crystal geometries in use, and a virtual library of expected $\gamma$ interactions is utilised with which the signals can be decomposed to extract the true positions with a resolution of approximately 2mm. For each interaction point the energy can be deduced from the amplitude, and the sum total of energies from all interactions within a given crystal can be compared with the energy of the core signal for consistency; it is possible during this process to apply offsets to force the segment energies to properly sum to this value.

The time of each interaction can also be extracted from each segment signal, which are aligned to the core signal (this calibration is typically not performed by the user). An additional utility of the information the PSA yields is $\gamma$-ray tracking: Position and
Figure 27: The uncertainty in the energy resulting from imperfect angular resolution, of a Doppler-corrected $\gamma$-ray of 1068keV, emitted at 0.47c, at a radius of 24cm. There is a substantial increase in this error as angular resolution is lost.

Energy of interactions can be used to accurately reconstruct a scattered $\gamma$-ray, allowing multiple $\gamma$-rays incident in a single crystal to be separated, and a single $\gamma$-ray that has scattered between germanium crystals to be identified and reconstructed. For low-energy photons, the photoelectric effect is prevalent; the energy is absorbed entirely in a single interaction. For $\gamma$-rays of a few hundred keV or more, and in particular those of interest to this study, it is more likely for the energy to be deposited in multiple interactions through Compton scattering, which can allow the $\gamma$-ray to leave the crystal before fully depositing its energy, and in some cases, deposit energy in many crystals. In an ordinary detector array, this situation would lead to a loss of the measurement entirely, and potentially disrupt the precision of other $\gamma$-rays measurements [39].

While several $\gamma$-ray tracking algorithms exist, the Mars-Gamma-Tracking (MGT) code is favoured for this experiment. The energy and position of each interaction is considered, with a few characteristic stipulations with regards to initial conditions: It is likely that the highest deposition of energy is the first interaction point, and interaction points from the same $\gamma$-ray are likely to be localised. Clusters of interaction points are identified.
Figure 28: An example of decomposition of segment signals and comparison with those calculated to deduce an interaction location \[41\].

by spatial coherency, and the energies of each point in a candidate event are evaluated against the Compton scattering formula (see Figures 28 and 29), so that a figure of merit can be assigned to each combination. The permutation with the highest figure of merit is chosen, and this process is repeated until all interaction points are assigned. The efficiency of the tracking algorithm is determined empirically by the integral of a $\gamma$-peak: $\frac{\text{counts}_{\text{tracked}}}{\text{counts}_{\text{untracked}}}$. 25 Crystals comprise the AGATA-0 demonstrator currently in operation, however due to a number of technical problems, 17 were available for this experiment.

5.6 Data Acquisition

The data acquisition (DAQ) is separated into two systems. The output of AGATA is processed by dedicated digital electronics, and is interpreted and recorded by NARVAL (Nouvelle Acquisition temps Réel Version 1.6 Avec Linux). This is an acquisition frame-
work that facilitates the use of ‘Actors’ across an acquisition network that allows the data to be handled and processed in a distributed manner that is well-suited to the demands of AGATA [43]. The raw signals (core, both high and low gain, as well as the individual pulse-shapes of each segment) are recorded immediately, and are also forwarded to a local computing cluster to be processed through pulse-shape-analysis (PSA) whilst the acquisition is running. This allows the data to be very quickly analysed after the data is recorded, as the PSA ordinarily requires considerable computing time to complete. The NARVAL system also allows the user to view a sample of the data as it is processed.

The second half of the DAQ is that attributed to the FRS and all ancillary detectors. Raw analogue signal processing is employed with analogue-digital-converters (ADCs) and multi-hit time-digital-converters (mhTDCs) to preserve the resolution of signals prior to the data recording. The FRS side of the DAQ is a multibranch system (MBS), by which multiple processors can be utilised to handle data collection. The data is stored in a list-mode data format (lmd), which refers to the internal structure of the data file. A remote event server can be utilised to forward a sample of incoming data to the user as it is recorded, which can be utilised in particular for the calibration of the fragment.

Figure 29: The dependency of interaction modes with germanium on incident γ energy [40].
Figure 30: A graphical representation of the clusterisation tracking technique. Phi describes the azimuthal angle of the incident $\gamma$-ray; and theta is the polar angle. Circles denote clusters that have been correctly identified, and squares are clusters that have been incorrectly identified [42].

separator and its detectors [44]; the processing necessary to handle the complete volume of data can exceed what is available, so the full dataset cannot be analysed ‘live’.

The two sides of the DAQ are synchronised by use of a common, 100 MHz clock, which allows timestamps to be used to merge the data files from AGATA and the FRS later. Trigger information from the FRS is interpreted by the AGATA Ancillary VME Adapter (AGAVA), which provides a global timestamp (GTS) value that can be associated with recorded events. An event is accepted on the basis of several different trigger conditions to allow detector information to be forwarded to the acquisition only when enough measurements have been made to perform an accurate Doppler correction. LYCCA, and AGATA information are both needed to perform a full Doppler-correction, and the triggers primarily reflect this requirement. Also of note is trigger 10, which requires only Scintillator 41, but is filtered through a reduction factor of $2^8$ prior to any further trigger request being forwarded to the rest of the system. This allows an approximate total beam rate to be monitored without requiring the entire detection system to record data at that rate. A complete list of the triggers available is given in Table 1.
<table>
<thead>
<tr>
<th>Trigger Number</th>
<th>Trigger Contents</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Pulser Only</td>
</tr>
<tr>
<td>2</td>
<td>LYCCA Calibration</td>
</tr>
<tr>
<td>3</td>
<td>AGATA Calibration</td>
</tr>
<tr>
<td>4</td>
<td>HECTOR Calibration</td>
</tr>
<tr>
<td>5</td>
<td>FRS from TB</td>
</tr>
<tr>
<td>6</td>
<td>Particle+HECTOR</td>
</tr>
<tr>
<td>7</td>
<td>Particle+AGATA</td>
</tr>
<tr>
<td>8</td>
<td>Particle+HECTOR+LYCCA</td>
</tr>
<tr>
<td>9</td>
<td>Particle+HECTOR+AGATA+LYCCA</td>
</tr>
<tr>
<td>10</td>
<td>Particle (Scintillator41 )</td>
</tr>
</tbody>
</table>

Table 1: List of possible triggers. “Particle“ denotes a trigger from Scintillator 41. Those in use during the experiment were 8, 9 and 10. Trigger 1 is used for calibration and testing purposes; and allows a pulser to trigger event acquisition. Similarly calibration triggers allow single detectors to trigger event acquisition.

6 Data Analysis

6.1 Data Format and Analysis Framework

The data is received for analysis in three parts: The MBS data in list-mode data format (.lmd), and the AGATA data in both AGATA data format (.adf), and the raw pulse-shape information, which can be replayed again later if a new calibration is required (for example, realignment of the timing signals of each segment). AGATA files are organised in a common structure for all runs, and also include the geometry of the array necessary for replay of the pulse-shapes. Each AGATA run is numbered in chronological order, and the associated AGATA run number is included in the MBS filename for easy association. This association can be verified by extracting the first and last timestamps of an .adf and .lmd file. An initial summary of the data, as described by the file sheets kept during the experiment is given in Table 2.

6.2 Merging and Post-Merging Analysis

The merging of the data is performed by a matching of timestamps within a window of 20 \( \mu \)s. A strict approach with no ‘shortcuts’ with regards to trigger-matching, and instead
<table>
<thead>
<tr>
<th>Fragment</th>
<th>AGATA Run</th>
<th>Measurement</th>
<th>Additional Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{208}$Pb</td>
<td>06</td>
<td>Calibration</td>
<td>Centered at S4</td>
</tr>
<tr>
<td>$^{208}$Pb</td>
<td>07</td>
<td>Calibration</td>
<td></td>
</tr>
<tr>
<td>$^{208}$Pb</td>
<td>08</td>
<td>Calibration</td>
<td></td>
</tr>
<tr>
<td>$^{206}$Hg</td>
<td>36</td>
<td>Isomer</td>
<td>Plastic stopper in place for isomer study</td>
</tr>
<tr>
<td>$^{206}$Hg</td>
<td>40</td>
<td>Coulex</td>
<td></td>
</tr>
<tr>
<td>$^{206}$Hg</td>
<td>41</td>
<td>Coulex</td>
<td>MBS files mislabelled</td>
</tr>
<tr>
<td>$^{206}$Hg</td>
<td>42</td>
<td>Coulex</td>
<td></td>
</tr>
<tr>
<td>$^{206}$Hg</td>
<td>43</td>
<td>Coulex</td>
<td></td>
</tr>
<tr>
<td>$^{206}$Hg</td>
<td>44</td>
<td>Coulex</td>
<td>Problem with AGATA DAQ</td>
</tr>
<tr>
<td>$^{206}$Hg</td>
<td>45</td>
<td>Coulex</td>
<td>Problems with AGATA crystals 01B, 00B, 01C</td>
</tr>
<tr>
<td>$^{206}$Hg</td>
<td>46</td>
<td>Coulex</td>
<td></td>
</tr>
<tr>
<td>$^{206}$Hg</td>
<td>47</td>
<td>Coulex</td>
<td></td>
</tr>
<tr>
<td>$^{206}$Hg</td>
<td>48</td>
<td>Coulex</td>
<td></td>
</tr>
<tr>
<td>$^{206}$Hg</td>
<td>65</td>
<td>Coulex</td>
<td>Provides the most data for the measurement of Hg$^{206}$.</td>
</tr>
<tr>
<td>$^{206}$Pb</td>
<td>22</td>
<td>Coulex</td>
<td>Coulex target in forward position</td>
</tr>
<tr>
<td>$^{206}$Pb</td>
<td>23</td>
<td>Isomer</td>
<td>Plastic stopper in place for isomer study</td>
</tr>
<tr>
<td>$^{206}$Pb</td>
<td>24</td>
<td>Coulex</td>
<td>Coulex target in central position</td>
</tr>
<tr>
<td>$^{206}$Pb</td>
<td>26</td>
<td>Coulex</td>
<td>Beam not properly centered</td>
</tr>
<tr>
<td>$^{206}$Pb</td>
<td>28</td>
<td>Coulex</td>
<td></td>
</tr>
<tr>
<td>$^{206}$Pb</td>
<td>29</td>
<td>Coulex</td>
<td>Target in forward position</td>
</tr>
<tr>
<td>$^{206}$Pb</td>
<td>56</td>
<td>Coulex</td>
<td></td>
</tr>
<tr>
<td>$^{206}$Pb</td>
<td>70</td>
<td>Coulex</td>
<td></td>
</tr>
</tbody>
</table>

Table 2: Summary of AGATA measurement runs for measurements of $^{206}$Hg and $^{206}$Pb.
<table>
<thead>
<tr>
<th>Fragment</th>
<th>AGATA Run</th>
<th>Measurement</th>
<th>Merged events</th>
<th>Total Particles (Trigger 10^6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>^206^Hg</td>
<td>36</td>
<td>Isomer</td>
<td>1100000</td>
<td></td>
</tr>
<tr>
<td>^206^Hg</td>
<td>40</td>
<td>Coulex</td>
<td>5100000</td>
<td>48,000,000</td>
</tr>
<tr>
<td>^206^Hg</td>
<td>41</td>
<td>Coulex</td>
<td>7300000</td>
<td>68,000,000</td>
</tr>
<tr>
<td>^206^Hg</td>
<td>43</td>
<td>Coulex</td>
<td>3900000</td>
<td>37,000,000</td>
</tr>
<tr>
<td>^206^Hg</td>
<td>47</td>
<td>Coulex</td>
<td>4400000</td>
<td>42,000,000</td>
</tr>
<tr>
<td>^206^Hg</td>
<td>65</td>
<td>Coulex</td>
<td>24300000</td>
<td>214,000,000</td>
</tr>
<tr>
<td>^206^Pb</td>
<td>23</td>
<td>Isomer</td>
<td></td>
<td></td>
</tr>
<tr>
<td>^206^Pb</td>
<td>28</td>
<td>Coulex</td>
<td>6590000</td>
<td>65,000,000</td>
</tr>
<tr>
<td>^206^Pb</td>
<td>56</td>
<td>Coulex</td>
<td>10010000</td>
<td>100,000,000</td>
</tr>
<tr>
<td>^206^Pb</td>
<td>70</td>
<td>Coulex</td>
<td>77700000</td>
<td></td>
</tr>
</tbody>
</table>

Table 3: While the data is intact and merges successfully, AGATA runs of Coulomb-excitation study with the target in the forward position are also discarded, in favour of simplifying an already complex analysis.

using this time stamp requirement intrinsically prevents any events from being improperly merged. Also revealed are misalignments between filenames, and inconsistencies in the naming scheme; if the merge rate is observed to be 0 for an entire file, it can be concluded that a problem with the AGAVA system occurred during the experiment, or more likely, the AGATA data file and the MBS data file were not recorded simultaneously. If a pair of files are indeed simply misaligned, they cannot be realigned without extensive examination of their contents on an event-by-event basis, in order to produce a reliable way of re-aligning them. In the case of this experiment, files are observed to be mostly consistent in their labeling, but not perfect. A revised summary of the data is shown in Table 3.

6.3 FRS Detectors

6.3.1 Scintillators

In the event that all signals are average and within reasonable ranges (checksums satisfied), an average of the left-left and right-right time differences can be taken for an accurate time-of-flight measurement. While in principle these signals could produce a measurement of the x-position of the interaction point of the particle within the scintillator material, the time cannot be calibrated linearly because of a clear change in timing
response at different locations in the detector. This intrinsically prevents a truly accurate calibration, and furthermore the intrinsic resolution is not sufficient to provide something that can be reliably correlated with other measurements of position.

$$T_{OF_{Sc21-Sc41}} = \frac{(T_{OF_{L-L}} + c_{LL}) + (T_{OF_{R-R}} + c_{RR})}{2}$$ (24)

Where $c_{LL}$ and $c_{RR}$ are arbitrary offsets to calibrate the values. Two scintillators are used at the S2 position, directly following the energy degrader and beam slits. These can be used as a ‘start’ for the time-of-flight measurement, and are denoted in order as Sc21 and Sc22. One scintillator detector is used as a time-of-flight ‘stop’ at S4, referred to as Sc41, and is located after the MUSIC detectors, and the first TPC and the beam slits. The distance along the beam axis between Sc21 and Sc41 is approximately 37.58 m.

![Figure 31: The calibration of the time-of-flight calculation is performed by plotting the simulated $\beta$ versus the measured difference between scintillator time signals multiplied by that $\beta$.](image)

Three calibration runs utilising the $^{208}$Pb primary beam were used to produce an accurate calibration for the whole experiment; by varying the thickness of the degrader, a range of resulting energies (and therefore velocities) were produced. The beam is centered in each case to within a few mm (as close as was practical) to ensure this is an accurate calibration for the fragment of interest, which ideally has a similar trajectory.

One of the most restricting factors in this experiment is the efficiency of the scintillator
<table>
<thead>
<tr>
<th>Fragments</th>
<th>Simulated ToF (ns)</th>
<th>Measured ToF (ns)</th>
<th>Measured $\beta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{206}$Hg</td>
<td>183.844</td>
<td>179.562</td>
<td>0.7058</td>
</tr>
<tr>
<td>$^{206}$Pb</td>
<td>182.712</td>
<td>178.142</td>
<td>0.7113</td>
</tr>
</tbody>
</table>

Table 4: For consistency, the calibration of the time-of-flight is common for all runs, and is calibrated using a primary beam setting.

<table>
<thead>
<tr>
<th>AGATA Run</th>
<th>Sc21</th>
<th>Sc22</th>
<th>Sc41</th>
<th>Total S2-S4 ToF</th>
</tr>
</thead>
<tbody>
<tr>
<td>40</td>
<td>0.90</td>
<td>0.88</td>
<td>0.89</td>
<td>0.76</td>
</tr>
<tr>
<td>41</td>
<td>0.91</td>
<td>0.87</td>
<td>0.89</td>
<td>0.78</td>
</tr>
<tr>
<td>43</td>
<td>0.90</td>
<td>0.88</td>
<td>0.87</td>
<td>0.76</td>
</tr>
<tr>
<td>47</td>
<td>0.93</td>
<td>0.89</td>
<td>0.93</td>
<td>0.74</td>
</tr>
<tr>
<td>65</td>
<td>0.36</td>
<td>0.43</td>
<td>0.70</td>
<td>0.41</td>
</tr>
</tbody>
</table>

Table 5: The efficiency of the time-of-flight system the FRS.

detectors, as they provide the most separation between different fragments. The measured efficiencies of the scintillators during early runs are in stark contrast to those during the later runs. Furthermore the quality of this information seems hindered, suggesting prolonged exposure to the highly-energetic heavy ions has caused damage over the course of the experiment. This is especially apparent in scintillator 21, which is exposed to potentially far higher intensities.

6.3.2 Time-Projection Chambers

Each TPC produces a total of eight signals from the anodes, which can be used to calculate the position of a particle incident on the detector. The time and amplitude of these signals, as well as the amplitude of the fiber scintillation grid, are used to calculate a two-fold checksum for each anode, which can be used to establish the validity of the signals. The conditions of the checksum are such that the amplitude and time of a signal are in agreement with the expected event (see Figure 32); a large amplitude and time suggests that either multiple energy depositions have occurred, resulting in a net charge induction that is not accurately proportional to either event. Similarly there is a lower threshold intended to eliminate noise at this stage. The calibration of the signals is linear (uses a gradient and offset), and an average is taken; an x-position is calculated from a weighted average of these resulting values, where a value is considered to have double the
weight if both checksums are passed.

![Checksum distribution](image.png)

**Figure 32**: An example checksum for TPC21. The hashed-red area is the range of acceptable values, and is fairly unambiguously presented in the data.

The TPCs are calibrated by means of a thin fiber grid with known dimensions being placed in front of the detector at a known position, and a diffuse beam is used to trigger as much of the detector as possible. The grid reduces the intensity of the beam behind it, displaying clearly the real scale of the detected signals. The consistency of the position measurement is checked against the LISE++ simulations (Figure 33).

While the TPCs are very useful throughout particle identification, as well as providing particle tracking information for use in the Doppler-correction, they are severely hindered by their extremely low efficiency. The low efficiency of each individual detector can be somewhat remedied by the redundancy of having multiple TPCs; as is the case when one signal is missing in a detector, the event is only completely lost when none of the signals in either detector are present. When more strict checksum conditions are introduced however, to compensate for higher beam rates at the S2 position, and the position at the focal plane is requested (more ideal for fragment separation), the efficiency drops further still (Table 6):

When a position at either S2 or S4 cannot be determined, the event is not immediately
Figure 33: Left: LISE++ simulation of the beam profile at TPC21 for the $^{206}$Hg setting. Right: TPC21 x position for the same setting. It should be noted that the reference frame used in the LISE++ simulation (left) is reversed.

<table>
<thead>
<tr>
<th>AGATA Run</th>
<th>TPC21</th>
<th>TPC22</th>
<th>S2 focal plane</th>
<th>TPC41</th>
<th>TPC42</th>
<th>S4 focal plane</th>
</tr>
</thead>
<tbody>
<tr>
<td>40</td>
<td>0.38</td>
<td>0.35</td>
<td>0.27</td>
<td>0.81</td>
<td>0.77</td>
<td>0.76</td>
</tr>
<tr>
<td>41</td>
<td>0.36</td>
<td>0.31</td>
<td>0.26</td>
<td>0.80</td>
<td>0.76</td>
<td>0.74</td>
</tr>
<tr>
<td>43</td>
<td>0.40</td>
<td>0.35</td>
<td>0.29</td>
<td>0.78</td>
<td>0.81</td>
<td>0.76</td>
</tr>
<tr>
<td>47</td>
<td>0.40</td>
<td>0.36</td>
<td>0.29</td>
<td>0.83</td>
<td>0.79</td>
<td>0.78</td>
</tr>
<tr>
<td>65</td>
<td>0.42</td>
<td>0.37</td>
<td>0.30</td>
<td>0.85</td>
<td>0.81</td>
<td>0.80</td>
</tr>
</tbody>
</table>

Table 6: The efficiency of the TPC particle tracking system throughout the FRS. AR40 is the first run of $^{206}$Hg Coulomb excitation, and AR65 is the last.

discarded, as it could easily have an otherwise complete 'set' of information necessary to contribute to the final $\gamma$ spectrum. Instead, each event is given a boolean 'flag' to indicate the validity of the position measurement, and those that have not been measured with a valid checksum are given a position of 0. This allows the event to be preserved in the absence of this position information, and the user can later determine whether or not to include it by examining the position-validity flag.

6.3.3 Multiple Sampling Ionization Chambers

MUSIC detectors are used to determine the Z of the secondary beam, and discriminate between fragments of varying charges. Each of the two MUSIC detectors provides eight signals, which are first tested for validity (must be non-zero, and within the range of the
detector), and the valid signals are then used along with the measured $\beta$ between Sc21 and Sc41 to calculate the $Z$ of the particle. Since energy deposition is related to the $Z^2/\beta^2$ of the particle, it is not affected by the accumulation or persistence of charge-states in the secondary beam, other than that the calculation of $\beta$ will be varied slightly by the altered trajectory through dipoles D3 and D4. The energy measured by a single MUSIC is given as a weighted average of the measurement for each anode, denoted $E_n$, as:

$$\Delta E = (\Delta E_1 \ldots \Delta E_N)^{1/N}. \quad (25)$$

The signals are first aligned with an offset, and then calibrated collectively; this alignment is to prevent the resulting calibration from relying on the initial 'skew' of the signals. If this is not done, and a particularly un-aligned signal is not present during an event, the resulting value of calculated energy deposition (and therefore $Z$) will be drastically altered. The calibration itself is separate for each MUSIC; energy loss is dependent on the $\beta$ of the particle, and a coefficient is calculated using a third-degree polynomial as $b_{poly} = T_0 + (T_1 \ast \beta) + (T_2 \ast \beta^2)$ where $T_n$ are the calibration coefficients for the detector as $\beta$ is the $v/c$ of the measured particle. The calibration is performed using the setup runs of the FRS, in which the primary beam is centered at different energies, allowing the same $Z$ to be seen at different values of $\beta$. The measured $Z$ can then be calculated as

$$Z = \sqrt{Z_{in} \sqrt{\Delta E_1/b_{poly1}} Z_{in} \sqrt{\Delta E_2/b_{poly2}}} \quad (26)$$

where $Z_{in}$ is the $Z$ of the primary beam (in this case 82), $E_1$ and $E_2$ are the calibrated energies of each MUSIC, and $b_{poly1}$ and $b_{poly2}$ denote the polynomial calibration on $\beta$ for each MUSIC. The efficiency of the MUSIC detectors is fairly high, with all anodes producing a signal simultaneously in all cases; the validity of this signal is also very reliable. The calculation of $Z$ is hindered primarily by the low efficiency of the scintillator detectors; without a measured value of $\beta$ it cannot be accurately determined. The value
of energy deposition (which has a higher efficiency) can be retained during data analysis to maximize statistics, however since $\beta$ is the primary source of particle identification, any events without it will be discarded anyway, and thus this is not a reasonable method of preserving statistics.

<table>
<thead>
<tr>
<th>AGATA Run</th>
<th>MUSIC1</th>
<th>MUSIC2</th>
<th>Combined</th>
</tr>
</thead>
<tbody>
<tr>
<td>40</td>
<td>0.92</td>
<td>0.92</td>
<td>0.92</td>
</tr>
<tr>
<td>41</td>
<td>0.92</td>
<td>0.92</td>
<td>0.92</td>
</tr>
<tr>
<td>43</td>
<td>0.91</td>
<td>0.91</td>
<td>0.91</td>
</tr>
<tr>
<td>47</td>
<td>0.93</td>
<td>0.93</td>
<td>0.93</td>
</tr>
<tr>
<td>65</td>
<td>0.92</td>
<td>0.93</td>
<td>0.93</td>
</tr>
</tbody>
</table>

Table 7: The efficiency of the MUSIC $\Delta E$ detectors.

6.3.4 AGATA

As described in Section 3.5, the inaccuracy in position measurement of the incident $\gamma$-ray contributes a substantial inaccuracy in the Doppler-correction, and as such, pulse-shape analysis and a $\gamma$-tracking algorithm must be employed to properly calculate the $\gamma$-spectrum in the lab frame. Energy, position and timing information from individual segments, as well as the core of each crystal are processed and sent to the Mars-Gamma Tracking code (MGT) by the merger. As expected, a correlation is observed between the number of interaction points contributing to a reconstructed $\gamma$-ray, and the energy of that $\gamma$-ray (Figure 34). The multiplicity does not increase beyond roughly 500 keV because as the number of interaction points within the crystal increases, the probability of an incomplete reconstruction due to an escaping $\gamma$-ray also increases.

Since the $\gamma$ tracking algorithm can be verified to reconstruct energies with reasonable precision, it is unnecessary to at any point eliminate events based on the number of interaction points contributing to the total energy of the $\gamma$. Single-interaction point events that are likely to be low energy background are correctly reconstructed as such, and thus do not affect the peak area (803 keV for $^{206}$Pb and 1068 keV for $^{206}$Hg). The data is not effectively cleaned by filtering on the number of interaction points contributing
Figure 34: $\gamma$-rays with higher energy are more likely to be scattered multiple times before total energy absorption.

to each $\gamma$.

The position information is provided by the pulse-shape analysis, and is not deduced during the merging of the ADF and MBS data. It is only necessary to correct for differences in the co-ordinates system utilised by the PSA, and that used in the rest of the analysis: In the co-ordinate system utilised by the PSA, the beam travels towards a positive value of x, and the z-axis points vertically upwards [45]. The FRS co-ordinate system describes the beam as travelling along the z-axis towards the positive, and the y-axis points vertically upwards. The particle-$\gamma$ time of each crystal can be individually aligned to provide marginal improvements on the precision of the $\gamma$-tracking algorithm, and also improvements to any gates applied to the particle-$\gamma$ time, such as those discussed in Section 5.9.

6.4 Isomeric Tagging

In addition to the thin gold target for Coulomb excitation, a thick plastic stopper was used for short runs of roughly one hour for each fragment setting. This allows the content of the secondary beam to be identified by examining the isomeric decay of fragments embedded
Figure 35: The hitpattern of AGATA, given in the FRS co-ordinate system (where z is along the beam axis, and y is vertical). A gate is used to limit events shown here to those with an interaction point within a radius of 26 cm from the target, effectively displaying only the 'front' of each crystal. This image is given in the FRS reference frame; looking along the beam-axis towards the end of the beamline. The grid-like structure seen in one crystal is due to crosstalk between the segments of that crystal.

in the plastic. Simply confirming the presence of known isomers is not the purpose, but rather the methodology by which the profile of the beam can be verified against the simulation of the setting with respect to position, velocity and energy measurements. The isomeric states in each fragment of interest allow each identification cut to be tested prior to being applied to the Coulomb excitation data. The isomeric ratio of each nucleus of interest must also be accurately calculated and accounted for during the calculation of the B(E2), as in all cases the isomers are situated at a higher excited state to those being measured. Particles in excited states above the ground state cannot Coulomb-excite from the 0+ ground state to the first excited 2+ as intended. In the setting of $^{206}$Hg, clear separation between fragments is not possible with a graphical cut in a Z vs A/Q plot. As seen in the Figure 36; there are three main fragments that can be discerned, however at all stages of separation, the identification of fragments is prone to contamination by charge-states of surrounding fragments. The lack of definition in the calculated value of A/Q stems from the use of $B_\rho$, which will change upon the acquisition of an electron
Figure 36: Correlating the measured Z and A/Q of the incoming beam is a commonly used approach in filtering out unwanted events.

during flight through the FRS. The solution is to focus identification on the time-of-flight, and the position of fragments after the *first* separation of Bρ by dipole magnets D1 and D2, as after the magnets D3 and D4, particles that would otherwise be separated by x-position will be re-aligned to the beam axis near the focal point, and thus inseparable from the nuclei of interest. The result of this approach is the plot shown in Figure 37, of time-of-flight vs x-position at the S2 focal plane. The structure observed here can be correlated well to that predicted by the LISE++ simulation, and the structure can subsequently be confirmed by performing graphical cuts on this plot, and examining the γ spectrum in AGATA. It should be noted that while there do appear to be some similarities between the structure present in the Z-vs-A/Q and S2x-vs-β plots, it can be demonstrated that these approaches are *not* equivalent. Shown in Figure 38 are the same identification plots after a cut has been made on the $^{206}$Hg fragment in the opposing identification plot. This reveals clearly that gating (selection of events) on the $^{206}$Hg in Z-vs-A/Q does not remove the $^{205}$Hg$^{+79}$, as can be seen in the subsequent S2x-vs-β plot in Figure 38. In each case, the lifetime of the isomeric state should be accounted for when
Figure 37: Fragments are identified by examining the isomeric $\gamma$ decays.

determining the particle-$\gamma$ time-gate applied; in the case of $^{206}$Hg the lifetime of the 5$^-$ isomeric state at 2.1 MeV has a lifetime of $2.15(21) \mu s$ [46]. The isomer study here is limited by the hardware trigger window, and the time window of the data-merger, which is $3 \mu s$. For the isomer runs the time window of the merger is increased to $20 \mu s$ to allow much longer lifetimes to be studied comprehensively. In the particle-$\gamma$ time spectrum this is seen as a 'soft gate' at $11 \mu s$, where the DAQ can either begin a new trigger, and thus a new event, or allow the current event to continue until either a new trigger comes, or the upper limit of the trigger window is reached. At $20 \mu s$ we see the upper limit of the event window in the merger; a particle-$\gamma$ time value will not be assigned beyond this point. The $\gamma$ spectra correlated against the particle-$\gamma$ time is shown in Figures 39 and 40 for AGATA’s core signals, and utilising the individual segment data in combination with the Mars-Gamma Tracking algorithm.

Similarly, the setting of $^{206}$Pb can be cleaned in the same manner as seen in Figure 39, however since this beam setting is significantly more pure, only the primary fragment of interest is clearly visible for identification. The isomeric state in this fragment is very long-lived, and the measurement of the isomeric ratio is described in Section 6.1.
6.5 LYCCA Time-of-Flight Detector Calibrations

The LYCCA time-of-flight detectors are three circular membranes of a plastic scintillator material, arranged around the target location for the purposes of measuring the a velocity of the particle directly applicable to Doppler-correction. The scintillations caused by the traversal of a charged particle are detected by a number of photomultiplier tubes; the precision of this measurement is intended to capitalize on the multitude of these detectors.

An initial calibration can be produced through the summation of PMT signals with respect to a reference signal, which is taken as an average of left and right signals from scintillator 41 (or whichever is available, if only one is present in the event). The accuracy of this reference signal is irrelevant, as it is merely needed as an identical basis to subtract the many absolute time-values from. An arbitrary offset is applied to each PMT signal to align them with respect to the reference signal. This averaging provides an easy boost to the precision. However, an accurate calibration of these detectors relies on an estimation of position which is provided externally by the rest of the particle-tracking system. The distance between the particle interaction-point with the scintillator material and each of the PMTs can be plotted against the time difference of that interaction point,
Figure 39: The Beta vs S2 x-position ID plot of $^{206}$Pb. Indicated in black is the gate used for the analysis of Coulomb excitation data.

shown in Figure 42. This facilitates a further calibration of each circular membrane, now accounting for the travel-time of the light inside the scintillator material. This method assumes only that the light travels a consistent path from each position to each PMT, and is thus only encounters problems when more than a single signal is detected for a given particle event. The location of each PMT is estimated assuming they are evenly spaced around a perfect circle, with the top centered perfectly over the axis of the beamline. With each PMT calibrated individually in this manner, a clear improvement to the timing resolution can be seen (see Figure 43), with the apparent intrinsic timing resolution of the scintillator material in principle restored. The resulting timing resolution of each LYCCA time-of-flight detector is shown in Table 8. Following this calibration of the timing signals themselves, the times of each of the three LYCCA detectors must be calibrated relative to one another to match the expected time-of-flight from the simulations of the experiment. Offsets are applied to each timing signal such that they can be subtracted appropriately ($\text{stop} - \text{start} = \text{time} - \text{of} - \text{flight}$). Due to apparent inconsistencies between runs, particularly over longer periods of time, this calibration is completely repeated for each
Figure 40: $\gamma$ energy versus particle-$\gamma$ time spectrum of $^{206}\text{Hg}$, with a gate on the $^{206}\text{Hg}$ structure in Beta vs S2 x-position. The long decay of the isomeric states can clearly be seen at larger values of particle-$\gamma$ time.

run to ensure absolute precision. A summary of the expected time-of-flight measurements between the various detector pairings is shown in Table 9.

<table>
<thead>
<tr>
<th>Detector</th>
<th>N PMTs</th>
<th>FWHM Resolution (ps)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Start</td>
<td>32</td>
<td>43.6(3)</td>
</tr>
<tr>
<td>Target</td>
<td>12</td>
<td>33.4(3)</td>
</tr>
<tr>
<td>Stop</td>
<td>32</td>
<td>96.0(7)</td>
</tr>
</tbody>
</table>

Table 8: Despite a position-time calibration not being possible for the ToF Target detector, its small size facilitates an comparable timing resolution to the fully calibrated membranes.

Furthermore, since for a given value of time, a distance from each PMT can be estimated, it is possible to utilise this calibration to then deduce a fine position measurement based solely on the timing signals of the PMTs of a circular membrane relative to a common reference signal. First, the PMTs are aligned to a common value (preferably 0 for ease of computation). An initial position in the center of the membrane is used, and the $\chi^2$ (variance) of the estimated distance is calculated for each PMT individually. Intervals of $1/7$ of the membrane width are then applied in positive and negative x and y, and the
Figure 41: $\gamma$ energy versus particle-$\gamma$ time spectrum of $^{206}$Hg, with a gate on the $^{206}$Hg structure in Beta vs S2 x-position. Here the Mars-Gamma Tracking algorithm is applied to the $\gamma$ spectrum.

$\chi^2$ of each is calculated, with the lowest value being chosen. This process is repeated in an iterative fashion, strictly moving towards a lower value of $\chi^2$, until no improvement can be made. The step size is halved, and the process is again repeated, until the position with the lowest $\chi^2$ is deduced for a step-size of 2 mm. The result of this is a position resolution close to that of the LYCCA DSSD modules, and the resulting x and y positions are demonstrated to be in very good agreement with the extrapolated positions at each of the ToF membranes. Since the ToF Target membrane cannot be accurately calibrated in the aforementioned manner (PMT time with respect to the distance from the particle position), a position measurement cannot be derived with this approach using the ToF Target detector.

It can be seen in Figure 44, that there is not a perfect 1:1 correlation between the position extrapolated from other position detectors, and that which is calculated by using the time-position calibration method. The pattern of this deviation implies there is a rotation or scaling factor that is not being accounted for in the calculation of position with the ToF Start detector.
Figure 42: Left: An example of an uncalibrated PMT from the LYCCA Start detector. Right: The same PMT is calibrated to a common zero with the other PMTs; the time signal is now adjusted to account for the location of the particle.

Figure 43: Left: The time signal of ToF Start detector PMT 15, uncalibrated (FWHM: 0.25 ns). Right: The time signal of ToF Start detector PMT 15, calibrated for distance (FWHM: 0.12 ns).

In the case of the ToF Stop this distortion is a mixture of rotation and scaling, which can be corrected for via re-calculation during the calculation of calibrated times and positions, equivalent to the translation matrix:

\[
\begin{vmatrix}
  x', y', z' \\
  x, y, z \\
\end{vmatrix} = \begin{vmatrix}
  s_x \cos \omega & -r_y \sin \omega & 0 \\
  r_x \sin \omega & s_y \cos \omega & 0 \\
  d_x & d_y & 1 \\
\end{vmatrix}
\] (27)

Where \( s \) is the scaling factor, \( r \) is the shear factor, \( \omega \) is the rotation angle (clockwise in the beam direction), and \( d \) is translation. The rotation is deduced by examining the angle between position measurements in different detectors for a narrowly selected range of values. The scaling factor is then deduced by taking the gradient of the difference.
Figure 44: There is a strong correlation between the extrapolated x position at the ToF Start membrane and the x position derived using the iterative PMT time-distance correlation method.

between the measured x position and the expected x position produced by extrapolations of other detector measurements. This issue of scaling in the calculated position can be caused by a number of factors, in particular varying responses in the PMTs (and associated electronics), or incorrect assumptions regarding the arrangement of the PMTs around the membrane; the PMTs are assumed to be arranged in a perfectly circular arrangement. The appropriate correction factors can be found by

The multiplicity of each PMT can also be addressed, as internal reflections within the material can produce more than one signal for each incident particle. Since particles can arrive in close proximity, and the timing signals from the LYCCA ToF PMTs are processed by a multi-hit TDC, it is important to provide the analysis code a way of interpreting this multitude of signals per event. Since the light traveling directly from the point of interaction to the PMT will always arrive first, the signal with the lowest time can be taken, however this does not account for scenarios where particles are arriving coincidentally close together. To provide a more substantial methodology, the complete
Figure 45: Hit-pattern of the LYCCA ToF Stop detector. The DSSD Wall pattern can still be seen, as these measurements are used when available as the starting points for the position algorithm. The beam is seen to spread significantly here, after it has interacted with the secondary target.

A set of signals can be shown for a given PMT, subtracted from a common reference (Trigger Sc41). Several formations representing each iteration of the reflection can be seen, and a range that is representative of the fastest signals can be selected. With this approach, if no signals are provided by a PMT that fall into this range, then it can be assumed that only reflections are present, and will not contribute to an accurate time measurement.

Since the time profile of the reflections can be seen to have consistent structure, it is (in principle) possible to utilise this information to produce a more precise measurement, however each reflection will require individual calibration; internal reflections are not utilised in this calibration.

<table>
<thead>
<tr>
<th>β notation</th>
<th>ToF Start</th>
<th>ToF Stop</th>
<th>$^{206}$Pb ToF (ns)</th>
<th>$^{206}$Hg ToF (ns)</th>
</tr>
</thead>
<tbody>
<tr>
<td>In</td>
<td>Start</td>
<td>Target</td>
<td>7.641</td>
<td>7.628</td>
</tr>
<tr>
<td>Average</td>
<td>Start</td>
<td>Stop</td>
<td>26.404</td>
<td>26.08</td>
</tr>
<tr>
<td>Outgoing</td>
<td>Target</td>
<td>Stop</td>
<td>34.044</td>
<td>33.708</td>
</tr>
</tbody>
</table>

Table 9: Time-of-flight values for each fragment as calculated in the LISE++ simulation.
Figure 46: Two different measurements of $\beta$ using different LYCCA start detectors (‘start’ and ’target’). Outlined in black is the most populated structure, which is taken for analysis; events outside this area account for roughly 21% of the data, and are discarded.

When evaluating the measured values of $\beta$, it is expected that there is a strong and consistent correlation between each method, with only small deviations relating to the angle incident on the target, which would cause a spread in momentum due to a change in the thickness of material traversed. Figure 46 shows the correlation between measurements made with two different start detectors. The difference between the two reveals differences in the momentum structure of the beam as it passes through the secondary target area. These structures could also be created by miscalculations, such as internal reflections mistakenly included in the measurement without being accounted for. Data that is recognizably incorrect in this measurement cannot be accurately Doppler corrected, and is removed from the analysis.

6.6 LYCCA E and $\Delta E$ Detectors

Post-secondary target identification of fragments can be provided be two measurements: An energy measurement of energy loss in the DSSD Wall, which is proportional to $Z^2/\beta^2$, and a total energy deposition measurement in the CsI modules positioned directly behind
the DSSD Wall, which is proportional to the mass of the particles. Each DSSD module provides 32 signals, arranged as 16 in x and 16 in y, which are used in combination to create the x and y position of the particle after interaction with the secondary target. The calibration of these is generally performed manually, however other methods do exist. In the simplest approach, the x and y strips are addressed separately, each as an individual element of 4096 channels, which can either be aligned to other strips in the wall, or can be calibrated for both alignment and gradient based on multiple runs. Since no dedicated runs with minimal momentum spread at similar beam energies were performed to facilitate this method of calibration, it is both limited by the energy spread of the beam for any given run, and the spread of statistics across the array. Those silicon strips near the edge of the array lack sufficient incident events to produce a reasonable profile of energy measurements to be properly calibrated using this ‘traditional’ approach. This limitation also applies to the calibration of the CsI detectors.

6.7 Tracking and Tracking Redundancies

Since the scattering angle of the particle is of vital importance to the precision of the Doppler-correction, it is important that careful attention be attributed to the reconstruction of the trajectory of the particle. Since the Target-DSSD is in close proximity to the secondary target, and provides a very reliable measurement of both x and y position due to its segmented design, this is the favoured choice for estimating the position of interaction with the target. Other available information however, can improve upon this further. TPCs 41 and 42 can be used in combination with the Target DSSD, to extrapolate the position assuming the particle is traveling in a straight line. This also allows the scattering angle to be determined with greater precision. The x position (or by the same method, y position) is extrapolated from any two detectors (with x and z positions denoted by subscripts 1 and 2) to a third given point at z-position $z_3$ as

$$x_3 = \frac{z_3 x_1 - x_2}{z_1 - z_2}. \tag{28}$$
For an overview of the arrangement and distances of relevant detectors used in the position-tracking system at S4, refer to Figure 19. This method is used to produce accurate position information at the target location for the purpose of Doppler-correction. It also provides information necessary for accurate calibration of the LYCCA ToF detectors as described in section 4.5. Utilising their respective PMT-distance-time calibrations, the LYCCA ToF Start and Stop detectors can each then be treated as position detectors, leaving the analysis with a multitude of position measurements available, which can utilised collectively to improve on the overall efficiency of the system. For each event, the available information is utilised to favour accuracy, with an alternative source of position being employed when the favoured detector does not produce a good signal. For the first measurement in position tracking, for the purpose of extrapolating the position of the particle at the secondary target, the detectors are prioritised as follows:

1. TPC41
2. TPC42
3. ToF Start
4. Target DSSD

For the second measurement in this extrapolation, the priority of these measurements is reversed. If only the Target DSSD measurement is present, and the particle position at the secondary target cannot be extrapolated, the particle is assumed to travel parallel to the beam axis for the remaining 58 mm between the Target DSSD and the secondary target. If no measurements are present at all, the particle is assumed to be traveling along the beam axis. The efficiency of the Target DSSD alone is approximately value, however this cannot provide a scattering angle, and thus the system is limited by the efficiency of those detectors working in tandem with this. Utilising this method brings the total efficiency of the system to 94% for the calculation of scattering angle, and 95% for the summation of all calculations of the particle position at the secondary target, where the
efficiency of the system using only one combination of any two detectors is at most 80%.

system is as follows:

6.8 Doppler Correction

The Doppler correction of the γ-ray spectrum accounts for the velocity of the emitting particle, which at relativistic speeds will produce a significant shift in the energy of the detected γ-ray. A successful Doppler Correction relies on the accurate measurement of two specific concepts, each of which poses its own challenges for a physicist: The velocity of the particle (β), and the angle between the particle and the γ ray. These can be broken down further into a number of individual measurements, which are listed as follows:

- Time-of-Flight of the particle across the target.
- Position of the particle upon emission of the γ ray.
- Outbound trajectory of the particle after emission of the γ ray.
- Position of the γ ray first interaction point.
- Alignment of the particle and γ position measurements.

The position of the particle at each required moment is provided relative to the beam axis by a number of detectors, discussed in the previous subsection; it should be noted that while this information may be precise within the reference frame of the detectors themselves, the issue of alignment to the reference frame of AGATA is not, and cannot be resolved by the particle-tracking system alone. While calculation of the β of the particle is discussed in section 4.5, and is aligned to the LISE++ simulation of the experiment, this may be inaccurate to the real value at the point of γ-ray emission, and therefore not the desired value to use for Doppler correction. The momentum spread of the beam is significant enough to make the alignment itself problematic, even with the full statistics of a longer run. Furthermore, both the momentum spread, and the energy lost within the target is far more than that which is attributed to the Coulomb excitation itself; particles
which did and did not Coulomb-excite cannot be distinguished with a measurement of the energy or velocity of the particles at any point. In light of this, rather than relying on a singular initial calibration of $\beta$, an approximate calibration is used, which is later refined using an offset to the time-of-flight.

$$E_{\text{emitted}} = E_{\text{lab}} \frac{1 - \beta \cos \theta}{\sqrt{1 - \beta^2}}$$

(29)

In order to deduce the correct $\beta$, it is necessary to use the only available indication of a correct value: The Coulomb excitation peak itself. When a $\beta$ is calculated; two quantities are used: The path length (provided by the particle-tracking system), and the time-of-flight. To apply an offset directly to the $\beta$ would not account for the path-length of each individual particle, so to preserve this component of the calculation and produce an accurate $\beta$, the offset must be applied as:

$$\beta = \frac{\text{path}}{\text{ToF} - \text{offset}} \frac{\text{path}}{\text{ToF}_c}$$

(30)

where $\text{ToF}_c$ is the time-of-flight for a particle traveling at $c$. The most direct way to address the determination of this offset is by means of testing a number of corrections within an acceptable range, which can be quickly compared to see the resulting trends in the $\gamma$ spectrum. When the offset is close to producing a correct value of $\beta$, the Doppler-correct spectrum will produce a peak at the correct energy. An example of this approach can be seen in Figure 47, where offsets ranging from -1 ns to +1 ns are applied prior to the Doppler-correction of the $^{206}$Hg $\gamma$ spectrum. Evidence that offsets to time-of-flight alone can been seen in Figure 48, implying that another offset is required for a precise Doppler correction.

Unfortunately this approach relies on several stipulations which can hinder its utility:

- The spectrum is already sufficiently cleaned enough for the Coulex peak to be visible.
Figure 47: 1000 different values of offset are used with the “Average” time-of-flight (taken between the first and last LYCCA ToF detectors), which are shown on the y-axis. The resulting $\beta$ for these offsets is shown to the left, and the $\gamma$ spectrum is shown on the right. Since the path length of each particle is accounted for in this calculation, the result is non-linear: Each particle $\beta$ is affected differently by the changing offset.

- The particle and gamma-interaction positions are known with enough precision to produce a peak above the surrounding background fluctuations.

- The measurements of particle and gamma interaction position are properly aligned, to produce the correct value of $\theta$.

An offset to the position of AGATA can be made during the Doppler-correction in the same manner as can be done for the $\beta$, however this approach will be intrinsically susceptible to the same restraints as applying a correction to the $\beta$; each can be refined independently with the assumption that the other is correct. Each can be employed sequentially, and an iterative method is used once reasonable gates are established in order to progress towards successful values of all offsets simultaneously. It should be noted that since the offsets are arbitrary for $\beta$, these must be set for each separate AGATA run individually, since the data already varies enough to make direct matching of the time-of-flight values impossible. When applying offsets to $\beta$, certain precautions must be adhered to in order to prevent the creation of artificial peaks by means of cumulative coincident noise; any peak from an individual contributing file must be representative of the final peak, and not only a single bin in width, or comparable to background fluctuations. Since AGATA can be assumed to be stationary throughout the experiment, any offsets to position to compensate for potential misalignment must be applied identically to all
Figure 48: A scan of $\beta$ values with no additional offsets. The Coulomb-excitation peak of $^{206}$Hg can be seen to focus at an energy just above the expected decay energy in the rest frame, which is marked by the red dashed line at 1068 keV. This is indicative of a systematic error elsewhere in the Doppler correction (position alignment).

A more thorough approach is to address the three variables most likely to influence the Doppler correction simultaneously. In particular $x$ and $z$ position are of interest, because of the way AGATA is mounted: The large framework is able to be shifted apart ($x$ in the frame used here), and while it encounters no trouble returning to a complete state, the precision of this configuration may not extend into the millimeter scale. Similarly, with the significant weight of the AGATA modules, the frame may not be perfectly rigid and stationary compared to its empty state. If any of these stipulations is inaccurate, then the most optimised offset will remain as zero, with the assumption that the peak will in fact be greater than random fluctuations in the background.

Each variable of interest is altered in small steps across a range of reasonable values. For the time-of-flight offset, this is a range that shifts the average $\beta$ from the simulated velocity incident on the target, to the simulated velocity leaving the target, since the
Figure 49: Optimisation of x and z offset for a fixed value of $\beta$. The height of each bin in this histogram is the integral of the spectrum at the energy of the peak (803 keV for $^{206}\text{Pb}$, for example) (+/- 5 keV). This figure is generated using the $^{206}\text{Hg}$ data, but will (in principle) produce the same result for all settings.

The projectile is expected to be able to Coulomb excite (and subsequently emit a detected $\gamma$-ray) at any point during its time within the target. This range of values is large when compared to the equivalent change in the corrected $\gamma$ energy spectrum, so the step size must be made appropriately small, so that the correct value is not missed during this process. For the x and z positions, the range of reasonable values is chosen to be up to 1 cm in any direction. As seen in Figure 25, an offset of 2mm can create a shift of more than 5 keV (depending on the angle $\theta$), so the step-size used in these offsets must also be significantly smaller. The result is shown in Figure 49, where the integral of the peak is seen to increase significantly for a particular range of position offsets. If these offsets are valid, this plot is expected to be consistent for all runs of all fragments while $\beta$ is correct.

200 values of each offset are computed, creating a total of 8 million corrected spectra. (Effectively a low-resolution plot similar to Figure 48 for each time-of-flight offset). In each case, the integral of the peak area is taken. For every value in the complete range of time-of-flight offsets, the x and z position offsets are recorded for the 5 highest peak-integrals (to prevent ‘fluke’ results from random cumulative bins). The complete list of
integrals can be examined for groups of high-integral-peaks, indicative of a successful Doppler-correction. Since between files both the physical $\beta$ itself, and the time-of-flight offsets are expected to change marginally, these must only approximately align, but spatial offsets in x, y and z for all files must be in perfect agreement.

Primarily owing to the use of this approach with the Doppler-correction of $^{206}$Pb, (as can be seen in Figure 49), the position offsets are derived:

- X offset = -5.7 mm
- Z offset = +8.0 mm

Once the Doppler-correction is approximately correct, the peak-to-background ratio can be addressed through improvements of the gating, and subsequently, further fine-tuning of the Doppler-correction can be performed in an iterative method.

The Doppler-correction can be tested for approximate accuracy by examining the x-ray emissions of the beam. The strong angular dependence of Doppler-shifted energies can be observed easily here, as shown in Figure 49: Those emitted in the stationary lab-frame can be seen to have no dependence with angle. In Figure 50, the Doppler-correction is applied, and the angular dependence is removed for those $\gamma$-rays originating from the beam particles, while those emitted in the lab-frame are now shifted. When the Doppler-correction is precise, there should be no angular dependence observed, though it can be anticipated that the uncertainty in the Doppler-correction will have a component of angular dependence.

While this approach does indicate an approximately-correct Doppler-correction, the energy resolution in this energy range, and the uncertainty in both particle and $\gamma$ position measurement, or more specifically the alignment of these two measurements, result in limitations to the utility. An inaccuracy of only 1 mm in this range can be extrapolated to an inaccuracy of 15 keV or more in the 1 MeV range of the peaks being measured. Since the absolute precision of energy measurements in AGATA do not scale to match the order of magnitude difference in energy, this approach cannot be relied upon for a
Figure 50: The x-rays emitted by $^{206}\text{Hg}$ can be seen shifted towards higher energies, particularly at higher angles from the emitted particle.

precise Doppler-correction. The full extent of the Doppler correction is shown in Figure 51; the $\gamma$ spectrum after the correction is correlated against the same spectrum before.

6.9 Background Reduction

In addition to producing a spectrum with an accurate Doppler correction, it is also important to reduce the background. Careful consideration is made to preserve statistics, whilst removing unwanted contaminants and unreacted beam content. Accurate gating once the Doppler correction is performed can be done computationally with respect to any single variable or calculable quantity. This is done by quantising the peak-to-background ratio with respect to the profile of a selected variable; a range of background is selected above and below the peak area, and an average of those bins is subtracted from an average of the bins within the selected peak area. This allows an estimate of peak counts above background (per bin) to be created. This process can be performed not only for a static 1-dimensional $\gamma$-spectrum, but with respect to any variable within the analysis that can be correlated with that spectrum, with an arbitrarily fine binning.

The $\gamma$-spectrum is plotted against a variable or quantity that is correlated, and the peak counts above the background are evaluated for each bin of the chosen variable.
Figure 51: The x-rays of $^{206}$Hg can now be seen corrected to their rest-frame energy of roughly 70 keV.

The purpose of this technique is to provide a robust method of deducing the best gates possible in each case, and as such the binning chosen for the variable can be more fine that is ordinarily practical for creating these gates by hand. The result is a profile of the peak counts above background (per bin) against the chosen variable. An example of this profile is shown in Figure 54. Where the peak is above the background, the value of the bin will be above zero. The integral of this profile is then evaluated in stages to determine the optimum gates.

When series of adjacent bins have a total integral of peak above background that is below zero, then they can be assumed to be detrimental to the gate, however if they exist between two bins whose collective integral exceeds that detriment, that it is worth it to include them in a contiguous gate. The highest bin is chosen as an initial starting point, and the profile of the peak integral above background is evaluated in groups of such bins, working outwards from the starting point. In each stage, the total integral between two bins that each have values above zero is summed. If that total is above zero, then the gain of peak outweighs the gain in background, and the gate can be expanded to include this area of the data, which will result in a net gain to the peak integral above background. If the integral is below zero, then the evaluated area is expanded to
Figure 52: On the x-axis: Doppler-corrected $\gamma$ spectrum, on the y-axis: Un-Doppler-corrected $\gamma$ spectrum. The red and green lines indicate the a 10 keV width centered on the $^{206}$Hg peak at 1068 keV, and the corresponding area of the spectrum on the un-corrected axis. Without a Doppler-correction, the peak is far too diffuse to be visible.

include the next bin above zero. If the evaluated area is expanded to the edge of the histogram, and the integral never rises above zero, then the evaluated area will result in a net loss for the peak, and must be discarded to improve the peak-to-background ratio. This computational technique is very fast, and allows a user to quickly create gates that are more precise than can be done by human judgment alone.

Amongst the most important gates is the identification of the secondary fragment in the FRS. This is done using both the usual Z vs AoQ graphical gate, as well as the Beta vs S2 x-position graphical gate described in the subsection “Isomeric Tagging”. The figure below demonstrates the effect on the $\gamma$-spectrum of the addition of Z and AoQ gates to this initial Beta vs S2 x-position graphical gate.

The gates utilised in standard particle identification (Z vs A/Q) are shown for $^{206}$Hg and $^{206}$Pb in Figures 55 and 56 respectively. The effect of these gates on the spectrum of $^{206}$Hg is shown in figure 57. In this figure, other gates such as the initial identification of the fragment via Beta vs S2 x-position, as well as a scattering angle and LYCCA EdE gates are already in place; a comprehensive demonstration of the effect of harsh gating is
difficult, as some gates reduce the data by an order of magnitude or more. Without any
gate on the incoming secondary beam, the structure of the peak is seen to be comparable
to background fluctuations; as such these gates are vital.

The scattering angle is of particular importance in an experiment utilising Coulomb
excitation, as it is an approximate measure of how closely the projectile fragment, and
the target interacted. A large scattering angle indicates the two particles came into close
contact, which is associated with an increased probability that the interaction involved
more than the desired electromagnetic exchange. The upper limit of scattering angle can
be calculated, and is known as the grazing angle.

Since the corresponding effective impact parameters can be calculated for both $^{206}$Hg
and $^{206}$Pb, once a gate is found for one fragment, it can simply be calculated for the other.
It is observed that the peak integral evaluation method yields similar gate widths for these
two fragments (see Table 10). The profile of the scattering angle, and the associated gate
for $^{206}$Hg is shown in Figure 58, along with the profile of the peak integral.

In the case of energy loss in the DSSD Wall, and total energy deposition in the CsI
detectors, this method can be used to guide a graphical cut made on a 2-dimensional his-
Figure 54: An example of integral evaluation: The area of the profile marked by the red-dashed line actually has an integral that is below zero. Despite those bins that do seem to contribute positively to the peak, taking this area of the data will result in a net loss of peak counts above background, so in this case it should instead be gated away.

togram. In each case, all other gates that contribute positively to the peak-to-background ratio are included, and the integral of the peak above the local background can be plotted against a profile of the variable to be tested, as described above. The resulting rectangular cut that is produced by using this method on $\Delta E$ and $E$ separately is applied, and a graphical gate is added on top, around whatever apparent structure remains in the data. The profile of these two LYCCA energy measurements, along with the resulting graphical gate are shown in Figures 61 and 62.

<table>
<thead>
<tr>
<th>Fragment</th>
<th>Grazing Angle (mrad)</th>
<th>Lower Limit (mrad)</th>
<th>Upper Limit (mrad)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{206}$Hg</td>
<td>21.3</td>
<td>6.8</td>
<td>14.4</td>
</tr>
<tr>
<td>$^{206}$Pb</td>
<td>21.8</td>
<td>3.7</td>
<td>15.0</td>
</tr>
</tbody>
</table>

Table 10: Gates described in this table are determined empirically by evaluating the range of the peak integral. The larger empirical gate for the $^{206}$Pb is likely due to the larger cross-section of Coulomb-excitation used to determine it, which will result in a peak that will be visible above background relatively more easily. The grazing angle should be considered a theoretical upper limit.
Figure 55: Z vs A/Q plot of the $^{206}\text{Hg}$ setting. The red lines indicate the gates implemented, derived using the background-profile integral evaluation method.

It should again be noted that the scales used for these (LYCCA E and $\delta E$) variables are arbitrary; the energy measurements of the modules are aligned, but not calibrated to a realistic energy scale, as it is not needed for this analysis. Once a graphical cut is made, the profile of peak counts above background can again be evaluated. If the integral evaluation algorithm determines that the edges of the data are the optimum gates, then the gate is either too strict, or no further improvement can be made. As this implies, provided moderation is used, this method can be applied iteratively against one variable at a time, continuously, until the peak-to-background ratio is completely optimised for all measured or calculated aspects of the data. The effect of these gates on the $\gamma$-spectrum is shown in Figure 60, and the final graphical gate created as a result of this method is shown in Figure 59.

Further gates include those on the position of the particle at various locations at S4. The large number of detectors providing position information allow the particles to be tracked with reasonable precision between the focal point at S4 to their interaction point with the secondary target. The evaluation of the peak integral reveals that there is a consistent, minor improvement to be found in removing the edges of the beam (where
Figure 56: Z vs A/Q plot of the $^{206}$Pb setting. The red lines indicate the gates implemented, derived using the background-profile integral evaluation method.

the purity is expected to be lower), as is shown in Figure 61. The result of this gate on the spectrum is shown in Figure 62.

Another important gate is the removal of the prompt-flash, and events that occur long after the beam has passed the secondary target. As the beam travels from the final dipole magnet D4, it interacts with a lot of detector material before it reaches the secondary target. Any induced $\gamma$ emissions will arrive ahead of the beam, creating this flash. This unwanted information is removed through the use of a particle-$\gamma$ time gate (Figure 63); the time between the detection of a $\gamma$-ray event and the trigger (Sc41) is reconstructed, and a time window of approximately 50-70 ns is used. The effect of this gate on the $\gamma$ spectrum is shown in Figure 64.

A complete summary of the gates utilised:

- Beta vs S2 x-position graphical cut
- Z vs AoQ graphical cut
- Scattering angle / “effective impact parameter”
- LYCCA E$\Delta$E graphical cut

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Figure 57: The $\gamma$ spectrum of $^{206}\text{Hg}$ with various Z and A/Q gates. In blue, neither gate is used. In pink, the Z gate is applied. In red, the A/Q gate is applied. In green, both gates are applied. It should be noted that the significant reduction in statistics when the A/Q gate is applied is in part due to the low efficiency of this calculation, which relies on a measurement of $\beta$, as well as x-position at both S2 and S4.

- x-position at target position
- y-position at target position
- x-position at S4 focal plane
- $\beta$ value used for Doppler-correction
- Particle-$\gamma$ time

6.10 Background Subtraction

Once a peak is found, and the Doppler-correction is set, it is inevitable that some background will remain. These $\gamma$-rays are introduced into the analysis at this late stage through random coincidences, as well as emissions from interactions within the beam itself, as well as between the beam particles and detector material in the immediate vicinity.
Figure 58: Left: Scattering angle measured by use of the particle-tracking system. Right: A profile of the integral of the peak above background with respect to the same measurement. The gate utilised is marked in red on both figures.

Figure 59: The graphical cut here is deduced by determining optimum linear cuts with respect to both $E$ and $\Delta E$. A comparable gate is used in both the $^{206}\text{Hg}$ and $^{206}\text{Pb}$ settings.

of the target. In order to eliminate this background and leave only the Coulomb-excitation interaction of the beam with the target, a background subtraction is performed. For each setting, a short run is performed with the Coulomb excitation target removed, allowing the profile of the background from all other sources to be analysed.

This “empty-target” data is processed in precisely the same manner as the typical data, ideally with settings primarily based on the Coulex runs that were performed immediately before or afterwards. $\gamma$-tracking and Doppler-corrections are also performed in the same way, using a set of time-of-flight offsets such that the mean $\beta$ value is the same as that of the Coulex data. The gates used are also identical, and adjusted in the case of
Figure 60: The $\gamma$ spectrum of $^{206}$Hg without any E or $\delta$E gates. In green, the total energy deposition (CsI) gate is applied. In red, the $\delta$E (DSSD Wall) gate is applied. In pink, the final graphical gate on E$\delta$E is applied.

the LYCCA E and $\delta$E gates to compensate for the lack of energy loss in the target.

Since the statistics of the empty-target runs are not comparable to that of the full statistics of the Coulex data, the $\gamma$-spectrum must be scaled, such that the total integral of the background area is equal. This background can be directly subtracted from the Coulex $\gamma$-spectrum, but a better approach is to fit this background in order to remove the granularity resulting from limited statistics. A polynomial line is fitted to the background, seen in Figure 65, and is subsequently scaled to match the magnitude of the peak spectrum.

Using the same strict gates as with the regular Coulomb excitation data, problems may arise from a lack of sufficient statistics to produce a reasonable spectra. In Figure 65, while the profile is representative of the desired physical spectra, increased counts in the low and high energy are observed to be inconsistent with the spectrum with the gold target in place. In this situation, the Coulomb excitation data itself can be fit in the same manner, to produce a background line above which anomalous features (such as the Doppler-corrected peak of interest) can still be be seen. An example of this is shown in
Figure 61: Left: The x-position of the particle extrapolated at the target position. Consistently, the beam of $^{206}$Hg is incident upon the target slightly to one side of the beam-axis. Right: The peak integral across the same profile, with the gate used marked in red.

Figure 62: The $\gamma$ spectrum of $^{206}$Hg with (red) and without (blue) the gate on particle x-position extrapolated at the target location.

Figure 66.

6.11 Isomeric Ratios

In order to calculate an accurate B(E2), it is important to know how many particles are able to interact with the target as intended. Existing data shows clearly the presence of isomeric states above the $2^+$ state being studied with lifetimes that are comparable to, or greater than the flight-time through the FRS. Therefore, it is necessary to make a correction to the calculation, accounting for particles that will arrive at the secondary
Figure 63: An identical gate is used in both $^{206}\text{Hg}$ and $^{206}\text{Pb}$ settings for particle-gamma time. The width of this gate is 66 ns.

target in a highly-excited state, unable to decay through the $2^+$ as a result of Coulomb excitation. The key to this correction is the measurement of the isomeric ratio. The isomeric ratio is defined as the number of ions detected in the isomeric state relative to the total number of ions detected.

$$R = \frac{\epsilon_{abs}N_\gamma(1 + \alpha_T)}{bTN_{ion}} \frac{1}{f_1f_2f_3f_4}$$

(31)

Where $\alpha_T$ and $b_t$ are the conversion coefficients and (absolute) branching ratio of a given transition, and $\epsilon_{abs}$ is the absolute efficiency of the detection system. The calculation is repeated for each decay below the isomeric state in question, and an average can be taken. $N_{ion}$ is the total number of ions contributing to the measurement. Also included are $f_{1,2,3,4}$, which are correction factors that must be accounted for. The decay of implanted ions follows the relation:

$$A(t) = A_0(2)^{-t/\tau}$$

(32)

Where $A(t)$ is the number of decays per second at time $t$, and $A_0$ is the decays per
Figure 64: The $\gamma$ spectrum shown with and without the $\gamma$-particle time gate. In blue, no gate is applied; in red, a time-window of approximately 66 ns is used to eliminate the prompt-flash.

second at t=0 s. The half-life of the state is $t_{1/2}$ while the nucleus is in a fully-stripped charge state, and the time elapsed since the excitation of the ion is t. The first correction is that of the time-of-flight of the particle, and accounts for two effects: The particle begins decaying immediately after it is formed by fragmentation with the primary target, and the particle is traveling at relativistic speeds through the FRS, and thus does not decay as much in the lab frame as it otherwise would at rest over the same period of time. The exact velocity of the particle varies as it travels through the FRS, and so a corresponding factor is used to correct for the exponential decay during the different stages of the particle’s path. This correction factor $f_1$ can be written as

$$f_1 = e^{-\lambda_0 \left( \frac{\text{ToF}_{1a}}{t_{1a}} \right)} + e^{-\lambda_0 \left( \frac{\text{ToF}_{1b}}{t_{1b}} \right)} + e^{-\lambda_0 \left( \frac{\text{ToF}_2}{t_2} \right)}$$  \hspace{1cm} (33)$$

where ToF$_{1a,1b,2}$ are the time-of-flights between the primary production target and Sc21 (1a), between Sc21 and Sc41 (1b), and Sc41 and the secondary target (2). The time-of-flight between Sc21 and Sc41 is of course a measured value, however the other values must be provided by the LISE++ simulation. $\lambda_0$ is the decay constant associated with the isomeric state at rest, with a parent nucleus that is in a fully-stripped charge state.
Figure 65: The $\gamma$-spectrum of $^{206}$Hg with the target removed. The red line indicates a polynomial fit to this background profile that can be subtracted from the spectrum of Coulomb excitation.

state:

$$\lambda_0 = \frac{\ln 2}{t_{1/2}} \sum n_i \left( \frac{b_{ti}}{1 + \alpha_{ti}} \right)$$ (34)

$b_{ti}$ and $\alpha_{ti}$ are the branching ratio and conversion coefficient of the transition. This factor accounts for the portion of particles in the excited state that will not decay via $\gamma$ emission, and thus cannot be detected in the isomeric state.

The second correction is to correct for the limited time window of $\gamma$-ray detection; once the particles are implanted in the plastic stopper, there is still a finite amount of time in which the $\gamma$-rays can be detected and still be accurately associated with particle information from the other detectors. Without this information, no reasonable trigger can be chosen (single-$\gamma$ events are too numerous). On this principle, a number of decays will inevitably be lost, and this fraction can be estimated as:

$$f_2 = e^{-t_{1/2}t_i} - e^{-t_{1/2}t_f}$$ (35)

Where $t_i$ and $t_f$ are the initial and final times of the detection window. The absolute
Figure 66: The $\gamma$-spectrum of $^{206}$Hg. The red line indicates a polynomial fit to this background profile that can be subtracted.

limits in this analysis are defined during the merging of the data, where an absolute limit of 20 $\mu$s is defined as the “merging window”; this constraint is introduced by the hardware of the data acquisition. In order to eliminate background, and improve the peak-to-background ratio for the measurement of the isomeric ratio, this time-window will be altered to better reflect the decay of the isomeric state, and remove the more prompt content from the spectrum.

The third correction regards the finite detection window available to AGATA; upon the arrival of the beam, a large number of $\gamma$-rays are also present, which can hinder the successful detection of $\gamma$-rays from particles embedded in the target. This prompt-flash of $\gamma$-rays is caused by primary and secondary Bremsstrahlung radiation, radiative electron capture, and x-ray emissions. As such, the dead-time of the detector array must be accounted for.

$$f_3 = 1 - \frac{N_p}{N_{ion}N_c} \quad (36)$$

Where $N_p$ is the number of prompt events, and $N_c$ is the number of active detectors. In this experiment, 17 AGATA crystals were available. The product of this value with
Material Depth (mm) | $^{206}\text{Hg}$ Particles Destroyed % | $^{206}\text{Pb}$ Particles Destroyed %
--- | --- | ---
1 | 2.56 | 2.56
2 | 5.05 | 5.03
5 | 12.00 | 11.95

Table 11: It can be estimated that approximated 24% of particles are destroyed in the thick plastic stopper.

The absolute efficiency $\epsilon_{abs}$ provide the effective efficiency of the detection system. The number of prompt events can be measured as the number of counts in the decay peak during the prompt flash. Also included in this correction is the efficiency of the $\gamma$-tracking algorithm, if it is used. This value can be calculated by taking the ratio of the counts in the peak of the “core” spectrum and the counts in the peak of the tracked spectrum.

$$f_4 = 1 - \frac{N_{\text{survived}}}{N_{\text{destroyed}}}$$  \hspace{1cm} (37)

The final correction, shown above, is to address the number of particles that react with the plastic stopper upon implantation, rather than simply being embedded, and decaying without additional reactions. This would otherwise lead to an underestimate of the isomeric ratio, as the number of participating ions in the measurement is not as high as the number of ions incident on the secondary target material. This quantity is largely provided by an estimation from the LISE++ simulation, whereby a fraction of the plastic stopper material is placed, and the transmission statistics can be examined. From this partial estimation, the fraction of ions reacting in the complete thickness of the plastic stopper used (25mm) can be extrapolated. The values for $^{206}\text{Hg}$ and $^{206}\text{Pb}$ are shown in Table 11:
Table 12: This approach allows the consistency of the tracking algorithm to be verified against the ‘core’ detector signals, where AGATA is used as a typical HPGe detector.

### 7 Results

#### 7.1 Isomeric Ratios

Isomeric ratio contributions can be determined for each transition in $^{206}$Hg and $^{206}$Pb for a range of time windows and particle gates. This allows the consistency of the $\gamma$-tracking algorithm to be determined under different conditions of $\gamma$-ray multiplicity. The spectra of isomeric states in the $^{206}$Hg and $^{206}$Pb beam settings are shown in Figures 68 and 69 respectively. The following table summarises the measurements made for transitions of fragments in the $^{206}$Hg isomer study, using a time-window with a fixed width of 2000 ns, with varying offsets from the prompt-peak. This long time-scale is chosen because of the lifetime of the 5- state: $2.15(21) \mu$s [9].

The lifetime of the state can also be measured by examining the decay profile of the peak area, as seen in Figure 67. A simple exponential decay is fitted, and the resulting measurement is $2.19(7) \mu$s, which is close to the previously measured value of $2.15(21) \mu$s [46].

Isomeric contributions can also be considered from the 10+ state, which is another known isomeric state in $^{206}$Hg. Since this state has a relatively short lifetime of $92(8)$ ns [9], the same time windows cannot be used, and a much shorter study is employed. A time window of 1000 ns is used, positioned close to the prompt events of the $\gamma$ spectrum. Since a filter is used to eliminate some noise in AGATA, which prevents the transmission of signals that correspond to single-core events of only (approximately) 200 keV or less, the 100 keV transition between the 10+ and 8+ states in $^{206}$Hg cannot be properly measured.
Figure 67: The exponential decay line fitted to the data is shown in red, and indicates a half-life of 2.19(7) µs.

<table>
<thead>
<tr>
<th>Transition</th>
<th>$\gamma$ energy (keV)</th>
<th>Peak Integral</th>
<th>Isomeric Ratio Contribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{10+} \rightarrow ^{7-}$</td>
<td>1257</td>
<td>73</td>
<td>0.023(6)</td>
</tr>
<tr>
<td>$^{8+} \rightarrow ^{7-}$</td>
<td>1157</td>
<td>290</td>
<td>0.028(4)</td>
</tr>
<tr>
<td>$^{7-} \rightarrow ^{5-}$</td>
<td>364</td>
<td>73</td>
<td>0.039(7)</td>
</tr>
</tbody>
</table>

Table 13: Isomeric decay statistics of the $^{10-}$ state in $^{206}$Hg.

This transition is highly converted however, with a conversion factor of 5.54 [47].

The contributions from each transition are adjusted according to their branching ratios, and an absolute isomeric ratio is calculated for the $^{10+}$ state. Since the decay from the $^{5-}$ state is a simple two-stage cascade with no alternate (known) branches, this calculation is simpler. It should be noted that while there are no known transition that circumvent the isomeric state, the existence of such a transition would be intrinsically accounted for with this methodology for determining the isomeric state at the secondary target. A summary of contributions from each isomeric state is shown in Table 14.

<table>
<thead>
<tr>
<th>Isomeric state</th>
<th>Energy Level (keV)</th>
<th>Isomeric Ratio (core)</th>
<th>Isomeric Ratio (Tracked)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{5-}$</td>
<td>2102</td>
<td>0.24(2)</td>
<td>0.24(2)</td>
</tr>
<tr>
<td>$^{10+}$</td>
<td>3723</td>
<td>0.029(2)</td>
<td>0.027(3)</td>
</tr>
</tbody>
</table>

Table 14: The total isomeric ratio is the sum of ratios of consecutive decays.
Figure 68: The $\gamma$ spectrum of $^{206}$Hg embedded in the thick plastic target for isomer study. A time window of 500 ns is used to create this spectrum. The lines of interest for this measurement of the isomeric ratio of the $10^+$ state are at 364 keV, 1157 keV, and 1257 keV.

Both of these isomeric ratios, as well as the ratio between them can be compared to previously measured values. For the tracked spectrum, $\text{IR}(10^+)/\text{IR}(5^-)=0.11(2)$, and for the core spectrum, $\text{IR}(10^+)/\text{IR}(5^-)=0.12(2)$. This provides a minor improvement on the precision of the previously measured value of 0.10(4) [46].

The measurement of the 'calibration point' transition strength must also be corrected in the same manner; an isomeric state exists in $^{206}$Pb at an energy of 2.2 MeV, with a lifetime of approximately 125 $\mu$s [9]. This state decays either directly to the $2^+$ state via a single transition of 881 keV, or through two transitions of 343 keV, and 537 keV. The transition of 343 keV is in an area of the spectrum where the background is particularly high, and they are not observed convincingly. A relevant level scheme of $^{206}$Pb is shown in Figure 8.

The measured isomeric ratio of the $7^-$ state in $^{206}$Pb is 0.209 (20) when using AGATA as a typical $\gamma$-ray detector, and is 0.210 (21) when using the Mars-Gamma Tracking
Figure 69: The isomeric spectrum of $^{206}$Pb. Labeled at the four observed transitions below the isomeric state at 2.2 MeV.

<table>
<thead>
<tr>
<th>Transition</th>
<th>$\gamma$ energy (keV)</th>
<th>Branching Ratio</th>
<th>Peak Integral</th>
<th>Isomeric Ratio Contribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>$7^- \rightarrow 4^+$</td>
<td>516</td>
<td>1.0</td>
<td>273</td>
<td>143</td>
</tr>
<tr>
<td>$3^+ \rightarrow 2^+$</td>
<td>537</td>
<td>0.314</td>
<td>112</td>
<td>37</td>
</tr>
<tr>
<td>$4^+ \rightarrow 2^+$</td>
<td>881</td>
<td>0.686</td>
<td>235</td>
<td>116</td>
</tr>
<tr>
<td>$2^+ \rightarrow 0^+$</td>
<td>803</td>
<td>1.0</td>
<td>256</td>
<td>215</td>
</tr>
</tbody>
</table>

Table 15: The isomeric ratio of $^{206}$Pb is consistent between tracked and non-tracked spectra.

algorithm. The increased error in the latter value is due to the decreased efficiency when using the $\gamma$-tracking, but the consistency is important to demonstrate it is working correctly. To account for the very long lifetime of this state (125 $\mu$s [9]), a long particle-$\gamma$ time gate was used, of 10 $\mu$s.

For all measurements and calculations of isomeric ratios, the efficiency of AGATA is required. Efficiency of the AGATA demonstrator with 21 crystals was measured to be 3.3% [48]; since the array used in this experimental only consisted of 17 crystals, this value was corrected to 2.6%. This value is further adjusted for the dead-time of the ancillary detectors during the calculation of isomeric ratios.
7.2 Transition Strengths

The strength of a transition can be defined to describe either the excitation to upper state, or the decay from that state. The transition strength is also closely related to the cross-section of excitation for a given state, and thus by measuring the cross-section of an excitation, the transition strength can be deduced. For the technique of Coulomb excitation, the number of observed Coulex events can be summarised as follows:

\[ N_{\text{coulomb}} = N_{\text{beam}}\sigma N_{\text{target}} \]  \hspace{1cm} (38)

Where \( N_{\text{coulomb}} \) is the number of instances of Coulomb excitation, \( N_{\text{beam}} \) is the number of beam particles, \( N_{\text{target}} \) is the number of target particles per unit area, and \( \sigma \) is the cross-section of the interaction (area). If a cross-section is known, and \( N_{\text{coulomb}} \) is measured, then a new, unknown cross-section can be deduced by measuring \( N_{\text{coulomb}} \), while the conditions and method of the measurement remains consistent. In this case, the known value is that of \( ^{206}\text{Pb} \), and the unknown value is that of \( ^{206}\text{Hg} \). The number of Coulex events detected differs by the efficiency of the detection system.

\[ N_\gamma = \epsilon_\gamma N_{\text{beam}}\sigma \]  \hspace{1cm} (39)

Where \( N_\gamma \) is the number of \( \gamma \)-ray events associated with the Coulomb excitation of the fragment of interest, and \( \epsilon_\gamma \) is the absolute efficiency of the detection system. This value of efficiency includes all aspects of the system, including (for instance) the efficiency of the \( \gamma \)-tracking algorithm used to reconstruct the spectrum, the efficiency of the \( \gamma \)-ray detection system itself, and any and all ancillary detectors used in conjunction. \( N_{\text{beam}} \) should represent the total number of particles available for Coulomb excitation, (in this case it is the number described by trigger 10, multiplied by the reduction factor \( 2^8 \)). Further corrections must be applied to refine this calculation.
\[
\frac{\sigma_{hg}}{\sigma_{pb}} = \frac{\epsilon_{Pb} N_{\gamma Hg} N_{\text{particlesPb}}}{\epsilon_{Hg} N_{\gamma Pb} N_{\text{particlesHg}}} \epsilon_{Hg}
\]  

(40)

By taking the ratio of two measurements, as shown above, unknown values, such as many contributing factors to the efficiency of the detection system are eliminated entirely, leaving only the relative efficiency of the $\gamma$ detectors, relating to the efficiency-curve in relation to the energy of each $\gamma$-ray transition. The isomeric ratio of each fragment must also be considered, as those particles arriving at the secondary target in an isomeric state cannot Coulomb excite, and thus should not be included in the calculation of the transition strength. The final spectrum of $^{206}$Hg is shown in Figure 72, and again in Figure 73 with a background subtraction applied. Similarly, the peak of $^{206}$Pb that will serve as a calibration point for this measurement is shown in Figure 70, and again in Figure 71 with a background subtraction applied.

![Figure 70: The peak of Coulomb excitation of $^{206}$Pb at 803 keV.](image)

Once a cross-section is calculated for the nucleus of interest, the transition strength can be deduced using a simulation code Dweiko [49]. In this case, the code is provided with the mass and proton number of the interacting nuclei, the energy and spin of the first
Figure 71: The peak of $^{206}\text{Pb}$ at 803 keV with a background subtraction applied. The background subtracted was calculated with a polynomial line based on the spectrum of an empty-target run of the same setting.

excited state in $^{206}\text{Hg}$, and finally the transition strength in units of $e^2b^2$. The produced value is the cross-section of the interaction. By testing a progressively more precise range of values of transition strength, the correct measured experimental value can be deduced for the measured cross-section. A summary of the results is shown in the table below:

<table>
<thead>
<tr>
<th>Fragment</th>
<th>Particles (millions)</th>
<th>Peak Counts</th>
<th>Isomeric Ratio</th>
<th>Cross-Section (mb)</th>
<th>B(E2) ($e^2b^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{206}\text{Pb}$</td>
<td>166</td>
<td>120</td>
<td>0.21(2)</td>
<td>124</td>
<td>0.101 (3)</td>
</tr>
<tr>
<td>$^{206}\text{Hg}$</td>
<td>410</td>
<td>95</td>
<td>0.27(3)</td>
<td>50</td>
<td>0.040$^{(2)},^{(15)}$</td>
</tr>
</tbody>
</table>

### 7.3 Additional Observations

In addition to the observed and measured transition from the $2^+$ to the $0^+$ state in $^{206}\text{Hg}$, it is also expected that a portion of the $^{206}\text{Hg}$ particles may also exhibit other behaviour upon interaction with the secondary target. Those that arrive in an isomeric state may excite further before decaying, and any undocumented states may also be excited, and in turn decay. One such state is the $3^-$ state, which has yet to have been observed in this
Figure 72: The peak of Coulomb excitation of $^{206}\text{Hg}$ at 1068 keV.

This 3$^-$ state is collective in nature, and has been documented in other even-even nuclei in the area, including those along the Z=80 line [50][51][52][53][54]. Both the transition strengths and energies of these have been measured. It is expected that a similar state exist in $^{206}\text{Hg}$, with a similar energy of roughly 2.7 MeV. A small peak is observed in this area, which could be this transition, see Figure 74.

An upper-limit for the cross-section of this transition can be estimated from a calculation of the transition strength made, using the Dweiko code. Based on the systematics of B(E3) measurements of other Hg isotopes leading to the N=126 shell closure, (see Figure 75) an estimate of the transition strength can be made. 22 Weisskopf units will yield a transition strength of 0.5 e$^2$b$^3$, which indicates a cross-section for Coulomb-excitation of 37 mb. For the number of particles in this analysis, it is therefore expected that the upper-limit of the peak integral for this transition is just 7 counts. It must also be noted however, that such a state would also be able to decay via the 2$^+$ state at 1068 keV via an M1 transition of the energy difference between these two states (approximately 1.7 MeV). This M1 transition would be highly favoured over the E3 decay implied here. The observation here is therefore unlikely to be the decay of the 3$^-$ state in $^{206}\text{Hg}$.
8 Interpretation and Conclusion

The transition strength can be expressed in a number of different ways, however amongst the most common representations is with the Weisskopf estimate. This unit describes a transition with a single contributing nucleon situated in an average central potential within the nucleus; 1 W.u. describes this scenario. Higher values imply contributions from other nucleons, and so the transition strength, when expressed in these units, can be a simple way of estimating the number of nucleons involved in a given decay. For an E2 transition, the formulation is as follows: [56]

$$B(E2)_W = 0.0594A^{4/3}e^2fm^4$$  \hspace{1cm} (41)

Where B(E2) is expressed in units of e²fm⁴ and A is the mass. The units of the cross-section of Coulomb excitation must be handled carefully here, as while the transition probability can be obtained via a linear relation, the reduced matrix element of interest
Figure 74: A small peak appears to be observed in the region of the anticipated $3^- \rightarrow 0^+$ transition.

is part of an exponential term in the equation relating it to the cross-section:

$$B_{\lambda}(E; J_i \rightarrow J_f) = \frac{L[(2L + 1)!]!!}{8\pi(L + 1)} (\frac{\hbar c}{E_\gamma})^{2L+1} P_\gamma(E; \lambda; J_i \rightarrow J_f) \tag{42}$$

$$B_{\lambda}(E; J_i \rightarrow J_f) = (2J_i + 1)^{-1} |\langle \Psi_f | M(E; \lambda) | \Psi_i \rangle|^2 \tag{43}$$

where P is the probability of transition by $\gamma$-ray. The second formula shown above (42) shows the reduced matrix element in relation to the measured transition strength. It can also be seen from this equation that as a result of the spin expression, the transition strength of $0^+ \rightarrow 2^+$ is a factor of 5 greater than that of the decay $2^+ \rightarrow 0^+$. During the calculation of the reduced matrix element, it is important therefore to recognize that the measured value is the decay, rather than the excitation. For the value determined by this measurement, the Weisskopf estimate of the decay is $1.109^{631}_{409}$. This low value suggests that the $2^+$ state is notably uncollective in nature; the excitation of the proton from the $s_{1/2}$ to the $d_{3/2}$ subshell is the principally responsible for the energy of the state.

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In order to properly understand the result, it must be put in the proper context. On the line of systematic measurements of B(E2) values in even-even Hg nuclei, the value describing $^{206}$Hg is of particular interest, as it is where this line of systematics reaches the closed shell at N=126. The result can also be compared to an estimate of the B(E2) using the D1S Gogny force, which uses a parametrized finite-range interaction to simulate the behaviour of nucleons more realistically at non-contact and long ranges [14], which makes it a more realistic approach to the study of nuclear structure than calculations using a zero-range, density dependent term. This force has been shown to have some consistency, in particular in light nuclei [57], however it is known to be poorly constrained towards heavier nuclei, such as those in the region of $^{208}$Pb [58]. The estimation by the D1S force is 2.49 W.u [12].

In addition to the D1S force, the Gogny 5-Dimension-Collective-Hamiltonian (CHFB-5DCH) variation is also considered [59]. This method also uses the D1S parametrizations, and employs an additional six kinetic terms and three associated rotational moments of inertia, as well as three mass parameters to describe the deformation and characteristics of single-particle states. This approach does not describe spherical nuclei well, and in
a comprehensive study, nuclei at closed shells, in particular doubly-magic nuclei are not considered as the mapping of CHFB to the collective Hamiltonian breaks down. In principle this approach is still reliable for depicting deformed $2^+$ excitations in nuclei with one open shell, however it does not achieve the same precision found in more deformed nuclei. The Gogny-CHFB-5DCH calculates the value of B(E2) in $^{206}$Hg to be 5.26 W.u [13].

![Figure 76: The level scheme b shows the effect of introducing additional measurements to a improvement to the level scheme indicated by a. The ordering of the $4^+$, $5^-$ and $7^-$ states is vital to accurately describe the observed experimental data [60].](image)

An additional comparison can be made to the calculated value of B(E2) for $^{206}$Hg made by Grawe [61] of 5.20 W.u. This theoretical calculation utilises a modified parametrization of the shell model, specifically refining the Ryström interaction in three places to better describe the ordering and energy levels in the area, using data from B. Fornal et al. [62]. The $(d_{3/2} h_{11/2})_7^-$ two-body matrix-element was increased by +135 keV to more effectively explain the energy structure of $^{206}$Hg. The $(s_{1/2} d_{5/2})$ monopole was increased
by +230 keV to describe the structure of $^{204}$Pt better, as can be seen in Figure 76. An unmodified shell model calculation does not correctly order the $4^+$, $7^-$ and $5^-$ states in this nucleus; experimental data shows that there is an isomeric state above the $2^+$ first excited state, with a lifetime of 5.5 µs. A $4^+$ state would decay to the $2^+$ state via an E2 transition, which would render the state far more short-lived. The modified version of the shell model calculation more accurately describes this structure. The third adjustment is the increase of the \((s_{1/2} h_{11/2} ; d_{3/2} h_{11/2})_6\) two-body matrix element to +160 keV. These changes also have consequences for the description of other nuclei in the area: Figure 77 shows the level schemes for $^{205}$Au and $^{203}$Ir which have also been improved. In particular the ordering of the $23/2^+$ and the $19/2^+$ states in $^{203}$Ir are indicated experimentally by a low-energy $\gamma$ decay from a state with a lifetime of 798 ns [63]. In the unmodified shell model, this transition would be drastically hindered by the re-ordering of these states, resulting in a very long lifetime that does not match the data. This calculation has been shown to have some limited success in describing the energy and transition strengths of known states in the vicinity of $^{208}$Pb [60].

![Figure 77](image.png)

**Figure 77:** The level scheme produced by the unmodified calculation is indicated by a, the modified is indicated by b. The modified shell-model calculation more accurately describes the structure of $^{203}$Ir [60].

Also included for comparison is the unmodified version of the shell model calculation,
which estimates a significantly higher estimation of B(E2) for $^{206}$Hg of 15.06 W.u. Experimental data on B(E2) values in even-even isotopes of mercury is established for N=116 to N=124 [50][51][52][53][54]. A summary of these values, as well as the calculated values of B(E2) in $^{206}$Hg is shown in Figure 78.

![Figure 78](image)

Figure 78: The lowest estimation by theoretical calculation is 2.49 W.u., is closest, but outside the uppermost error range of the measured result.

All theoretical calculations thus far overestimate the value of B(E2) in $^{206}$Hg. Further parametrization of existing calculations has been shown to clearly improve our description of nuclei in the light of experimental data (Table 16), however there are a number of discrepancies, and in particular this result shows a stark contrast between expectations and the measured value of 1.109$^{63\pm409}$ W.u., which describes the state as notably uncollective in nature. It is clear that no single theoretical model describes this region of the nuclear chart well in every aspect, and in order to address this, further work is needed in the parametrization of such calculations to refine our mathematical description of the nucleus, and adhere better to empirical measurements. It is apparent that this experimental effort is close to the limit of current experimental techniques and technology; more experimental work will be required to improve the precision of this technique if further measurements
Table 16: B(E2) measurements and calculations for a number of A=206 nuclei. Parametrization of theoretical calculations result in clear improvements in many cases.

<table>
<thead>
<tr>
<th>Nucleus</th>
<th>Transition</th>
<th>B(EL) (W.u.)</th>
<th>( \text{exp} )</th>
<th>SM\text{standard}</th>
<th>SM\text{mod}</th>
</tr>
</thead>
<tbody>
<tr>
<td>(^{206}\text{Hg})</td>
<td>B(E2 : (2^+ \rightarrow 0^+))</td>
<td>(1.109^{(631)}_{(410)})</td>
<td>15.055</td>
<td>5.2</td>
<td></td>
</tr>
<tr>
<td>(^{206}\text{Hg})</td>
<td>B(E3 : (10^+ \rightarrow 7^-))</td>
<td>0.25(3)</td>
<td>0.17</td>
<td>0.21</td>
<td></td>
</tr>
<tr>
<td>(^{206}\text{Hg})</td>
<td>B(E2 : (10^+ \rightarrow 8^+))</td>
<td>0.94(15)</td>
<td>0.87</td>
<td>0.87</td>
<td></td>
</tr>
<tr>
<td>(^{206}\text{Hg})</td>
<td>B(E3 : (5^- \rightarrow 2^+))</td>
<td>0.18(2)</td>
<td>1.17</td>
<td>0.90</td>
<td></td>
</tr>
<tr>
<td>(^{205}\text{Au})</td>
<td>B(E3 : (19/2^+ \rightarrow 13/2^-))</td>
<td>0.3(1)</td>
<td>0.004</td>
<td>0.2</td>
<td></td>
</tr>
<tr>
<td>(^{205}\text{Au})</td>
<td>B(E3 : (19/2^+ \rightarrow 15/2^-))</td>
<td>0.3(1)</td>
<td>1.0</td>
<td>1.10</td>
<td></td>
</tr>
<tr>
<td>(^{205}\text{Au})</td>
<td>B(E2 : (19/2^+ \rightarrow 15/2^+))</td>
<td>1.2(2)</td>
<td>2.99</td>
<td>1.70</td>
<td></td>
</tr>
<tr>
<td>(^{205}\text{Au})</td>
<td>B(M4 : (11/2^- \rightarrow 3/2^+))</td>
<td>(\leq 1.7(7))</td>
<td>1.92</td>
<td>2.46</td>
<td></td>
</tr>
<tr>
<td>(^{204}\text{Pt})</td>
<td>B(E3 : (10^+ \rightarrow 7^-))</td>
<td>0.19(3)</td>
<td>0.21</td>
<td>0.21</td>
<td></td>
</tr>
<tr>
<td>(^{204}\text{Pt})</td>
<td>B(E2 : (10^+ \rightarrow 8^+))</td>
<td>0.80(8)</td>
<td>2.64</td>
<td>1.22</td>
<td></td>
</tr>
<tr>
<td>(^{204}\text{Pt})</td>
<td>B(E2 : (7^- \rightarrow 5^-))</td>
<td>0.0034</td>
<td>1.21</td>
<td>0.0037</td>
<td></td>
</tr>
<tr>
<td>(^{204}\text{Pt})</td>
<td>B(E3 : (5^- \rightarrow 2^+))</td>
<td>0.039(5)</td>
<td>0.713</td>
<td>0.612</td>
<td></td>
</tr>
</tbody>
</table>

The effectiveness of \(\gamma\)-ray tracking is hindered by detectors that are not surrounded by other detectors. More clustered formations are heavily favoured to facilitate the tracking of \(\gamma\)-rays between crystals, that would otherwise be lost and incorrectly reconstructed. It is therefore clear that the expansion of AGATA will facilitate more effective and more precise measurements of transition strengths via Coulomb excitation. In addition, the purity of secondary beam settings for this experiment is another significant limitation with regards to the accuracy of this measurement. Further from the line of stability, the separation of fragments relies on the double-\(B\rho\) cuts from the FRS, however the extent of these cuts are bound to the technical limitations of the separator itself; the strength and angle of the magnets, as well as the energy of the incoming beam. It should also be noted that momentum spread is increased with the use of energy degraders, and the solution to this issue is therefore not so straightforward.

With further development in this area, the continuation of systematic measurements
can further constrain theoretical calculations, with the measurement of other B(E2) transition strengths of even-even nuclei surrounding the doubly-magic $^{208}\text{Pb}$. Potential candidates to compliment this measurement include the nuclei $^{208,210,212}\text{Po}$, as well as continuing the systematics of the Pb line to $^{210}\text{Pb}$. Some work has been done to examine the isotopes of Polonium, however unforeseen technical constraints hindered the experiment in early 2014. As part of the PRESPEC campaign, the experimental measurement of the B(E2) of $^{206}\text{Hg}$ serves as a demonstration of the limitations and capabilities of new detector technologies. This campaign, spanning from 2010 to 2014, serves as preparation to develop and refine the spectroscopy to be employed with HISPEC/DESPEC at NUSTAR/FAIR in years to come; as such this work is a small part of a larger endeavour to expand our collective understanding of the nucleus, and the nuances and details of its structure.
9 Bibliography

References


10 Appendix
ISOMERIC RATIOS IN $^{206}$Hg

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P.H. Regan$^a$, P. Reiter$^g$, H. Schaffner$^c$, P. Singh$^{b,c}$, C. Stahl$^b$
R. Stegmann$^b$, O. Stezowski$^a$, J. Taprogge$^j$, P. Thöle$^g$
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$^{206}$Hg was populated in the fragmentation of an $E/A = 1$ GeV $^{208}$Pb beam at GSI. It was part of a campaign to study nuclei around $^{208}$Pb via relativistic Coulomb excitation. The observation of the known isomeric states confirmed the identification of the fragmentation products. The isomeric decays were also used to prove that the correlations between beam identification detectors and the AGATA $\gamma$-ray tracking array worked properly and that the tracking efficiency was independent of the time relative to the prompt flash.

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1. Introduction

Spectroscopic data from doubly-magic nuclei and their nearest neighbours are essential to constrain nuclear models. One of the key observables is the $B(E2; 2^+ \rightarrow 0^+)$ transition strength which provides information about collectivity. Under the umbrella of the PReSPEC-AGATA campaign, an experiment to measure $B(E2)$ values in even-mass nuclei around $^{208}$Pb was performed at GSI, Darmstadt, Germany. Relativistic energy Coulomb excitation was employed. As a first step of such measurements, the secondary beam composition has to be determined. Isomeric decay studies are ideally suited for this, allowing unambiguous isotopic identification. In addition, the isomeric and ground-state content of the nucleus of interest can be determined. In the present contribution, isomeric decays were used to check the performance of the $\gamma$-ray detection system.

2. Experimental details

The experiment took place during October 2012. A primary beam of $E/A = 1$ GeV $^{208}$Pb was fragmented on a 2.5 g/cm$^2$ Be target. The secondary fragments were selected and identified using the FRS fragment separator. Settings centred on $^{198,200,202,206,208}$Pb, $^{206}$Hg and $^{200}$Pt were used. In the isomeric decay part of the experiment, the secondary gold Coulomb excitation target was removed and the nuclei were stopped in a 10 mm thick perspex stopper. The stopper was positioned 15 cm downstream from the centre of the AGATA tracking array [1]. The identification information from the FRS detectors was correlated with the $\gamma$-ray information provided by AGATA; 17 crystals were available for this experiment.

The identification of the fragmentation products is shown in Fig. 1. The observation of isomeric decays in $^{205}$Hg and $^{206}$Hg provide verification of consistency with the simulation.
3. Results and conclusions

The isomeric ratio is defined as the number of nuclei in the isomeric state divided with the total number of nuclei. It can be experimentally determined using the equation: $\text{IR} = \frac{N_\gamma (1+\alpha)}{N_{\text{ion}} \epsilon b FG}$, where $N_\gamma$ is the number of $\gamma$-rays detected, $\alpha$ is the conversion electron coefficient, $b$ is the branching ratio, $\epsilon$ is the $\gamma$-ray detection efficiency, $N_{\text{ion}}$ is the total number of ions, $F$ is a correction factor for the in-flight isomeric decays and $G$ considers that we measure only for a limited time window. For details of the procedure, see Ref. [2].

In the case of $^{206}\text{Hg}$, there are two known isomeric states: a $5^-$ state with a half-life of $2.15(21) \mu\text{s}$ [3, 4], and a $10^+$ state at a higher excitation energy, with a half-life of just $92(8) \text{ns}$ [3, 5]. In this experiment, 1.5 million $^{206}\text{Hg}$ ions were implanted. The delayed $\gamma$-ray spectrum associated with this nucleus, together with the relevant level scheme, is shown in Fig. 2. The half-life of the $5^-$ isomer was measured as $T_{1/2} = 2.19(7) \mu\text{s}$, which is in good agreement with the Nuclear Data Sheet value [3], and is slightly longer than the $2.09(2) \mu\text{s}$ obtained from a similar fragmentation experiment [6].

The isomeric ratio was determined in two different ways:

(i) using the AGATA detectors as traditional HPGe detectors, i.e. using only the core signals from the Ge crystals, and

(ii) using the Mars-Gamma Tracking algorithm (MGT), reconstructing each event from individual interactions within the array.

In order to investigate the functionality of the system and the tracking efficiency dependence on the $\gamma$-ray rate, the isomeric ratio was determined using different delayed time windows. The $\gamma$-ray rate during the considered time window changed by a factor of three.
The results were normalised to 1.00 in order to remove systematic uncertainties related to the level scheme as well as detection efficiency. The normalised isomeric ratio results are given in Table I. The extracted isomeric ratio, as it is supposed to be, is independent of the time window in which delayed $\gamma$ rays are accepted. This proves that the correlations between the two subsystems (FRS and AGATA) worked correctly and that the tracking efficiency was independent of the time relative to the prompt flash. In addition, the ratio of the two isomeric ratios was determined as: $\text{IR}(10^+)/\text{IR}(5^-) = 0.11(2)$. This compares well, and improves the precision of the previously measured value of 0.10(4) [6].

**TABLE I**

Normalised isomeric ratios for the $5^-$ isomer in $^{206}$Hg, determined with and without tracking and for different time windows.

<table>
<thead>
<tr>
<th>Time window [ns]</th>
<th>IR (non-tracked) [%]</th>
<th>IR (tracked) [%]</th>
</tr>
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<tr>
<td>384–2384</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>1384–3384</td>
<td>1.02(3)</td>
<td>1.01(4)</td>
</tr>
<tr>
<td>2384–4384</td>
<td>1.00(3)</td>
<td>1.02(4)</td>
</tr>
<tr>
<td>3384–5384</td>
<td>1.03(3)</td>
<td>0.99(4)</td>
</tr>
</tbody>
</table>

The Coulomb excitation data is being presently analysed. Combined with the beam composition information provided by the isomeric states, the $B(E2)$ values will be determined in the future.
REFERENCES

10.1 Half-lives of transitions

![Graph showing half-lives of electric transitions (Weisskopf estimate) corrected for internal conversion.](image)

Image source: Table of Isotopes [64].
Figure 2. Half-lives of magnetic transitions (Weisskopf estimate) corrected for internal conversion

Image source: Table of Isotopes [64].
### Table 10.2: Table of Beam Settings

<table>
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<tr>
<th></th>
<th>Z</th>
<th>B</th>
<th>G</th>
<th>H</th>
<th>X</th>
<th>Y</th>
<th>T</th>
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<td><strong>Primary Beam</strong></td>
<td>208Pb</td>
<td>208Pb</td>
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<tr>
<td><strong>SC21 Thickness</strong></td>
<td>3.27mm</td>
<td>3.27mm</td>
<td>3.27mm</td>
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<td>3.27mm</td>
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<td><strong>∆E Sc21 [GeV]</strong></td>
<td>5.362</td>
<td>5.436</td>
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<td><strong>∆E Finger</strong></td>
<td>1.286</td>
<td>1.591</td>
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<td>6.1 g/cm²</td>
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<td>5.2 g/cm²</td>
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<tr>
<td><strong>Brho S2-S4</strong></td>
<td>13.9141</td>
<td>13.6612</td>
<td>8.4820</td>
<td>6.9685</td>
<td>7.5155</td>
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<td>-</td>
<td>0.290</td>
<td>0.334</td>
<td>0.315</td>
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<td>1.245</td>
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<td>-</td>
<td>-</td>
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<td>1.302</td>
<td>1.210</td>
<td>1.156</td>
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<td><strong>∆E TPC42</strong></td>
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<td>-</td>
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<td>0.302</td>
<td>0.363</td>
<td>0.336</td>
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<tr>
<td><strong>∆E Sc41</strong></td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>7.172</td>
<td>9.108</td>
<td>8.200</td>
<td>7.704</td>
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<tr>
<td><strong>ToF Sc21-Sc41</strong></td>
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<td>-</td>
<td>-</td>
<td>189.200</td>
<td>181.600</td>
<td>176.8</td>
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<tr>
<td><strong>S4 Degrader Thickness</strong></td>
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<td>-</td>
<td>-</td>
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<td>-</td>
<td>-</td>
</tr>
<tr>
<td><strong>Energy at secondary target</strong></td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>140 MeV/u</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td><strong>Secondary Target</strong></td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>Au</td>
<td>-</td>
<td>-</td>
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<tr>
<td><strong>Secondary Target thickness</strong></td>
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<td>-</td>
<td>-</td>
<td>-</td>
<td>400 mg/cm²</td>
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<td><strong>∆E Target-ToF</strong></td>
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<td>-</td>
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<td>1.931</td>
<td>1.552</td>
<td>1.395</td>
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<tr>
<td><strong>LYCCA ToF</strong></td>
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<tr>
<td><strong>Target ToF</strong></td>
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<td><strong>∆E STOP</strong></td>
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<td>-</td>
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<td>7.890</td>
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<td><strong>∆E LYCCA DSSD</strong></td>
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<td>3.552</td>
<td>2.123</td>
<td>1.797</td>
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<tr>
<td><strong>∆E LYCCA CsI</strong></td>
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<td>6.735</td>
<td>25.594</td>
<td>36.386</td>
<td>39.397</td>
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</table>

Table 17: These tables display information used to create the beam settings necessary for FRS ID calibration, as well as the experimental runs.
<table>
<thead>
<tr>
<th>Component</th>
<th>Quantity</th>
<th>Unit</th>
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<tr>
<td>Primary Beam</td>
<td>200%±</td>
<td></td>
</tr>
<tr>
<td>Intensity</td>
<td>200%</td>
<td>200%</td>
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<tr>
<td>Beam</td>
<td>Primary Beam</td>
<td>100%</td>
</tr>
<tr>
<td>Energy</td>
<td>140.5 MeV/u</td>
<td>140.5 MeV/u</td>
</tr>
<tr>
<td>Beam</td>
<td>Secondary Target</td>
<td>100%</td>
</tr>
<tr>
<td>Energy</td>
<td>140 MeV</td>
<td>140 MeV</td>
</tr>
</tbody>
</table>

Additional Notes:

- Beam intensity was monitored using a Hall probe (MU4) and a beta detector.
- Secondary beam thickness was recorded using a ToF (TA-S2) detector.
- Target thickness was measured using a cesium iodide (CsI) detector.
- Secondary target thickness was measured using a TPC detector.
- SC21 thickness was also recorded.