

On Capacity and Bounds for the Gaussian CZIC and CBZIC with Linear Coded Approximation

Aaqib Patel, *Student Member, IEEE*, Fernando Reátegui del Águila, Md. Zafar Ali Khan, *Member, IEEE*, Muhammad Ali Imran *Senior Member, IEEE*, S. N. Merchant, *Senior Member, IEEE*, U. B. Desai, *Senior Member, IEEE* and Rahim Tafazolli, *Senior Member, IEEE*

Abstract—In this paper, we study the Gaussian Cognitive Z-interference channel (GCZIC) and its multiuser extension the Gaussian Cognitive Z-broadcast interference channel (GBZIC). We review some known capacity results and bounds for the GCZIC for various levels of interference. We derive a new improved inner bound for the CZIC under conditions which intersect with those for which the capacity is not known. Then we derive the capacity results and bounds for the CBZIC when the broadcast component of the channel is a degraded broadcast channel.

Index Terms—Gaussian Cognitive Interference channels, Z-Interference Channels, Degraded Broadcast Z-Interference Channels, Linear Coded Approximation.

I. INTRODUCTION

Cognitive radio (CR) is seen as a promising technology to solve the problem of spectrum scarcity. CR is an intelligent radio that exists along with traditional licensed radios known as Primary users (PU) and transmits either simultaneously with the PUs or when the PU is idle. When transmitting simultaneously, the CRs ensure that the interference to the PUs is either limited (underlay) or mitigated (overlay) [1]. Hence, the CR ensures that PU is not at a loss. An important question then arises that, what is the limit on the data rates that a CR can achieve in existing together with the PU? Answering this question will lead us to also answer how much spectral efficiency increase the CR can offer, which will help us to study the overall improvement in spectrum usage possible with the help of CR.

With this in mind, we study a class of interference mitigating CR and study the benefits that one can achieve. Interference mitigation or overlay based CR assumes that the message and or codebooks of the PU is known to the CR when the CR is transmitting its own message, either causally or non causally. Consequently, this message is also incorporated by the CR in coding the final message that it sends. This has been studied extensively in literature [1]–[5].

In general, the transmissions of the PU and CR interfere with each other. However, in our case, we review a special case of interference, wherein the interference from the PU

transmitter to the CR receiver is zero¹. This channel is called as the cognitive Z-interference channel or CZIC. Also, more specifically we look at the Gaussian case, where the transmissions are corrupted with AWGN noise.

It is well known that the capacity regions in the standard interference channels or Gaussian Interference channels is dependent on the levels of interference caused by one or more of the transmitters [6]–[9]. Also for the GCZIC capacity results and bounds are provided when one link is assumed noiseless [10], [11]. Also for the GCZIC [9], [12] and GCBZIC for the degraded case [13] the capacity and bounds are found for various regimes of interference coefficients. We review some known capacity results and bounds for various ranges of the interference coefficients.

For those ranges where the capacity is not known, we provide an improved inner bound. This improved inner bound is obtained by assuming that the message of the PU is linearly incorporated by CR in encoding the message at its transmitter. The rationale behind choosing a linear approximation is that in most capacity achieving strategies or establishing inner bound, the optimal thing is to choose an auxiliary random variable either on the basis of rate splitting or superposition coding [14]–[16]. We show that the linear approximation results in a modified channel which is the standard Gaussian interference channel (GIC). The capacity of such channel in the case of strong interference is known [6], [14], [17]. The strong interference condition in the modified channel intersects with the condition in the original CZIC for which the capacity is not characterized. Then we move ahead and define an extension of the CZIC, viz. the Cognitive Z-Broadcast interference channel. We derive, capacity results in some cases and bound in some others.

The rest of the paper is divided as follows. The system model for the CZIC and its modified version are provided in Section II. In Section III we review some of the existing results and bounds. We then derive an larger inner bound based on the linear approximation in Section IV. The we define the GCBZIC in Section V and derive some capacity results and bounds. In Section VI we conjecture an alternative inner bound based on the linear coded approximation. Finally we draw conclusions in Section VII.

Aaqib Patel and S. N. Merchant are with the Dept. of Electrical Engineering, Indian Institute of Technology, Bombay; Email: aaqib,merchant@ee.iitb.ac.in.

Md. Zafar Ali Khan and U. B. Desai are with the Dept. of Electrical Engineering, Indian Institute of Technology, Hyderabad AP 502205, India. Email: zafar,ubdesai@iith.ac.in

Fernando Reátegui del Águila, Muhammad Ali Imran and Rahim Tafazolli are with the University of Surrey, Guildford. United Kingdom. Email: f.reategui,m.imran,r.tafazolli@surrey.ac.uk

¹This might be the case when PU is a uni-cast whereas the CR is a broadcast or a multicast

II. GAUSSIAN COGNITIVE Z-INTERFERENCE CHANNEL AND LINEAR CODED APPROXIMATION

The Gaussian cognitive Z-interference channel (GCZIC) is as shown Fig. 1a. It consists of two inputs X_1 and X_2 and two outputs Y_1 and Y_2 . The message at transmitter 1 is encoded in X_1 and is intended to be sent at receiver 2 where it is received as Y_1 . Similarly, the message at transmitter 2 is encoded in X_1 and is intended to be sent at receiver 2 where it is received as Y_2 . Note that, the transceiver 1 denotes the cognitive radio (CR), whereas the transceiver 2 denotes the primary user (PU). The message encoded at the PU, that is X_2 , is known non-causally at the CR transmitter. Consequently, the CR encodes its message X_1 , incorporating the PUs message, i.e. $X_1 = f(X_2)$, where f is some function. The transmissions are corrupted at both the receivers by AWGN noise Z_1 and Z_2 which are independent and are zero mean and unit variance Gaussian Random variables, denoted as $Z_1, Z_2 \sim \mathcal{N}(0, 1)$. The base band equations are as follows

$$\begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ b & 1 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} + \begin{bmatrix} Z_1 \\ Z_2 \end{bmatrix} \quad (1)$$

Let, R_2 and R_1 denote the rates on which the messages of the CR and PU are encoded and transmitted. An achievable rate region for the Gaussian channel is the set of rate pairs (R_2, R_1) such that any message transmitted at these rates are recovered at the receivers with arbitrary small probability of error.

If function f is linear then, $X_1 = cX_2 + W$ where c is some positive real number and W is that part of message encoded at transmitter 1 that is solely intended for receiver 1. We refer to this as the linear coded approximation for X_1 in terms of X_2 . We then have,

$$\begin{aligned} \begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix} &= \begin{bmatrix} 1 & 0 \\ b & 1 \end{bmatrix} \begin{bmatrix} c & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} X_2 \\ W \end{bmatrix} + \begin{bmatrix} Z_1 \\ Z_2 \end{bmatrix} \\ \begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix} &= \begin{bmatrix} c & 1 \\ cb+1 & b \end{bmatrix} \begin{bmatrix} X_2 \\ W \end{bmatrix} + \begin{bmatrix} Z_1 \\ Z_2 \end{bmatrix} \end{aligned} \quad (2)$$

The baseband equation for the modified channel based on linear approximation is represented in (2). Note that, from Fig. 1b the modified CZIC (MCZIC) is the standard interference channel.

In the next section we provide the basic capacity results, inner and outer bounds for the CZIC.

III. KNOWN RESULTS AND BOUNDS FOR CZIC

There has been extensive work done on various CZICs depending upon the level of interference caused by the CR to the PU and capacity regions are characterized for good portion of such cases. However, capacity remains elusive for a certain values of interference, In this following theorem, the known capacity regions are brought out.

Theorem 1 ([12], TABLE I). *The capacity region of Gaussian CZIC, for*

- $a \leq 1$, is the region given by

$$\begin{aligned} R_2 &\leq \frac{1}{2} \log \left(1 + \frac{(\sqrt{P_2} + b\sqrt{\alpha P_1})^2}{1 + b^2 \bar{\alpha} P_1} \right) \\ R_1 &\leq \frac{1}{2} \log (1 + \bar{\alpha} P_1) \end{aligned} \quad (3)$$

- $1 \leq b \leq \sqrt{1 + \frac{P_1}{1 + P_2}}$, is the region given by

$$\begin{aligned} R_2 + R_1 &\leq \frac{1}{2} \log \left(1 + P_2 + b^2 P_1 + 2b\sqrt{\alpha P_2 P_1} \right) \\ R_1 &\leq \frac{1}{2} \log (1 + \bar{\alpha} P_1) \end{aligned} \quad (4)$$

- $b \geq \sqrt{1 + P_1}$, is the region given by

$$\begin{aligned} R_2 + R_1 &\leq \frac{1}{2} \log \left(1 + P_2 + b^2 P_1 + 2b\sqrt{\alpha P_2 P_1} \right) \\ R_1 &\leq \frac{1}{2} \log \left(1 + \frac{\bar{\alpha} P_1}{1 + \alpha P_1} \right) \end{aligned} \quad (5)$$

- $b \geq \sqrt{P_2 P_1} + \sqrt{1 + P_2 + P_2 P_1}$

$$\begin{aligned} R_2 &\leq \frac{1}{2} \log \left(1 + (\sqrt{P_2} + b\sqrt{\alpha P_1})^2 \right) \\ R_1 &\leq \frac{1}{2} \log \left(1 + \frac{\bar{\alpha} P_1}{1 + \alpha P_1} \right) \end{aligned} \quad (6)$$

The only range of b for which the capacity is not known is $\sqrt{1 + \frac{P_1}{1 + P_2}} < b < \sqrt{1 + P_2}$. For this the outer and inner bounds are specified. First the outer bound is provided

Lemma 1 (Lemma 1, [12]). *For $\sqrt{1 + \frac{P_1}{1 + P_2}} < b < \sqrt{1 + P_2}$, an achievable rate pair (R_2, R_2) satisfies the following constraints*

$$R_1 \leq \frac{1}{2} \log \left(\frac{1 + P_1}{1 + P_1(1 - \rho_2^2)} \right) \quad (7)$$

$$\begin{aligned} R_2 + R_1 &\leq \frac{1}{2} \log \left(1 + (\sqrt{(1 - \rho_1^2)P_2} + b\sqrt{(1 - \rho_2^2)P_1})^2 \right) \\ &\quad + \frac{1}{2} \log \left(\frac{1 + P_1}{1 + P_1(1 - \rho_2^2)} \right) \end{aligned} \quad (8)$$

$$R_2 + R_2 \leq \frac{1}{2} \log \left(1 + P_2 + b^2 P_1 + 2b\sqrt{\rho_{12}^2 P_2 P_1} \right) \quad (9)$$

$$R_1 \leq \frac{1}{2} \log (1 + (1 - \rho_{12}^2)P_1) \quad (10)$$

where $|\rho_{12} - \rho_1 \rho_2| \leq \sqrt{(1 - \rho_1^2)(1 - \rho_2^2)}$, and $|\rho_i| \leq 1, i = 1, 2$.

Next an achievable region is provided

Lemma 2. *Any rate pair (R_2, R_2) satisfying*

$$R_2 \leq \frac{1}{2} \log \left(1 + (\sqrt{P_2} + b\sqrt{\alpha P_1})^2 \right)$$

$$R_1 \leq \frac{1}{2} \log \left(1 + \frac{\bar{\alpha} P_1}{1 + \alpha P_1} \right)$$

$$R_2 + R_1 \leq \frac{1}{2} \log \left(1 + P_2 + b^2 P_1 + 2b\sqrt{\alpha P_2 P_1} \right) \quad (11)$$

with $\alpha \in [0, 1]$ is achievable for the GCZIC.

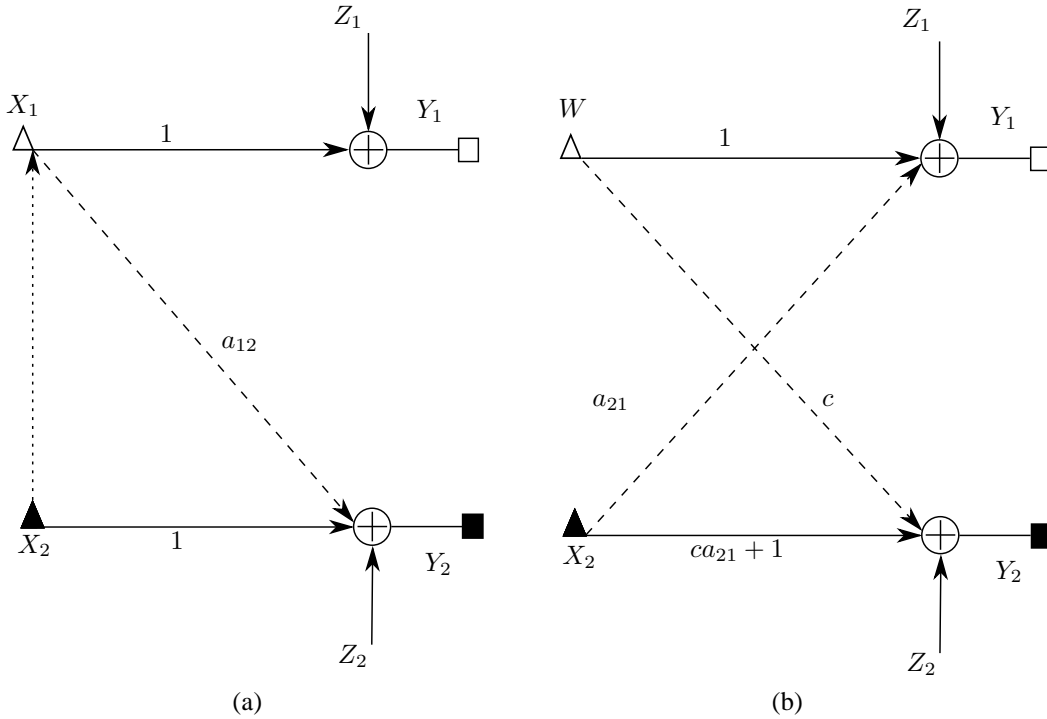


Fig. 1: The cognitive Z-Interference channel and its linear coded approximation.

IV. CAPACITY OF LINEAR CODED CZIC UNDER CERTAIN ASSUMPTION

The GCZIC after linear approximation of X_1 in terms of X_2 appears as shown in Fig. 1, which is the standard Gaussian interference channel (GIC). We first work out the power constraints on transmitter 1 and 2. Note that, in the original model, some power from transmitter 1 will be used to transmit message for receiver 3. In our equivalent linear approximation model, the transmitter 1 solely transmits for receivers 1 and 2 where receiver 3 either treats this transmission as noise or decodes it out. On the other hand transmitter 2 transmits solely for receiver 3 and receivers 1 and 2 treat it as noise or decode it out. The new power constraints for transmitters 1 and 2 are worked out as follows.

$$\begin{aligned} \mathbb{E}[X_1^2] &\leq P_1 \\ \mathbb{E}[(cX_2 + W)^2] &\leq P_1 \end{aligned} \quad (12)$$

Now assuming the distribution of either X_2 , W or both is of zero mean, which is justified as channel is a continuous real valued channel, we arrive at,

$$\mathbb{E}[W^2] = P_1 - c^2P_2; \quad (13)$$

This means an additional c^2P_2 amount of power is spent by the transmitter 1 in the original model for the transmission of transmitter 2 message. Hence in the new model, the power constraints \tilde{P}_1 and \tilde{P}_2 for transmitters 1 and 2 respectively are as follows

$$\tilde{P}_1 = P_1 - c^2P_2 \quad (14)$$

$$\tilde{P}_2 = P_2 + c^2P_2. \quad (15)$$

The capacity for the GIC channel is known in case of strong interference. The GIC given by (2) has strong interference when [6](Chapter 6),

$$c^2(P_2 + c^2P_2) > P_1 - c^2P_2 \quad (16)$$

$$b^2(P_1 - c^2P_2) > (cb + 1)^2(P_2 + c^2P_2). \quad (17)$$

Now, for these conditions to hold simultaneously, certain conditions on the transmit power constraints and the interference coefficients in the original model have to be met. These are provided in Lemma 3.

Lemma 3. *For the strong interference conditions of the GCZIC mentioned in (17) to hold simultaneously, the following two conditions should be satisfied*

$$P_1 \ll P_2 \quad (18)$$

$$b > \sqrt{\frac{P_1}{P_2}}. \quad (19)$$

Proof: By simplifying (16) and the fact that c should cover the entire range of values such that either full power of transmitter 1 is used for transmission of X_2 or full power is used for transmission of W we get (18). Simplifying (17) and setting $c = \sqrt{\frac{P_1}{P_2}}$ we get the condition (19) respectively. ■

Now we have the following theorem

Theorem 2. *For the modified GCZIC, if the conditions specified in Lemma 3 are specified then the capacity region is given*

by

$$R_2 < \frac{1}{2} \log(1 + (cb + 1)^2(P_2 + c^2P_2)) \quad (20)$$

$$R_1 < \frac{1}{2} \log(1 + P_1 - c^2P_2) \quad (21)$$

$$R_2 + R_1 < \min \left\{ \frac{1}{2} \log(1 + c^2(P_2 + c^2P_2) + P_1 - c^2P_2), \right. \\ \left. \frac{1}{2} \log(1 + b^2(P_1 - c^2P_2) + (cb + 1)^2(P_2 + c^2P_2)) \right\} \quad (22)$$

$$\text{for } c \in \left[0, \sqrt{\frac{P_2}{P_1}} \right]$$

Proof: This follows from the standard result of the strong interference capacity, which is obtained by intersection of the two component MACs. Details of the proof can be found in [6](chapter,6). ■

Now, the key point is that since we have approximated the original channel with linear approximation, then this is a lower bound on the capacity of the GCZIC. This bound is true for conditions specified in Lemma 3. So it is also true for some P_1, P_2 , that satisfy $\sqrt{1 + \frac{P_1}{1 + P_2}} < b < \sqrt{1 + P_2}$. Hence, we have obtained a new inner bound for this case. To compare with Lemma 2, we bring in the parameter $\alpha \in [0, 1]$, which denotes the fraction of power spent by the CR for the transmission of the PU message. Thus,

$$(1 - \alpha)P_1 = P_1 - c^2P_2 \\ c = \sqrt{\frac{\alpha P_1}{P_2}} \quad (23)$$

Hence, we have the following theorem.

Theorem 3. *An achievable region for the CZIC, given the conditions in Lemma 3 are satisfied, is the rate pair (R_2, R_1) satisfying*

$$R_1 < \frac{1}{2} \log(1 + (1 - \alpha)P_1) \\ R_1 + R_2 < \frac{1}{2} \log \left(1 + P_1 \left(1 + \alpha^2 \frac{P_1}{P_2} \right) \right) \quad (24)$$

for $\alpha \in [0, 1]$

Proof: The condition (20) mentioned in Theorem 2 is redundant when compared to the first term in (22). Also, the second term in (22) is redundant when compared to the first term. Finally, using (23) we get the desired region. ■

The constraint on R_1 specified in Theorem 3 is strictly greater than the constraint on R_1 specified in Lemma 1, which implies that our inner bound beats that specified in [12].

V. GAUSSIAN COGNITIVE BROADCAST Z-INTERFERENCE CHANNEL - GCBZIC

In this section, we extend the GCZIC to a more generalized version called as Gaussian Cognitive Z- Broadcast interference channel or GCBZIC. We provide achievable rates and capacity in some cases for GCBZIC. We assume that the broadcast component of the channel is a degraded broadcast channel. The channel model is shown in Fig. 2 In the Gaussian case

the received signal at each receiver are given by

$$Y_1 = X_1 + Z_1 \\ Y_2 = aX_1 + Z_2 \\ Y_3 = bX_1 + X_2 + Z_3. \quad (25)$$

The channel coefficients a and b are fixed and known at all transmitters and receivers and are assumed to be positive. Each Z_j in (25) is Gaussian noise with unit variance. Independence is assumed between noise variables and between different instances of a given variable. The power is constrained at each transmitter and is assumed to comply with $\sum_{t=1}^n E[X_{i,t}^2] \leq nP_i$.

Due to the degradedness condition, $X_1 \rightarrow Y_1 \rightarrow Y_2$ form a Markov chain, channel coefficient $a \leq 1$. The CBZIC in strong interference complies with the Markov chain $X_1 \rightarrow (X_2, Y_3) \rightarrow Y_1$ [18], which implies that

$$I(V; Y_3 | X_2) \geq I(V; Y_1), \quad (26)$$

for all input distributions $p(x_2)p(v)p(x_1|v)$. It can be readily verified that $b \geq 1$ complies with (26) in strong interference. We are interested to see the set of the rates R_{11} , that the transmitter 1 transmits to receiver 1, R_{12} , that the transmitter 1 transmits to receiver 2 and R_2 that the transmitter 2 transmits to receiver 3 that are simultaneously achievable such that the probability of error is arbitrarily small. The closure of the set of all such achievable pairs is the capacity region. We derive an achievable rate region in Lemma 4 and an outer bound in Lemma 5. The proofs of Lemmas 4 and 5 are omitted here due to lack of space. For details of proof, the reader is referred to [19].

Lemma 4. *For the Gaussian CBZIC with channel coefficients $a \leq 1 \leq b$, and real numbers $\alpha, \beta \in [0, 1]$, a set of rate triples (R_{11}, R_{12}, R_2) is achievable if satisfies:*

$$R_{12} \leq \mathcal{C} \left(\frac{a^2 \alpha \bar{\beta} P_1}{1 + a^2 \alpha \beta P_1 + a^2 \bar{\alpha} P_1} \right), \\ R_{11} \leq \mathcal{C} \left(\frac{\alpha \beta P_1}{1 + \bar{\alpha} P_1} \right), \\ R_2 \leq \mathcal{C} \left((\sqrt{P_2} + b\sqrt{\bar{\alpha} P_1})^2 \right), \\ R_{11} + R_2 \leq \mathcal{C} \left(b^2 \alpha \beta P_1 + (\sqrt{P_2} + b\sqrt{\bar{\alpha} P_1})^2 \right), \\ R_{11} + R_{12} + R_2 \leq \mathcal{C} \left(b^2 P_1 + P_2 + 2b\sqrt{\bar{\alpha} P_1 P_2} \right), \quad (27)$$

where $\mathcal{C}(x) = \frac{1}{2} \log_2(1 + x)$.

Lemma 5. *For the Gaussian CBZIC with channel coefficients $a \leq 1 \leq b$, and real numbers $\alpha, \beta, \gamma \in [0, 1]$, an achievable rate triple (R_{11}, R_{12}, R_2) is outer bounded by*

$$R_{12} \leq \mathcal{C} \left(\frac{a^2 \alpha \bar{\beta} P_1}{1 + a^2 \alpha \beta P_1 + a^2 \bar{\alpha} P_1} \right), \\ R_{11} \leq \mathcal{C} \left(\frac{\alpha \beta P_1}{1 + \bar{\alpha} P_1} \right), \quad R_2 \leq \mathcal{C} \left((\sqrt{\gamma P_2} + b\sqrt{\bar{\alpha} P_1})^2 \right), \\ R_{11} + R_{12} + R_2 \leq \mathcal{C} \left(b^2 P_1 + P_2 + 2b(\sqrt{\alpha \bar{\alpha} \gamma \bar{\gamma}}) \sqrt{P_1 P_2} \right), \quad (28)$$

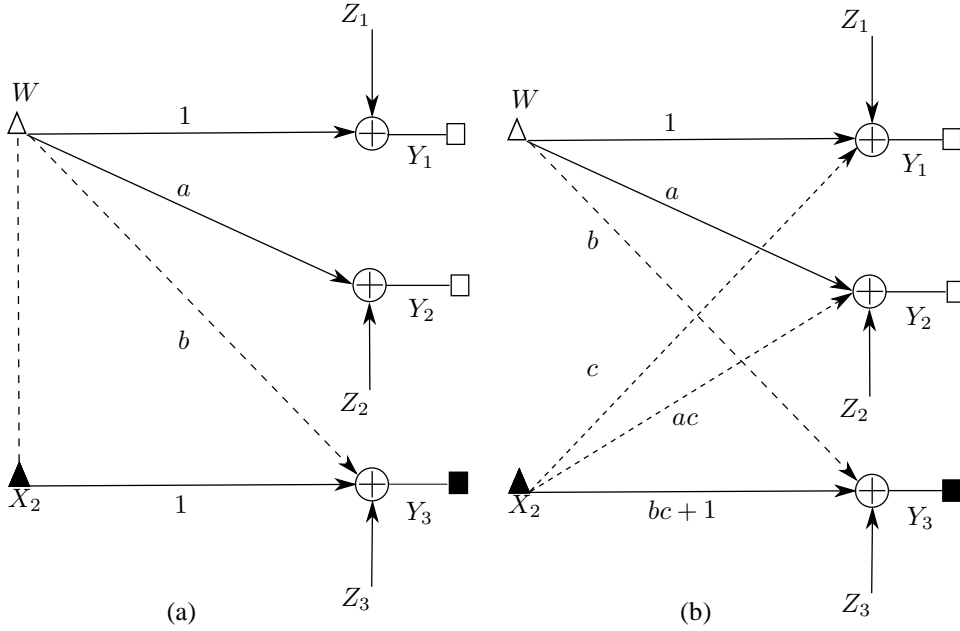


Fig. 2: (a)-The cognitive Z Broadcast Interference channel and (b)- its linear coded approximation.

In the following we present some cases for which the outer bound of Lemma 5 meets the inner bound of Lemma 4.

Theorem 4. *The capacity region of a Gaussian CBZIC with $a \leq 1$ and $b \geq \sqrt{P_1 P_2} + \sqrt{P_1 P_2 + P_2 + 1}$ is given by all the rate triples (R_{11}, R_{12}, R_2) that satisfy*

$$\begin{aligned} R_{12} &\leq \mathcal{C}\left(\frac{a^2 \alpha \bar{\beta} P_1}{1 + a^2 \alpha \beta P_1 + a^2 \bar{\alpha} P_1}\right), \\ R_{11} &\leq \mathcal{C}\left(\frac{\alpha \beta P_1}{1 + \bar{\alpha} P_1}\right), \quad R_2 \leq \mathcal{C}\left((\sqrt{P_2} + b\sqrt{\bar{\alpha} P_1})^2\right), \end{aligned} \quad (29)$$

for $\alpha, \beta \in [0, 1]$.

Proof: The proof follows after realizing that R_2 is not redundant in the inner bound for $b \geq \sqrt{1 + P_2}$ which forces $\bar{\gamma} = 1$, otherwise the inner bound would be greater than the outer bound. This complies with the fact that V and X_2 are independent in the encoding process. Furthermore the two sum rates in (27) are redundant for $b \geq \sqrt{P_1 P_2} + \sqrt{P_1 P_2 + P_2 + 1}$ completing the proof. ■

Theorem 5. *For the CBZIC with $0 \leq a \leq 1$, $\max\{1, b_0\} \leq b \leq b_1$, and $\alpha \in [0, 1]$, the rate triples (R_{11}, R_{12}, R_2) in $\cup_{i=1}^3 \mathcal{R}_i$ are on the boundary of the capacity region, where \mathcal{R}_1 is given by the region*

$$\begin{aligned} R_{11} &= \frac{1}{2} \log\left(1 + \frac{\alpha \beta P_1}{1 + \bar{\alpha} P_1}\right) \\ R_{12} &= \frac{1}{2} \log\left(1 + \frac{a^2 \alpha \bar{\beta} P_1}{1 + a^2 \alpha \beta P_1 + a^2 \bar{\alpha} P_1}\right) \\ R_2 &\leq \frac{1}{2} \log\left(1 + b^2 \alpha \beta P_1 + (\sqrt{P_2} + b\sqrt{\bar{\alpha} P_1})^2\right) \\ &\quad - \frac{1}{2} \log\left(1 + \frac{\alpha \beta P_1}{1 + \bar{\alpha} P_1}\right), \end{aligned} \quad (30)$$

for $\beta \in [0, 1]$; \mathcal{R}_2 is given by the region

$$\begin{aligned} R_{11} &= \frac{1}{2} \log\left(1 + \frac{\alpha \beta P_1}{1 + \bar{\alpha} P_1}\right) \\ R_{12} &\leq \frac{1}{2} \log\left(1 + \frac{a^2 \alpha \bar{\beta} P_1}{1 + a^2 \alpha \beta P_1 + a^2 \bar{\alpha} P_1}\right) \\ R_2 &= \frac{1}{2} \log\left(1 + (\sqrt{P_2} + b\sqrt{\bar{\alpha} P_1})^2\right), \end{aligned} \quad (31)$$

for $\beta \in [0, \beta_0]$; and \mathcal{R}_3 is given by the region

$$\begin{aligned} R_{11} &= \frac{1}{2} \log\left(1 + \frac{\alpha \beta P_1}{1 + \bar{\alpha} P_1}\right) \quad R_{12} = 0 \\ R_2 &= \frac{1}{2} \log\left(1 + b^2 P_1 + P_2 + 2b\sqrt{\bar{\alpha} P_1 P_2}\right) \\ &\quad - \frac{1}{2} \log\left(1 + \frac{\alpha \beta P_1}{1 + \bar{\alpha} P_1}\right), \end{aligned} \quad (32)$$

for $\beta \in [\beta_0, 1]$; and we define:

$$b_0 = \left(a\sqrt{\bar{\alpha} P_1 P_2} + \sqrt{a^2 \bar{\alpha} P_1 P_2 + 1 + P_2}\right) a \quad (33)$$

$$b_1 = \sqrt{\bar{\alpha} P_1 P_2} + \sqrt{\bar{\alpha} P_1 P_2 + 1 + P_2} \quad (34)$$

$$\beta_0 = \frac{b^2(1 + \bar{\alpha} P_1)}{1 + (\sqrt{P_2} + b\sqrt{\bar{\alpha} P_1})^2} \quad (35)$$

$$\beta_1 = \frac{\beta}{\beta_0}. \quad (36)$$

Proof: For $b \geq b_0$ it can be verified that the sum rate in (27) is redundant. The inner bound for fixed R_{11} and fixed α , turns out to be the union of regions for $\theta \in [\beta, 1]$ such that

$$\begin{aligned} R_{11} &= \frac{1}{2} \log\left(1 + \frac{\alpha \beta P_1}{1 + \bar{\alpha} P_1}\right), \\ R_{12} &\leq \frac{1}{2} \log\left(1 + \frac{a^2 \alpha \bar{\theta} P_1}{1 + a^2 \alpha \theta P_1 + a^2 \bar{\alpha} P_1}\right), \end{aligned} \quad (37)$$

$$\begin{aligned}
R_2 &\leq \frac{1}{2} \log \left(1 + \left(\sqrt{P_2} + b\sqrt{\bar{\alpha}P_1} \right)^2 \right), \\
R_2 &\leq \frac{1}{2} \log \left(1 + b^2\alpha\theta P_1 + \left(\sqrt{P_2} + b\sqrt{\bar{\alpha}P_1} \right)^2 \right) \\
&\quad - \frac{1}{2} \log \left(1 + \frac{\alpha\beta P_1}{1 + \bar{\alpha}P_1} \right). \tag{38}
\end{aligned}$$

Similarly, the outer bound can be written as

$$\begin{aligned}
R_{11} &= \frac{1}{2} \log \left(1 + \frac{\alpha\beta P_1}{1 + \bar{\alpha}P_1} \right), \\
R_{12} &\leq \frac{1}{2} \log \left(1 + \frac{a^2\alpha\bar{\beta}P_1}{1 + a^2\alpha\beta P_1 + a^2\bar{\alpha}P_1} \right), \tag{39}
\end{aligned}$$

$$R_2 \leq \frac{1}{2} \log \left(1 + \left(\sqrt{P_2} + b\sqrt{\bar{\alpha}P_1} \right)^2 \right), \tag{40}$$

$$\begin{aligned}
R_{12} + R_2 &\leq \frac{1}{2} \log \left(1 + b^2P_1 + P_2 + 2b\sqrt{\bar{\alpha}P_1P_2} \right) \\
&\quad - \frac{1}{2} \log \left(1 + \frac{\alpha\beta P_1}{1 + \bar{\alpha}P_1} \right),
\end{aligned}$$

for $\beta \leq \beta_0$, and as

$$\begin{aligned}
R_{11} &= \frac{1}{2} \log \left(1 + \frac{\alpha\beta P_1}{1 + \bar{\alpha}P_1} \right), \\
R_{12} &\leq \frac{1}{2} \log \left(1 + \frac{a^2\alpha\bar{\beta}P_1}{1 + a^2\alpha\beta P_1 + a^2\bar{\alpha}P_1} \right), \\
R_{12} + R_2 &\leq \frac{1}{2} \log \left(1 + b^2P_1 + P_2 + 2b\sqrt{\bar{\alpha}P_1P_2} \right) \\
&\quad - \frac{1}{2} \log \left(1 + \frac{\alpha\beta P_1}{1 + \bar{\alpha}P_1} \right), \tag{41}
\end{aligned}$$

for $\beta \geq \beta_0$. The inner and outer bounds intersect when: (a) (37) = (39), or (b) (38) \geq (40), or (c) (38) \geq (41). For (a) we get $\theta = \beta$, for (b) we get $\theta \geq \beta/\beta_0 = \beta_1$ and $\beta \leq \beta_0$, and for (c) we get $\theta = 1$ and $\beta \geq \beta_0$, which completes the proof. ■

VI. LINEAR CODED APPROXIMATION TO THE GCBZIC

The linear approximation of X_1 in terms of X_2 is shown in Fig. 2. The baseband equations of the original CBZIC are

$$\begin{bmatrix} Y_1 \\ Y_2 \\ Y_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ a & 0 \\ b & 1 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} + \begin{bmatrix} Z_1 \\ Z_2 \\ Z_3 \end{bmatrix}$$

Plugging in the linear approximation of $X_1 = cX_2 + W$ we have,

$$\begin{bmatrix} Y_1 \\ Y_2 \\ Y_3 \end{bmatrix} = \begin{bmatrix} c & 1 \\ ac & a \\ bc + 1 & b \end{bmatrix} \begin{bmatrix} X_2 \\ W \end{bmatrix} + \begin{bmatrix} Z_1 \\ Z_2 \\ Z_3 \end{bmatrix} \tag{42}$$

We call this channel the Gaussian Broadcast interference channel. The capacity of this channel is still an open problem. However, the capacity region of this channel or an inner bound will also be an inner bound for the CBZIC channel.

VII. CONCLUSION

We reviewed known results and bounds for the cognitive z-interference channel. We have shown a new improved inner

bound for a case of CZIC. Also we have defined Cognitive Z-Broadcast interference channel and shown capacity result and bounds in certain cases. Finally we have conjectured an inner bound based on the capacity of the linear coded cognitive z-broadcast.

ACKNOWLEDGMENT

This work was sponsored by IUATC group.

REFERENCES

- [1] A. Goldsmith, S. Jafar, I. Maric, and S. Srinivasa, "Breaking spectrum gridlock with cognitive radios: An information theoretic perspective," *Proceedings of the IEEE*, vol. 97, pp. 894–914, may 2009.
- [2] N. Devroye, P. Mitran, and V. Tarokh, "Achievable rates in cognitive radio channels," *Information Theory, IEEE Transactions on*, vol. 52, pp. 1813–1827, may 2006.
- [3] S. Srinivasa and S. Jafar, "The throughput potential of cognitive radio: A theoretical perspective," in *Signals, Systems and Computers, 2006. ACSSC '06. Fortieth Asilomar Conference on*, pp. 221–225, 29 2006-nov. 1 2006.
- [4] W. Wu, S. Vishwanath, and A. Arapostathis, "Capacity of a class of cognitive radio channels: Interference channels with degraded message sets," *Information Theory, IEEE Transactions on*, vol. 53, pp. 4391–4399, nov. 2007.
- [5] A. Jovicic and P. Viswanath, "Cognitive radio: An information-theoretic perspective," *Information Theory, IEEE Transactions on*, vol. 55, pp. 3945–3958, sept. 2009.
- [6] A. E. Gamal and Y.-H. Kim, *Network Information Theory*. New York, NY, USA: Cambridge University Press, 2012.
- [7] S. Rini, E. Kurniawan, and A. Goldsmith, "Primary rate-splitting achieves capacity for the gaussian cognitive interference channel," *CoRR*, vol. abs/1204.2083, 2012.
- [8] S. Rini, D. Tuninetti, and N. Devroye, "Inner and outer bounds for the gaussian cognitive interference channel and new capacity results," *IEEE Trans. Inf. Theor.*, vol. 58, pp. 820–848, Feb. 2012.
- [9] S. Rini, D. Tuninetti, and N. Devroye, "Inner and outer bounds for the gaussian cognitive interference channel and new capacity results," *IEEE Trans. Inf. Theor.*, vol. 58, pp. 820–848, Feb. 2012.
- [10] N. Liu, I. Maric, A. J. Goldsmith, and S. Shamai, "Bounds and capacity results for the cognitive z-interference channel," in *Proceedings of the 2009 IEEE international conference on Symposium on Information Theory - Volume 4, ISIT'09*, (Piscataway, NJ, USA), pp. 2422–2426, IEEE Press, 2009.
- [11] N. Liu, I. Maric, A. J. Goldsmith, and S. S. (Shitz), "Capacity bounds and exact results for the cognitive z-interference channel," *IEEE Transactions on Information Theory*, vol. 59, no. 2, pp. 886–893, 2013.
- [12] M. Vaezi and M. Vu, "Superposition coding-based bounds and capacity for the cognitive z-interference channels," *CoRR*, vol. abs/1101.1920, 2011.
- [13] X. Shang and H. V. Poor, "On the capacity of type i broadcast-z-interference channels," *IEEE Trans. Inf. Theor.*, vol. 57, pp. 2648–2666, May 2011.
- [14] T. Han and K. Kobayashi, "A New Achievable Rate Region for the Interference Channel," *Information Theory, IEEE Transactions on*, vol. 27, no. 1, pp. 49–60, 1981.
- [15] T. Cover, "Comments on broadcast channels," *Information Theory, IEEE Transactions on*, vol. 44, pp. 2524–2530, oct. 1998.
- [16] H.-F. Chong, M. Motani, H. K. Garg, and H. El Gamal, "On the han kobayashi region for the interference channel," *IEEE Trans. Inf. Theor.*, vol. 54, pp. 3188–3195, July 2008.
- [17] A. Carleial, "A case where interference does not reduce capacity (corresp.)," *Information Theory, IEEE Transactions on*, vol. 21, pp. 569–570, sep 1975.
- [18] H.-F. Chong, M. Motani, and H. K. Garg, "Capacity theorems for the z channel," *IEEE Trans. Inf. Theor.*, vol. 53, pp. 1348–1365, Apr. 2007.
- [19] F. R. Aguila, M. A. Imran, and R. Tafazolli, "On a class of cognitive broadcast z-interference channel in strong interference," *Technical report*.