MEASUREMENT OF CARDIOVASCULAR STATE USING ATTRACTOR RECONSTRUCTION ANALYSIS

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ABSTRACT

Attractor reconstruction (AR) analysis has been used previously to quantify the variability in arterial blood pressure (ABP) signals. Since ABP signals are only available in a minority of clinical scenarios, we sought to determine whether AR could also be performed on more widely available photoplethysmogram (PPG) signals. AR analysis was performed on simultaneous ABP and PPG signals before, during and after a change in cardiovascular state. A novel quality metric was used to eliminate windows of low quality AR. A high level of agreement was found between the detected periodicity of each signal, τ_{opt}, a measure of the heart rate. The remaining cardiovascular parameters derived using AR analysis exhibited similar trends between the two signals in response to the change in state, although there was poor agreement between their absolute values. This demonstrates the feasibility of applying AR to the PPG signal, providing opportunity to increase the range of patients in whom cardiovascular state can be measured using AR analysis.

Index Terms— attractor reconstruction (AR), arterial blood pressure (ABP), photoplethysmogram (PPG), eHealth

1. INTRODUCTION

Clinical deteriorations of hospital patients must be recognised early to maintain patient safety and minimise treatment costs. Deteriorations such as cardiac arrests, critical illnesses requiring intensive care treatment, and deaths, are commonly preceded by changes in physiological parameters [1]. The earlier these changes are recognised and responded to, the more likely it is that such deteriorations can be prevented.

Physiological parameters such as heart rate and blood pressure are used to assess the likelihood of deterioration. However, they are maintained within normal ranges until late in the progression of a deterioration by the body's regulatory mechanisms. Conversely, changes in the variability of such parameters may occur prior to derangement of their absolute values [2]. Consequently, measures such as heart rate variability (HRV) and respiratory rate variability (RRV) have been developed to quantify the variability in the period of cardiac and respiratory signals [3].

A novel technique, attractor reconstruction (AR) analysis, quantifies not only variability in the period of signals, but also in their morphology [4]. Consequently it may provide increased diagnostic value, since the physiological changes which precede deteriorations alter the morphology of these signals, as well as their period. AR analysis has previously been applied to arterial blood pressure (ABP) signals obtained from animals and healthy volunteers. However, ABP is measured by invasive insertion of a cannula, limiting its routine use to those patients known to be critically ill. The utility of AR analysis for prediction of deteriorations would be greatly increased if it could be applied to a physiological signal which is easily and routinely measured.

The photoplethysmogram (PPG) signal provides a potential solution since it is easily measured by a non-invasive pulse oximeter. It is closely related to the ABP signal, being a measure of the volume of blood within a sample of arteries, rather than the pressure within the arteries. The PPG is widely measured in a wide range of clinical scenarios to provide blood oxygen saturation measurements, making it easily obtainable. Furthermore, it is modulated by the cardiac, vascular, respiratory and autonomic nervous systems. Therefore, AR analysis of the PPG could potentially identify the onset of deteriorations from changes in the state of any of these systems.

The aim of the present study was to determine whether the PPG signal could be used instead of the ABP signal for measurement of cardiovascular state using AR analysis. AR analysis was performed on simultaneous ABP and PPG signals recorded during a pharmacologically-induced change in cardiovascular state, and the derived cardiovascular parameters were compared.

2. ATTRACTOR RECONSTRUCTION (AR) ANALYSIS

AR analysis consists of two components as described below.

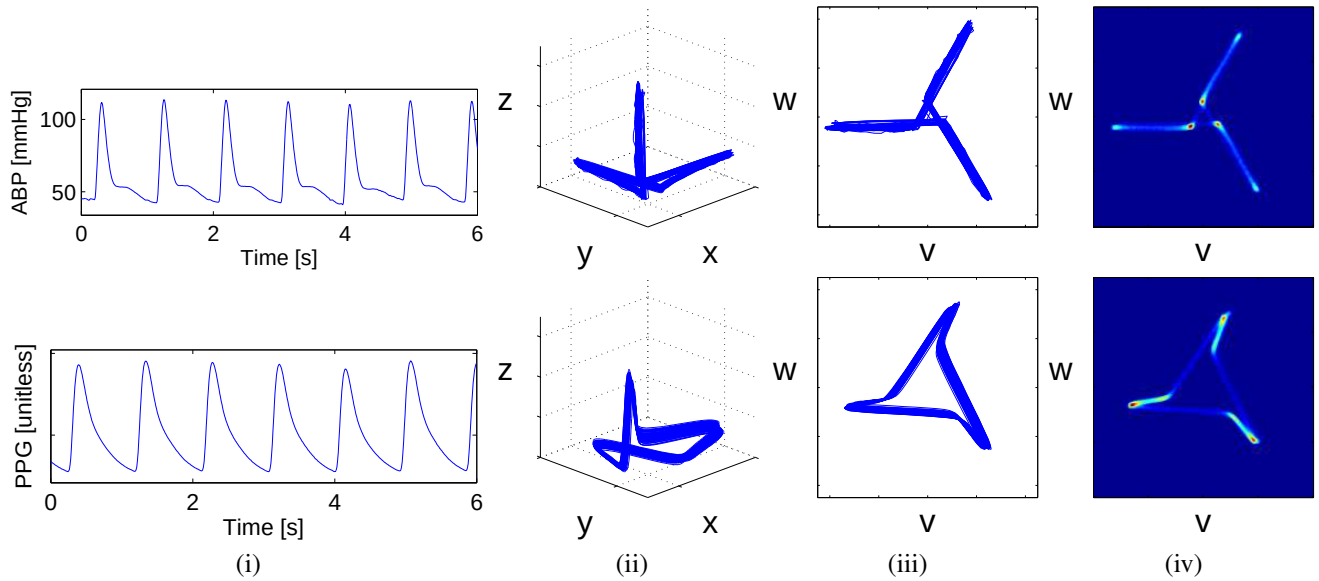

Fig. 1. Attractor reconstruction (AR) performed on simultaneous arterial blood pressure (ABP, above) and photoplethysmogram (PPG, below) signals. From left to right: (i) raw signal, (ii) 3D attractor calculated using Takens' Embedding Theorem, (iii) 2D attractor, $A(\tau_{opt})$, calculated from the projection of the 3D attractor onto a plane perpendicular to the vector (1,1,1), (iv) the 2D attractor density, $D(\tau_{opt})$. Accurate estimation of τ_{opt} is required to achieve the reliable AR shown in (ii) to (iv).

2.1. Attractor Reconstruction (AR)

The goal of AR is to represent a periodic or quasiperiodic dynamical system, in this case a physiological time series, as an attractor from which the variability of the dynamical system can be quantified. AR has been described previously, so is recapped only briefly [4]. The four steps are demonstrated in Figure 1.

Firstly, Takens' Embedding Theorem is used to represent the time series in a three-dimensional phase space [5]. This approach is particularly suitable to physiological systems, where only a subset of the dynamical variables can be measured [6]. Given a time series $x(t)$, two additional variables,

$$y(t) = x(t-\tau) \quad \text{and} \quad z(t) = x(t-2\tau) \quad ,$$

are defined, where τ is a time delay. $x(t)$ has now been transformed to an (x, y, z) phase space.

Secondly, the influence of baseline variation of $x(t)$ on the attractor is removed. This is achieved by projecting the attractor onto a plane perpendicular to the vector $(1, 1, 1)$, the (v, w) plane, defined as

$$v = \frac{1}{\sqrt{6}}(x + y - 2z), \quad w = \frac{1}{\sqrt{2}}(x - y).$$

A 2D attractor, $A(\tau)$, is now generated in (v, w) space. If $x(t)$ is periodic with period T, and $\tau = T/3$ or $\tau = 2T/3$, then $A(\tau)$ has threefold rotational symmetry about the origin.

Thirdly, the value of τ is optimised to give the maximum threefold rotational symmetry of $A(\tau)$. To do so, $A(\tau)$ is characterised using a density function, $D(\tau)$. $D(\tau)$ is rotated by $2\pi/3$ and $4\pi/3$ to give $D_2(\tau)$ and $D_3(\tau)$ respectively. A rotationally symmetric density function, $D_s(\tau) = D(\tau) + D_2(\tau) + D_3(\tau)$, is calculated. The optimal value of τ, τ_{opt}, is chosen to minimise the symmetry measure,

$$S(D(\tau)) = ||D(\tau) - D_s(\tau)||_2 \quad . \tag{1}$$

When applied to cardiac signals, $3\tau_{opt}$ is the average period between consecutive heart beats, as shown in Figure 2. The second local minimum of $S(D(\tau))$, $\tau_{opt2} \approx 2\tau_{opt}$.

Finally, the 2D attractor density, $D(\tau_{opt})$, is reconstructed.

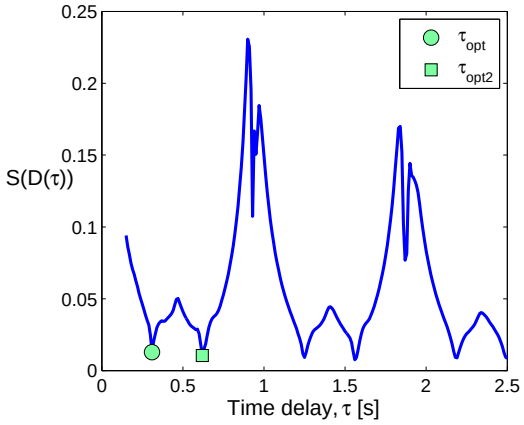

Fig. 2. Optimisation of τ to minimise the symmetry measure, $S(D(\tau))$. When evaluated on a periodic signal, τ_{opt} and τ_{opt2} correspond to $T/3$ and $2T/3$ respectively.

Parameter	Abbreviation	Explanation
		Attractor Reconstruction (AR)
Optimal time delay	τ_{opt}	If $x(t)$ is periodic with period T, then $3\tau_{opt} = T$. When applied to a signal dominated by cardiac modulation, $60/(3\tau_{opt})$ is the heart rate measured in beats per minute (bpm).
		Observation 1: If $x(t)$ is periodic, $D(\tau_{opt})$ has threefold rotational symmetry about the origin.
Symmetry measure	$S(D(\tau_{opt}))$	The rotational symmetry of the attractor at τ_{opt}, defined in (1). A value of 0 indicates perfect threefold rotational symmetry, whilst greater values indicate reduced symmetry.
Second symmetry measure	$S(D(2\tau_{opt}))$	The rotational symmetry of the attractor at $2\tau_{opt}$.
Ratio of symmetry measures	$SD1/SD2$	$S(D(\tau_{opt}))/S(D(2\tau_{opt}))$. A value of 1 indicates a high level of periodicity.
Ratio of time delays	τ_{opt}/τ_{opt2}	If $x(t)$ is periodic with period T, then $\tau_{opt2} \approx 2\tau_{opt}$. Therefore, a value of 0.5 indicates a high level of periodicity.
		Observation 2: If $x(t)$ is stable, the geometry of $D(\tau_{opt})$ remains stable over time.
Spread	r	The radius of the circle centred on $(0, 0)$ which encloses 95% of A.
Angle	θ	The mean of the polar angles of the points in A enclosed by the circle specified by r.
Angular spread	θ_s	The polar angles of the points in A enclosed by the circle specified by r are found. θ_s is the mean difference between each of these angles and the closest of the polar angles $\{\pi/2, -\pi/6, -5\pi/6\}$ when using $A(\tau_{opt})$, or $\{\pi/6, 5\pi/6, -\pi/2\}$ when using $A(\tau_{opt2})$. A value of 0 indicates that A lies completely on the three lines of rotational symmetry. The maximum possible value is $\pi/3$.

Table 1. Parameters calculated in attractor analysis (AA).

2.2. Attractor Analysis (AA)

Once an attractor has been generated, attractor analysis (AA) is used to extract measurements of its variability. AA is based on two observations. Firstly, if the time series from which the density $D(\tau_{opt})$ is generated, $x(t)$, is periodic, then $D(\tau_{opt})$ has threefold rotational symmetry about the origin. Secondly, if $x(t)$ is stable over time, then when separated into multiple time series of shorter duration, $x_1(t), ..., x_n(t)$, the geometries of the density functions generated from each shorter time series, $D_1(\tau_{opt}), ..., D_n(\tau_{opt})$, also remain stable over time. AA is used to determine how closely a time series conforms to these observations. The AA parameters extracted in this study are defined in Table 1, building on previous work [4].

3. METHODS

3.1. Clinical dataset

The dataset used in this study is a subset of one described previously [7, 8] and is only summarised here. Simultaneous ABP and PPG signals were acquired from 6 critically ill patients with a median (lower - upper quartiles) age of 56 (45 - 62) years, 5 of whom were male. ABP signals were obtained from either the radial or iliac artery, and PPG from the finger. The total duration of recordings was 7.1 hours.

Throughout the recording each patient was receiving continuous infusion of norepinephrine, a drug which alters cardiovascular state. The dosage of norepinephrine was doubled during the recording for a median of 10 (6-11) mins (minimum 5 mins). This provided a step-change in cardiovascular properties since the half-life of norepinephrine is 1-2 mins.

3.2. Data processing

100 Hz ABP and PPG signals were segmented into overlapping windows of duration 100 s with 90 s overlap for analysis. Data in each window was normalised to have a specified mean and standard deviation. A value for each of the variables in Table 1 was calculated using each window of data.

3.3. Statistical analysis

A commonly used method for assessing agreement between two methods of clinical measurement is that described by Bland and Altman [9]. Agreement is measured by assessing the bias (the difference between the means of the variable estimated using each signal), and limits of agreement (LOA = $bias \pm 1.96SD$, where SD is the standard deviation of the differences between the variable estimates from each signal). If the differences are normally distributed, the LOA indicate the limits within which 95% of the data lie. The mean and SD of the means of each variable are also reported.

4. RESULTS

τ_{opt} was compared to determine whether AR detected the same period when applied to ABP and PPG signals. There

Variable	Mean (±SD) of means	Bias	LOA
τ_{opt} [s]	0.225 ± 0.046	-0.001	-0.013 to 0.011
$S(D(\tau_{opt})) \times 10^3$	19.7 ± 5.9	-0.1	-21.3 to 21.2
$S(D(2\tau_{opt})) \times 10^3$	14.6 ± 4.1	1.1	-13.7 to 16.0
$S(D(\tau_{opt}))/S(D(2\tau_{opt}))$	1.37 ± 0.29	-0.09	-0.78 to 0.61
τ_{opt}/τ_{opt2}	0.504 ± 0.016	-0.006	-0.052 to 0.040
r	2.36 ± 0.32	0.04	-0.66 to 0.75
θ [rad]	-0.552 ± 0.124	-0.143	-0.398 to 0.113
θ_s [rad]	0.283 ± 0.032	0.023	-0.124 to 0.170

Table 2. Agreement of variables calculated from ABP and PPG signals (to 3sf.).

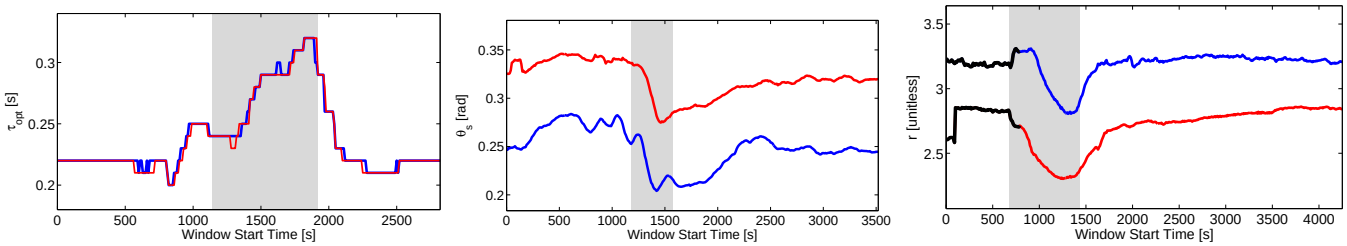

Fig. 3. Trends in variables extracted by attractor analysis (AA) from simultaneous PPG (blue) and ABP (red) signals in response to changes in vascular tone (the time of dosage increase shown in grey). Data where the corresponding $\theta_s \geq 0.425$, indicating unreliable attractor reconstruction (AR), are shown in black. Each example is from a different patient.

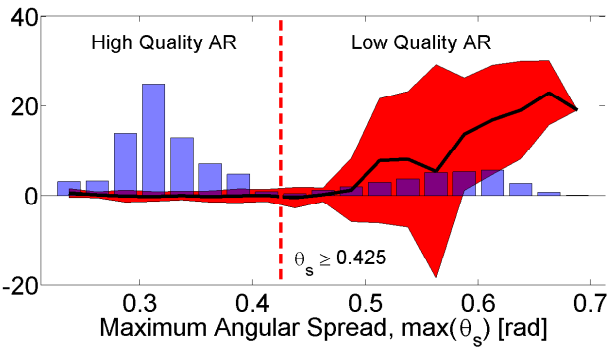

Fig. 4. Determination of an Attractor Quality Index (AQI). The bias (black line) and limits of agreement (LOA, red shaded area) of τ_{opt} are shown against the maximum angular spread, $\max(\theta_s)$ of the attractors generated from each window of simultaneous ABP and PPG signals. Blue bars show the percentage of the analysed windows.

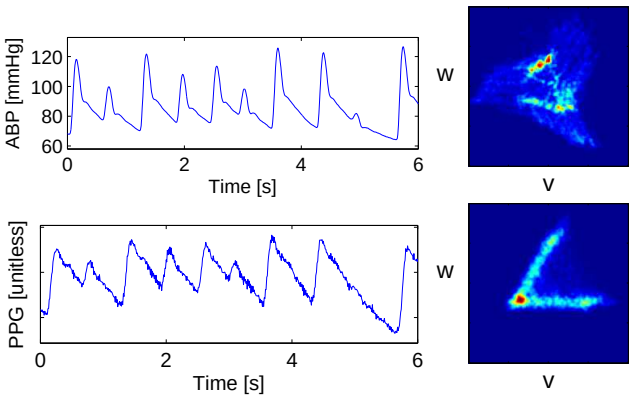

Fig. 5. Aperiodic arterial blood pressure (ABP, above) and photoplethysmogram (PPG, below) signals from which reliable attractors could not be reconstructed.

was poor agreement between the τ_{opt} values, with a bias (LOA) of 0.031 (-0.111 to 0.173) s. This was due in part to τ_{opt} being inaccurately extracted from one patient's data, giving unreliable AR as shown in Figure 5. This was because the signals were highly aperiodic due to a cardiac arrhythmia.

To resolve this inadequacy, θ_s was used to discriminate between windows on which AR could or could not be reliably performed. $\theta_s < 0.425$ rad was chosen as a cut-off below which windows AR would be considered to be reliable (see Figure 4). θ_s calculated from both the ABP and PPG were less than 0.425 rad for 70% of the windows. In these windows, the bias (LOA) of τ_{opt} was -0.001 (-0.013 to 0.011) s, compared to 0.107 (-0.082 to 0.297) s in the remaining 30% of windows.

The results of a comparison of the agreement of the AA parameters calculated from ABP and PPG signals are given in Table 2. Data from windows with $\theta_s \geq 0.425$ have been excluded. The LOA were not comparable to the mean of means for all AA parameters except τ_{opt}/τ_{opt2}, suggesting that ABP and PPG signals cannot be used interchangably to measure

their absolute values. However, similar trends were observed in some of the variables, as shown in Figure 3.

5. DISCUSSION

Advanced warning of clinical deteriorations can be provided by assessing the variability of a patient's physiological signals [2]. Current methods, such as HRV and RRV, discard most of the information contained within the signals. A novel technique, AR analysis, utilises all available information to calculate the variability of signals. As we have previously applied AR analysis to ABP signals, which are not available in most clinical scenarios, we considered whether the approach could be applied to the more widely available PPG signal.

In this study we have demonstrated the application of AR analysis to the PPG signal. AA parameters derived from the PPG were compared to those derived from simultaneous ABP signals during a pharmacologically-induced change in arterial tone. A novel quality metric, θ_s, was used to discriminate between reliable and unreliable AR. A good agreement was found between ABP- and PPG-derived values of τ_{opt}, a measure of the heart rate. Other AA parameters exhibited similar trends between the two signals in response to the change in tone. Therefore, AR analysis of the PPG may facilitate identification of the changes in cardiovascular state which occur during the early stages of deterioration. This could prompt earlier clinical intervention to prevent further deterioration.

The range of AA parameters has been extended in this study. However, there is much scope for increasing the range of AA parameters further, thereby enhancing the cardiovascular assessment afforded by AR analysis.

6. CONCLUSIONS

This study demonstrates the feasibility of applying AR analysis to the PPG signal. This increases the scope for the use of AR analysis to measure cardiovascular state in a wider range of clinical scenarios. A good agreement was observed between the measure of the signal's periodicity, τ_{opt}, which is fundamental to AR analysis, derived from the PPG and ABP signals. Furthermore, similar trends were observed in other cardiovascular parameters extracted from the two signals in response to a change in arterial tone. This suggests that AR analysis could be used to track changes in cardiovascular state using either the PPG signal or the ABP signal.

7. ACKNOWLEDGMENTS

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