Love for Quality, Comparative Advantage, and Trade*

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Abstract

We propose a Ricardian trade model with horizontal and vertical differentiation, where willingness to pay for quality rises with individuals’ incomes, and productivity differentials across countries are stronger for high-quality varieties of goods. Our theory predicts that the scope for trade widens and international specialisation intensifies as incomes grow and wealthier consumers raise the quality of their consumption baskets. This implies that comparative advantages strengthen gradually over the path of development as a by-product of the process of quality upgrading. The evolution of comparative advantages leads to specific trade patterns that change over the growth path, by linking richer importers to more specialised exporters. We provide empirical support for this prediction, showing that the share of imports originating from exporters exhibiting a comparative advantage in a specific product correlates positively with the importer’s GDP per head.

Keywords: International Trade, Nonhomothetic Preferences, Quality Ladders.

JEL Classifications: F11, F43, O40

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1 Introduction

Income is a key determinant of consumer choice. A crucial dimension through which purchasing power influences this choice is the quality of consumption. People with very different incomes tend to consume commodities within the same category of goods, such as clothes, cars, wines, etc. However, the actual quality of the consumed commodities differs substantially when comparing poorer to richer households. The same reasoning naturally extends to countries with different levels of income per capita. In this case, the quality dimension of consumption entails important implications on the evolution of trade flows.

Several recent studies have investigated the links between quality of consumption and international trade. One strand of literature has centred their attention on the demand side, finding a strong positive correlation between quality of imports and the importer’s income per head [Hallak (2006), Fieler (2012)].\footnote{See also related evidence in Choi et al. (2009), Francois and Kaplan (1996) and Dalgin et al. (2008).} Another set of papers has focused instead on whether exporters adjust the quality of their production to serve markets with different income levels. The evidence here also points towards the presence of nonhomothetic preferences along the quality dimension, showing that producers sell higher quality versions of their output to richer importers.\footnote{For example, Verhoogen (2008) and Iacovone and Javorcik (2008) provide evidence of Mexican manufacturing plants selling higher qualities in the US than in their local markets. Brooks (2006) establishes the same results for Colombian manufacturing plants, and Manova and Zhang (2012) show that Chinese firms ship higher qualities of their exports to richer importers. Analogous evidence is provided by Bastos and Silva (2010) for Portuguese firms, and by Crino and Epifani (2012) for Italian ones.}

These empirical findings have motivated a number of models that yield trade patterns where richer importers buy high-quality versions of goods, while exporters differentiate the quality of their output by income at destination [Hallak (2010), Fajgelbaum, Grossman and Helpman (2011), Jaimovich and Merella (2012)]. Yet, this literature has approached the determinants of countries’ sectoral specialisation as a phenomenon that is independent of the process of quality upgrading resulting from higher consumer incomes. In this paper, we propose a theory where quality upgrading in consumption becomes the central driving force behind a general process of sectoral specialisation and comparative advantage intensification. The crucial novel feature of our theory is that quality upgrading by consumers leads to a strengthening in countries’ specialisation in the sectors where they exhibit a relative cost advantage. Therefore, the quality of the goods consumed and exchanged in world markets becomes a first-order determinant of the evolution of countries’ sectoral specialisation, and of the intensity of the trade links that importers establish with different
exporters.

Our theory is grounded on the hypothesis that productivity differentials are stronger for higher-quality goods, combined with the notion that willingness to pay for quality rises with income. Within this framework, we show that international specialisation and sectoral trade intensify over the growth path. The evolution of trade flows featured by our model presents novel specificities that stem from the interaction between nonhomothetic preferences and the deepening of sectoral productivity differentials at higher levels of quality. In particular, the process of quality upgrading with rising income sets in motion simultaneous demand-driven and supply-driven factors, which together lead to a rise in specialisation by importers and exporters. Import and export specialisation arise as intertwined phenomena because, as countries become richer, consumers shift their spending towards high-quality goods, which are exactly those that tend to display greater scope for international trade.

We model a world economy with a continuum of horizontally differentiated goods, each of them available in a continuum of vertically ordered quality levels. Each country produces a particular variety of each good. The production technology differs both across countries and sectors. We assume that some countries are intrinsically better than others in producing certain types of goods. In addition, these intrinsic productivity differentials on the horizontal dimension tend to become increasingly pronounced along the vertical dimension. These assumptions lead to an intensifying process of sectoral specialisation as production moves up on the quality ladders of each good. For example, a country may have a cost advantage in producing wine, while another country may have it in whisky. This would naturally lead them to exchange these two goods. Yet, in our model, productivity differences in the wine and whisky industries do not remain constant along the quality space, but become more intense as production moves up towards higher quality versions of these goods. As a result, the scope for international trade turns out to be wider for high-quality wines and whiskies than for low-quality ones.

A key feature of our model is the embedded link between nonhomotheticities in quality and international trade at the sectoral level. More precisely, as richer individuals upgrade the quality of their consumption baskets, sectoral productivity differentials across countries become stronger, leading to the intensification of some trading partnerships and the weakening of others. In that respect, our model suggests that the study of the evolution of trade links may require considering a more flexible concept of comparative advantage than the one traditionally used in the literature, so as to encompass quality upgrading as an inherent part of it. In the literature of Ricardian trade, the comparative advantage is univocally determined by exporters’ technologies. This paper
instead sustains that both the importers’ incomes and the exporters’ sectoral productivities must be jointly taken into account in order to establish a rank of comparative advantage. This is because the degree of comparative advantage between any two countries is crucially affected by the quality of consumption. As a consequence, richer and poorer importers may end up establishing trade links of substantially different intensity with the same set of exporters, simply because the gaps between their willingness-to-pay for quality may translate into unequal degrees of comparative advantage across their trade partners.

The conditionality of comparative advantage on importers incomes entails novel testable predictions on the evolution of sectoral trade flows. In particular, our model predicts that the share of imports originating from exporters exhibiting a cost advantage in a given good must grow with the income per head of the importer. This is the result of richer importers buying high-quality versions of goods, which are those for which cost differentials across countries are relatively more pronounced. Using bilateral trade data at the sectoral level, we provide evidence consistent with the prediction that richer economies are more likely to buy their imports from producers who display a comparative advantage in the imported goods.

Finally, our theory also has implications in terms of policy, particularly with regard to stimulating the growth of a specific industry in the economy through import tariffs or subsidies to local producers. Using simple comparative statics, we show that the gains from free trade are stronger for more developed economies, as their consumers suffer a greater welfare loss when the tariff is imposed on more efficient producers. In addition, our results suggest that subsidies have a larger impact at fostering local production when introduced in developing countries.

Related Literature

Nonhomothetic preferences are by now a widespread modelling choice in the trade literature. However, most of the past trade literature with nonhomotheticities has focused either on vertical differentiation \[\text{e.g., Flam and Helpman (1987), Stokey (1991) and Murphy and Shleifer (1997)}\] or horizontal differentiation in consumption \[\text{e.g., Markusen (1986), Bergstrand (1990) and Matsuyama (2000)}\]. Two recent articles have combined vertical and horizontal differentiation with

\[\text{3For some recent contributions with horizontal differentiation and nonhomothetic preferences see: Foellmi, Hegenstrick and Zweimuller (2012) and Tarasov (2012), where consumers are subject to a discrete consumption choice; Fieler (2011) who ties the income elasticity of consumption goods across different industries to the elasticity of substitution of goods within the same industry; Simonovska (2015) who fixes a bounded level of utility for each differentiated good; Breinlich and Cuñat (2013) who combine a Stone-Geary representation with Armington aggregators of country-specific varieties; and Melitz and Ottaviano (2008), Zhelobodko et al. (2012) and Dhingra and} \]

Fajgelbaum et al. (2011) analyse how differences in income distributions between economies with access to the same technologies determine trade flows in the presence of increasing returns and trade costs. Like ours, their paper leads to an endogenous emergence of comparative advantages, which may have remained initially latent (in their case, this could be either due to trade costs being too high to allow trade, or countries’ income distributions being too similar to induce specialisation via a home-market effect). Our paper, instead, sticks to the Ricardian tradition where trade and specialisation stem from cross-country differences in sectoral technologies featuring constant returns to scale. In particular, in our model, comparative advantages and trade emerge gradually, not because trade costs may initially hinder the scope for exchange in the presence of increasing returns to scale, but because the demand for commodities displaying wider heterogeneity in cost of production (the high-quality goods) expands as incomes rise.\footnote{In this regard, an important feature present in our model is that high-quality versions of goods are inherently more tradable than low-quality ones, while this is not necessarily the case in Fajgelbaum et al. (2011) unless they specifically assume quality-specific trade costs that are restricted to be relatively lower for high-quality varieties.}

Jaimovich and Merella (2012) also propose a nonhomothetic preference specification where budget reallocations take place both within and across horizontally differentiated goods. That paper, however, remained within a standard Ricardian framework where absolute and comparative advantages are determined from the outset, and purely by technological conditions. Hence, nonhomothetic preferences play no essential role there in determining export and import specialisation at different levels of development. By contrast, it is the interaction between rising differences in productivity at higher quality levels and nonhomotheticities in quality that generates our novel results in terms of co-evolution of export and import specialisation.

A key assumption in our theory is the widening in productivity differentials at higher levels of quality. To the best of our knowledge, Alcala (2012) is the only other paper that has explicitly introduced a similar feature into a Ricardian model of trade. An important difference between the two papers is that Alcala’s keeps the homothetic demand structure presented in Dornbusch, Fisher and Samuelson (1977) essentially intact. Nonhomotheticities in demand are actually crucial to our story and, in particular, to its main predictions regarding the evolution of trade flows and specialisation at different levels of income.

Finally, Fieler (2011) also studies the interplay between nonhomothetic demand and Ricardian

\cite{Morrow2012}, who adopt non-homothetic specifications of preferences delivering linear demand systems.
technological disparities. She shows that, when productivity differences are stronger for goods with high income elasticity, her model matches quite closely key features of North-North and North-South trade. While her model exhibits horizontal differentiation, it does not display vertical differentiation, which is a crucial dimension exploited by our model. Our mechanism differs from hers in that the effects of demand on trade flows stem from the (vertical) reallocation of consumer spending within categories of goods rather than (horizontally) across them. In particular, our results hinge on richer consumers switching their good-specific expenditure shares from lower-quality to higher-quality versions of the goods. It is in fact this within-good substitution process that leads to our main predictions where spending shares across different exporters of the same good change with the income of the importer.

The rest of the paper is organised as follows. Section 2 studies a world economy with a continuum of countries where all economies have the same level of income per head in equilibrium. Section 3 generalises the main results to a world economy where some countries are richer than others. Section 4 presents some empirical results consistent with the main predictions of our model. Section 5 provides some further discussion in terms of policy implications. Section 6 concludes. All relevant proofs can be found in the Appendices.

2 A world economy with equally rich countries

We study a world economy with a unit continuum of countries indexed by \( v \). In each country there is a continuum of individuals with unit mass. Each individual is endowed with one unit of labour time. We assume labour is immobile across countries. In addition, we assume all countries are open to international trade, and there are no trading costs of any sort.

All countries share a common commodity space defined along three distinct dimensions: a horizontal, a varietal, and a vertical dimension. Concerning the horizontal dimension, there exists a unit continuum of differentiated goods, indexed by \( z \). In terms of the varietal dimension, we assume that each country \( v \) produces a specific variety \( v \) of each good \( z \). Finally, our vertical dimension refers to the intrinsic quality of the commodity: we assume that a continuum of different qualities \( q \geq 1 \) are potentially available for each good \( z \).\(^5\)

\(^5\)To fix ideas, the horizontal dimension refers to different types of goods, such as cars, wines, coffee beans, etc. The varietal dimension refers to the different varieties of any given type of good, originating from different countries, such as Spanish and French wines (differing, for instance, in specific traits like the types of grapes and regional vinification techniques). The vertical dimension refers to the intrinsic quality of each specific commodity (e.g., the ageing and the grapes selection in the winemaking).
Our model will display two main distinctive features. First, productivity differentials across countries will rise with the quality level of the commodities being produced. Second, richer individuals will choose to consume higher-quality commodities than poorer ones. The next two subsections specify the functional forms of production technologies and consumer utility that we adopt to generate these two features.

2.1 Production technologies

In each country $v$ there exists a continuum of firms that may transform local labour into a variety $v$ of good $z$. Production technologies are idiosyncratic both to the sector $z$ and to the country $v$. In order to produce one unit of commodity $z$ at the quality level $q$, a firm from country $v$ needs to use $\Gamma_{z,v}(q)$ units of labour, where:

$$\Gamma_{z,v}(q) = \frac{A}{1 + \kappa} q^{\eta_{z,v}}. \quad (1)$$

Unit labour requirements contain two key technological parameters. The first is $\kappa > 0$, which applies identically to all sectors and countries, and we interpret it as the worldwide total factor productivity level. As such, in our model, increases in $\kappa$ will capture the effects of aggregate growth and rising real incomes. The second is $\eta_{z,v}$, which may differ both across $z$ and $v$, and governs the elasticity of the labour requirements with respect to quality upgrading. In what follows, we assume that each parameter $\eta_{z,v}$ is independently drawn from a probability density function with uniform distribution over the interval $[\underline{\eta}, \overline{\eta}]$. In addition, we assume that $\underline{\eta} > 1$. Hence, $\Gamma_{z,v}(q)$ are always strictly increasing and convex in $q$. The term $A = e^{-\frac{\eta(\overline{\eta} - 1)}{(\overline{\eta} - 1)}}$ is simply a scale factor between labour input units and quality units, introduced for mathematical convenience.

An important feature implicit in the functional form of (1) is that cross-country sectoral productivity differentials will widen with the level of quality of production. This feature will in turn imply that the cost advantage of countries with better sectoral productivity draws will expand at higher levels of quality of production.

Let $w_v$ denote henceforth the wage per unit of labour time in country $v$. We assume that, in all countries and all sectors, firms face no entry costs. In equilibrium, all commodities will then be priced exactly at their unit cost. Hence, the variety of good $z$ in quality $q$ produced by country $v$ will be sold (internationally) at price:

$$p_{z,v}(q) = \frac{A w_v}{1 + \kappa} q^{\eta_{z,v}}. \quad (2)$$

Notice from (2) that changes in $\kappa$ leave all relative prices unaltered. In this regard, we may
consider a rise in total factor productivity \( \kappa \) as resulting in a pure increase in real income, entailing no substitution effect across the different commodities.

### 2.2 Utility function and budget constraint

To simplify the analysis, we introduce the following assumption concerning consumer choice:

**Assumption 1 (Selection of quality)** Individuals consume a strictly positive amount of (at most) one quality version of each good \( z \) produced by country \( v \).

Assumption 1 is analogous to assuming an infinite elasticity of substitution across different quality versions of the good \( z \) sourced from country \( v \). Henceforth, to ease notation, we denote the selected quality of the good \( z \) sourced from country \( v \) by \( q_{z,v} \). In addition, we denote by \( c_{z,v} \) the consumed physical quantity of the selected quality \( q_{z,v} \).

Utility is defined over the consumed quantities \( c_{z,v} \) in the selected qualities \( q_{z,v} \). Formally:

\[
U = \left[ \int_Z \left( \int_V \ln \left( c_{z,v}^{q_{z,v}} \right) \, dv \right)^{\sigma} \, dz \right]^{\frac{1}{\sigma}}, \quad \text{where} \quad \sigma < 0.
\] (3)

Individuals choose the quantity to consume for each selected quality, subject to the budget constraint:

\[
\int_Z \left[ \int_V p_{z,v} (q_{z,v}) \, c_{z,v} \, dv \right] \, dz \leq w,
\] (4)

where each \( p_{z,v} (q_{z,v}) \) in (4) is given by the price functions (2) when \( q \) is equal to the selected quality \( q_{z,v} \).

The utility function (3) displays a number of features that are worth discussing in detail. Firstly, considering the quality dimension in isolation, the exponential terms \( (c_{z,v})^{q_{z,v}} \) in (3) are instrumental to obtaining our desired non-homothetic behaviour along the quality space. The exponential form implies that, whenever \( c_{z,v} > 1 \), the magnifying effect of quality becomes increasingly important as \( c_{z,v} \) rises. Such non-homothetic feature in turn leads to a solution of the consumer problem where higher incomes will translate into quality upgrading of consumption. Secondly, abstracting now from the quality dimension, (3) features two nested CES functions. On the one hand, for each good \( z \), the (inner) logarithmic function implies a unit elasticity of substitution across varieties of the same good \( z \). On the other hand, the parameter \( \sigma < 0 \) governs the elasticity of substitution across goods, which is equal to \( 1 / (1 - \sigma) < 1 \). Thus, the elasticity of substitution across different goods is smaller than within goods (i.e., across the different varieties of the same good).
2.3 Utility maximisation

Consider a representative individual (in a generic country) with income \( w \). The consumer’s problem requires maximising (3) subject to (4). This is a problem that could be in principle solved in terms of physical quantities of consumption for each good. However, Assumption 1 allows us to easily re-state the problem in terms of two other variables that we will henceforth use: selected qualities and budget allocations. More precisely, denoting by \( \beta_{z,v} \) the share of income spent in the good \( z \) sourced from country \( v \), by using Assumption 1 we may write:

\[
C_{z,v} = \frac{\beta_{z,v} w}{p_{z,v}(q_{z,v})},
\]

where, again, the expression \( p_{z,v}(q_{z,v}) \) in (5) is given by the price functions (2) with \( q = q_{z,v} \).

In the next sections, we will study how the intensity of sectoral trade partnerships change at different levels of consumer income within a full general equilibrium framework. However, before moving on to that framework, it proves useful to present the formal solution of the consumer problem in the specific case when the wage is the same for all countries; that is, when \( w_v = w \) for all \( v \). This will allow us to convey some preliminary intuition for the mechanism underlying the general equilibrium results presented later on.\(^6\)

**Lemma 1 (Optimal selected quality and budget allocation)** When all countries have the same wage, for each good \( z \) produced in country \( v \) the consumer chooses the level of quality:

\[
q_{z,v} = \left( \frac{(1 + \kappa)/A}{e^{q_{z,v}Q}} \right)^{1/(\eta_{z,v}-1)},
\]

and spends the share of income:

\[
\beta_{z,v} = \left( \frac{(1 + \kappa)/A}{(eQ)^{\eta_{z,v}}} \right)^{1/(\eta_{z,v}-1)},
\]

where the variable \( Q \equiv \int_{z} \int_{v} q_{z,v} dv dz \) in the denominator of (6) and (7) denotes the average quality of the optimal consumption bundle chosen by the consumer.

Lemma 1 characterises the solution of the consumer’s problem in terms of two sets of variables: the expressions in (6), which stipulate the quality level in which each variety of every good is optimally consumed; the expressions in (7), describing the optimal expenditure shares allocated to those commodities. An important implication of Lemma 1 is the implicit link between optimal budget shares and optimal qualities. In particular, plugging (6) into (7) yields \( \beta_{z,v} = q_{z,v}/Q \).

\(^6\)The next section shows that, in this symmetric specification of the model, all wages will in any case turn out to be equal in equilibrium. As a consequence, there is no loss of generality by preliminarily proceeding to study the optimum of the consumer problem when \( w_v = w \) for all \( v \).
Lemma 2 (Nonhomotheticity in quality of consumption) The selected quality of the consumed goods rises as the real income of the consumer increases; that is: $\partial q_{z,v}/\partial \kappa > 0$. Furthermore, the process of quality upgrading is more pronounced for the varieties of the good sourced from countries that can more easily improve its quality; that is: $\partial^2 q_{z,v} / (\partial \kappa \partial \eta_{z,v}) < 0$.

Lemma 2 summarises the key nonhomothetic aspect present in our model: quality upgrading of consumption. From the result $\partial q_{z,v}/\partial \kappa > 0$ it follows that, as real incomes grow with a rising $\kappa$, individuals substitute lower-quality versions of every good $z$ by better versions of them. Moreover, the cross-derivative $\partial^2 q_{z,v} / (\partial \kappa \partial \eta_{z,v}) < 0$ implies that the quality rise is faster for commodities supplied by countries that received better sectoral productivity draws ($i.e.$, lower values of $\eta$).

Jointly considered, the two lemmas underlie the main source of interaction between supply and demand sides that we will exploit in our general equilibrium analysis: as $\kappa$ grows, producers better able at upgrading quality in a particular sector will gradually attract larger world expenditure shares in that sector.

2.4 General equilibrium

In equilibrium, total world spending on commodities produced in country $v$ must equal the total labour income in country $v$. Denoting by $\beta^i_{z,v}$ the expenditure share by importer $i$ in the variety of good $z$ produced in country $v$, we may write down the market clearing condition as follows:

$$\int_{z} \int_{v} \beta^i_{z,v} w_i \, dz = w_v, \quad (8)$$

where $w_i$ refers to the income of country $i$.

More formally, an equilibrium in the world economy is given by a set of wages $w_v$ for each country $v$ such that: $i$) prices of all traded commodities are determined by (2); $ii$) all consumers in the world choose their commodity spending by maximising (3) subject to (4); and $iii$) the market clearing conditions stipulated in (8) hold simultaneously for all countries.

In this world economy, the ex-ante symmetry across countries implies that, in equilibrium, all country wages $w_v$ will always turn out to be equal to each other. Thus, we can simply write that $w_v = w$, for any level of $\kappa > 0$. The reason for this result is the following: as $\kappa$ rises, and real income-effect. In fact, a rise in $\kappa$ entails the same effects as an exogenous increase of $w$ (in that regard, the parameter $\kappa$ plays a role that is isomorphic to that of the units of efficiency labour available to each individual).

8 For a formal proof of this result, see Proposition 5 in the Online Appendix.
incomes accordingly increase, aggregate demands and supplies grow together at identical speed in all countries. As a consequence, markets clearing conditions in (8) will constantly hold true without the need of any adjustment in relative wages across economies.

The fact that relative wages remain constant over the path of development conceals the fact that, as $\kappa$ increases, economies actually experience significant changes in their consumption and production structures at the sectoral level. Such sectoral reallocations stem from the interplay of demand and supply side factors. On the demand side, as real incomes grow with a rising $\kappa$, individuals consume higher quality versions of each commodity – as can be observed from (6). On the supply side, heterogeneities in sectoral labour productivities across countries become stronger as producers raise the quality of their output – as can be gleaned from (1). As we will formally show next, the interplay between income-dependent willingness to pay for quality and intensification of sectoral productivity differences at higher levels of quality leads to a process of ever increasing sectoral specialisation as $\kappa$ rises.

### 2.5 Sectoral specialisation

We study now the effects of the above-mentioned sectoral reallocations on the sectoral trade flows. With regards to the demand side of the economy, we examine the import penetration (IP) of good $z$ sourced from country $v$ by country $i$, defined as:

$$ IP_{i, z, v}^i \equiv \frac{M_{i, z, v}^i}{M_{z, i}^i} $$

(9)

where $M_{i, z, v}^i$ denotes the value of imports of good $z$ sourced from country $v$, and $M_{z, i}^i$ is the total value of imports of good $z$ (the superindex $i$ refers to the country that purchases these goods).

For the supply side, we consider the revealed comparative advantage (RCA) of country $v$ in sector $z$. Formally:

$$ RCA_{z, v} \equiv \frac{X_{z, v}}{X_{v}} \frac{X_{v}}{W} $$

(10)

In the numerator of (10), $X_{z, v}$ denotes the value of exports of good $z$ by country $v$, and $X_{v}$ the value of total exports by country $v$. In the denominator of (10), $W_{z}$ refers to the value of exports of good $z$ worldwide, and $W$ represents the value of total exports in the world.

**Lemma 3 (Import penetration and revealed comparative advantage)** In a world economy where all countries have the same income, for every variety $v$ of good $z$, the measures of import penetration and revealed comparative advantage equal the share of income spent on that commodity. Formally:
1. For any importer $i$ of variety $v$ of good $z$: $IP^i_{z,v} = \beta_{z,v}$.

2. For any exporter $v$ of good $z$: $RCA_{z,v} = \beta_{z,v}$.

In our symmetric world economy, the revealed comparative advantage of country $v$ in good $z$ and the import penetration of variety $v$ of good $z$ mirror one another. This is the result of all countries displaying the same income (coupled with individuals having the same preferences worldwide and the absence of trade costs) which entails that all consumers will choose exactly the same optimal consumption bundle. The next proposition shows how these measures of bilateral trade flows relate to countries’ sectoral productivity draws, and how they evolve as real incomes rise in the world economy.

**Proposition 1** In a world economy where all countries have the same income, the degree of specialisation in sector $z$ is larger for countries that received better sectoral productivity draws in that sector. In addition, sectoral specialisation intensifies as the real incomes of individuals increase. Formally, for any sector $z$ and any pair of countries $v$ and $v'$ such that $\eta_{z,v} < \eta_{z,v'}$:  

1. $\beta_{z,v} > \beta_{z,v'}$. Therefore, $RCA_{z,v} > RCA_{z,v'}$, and $IP^i_{z,v} > IP^i_{z,v'}$ for any importer $i$.

2. $\frac{\partial \beta_{z,v}}{\partial \kappa} > \frac{\partial \beta_{z,v'}}{\partial \kappa}$. Thus, $\frac{\partial (RCA_{z,v} - RCA_{z,v'})}{\partial \kappa} > 0$, and $\frac{\partial (IP^i_{z,v} - IP^i_{z,v'})}{\partial \kappa} > 0$ for any importer $i$.

Proposition 1 merges together supply side and demand side results. It firstly describes how export and import specialisation relate to the sectoral productivity draws, and secondly it shows how both measures evolve as real incomes grow with a rising value of $\kappa$.

From a supply side perspective, Proposition 1 states that the RCA in sector $z$ are monotonically linked to the sectoral productivity draws $\eta_{z,v}$: countries that receive better draws for sector $z$ exhibit a higher RCA in that sector. More importantly, the second result in the proposition shows that this gap further intensifies as $\kappa$ rises. This last result is what we interpret as increasing export specialisation along the growth path.

From a demand side perspective, Proposition 1 may be interpreted in terms of increasing import specialisation along the growth path. More precisely, the result that $\partial (IP^i_{z,v} - IP^i_{z,v'})/\partial \kappa > 0$ means that, as consumers get richer, we observe a process of growing import penetration of the varieties of $z$ produced by exporters who enjoy a higher RCA in sector $z$.

The joint consideration of these two arguments suggests that, over the path of development, countries with a cost advantage in a given sector will increasingly specialise in that sector. At the same time, these countries will also attract a growing share of the world spending in that
particular sector. Intuitively, as world consumers raise the quality of their consumption when $\kappa$ grows, sectoral productivity differentials across countries widen up, leading to an increase in sectoral trade specialisation. Interestingly, this process takes place both at the importer and at the exporter level. In this regard, a central prediction of our model is the implicit secular tendency of sectoral trade flows to gravitate towards exporters with a rising cost advantage in the sector. This, in turn, means that while some bilateral sectoral trade links will intensify during the path of development, others will gradually fade.$^9$

3 A world economy with cross-country inequality

The previous section has dealt with a world economy where all countries exhibit the same real income, while we let the worldwide total factor productivity parameter $\kappa$ increase. Such an analytical framework allowed us to portray the behaviour of sectoral trade flows (and sectoral specialisation patterns) within a world economy where countries shared a common growth path.

In this section, we slightly modify the previous setup to give room for cross-country inequality. To keep the focus as clean as possible (departing from Section 2) we now hold constant the parameter $\kappa$. More importantly, we no longer force sectoral productivity differentials to be drawn from the same probability distribution function, which was the ultimate reason leading to equal equilibrium wages. This alternative setup allows us to generalise the previous results concerning export specialisation to a case in which productivity differentials and cost differentials may not always coincide (as a result of equilibrium wages that differ across countries). In addition, introducing cross-country inequality leads to more powerful predictions concerning import penetration (of the different export sources) at different income levels, which we will contrast with cross-sectional data of bilateral trade flows in Section 4.

We keep the same commodity space and preference structure as those previously used in Section 2. However, we now assume that the world is composed by two subsets of countries. We will refer to the two subsets as region $H$ and region $L$ and, whenever it proves convenient, to a generic country by $h$ or $l$, respectively. We let countries in $H$ and $L$ differ from each other in that they face different random generating processes for their sectoral productivity parameters. For any country $h$, we assume that $\eta_{z,h}$ for each good $z$ is independently drawn from a uniform density

$^9$The equilibrium characterised in this section has the particular feature that revealed comparative advantages coincide with the import penetrations. This is clearly a very specific result that hinges on the assumed symmetry in the distributions of sector-specific productivities across countries. The next section shows that this is no longer the case when we introduce some asymmetry across countries.
function with support $[\eta, \overline{\eta}]$, where $\overline{\eta} > 1$, just like before. Instead, for any country $l$, we assume that $\eta_{z,l} = \overline{\eta}$ for every good $z$.\footnote{None of our results hinge upon countries in region $L$ drawing their sectoral productivity parameters from a degenerate distribution. In the Appendix B, we extend the results of this section to a world economy with multiple regions, and where all sectoral productivities are drawn from non-degenerate uniform distributions.}

This alternative setup still features the fact that sectoral productivity differentials become increasingly pronounced at higher levels of quality. In addition, it also allows for the presence of absolute advantages (at the aggregate level) across regions.\footnote{Another way to introduce a source of absolute advantages into our framework would be by letting total factor productivity be higher in region $H$ than in region $L$, namely: $\kappa_h > \kappa_l$. Adding $\kappa_h > \kappa_l$ to the aggregate productivity gap resulting from the regionally different random generating processes for $\eta_{z,v}$ would just reinforce the equilibrium wage differential between $H$ and $L$, while it would not qualitatively change any of the main results obtained in this section.}

The \textit{ex-ante} symmetry across countries from the \textit{same} region implies now that, in equilibrium, wages of countries within that region must be equal. By contrast, wages in region $H$ must necessarily be higher than in region $L$. More formally, in equilibrium: $w_h = w_{h'}$ for any pair of countries $h, h' \in H$, and $w_l = w_{l'}$ for any $l, l' \in L$, where $w_h > w_l$.\footnote{For a formal proof of this result, see Proposition 6 in the Online Appendix.}

The intuition for $w_h > w_l$ is analogous to all Ricardian models of trade with absolute and comparative advantages. Essentially, region $H$ (which displays an absolute advantage over region $L$) will enjoy higher wages than region $L$, since this is necessary to lower the production costs in $L$, thereby allowing countries in $L$ to export enough to countries in $H$ and keep the trade balance in equilibrium. Henceforth, without loss of generality, we take the wage in region $L$ as the \textit{numeraire} of the economy. We accordingly set $w_l = 1$, with $w_h$ hereafter denoting the relative wage between region $H$ and region $L$.

The first set of results that differ qualitatively from those obtained in a world economy with symmetric countries are to do with quality of consumption and quality of production. Considering the former, nonhomothetic preferences on the quality dimension imply that consumers in $H$ purchase higher quality consumption bundles than consumers in $L$. For the latter, the difference in wage between the two regions will distort the monotonicity between the monetary cost of production and sectoral productivity draws ($\eta_{z,v}$) present throughout Section 2.

\textbf{Proposition 2} \textit{In a two-region world economy with $w_h > 1$:}

\begin{enumerate}
\item \textit{Consumers from region $H$ select higher quality versions than consumers from region $L$.}
\end{enumerate}
2. All consumers set the level of quality highest for the varieties sourced from countries in region $H$ that received the best possible sectoral productivity draw, $\eta_{z,h} = \eta$, and lowest for the varieties sourced from countries in $H$ that received the worst possible sectoral productivity draw, $\eta_{z,h} = \bar{\eta}$. Furthermore, the quality level of varieties sourced from countries in region $L$ lies within those two extreme levels.

3. All consumers choose higher qualities for the varieties sourced from countries in region $H$ that received better sectoral productivity draws.

The first result stems again from the rising willingness-to-pay for quality implied by (3): richer consumers substitute lower-quality versions of each good $z$ by higher-quality versions of them. The second result shows that the highest quality of each good $z$, purchased by any consumer, is produced in the country in region $H$ that received the best possible draw, $\eta_{z,h} = \eta$. Conversely, the lowest quality of each good $z$, purchased by any consumer, is produced in the country in region $H$ that received the worst possible draw, $\eta_{z,h} = \bar{\eta}$. Notice that, although all countries in region $L$ also receive draws equal to $\bar{\eta}$, the lower labour cost there allows them to sell higher qualities than the least efficient producers in region $H$. Finally, the third result shows that, when considering only commodities produced in region $H$, the quality of consumption is a monotonically decreasing function of the elasticities of quality upgrading $\eta_{z,h}$. Intuitively, since all countries in region $H$ have the same wage, a larger $\eta_{z,h}$ maps monotonically into a higher production cost (for a given the level of quality), thus consumers worldwide find it optimal to demand higher quality varieties from countries with lower draws of $\eta_{z,h}$.

### 3.1 Export specialisation

Assume henceforth that a fraction $\lambda \in (0, 1)$ of all countries in the world belong to region $H$. We proceed now to study the patterns of exporters’ specialisation in this world economy with cross-country inequality. We let $\beta_{z,h}^H$ and $\beta_{z,h}^L$ denote henceforth the expenditure share in the variety of good $z$ produced in country $h$ by a consumer from region $H$ and $L$, respectively.

**Lemma 4 (Revealed comparative advantage in a world with cross-country inequality)**

In a two-region world economy with $w_h > 1$, the measures of revealed comparative advantage for a generic good $z$ are

1. for any country $l$:

$$RCA_{z,l} = 1;$$  \hspace{1cm} (11)
2. for any country $h$:

$$RCA_{z,h} = \frac{\lambda \beta^H_{z,h} w_h + (1 - \lambda) \beta^L_{z,h}}{w_h}. \quad (12)$$

The result in (11) states that in every country in region $L$ the revealed comparative advantages are identical for all goods; this is the consequence of all sectors in those countries receiving the exact same draw, $\eta_{z,l} = \bar{\eta}$. Revealed comparative advantages do vary though across countries in region $H$. In particular, since $\beta^H_{z,h}$ and $\beta^L_{z,h}$ are decreasing functions of the sectoral productivity draws $\eta_{z,h}$, (12) implies that the $RCA_{z,h}$ is also a decreasing function of $\eta_{z,h}$. Moreover, such monotonicity of the demand intensities also means that the revealed comparative advantage of the country belonging to $H$ with draw $\bar{\eta}$ will turn out to be lower than that of any country in $L$. Similarly, the revealed comparative advantage of the country belonging to $H$ with draw $\eta$ will be higher than that of any country in $L$. These results are summarised in the following proposition.

Proposition 3 Let $RCA_{z,\bar{\eta}}$ and $RCA_{z,\eta}$ denote the revealed comparative advantage in sector $z$ of countries in region $H$ that received the best possible productivity draw, $\eta_{z,h} = \bar{\eta}$, and the worst possible productivity draw, $\eta_{z,h} = \eta$, respectively. Then:

1. $RCA_{z,\bar{\eta}} < RCA_{z,l} < RCA_{z,\eta}$;

2. The revealed comparative advantage in sector $z$ of country $h$ is a decreasing function of the sectoral productivity draw: $\partial (RCA_{z,h}) / \partial \eta_{z,h} < 0$.

The main result to draw from Proposition 3 is that the country (in region $H$) receiving the best possible draw in sector $z$ will display as well the highest revealed comparative advantage in that sector. Notice that these countries are also those supplying the highest quality varieties of good $z$, as shown in Proposition 2. Therefore, like in Section 2, countries offering the top quality varieties in a given sector also exhibit the strongest degree of export specialisation in that sector. Finally, note that Proposition 3 also implies that there exists a subset of countries in $H$ exhibiting a lower RCA in sector $z$ than countries in $L$. The reason is that $w_h > 1$ creates a wedge between the absolute and the comparative advantage, allowing countries in $L$ to supply more competitively the relatively low-quality varieties of good $z$.

3.2 Import specialisation

We turn now to study the implications of this version of the model in terms of import specialisation. For any destination country in region $j = H, L$, the import penetration of good $z$ originating from
country \( v \) is given by \( IP_{j,z,v}^i = \beta_{z,v}^i / \int_{v} \beta_{z,v}^i dv \). Since the budget constraint implies \( \int_{v} \beta_{z,v}^i dv = 1 \), we may then track the behaviour of \( IP_{j,z,v}^i \) simply by looking at the demand intensity \( \beta_{z,v}^i \).

**Proposition 4** Let \( IP_{z,H}^j \) and \( IP_{z,L}^j \) denote the import penetration in sector \( z \) in region \( j = H, L \) by countries in region \( H \) that received best possible productivity draw, \( \eta_{z,h} = \eta \), and the worst possible productivity draw, \( \eta_{z,h} = \bar{\eta} \), respectively. Then:

1. \( IP_{z,H}^j < IP_{z,L}^j < IP_{z,\bar{\eta}}^j \), where \( IP_{z,\bar{\eta}}^j \) is the import penetration in sector \( z \) in region \( j = H, L \) by countries in region \( L \). Moreover, for imports sourced from region \( H \), the import penetration in sector \( z \) by a country \( h \) is decreasing in its sectoral productivity draw: \( \partial IP_{z,h}^j / \partial \eta_{z,h} < 0 \).

2. The difference in import penetration in any given sector \( z \) between a country from \( H \) that received the best possible productivity draw and any other producer of good \( z \) is always larger in region \( H \) than in region \( L \). Formally:

\[
IP_{z,H}^H - IP_{z,H}^L > IP_{z,\bar{\eta}}^H - IP_{z,\bar{\eta}}^L, \quad \text{whenever} \quad \eta_{z,h} > \bar{\eta};
\]

\[
IP_{z,H}^H - IP_{z,L}^H > IP_{z,\bar{\eta}}^H - IP_{z,\bar{\eta}}^L.
\]

The first part of Proposition 4 can be seen simply as the demand-side counterpart of Proposition 3: importers source a larger share of good \( z \) from exporters with a cost advantage in sector \( z \) (this is, ultimately, what turns these exporters into the ones exhibiting the greatest RCA in that sector).

More interestingly, the second part of Proposition 4 states that import specialisation in those exporters is stronger for richer importers (that is, for countries in region \( H \)).

The intuition for this result rests on the specific nonhomothetic structure of (3). As shown in Proposition 2, richer importers buy high-quality varieties, which are exactly those for which the cost advantage of countries receiving better sectoral productivity draws widens. In addition, since the preference structure in (3) also implies that high-quality varieties attract growing consumer expenditure shares, richer importers tend to spend *proportionally* more in commodities sourced from exporters that exhibit a stronger cost advantage in higher-quality varieties.

### 3.3 Discussion: Sectoral trade flows

The previous subsections have dealt separately with the behaviour of exporters facing importers with heterogeneous income, and with the behaviour of importers facing exporters with heterogeneous cost advantages. The joint consideration of these results yields an additional important prediction. To illustrate this prediction, we focus now on two intertwined demand-supply relationships implicit in our model: i) the link between the sectoral productivity draw \( \eta_{z,v} \) and the RCA
of exporter \( v \) in sector \( z \); ii) the link between \( \eta_{z,v} \) and the import penetration by exporter \( v \) in the total consumption of good \( z \) in a generic destination country \( i \).

Firstly, Proposition 3 implies that, for a given income of the exporter \( w_v \), better productivity draws in sector \( z \) lead to a greater RCA in that sector: \( \partial RCA_{z,v} / \partial \eta_{z,v} < 0 \). Secondly, Proposition 4 adds to this result that (again for a given level of \( w_v \)) the import penetration in sector \( z \) in any destination country \( i \) is larger in the case of exporters that received better productivity draws in sector \( z \): \( \partial IP_{i,z,v} / \partial \eta_{z,v} < 0 \). Furthermore, that proposition also shows that the association between \( IP_{i,z,v} \) and \( \eta_{z,v} \) becomes stronger in richer importers: \( \partial (\partial IP_{i,z,v} / \partial \eta_{z,v}) / \partial w_i < 0 \).

The above results can in turn be translated into relations between \( IP_{i,z,v} \) and \( RCA_{z,v} \). In particular, when holding fixed the income of the exporter \( w_v \), the model delivers: \( \partial IP_{i,z,v} / \partial RCA_{z,v} > 0 \) and \( \partial (\partial IP_{i,z,v} / \partial RCA_{z,v}) / \partial w_i > 0 \). The first of these predictions is simply stating that import penetrations by exporters with a higher RCA will be larger in all importers. More interestingly, the second entails that, as we move from poorer to richer importers, the positive association between import penetration and RCA becomes even stronger.

The economic intuition behind this last result is analogous to the one discussed in Section 2 for the case of growing world incomes with a rising \( \kappa \). However, with cross-country inequality this intuition becomes even more apparent because importers with heterogeneous incomes choose different quality levels of all varieties, which in turn implies different distributions of budget shares across the same set of exporters. More precisely, since richer consumers purchase higher-quality varieties of each good \( z \), the most productive suppliers of each good \( z \) turn out to be better able to exploit their widening cost advantage when dealing with richer importers.

Our model thus delivers a mechanism entailing a simultaneous rise in sectoral trade specialisation by importers and exporters at higher incomes: when measured at the sectoral level, richer importers tend to increasingly specialise their consumption in those varieties of goods supplied by the exporters who display a stronger revealed comparative advantage in the sector. In the next section we provide evidence consistent with this prediction using bilateral trade flows at the sectoral level.

### 4 Empirical analysis

Our theory rests crucially on two fundamental assumptions: one related to cross-country heterogeneities in sectoral production functions; the other one related to nonhomotheticities in the consumers’ preference structure. In terms of technologies, we have assumed that cross-country sectoral productivity differentials widen at higher levels of quality of production. Concerning pref-
ferences, we postulated a utility function where richer individuals choose a consumption basket comprising higher-quality varieties of all available goods.

Taken independently, each of these two assumptions lead to clear testable predictions in terms of trade flows, which in fact our theory shares with several other papers in the trade literature. First, our model implies that the degree of specialisation of countries in particular goods and the level of quality of their exports of those goods should display a positive correlation. Second, our model implies that richer consumers buy their imports in higher quality levels than poorer consumers do.

Besides these two results, the most interesting testable prediction of our model stems from the interaction between the above-mentioned assumptions. When consumers’ taste for quality rises with their income and sectoral cost advantages deepen at higher levels of quality, richer countries will purchase a larger share of their imports of every good from economies displaying a stronger revealed comparative advantage in the sector producing the good. In other words, our model yields novel predictions regarding import specialisation at different income levels, which link richer importers more intensely to highly specialised exporters in each of the sectors.

4.1 Baseline regression structure

Recall from Section 3.3 that, once we condition on the income of the exporter $w_v$, our model delivers the following two results:

13 Several papers provide evidence consistent with this prediction. For example, Alcala (2012) shows that import prices by the US in the apparel industry tend to be higher for imports sourced from exporter displaying a higher revealed comparative advantage in that industry. In a previous working paper version, Jaimovich and Merella (2013), we show that a similar correlation is found considering all 5000 products categorised according to the 6-digit Harmonised System (HS-6), and all pairs of bilateral sectoral trade flows in the world. Furthermore, empirical results consistent with this assumption can also be found in articles using firm-level data. For example, Kugler and Verhoogen (2012) find a positive correlation between output prices in narrowly defined products and plant size for Colombian manufacturing firms, while Manova and Zhang (2012) report a positive correlation between unit values and total export sales by Chinese firms. Similarly, Crino and Epifani (2012) find that Italian manufacturing firms exhibiting higher TFP tend to concentrate their production in high-quality varieties and export relatively more to richer destinations.

14 There is also vast evidence supporting this prediction: e.g., Hallak (2006, 2010), Choi et al. (2009), Fieler (2012), Feenstra and Romalis (2012), Crozet et al. (2012), Chen and Juvenal (2014), Flach (2014). In particular, Fieler (2012) shows that import prices correlate positively with the level of income per head of the importer, even when looking at products originating from the same exporter and HS-6 category. The use of unit values as proxy for quality dates back to Schott (2004). See Khandelwal (2010) and Hallak and Schott (2011) for some innovative methods to infer quality from prices, taking into account both horizontal and vertical differentiation of products.
(i) The import penetration in sector $z$ of country $i$ is larger for exporters that exhibit a higher RCA in $z$.

(ii) If we compare importers with different incomes, the import penetration in sector $z$ by exporters that exhibit a higher RCA in $z$ is relatively larger in richer importers.

Result (i) is simply saying that all importers tend to buy more of good $z$ from exporters displaying a revealed comparative advantage in sector $z$. In that regard, result (i) is not really informative about the interplay between our nonhomothetic preferences and the cost advantage of exporters more specialised in a good $z$ intensifying at the high-quality versions of that good. Result (ii), instead, is the direct consequence of that particular mechanism. More precisely, this result is suggestive of a positive correlation between import penetration and the exporter’s revealed comparative advantage of varying magnitude depending on the level of GDP per capita of the importer. This feature could be captured by a regression that allows for heterogeneous intensity of import penetration at different levels of importer income, such as one including an interaction term between exporter’s RCA and importer’s income per head like:

$$
\log (IP_{iz;v}^i) = \rho \log (RCA_{iz;v}) + \theta [\log (RCA_{iz;v}) \times \log (w_i)] + \varphi \log (w_i) + \psi \log (w_v) + u_{iz;v}^i. 
$$

(13)

A regression of this type should yield an estimated value of $\theta > 0$ to be consistent with our model. The theoretical rationale for this prediction lies in the interaction between richer consumers buying higher quality versions of the traded goods, and exporters with a stronger RCA in a sector being increasingly productive at delivering high quality versions of these goods. Notice that (13) includes the exporter’s per-capita GDP, $w_v$, as additional regressor. This is done in order to account for the fact that prices at which exporters sell their output may differ simply owing to differences in local wages – more precisely, in terms of our model results, once we condition on $w_v$, we are able to maintain the monotonicity between $RCA_{iz;v}$ and $\eta_{iz;v}$, that we exploit in results (i) and (ii) above.

In terms of actual implementation, our regression needs to include a number of additional controls. In particular, we consider:

- **Importer fixed effects.** In our main regression, given by (14) below, we substitute the importer’s per-capita GDP ($w_i$) by a set of importers fixed effects. Since we are using a cross section of countries, these suffice to control for importer income. In addition, our model assumes identical trade openness and barriers across all importers, which in practice does not seem a tenable assumption. Including importer fixed effects partly controls for some of these factors as well.
• **Exporter fixed effects.** Similarly, we substitute exporter’s per-capita GDP \((w_e)\) by a set of exporters fixed effects. Like with importer fixed effects, the exporter fixed effects control for additional effects, possibly present in practice, that are assumed away by our model (e.g., differences in openness across exporters).

• **Product fixed effects.** Our model assumes symmetry of technologies for all sectors, while it also assumes no differential trade costs or barriers across sectors. In practice, these assumptions do not seem tenable either. In our main regression (14) we thus include product fixed effects to control for some of these factors.\(^{15}\)

• **Gravity terms.** Our model assumes away any sort of trade costs or frictions that are partner-specific, hence (13) applies identically to any importer-exporter transaction. In practice, not only there are trade costs and frictions, but also they affect different partners differently. In our main regression (14) we include the standard gravity terms to control for some of these factors.

In Table 1.A, we therefore show the results of regression (14) using sectoral bilateral trade data for year 2009, where we include product dummies \((\delta_z)\), importer dummies \((\mu_i)\), exporter dummies \((\varepsilon_v)\), and a set of bilateral gravity terms \((G_{i,v})\) taken from Mayer and Zignago (2006):\(^{16}\)

\[
\log(IP_{z,v}^i) = \rho \log(RCA_{z,v}) + \theta [\log(w_i) \times \log(RCA_{z,v})]
+ G_{i,v} + \delta_z + \mu_i + \varepsilon_v + \nu_{z,v,i}. \tag{14}
\]

Before moving on to the estimation results, we should conclude this subsection by stressing that our regression analysis aims is capturing only a partial correlation coefficient, possibly becoming stronger at higher levels of importer income per head. The coefficient \(\theta > 0\) is indeed indicative of

\(^{15}\)In some specifications, we substitute the importer fixed effects and product fixed effects by importer-product fixed effects. These can account for differences in sectoral market structures and sectoral trade barriers across importers. In addition, they may also account for heterogeneity in importers preferences for different goods, which is assumed away by our common utility function.

\(^{16}\)Import penetration, as defined by (9), and revealed comparative advantage, as defined by (10), are both computed using the dataset compiled by Gaulier and Zignago (2010). This database reports monetary values of bilateral trade (measured FOB in US dollars) for years 1995 to 2009 for more than 5000 products categorised according to the 6-digit Harmonised System (HS-6). As robustness checks, we have also run the regressions reported in Table 1.A separately for all the years in the sample. All their estimates results are of very similar in magnitude to those of year 2009, and available upon request. Notice, also, that the fact that we are looking at a cross-section of countries implicitly works as holding fixed \(\kappa\) in the model. If we were using instead a panel of countries, we could remove the effect of a time-varying \(\kappa\) simply by including time fixed effects in the regression.
such varying partial correlation. We rely on our model to interpret this result as emerging from the interplay between nonhomotheticities in quality and widening productivity differentials at higher levels of the quality ladder. However, as a simple set of correlations, the coefficients in (14) cannot be directly linked to fundamental parameters of the model. Also for this reason, we cannot make use of those estimates to back out the values of those parameters and thus construct quantitative counterfactuals with them.

4.2 Baseline regression results

Before strictly running regression (14), we firstly regress the dependent variable against only the RCA of exporter \(v\) in good \(z\) (together with product, importer and exporter dummies). Column (1) of Table 1.A shows (quite expectably) that those two variables are positively correlated. Secondly, in column (2), we report the results of the regression that includes the interaction term. We can see that the estimated \(\theta\) is positive and highly significant, consistent with our theory. Finally, in column (3), we add the six traditional gravity terms, and we can observe the previous results remain essentially intact. We can also observe that the estimates for each of the gravity terms are significant, and they all carry the expected sign.

Notice that regression (14) includes exporter fixed effects \((\varepsilon_v)\). This implies that our regressions are actually comparing different degrees of export specialisation across products for a given exporter, and the different degrees of import penetration of the exporter across its exports destinations. As such, exporter dummies would control for the fact that a country with higher total factor productivity may be commanding larger market shares and may be specialising in higher quality varieties of goods, which are exactly the varieties mostly purchased by richer importers.

4.3 Robustness checks and Linder term

Table 1.B presents some additional regressions as robustness checks. First, in column (1) we show the results of a regression analogous to column (3) in Table 1.A, but where we control for product-importer fixed effects, instead of product \((\delta_z)\) and importer \((\mu_i)\) fixed effects separately. After including the set of product-importer dummies, the estimated coefficient for the interaction term remains essentially intact, as well as its significance level. Next, in columns (2) and (3) we exclude from the importers sample the OECD countries and the high-income countries as classified by the World Bank, respectively. The idea behind these restricted-sample regressions is to see whether our previous results are driven only by the behaviour of the richest importers. As we can observe,
in both cases our correlation of interest remains still positive and highly significant.\textsuperscript{17}

Our paper emphasises the interplay between nonhomothetic preferences with respect to quality and increasing sectoral specialisation at higher qualities of production (owing to wider cross-country sectoral productivity differentials at higher layers of quality). The interaction term in (14) intends to reflect the impact of such mechanism on the intensity of bilateral trade links at different levels of income per head of the importer. Some recent articles in the trade literature with nonhomothetic preferences have argued that richer countries exhibit a comparative advantage in higher-quality varieties of goods – see Hallak (2010) and Fajgelbaum\textit{ et al.} (2011).\textsuperscript{18} If that is actually the case in reality and, moreover, if the share of imports to GDP grows with the importer’s income per capita (as it has been widely documented in the trade literature), then our interaction term in (14) may end up capturing (at least partially) a different type of effect: the fact that richer importers, who tend to source a larger fraction of their final demand from abroad, establish stronger trade links

\textsuperscript{17}The estimate associated to ‘common currency’ falls essentially to zero in columns (2) and (3). This is because when we remove the Euro-area countries from the sample, we lose practically all its source of variation.

\textsuperscript{18}In fact, this result is also present in our model when we extend our basic setup in Section 3 to allow for cross-country income inequality.
with richer countries, since these tend to specialise in higher-quality varieties which are in turn those demanded by richer importers. In order to deal with this concern, the regression in column (4) adds a Linder term among the regressors. In particular, we include as independent variable the absolute difference between the log income per head of the importer and exporter: $|\ln y_{impo} - \ln y_{expo}|$.\(^{19}\)

This regressor should absorb the above-mentioned concern. The results in column (4) indeed show that the Linder term carries a negative and highly significant coefficient, which is consistent with the evidence of the Linder hypothesis holding at the sectoral level previously found in Hallak (2010). Nevertheless, the estimate of the coefficient associated to the interaction term remains positive and highly significant. This last result suggests that our mechanism explaining the intensity of sectoral trade links by export source at different levels of income of importers is playing a role alongside the traditional Linder-type effect.\(^{20}\)

In our theory, both the measures of import penetration and revealed comparative advantage are endogenous variables, determined simultaneously as general equilibrium outcomes of the model. For this reason, we cannot interpret those estimates for RCA and the interaction term in Table 1.A and Table 1.B as quantifying a causal effect on the intensity of sectoral import penetration at different levels of importer income per head. However, it still proves interesting to use the estimates in column (4) of Table 1.B to get a feeling of the magnitudes of the correlations arising from the mechanism proposed by our model relative to those captured by the Linder term.

Our mechanism entails a greater intensity of sectoral bilateral trade between richer importers and exporters displaying a stronger RCA in the sector. For that reason, in what follows, we quantify the difference in the correlation between these two variables for a rich importer and a poor importer, at the level of the logarithm of the RCA corresponding to its 90th percentile (this value equals 1.07).\(^{21}\) Computing the difference in magnitude yielded by the interaction term for the

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\(^{19}\)When we use $(\ln y_{impo} - \ln y_{expo})^2$ instead, the results remain qualitatively the same as in column (4).

\(^{20}\)In practical terms, one additional concern may be raised: the possibility that high-quality varieties of goods face lower trade frictions than lower-quality ones. If this were true, then our interaction term in (14) might also be capturing a different type of effect: the fact that richer economies tend to consume higher-quality varieties, and that those varieties are traded more intensely as a result of lower frictions. Since we cannot observe unit trade costs at different layers of quality, and our regressions exploit within-product variation of import shares by source, we cannot envisage a practical way to directly gauge the severity of this concern. Notice, however, that if this issue were quantitatively significant, we should expect to find very different estimate for ‘distance’ and ‘contiguity’ in column (1) and column (3), since the latter excludes richer importers. Indeed, the fact that both regressions yield similar estimates suggests that, once we control for all the importer and exporter characteristics, we do not observe huge differences in the effects of sector-specific trade frictions across richer and poorer importers.

\(^{21}\)The median number of exporters by product in our sample is 80, therefore the 90th percentile value of the RCA
importer in the 90th percentile of the GDP per head in PPP (which corresponds to Belgium with 34,625) and that one for 10th percentile (which corresponds to Mali with 999), we obtain that the 90th-percentile exporter in a given sector (measured by the RCA) exhibits an income penetration that is approximately 32.6% larger in the high-income importer relative to the low-income importer. Similarly, using the estimate for the Linder term (−0.117) with the absolute difference between the logarithm of Belgium’s and Mali’s GDP per head in PPP, we obtain that economies in the top 90th and bottom 10th percentile of income tend to exhibit import penetrations approximately 41.5% lower than those displayed by equally rich countries. These simple computations suggest that both our proposed mechanism and the standard Linder effect seem to be driving important quantitative effects in terms of the correlations between sectoral bilateral trade links and income per head observed in the data.

4.4 Sectoral and product level regressions

The regressions in Table 1.A pool together approximately 5000 different 6-digit products, implicitly assuming the same coefficients for all of them. This might actually be a strong assumption to make. In Table 2.A we split the set of HS 6-digit products according to fourteen separate subgroups at seems a sensible benchmark to look at for a ‘highly specialised exporter’ of the product.
Table 2.A

<table>
<thead>
<tr>
<th></th>
<th>animal &amp; anim. prod.</th>
<th>vegetable products</th>
<th>foodstuff</th>
<th>mineral products</th>
<th>chem. &amp; allied ind.</th>
<th>plastic &amp; rubbers</th>
<th>skin, leath. &amp; furs</th>
</tr>
</thead>
<tbody>
<tr>
<td>log RCA</td>
<td>-0.322***</td>
<td>-0.298***</td>
<td>-0.344***</td>
<td>-0.269***</td>
<td>-0.500***</td>
<td>-0.548***</td>
<td>-0.622***</td>
</tr>
<tr>
<td></td>
<td>(0.106)</td>
<td>(0.104)</td>
<td>(0.096)</td>
<td>(0.145)</td>
<td>(0.138)</td>
<td>(0.138)</td>
<td>(0.155)</td>
</tr>
<tr>
<td>interaction term</td>
<td>0.073***</td>
<td>0.079***</td>
<td>0.089***</td>
<td>0.075***</td>
<td>0.107***</td>
<td>0.118***</td>
<td>0.120***</td>
</tr>
<tr>
<td></td>
<td>(0.012)</td>
<td>(0.011)</td>
<td>(0.011)</td>
<td>(0.015)</td>
<td>(0.015)</td>
<td>(0.015)</td>
<td>(0.016)</td>
</tr>
<tr>
<td>Observations</td>
<td>105,332</td>
<td>210,866</td>
<td>215,975</td>
<td>72,839</td>
<td>602,592</td>
<td>317,328</td>
<td>66,347</td>
</tr>
<tr>
<td>Adj. R squared</td>
<td>0.44</td>
<td>0.49</td>
<td>0.50</td>
<td>0.46</td>
<td>0.49</td>
<td>0.52</td>
<td>0.60</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>wood &amp; wood prod.</th>
<th>textiles</th>
<th>footwear</th>
<th>stone &amp; glass</th>
<th>metals</th>
<th>machinery &amp; electrical</th>
<th>transport</th>
</tr>
</thead>
<tbody>
<tr>
<td>log RCA</td>
<td>-0.444***</td>
<td>-0.411***</td>
<td>-0.644***</td>
<td>-0.527***</td>
<td>-0.541***</td>
<td>-0.711***</td>
<td>-0.554***</td>
</tr>
<tr>
<td></td>
<td>(0.105)</td>
<td>(0.166)</td>
<td>(0.155)</td>
<td>(0.131)</td>
<td>(0.130)</td>
<td>(0.131)</td>
<td>(0.112)</td>
</tr>
<tr>
<td>interaction term</td>
<td>0.101***</td>
<td>0.090***</td>
<td>0.119***</td>
<td>0.107***</td>
<td>0.111***</td>
<td>0.134***</td>
<td>0.114***</td>
</tr>
<tr>
<td></td>
<td>(0.012)</td>
<td>(0.019)</td>
<td>(0.016)</td>
<td>(0.015)</td>
<td>(0.015)</td>
<td>(0.014)</td>
<td>(0.013)</td>
</tr>
<tr>
<td>Observations</td>
<td>252,135</td>
<td>795,926</td>
<td>75,522</td>
<td>209,397</td>
<td>630,910</td>
<td>1,296,090</td>
<td>176,916</td>
</tr>
<tr>
<td>Adj. R squared</td>
<td>0.53</td>
<td>0.55</td>
<td>0.61</td>
<td>0.53</td>
<td>0.50</td>
<td>0.55</td>
<td>0.53</td>
</tr>
</tbody>
</table>

Robust absolute standard errors clustered at the importer-exporter level in parentheses. All data corresponds to year 2009. All regressions include product, exporter and importer dummies, and the set of gravity terms used before in Table 1.A taken from Mayer & Zignano (2006). *** significant 1%.

Table 2.B

Independent regressions for each HS 6-digit product

<table>
<thead>
<tr>
<th>% positive coefficients</th>
<th>% negative coefficients</th>
<th>median coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>insignificant</td>
<td>significant 10%</td>
<td>significant 1%</td>
</tr>
<tr>
<td>29.8%</td>
<td>15.7%</td>
<td>38.0%</td>
</tr>
<tr>
<td>83.5%</td>
<td>14.3%</td>
<td>1.6%</td>
</tr>
<tr>
<td></td>
<td>0.5%</td>
<td>0.076</td>
</tr>
</tbody>
</table>

Total number of different products was 4904 (98 products were lost due to insufficient observations). Data corresponds to year 2009. Regressions include importer dummies and the set of gravity terms used in Table 1.A taken from Mayer & Zignano (2006).

The subgroups are formed by merging together subgroups at 2-digit aggregation level, according to http://www.foreign-trade.com/reference/hscode.htm. We excluded the subgroups ‘Miscellaneous’ and ‘Service’.

In the sake of brevity, we report only the estimates for $\rho$ and $\theta$ in (14). As we can observe, the estimates for each subgroup follow a similar pattern as those in Table 1.A: the estimate for the interaction term is always positive and highly significant for each subgroup.

Lastly, as further robustness check, in Table 2.B we report the percentage of positive and negative estimates obtained for $\theta$ when we run a separate regression for each of the products in the HS 6-digit categorisation. These results again tend to confirm those obtained before Table 1.A.
4.5 A comparison with the empirical predictions in the existing literature

Three recent related articles have also incorporated nonhomothetic preferences into general equilibrium trade models, and study the ensuing patterns of trade flows: Fieler (2011), Fajgelbaum, Grossman and Helpman (2011) –FGH–, and Jaimovich and Merella (2012). In the Introduction, we summarised mostly the main theoretical differences between our framework and theirs. We now discuss briefly how some of our empirical predictions differ from theirs, and how these differences may be discerned in the data.

Fieler (2011) focuses on the bilateral trade flows of horizontally differentiated goods displaying heterogeneous income demand elasticities.\(^{23}\) She finds that including intersectoral nonhomotheticities into a model à la Eaton and Kortum (2002), coupled with productivity dispersions across countries that correlate positively with income demand elasticities, can substantially improve its quantitative predictions on aggregate trade flows. Her empirical predictions then encompass cross-country variation of aggregate trade flows at different income levels as a result of intersectoral changes in trade, while her paper is silent about intrasectoral variations in trade flows. This last source of adjustment is exactly what regression (14) aims at. More precisely, our regressions are exploiting within-product variation of export sources by importer, abstracting from intersectoral changes in trade flows. The main novel empirical finding is that, looking at each particular product category in isolation, we can observe that richer importers source a larger share of their imports from those exporters that display a stronger degree of specialisation in the sector producing that product.

FGH shares with our framework the introduction of nonhomothetic preferences in a context with vertical and horizontal differentiation. Both papers lead to a rise in international specialisation as incomes increase. The underlying driving forces however differ. In FGH, the main driving force is the exploitation of a home-market effect, in the spirit of Linder (1961).\(^{24}\) In our paper, instead, the leading aspect is the deepening of heterogeneities in the cost of production across countries at higher levels of quality. More importantly, our mechanism leads to some testable predictions that cannot be straightforwardly rationalised by FGH. In particular, FGH leads to patterns of productive specialisation that take place only along the quality dimension: richer countries are net

---

\(^{23}\) See also Hunter (1991) and Francois and Kaplan (1996) for partial equilibrium frameworks assessing the relevance of intersectoral differences in income demand elasticities in explaining trade patterns.

\(^{24}\) Hallak (2010) provides a partial equilibrium model with a home-market effect that also builds on the original hypothesis in Linder (1961). In his model, countries of similar incomes trade more with each other, when considering sectoral level trade flows. He also provides empirical evidence for this prediction.
exporters of high-quality varieties, while poorer countries are net exporter of low-quality ones. Yet, those patterns of specialisation in quality cannot be ascribed to any specific sector. By contrast, in our model, for each particular sector, richer importers will end up establishing stronger trade links with those exporters more intensely specialised in the sector. From a strict empirical viewpoint, the mechanism suggested by FGH is then reflected in the Linder term included in column (4) of Table 2.A. However, even when we take into account that richer importers trade more with richer exporters, this factor does not fully explain the fact that richer economies source a larger fraction of their imports of each product from exporters exhibiting a comparative advantage in those products. In that regard, our mechanism seems to play an important role in the determination of sectoral trade links, alongside the more traditional Linder home-market effect.

Nonhomotheticities along both the vertical and horizontal dimensions is also a feature present in Jaimovich and Merella (2012). The main distinction between that model and the one presented here lies in the technological structure. Jaimovich and Merella (2012) remained within a traditional Ricardian framework where comparative advantages apply only at the sectoral level. There, richer economies specialise in goods with longer quality ladders and poorer ones in those with shorter ladders. The model presented here, instead, exploits an intrasectoral comparative advantage, as the result of widening productivity differences at higher layers of quality. This, in turn, delivers predictions for the degree of specialisation within the same product category, which cannot be rationalised by a model featuring full sectoral specialisation by a single country, like in Jaimovich and Merella (2012). In particular, that model is unable to account for some of the novel empirical findings that we delineate here: i.e., the intensity of sectoral specialization by exporters at different levels of quality of production, and the varying intensity of import penetration at different levels of importers’ income.

5 Further Discussion

The next subsections develop two simple extensions to our model in Section 3, in order to study the impact of different policies aimed at promoting the production and size of a particular sector in the economy. We first study the case of import tariffs, then the case of a subsidy to local producers. In the sake of brevity, we relegate the formal analysis of both subsections to the Online Appendix, in Section A.3 and Section A.4 respectively.
5.1 Trade frictions and consumer loss

Although the main focus of the paper is on the behaviour of sectoral import shares, our model carries also implications regarding trade frictions and consumer welfare. In particular, in our framework, import restrictions entail a more severe welfare loss for richer countries than for poorer ones. Intuitively, our nonhomothetic structure implies that richer importers choose higher-quality bundles of goods and devote a larger share of income to goods sourced from exporters who can more efficiently increase quality of production. Therefore, since comparative advantages deepen and gains from trade expand at higher levels of quality, it is rich consumers those who benefit most from frictionless trade.

To illustrate the argument succinctly, it proves more convenient to consider a simplified version of our model with only two levels of sectoral productivity draws $\eta_{z,v} \in \{\overline{\eta}, \overline{\eta}\}$ and a discrete number of countries. In particular, we let region $L$ and region $H$ comprise now two countries each: $L = \{l_1, l_2\}$ and $H = \{h_1, h_2\}$. Like before, countries in $L$ always receive the bad sectoral draw $\overline{\eta}$ in all sectors. Instead, for region $H$, we assume that in each sector $z$ one country receives $\eta_{z,v} = \overline{\eta}$ and the other one $\eta_{z,v} = \eta$. To keep the symmetry we had in Section 3, suppose that $h_1$ and $h_2$ have both an equal mass of sectors with good and bad sectoral productivity draws.$^{25}$

Consider first a country from region $L$. This country (by assumption) receives the bad productivity draw, $\overline{\eta}$, in sector $z$. Suppose that, for some reason, this country wishes to discourage imports of good $z$, and thus imposes a tariff on those goods.$^{26}$ Since countries in region $L$ are poorer, the welfare loss to local consumers owing to the tariff will not be too large. The reason for this is that individuals in region $L$ tend to purchase lower-quality varieties of $z$, and for these varieties the productivity gap relative to the most efficient producer in sector $z$ remains relatively narrow. Consider now the country in region $H$ that received the bad productivity draw in sector $z$. In this case, the welfare loss to local consumer resulting from a tariff on imports of good $z$ becomes more severe. Since richer consumers are those who intend to purchase higher-quality versions of good $z$, they end up being harmed relatively more by tariffs imposed on sectors where there are other countries that can more easily upgrade quality. In that respect, our model suggests that gains from trade are especially stronger for richer consumers and, therefore, high-income countries should display a more negative stance towards trade barriers to imports.

$^{25}$That is, for each sector $z$, there is always only one country in $H$ with draw $\overline{\eta}$ (and only one with draw $\eta$), while the mass of sectors that received a draw $\overline{\eta}$ is equal to 0.5 both in $h_1$ and in $h_2$.

$^{26}$This could be the result, for example, of policymakers of country $i$ believing sector $z$ represents an important sector where to develop enough local production, hence it needs protection from more efficient foreign producers.
In order to offer a hint of the relative magnitude of welfare loss between richer and poorer importers, in the Online Appendix A.3 we exploit a pooled estimation of the log of unit values (used as a proxy for quality) on the log of importer’s income to back out two figures. First, we pinpoint several implied values of the sectoral productivity draws. Then we derive the respective welfare loss differential, due to import tariff, that each of those draws would generate when comparing individuals from a first country with those from a second country being 10% richer than the first.

The fact that the consumer welfare loss owing to the tariff is greater in richer economies rests crucially on our specific non-homothetic structure of preferences. In particular, under homothetic preferences, willingness to pay for higher quality will not rise with income. In such a case, all consumers, regardless of their income, will suffer a welfare loss of equal magnitude after the imposition of an import tariff. In Section A.5 of the Online Appendix we show formally how the unequal welfare effects of an import tariff vanish away in the presence of homothetic preferences.

5.2 Sectoral subsidy and comparative advantage

The previous subsection has illustrated the differential welfare effects of a sector-specific import tariff across richer and poorer importers. One could rationalise this tariff as the outcome of a policy that aims at promoting some particular sector of the economy. An alternative (and, possibly, more direct) policy to foster sector $z$ is simply to subsidize the local producers in that sector.

Consider again the simplified model introduced in the previous subsection, and suppose that a country with a bad productivity draw in sector $z$ (i.e., a country $v$ with $\eta_{z,v} = \bar{\eta}$) introduces a subsidy for local producers of good $z$, with the intention of expanding the size of this sector. In our model, such a subsidy turns out to be more effective in increasing the share of sector $z$ in the GDP in poorer economies than in richer ones. The reason for this is again related to our nonhomothetic preference structure. In our model, in order to absorb a larger share of demand in sector $z$, a country must be able to offer higher-quality varieties of good $z$ more cheaply than their competitors. When we compare a country from region $L$ with the country from region $H$ that received the bad draw in sector $z$, it turns out that the impact of the subsidy in fostering sector $z$ is stronger in the former. The intuition for this result is that, given our non-homothetic structure of preferences, higher qualities are instrumental to attracting larger consumer spending shares. Therefore, the expansionary effect of the subsidy turns out to be larger in $L$ than in a country from region $H$ with the same draw of $\eta_{z,v} = \bar{\eta}$, as in the former the effect of the subsidy on quality expansion is compounded with the lower labour cost in $L$.\textsuperscript{27}

\textsuperscript{27}In Section A.5 of the Online Appendix we show formally how this result disappears when we substitute our
6 Conclusion

We presented a Ricardian model of trade with the distinctive feature that comparative advantages reveal themselves gradually over the course of development. The key factors behind this process are the individuals’ upgrading in quality of consumption combined with sectoral productivity differentials that widen up at higher levels of quality. As incomes grow and wealthier consumers raise the quality of their consumption baskets, cost differentials between countries become more pronounced. The emergence of such heterogeneities, in turn, alters sectoral trade flows, as each economy gradually further specialises in producing the subset of goods for which they enjoy a rising comparative advantage.

Our theory yielded a number of implications that find empirical support. Using bilateral trade data at the product level, we showed that the share of imports originating from exporters more intensely specialised in a given product correlates positively with GDP per head of the importer. This is consistent with richer consumers buying a larger share of their consumption of specific goods from countries exhibiting a comparative advantage in the sectors producing those goods.

Our core model assumed away any sort of trade frictions. This was in a sense a deliberate choice, so as to illustrate our proposed mechanism as cleanly as possible. In this respect, we extended our analysis in two directions, discussing some interesting policy implications of our theory in the presence of frictions. First, gains from free trade are stronger for more developed economies. Second, sectoral subsidies to local producers are more effective in stimulating their production and exports when introduced in developing countries. These findings seem to fit well with some recent claims suggesting that policy interventions may help developing countries in becoming stronger competitors in sectors where they previously enjoyed no comparative advantage.

\[\text{nons homothetic preferences by homothetic preferences.}\]
Appendices

A Omitted proofs

Formal Solution of the Consumer Optimisation Problem. By using the expression (5) for physical consumption and the price functions (2), the consumer optimisation problem can be re-stated as one where the consumer must choose the optimal quality $q_{z,v}$ and optimal budget allocation $\beta_{z,v}$ for each commodity $(z,v) \in Z \times V$. In particular, using the index $i \in V$ to denote country of origin of the consumer, the optimisation problem can be thus re-stated as follows:

$$\max_{\{q^i_{z,v}, \beta^i_{z,v}\} \in \mathbb{Z} \times \mathbb{V}} U = \left\{ \int_Z \left[ \int_V q^i_{z,v} \ln \left( \frac{1 + \kappa}{A} \frac{\beta^i_{z,v} w_i}{(q^i_{z,v})^{\eta_{z,v}} w_v} \right) \, dv \right]^\sigma \, dz \right\}^\frac{1}{\sigma} \tag{15}$$

subject to: $\int_Z \int_V \beta^i_{z,v} \, dv \, dz \leq 1$, and $q^i_{z,v} \geq 1$.

Denoting by $\nu^i$ the Lagrange multiplier associated to the budget constraint, and by $\delta^i_{z,v}$ the Lagrange multipliers associated to each constraint $q^i_{z,v} \geq 1$, we may derive the first-order conditions:

$$\ln \beta^i_{z,v} - \eta_{z,v} \ln q^i_{z,v} + \ln (1 + \kappa) - \ln A + \ln \left( \frac{w_i}{w_v} \right) - \eta_{z,v} + \delta^i_{z,v} = 0, \tag{16}$$

$$\frac{1}{\Omega \cdot \Lambda_z} \frac{q^i_{z,v}}{\beta^i_{z,v}} - \nu^i = 0, \tag{17}$$

$$q^i_{z,v} - 1 \geq 0, \quad \delta^i_{z,v} \geq 0, \quad \text{and} \quad (q^i_{z,v} - 1) \delta^i_{z,v} = 0, \tag{18}$$

$$1 - \int_Z \int_V \beta^i_{z,v} \, dv \, dz \geq 0, \quad \nu^i \geq 0, \quad \text{and} \quad (1 - \int_Z \int_V \beta^i_{z,v} \, dv \, dz) \nu^i = 0. \tag{19}$$

where:

$$\Omega \equiv \left\{ \int_Z \left[ \int_V q^i_{z,v} \ln \left( \frac{1 + \kappa}{A} \frac{\beta^i_{z,v} w_i}{(q^i_{z,v})^{\eta_{z,v}} w_v} \right) \, dv \right]^\sigma \, dz \right\}^\frac{1}{\sigma},$$

$$\Lambda_z \equiv \left[ \int_V q^i_{z,v} \ln \left( \frac{1 + \kappa}{A} \frac{\beta^i_{z,v} w_i}{(q^i_{z,v})^{\eta_{z,v}} w_v} \right) \, dv \right]^{1-\sigma}.$$

Note that, although $\Lambda_z$ in (17) are indexed by $z$, in the optimum all $\Lambda_z$ will turn out to be equal.\textsuperscript{28}

Hence, we may write that, in the optimum, $\Lambda_z = \Lambda$ for all $z$, and define:

$$\mu^i \equiv (\Omega \cdot \Lambda) \nu^i,$$

\textsuperscript{28} The result $\Lambda_z = \Lambda$ for all $z$ stems from the assumed iid draws of $\eta_{z,v}$ with a continuum of countries and goods. The combination of these assumptions implies that all goods $z$ will display (ex post) an identical distribution of $\eta_{z,v}$ over the space of countries $v$. Such ex post symmetry in the distribution of $\eta_{z,v}$ across goods, in turn, leads consumers to optimally set $\Lambda_z = \Lambda$ for all $z$. 
which in turn allows us to re-write (17) as \( q_{z,v}^i = \mu^i \beta_{z,v}^i \). Hence, integrating both sides of the equation over \( V \) and \( Z \), and noting that in the optimum the first expression in (19) always hold with equality, we may obtain:

\[
\int_Z \int_V q_{z,v}^i \, dv \, dz = \mu^i;
\]

which in turn implies that:

\[
\beta_{z,v}^i = \frac{q_{z,v}^i}{\mu^i}.
\]

Finally, note that (20) can be interpreted as the average quality of the optimal consumption basket. Denoting this by \( Q^i \equiv \mu^i \), from (21) it straightforwardly follows that \( q_{z,v}^i = q_{z,v}^i / Q^i \).

**Proof of Lemma 1.** We first show that when \( w_v = w \) for all \( v \), none of the constraints \( q_{z,v} \geq 1 \) of (15) binds in the optimum. For this, note that given the expressions in (16) and (21), whenever \( w_v = w \) for all \( v \), it must be the case that \( q_{z,v}^i \geq q_{z',v}^i \Leftrightarrow \eta_{z',v} \leq \eta_{z,v} \). Thus, if in the optimum \( q_{z,v}^i > 1 \) holds for a pair \((z', v')\) with \( \eta_{z',v} = \eta \), then \( q_{z,v}^i > 1 \) must be true for all pairs \((z, v)\).

Then, in order to prove that \( q_{z,v}^i > 1 \) holds for all \((z, v)\), it suffices to prove the following: even when all \( z_v = \eta \), except for one single good-variety \( (z_0^0, v_0^0) \) for which \( \eta_{z_0^0, v_0^0} = \eta \), the problem (15) yields \( q_{z_0^0, v_0^0}^i > 1 \). If this is the case, then \( q_{z_0^0, v_0^0}^i > 1 \) will actually hold true for any distribution of the productivity draws \( \eta_{z,v} \) with support in the interval \([\eta, \bar{\eta}]\), which includes the uniform distribution as one special case.

When all \( \eta_{z,v} = \eta \), except for a single \( (z_0^0, v_0^0) \) with \( \eta_{z_0^0, v_0^0} = \eta \), it follows that when \( q_{z_0^0, v_0^0}^i = 1 \):

\[
q_{z,v}^i = e^{-\frac{\lambda^i}{2\bar{\eta}}} \left( \frac{1 + \kappa}{A \bar{\eta}} \right)^{\frac{1}{q-1}}, \quad \text{for all } (z, v) \in Z \times V \text{ other than } (z_0^0, v_0^0).
\]

Integrating (22) across the space \( Z \) and \( V \), we obtain \( \mu^i = e^{-\frac{\eta}{2\bar{\eta}}} \left[ (1 + \kappa) / (A \bar{\eta}) \right]^{1/(q-1)} \), which in turn yields:

\[
\mu^i = \frac{1}{e} \left( \frac{1 + \kappa}{A} \right)^{\frac{1}{q-1}}.
\]

Now, plugging (23) into (16) and (21), computed for \( (z'', v'') \), while using the fact that \( \beta_{z,v}^i = 1/\mu^i \) when \( q_{z,v}^i = 1 \):

\[
\ln (1 + \kappa) - \ln A - [\ln (1 + \kappa) - \ln A] / \eta + \ln e - \eta + \delta_{z,v}^{z'',v''} = 0.
\]

Hence, considering the definition of \( A \equiv e^{-\frac{\eta}{2\bar{\eta}}(q-1)/(q-1)} \), (24) reduces to

\[
\ln (1 + \kappa) + \delta_{z,v}^{z'',v''} \frac{\eta}{q-1} = 0.
\]
However, \((25)\) cannot be true for any \(\kappa > 0\). As a consequence, it must be true that \(q_{z,v} > 1\) for all \(\kappa > 0\), implying in turn that \(q_{z,v} > 1\) must hold under any distribution of \(\eta_{z,v}\) with support within the interval \([\underline{\eta}, \overline{\eta}]\) when \(w_v = w\) for all \(v\). Now, taking into account the above result, we can use \((20), (21)\) and \((16)\), setting \(\delta^i_{z,v} = 0\) for all \((z, v) \in \mathbb{Z} \times \mathbb{V}\), to obtain \((6)\) and \((7)\).

**Proof of Lemma 2.** When \(w_v = w\) for all \(v \in \mathbb{V}\), since \(\delta^i_{z,v} = 0\) for all \((z, v) \in \mathbb{Z} \times \mathbb{V}\), using \((21)\) into \((16)\) leads to \(\ln (1 + \kappa) - \ln A - \ln \mu^i = \eta_{z,v} + (\eta_{z,v} - 1) \ln q^i_{z,v}\) for all \((z, v) \in \mathbb{Z} \times \mathbb{V}\). Defining now \(\Upsilon^i(\kappa) \equiv \ln (1 + \kappa) - \ln A - \ln \mu^i\), we can observe that:

\[
\frac{\partial \Upsilon^i}{\partial \kappa} = \frac{(\eta_{z,v} - 1) \partial q^i_{z,v}}{q^i_{z,v}}.
\]

But, given that \((\eta_{z,v} - 1) > 0\), then all \(\partial q^i_{z,v}/\partial \kappa\) must necessarily carry the same sign. Suppose then that \(\partial q^i_{z,v}/\partial \kappa \leq 0\), for all \((z, v) \in \mathbb{Z} \times \mathbb{V}\). Recalling \((20)\), it follows that \(\partial \mu^i/\partial \kappa \leq 0\) as well. But, since \(\partial \Upsilon^i/\partial \kappa = (1 + \kappa)^{-1} - (\mu^i)^{-1} \partial \mu^i/\partial \kappa\), the fact that \(\partial \mu^i/\partial \kappa \leq 0\) implies that \(\partial \Upsilon^i/\partial \kappa > 0\), which in turn contradicts the fact that \(\partial q^i_{z,v}/\partial \kappa \leq 0\) for all \((z, v) \in \mathbb{Z} \times \mathbb{V}\). As a result, it must be the case that \(\partial q^i_{z,v}/\partial \kappa > 0\) for all \((z, v) \in \mathbb{Z} \times \mathbb{V}\). Finally, the result \(\partial^2 q^i_{z,v}/(\partial \kappa \partial \eta_{z,v}) < 0\) follows immediately from the expression in \((26)\), after noting that \(\partial (\partial \Upsilon^i/\partial \kappa) / \partial \eta_{z,v} = 0\).

**Proof of Lemma 3.** Notice first that \(M^i_{z,v} \equiv \beta^i_{z,v}\) and \(M^i_z \equiv \int_{\mathbb{V}} \beta^i_{z,v} \, dv\). Also, when all countries in the world have the same wage (and, therefore, the same income), in the optimum \(\beta^i_{z,v} = \beta_{z,v}\) for all importers. Moreover, the symmetry in the distribution of draws \(\eta_{z,v}\), also implies that, in the optimum, \(M^i_z = 1\). Therefore, using \((9)\), \(IP^i_{z,v} = \beta_{z,v}\).

To compute the RCA, note that \(X_{z,v} \equiv \int_{\mathbb{V}} \beta^i_{z,v} \, di\), \(X_v \equiv \int_{\mathbb{Z}} \int_{\mathbb{V}} \beta^i_{z,v} \, di \, dz\), \(W_z \equiv \int_{\mathbb{V}} X_{z,v} \, dv\) and \(W \equiv \int_{\mathbb{Z}} W_z \, dz\). Then, using the fact that \(\beta^i_{z,v} = \beta_{z,v}\) for all importers, then \(X_{z,v} = \int_{\mathbb{V}} \beta_{z,v} \, di = \beta_{z,v}\). Moreover, the budget constraint in turn implies that \(X_v = \int_{\mathbb{Z}} \beta_{z,v} \, dz = 1\). Also, the symmetry in the distribution of draws \(\eta_{z,v}\) implies that the aggregate world spending in good \(z\) will be equal for all goods, thus \(W_z = \int_{\mathbb{V}} \beta_{z,v} \, dv = 1\). Plugging in all these results into \((10)\), and using the fact that \(W = 1\), the claimed \(RCA_{z,v} = \beta_{z,v}\) result follows.

**Proof of Proposition 1.** Preliminarily, notice that \((20)\) together with \((21)\) yields:

\[
\beta_{z',v'} = \frac{q_{z',v'}}{\int_{\mathbb{Z}} \int_{\mathbb{V}} q_{z,v} \, dv \, dz}.
\]

From \((16)\), together with Lemma 1 and Proposition 5, we have:

\[
(\eta_{z,v} - 1) \ln q_{z,v} + \eta_{z,v} = \ln (1 + \kappa) - \ln A - \ln \mu;
\]

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thus, computing (28) for any pair of commodities \((z', v')\) and \((z, v)\) yields:

\[
(\eta_{z', v'} - 1) \ln q_{z', v'} + \eta_{z', v'} = (\eta_{z, v} - 1) \ln q_{z, v} + \eta_{z, v}.
\]

(29)

Hence, (29) implies that \(q_{z', v'} > q_{z, v} \iff \eta_{z', v'} < \eta_{z, v}\). By considering this result in conjunction with (27), our claim immediately follows.

Furthermore, differentiating (29) with respect to \(\kappa\) yields:

\[
\frac{dq_{z', v'}}{d\kappa} = \eta_{z, v} - 1 \frac{q_{z', v'}}{q_{z, v} - 1} \frac{dq_{z, v}}{d\kappa}.
\]

(30)

Using (20), (28) and (30):

\[
\frac{dq_{z', v'}}{d\kappa} = \frac{A}{1 + \kappa} \left( \int_{Z} \int_{V} \frac{\eta_{z, v} - 1}{q_{z, v} - 1} q_{z, v} dv dz \right) > 0
\]

(31)

Moreover, from (27), and considering (30) and (31):

\[
\frac{d\beta_{z', v'}}{d\kappa} = 1 \frac{dq_{z', v'}}{d\kappa} \left( \int_{Z} \int_{V} \frac{\eta_{z, v} - \eta_{z', v'}}{\eta_{z, v} - 1} q_{z, v} dv dz \right)
\]

(32)

It is then easy to observe that (30) implies that \(dq_{z', v'}/d\kappa > dq_{z, v}/d\kappa\) when \(\eta_{z', v'} < \eta_{z, v}\). By considering this result in conjunction with (32) our claim immediately follows.

**Proof of Proposition 2.**

**Part (i).** From the FOC (16)-(19) we may obtain that for a consumer in any country in region \(L\) the following conditions must hold:

\[-(\overline{\eta} - 1) \ln q_{z, l}^L - \ln \mu^L + \ln (1 + \kappa) - \ln A - \overline{\eta} + \delta_{z, l}^L = 0, \text{ for all } (z, l) \in Z \times L;\]

(33)

\[-(\eta_{z, h} - 1) \ln q_{z, h}^L - \ln \mu^L + \ln (1 + \kappa) - \ln A - \ln w_h - \eta_{z, h} + \delta_{z, h}^L = 0, \text{ for all } (z, h) \in Z \times H.\]

(34)

Similarly, for a consumer in any country in region \(H\), it must be true that:

\[-(\overline{\eta} - 1) \ln q_{z, l}^H - \ln \mu^H + \ln (1 + \kappa) - \ln A - \overline{\eta} + \delta_{z, l}^H = 0, \text{ for all } (z, l) \in Z \times L;\]

(35)

\[-(\eta_{z, h} - 1) \ln q_{z, h}^H - \ln \mu^H + \ln (1 + \kappa) - \ln A - \eta_{z, h} + \delta_{z, h}^H = 0, \text{ for all } (z, h) \in Z \times H.\]

(36)

Suppose now there exists some \((z', v') \in Z \times V\) for which \(q_{z', v'}^L > q_{z', v'}^H\). Then, combining either the pair of equations (33) and (35), or the pair of equations (34) and (36), in both cases we would obtain that:

\[
\ln \left( \frac{\mu^H}{\mu^L w_h} \right) = (\eta_{z', v'} - 1) \ln \left( \frac{q_{z', v'}^L}{q_{z', v'}^H} \right) + \delta_{z', v'}^H > 0.
\]

(37)
Expression (37) implies, in turn, that $1 < \mu^L < w_h \mu^L < \mu^H$. From (20), it follows that there must exist some $(z'', v'') \in \mathbb{Z} \times \mathbb{V}$ for which $q_{z'',v''}^L < q_{z'',v''}^H$. Using the same line of reasoning, we now obtain:

$$\ln \left( \frac{\mu^L w_h}{\mu^H} \right) = \left( \eta_{z'',v''} - 1 \right) \ln \left( \frac{q_{z'',v''}^H}{q_{z'',v''}^L} \right) + \delta_{z'',v''}^L > 0,$$

which contradicts (37). As a consequence, it must be the case that $q_{z,v}^H \geq q_{z,v}^L$ for all $(z,v) \in \mathbb{Z} \times \mathbb{V}$.

Now, suppose $q_{z',v'}^H = q_{z',v'}^L > 1$ for some $(z', v') \in \mathbb{Z} \times \mathbb{V}$. Again, combining either the pair of equations (33) and (35), or the pair of equations (34) and (36), we obtain:

$$\ln \left( \frac{\mu^L w_h}{\mu^H} \right) = 0. \tag{38}$$

Expression (38) implies, in turn, that $1 < \mu^L < w_h \mu^L = \mu^H$. Hence, there must exist again some $(z'', v'') \in \mathbb{Z} \times \mathbb{V}$ for which $q_{z'',v''}^L < q_{z'',v''}^H$. Using the same line of reasoning, we now obtain:

$$\ln \left( \frac{\mu^L w_h}{\mu^H} \right) = \left( \eta_{z'',v''} - 1 \right) \ln \left( \frac{q_{z'',v''}^H}{q_{z'',v''}^L} \right) > 0,$$

which contradicts (38). Therefore, it must be true that $q_{z,v}^H > q_{z,v}^L$ for all $(z,v) \in \mathbb{Z} \times \mathbb{V}$, whenever $q_{z,v}^H > 1$.

**Part (ii).** The proof that $q_{z,l}^l = q_{z,l}^L$ for all $(z,l) \in \mathbb{Z} \times L$ follows straightforwardly from (33) and (35). For the second argument, let $i = L$, and consider the commodity $(z', h') \in \mathbb{Z} \times H$ such that $q_{z',h'}^L = q_{z',h'}^L > 1$. Using (33) and (34) we obtain, respectively:

$$- (\bar{\eta} - 1) \ln q_{L}^L - \ln \mu^L + \ln (1 + \kappa) - \ln A - \bar{\eta} = 0,$$

and:

$$- (\eta_{z',h'} - 1) \ln q_{L}^L - \ln \mu^L + \ln (1 + \kappa) - \ln A - \ln w_h - \eta_{z',h'} = 0.$$

This, in turn, leads to:

$$(\bar{\eta} - 1) \ln q_{L}^L + \bar{\eta} = (\eta_{z',h'} - 1) \ln q_{L}^L + \ln w_h + \eta_{z',h'} \tag{39}.$$
necessarily be that \( \hat{\eta} > \eta \). Thus, given the fact that \( \partial q_{z,h}^i / \partial \eta_{z,h} < 0 \) whenever \( q_{z,h}^i > 1 \), the result \( q_{z,\hat{\eta}}^L < q_{L}^L < q_{z,\eta}^L \) immediately follows. An analogous reasoning, letting \( i = H \), may be followed to prove that \( q_{H,\hat{\eta}}^L < q_{L}^H < q_{H,\eta}^L \).

**Part (iii).** The claim follows by differentiation of (34) and (36). This yields \( \partial q_{z,h}^i / \partial \eta_{z,h} = -q_{z,h}^i (1 + \ln q_{z,h}^i) / (\eta_{z,h} - 1) < 0 \) whenever \( q_{z,h}^i > 1 \), while \( \partial q_{z,h}^i / \partial \eta_{z,h} = 0 \) whenever \( q_{z,h}^i = 1 \). ■

**Proof of Proposition 4.** Part (i). Consider (11) and (12), note that total exports by sector \( z \) from country \( v \) are \( X_{z,v} = \lambda \beta_{z,v}^H w_h + (1 - \lambda) \beta_{z,v}^L \), hence aggregate exports by country \( v \) are \( X_v = \lambda w_h \int_Z \beta_{z,v}^H dz + (1 - \lambda) \int_Z \beta_{z,v}^L dz \). Now, notice that since \( \eta_{z,l} = \bar{\eta} \), we must have that \( \beta_{z,l}^H = \beta_{z,l}^H \) and \( \beta_{z,l}^L = \beta_{z,l}^L \), for all \((z,l) \in Z \times L\). Plugging these expressions into (10) then yields (11). Moreover, since all \( h \) obtain their draws of \( \eta_{z,h} \) from independent \( U[\eta, \bar{\eta}] \) distributions, and since all \( \beta_{z,h}^H \) are well-defined functions of \( \eta_{z,h} \), by the law of large numbers it follows that \( \int_Z \beta_{z,h}^H dz \) and \( \int_Z \beta_{z,h}^L dz \) must both yield an identical value for every country \( h \in H \). Using these expressions, in conjunction with those for \( X_{z,v} \) and \( X_v \), and denoting \( \beta_{z,h}^H = \int_Z \beta_{z,h}^H dz \) and \( \beta_{z,h}^L = \int_Z \beta_{z,h}^L dz \), into (10) then leads to (12). ■

**Proof of Proposition 3.** The proof follows from noting that: (a) both \( \beta_{z,h}^H \) and \( \beta_{z,h}^L \) in (12) are functions of \( \eta_{z,h} \); (b) Proposition 2 implies that \( \partial \beta_{z,h}^H / \partial \eta_{z,h} < 0 \) and \( \partial \beta_{z,h}^L / \partial \eta_{z,h} < 0 \); (c) \( \beta_{z,h}^H \) and \( \beta_{z,h}^L \) represent average demand intensities, hence \( \beta_{z,\hat{\eta}}^H < \beta_{z,\eta}^H \) and \( \beta_{z,\hat{\eta}}^L < \beta_{z,\eta}^L \); and (d) from (11), it follows that \( RCA_{z,h} = RCA_{z,l} \) only if \( \beta_{z,h}^H = \beta_{z,l}^H \) and \( \beta_{z,h}^L = \beta_{z,l}^L \). ■

**Proof of Proposition 4.** Part (i). Our claim immediately follows from part (iii) of Proposition 2 in conjunction with (21), since the distribution of demand intensities across goods and varieties mirrors that of the levels of the optimally selected qualities.

**Part (ii).** Using (35) and (36), together with (21), for a consumer from \( H \) we get:

\[
\ln (1 + \kappa) - \ln A = (\eta_{z,h} - 1) \ln \beta_{z,h}^H + \eta_{z,h} \ln \mu^H + \eta_{z,h}, \quad \text{for all } (z, h) \in Z \times H.
\]

\[
= (\bar{\eta} - 1) \ln \beta_{z,h}^H + \bar{\eta} \ln \mu^H - \ln w_h + \bar{\eta}, \quad \text{for all } (z, l) \in Z \times L.
\]

Similarly, considering (33) and (34) together with (21), in the case of a consumer from \( L \) we obtain:

\[
\ln (1 + \kappa) - \ln A = (\eta_{z,h} - 1) \ln \beta_{z,h}^L + \eta_{z,h} \ln \mu^L + \ln w_h + \eta_{z,h} - \delta_{z,h}, \quad \text{for all } (z, h) \in Z \times H.
\]

\[
= (\bar{\eta} - 1) \ln \beta_{z,l}^L + \bar{\eta} \ln \mu^L + \bar{\eta} - \delta_{L z,h}, \quad \text{for all } (z, l) \in Z \times L.
\]

On the one hand, equating the first expression of the each case, simplifying and rearranging:

\[
\ln \beta_{z,h}^H - \ln \beta_{z,h}^L = \frac{\eta_{z,h} (\ln \mu^L - \ln \mu^H) + \ln w_h - \delta_{z,h}}{(\eta_{z,h} - 1)} = k_{z,h}.
\]
Getting rid of the logs, we then obtain \( \beta_{z,h}^H = e^{k_{z,h}} \beta_{z,h}^L \), and hence:

\[
\beta_{z,h}^H - \beta_{z,h}^L = (e^{k_{z,h}} - 1) \beta_{z,h}^L.
\]

Consider now two producers \( h', h'' \in H \) such that \( \eta_{z,h''} < \eta_{z,h'} \). Since \( \beta_{z,h'}^L - \beta_{z,h''}^L < \beta_{z,h'}^H - \beta_{z,h''}^H \), and from part (i) of this proof it follows that \( \beta_{z,h''} < \beta_{z,h'}^L \), we are left to prove that \( k_{z,h''} \leq k_{z,h'} \). Suppose \( k_{z,h''} > k_{z,h'} \). Since by assumption \( \eta_{z,h''} < \eta_{z,h'} \), and part (ii) of Proposition 2 implies \( \delta_{z,h''}^L \leq \delta_{z,h'}^L \), a necessary condition for this to hold is \( (\eta_{z,h''} - \eta_{z,h'}) \ln \mu^L - \ln \mu^H \geq 0 \). But this is impossible, since \( \ln \mu^H < \ln \mu^L \). So it must be that \( k_{z,h''} \leq k_{z,h'} \), and from part (ii) of this proof it follows that \( \delta_{z,h''}^L < \delta_{z,h'}^L \), we are left to prove that \( \beta_{z,h''}^L - \beta_{z,h'}^L \leq \beta_{z,h''}^H - \beta_{z,h'}^H \).

On the other hand, equating the second expression of each case, simplifying and rearranging:

\[
\ln \beta_{z,l}^H - \ln \beta_{z,l}^L = \frac{\eta (\ln \mu^L - \ln \mu^H) + \ln w_h - \delta_{z,l}^L}{\eta - 1} \equiv k_{z,l}.
\]

We can thus write \( \beta_{z,l}^H - \beta_{z,l}^L = (e^{k_{z,l}} - 1) \beta_{z,l}^L \) and, following an analogous reasoning, it is straightforward to obtain \( \beta_{z,l}^H < \beta_{z,l}^L \) (and \( \beta_{z,l}^L < \beta_{z,l}^H \)).

\[\Box\]

### B Cross-country inequality in a multi-region world

We now consider a setup where the world is composed by \( K > 2 \) regions, indexed by \( k = 1, \ldots, K \). We let \( \mathcal{V}_k \) denote the subset of countries from region \( k \), where \( \mathcal{V}_k \) has Lebesgue measure \( \lambda_k > 0 \). In addition, we let each country in region \( k \) be denoted by a particular \( v_k \). (All the results discussed in this section are formalised in the Online Appendix, Section B.)

We assume that for any \( v_k \) and every \( z \), each \( \eta_{z,v_k} \) is independently drawn from a uniform distribution with support over \([\eta_k, \bar{\eta}]\), where \( \eta_k < \bar{\eta} \). To keep the consistency with the previous sections, let \( \eta_k = \bar{\eta} \) when \( k = 1 \). In addition, let \( \eta_{k'} < \eta_{k''} \) for any two regions \( k' < k'' \). In other words, we are indexing regions \( k = 1, \ldots, K \) in terms of first-order stochastic dominance of their respective uniform distributions. All uniform distributions are assumed to share the same upper-bound \( \bar{\eta} \), while they differ in their lower-bounds \( \eta_k \).

In this extended setup, equilibrium wages display an analogous structure as the one described in Proposition 6. Namely, in equilibrium, the wage in each \( v_k \) is \( w_k \). In addition, equilibrium wages are such that \( w_1 = \cdots > w_{k'} = \cdots > w_K \), where \( 1 < k' < K \).

Notice that, since all individuals from the same region earn the same wages, they choose identical consumption profiles. We then let \( \beta_{z,v_k}^{ij} \) denote the demand intensity by a consumer from region \( \mathcal{V}_j \) for good \((z, v_k)\). Once again, this immediately implies that \( IP_{z,v}^{j} = \beta_{z,v}^{ij} \). Furthermore,
it follows that, for a country $v_k$:

$$X_{z,v_k} = \sum_{j=1}^{K} \lambda_j w_j \beta_{z,v_k}^j.$$ 

In equilibrium, it must be the case that $X_{v_k} = w_k$ for all $v_k \in \mathcal{V}_k$. In addition, $W_z$ equal for all $z$ is still true in this extended setup. As a result, the RCA of country $v_k$ in good $z$ is given by:

$$RCA_{z,v_k} = \frac{\sum_{j=1}^{K} \lambda_j w_j \beta_{z,v_k}^j}{w_k}.$$ (40)

Since wages differ across regions, once again, we cannot find a monotonic relationship between $RCA_{z,v_k}$ in (40) and the productivity draws $\eta_{z,v_k}$ when all countries in the world are pooled together. However, we can still find a result analogous to Proposition 3. In particular, it is still true that the highest value of $RCA_{z,v_k}$ corresponds to the country in region $\mathcal{V}_1$ receiving the best possible draw in sector $z$. That is, $RCA_{z,v_k}$ is the highest for some country $v_1$ with $\eta_{z,v_1} = \eta$.

Lastly, concerning import penetration, this extension also yields a result that is analogous to that in Proposition 4. Following the notation in Proposition 4, we can show that $\beta_{z,\eta}^1 > \ldots > \beta_{z,\eta}^{k'} > \ldots > \beta_{z,\eta}^K$, where $1 < k' < K$. Again, this result stems from our nonhomothetic structure along the quality dimension, which implies that richer consumers allocate a larger share of their spending in good $z$ to the producers who can most efficiently offer higher qualities versions of $z$. 

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References


Online Appendix

A Additional theoretical results

A.1 Existence and uniqueness of equilibrium

Proposition 5 Suppose that, for each commodity \((z, v) \in \mathbb{Z} \times \mathbb{V}\), \(\eta_{z,v}\) is independently drawn from a uniform density function with support \([\underline{\eta}, \overline{\eta}]\). Then, for any \(\kappa > 0\), in equilibrium: \(w_v = w\) for all \(v \in \mathbb{V}\).

Proof. Existence of equilibrium: As a first step, we prove that \(w_v = w\) for all \(v \in \mathbb{V}\) is an equilibrium of the model. Firstly, notice that when \(w_i = w\) for all \(i \in \mathbb{V}\), the Lagrange multipliers will be identical for all countries, and in particular we may write \(\mu^i = \mu\) for all \(i \in \mathbb{V}\). Secondly, using Lemma 1, when \(w_v = w\) for all \(v \in \mathbb{V}\), conditions in (16) together with (21) and \(\mu^i = \mu\) for all \(i \in \mathbb{V}\), lead to:

\[
q^i_{z,v} = q_{z,v} = \left( \frac{1 + \kappa}{A e^{\eta_{z,v} \mu}} \right)^{1/(\eta_{z,v} - 1)},
\]

\[
\beta^i_{z,v} = \beta_{z,v} = \left( \frac{1 + \kappa}{A (e \mu)^{\eta_{z,v}}} \right)^{1/(\eta_{z,v} - 1)}.
\]

Now, recall that each \(\eta_{z,v}\) is drawn from an independent uniform probability distribution with support \([\underline{\eta}, \overline{\eta}]\). Hence, by the law of large numbers, for each country \(v \in \mathbb{V}\), the (infinite) sequence of draws \(\{\eta_{z,v}\}_{z \in \mathbb{Z}}\) will also be uniformly distributed over \([\underline{\eta}, \overline{\eta}]\) along the goods space. This implies that, integrating over \(\mathbb{Z}\) and bearing in mind (42), \(\int_{\mathbb{Z}} \beta^i_{z,v} dz = \int_{\mathbb{Z}} \beta_{z,v} dz = \beta_v = \beta > 0\), for each good \(v \in \mathbb{V}\). Next, replacing \(\int_{\mathbb{Z}} \beta^i_{z,v} dz = \beta\) into (19), and swapping the order of integration, we obtain \(\int_{\mathbb{V}} \beta dv = 1\), which in turn implies that \(\beta = 1\) since \(\mathbb{V}\) has unit mass. Then, it is easy to check that all conditions (8) hold simultaneously when \(w_v = w\) for all \(v \in \mathbb{V}\).

Equilibrium uniqueness: We now proceed to prove the above equilibrium is unique. Normalise \(w = 1\), and suppose for a subset \(\mathcal{J} \subset \mathbb{V}\) of countries with measure \(\lambda_j > 0\) we have \(w_j > 1\), while for a (disjoint) subset \(\mathcal{K} \subset \mathbb{V}\) of countries with measure \(\lambda_k > 0\) we have \(w_k < 1\). Denote finally by \(\mathcal{I} \subset \mathbb{V}\) the (complementary) subset of countries with \(w_i = 1\). Consider some \(k \in \mathcal{K}, i \in \mathcal{I}\), and \(j \in \mathcal{J}\), and take \((z_k, k), (z_i, i), (z_j, j)\) such that: \(\eta_{z_k, k} = \eta_{z_i, i} = \eta_{z_j, j} = \eta\). Notice that, due to the law of large numbers, for any \(\eta \in [\underline{\eta}, \overline{\eta}]\) the measure of good-variety couples for which the last condition is satisfied is the same in \(k, i\) and \(j\).

As a first step, take country \(i \in \mathcal{I}\). (16) and (17) imply that, for \((z_k, k), (z_i, i)\) and \((z_j, j)\), we must
have, respectively:

\[ \ln (1 + \kappa) - \ln A = \eta \ln (\mu^i) + \ln (w_k) + (\eta - 1) \ln (\beta_{z_{ij},k}^i) + \eta - \delta_{z_{ij},k} \]
\[ = \eta \ln (\mu^i) + (\eta - 1) \ln (\beta_{z_{ij},i}^i) + \eta - \delta_{z_{ij},i} \]
\[ = \eta \ln (\mu^i) + \ln (w_j) + (\eta - 1) \ln (\beta_{z_{ij},j}^i) + \eta - \delta_{z_{ij},j}. \]

Notice also from (18) and (21) that if \( \delta^i_{z,v} > 0 \), then \( \ln \beta^i_{z,v} = -\ln \mu^i \), whereas if \( \delta^i_{z,v} = 0 \), then \( \ln \beta^i_{z,v} = -\ln \mu^i \). Then, \( \beta^i_{z_{ij},k} \geq \beta^i_{z_{ij},i} \geq \beta^i_{z_{ij},j} \).

As a second step, take country \( k \in K \). (16) and (17) imply that, for \((z_k, k), (z_i, i)\) and \((z_j, j)\), we must have, respectively:

\[ \ln (1 + \kappa) - \ln A = \eta \ln(\mu^k) + (\eta - 1) \ln (\beta_{z_{ij},k}^k) + \eta - \delta_{z_{ij},k} \]
\[ = \eta \ln(\mu^k) + \ln \left( \frac{1}{w_k} \right) + (\eta - 1) \ln (\beta_{z_{ij},i}^k) + \eta - \delta_{z_{ij},i} \]
\[ = \eta \ln(\mu^k) + \ln \left( \frac{w_j}{w_k} \right) + (\eta - 1) \ln (\beta_{z_{ij},j}^k) + \eta - \delta_{z_{ij},j}. \]

Following an analogous reasoning as before, it follows that \( \beta^k_{z_{ij},k} \geq \beta^k_{z_{ij},i} \geq \beta^k_{z_{ij},j} \).

As a third step, take country \( j \in J \), and notice \( w_j > 1 \). (16) and (17) imply that, for \((z_k, k), (z_i, i)\) and \((z_j, j)\), we must have, respectively:

\[ \ln (1 + \kappa) - \ln A = \eta \ln (\mu^j) + \ln \left( \frac{w_k}{w_j} \right) + (\eta - 1) \ln (\beta_{z_{ij},k}^j) + \eta - \delta_{z_{ij},k} \]
\[ = \eta \ln (\mu^j) + \ln \left( \frac{1}{w_j} \right) + (\eta - 1) \ln (\beta_{z_{ij},i}^j) + \eta - \delta_{z_{ij},i} \]
\[ = \eta \ln (\mu^j) + (\eta - 1) \ln (\beta_{z_{ij},j}^j) + \eta - \delta_{z_{ij},j}. \]

Again, an analogous reasoning as in the previous cases leads to \( \beta^j_{z_{ij},k} \geq \beta^j_{z_{ij},i} \geq \beta^j_{z_{ij},j} \).

Finally, integrate among the good space \( Z \) and country space \( V \). The above results lead to:

\[ \lambda^i w_j \int_Z \beta^i_{z_{ij},k} \, dz + \lambda^k w_k \int_Z \beta^k_{z_{ij},k} \, dz + (1 - \lambda^j - \lambda^k) \int_Z \beta^i_{z_{ij},k} \, dz \geq \]
\[ \lambda^j w_j \int_Z \beta^j_{z_{ij},i} + \lambda^k w_k \int_Z \beta^k_{z_{ij},i} + (1 - \lambda^i - \lambda^k) \int_Z \beta^i_{z_{ij},i} \, dz \geq \]
\[ \lambda^i w_j \int_Z \beta^i_{z_{ij},j} + \lambda^k w_k \int_Z \beta^k_{z_{ij},j} + (1 - \lambda^i - \lambda^k) \int_Z \beta^i_{z_{ij},j} \, dz. \] (43)

Note that the first line in (43) equals the world spending on commodities produced in \( k \), the second equals the world spending on commodities produced in \( i \), and the third equals the world spending on commodities produced in \( j \). However, when \( w_k < 1 < w_j \), those inequalities are inconsistent with market clearing conditions (8). As a result, there cannot exist an equilibrium with measure \( \lambda_j > 0 \) of countries with \( w_j > 1 \) and/or a measure \( \lambda_k > 0 \) of countries with \( w_k < 1 \).
Proposition 6 Suppose that the set \( \mathcal{V} \) is composed by two disjoint subsets with positive measure: \( H \) and \( L \). Assume that: a) for any \((z, h) \in \mathbb{Z} \times H\), \( \eta_{z, h} \) is independently drawn uniform density function with support \([\eta, \overline{\eta}]\); b) for any \((z, l) \in \mathbb{Z} \times L\), \( \eta_{z, l} = \overline{\eta} \). Then, for any \(h, h', h'' \in H \) and \(l, l', l'' \in L\): (i) \( w_{h'} = w_{l''} \); (ii) \( w_{v} = w_{l''} \); (iii) \( w_{h} > w_{l} \).

Proof. We prove the proposition in different steps. We first prove that, if an equilibrium exists, then it must necessarily be the case that, for any \(h, h', h'' \in H \) and \(l, l', l'' \in L\): 1) \( w_{h} \neq w_{l} \); 2) \( w_{h'} = w_{l''} \) and \( w_{v} = w_{l''} \); 3) \( w_{h}/w_{l} > 1 \); 4) \( w_{h}/w_{l} < \infty \). Lastly, we prove that a unique equilibrium exists, with: 5) \( 1 < w_{h}/w_{l} < \infty \).

Preliminarily, consider a generic country \( i \in \mathcal{V} \), and compute the aggregate demand by \( i \) for goods produced in country \( v \in \mathcal{V} \). From the first-order conditions, it follows that:

\[
\beta^i_{z, v} = \max \left\{ \left[ \frac{(1 + \kappa) (w_i/w_v)}{A (e\mu^i)^\eta w_h} \right]^\frac{1}{\eta w_h - 1}, \frac{1}{\mu^i} \right\}. \tag{44}
\]

Hence, total demand by \( i \) for goods produced in \( h \in H \) and in \( l \in L \) are respectively given by:

\[
\int_{\mathbb{Z}} \beta^i_{z, h} w_i \, dz = w_i \int_{\eta}^{\overline{\eta}} \max \left\{ \left( \frac{1 + \kappa}{A (e\mu^i)^\eta} w_h \right)^{1/(\eta-1)}, \frac{1}{\mu^i} \right\} \frac{1}{\eta - \eta} \, d\eta, \quad \text{for any } h \in H, \tag{45}
\]

and

\[
\int_{\mathbb{Z}} \beta^i_{z, l} w_i \, dz = w_i \max \left\{ \left( \frac{1 + \kappa}{A (e\mu^i)^\eta} w_l \right)^{1/(\eta-1)}, \frac{1}{\mu^i} \right\}, \quad \text{for any } l \in L. \tag{46}
\]

Step 1. Suppose now that, in equilibrium, \( w_i = w \) for all \( i \in \mathcal{V} \). Recalling the proof of Lemma 1, we can observe that the constraints \( q^i_{z, v} \geq 1 \) will not bind in this case. Demand intensities in (44) are then given by \( \beta^i_{z, v} = \beta_{z, v} = (e \cdot \mu)^{-\eta_{z, v}/(\eta_{z, v} - 1)} [(1 + \kappa)/A]^{1/(\eta_{z, v} - 1)} \) for all \( i \in \mathcal{V} \).

As a result, the value in (45) must be strictly larger than the value in (46), since the term \([(1 + \kappa)/A]^{1/(\eta-1)}/\mu^{\eta/(\eta-1)}\) is strictly decreasing in \( \eta \). As a consequence, given that \( i \) represents a generic country in \( \mathcal{V} \), integrating over the set \( \mathcal{V} \), it follows that the world demand for goods produced in a country from \( H \) will be strictly larger than the world demand for goods produced in a country from \( L \). But this is inconsistent with the market clearing conditions, which require that world demand is equal for all \( v \in \mathcal{V} \). Hence, \( w_i = w \) for all \( v \in \mathcal{V} \) cannot hold in equilibrium.

Step 2. Suppose that, in equilibrium, \( w_{h'} > w_{h''} \) for some \( h', h'' \in H \). Computing (45) respectively for \( h' \) and \( h'' \) yields:

\[
w_i \int_{\eta}^{\overline{\eta}} \max \left\{ \left( \frac{1 + \kappa}{A (e\mu^i)^\eta} w_{h'} \right)^{1/(\eta-1)}, \frac{1}{\mu^i} \right\} \frac{1}{\eta - \eta} \, d\eta \leq w_i \int_{\eta}^{\overline{\eta}} \max \left\{ \left( \frac{1 + \kappa}{A (e\mu^i)^\eta} w_{h''} \right)^{1/(\eta-1)}, \frac{1}{\mu^i} \right\} \frac{1}{\eta - \eta} \, d\eta.
\]
Now, since $i$ represents a generic country in $\mathbb{V}$, integrating over the set $\mathbb{V}$, it follows that the world demand for goods produced in country $h'$ will be no larger than the world demand for goods produced in country $h''$. But this is inconsistent with the market clearing conditions, which require that world demand for goods produced in country $h'$ must be strictly larger than world demand for goods produced in country $h''$. Furthermore, an analogous reasoning rules out $w_{h'} < w_{h''}$. As a consequence, it must be the case that, if an equilibrium exists, it must be characterised by $w_{h'} = w_{h''}$ for any $h', h'' \in H$. (Similarly, it can be proved that, if an equilibrium exists, it must be characterised by $w_{l'} = w_{l''}$ for any $l', l'' \in L$.)

**Step 3.** Suppose that $w_h < w_l$. Since $\{(1 + \kappa) / A \} (w_i / w_v) / (\mu^i) \frac{1}{\eta-1}$ is strictly decreasing in $\eta$, it follows that the value in (46) is no larger than the value in (45). Moreover, since $i$ represents a generic country in $\mathbb{V}$, integrating over the set $\mathbb{V}$, we obtain that the world demand for goods produced in a country from region $L$ is no larger than world demand for goods produced in a country from region $H$. But this is inconsistent with the market clearing conditions when $w_h < w_l$, which require that world demand for goods produced in a country from region $L$ must be strictly larger than world demand for goods produced in a country from region $H$.

**Step 4.** As a result of steps 1, 2 and 3, our only remaining candidate for an equilibrium is then $w_h > w_l$. From (45), it follows that the aggregate demand by any $h' \in H$ for goods produced in region $H$ coincides with its aggregate supply to the same region. Hence, there must be no net surplus within region $H$. Analogously, from (46) it follows that there must be no net surplus within region $L$. As a result, a necessary condition for market clearing is that the aggregate demand by region $L$ for goods produced in region $H$ must equal the aggregate demand by region $H$ for goods produced in region $L$. Formally:

$$\int_L \int_H \int_Z \beta_{z,h}^{l'} w_{l'} dz dh dl' = \int_H \int_L \int_Z \beta_{z,l}^{h'} w_{h'} dz dl dh' \quad (47)$$

Suppose now that $w_h \to \infty$. Then, on the one hand, from (45) we obtain the aggregate demand by $l' \in L$ for goods produced in region $H$ would be equal to a finite (non-negative) number. Since this would hold true for every $l' \in L$, then the aggregate demand by region $L$ for goods produced in region $H$—left-hand side of (47)—would be equal to a finite (non-negative) number. On the other hand, from (45) it follows that when $w_h \to \infty$ the aggregate demand by $h' \in H$ for goods produced in any $l \in L$ would tend to infinity. Since this would hold true for every $h' \in H$ and $l \in L$, then the aggregate demand by region $H$ for goods produced in region $L$—right-hand side of (47)—would also tend to infinity. But this then is inconsistent with the equality required by condition (47). Hence, if an equilibrium exists, it must be then characterised by $w_l < w_h < \infty$. 

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Step 5. Finally, we prove now that there exists an equilibrium $1 < w_H/w_L < \infty$, and this equilibrium is unique. Recall that $w_H/w_L$ represents the relative wage between region $H$ and region $L$. Step 1 shows that, should the relative wage equal one, then the world demand for goods produced in a country from $H$ would be strictly larger than the world demand for goods produced in a country from $L$. Step 4 shows instead that, should $w_H \to \infty$, then the world demand for goods produced in a country from $H$ would be strictly smaller than the world demand for goods produced in a country from $L$. Consider now (44) for any $v = h$, and notice that the demand intensities $\beta^i_{z,h}$ are all non-increasing in $w_H/w_L$. In addition, consider (44) for any $v = l$, and notice that in this case the $\beta^i_{z,l}$ are all non-decreasing in $w_H/w_L$, while they are strictly increasing in $w_H/w_L$ for at least some $z \in \mathbb{Z}$ when $i \in H$. Therefore, taking all this into account, together with the expressions in (45) and (46), it follows that the world demand for goods produced in a country from $L$ may increase with $w_H/w_L$, while world demand for goods produced in a country from $H$ will decrease with $w_H/w_L$. Hence, by continuity, there must necessarily exist some $1 < w_H/w_L < \infty$ consistent with all market clearing conditions holding simultaneously. In addition, this equilibrium must then also be unique.

A.2 Formalisation of results discussed in Appendix B

Proposition 7 Suppose that the set $\mathcal{V}$ is composed by $K$ disjoint subsets, indexed by $k = 1, \ldots, K$, each denoted by $\mathcal{V}_k \subset \mathcal{V}$ and with Lebesgue measure $\lambda_k > 0$. Assume that for any country $v_k \in \mathcal{V}_k$ each $\eta_{z,v_k}$ is independently drawn from a uniform distribution with support $[\eta_k, \bar{\eta}]$, with $\eta_k < \eta_{k''}$ for $k' < k''$. Then: $w_1 > \ldots > w_{k'} > \ldots > w_K$, where $1 < k' < K$.

Proof. Combining (16) and (17), yields:

$$
\beta^i_{z,v} = \max \left\{ \left[ \left( \frac{1 + \kappa}{A} \right) \frac{w_i}{w_v} \right] \left( e \cdot \mu^i \right)^{-\eta_{z,v}} \right\}^{1/(\eta_{z,v} - 1)} \left( \frac{1}{\mu^i} \right) \equiv \beta^i (\eta_{z,v}, w_v). \quad (48)
$$

Notice from (48) that $\partial \beta^i (\eta_{z,v}, w_v) / \partial \eta_{z,v} \leq 0$ and $\partial \beta^i (\eta_{z,v}, w_v) / \partial w_v \leq 0$.

Consider now two generic regions $k' < k''$, and suppose that $w_{k'} \leq w_{k''}$. Since the distribution of $\eta_{z,k'}$ FOSD the distribution of $\eta_{z,k''}$, then it follows that $\int_{\mathbb{Z}} \beta^i_{z,k'} dz \geq \int_{\mathbb{Z}} \beta^i_{z,k''} dz$. Moreover, recalling the proof of Lemma 1 it follows that the $\beta^i_{z,v}$ in (48) must be strictly decreasing in $\eta_{z,v}$ and in $w_v$ at least in one of all the regions in the world.\footnote{More precisely, it must be that the $\beta^i_{z,v}$ in (48) are strictly decreasing in $\eta_{z,v}$ and $w_v$ at least in region $k''$, such that $w_{k''} \in \max\{w_1, \ldots, w_K\}$. That is, the region (or regions) exhibiting with the highest wage.}

As a result, there will exist a positive measure of countries for which $\int_{\mathbb{Z}} \beta^i_{z,k'} dz > \int_{\mathbb{Z}} \beta^i_{z,k''} dz$ when $w_{k'} \leq w_{k''}$. Therefore, integrating
over the set $\mathcal{V}$, we obtain that $\int_{\mathcal{V}} \int_{\mathcal{Z}} \beta_{z,k}^i dz > \int_{\mathcal{V}} \int_{\mathcal{Z}} \beta_{z,k'}^i dz$. That is, the world demand for goods produced in a country from region $k'$ is strictly larger than world demand for goods produced in a country from region $k''$. But this is inconsistent with the market clearing conditions when $w_{k'} \leq w_{k''}$, which require that world demand for goods produced in a country from region $k'$ must be no larger than world demand for goods produced in a country from region $k''$. As a consequence, it must be that $w_{k'} > w_{k''}$.

**Proposition 8** For country $v_1 \in \mathcal{V}_1$ such that $\eta_{z,v_1} = \eta$ and any country $v_k \in \mathcal{V}_k$ such that $\eta_{z,v_k} = \eta_k$ and $k \neq 1$: $RCA_{z,v_1} > RCA_{z,v_k}$, for any $z \in \mathcal{Z}$.

**Proof.** Countries with identical incomes have identical budget shares. Let $\beta_{z,v}^j$ denote the common budget share for $(z, v)$ in $j$. Then, from the definition of total production of good $z$ by country $v$, we have that $X_{z,v} = \sum_{j=1}^{K} \lambda_j \beta_j^z (\eta_{z,v}, w_v) w_j$. Notice also that $X_v = w_v$ and $W_z/W = 1$. Hence, (10) yields:

$$RCA_{z,v} = \frac{\sum_{j=1}^{K} \lambda_j \beta_j^z (\eta_{z,v}, w_v) w_j}{w_v}. \quad (49)$$

Consider a generic good $z \in \mathcal{Z}$ and, without loss of generality, select countries: $v_1 \in \mathcal{V}_1$ such that $\eta_{z,v_1} = \eta$; and $v_k \in \mathcal{V}_k$ from any region $k \in (1, K]$ such that $\eta_{z,v_k} = \eta_k$. From (49) we obtain that $RCA_{z,v_1} > RCA_{z,v_k}$ requires:

$$\frac{\sum_{j=1}^{K} \lambda_j \beta_j^z (\eta, w_1) w_j}{w_1} > \frac{\sum_{j=1}^{K} \lambda_j \beta_j^z (\eta_k, w_k) w_j}{w_k}. \quad (50)$$

Notice too that market clearing conditions imply:

$$\int_{\mathcal{Z}} \left[ \sum_{j=1}^{K} \lambda_j \beta_j^z (\eta_{z,1}, w_1) w_j \right] dz = w_1 \quad \text{and} \quad \int_{\mathcal{Z}} \left[ \sum_{j=1}^{K} \lambda_j \beta_j^z (\eta_{z,k}, w_k) w_j \right] dz = w_k.$$

Therefore, it follows that $\int_{\mathcal{Z}} RCA_{z,v_1} dz = \int_{\mathcal{Z}} RCA_{z,v_k} dz = 1$. We can transform the integrals over $z$ in integrals over $\eta$, to obtain:

$$\frac{1}{\eta - \eta_1} \int_\eta^{\eta_1} [RCA_{\eta,v_1}] d\eta = 1, \quad (51)$$

$$\frac{1}{\eta - \eta_k} \int_\eta^{\eta_k} [RCA_{\eta,v_k}] d\eta = 1 \quad (52)$$

Recall that $\partial \beta_j^z (\cdot) / \partial \eta < 0$, implying that $\partial (RCA_{\eta,v}) / \partial \eta < 0$. Moreover, since $w_k < w_1$, notice that it must be the case that $RCA_{\eta,v_k} > RCA_{\eta,v_1}$ for any $\eta \in [\eta_k, \eta]$. Now, suppose that
Proposition 9 Let $\beta^{j}_{z, v}$ denote the demand intensity by a consumer from region $j \in V_j$ for the variety of good $z$ produced in country $v_1$ such that $\eta_{z, v_1} = \eta$. Then: $\beta^{1}_{z, \eta} > ... > \beta^{j}_{z, \eta} > ... > \beta^{K}_{z, \eta}$, where $1 < j' < K$.

Proof. Consider a pair of generic consumers from regions $j'$ and $j''$, where $j' < j''$. In addition, consider a pair of generic exporters from countries $v_{k'}$ and $v_{k''}$, where $k' \leq k''$. Following an analogous procedure as in the proof of Proposition 4, combining (16) and (17) of consumers $j'$ and $j''$ for the varieties of good $z$ produced in $v_{k'}$ and $v_{k''}$, we may obtain:

$$\left( \eta_{z, v_{k'}} - \eta_{z, v_{k''}} \right) \ln \left( \frac{\mu^{j'}}{\mu^{j''}} \right) + \left( \delta^{j''}_{z, v_{k'}} - \delta^{j'}_{z, v_{k'}} \right) + \left( \delta^{j''}_{z, v_{k''}} - \delta^{j'}_{z, v_{k''}} \right) = \left( \eta_{z, v_{k'}} - 1 \right) \ln \left( \frac{\beta^{j'}_{z, v_{k'}}}{\beta^{j''}_{z, v_{k'}}} \right) + \left( \eta_{z, v_{k''}} - 1 \right) \ln \left( \frac{\beta^{j'}_{z, v_{k''}}}{\beta^{j''}_{z, v_{k''}}} \right).$$

(53)

Since $\ln \left( \frac{\mu^{j'}}{\mu^{j''}} \right) > 0$ and $\delta^{j''}_{z, v_{k'}} \geq \delta^{j'}_{z, v_{k'}}$, from (53) it follows that $\frac{\beta^{j'}_{z, v_{k'}}}{\beta^{j''}_{z, v_{k'}}} > \frac{\beta^{j'}_{z, v_{k''}}}{\beta^{j''}_{z, v_{k''}}}$ when $\eta_{z, v_{k'}} < \eta_{z, v_{k''}}$. Now, let $k' = 1$ and pick $z$ such that $\eta_{z, v_1} = \eta$. Next, suppose $\beta^{j'}_{z, \eta} \leq \beta^{j''}_{z, \eta}$. Then, we must have that $\beta^{j'}_{z, v} \leq \beta^{j''}_{z, v}$ for all $(z, v) \in \mathbb{Z} \times \mathbb{V}$, with strict inequality for all $(z, v)$ such that $\eta_{z, v} > \eta$. However, since the budget constraints of consumer $j'$ and $j''$ require that $\int_{\mathbb{Z}} \int_{\mathbb{V}} \beta^{j'}_{z, v} dv dz = \int_{\mathbb{Z}} \int_{\mathbb{V}} \beta^{j''}_{z, v} dv dz$, then $\beta^{j'}_{z, \eta} \leq \beta^{j''}_{z, \eta}$ cannot possibly be true. ■

A.3 Trade frictions and consumer loss

Consider country $i$ as an importer of good $z$. We assume this commodity is subject to a tariff $t_z > 0$ applied on the (free-on-board) price of imports, regardless of the quality level in which it is imported by $i$. Since the tariff is applied only to one (atomless) sector within a continuum of sectors, we can disregard general equilibrium effects as they would be negligible. Given the tariff $t_z$, the final price at which good $z$ sourced from a generic country $v \neq i$ will be sold to consumers in country $i$ will be:

$$p^i_{z, v} (q) = (1 + t_z) A q_{z, v} w_v / (1 + \kappa).$$

(54)

Recall the utility function of the individual (3), and focus on the sub-utility derived from the consumption of good $z$ sourced from country $v$. Let us write this sub-utility as $u_{z, v} = \ln (c_{z, v})^{q_{z, v}}.$

RCA$_{\eta_k, v_k} \geq$ RCA$_{\eta_{v_1}}$, then bearing in mind that $\partial^2 \beta^j (\cdot) / (\partial \eta)^2 > 0$ and $\partial^2 \beta^j (\cdot) / (\partial \eta \partial w_v) > 0$, we can observe that when (52) holds true then

$$\frac{1}{\eta - \eta} \int_{\eta}^{\eta} [RCA_{\eta, v_1}] d\eta < 1,$$

which contradicts (51). Therefore, it must be the case that $RCA_{\eta_k, v_k} < RCA_{\eta_{v_1}}.$ ■
Bearing in mind (54), and considering that, in the optimum:

\[ q_i^{z,v}(t_z) = \left(1 + \frac{\kappa}{1 + t_z A Q_i e^{n_{z,v}}} \frac{w_i}{w_v} \right)^{1/(n_{z,v} - 1)}, \]  

and \( \beta_{z,v}^i = q_i^{z,v}/Q_i \), then \( u_{z,v} \) boils down to:

\[ u_{z,v} = \eta_{z,v} \left( 1 + \frac{\kappa}{1 + t_z A Q_i e^{n_{z,v}}} \frac{w_i}{w_v} \right)^{1/(n_{z,v} - 1)}. \]  

Let us denote by \( \Gamma_v^i(t_z) \) the utility loss due to imposing an import tariff \( t_z > 0 \) relative to the case where \( t_z = 0 \). From (56), we get:

\[ \Gamma_v^i(t_z) = \eta_{z,v} \left( 1 + \frac{\kappa}{A Q_i e^{n_{z,v}} w_v} \right)^{1/(n_{z,v} - 1)} \left[ 1 - (1 + t_z)^{-\frac{1}{n_{z,v} - 1}} \right]. \]  

It is plain from (57) that \( \Gamma_v^i(t_z) > 0 \) whenever \( t_z > 0 \), and that \( \partial \Gamma_v^i(\cdot)/\partial t_z > 0 \).

More interesting is studying how the tariff loss function behaves at different levels of income of the importer. Using (57), we may compute the elasticity of \( \Gamma_v^i(t_z) \) with respect to \( w_i \), to obtain:

\[ \frac{\partial \ln \Gamma_v^i(t_z)}{\partial \ln w_i} = \frac{1}{\eta_{z,v} - 1} > 0. \]  

This result implies that the utility loss due to the tariff is greater for richer consumers. Moreover, this difference in welfare loss between richer and poorer importers becomes greater when the tariff is imposed on more efficient producers of good \( z \) (i.e., when the tariff is imposed on countries that received a lower \( n_{z,v} \)).

Our model then yields the following two qualitative welfare loss results. First, the consumer loss due to import tariffs is always greater for richer importers. Second, the loss disparity between richer and poorer importers gets larger when the tariff is imposed on more efficient producers of good \( z \). These two results crucially rest on our nonhomothetic preference structure (in Section A.5 below we show that these two results vanish away in the presence of homothetic preferences).

Lastly, we can use some of the above expressions to get a sense of the relative magnitude of welfare loss due to the tariff implied by our model. Bearing in mind (58), from (55) we can observe that:

\[ \frac{\partial \ln q_i^{z,v}}{\partial \ln w_i} = \frac{1}{\eta_{z,v} - 1} = \frac{\partial \ln \Gamma_v^i(t_z)}{\partial \ln w_i}. \]  

Interestingly, the magnitude of the left-hand side of this expression can be obtained from the data by using unit values as a proxy for \( q_i^{z,v} \). The pooled estimation of the log of unit values on the log of importer’s income delivers a mean value of 0.075.\(^{30}\) This implies that a 10% richer importer

\(^{30}\) We conduct our pooled regression using the data on value of imports and quantity of imports by product at the 6-digit Harmonised System (HS-6) level of disaggregation in year 2009. Our regression includes a full set of product-exporter dummies. Full details of this regression are available from the authors upon request.
will suffer a 0.75% higher welfare loss when importing good \( z \) from the average producer.\(^{31}\) If we add (subtract) one standard deviation to the estimate, the welfare loss suffered by a 10% richer importer rises (declines) to 0.85% (0.65%).\(^{32}\)

**A.4 Sectoral subsidy and comparative advantage**

Consider country \( v \) as a producer of good \( z \), and assume that local producers of good \( z \) receive a proportional subsidy \( \sigma_{z,v} \) regardless of the quality level of their output. Since the subsidy is applied only to one (atomless) sector within a continuum of sectors, we can disregard again general equilibrium effects as they would be negligible. Following an analogous reasoning as before, they will sell their output to consumers of country \( i \) at price 

\[
p_{z,v}(q) = (1 - \sigma_{z,v}) A q^{\eta_{z,v}} w_v / (1 + \kappa).
\]

This, in turn, implies that the share of income of consumers in country \( i \) spent on good \( z \) produced in country \( v \) is given by:\(^{33}\)

\[
\beta_{z,v}^i = \left[ \frac{(1 + \kappa)}{(1 - \sigma_{z,v}) A (\epsilon \mu)^{\eta_{z,v}} w_v} \right]^{1/(\eta_{z,v} - 1)}.
\]  

Let \( S_{z,v} \) denote the share of sector \( z \) in the total GDP of country \( v \). Bearing in mind that, in this simplified version of the model, there are two countries in region \( L \) (with income \( w_L \)) and two countries in region \( H \) (with income \( w_H \)), it follows that:

\[
S_{z,v} = \frac{2}{w_v} (w_H \beta_{z,v}^H + w_L \beta_{z,v}^L).
\]  

Differentiating (60) with respect to \( \sigma_{z,v} \) yields:

\[
\frac{d \beta_{z,v}^i}{d \sigma_{z,v}} = \frac{\beta_{z,v}^i}{(\eta_{z,v} - 1) (1 - \sigma_{z,v})} > 0.
\]  

The impact of a subsidy to sector \( z \) in country \( v \) on its the GDP share can be obtained by differentiating (61) with respect to \( \sigma_{z,v} \) while bearing in mind (62). Thus,

\[
\frac{d S_{z,v}}{d \sigma_{z,v}} = \frac{2}{w_v} \left( w_H \frac{d \beta_{z,v}^H}{d \sigma_{z,v}} + w_L \frac{d \beta_{z,v}^L}{d \sigma_{z,v}} \right).
\]

\(^{31}\) Using (59), we can back out the implied value of the sectoral productivity draw for the average producer, \( \bar{\eta} = 14.33 \). Performing a similar pooled regression analysis as the one we do here, but including a panel of transactions instead of data for year 2009 only, Fieler (2012, Table 2) obtains an estimate equal to 0.06. According to her estimate (which is just slightly smaller in magnitude than ours), the sectoral productivity of average producer would be \( \bar{\eta} = 17.67 \).

\(^{32}\) Again, using (59), the implied value of the sectoral productivity draw for the more (less) productive exporter when we add (subtract) one standard deviation to the estimated value is \( \bar{\eta} = 12.76 \) (\( \bar{\eta} = 16.38 \)). Repeating this exercise for a two standard deviations difference, we obtain a 1.05% (0.45%) higher welfare loss, corresponding to sectoral a sectoral productivity draw for the exporter of \( \bar{\eta} = 10.52 \) (\( \bar{\eta} = 23.22 \)).

\(^{33}\) Notice that when \( i = v \) then \( \beta_{z,v}^i \) refers to domestic sales of good \( z \).
Let us now compare the impact of the subsidy $\sigma_{z,v}$ on the sectoral share $S_{z,v}$ for the case of a country in region $L$ (which, by construction, must have received $\bar{\eta}$ as productivity draw in sector $z$) and the country in region $H$ that received $\bar{\eta}$ as productivity draw in sector $z$. That is, we are computing the derivative (63) for two economies with different wages in the denominator ($w_l$ and $w_h$, respectively), but both sharing the same elasticity of quality upgrading $\eta_{z,v} = \bar{\eta}$. Notice now that both $d\beta_{z,v}^H/\sigma_{z,v}$ and $d\beta_{z,v}^L/\sigma_{z,v}$ are always larger in a country from region $L$ than in the country from region $H$ that received the productivity draw $\eta_{z,v} = \bar{\eta}$. Therefore, the effect of $\sigma_{z,v}$ on $S_{z,v}$ will be larger in the country from region $L$ than in the country from region $H$ that received the productivity draw $\eta_{z,v} = \bar{\eta}$. This uneven effect of the subsidy across producers with different incomes rests crucially on our non-homothetic preference structure, as it is shown next in Section A.5.

### A.5 Homothetic preferences

We now introduce an alternative preference specification, designed to deliver homothetic demand schedules. For the remaining of this appendix, to streamline the illustration it proves convenient to exploit the ordinal nature of the quality ladders and apply the following monotonic transformation to the quality index: $\tilde{q}_{z,v} = \ln q_{z,v}$. This transformation comes at no loss of generality since the result derived here would obtain even without such transformation.\(^{34}\)

Suppose that preferences, while retaining the same structure across goods, are for each good now represented by the sub-utility index:

$$u_{z,v} = \ln(\tilde{q}_{z,v} c_{z,v}).$$

This index replaces the expression $\ln \left( c_{z,v}^{q_{z,v}} \right)$ in (3). The rest of the model remains unchanged. Individuals choose the optimal values of quality and consumption to maximise that utility function subject to (4). There, the pricing function in terms of $\tilde{q}_{z,v}$ becomes:

$$p_{z,v}(\tilde{q}_{z,v}) = \frac{A w_v}{1 + \kappa} (\tilde{q}_{z,v})^{\eta_{z,v}} = \frac{A w_v}{1 + \kappa} (e^{\tilde{q}_{z,v}})^{\eta_{z,v}} = \frac{A w_v}{1 + \kappa} e^{\eta_{z,v} \tilde{q}_{z,v}}. \quad (64)$$

Following an analogous reasoning as the one used in Appendix A to solve the original consumer $i$ optimisation problem, we obtain the the (relevant) first-order conditions:

$$\frac{1}{\tilde{q}_{z,v}} - \eta_{z,v} = 0; \quad (65)$$

$$\frac{1}{\Omega \cdot \Lambda} \frac{1}{\beta_{z,v}^i} - \nu^i = 0. \quad (66)$$

\(^{34}\)More precisely, all the homothetic results we show below will remain qualitatively unchanged if we keep $\tilde{q} = q$. These additional results are available from the authors upon request.
Note that (66) implies that budget shares are identical for all goods, wherever produced. Recalling that budget shares must sum up to one, from (65) and (66) we can thus obtain the following two expressions respectively identifying, for each good $z$ and country $v$, the optimal quality level and budget share:

$$q^i_{z,v} = \frac{1}{\eta_{z,v}}; \quad \text{and} \quad \beta^i_{z,v} = 1.$$

In the light of these findings, we can show that the results discussed in Appendices A.3 and A.4 vanish away once we modify the utility function to deliver homothetic preferences.

First, consider again (54), now expressed in terms of $\tilde{q}^i_{z,v}$:

$$p_{z,v}(\tilde{q}_{z,v}) = (1 + t_z) Ae^{\tilde{q}_{z,v} / w_v / (1 + \kappa)}. \quad (67)$$

Using the sub-utility index $u_{z,v} = \ln(\tilde{q}_{z,v}^i e^i_{z,v})$, replacing $e_{z,v}^i = \beta_{z,v}^i w_v / p_{z,v}(\tilde{q}_{z,v})$ and then $p_{z,v}(\tilde{q}_{z,v})$ by (67) and $\tilde{q}_{z,v}$ and $\beta_{z,v}^i$ by their optimal values, we obtain the welfare loss function:

$$u_{z,v} = -\ln \eta_{z,v} + \ln \left( \frac{1 + \kappa}{1 + t_z e A w_v} \right).$$

Differentiating with respect to $t_z$ yields $\partial u_{z,v} / \partial t_z = -(1 + t_z)^{-1} < 0$, from which it is easy to observe that the value of the derivative in (58) in this case equals zero.\(^{35}\) This implies that the utility loss due to the tariff is now independent of consumers’ income. In other words, differently from our model with nonhomothetic preferences, in the presence of homothetic preferences (i.e., when willingness to pay for quality is constant), the utility loss due to the import tariff is the same for all individuals, regardless their income level.

Second, recalling the definition of $S_{z,v}$ in (61), from the fact that $\beta_{z,v} = 1$ it straightforwardly follows that the derivatives in (62), and therefore in (63), are in this case equal to zero. This implies that the effect of a subsidy on the share of sector $z$ in the total GDP is now independent of the region to which country $v$ belongs.

\(^{35}\) Note that the welfare loss function is this case reads $\Gamma_v^i(t_z) = \ln(1 + t_z)$, from which $\partial \Gamma_v^i(t_z) / \partial w_i = 0$ straightforwardly obtains.