A Model for the Backscattering From a Canonical Ship in SAR Imagery

Pasquale Iervolino, Student Member, IEEE, Raffaella Guida, Member, IEEE, and Philip Whittaker

Abstract—Synthetic aperture radar (SAR) sensors represent one of the most effective means to support activities in the sector of maritime surveillance. In the field of ship detection, many SAR-based algorithms have been proposed recently, but none of them has ever considered the electromagnetic aspects behind the interactions of SAR signals with the ship and surrounding waters, with the detection step and rate strongly influenced by relative thresholding techniques applied to the SAR amplitude or intensity image. This paper introduces a novel model to evaluate the radar cross section (RCS) backscattered from a canonical ship adapted, to the case at issue, from similar existing models developed for, and applied to, urban areas. The RCS is modeled using the Kirchhoff approximation (KA) within the geometrical optics (GO) solution and, following some assumptions on the scene parameters, derived by empirical observations; its probability density function is derived for all polarizations. An analysis of the sensitiveness of the RCS to the uncertainty on the input scene parameters is then performed. The new model is validated on two different TerraSAR-X images acquired in November 2012 over the Solent area in the U.K.: the RCS relevant to several isolated ships is measured and compared with the expected value deriving from the theoretical model here introduced. Results are widely discussed and ranges of applicability finally suggested.

Index Terms—Electromagnetic modelling, radar cross-sections, radar detection, synthetic aperture radar (SAR).

I. INTRODUCTION

MARITIME surveillance is a topic of great importance and growing interest during the recent years. It is estimated that oceans cover fully 70% of the Earth’s surface and support more than 80% of the global trade [1]. With regard to the European Union (EU), almost 90% of the external freight trade is seaborne, short sea shipping represents 40% on the intra-EU exchanges, and more than 400 million passengers embark and disembark in European ports each year [2]. Looking at the U.K., 50% of the energy supplier is provided by oil shipping. Moreover, Britain’s Sea Trade brings substantial revenue to the Treasury, with a projected value of 700 billion by the end of 2017 [3]. In this context, the U.K. National Maritime Information Centre (NMIC) has the role to monitor and track maritime activity around the U.K. and areas of national interest [4]. Furthermore, NMIC has to support both government and industry decisions in time of need, enabling a better understanding of maritime security issues [4].

One of the main applications of the maritime surveillance is the ship detection. In this context, there is a need for persistent wide-area (global coverage) surveillance. The requirements, which an identification system should provide, are: high probability of detection and low probability of false alarm; accurate geo-locating; ship identification; and ability to operate in all weather and day conditions [5].

While there are several ways to monitor and track ships, there is no single mean which can meet all the above requirements. As part of the coastal-based surveillance systems, the automatic identification system (AIS) represents nowadays the most used technique. AIS systems were originally designed for safety reasons and mainly for collision avoidance. They are basically shipborne systems by which ships inform each other and the coastal receivers about their position, course, speed, and name. However, the system is able to cover up to only about 40 km off the coast and obviously requires that the on-board AIS works correctly [6]. The main drawbacks concerning land-based AIS systems are limited spatial coverage, impossibility of berth-to-berth tracking, and need for land-based means. In order to overcome the limitation on the spatial coverage, it has been recently demonstrated that AIS signals can be well received using spaceborne systems [6]. The detection of the AIS signal transmitted from a ship is possible in the entire radio visibility range of a satellite equipped with an AIS payload. For example, for a satellite at an altitude of 650 Km, the average field of view is above 20 million square kilometers [6]. In 2011, a SatAIS system was launched by the German Aerospace Centre (DLR) to enable observations of worldwide ship movements by means of AIS [6]. In the next couple of years, space agencies are planning to launch further SatAIS payloads to enhance the reception of AIS messages as the U.K. NovaSAR-S [7].

A useful support to the ship identification techniques based on AIS could be brought by synthetic aperture radar (SAR) sensors. A spaceborne SAR can be considered a complementary means to the traditional ones, thanks to its peculiar ability to acquire images independently from daylight, meteorological conditions, and national borders. SAR sensors are very useful in the detection of noncooperative ships and in the tracking of small ships without AIS on-board [8]. Modern SAR sensors offer wide spatial coverage and, if operated in constellation, allow for a reduction in the revisit time and, consequently, a better control of open sea areas. The main issue concerned with spaceborne SAR sensors is the impossibility to identify ships,
Although high-resolution SAR (HR-SAR) and its submeter resolution can recognize at least the ship type [9].

So far, traditional SAR ship-detection algorithms have been based on constant false alarm rate (CFAR) methods [10]: the sea background is characterized statistically and then the detector looks for individual pixels (or small group of pixels) whose brightness values are statistically unusual [10]. A commonly used statistical model for CFAR detector is the Gaussian distribution. This is often applicable according to the central limit theorem stating that the average of a large number of identically distributed random variables tends to have a Gaussian distribution [11]. Obviously, the more independent the scatterers are in a resolution cell, the more adequate the Gaussian model is. With this underlying model, closed forms of CFAR ship-detection algorithms can be found with a substantial computational saving compared to others based on different distributions [6], [11].

In modern HR-SARs, the spatial resolution is tremendously improved and, consequently, the number of independent scatterers in the resolution cell is decreasing. In this framework, the Gaussian distribution is unlikely to stay the one which better models the sea clutter and, indeed, previous works prove that the K distribution and the Generalized Gamma distribution fit much better [12], [13]. Some CFAR algorithms compute the statistics of the sea clutter globally (defining, in turn, a global threshold), while others evaluate the clutter distribution parameters locally to take into account the variability of the sea (and propose an adaptive threshold). The adaptive threshold algorithms are obviously less efficient in terms of computational load but, generally, present better performance than the Global Threshold algorithms [10].

The main limitation of CFAR algorithms is that targets with intensity values very similar to those of sea clutter might not be detected; moreover, the parameters’ estimation distribution and the threshold definition are computationally expensive procedures for non-Gaussian models. A different ship-detection approach relies on the subaperture decomposition where no assumption about the sea background is needed [14]–[17]. The detection is performed based on the different behaviors of targets and clutters regarding the sublook coherence: the original SAR image is decomposed into sublooks (generally between two and four) and then the coherence index between the different sublook images is computed. The random behavior of the sea clutter makes the coherence of its pixels close to 0, while, on the other hand, the targets present a more deterministic behavior and a coherence index closer to 1. In this way, the signal-to-clutter ratio can be increased up to 2 dB [14]. Consequently, from one hand, the targets can be detected easily, but, from the other hand, the spatial resolution is reduced depending on the number of subapertures employed [14]. In [17], the spatial resolution loss is reduced by computing the cross-correlation over partially overlapping subapertures (30%). In [18] and [19], instead, a novel prescreening algorithm, based on the Wavelet Transform (WT), is presented. This approach shows some similarities with the subaperture method. The WT studies a complex phenomenon dividing it into different simpler pieces by projecting the input signal in a particular function space (wavelet space). In this way, the WT is able to characterize the local regularity of the signal: the existence of discontinuities in the original images produces large wavelet coefficients, while homogeneous areas, on the contrary, present small coefficients [18]. In [20] and [22], instead, polarimetric analyses are employed to detect ship targets over the sea clutter pointing out that, generally, SAR polarimetry improves ship-detection performance.

In all the papers cited, the modeling of the ship, whether statistical or analytical, is generally neglected to keep the overall model simple. As a consequence, the detector may result in a higher false-alarm rate [10] since intrinsic properties of a ship, such as its peculiar way to backscatter the transmitted signal, are completely ignored. An extremely simple model for the ship backscattering is introduced in [23] and more recently exploited in [24] to build a generalized likelihood ratio test (GLRT). The model assumes that the ship pixels are independent and Gaussian distributed. No evidence is provided to support this assumption and the same distribution is employed for both the sea clutter and the ship but with a different standard deviation.

In [11], instead, the authors show that the overall ship-detection performance improves with the inclusion of a proper scattering evaluation block in the detection chain, where the electromagnetic field backscattered from a canonical ship is considered for the first time and used to reject nonparallelepiped-like targets to improve the final detection performance.

In this paper, a complete statistical and analytical evaluation of the most representative backscattering contributions from canonical ship-sea configurations is presented and tested on spaceborne SAR imagery. The work leads to the identification of a suitable statistical distribution to characterize the backscattering return from a ship within certain assumptions. In future, the retrieved distribution can be employed to build a GLRT-based detector and improve the performance of SAR ship-detection algorithms by considering the model for both the ship and the sea clutter.

This paper is organized as follows. In Section II, the electromagnetic model characterizing the different scattering contributions of the canonical ship is presented. In Section III, the canonical ship model is derived and its distribution is shown for all combinations of transmitted/received polarizations. The goodness-of-fit (GoF) test is performed in Section IV to derive the best distribution in modeling the radar cross section (RCS) relevant to the double scattering from the ship. In Section V, the analysis of the sensitivity of the RCS to the errors on the knowledge of the model parameters is presented. In Section VI, the proposed model is validated on actual spaceborne SAR images. Finally, in Section VII, some concluding remarks are reported.

II. ELECTROMAGNETIC MODEL

The modeling of the electromagnetic field backscattered from ships is still poorly considered in ship-detection algorithms, mainly because of the natural complexity behind a reliable model. This section aims to show that the scenario ship sea has many similarities with urban settlements for which scattering models have been introduced, and successfully inverted, in the last years for different kinds of applications, see [25]–[27].
Actually, it is even simpler to model in some situations as, e.g., multiple scattering due to the interaction of the backscattered signal with other ships does not arise in open ocean. Moreover, the isolation of ships makes their backscattering contributions easier to detect in SAR images. With regard to the nonstationarity of the target and the dynamic scenery in which it is placed, from the literature, it is well known that the motion effects of both capillary and gravitational waves, as well as the possible motion of the ship itself, lead to a change in the Doppler frequencies resulting in azimuth image shift and smearing [10]. For example, as a consequence of its motion, a ship appears shifted along the azimuth direction in the focused SAR image, far from the position of its wake. However, the study of these undesired effects, and their inclusion in the model here being proposed, is at the moment a challenge (see [28]) and led the authors to assume the ocean surface stationary and the ship still. These assumptions will simplify the following analysis without invalidating the model as, essentially, the composition of the signal backscattered from a canonical ship does not change. These considerations brought the authors to reconsider the scattering models introduced in [25] and, after proper modification, adapt them to this new scenario. In addition, further assumptions about the canonical shape and size of the ship are made to reduce the complexity of the problem.

1) The ship is a perfect parallelepiped (hence, superimposed structures and tips are ignored).
2) Its hull is completely smooth.
3) Its dimensions are much larger than the working radar wavelength.

Moreover, we suppose that the ships are isolated (i.e., in open ocean and far from other ships), so that multiple scattering does not arise. We also neglect any diffraction effects. The diffraction contributions are due to the finite dimensions of the scattering surfaces (in this case, the ship hull); hence, they can be modeled as contributions from horizontal and vertical edges of the ship. Since the ships dimensions are very large in terms of wavelength in high regime frequency, edge diffractions are expected to be small with respect to the other reflection contributions (single and multiple scattering), and errors caused by neglecting diffractions are certainly smaller than those caused by simplifying hypotheses on the ship geometry [25].

Within these hypotheses, a real cargo such as the Celtic Fortune in Fig. 1(a) would be more easily modeled with a parallelepiped-like canonical ship forming a perfect dihedral with the sea surface, as the one in Fig. 1(b) drawn with AUTOCAD software. However, the deviation of the angle formed by sea and ship from the right angle of a perfect dihedral is here assumed negligible (as it happens in many real cases). The canonical ship is regarded as a metallic object which is decomposed by using a series of rectangular facets. Consequently, the total component field can be obtained with a vectorial summation of all integral radiation on each facet [29]. In addition, the computation of the RCS of large and complex targets involves scattering mechanisms of different orders [30]. The corresponding contributions on a SAR image can be mapped or pictorially represented knowing the elementary shape of the ship and some radar parameters. In Fig. 2, where the illumination comes from the left side and \( \vartheta \) is the radar look angle, the following contributions are expected to be found: first, the layover area (single scattering mechanism from the top and lateral side of the ship plus single scattering from the sea), followed by the double-reflection contribution (located in the vertex O), then the single scattering from the top together with the triple scattering (from vertex O to vertex A), the single scattering from the top alone and, finally, the dark shadow area [31]. It has already been demonstrated in [11] that such a geometrical model is a good discriminator between man-made objects over the sea of different shapes.
In order to consider an analytical, closed-form expression for the different scattering contributions, some further assumptions are made.

1) The sea clutter is modeled via a Gaussian stochastic process with Gaussian autocorrelation function (however, more involved stochastic processes can be easily considered in the following derivation [32]).

2) The water is considered infinitively deep, still in terms of working wavelength, so that multiple bounces do not arise from beneath the water surface [27].

Within these hypotheses and using the Kirchhoff approximation (KA), it is possible to evaluate the scattered field at each bounce with physical optic (PO) or geometrical optic (GO) solutions according to the sea roughness. In particular, PO approximation is applied if \( k \sigma_{dev} \ll 1 \), where \( k \) is the radar wavenumber and \( \sigma_{dev} \) is the standard deviation describing the stochastic process of the sea. Vice versa, if \( k \sigma_{dev} \gg 1 \), GO approximation is applied [25]–[27].

Since all the double-reflection rays present the same delay [31], the double reflection is the dominant scattering contribution. The triple and single scattering contributions, instead, are mixed each other and they are not easily detectable on the SAR images. For these reasons, the following analysis is limited to the double-reflection contribution. For the sake of simplicity, the final formulations of the RCS, already computed in [25], are here reported for both GO–PO and GO–GO approximations. For the GO–PO, the RCS is given by

\[
\sigma = h |S_{pq}|^2 l \tan \theta \cos \varphi \exp \left( -4k^2 \sigma_{dev}^2 \cos^2 \varphi \right) \sum_{m=1}^{\infty} \frac{(2k \sigma_{dev} \cos \varphi)^2 m^2 k^2 L^2}{4m} \exp \left( \frac{(2k \sin \varphi \sin \theta)^2}{4m} \right).
\]

Alternatively, for the GO–GO, the RCS is given by

\[
\sigma = \frac{h |S_{pq}|^2 l \tan \varphi \cos \varphi (1 + \tan^2 \varphi \sin^2 \varphi) \exp \left( -\frac{\tan \varphi \sin \varphi}{2 \sigma_{dev}^2 L^2} \right)}{8\pi^2 \sigma_{dev}^2 (2/L^2) \cos^2 \theta}.
\]

In (1) and (2), \( \sigma \) represents the RCS relevant to the double-reflection contribution; \( S_{pq} \) is the generic element of the scattering matrix with \( p \) and \( q \) standing for horizontal \( H \) or vertical \( V \) polarization, respectively; \( l \) is the length of the portion of the ship belonging to the resolution cell, assuming the ship length larger than the SAR spatial resolution; \( \sigma_{dev} \) and \( L \) are the standard deviation and the correlation length, respectively, of the stochastic process representing the sea clutter; \( \varphi \) is the angle between the sensor line of flight and the ship hull to the water surface; \( \theta \) is the SAR look angle and \( h \) is the portion of the ship height forming the dihedral surface between the sea and the ship hull. In nautical terms, the latter is also known as freeboard and represents the distance from the waterline to the upper deck of the ship. The distance between the waterline and the bottom of the hull, instead, is known as draught [33]. However, due to the hypothesis e), the draught does not contribute to the double scattering [(1) and (2)] and is therefore neglected in the following. The freeboard and the draught of a ship are, finally, shown in Fig. 3.

In (1) and (2), \( S_{pq} \) depends on the dielectric constant of the sea \( (\varepsilon_{SW}) \), the dielectric constant of the hull \( (\varepsilon_{HULL}) \), \( \varphi \), \( \theta \), \( k \), and the Fresnel coefficient according to the polarization of the propagating wave. The equations to compute \( S_{pq} \), for both GO–GO and GO–PO solutions and for each polarization, are reported in [25].

In the next section, some assumptions on the parameters involved in the electromagnetic model are made to model the distribution of the RCS values for the double-reflection contribution.

III. COMPUTATION OF THE RCS DISTRIBUTION

In order to compute the RCS relevant to the double-reflection contribution, the parameters involved in (1) and (2) have to be known. Unfortunately, this \textit{a priori} knowledge is not completely available. Only some parameters are \textit{a priori} known (the radar look angle and the wavelength), others can be retrieved directly either from the SAR image (the sea roughness parameters) or from the literature (dielectric constant of the sea), while for the remaining ones (the orientation angle, the dielectric constant of the hull, and the freeboard height), suitable probability distribution functions can be estimated bringing, in turn, to a probability density function for the RCS too.

Due to the wind, the sea surface is never completely smooth and presents several capillary waves [28]. For this reason, the GO–GO approximation has been chosen and is the unique solution being analyzed in the following.

A. Estimation of the Input Parameters

As anticipated, some parameters are \textit{a priori} known. In particular, \( \varphi \) and \( k \) can be retrieved from the ancillary data of the SAR sensor, while \( \varepsilon_{SW} \) is computed according to the model presented in [34] where the real \( (\varepsilon'_{SW}) \) and the imaginary \( (\varepsilon''_{SW}) \) part of the saline water are given by

\[
\varepsilon'_{SW} = \varepsilon_{SW0} - \varepsilon_{SW\infty} \frac{1 + (2\pi f \tau_{SW})^2}{1 + (2\pi f \tau_{SW})^2 + \sigma_{SW}^2} + \frac{2\pi f \tau_{SW}}{2\pi f_{0} f} (\varepsilon_{SW0} - \varepsilon_{SW\infty}) \tag{3}
\]

where \( f \) is the working frequency, \( \varepsilon_0 \) is the permittivity of free space, \( \varepsilon_{SW\infty} \) is a constant equal to 4.9, \( \sigma_{SW} \) is the conductivity of the sea depending on the salinity \( S \), \( \varepsilon_{SW0}(T,S) \) and \( \tau_{SW}(S,T) \) are quantities depending on both the salinity \( S \) and the sea temperature \( T \). The average information about the salinity and the temperature of the North Sea (where the datasets used in this study were acquired) are retrieved from
The relative complex dielectric constant of the sea is

$$\varepsilon_{\text{SW}} = \varepsilon'_{\text{SW}} + j\varepsilon''_{\text{SW}} = 71.82 - j37.78. \quad (4)$$

The roughness parameters ($\sigma_{\text{dev}}/L$) can be estimated, instead, by minimizing the absolute error between the RCS relevant to the single scattering from the sea of the sea surface measured on the SAR images and the expected RCS within the GO solution, as shown in [36].

With regard to the angle $\varphi$, the authors assume to work in the worst-case scenario where the orientation angle can be retrieved neither from the ship signature nor from a visible wake. This consideration drives the choice of describing the angle $\varphi$ statistically as uniformly distributed between $0^\circ$ and $45^\circ$ [$\varphi \sim U (0; 45)$]. Obviously, when the heading angle is greater than $45^\circ$, one side of the hull will always present an angle with the projection of the sensor on the sea smaller than $45^\circ$ and, consequently, $\varphi \sim U (0; 45)$ takes into account all the possible scenarios.

The range of values for the freeboard height is selected according to the amendments of the 1974 SOLAS Convention which regulates the freeboard of the ships of 24-m length or more [37]. Ships are divided into two categories (Type A and Type B) and for each ship length (from 24 to 365 m), the freeboard is provided. Freeboard values are included between 0.2 and 5.3 m [37]. Having the latter as the only available information about the size of the ship, $h$ is consequently selected which is uniformly distributed between 0.2 and 5.3 m [$h \sim U (0.2; 5.3)$m].

The dielectric constant of the hull is chosen by performing a weighted average of several dielectric constants of materials which mainly compose the structure of a ship. First of all, it is assumed that the canonical ship is made mostly of steel (with a percentage uniformly distributed between 60% and 90%, values based on authors’ empirical evaluations) and for the remaining part of a mixture (equally distributed) of glass, aluminum, and fused silica. Within these hypotheses, $\varepsilon_{\text{HULL}}$ can be computed as already done in [27] and [38]

$$\varepsilon_{\text{HULL}} = p\varepsilon_{\text{st}} + \frac{q}{3}(\varepsilon_a + \varepsilon_g + \varepsilon_{\text{st}}) \quad (5)$$

where $p \sim U (0.6; 0.9)$, $q = 1 - p$ and $\varepsilon_{\text{st}}$, $\varepsilon_a$, $\varepsilon_g$, and $\varepsilon_{\text{st}}$ are the complex relative dielectric constant of steel, aluminum, glass, and fused silica, respectively. Their values are listed in Table I at S, C, and X bands, according to [38] and [39].

In Table II, the way to estimate all the parameters, needed to compute the RCS in (2), is summarized.

### Table I

<table>
<thead>
<tr>
<th>Material</th>
<th>$\varepsilon_{\text{Req}}$</th>
<th>S</th>
<th>C</th>
<th>X</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel</td>
<td>$3.1 - j1.12 \times 10^6$</td>
<td>3.1</td>
<td>3.1</td>
<td>3.1</td>
</tr>
<tr>
<td>Aluminum</td>
<td>$9.4 - j1.19 \times 10^8$</td>
<td>9.4</td>
<td>9.4</td>
<td>9.4</td>
</tr>
<tr>
<td>Glass</td>
<td>$6.2 - j0.021$</td>
<td>6.2</td>
<td>6.2</td>
<td>6.2</td>
</tr>
<tr>
<td>Fused silica</td>
<td>$3.8 - j0.0008$</td>
<td>3.8</td>
<td>3.8</td>
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</tr>
</tbody>
</table>

### Table II

<table>
<thead>
<tr>
<th>Parameters Needed to Compute the RCS of the Ship</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Estimation</strong></td>
</tr>
<tr>
<td><strong>Dielectric constant of the sea $\varepsilon_{\text{st}}$</strong></td>
</tr>
<tr>
<td>Roughness parameters $\sigma_{\text{dev}}/L$</td>
</tr>
<tr>
<td>Orientation angle $\varphi$ (deg)</td>
</tr>
<tr>
<td>Freeboard height $h$ (m)</td>
</tr>
</tbody>
</table>

Fig. 4. Histogram of the RCS values relevant to the double-reflection contribution for HH polarization at X band in $\text{m}^2$ at the top (mean = 0.24 $\text{m}^2$ and std = 0.30 $\text{m}^2$) and in $\text{dBsm}$ at the bottom.

### Table II

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Estimation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Radar look angle $\theta$ (deg)</td>
<td>$U (0.45)$</td>
</tr>
<tr>
<td>Radar working frequency $f$ (Hz)</td>
<td>$U (0.2; 5.3)$</td>
</tr>
<tr>
<td>Radar ancillary data</td>
<td>$U (0.2; 5.3)$</td>
</tr>
<tr>
<td>Dielectric constant of the sea $\varepsilon_{\text{st}}$</td>
<td>$U (0.2; 5.3)$</td>
</tr>
</tbody>
</table>

#### B. Distribution of the RCS Values

Once all parameters are estimated or statistically modeled, the RCS relevant to the double-reflection contribution of the canonical ship can be modeled too. Equation (2) has been implemented using a MATLAB script with $10^6$ samples for each polarization (HH, V, and HV). In Fig. 4, the histograms of the RCS values in $\text{m}^2$ and $\text{dBsm}$ are shown for HH polarization at X band (9.65 GHz). Similarly, in Figs. 5 and 6, the histograms for the VV and HV polarizations are reported.

The greatest values of the RCS are obtained when the freeboard is high and when the ship is parallel or near parallel to the SAR flight direction (low $\varphi$ angle). Instead, when the freeboard is small or the angle $\varphi$ great, the RCS is reduced severely resulting in missing targets in ship-detection algorithms. All the assumptions made on the input parameters make the canonical ship a heterogeneous target and this is witnessed by the large standard deviation (compared to the mean value) of the RCS, as reported in Figs. 4–6. HH and VV distributions are quite similar in shape, mean, and standard deviation, while the HV distribution presents values three orders of magnitude smaller. This is why, according to the model presented in literature [25], the RCS relevant to the double-reflection line of the cross-polarized channel is much weaker due to the scattering coefficient $S_{\text{HHV}}$ being proportional to $\sin 2\varphi$. As a consequence, there is no cross-polarized double-reflection...
component in the case of an ideal dihedral perfectly aligned with the sensor azimuth direction ($\phi = 0$). Furthermore, in many real cases, when $\phi \neq 0$, the double reflection component may be lower than the noise floor level and undetectable on SAR images. However, it has been demonstrated that on real SAR images, the RCS relevant to the cross-polarized channels is not negligible and can be useful to improve the performance of the SAR detectors [10], [20]. For these reasons, and for the sake of completeness, the authors have added here also the results relative to the cross-polarized channel.

Similar distributions can be computed also at C and S bands. In the next section, the distributions retrieved at X band are compared, for each polarization, with standard distributions to find the best-fitting distribution.

In this section, the probability distribution function (pdf) and the cumulative distribution function (cdf) for each polarization (HH, VV, and HV) are compared to those ones of standard distributions. In particular, the Inverse Gaussian, the $G^0$, the Gamma, the Rayleigh, and the Weibull distributions, [40], [41], are analyzed. The pdfs of all the aforementioned distributions are reported in Table III.

The parameters of all the distributions (except those of the $G^0$ distribution) are estimated through the maximum likelihood method and using the MATLAB mle function. The shape ($\tilde{\alpha}$) and the scale ($\tilde{\gamma}$) parameters of the $G^0$ distribution, instead, are estimated through the mixed estimator method introduced in [41], according to the following equations:

\[
Q = \frac{2}{\sqrt{\pi}} \left( \frac{\sqrt{0.5} - 1}{\sqrt{\pi}} \right)^{0.5} \frac{\Gamma(-\tilde{\alpha})}{\Gamma(-\tilde{\alpha} - 0.5)}
\]

\[
\tilde{\gamma} = \frac{4}{m_1^2} \left( \frac{\Gamma(-\tilde{\alpha})}{\Gamma(-\tilde{\alpha} - 0.5)} \right)^2
\]

where $Q$ is the median of $\sigma/E[\sigma]$, $m_1$ is the first $\sigma$ moment, and $\Gamma(\cdot)$ is the gamma function.

In Figs. 7–9, the pdfs and the cdfs are shown for all the aforementioned distributions and compared with the histogram data at X band for HH, VV, and HV polarization, respectively. It is possible to note that the distribution with one parameter (Rayleigh, plotted in cyan) is not able to fit the heterogeneity of the data for all polarizations. The best fittings, instead, are obtained using the Gamma (in purple) and the Weibull (in green) for all the distributions. In order to verify if and how well these distributions approximate the histograms of the RCS data, a GoF test has been performed. In Table IV, the $\chi^2$ GoF test [42] is performed considering a significance level of 5% for all the distributions for each polarization and, consequently, the $p$ value is computed. The test is passed if $p \geq 0.05$.

From the analysis of the results in Table IV, the Gamma distribution results the best-fitting distribution for the copolarized
channels where a p value of 52.54% and 39.01% is obtained for HH and VV polarizations, respectively. No other distribution passes the χ² GoF test. In the cross-polarized channel (HV), the Gamma distribution still passes the test (p = 10.03%), but it is no longer the best distribution. The Weibull distribution indeed presents a higher p value (63.15%), while all the other distributions fail the test at the same way as the copolarized channels.

Once the best-fitting distribution is found, a fidelity region (FR) may be chosen. The FR represents the interval \([σ_{α_l}; σ_{1−α_u}]\) of the most probable σ values. In particular, \(σ_{α_l}\) and \(σ_{1−α_u}\) represent the percentile \(α_l\)th and \(1−α_u\)th of σ. In formula

\[
\begin{align*}
α_l: \Pr(σ ≤ σ_{α_l}) &= F_σ(σ_{α_l}) = α_l \\
1−α_u: \Pr(σ ≤ σ_{1−α_u}) &= F_σ(σ_{1−α_u}) = 1−α_u
\end{align*}
\]

(7)

Where \(F_σ(·)\) is the cdf of σ. In particular, the lower threshold can be chosen according to the sensitivity of the SAR antenna and it can be set 3dB greater than the system noise equivalent sigma zero (NESZ). In formula

\[
\Pr(σ^0 ≤ NESZ + 3) = F_{σ^0}(NESZ + 3)
\]

(8)

where \(σ^0\) is the normalized RCS and it is linked to σ by the flowing [43]:

\[
σ^0 = \frac{σ}{ΔxΔr/sinθ}
\]

(9)

where Δx and Δr are the spatial resolution in azimuth and slant range, respectively.

For example, by considering the distribution of the RCS values at X band and HH polarization, choosing the Gamma distribution to approximate the RCS data, assuming a typical NESZ = -23 dB for the TerraSAR-X platform [44] and setting \(α_u = 0.01\), it results that \(σ_{α_l} = 9.10 \cdot 10^{-2} m^2\) and \(σ_{1−α_u} = 1.26 m^2\) and, consequently, \(FR = [9.10 \cdot 10^{-2}; 1.26] m^2\).

Different choices may be suggested to set the lower and upper thresholds. However, as a general guideline, the authors advice to perform a sharper cut to the lower tail because, in that region of RCS values, the sea clutter and the SAR azimuth ambiguities are normally included.
V. UNCERTAINTY ON INPUT PARAMETERS AND MODEL INACCURACY

In this section, the accuracy of the RCS relevant to the double-reflection contribution from (2) is analyzed. According to the proposed model, the error sources are the uncertainty on the knowledge of the input parameters and the inaccuracy of the model itself in describing all the details of a complex reflecting object as a ship.

A. Uncertainty on Input Parameters

With regard to (2), the parameters that are a priori known (\( \varphi \) and \( k \)) and retrieved from the literature (\( \varepsilon_{SW} \)) are not considered as sources of error. Vice versa, the uncertainty on the estimated value of the unknown parameters (\( h, \varphi \) and \( \varepsilon_{HULL} \)) and the parameters that are measured directly on the SAR images (\( \sigma_{dev}/L \)) is considered in the following, where each source of error is regarded separately from the other ones.

Let us first consider the uncertainty \( \Delta \sigma \) on the estimated value \( \sigma \), caused by an uncertainty \( \Delta h \) on \( h \):

\[
\Delta \sigma = \left| \frac{\partial \sigma}{\partial h} \right| \Delta h = \frac{\sigma}{h} \Delta h \Rightarrow \frac{\Delta \sigma}{\sigma} = \frac{\Delta h}{h}.
\]

Equation (10) suggests that the relative uncertainty on \( \sigma \) is equal to the relative uncertainty on \( h \); in other words, if \( h \) has been estimated with a certain error, the computed \( \sigma \) will present an error of the same order.

As regards the uncertainties on \( \varphi \) and \( \varepsilon_{HULL} \), instead, deriving their analytical expressions is less useful. Precisely, even if the relative derivatives can still be computed, the retrieved analytical expression would be so involved that useful considerations about the influence on \( \sigma \) estimation could not be carried on. For this reason, the analytical expressions in closed form of the errors have not been computed, but they have been evaluated with the support of a MATLAB code. For the sake of brevity, the authors report here only the graphical representation of the results. In the MATLAB code employed, the a priori known parameters, \( \sigma_{dev}/L \) and \( \varepsilon_{SW} \) are set according to the indications in Table II and the radar parameters of the datasets that will be introduced in the next section. The unknown parameters, instead, are set equal to their mean values according to the distribution functions reported in Table II.

Again, considering the uncertainty on the orientation angle \( \varphi \), it can be written as

\[
\Delta \sigma = \left| \frac{\partial \sigma}{\partial \varphi} \right| \Delta \varphi.
\]

In Fig. 10, \( \left| \frac{\partial \sigma}{\partial \varphi} \right| \) is shown at X band for HH, VV, and HV polarizations. Copolarized channels present the worst case when \( \varphi \) is about 15°, where even a minimum error on the knowledge of the orientation angle results in a completely wrong estimation of the RCS. The best range of value, instead, is included between \( \varphi = 35° \) and \( \varphi = 45° \) where a nonperfect knowledge of the orientation angle does not affect the estimation of the RCS. It is important to underline that, in this same range, the performances of the ship-detection algorithm are worse because most of the incidence radiation from SAR is reflected in the specular direction and, consequently, the ship could appear as dark as the sea clutter in the final SAR image. The cross-polarized channel, instead, presents two relative maxima (when \( \varphi = 10° \) and \( \varphi = 30° \)), while the best case is represented by ships with orientation angle around 20°.

The analysis concerning the dielectric constant of the hull is divided into two parts to consider separately the permittivity and the conductivity. The permittivity is supposed to be unknown in the first part, and the conductivity is supposed to be unknown in the second one, as already done in [26]. However, a general equation can be derived for the uncertainty \( \Delta \sigma \) based on the uncertainty on the permittivity/conductivity of \( \varepsilon_{HULL} \):

\[
\Delta \sigma = \left| \frac{\partial \sigma}{\partial \varepsilon_x} \right| \Delta \varepsilon_x.
\]

where \( \varepsilon_x \) is the real or the imaginary part of \( \varepsilon_{HULL} \) according to the case at issue. In Figs. 11 and 12, \( \left| \frac{\partial \sigma}{\partial \varepsilon_x} \right| \) is shown at X band for HH, VV, and HV polarizations for the real and the imaginary part of \( \varepsilon_{HULL} \), respectively. In Fig. 12, the plot is given in semi-logarithmic scale due to the wide variability in the imaginary part of \( \varepsilon_{HULL} \). From the analysis of the real part of the dielectric constant (Fig. 11), the range of variability of \( \left| \frac{\partial \sigma}{\partial \varepsilon_x} \right| \) is several orders of magnitude smaller than the mean value of \( \sigma \) as it appears in the plots of Figs. 4–6 for each polarization. Consequently, the influence from a nonperfect knowledge of the hull permittivity is negligible for any ship. Moving to the imaginary part of the dielectric constant (Fig. 12), similar considerations can be drawn. The term \( \left| \frac{\partial \sigma}{\partial \varepsilon_x} \right| \) presents remarkable variations for small values of the imaginary part of \( \varepsilon_{HULL} \), but it approaches 0 for \( \text{Im}(\varepsilon_{HULL}) > 10^4 \) for both co- and cross-polarized channels. As a consequence, since the \( \text{Im}(\varepsilon_{HULL}) \) of the metals is much greater than \( 10^4 \) (as it is shown in Table I), the uncertainty relative to an imperfect
knowledge of the conductivity is null if the ship is mostly made by metal, hypothesis certainly verified in many real cases.

Let us finally consider the uncertainty $\Delta \sigma$ on the estimated value $\sigma$, caused by an uncertainty $\Delta (\sigma_{\text{dev}}/L)$ on the roughness ratio $\sigma_{\text{dev}}/L$

$$\Delta \sigma = \left| \frac{\partial \sigma}{\partial (\sigma_{\text{dev}}/L)} \right| \Delta \sigma_{\text{dev}}/L. \quad (13)$$

In Fig. 13, the term $\left| \frac{\partial \sigma}{\partial (\sigma_{\text{dev}}/L)} \right|$ is shown at X band for HH, VV, and HV polarizations, respectively. The trend of the function and the position of the relative minima and maxima are exactly the same for all the polarizations because the difference in polarization is given by the term $S_{pq}$, which represents only a scale factor for the derivative $\left| \frac{\partial \sigma}{\partial (\sigma_{\text{dev}}/L)} \right|$. The uncertainty $\Delta \sigma$ tends to zero when the sea surface is smooth ($\sigma_{\text{dev}}/L \to 0$) and when the sea surface is extremely rough ($\sigma_{\text{dev}}/L \to \infty$). The worst case occurs when $\sigma_{\text{dev}}/L = 0.10$, while the best case occurs when $\sigma_{\text{dev}}/L = 0.16$.

Finally, it is possible to write down the total uncertainty $\Delta \sigma_{\text{tot}}$ on the estimated value $\sigma$ for all the sources of error

$$\Delta \sigma_{\text{tot}} = \left| \frac{\partial \sigma}{\partial h} \right| \Delta h + \left| \frac{\partial \sigma}{\partial \varphi} \right| \Delta \varphi + \left| \frac{\partial \sigma}{\partial (\sigma_{\text{dev}}/L)} \right| \Delta (\sigma_{\text{dev}}/L) \Delta \sigma_{\text{dev}}/L. \quad (14)$$

Obviously, for the considerations carried out from Figs. 11 and 12, the third term of (14) can be neglected and, therefore, the only sources of uncertainty are the freeboard $h$, the orientation angle $\varphi$, and the ratio of the roughness parameters $\sigma_{\text{dev}}/L$.

**B. Model Inaccuracy**

In this section, the errors on the RCS due to approximations on the shape of the canonical ship are analyzed. The simple basic parallelepiped model assumed for the ship (described in Section II) is certainly a valuable starting basis, but it is not able to describe all the scattering mechanisms which occur in a real scenario.

Neglecting the superimposed structures of a ship (ship upper decks and masts), e.g., leads to an underestimation of the dihedral surface, which contributes to the double-reflection mechanism, with a consequent underestimation of the final
Fig. 14. HH intensity SAR image of the Isle of Wight in the slant range (r-axis)/azimuth (x-axis) plane acquired by the TerraSAR-X sensor on (a) 9th November and (b) 12th November. In both images, the red rectangles enclose the ship signatures with available AIS data. The green rectangle includes the signature of a ship which does not fulfill the proposed model.

TABLE VI

<table>
<thead>
<tr>
<th>TARGETS</th>
<th>b (m)</th>
<th>φ (deg)</th>
<th>$\sigma^H_1$ (m²)</th>
<th>$\sigma^H_2$ (m²)</th>
<th>$E_1$ (m²)</th>
<th>$\sigma_\psi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.2710</td>
<td>10°</td>
<td>0.2253</td>
<td>0.2467</td>
<td>-0.0214</td>
<td>-0.0950</td>
</tr>
<tr>
<td>2</td>
<td>0.5859</td>
<td>8°</td>
<td>0.1151</td>
<td>0.1721</td>
<td>-0.0570</td>
<td>-0.4952</td>
</tr>
<tr>
<td>3</td>
<td>0.5102</td>
<td>0°</td>
<td>0.1198</td>
<td>0.1309</td>
<td>-0.0111</td>
<td>-0.0927</td>
</tr>
<tr>
<td>4</td>
<td>1.3124</td>
<td>13°</td>
<td>0.1904</td>
<td>0.2495</td>
<td>-0.0591</td>
<td>-0.3104</td>
</tr>
<tr>
<td>5</td>
<td>2.2000</td>
<td>2°</td>
<td>0.5110</td>
<td>0.6627</td>
<td>-0.1517</td>
<td>-0.2969</td>
</tr>
<tr>
<td>6</td>
<td>1.1582</td>
<td>4°</td>
<td>0.2604</td>
<td>0.3435</td>
<td>-0.0831</td>
<td>-0.0319</td>
</tr>
<tr>
<td>7</td>
<td>0.9231</td>
<td>6°</td>
<td>0.1961</td>
<td>0.2751</td>
<td>-0.0790</td>
<td>-0.4029</td>
</tr>
<tr>
<td>8</td>
<td>0.2500</td>
<td>11°</td>
<td>0.0417</td>
<td>0.0958</td>
<td>-0.0591</td>
<td>-1.2974</td>
</tr>
</tbody>
</table>

RCS. Depending on the dimensions (length and heights) of masts and decks and the orientation angle of the ships, these contributions may be more or less relevant. In addition, the same superimposed structures may also originate strong trihedral reflection mechanisms [30] with an even worse estimation. Finally, a way to assess the inaccuracies deriving from the employment of the simplified ship model is provided in the next section, where the proposed model is compared with the RCS of several ships measured on real SAR images.

VI. VALIDATION RESULTS

The model proposed for the RCS of a canonical ship is tested on two different TerraSAR-X images acquired over the
Solent area (the channel between the Isle of Wight and the Portsmouth’s harbor), in the south of the U.K., in November 2012. The acquisition parameters of the two Stripmap images are reported in Table V.

Before processing the images, the absolute calibration is performed to minimize the radiometry differences and to compare the images [44]. The pixels intensities are scaled according to the following formula [44]:

$$\sigma^0 = k_s|DN|^2 \sin \theta - NESZ$$  

(15)

where $k_s$ is the absolute calibration factor, $|DN|$ is the amplitude of each pixel, and $NESZ$ is the NESZ of the SAR system.

Both $k_s$ and $NESZ$ are provided with the ancillary data of the images. In Fig. 14, the intensity of the SAR images is shown in the slant range/azimuth plane. Some AIS data from [45] are collected and used as ground truth to validate the electromagnetic model proposed. However, the available ground truth is not complete since more ship signatures are clearly detectable from both SAR images [11]. The RCS relevant to the double-reflection contribution of eight ships (four from the first SAR image and four from the second one) is measured on the SAR image by averaging the intensity of the double-reflection line, as already performed in [25] and [27]. In formula

$$\hat{\sigma}_j = \frac{1}{N_j} \sum \sigma_{ij} \quad j = 1, 2, \ldots, 8$$  

(16)

where $\hat{\sigma}_j$ is the RCS of the $j$th ship, $\sigma_{ij}$ is the intensity of the $i$th pixel of the double-reflection contribution of the $j$th ship, and $N_j$ is the number of resolution cells in the double-reflection line relative to the $j$th ship ($N_j = l_{ij}/\Delta x$ where $l_{ij}$ is the length of the $j$th ship). The mean operation let us mitigate the overall contributions of the superimposed structures leading to a less relevant underestimation of the RCS.

The measured RCS is affected by speckle noise and it is possible to evaluate the relative uncertainty $\Delta \hat{\sigma}$ [25], [26]

$$\Delta \hat{\sigma} \leq \frac{\hat{\sigma}_j}{\sqrt{N_j}} \quad j = 1, 2, \ldots, 8$$  

(17)

In (17), $\hat{\sigma}_j/\sqrt{N_j}$ represents the uncertainty in the worst case of fully developed speckle where each contribution is independent from the others (a collection of random variables that are independent and identically distributed).

The signatures of the eight ships under test are highlighted by red rectangles in Fig. 14. The measured RCSs ($\hat{\sigma}_j$), instead, are reported in Table VI and compared to the RCSs deriving from the electromagnetic model ($\sigma_j$). The angle $\varphi$ is computed from the ship bearing provided with the AIS data. The freeboard height $h$, instead, is evaluated from the ship length according to the 1974 SOLAS Convention [37] because AIS data provide only ship length, width, and draught. The values of $\varphi$ and $h$ are shown in Table VI for each ship signature analyzed. All the other parameters involved in the electromagnetic model are either retrieved from the ancillary data of the SAR sensor ($k$ and $\vartheta$) or set equal to the mean value of the distribution function shown in Table II. For each ship, the absolute ($E_j$) and the relative ($e_j$) errors of measurement are computed according to the following and reported in Table VI:

$$E_j = \sigma_j - \hat{\sigma}_j \quad j = 1, 2, \ldots, 8$$

$$e_j = \frac{\sigma_j - \hat{\sigma}_j}{\sigma_j} \quad j = 1, 2, \ldots, 8.$$  

(18)

Results highlight that the electromagnetic model always underestimates the measured RCS on real SAR images. In particular, the average absolute error of measurement is $-0.0646$ m$^2$, while the average relative error of measurement is $-0.4137$ so, in other words, the model underestimates the measured RCS of 1.5 dB on average. The discrepancy between the model and the measured RCSs may be caused by the simplified geometry of the canonical ship where no superimposed structure is taken into account.

Outcomes also show that all the measured RCSs are included in the FR identified in Section IV ($\sigma_{\alpha_1} = 9.10 \times 10^{-2} m^2$ and $\sigma_{1-\alpha_3} = 1.26 m^2$). Therefore, the matching between the measured RCSs and the proposed ship model with the Gamma distribution for the HH polarization can be considered suitable. As a counter-example, a region of interest, highlighted with a green rectangle [Fig. 14(b)], is selected in the second SAR image. It represents the signature of a ship whose RCS is greater than the upper bound of the FR chosen in the proposed model. A zoom of the ship signature is shown in Fig. 15. Unfortunately, AIS signal of this ship is not available and, therefore, it is not possible to retrieve any information about the shape and the size of the ship. However, from the analysis of Fig. 14, a big mast (at the back) and some superimposed structures are clearly identified. As already underlined in Section V-II, in this particular scenario, the electromagnetic model introduced leads to an underestimation of the RCS because it is not able to describe all the scattering mechanisms. The measured RCS is 3.21 m$^2$ but, excluding the mast contribution from the evaluation of the double-reflection contribution, the RCS is reduced to 1.07 m$^2$, thus falling in the selected FR of the model.

VII. CONCLUSION AND FUTURE EXTENSIONS

In this paper, a novel model-based approach for the RCS evaluation of a canonical ship has been presented. The best pdfs have been identified for each polarization at X band...
within the hypotheses introduced on the input parameters of the model: the Gamma and the Weibull distribution are the pdfs which best approximate the simulated RCS data for the co- and cross-polarized channels, respectively (see Table V). The same analysis may also be performed at C and S band.

The influence of an imperfect knowledge of the input parameters on the retrieval of the RCS of the canonical ship has been evaluated through an error budget analysis: the proposed model is affected by the uncertainties on the freeboard height, the orientation angle (see Fig. 10), and the ratio of the roughness parameters (see Fig. 13), while it is robust respect to the uncertainty on the dielectric material composing the hull of the ship (see Figs. 11 and 12).

In general, when a better knowledge on the input parameters is available, different distributions could be considered for them, leading to a different shape and distribution of the RCS values. For example, in specific areas characterized by high maritime traffic and/or geographical straits, ship routes may be more bounded. In these cases, the orientation angle can be more easily evaluated.

Preliminary results are promising as a good match between the measured RCS on real SAR images and the theoretical RCS has been found on a good number of different ships. The hypotheses made, in order to work with a simplified model of the ship, may lead to an underestimation of the real RCS due to superimposed structures and evaluated to be 1.5 dB on average (see Section VI-II). However, this underestimation of the RCS is a minor issue in the SAR ship-detection algorithms meaning that such targets can only be more easily detectable in real scenarios.

The model introduced is interesting especially in consideration of its applicability scenarios. The authors are already working at its inclusion in an SAR-based tool for ship detection. A likelihood-ratio test can indeed be performed at the detection stage leading to an improvement of the overall performance (lower false alarm rate and higher probability of detection) of the algorithms.

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REFERENCES


Pasquale Iervolino (S’12) was born in Naples, Italy, on August 16, 1985. He received the B.S. (cum laude) and the M.S. (cum laude) degrees in telecommunications engineering from the University of Naples Federico II, Naples, Italy, in 2008 and 2010, respectively. He is currently pursuing the Ph.D. degree at the Surrey Space Centre (SSC), University of Surrey, Guildford, U.K., in electronics engineering (remote sensing application group).

His research interests include microwave remote sensing, the development and inversion of scattering models from natural and man-made surfaces, and SAR ship-detection algorithms.

Raffaella Guida (S’04–M’08) was born in Naples, Italy, on October 24, 1975. She received the Laurea degree (cum laude) in telecommunications engineering and the Ph.D. degree in electronic and telecommunications engineering from the University of Naples Federico II, Naples, Italy, in 2003 and 2007, respectively.

In 2006, she received a 2-year research grant from the University of Naples Federico II to be spent at the Department of Electronic and Telecommunication Engineering on the topic of electromagnetic field propagation in urban environment. In 2008, she joined the Surrey Space Centre (SSC), University of Surrey, Guildford, U.K., as a Lecturer of Satellite Remote Sensing, where she is currently a Senior Lecturer and leads the Remote Sensing Applications Research Group. Her research interests include electromagnetics and microwave remote sensing, particularly in simulation and modeling of synthetic aperture radar signals relevant to natural surfaces and urban scenes, new remote sensing mission concepts and applications.

Dr. Raffaella is one of the recipients of the IEEE J-STARs Best Paper Award 2013.

Philip Whittaker received the B.Eng. degree in electronic engineering from the University of Sussex, Brighton, U.K., in 1993, the M.Sc. degree in telecommunications system engineering from the University of Kent, Canterbury, U.K., in 1997, and the Ph.D. degree in spectrum monitoring from low Earth orbit satellites from the University of Surrey, Guildford, U.K., in 2001.

Since 2001, he has been working with Surrey Satellite Technology Ltd., Guildford, U.K., where he has worked on platform RF systems, navigation payloads, and microwave remote sensing systems and is currently a Product Manager for synthetic aperture radar systems.
A Model for the Backscattering From a Canonical Ship in SAR Imagery

Pasquale Iervolino, Student Member, IEEE, Raffaella Guida, Member, IEEE, and Philip Whittaker

Abstract—Synthetic aperture radar (SAR) sensors represent one of the most effective means to support activities in the sector of maritime surveillance. In the field of ship detection, many SAR-based algorithms have been proposed recently, but none of them has ever considered the electromagnetic aspects behind the interactions of SAR signals with the ship and surrounding waters, with the detection step and rate strongly influenced by relative thresholding techniques applied to the SAR amplitude or intensity image. This paper introduces a novel model to evaluate the radar cross section (RCS) backscattered from a canonical ship adapted, to the case at issue, from similar existing models developed for, and applied to, urban areas. The RCS is modeled using the Kirchhoff approximation (KA) within the geometrical optics (GO) solution and, following some assumptions on the scene parameters, derived by empirical observations; its probability density function is derived for all polarizations. An analysis of the sensitiveness of the RCS to the uncertainty on the input scene parameters is then performed. The new model is validated on two different TerraSAR-X images acquired in November 2012 over the Solent area in the U.K.: the RCS relevant to several isolated ships is measured and compared with the expected value deriving from the theoretical model here introduced. Results are widely discussed and ranges of applicability finally suggested.

Index Terms—Electromagnetic modelling, radar cross-sections, radar detection, synthetic aperture radar (SAR).

I. INTRODUCTION

MARITIME surveillance is a topic of great importance and growing interest during the recent years. It is estimated that oceans cover fully 70% of the Earth’s surface and support more than 80% of the global trade [1]. With regard to the European Union (EU), almost 90% of the external freight trade is seaborne, short sea shipping represents 40% on the intra-EU exchanges, and more than 400 million passengers embark and disembark in European ports each year [2]. Looking at the U.K., 50% of the energy supplier is provided by oil shipping. Moreover, Britain’s Sea Trade brings substantial revenue to the Treasury, with a projected value of 700 billion by the end of 2017 [3]. In this context, the U.K. National Maritime Information Centre (NMIC) has the role to monitor and track maritime activity around the U.K. and areas of national interest [4]. Furthermore, NMIC has to support both government and industry decisions in time of need, enabling a better understanding of maritime security issues [4].

One of the main applications of the maritime surveillance is the ship detection. In this context, there is a need for persistent wide-area (global coverage) surveillance. The requirements, which an identification system should provide, are: high probability of detection and low probability of false alarm; accurate geo-locating; ship identification; and ability to operate in all weather and day conditions [5].

While there are several ways to monitor and track ships, there is no single means which can meet all the above requirements. As part of the coastal-based surveillance systems, the automatic identification system (AIS) represents nowadays the most used technique. AIS systems were originally designed for safety reasons and mainly for collision avoidance. They are basically shipborne systems by which ships inform each other and the coastal receivers about their position, course, speed, and name. However, the system is able to cover up to only about 40 km off the coast and obviously requires that the on-board AIS works correctly [6]. The main drawbacks concerning land-based AIS systems are limited spatial coverage, impossibility of berth-to-berth tracking, and need for land-based means. In order to overcome the limitation on the spatial coverage, it has been recently demonstrated that AIS signals can be well received using spaceborne systems [6]. The detection of the AIS signal transmitted from a ship is possible in the entire radio visibility range of a satellite equipped with an AIS payload. For example, for a satellite at an altitude of 650 Km, the average field of view is above 20 million square kilometers [6]. In 2011, a SatAIS system was launched by the German Aerospace Centre (DLR) to enable observations of worldwide ship movements by means of AIS [6]. In the next couple of years, space agencies are planning to launch further SatAIS payloads to enhance the reception of AIS messages as the U.K. NovaSAR-S [7].

A useful support to the ship identification techniques based on AIS could be brought by synthetic aperture radar (SAR) sensors. A spaceborne SAR can be considered a complementary means to the traditional ones, thanks to its peculiar ability to acquire images independently from daylight, meteorological conditions, and national borders. SAR sensors are very useful in the detection of noncooperative ships and in the tracking of small ships without AIS on-board [8]. Modern SAR sensors offer wide spatial coverage and, if operated in constellation, allow for a reduction in the revisit time and, consequently, a better control of open sea areas. The main issue concerned with spaceborne SAR sensors is the impossibility to identify ships,
although high-resolution SAR (HR-SAR) and its submeter resolution can recognize at least the ship type [9].

So far, traditional SAR ship-detection algorithms have been based on constant false alarm rate (CFAR) methods [10]: the sea background is characterized statistically and then the detector looks for individual pixels (or small group of pixels) whose brightness values are statistically unusual [10]. A commonly used statistical model for CFAR detector is the Gaussian distribution. This is often applicable according to the central limit theorem stating that the average of a large number of independently distributed random variables tends to have a Gaussian distribution [11]. Obviously, the more independent the scatterers are in a resolution cell, the more adequate the Gaussian model is. With this underlying model, closed forms of CFAR ship-detection algorithms can be found with a substantial computational saving compared to others based on different distributions [6], [11].

In modern HR SARs, the spatial resolution is tremendously improved and, consequently, the number of independent scatterers in the resolution cell is decreasing. In this framework, the Gaussian distribution is unlikely to stay the one which better models the sea clutter and, indeed, previous works prove that the K distribution and the Generalized Gamma distribution fit much better [12], [13]. Some CFAR algorithms compute the statistics of the sea clutter globally (defining, in turn, a global threshold), while others evaluate the clutter distribution parameters locally to take into account the variability of the sea (and propose an adaptive threshold). The adaptive threshold algorithms are obviously less efficient in terms of computational load but, generally, present better performance than the Global Threshold algorithms [10].

The main limitation of CFAR algorithms is that targets with intensity values very similar to those of sea clutter might not be detected; moreover, the parameters’ estimation distribution and the threshold definition are computationally expensive procedures for non-Gaussian models. A different ship-detection approach relies on the subaperture decomposition where no assumption about the sea background is needed [14]–[17]. The detection is performed based on the different behaviors of targets and clutter regarding the sublook coherence: the original SAR image is decomposed into sublooks (generally between two and four) and then the coherence index between the different sublook images is computed. The random behavior of the sea clutter makes the coherence of its pixels close to 0, while, on the other hand, the targets present a more deterministic behavior and a coherence index closer to 1. In this way, the signal-to-clutter ratio can be increased up to 2 dB [14]. Consequently, from one hand, the targets can be detected easily, but, from the other hand, the spatial resolution is reduced depending on the number of subapertures employed [14]. In [17], the spatial resolution loss is reduced by computing the cross-correlation over partially overlapping subapertures (30%). In [18] and [19], instead, a novel prescreening algorithm, based on the Wavelet Transform (WT), is presented. This approach shows some similarities with the subaperture method. The WT studies a complex phenomenon dividing it into different simpler pieces by projecting the input signal in a particular function space (wavelet space). In this way, the WT is able to characterize the local regularity of the signal: the existence of discontinuities in the original images produces large wavelet coefficients, while homogeneous areas, on the contrary, present small coefficients [18]. In [20] and [22], instead, polarimetric analyses are employed to detect ship targets over the sea clutter pointing out that, generally, SAR polarimetry improves ship-detection performance.

In all the papers cited, the modeling of the ship, whether statistical or analytical, is generally neglected to keep the overall model simple. As a consequence, the detector may result in a higher false-alarm rate [10] since intrinsic properties of a ship, such as its peculiar way to backscatter the transmitted signal, are completely ignored. An extremely simple model for the ship backscattering is introduced in [23] and more recently exploited in [24] to build a generalized likelihood ratio test (GLRT).

The model assumes that the ship pixels are independent and Gaussian distributed. No evidence is provided to support this assumption and the same distribution is employed for both the sea clutter and the ship but with a different standard deviation.

In [11], instead, the authors show that the overall ship-detection performance improves with the inclusion of a proper scattering evaluation block in the detection chain, where the electromagnetic field backscattered from a canonical ship is considered for the first time and used to reject nonparallelepiped-like targets to improve the final detection performance.

In this paper, a complete statistical and analytical evaluation of the most representative backscattering contributions from canonical ship-sea configurations is presented and tested on spaceborne SAR imagery. The work leads to the identification of a suitable statistical distribution to characterize the backscattering return from a ship within certain assumptions. In future, the retrieved distribution can be employed to build a GLRT-based detector and improve the performance of SAR ship-detection algorithms by considering the model for both the ship and the sea clutter.

This paper is organized as follows. In Section II, the electromagnetic model characterizing the different scattering contributions of the canonical ship is presented. In Section III, the canonical ship model is derived and its distribution is shown for all combinations of transmitted/received polarizations. The goodness-of-fit (GoF) test is performed in Section IV to derive the best distribution in modeling the radar cross section (RCS) relevant to the double scattering from the ship. In Section V, the analysis of the sensitivity of the RCS to the errors on the knowledge of the model parameters is presented. In Section VI, the proposed model is validated on actual spaceborne SAR images. Finally, in Section VII, some concluding remarks are reported.

II. ELECTROMAGNETIC MODEL

The modeling of the electromagnetic field backscattered from ships is still poorly considered in ship-detection algorithms, mainly because of the natural complexity behind a reliable model. This section aims to show that the scenario ship sea has many similarities with urban settlements for which scattering models have been introduced, and successfully inverted, in the last years for different kinds of applications, see [25]–[27].
Actually, it is even simpler to model in some situations as, e.g., multiple scattering due to the interaction of the backscattered signal with other ships does not arise in open ocean.

Moreover, the isolation of ships makes their backscattering contributions easier to detect in SAR images. With regard to the nonstationarity of the target and the dynamic scenery in which it is placed, from the literature, it is well known that the motion effects of both capillary and gravitational waves, as well as the possible motion of the ship itself, lead to a change in the Doppler frequencies resulting in azimuth image shift and smearing [10]. For example, as a consequence of its motion, a ship appears shifted along the azimuth direction in the focused SAR image, far from the position of its wake. However, the study of these undesired effects, and their inclusion in the model here being proposed, is at the moment a challenge (see [28]) and led the authors to assume the ocean surface stationary and the ship still. These assumptions will simplify the following analysis without invalidating the model as, essentially, the composition of the signal backscattered from a canonical ship does not change. These considerations brought the authors to reconsider the scattering models introduced in [25] and, after proper modification, adapt them to this new scenario. In addition, further assumptions about the canonical shape and size of the ship are made to reduce the complexity of the problem.

1) The ship is a perfect parallelepiped (hence, superimposed structures and tips are ignored).

2) Its hull is completely smooth.

3) Its dimensions are much larger than the working radar wavelength.

Moreover, we suppose that the ships are isolated (i.e., in open ocean and far from other ships), so that multiple scattering does not arise. We also neglect any diffraction effects. The diffraction contributions are due to the finite dimensions of the scattering surfaces (in this case, the ship hull); hence, they can be modeled as contributions from horizontal and vertical edges of the ship. Since the ships dimensions are very large in terms of wavelength in high regime frequency, edge diffractions are expected to be small with respect to the other reflection contributions (single and multiple scattering), and errors caused by neglecting diffractions are certainly smaller than those caused by simplifying hypotheses on the ship geometry [25].

Within these hypotheses, a real cargo such as the Celtic Fortune in Fig. 1(a) would be more easily modeled with a parallelepiped-like canonical ship forming a perfect dihedral with the sea surface, as the one in Fig. 1(b) drawn with AUTOCAD software. However, the deviation of the angle formed by sea and ship from the right angle of a perfect dihedral is here assumed negligible (as it happens in many real cases).

The canonical ship is regarded as a metallic object which is decomposed by using a series of rectangular facets. Consequently, the total component field can be obtained with a vectorial summation of all integral radiation on each facet [29]. In addition, the computation of the RCS of large and complex targets involves scattering mechanisms of different orders [30]. The corresponding contributions on a SAR image can be mapped or pictorially represented knowing the elementary shape of the ship and some radar parameters. In Fig. 2, where the illumination comes from the left side and \( \vartheta \) is the radar look angle, the different scattering contributions are shown for a canonical ship. According to [26] and [31], scanning the image from near-to-far range at constant azimuth, the following contributions are expected to be found: first, the layover area (single scattering mechanism from the top and lateral side of the ship plus single scattering from the sea), followed by the double-reflection contribution (located in the vertex O), then the single scattering from the top together with the triple scattering (from vertex O to vertex A), the single scattering from the top alone and, finally, the dark shadow area [31]. It has already been demonstrated in [11] that such a geometrical model is a good discriminator between man-made objects over the sea of different shapes.
In order to consider an analytical, closed-form expression for the different scattering contributions, some further assumptions are made.

1) The sea clutter is modeled via a Gaussian stochastic process with Gaussian autocorrelation function (however, more involved stochastic processes can be easily considered in the following derivation [32]).

2) The water is considered infinitely deep, still in terms of working wavelength, so that multiple bounces do not arise from beneath the water surface [27].

Within these hypotheses and using the Kirchhoff approximation (KA), it is possible to evaluate the scattered field at each bounce with physical optic (PO) or geometrical optic (GO) solutions according to the sea roughness. In particular, PO approximation is applied if $k\sigma_{dev} \ll 1$, where $k$ is the radar wavenumber and $\sigma_{dev}$ is the standard deviation describing the stochastic process of the sea. Vice versa, if $k\sigma_{dev} \gg 1$, GO approximation is applied [25]–[27].

Since all the double-reflection rays present the same time delay [31], the double reflection is the dominant scattering contribution. The triple and single scattering contributions, instead, are mixed each other and they are not easily detectable on the SAR images. For these reasons, the following analysis is limited to the double-reflection contribution. For the sake of simplicity, the final formulations of the RCS, already computed in [25], are here reported for both GO–PO and GO–GO approximations. For the GO–PO, the RCS is given by

$$
\sigma = h|S_{pq}|^2 l \tan \vartheta \cos \varphi \exp \left(-4k^2\sigma_{dev}^2\cos^2 \vartheta \right) \times \sum_{m=-\infty}^{\infty} \frac{(2k\sigma_{dev} \cos \vartheta)^m}{m!} \frac{k^2 L^2}{4m} \exp \left(\frac{(2k\sin\varphi \sin\vartheta)^2}{4m}\right).
$$

(1)

Alternatively, for the GO–GO, the RCS is given by

$$
\sigma = \frac{h|S_{pq}|^2 l \tan \vartheta \cos \varphi \left(1 + \tan^2 \vartheta \sin^2 \varphi \right)}{8\pi^2\sigma_{dev}^2 (2/L^2) \cos^2 \vartheta \exp}.\sin^2 \vartheta \right) \times \sum_{m=-\infty}^{\infty} \frac{(2k\sigma_{dev} \cos \vartheta)^m}{m!} \frac{k^2 L^2}{4m} \exp \left(\frac{(2k\sin\varphi \sin\vartheta)^2}{4m}\right).
$$

(2)

In (1) and (2), $\sigma$ represents the RCS relevant to the double-reflection contribution; $S_{pq}$ is the generic element of the scattering matrix with $p$ and $q$ standing for horizontal H, or vertical V polarization, respectively; $l$ is the length of the portion of the ship belonging to the resolution cell, assuming the ship length larger than the SAR spatial resolution; $\sigma_{dev}$ and $L$ are the standard deviation and the correlation length, respectively, of the stochastic process representing the sea clutter; $\varphi$ is the angle between the sensor line of flight and the ship hull to the water surface; $\vartheta$ is the ship look angle and $h$ is the portion of the ship height forming the dihedral surface between the sea and the ship hull. In nautical terms, the latter is also known as freeboard and represents the distance from the waterline to the upper deck of the ship. The distance between the waterline and the bottom of the hull, instead, is known as draught [33]. However, due to the hypothesis e), the draught does not contribute to the double scattering [(1) and (2)] and is therefore neglected in the following. The freeboard and the draught of a ship are, finally, shown in Fig. 3.

In (1) and (2), $S_{pq}$ depends on the dielectric constant of the sea ($\varepsilon_{SW}$), the dielectric constant of the hull ($\varepsilon_{HULL}$), $\varphi$, $\vartheta$, $k$, and the Fresnel coefficient according to the polarization of the propagating wave. The equations to compute $S_{pq}$, for both GO–GO and GO–PO solutions and for each polarization, are reported in [25].

In the next section, some assumptions on the parameters involved in the electromagnetic model are made to model the distribution of the RCS values for the double-reflection contribution.

### III. Computation of the RCS Distribution

In order to compute the RCS relevant to the double-reflection contribution, the parameters involved in (1) and (2) have to be known. Unfortunately, this a priori knowledge is not completely available. Only some parameters are a priori known (the radar look angle and the wavelength), others can be retrieved directly either from the SAR image (the sea roughness parameters) or from the literature (dielectric constant of the sea), while for the remaining ones (the orientation angle, the dielectric constant of the hull, and the freeboard height), suitable probability distribution functions can be estimated bringing, in turn, to a probability density function for the RCS too.

Due to the wind, the sea surface is never completely smooth and presents several capillary waves [28]. For this reason, the GO–GO approximation has been chosen and is the unique solution being analyzed in the following.

#### A. Estimation of the Input Parameters

As anticipated, some parameters are a priori known. In particular, $\vartheta$ and $k$ can be retrieved from the ancillary data of the SAR sensor, while $\varepsilon_{SW}$ is computed according to the model presented in [34] where the real ($\varepsilon'_SW$) and the imaginary ($\varepsilon''_SW$) parts of the saline water are given by

$$
\varepsilon'_{SW} = \varepsilon_{SW\infty} + \frac{\varepsilon_{SW0} - \varepsilon_{SW\infty}}{1 + (2\pi f \tau_{SW})^2} + \frac{\sigma_{SW}}{2\pi \varepsilon_0 f}
$$

$$
\varepsilon''_{SW} = \frac{2\pi f \tau_{SW} (\varepsilon_{SW0} - \varepsilon_{SW\infty})}{1 + (2\pi f \tau_{SW})^2} + \frac{\sigma_{SW}}{2\pi \varepsilon_0 f}
$$

(3)

where $f$ is the working frequency, $\varepsilon_0$ is the permittivity of free space, $\varepsilon_{SW\infty}$ is a constant equal to 4.9, $\sigma_{SW}$ ($S$) is the conductivity of the sea depending on the salinity $S$, $\varepsilon_{SW0}$ ($T$, $S$) and $\tau_{SW}$ ($S$, $T$) are quantities depending on both the salinity $S$ and the sea temperature $T$. The average information about the salinity and the temperature of the North Sea (where the datasets used in this study were acquired) are retrieved from...
The roughness parameters $\sigma_{\text{deg}}/L$ can be estimated, instead, by minimizing the absolute error between the RCS (relevant to the single scattering from the sea) of the sea surface measured on the SAR images and the expected RCS within the GO solution, as shown in [36].

With regard to the angle $\varphi$, the authors assume to work in the worst-case scenario where the orientation angle can be retrieved neither from the ship signature nor from a visible wake. This consideration drives the choice of describing the angle $\varphi$ statistically as uniformly distributed between $0^\circ$ and $45^\circ$ [27]. Obviously, when the heading angle is greater than $45^\circ$, one side of the hull will always present an angle with the projection of the sensor on the sea smaller than $45^\circ$ and, consequently, $\varphi \sim U(0;45)$ takes into account all the possible scenarios. The range of values for the freeboard height is selected according to the amendments of the 1974 SOLAS Convention which regulates the freeboard of the ships of 24-m length or more [37]. Ships are divided into two categories (Type A and Type B) and for each ship length (from 24 to 365 m), the freeboard is provided. Freeboard values are included between 0.2 and 5.3 m [37]. Having the latter as the only available information about the size of the ship, $h$ is consequently selected which is uniformly distributed between 0.2 and 5.3 m [$h \sim U(0.2;5.3)$].

The dielectric constant of the hull is chosen by performing a weighted average of several dielectric constants of materials which mainly compose the structure of a ship. First of all, it is assumed that the canonical ship is made mostly of steel (with a percentage uniformly distributed between 60% and 90%, values based on authors’ empirical evaluations) and for the remaining part of a mixture (equally distributed) of glass, aluminum, and fused silica. Within these hypotheses, $\varepsilon_{\text{HULL}}$ can be computed as already done in [27] and [38]

$$\varepsilon_{\text{HULL}} = p\varepsilon_{st} + \frac{q}{3}(\varepsilon_a + \varepsilon_g + \varepsilon_{sl})$$  (5)

where $p \sim U(0.6;0.9)$, $q = 1 - p$ and $\varepsilon_{st}$, $\varepsilon_a$, $\varepsilon_g$, and $\varepsilon_{sl}$ are the complex relative dielectric constant of steel, aluminum, glass, and fused silica, respectively. Their values are listed in Table I at S, C, and X bands, according to [38] and [39].

In Table II, the way to estimate all the parameters, needed to compute the RCS in (2), is summarized.
component in the case of an ideal dihedral perfectly aligned with the sensor azimuth direction ($\varphi = 0$). Furthermore, in many real cases, when $\varphi \neq 0$, the double reflection component may be lower than the noise floor level and undetectable on SAR images. However, it has been demonstrated that on real SAR images, the RCS relevant to the cross-polarized channel is not negligible and can be useful to improve the performance of the SAR detectors \cite{10, 20}. For these reasons, and for the sake of completeness, the authors have added here also the results relative to the cross-polarized channel.

Similar distributions can be computed also at C and S bands. In the next section, the distributions retrieved at X band are compared, for each polarization, with standard distributions to find the best-fitting distribution.

In this section, the probability distribution function (pdf) and the cumulative distribution function (cdf) for each polarization (HH, VV, and HV) are compared to those of standard distributions. In particular, the inverse Gaussian, the G$^0$, the Gamma, the Rayleigh, and the Weibull distributions, \cite{40, 41}, are analyzed. The pdfs of all the aforementioned distributions are reported in Table III.

The parameters of all the distributions (except those of the G$^0$ distribution) are estimated through the maximum likelihood method and using the MATLAB \textit{mle} function. The shape ($\hat{\alpha}$) and the scale ($\hat{\gamma}$) parameters of the G$^0$ distribution, instead, are estimated through the mixed estimator method introduced in \cite{41}, according to the following equations:

\begin{equation}
Q = \frac{2}{\sqrt{\pi}} \left( \frac{\sqrt{0.5} - 1}{\sqrt{0.5} - 0.5} \right)^{0.5} \left( \frac{\Gamma(-\hat{\alpha})}{\Gamma(-\hat{\alpha} - 0.5)} \right)
\end{equation}

\begin{equation}
\hat{\gamma} = \left( \frac{m_1}{m_1^2} \left( \frac{\Gamma(-\hat{\alpha})}{\Gamma(-\hat{\alpha} - 0.5)} \right)^2 \right)
\end{equation}

where $Q$ is the median of $\sigma/E[\sigma]$, $m_1$ is the first $\sigma$ moment, and $\Gamma(\cdot)$ is the gamma function.

In Figs. 7–9, the pdfs and the cdfs are shown for all the aforementioned distributions and compared with the histogram data at X band for HH, VV, and HV polarization, respectively. It is possible to note that the distribution with one parameter (Rayleigh, plotted in cyan) is not able to fit the heterogeneity of the data for all polarizations. The best fittings, instead, are obtained using the Gamma (in purple) and the Weibull (in green) for all the distributions. In order to verify if and how well these distributions approximate the histograms of the RCS data, a GoF test has been performed. In Table IV, the $\chi^2$ GoF test \cite{42} is performed considering a significance level of 5% for all the distributions for each polarization and, consequently, the $p$ value is computed. The test is passed if $p \geq 0.05$.

From the analysis of the results in Table IV, the Gamma distribution results the best-fitting distribution for the copolarized
channels where a \( p \text{ value} \) of 52.54\% and 39.01\% is obtained for HH and VV polarizations, respectively. No other distribution passes the \( \chi^2 \) GoF test. In the cross-polarized channel (HV), the Gamma distribution still passes the test (\( p = 10.03\% \)), but it is no longer the best distribution. The Weibull distribution indeed presents a higher \( p \text{ value} \) (63.15\%), while all the other distributions fail the test at the same way as the copolarized channels.

Once the best-fitting distribution is found, a fidelity region (FR) may be chosen. The FR represents the interval \([\sigma_{\alpha_l}; \sigma_{1-\alpha_u}]\) of the most probable \( \sigma \) values. In particular, \( \sigma_{\alpha_l} \) and \( \sigma_{1-\alpha_u} \) represent the percentile (\( \alpha_l \))th and (1 - \( \alpha_u \))th of \( \sigma \). In formula

\[
\alpha_l: \Pr(\sigma \leq \sigma_{\alpha_l}) = F_\sigma(\sigma_{\alpha_l}) = \alpha_l \\
1 - \alpha_u: \Pr(\sigma \leq \sigma_{1-\alpha_u}) = F_\sigma(\sigma_{1-\alpha_u}) = 1 - \alpha_u
\]  

(7)

Where \( F_\sigma(\cdot) \) is the cdf of \( \sigma \). In particular, the lower threshold can be chosen according to the sensitivity of the SAR antenna and it can be set 3dB greater than the system noise equivalent sigma zero (NESZ). In formula

\[
\Pr(\sigma^0 \leq NESZ + 3) = F_{\sigma^0}(NESZ + 3)
\]  

(8)

where \( \sigma^0 \) is the normalized RCS and it is linked to \( \sigma \) by the flowing [43]:

\[
\sigma^0 = \frac{\Delta x \Delta r'}{\sin \theta}
\]  

(9)

where \( \Delta x \) and \( \Delta r \) are the spatial resolution in azimuth and slant range, respectively.

For example, by considering the distribution of the RCS values at X band and HH polarization, choosing the Gamma distribution to approximate the RCS data, assuming a typical NESZ = -23 dB for the TerraSAR-X platform [44] and setting \( \alpha_u = 0.01 \), it results that \( \sigma_{\alpha_l} = 9.10 \cdot 10^{-2} \)  \( m^2 \) and \( \sigma_{1-\alpha_u} = 1.26 \)  \( m^2 \) and, consequently, \( FR = [9.10 \cdot 10^{-2}; 1.26] \)  \( m^2 \).

Different choices may be suggested to set the lower and upper thresholds. However, as a general guideline, the authors advice to perform a sharper cut to the lower tail because, in that region of RCS values, the sea clutter and the SAR azimuth ambiguities are normally included.


V. Uncertainty on Input Parameters and Model Inaccuracy

In this section, the accuracy of the RCS relevant to the double-reflection contribution from (2) is analyzed. According to the proposed model, the error sources are the uncertainty on the knowledge of the input parameters and the inaccuracy of the model itself in describing all the details of a complex reflecting object as a ship.

A. Uncertainty on Input Parameters

With regard to (2), the parameters that are a priori known \((\vartheta \text{ and } k)\) and retrieved from the literature \((\varepsilon_{SW})\) are not considered as sources of error. Vice versa, the uncertainty on the estimated value \(\sigma\) of the unknown parameters \((h, \varphi \text{ and } \varepsilon_{HULL})\) and the parameters that are measured directly on the SAR images \((\sigma_{dev}/L)\) is considered in the following, where each source of error is regarded separately from the other ones.

Let us first consider the uncertainty \(\Delta \sigma\) on the estimated value \(\sigma\), caused by an uncertainty \(\Delta h\) on \(h\):

\[
\Delta \sigma = \left| \frac{\partial \sigma}{\partial h} \right| \Delta h = \frac{\sigma}{h} \Delta h \Rightarrow \frac{\Delta \sigma}{\sigma} = \frac{\Delta h}{h}. \tag{10}
\]

Equation (10) suggests that the relative uncertainty on \(\sigma\) is equal to the relative uncertainty on \(h\); in other words, if \(h\) has been estimated with a certain error, the computed \(\sigma\) will present an error of the same order.

As regards the uncertainties on \(\varphi\) and \(\varepsilon_{HULL}\), instead, deriving their analytical expressions is less useful. Precisely, even if the relative derivatives can still be computed, the retrieved analytical expression would be so involved that useful considerations about the influence on \(\sigma\) estimation could not be carried on. For this reason, the analytical expressions in closed form of the errors have not been computed, but they have been evaluated with the support of a MATLAB code. For the sake of brevity, the authors report here only the graphical representation of the results. In the MATLAB code employed, the \(a\) priori known parameters, \(\sigma_{dev}/L\), and \(\varepsilon_{SW}\) are set according to the indications in Table II and the radar parameters of the datasets that will be introduced in the next section. The unknown parameters, instead, are set equal to their mean values according to the distribution functions reported in Table II.

Again, considering the uncertainty on the orientation angle \(\varphi\), it can be written as

\[
\Delta \sigma = \left| \frac{\partial \sigma}{\partial \varphi} \right| \Delta \varphi. \tag{11}
\]

In Fig. 10, \(\left| \frac{\partial \sigma}{\partial \varphi} \right|\) is shown at X band for HH, VV, and HV polarizations. Copolarized channels present the worst case when \(\varphi\) is about 15°, where even a minimum error on the knowledge of the orientation angle results in a completely wrong estimation of the RCS. The best range of value, instead, is included between \(\varphi = 35°\) and \(\varphi = 45°\), where a nonperfect knowledge of the orientation angle does not affect the estimation of the RCS. It is important to underline that, in this range, the performances of the ship-detection algorithm are worse because most of the incidence radiation from SAR is reflected in the specular direction and, consequently, the ship could appear as dark as the sea clutter in the final SAR image. The cross-polarized channel, instead, presents two relative maxima (when \(\varphi = 10°\) and \(\varphi = 30°\)), while the best case is represented by ships with orientation angle around 20°.

The analysis concerning the dielectric constant of the hull is divided into two parts to consider separately the permittivity and the conductivity. The permittivity is supposed to be known in the first part, and the conductivity is supposed to be unknown in the second one, as already done in [26]. However, a general equation can be derived for the uncertainty \(\Delta \sigma\) based on the uncertainty on the permittivity/conductivity of \(\varepsilon_{HULL}\):

\[
\Delta \sigma = \left| \frac{\partial \sigma}{\partial \varepsilon} \right| \Delta \varepsilon_{x_x} \tag{12}
\]

where \(\varepsilon_{x_x}\) is the real or the imaginary part of \(\varepsilon_{HULL}\) according to the case at issue. In Figs. 11 and 12, \(\left| \frac{\partial \sigma}{\partial \varepsilon} \right|\) is shown at X band for HH, VV, and HV polarizations for the real and the imaginary part of \(\varepsilon_{HULL}\), respectively. In Fig. 12, the plot is given in semi-logarithmic scale due to the wide variability in the imaginary part of \(\varepsilon_{HULL}\). From the analysis of the real part of the dielectric constant (Fig. 11), the range of variability of \(\left| \frac{\partial \sigma}{\partial \varepsilon} \right|\) is several orders of magnitude smaller than the mean value of \(\sigma\) as it appears in the plots of Figs. 4–6 for each polarization. Consequently, the influence from a nonperfect knowledge of the hull permittivity is negligible for any ship. Moving to the imaginary part of the dielectric constant (Fig. 12), similar considerations can be drawn. The term \(\left| \frac{\partial \sigma}{\partial \varepsilon_{x_x}} \right|\) presents remarkable variations for small values of the imaginary part of \(\varepsilon_{HULL}\), but it approaches 0 for \(\text{Im}(\varepsilon_{HULL}) > 10^3\) for both co- and cross-polarized channels. As a consequence, since the \(\text{Im}(\varepsilon_{HULL})\) of the metals is much greater than \(10^3\) (as it is shown in Table I), the uncertainty relative to an imperfect...
Fig. 11. Plots of the uncertainty relative to the real part of the dielectric constant of the hull $\varepsilon_{\text{HULL}}$ for each polarization at X band.

Fig. 12. Plots of the uncertainty relative to the imaginary part of the dielectric constant of the hull $\varepsilon_{\text{HULL}}$ for each polarization at X band.

In Fig. 13, the term $\left| \frac{\partial \sigma}{\partial (\sigma_{\text{dev}}/L)} \right|$ is shown at X band for HH, VV, and HV polarizations, respectively. The trend of the function and the position of the relative minima and maxima are exactly the same for all the polarizations because the difference in polarization is given by the term $S_{pq}$, which represents only a scale factor for the derivative $\frac{\partial \sigma}{\partial (\sigma_{\text{dev}}/L)}$. The uncertainty $\Delta \sigma$ tends to zero when the sea surface is smooth ($\sigma_{\text{dev}}/L \rightarrow 0$) and when the sea surface is extremely rough ($\sigma_{\text{dev}}/L \rightarrow \infty$). The worst case occurs when $\sigma_{\text{dev}}/L = 0.10$, while the best case occurs when $\sigma_{\text{dev}}/L = 0.16$.

Finally, it is possible to write down the total uncertainty $\Delta \sigma_{\text{tot}}$ on the estimated value $\sigma$ for all the sources of error:

$$\Delta \sigma_{\text{tot}} = \left| \frac{\partial \sigma}{\partial h} \right| \Delta h + \left| \frac{\partial \sigma}{\partial \phi} \right| \Delta \phi + \left| \frac{\partial \sigma}{\partial \varepsilon} \right| \Delta \varepsilon + \frac{\partial \sigma}{\partial (\sigma_{\text{dev}}/L)} \left| \frac{\partial (\sigma_{\text{dev}}/L)}{\sigma_{\text{dev}}/L} \right| \Delta \sigma_{\text{dev}}/L.$$  

(14)

Obviously, for the considerations carried out from Figs. 11 and 12, the third term of (14) can be neglected and, therefore, the only sources of uncertainty are the freeboard $h$, the orientation angle $\phi$, and the ratio of the roughness parameters $\sigma_{\text{dev}}/L$.

### B. Model Inaccuracy

In this section, the errors on the RCS due to approximations on the shape of the canonical ship are analyzed. The simple basic parallelepiped model assumed for the ship (described in Section II) is certainly a valuable starting basis, but it is not able to describe all the scattering mechanisms which occur in a real scenario.

Neglecting the superimposed structures of a ship (ship upper decks and masts), e.g., leads to an underestimation of the dihedral surface, which contributes to the double-reflection mechanism, with a consequent underestimation of the final...
RCS. Depending on the dimensions (length and heights) of masts and decks and the orientation angle of the ships, these contributions may be more or less relevant. In addition, the same superimposed structures may also originate strong trihedral reflection mechanisms [30] with an even worse estimation. Finally, a way to assess the inaccuracies deriving from the employment of the simplified ship model is provided in the next section, where the proposed model is compared with the RCS of several ships measured on real SAR images.

VI. VALIDATION RESULTS

The model proposed for the RCS of a canonical ship is tested on two different TerraSAR-X images acquired over the
Solent area (the channel between the Isle of Wight and the Portsmouth’s harbor), in the south of the U.K., in November 2012. The acquisition parameters of the two Stripmap images are reported in Table V.

Before processing the images, the absolute calibration is performed to minimize the radiometry differences and to compare the images [44]. The pixels intensities are scaled according to the following formula [44]:

$$\sigma^0 = k_s|DN|^2\sin\theta - NESZ$$  \hspace{1cm} (15)

where $k_s$ is the absolute calibration factor, $|DN|$ is the amplitude of each pixel, and $NESZ$ is the NESZ of the SAR system. Both $k_s$ and $NESZ$ are provided with the ancillary data of the images. In Fig. 14, the intensity of the SAR images is shown in the slant range/azimuth plane. Some AIS data from [45] are collected and used as ground truth to validate the electromagnetic model proposed. However, the available ground truth is not complete since more ship signatures are clearly detectable from both SAR images [11]. The RCS relevant to the double-reflection contribution of eight ships (four from the first SAR image and four from the second one) is measured on the SAR image by averaging the intensity of the double-reflection line, as already performed in [25] and [27]. In formula

$$\hat{\sigma}_j = \frac{1}{N_j} \sum_{i=1}^{N_j} \hat{\sigma}_{ij} \quad j = 1, 2, \ldots, 8$$  \hspace{1cm} (16)

where $\hat{\sigma}_j$ is the RCS of the jth ship, $\hat{\sigma}_{ij}$ is the intensity of the $i$th pixel of the double-reflection contribution of the $j$th ship, and $N_j$ is the number of resolution cells in the double-reflection line relative to the jth ship ($N_j = l_{ij}/\Delta x$ where $l_{ij}$ is the length of the jth ship). The mean operation let us mitigate the overall contributions of the superimposed structures leading to a less relevant underestimation of the RCS.

The measured RCS is affected by speckle noise and it is possible to evaluate the relative uncertainty $\Delta \hat{\sigma}$ [25], [26]

$$\Delta \hat{\sigma} \leq \frac{\hat{\sigma}_j}{\sqrt{N_j}} \quad j = 1, 2, \ldots, 8.$$  \hspace{1cm} (17)

In (17), $\hat{\sigma}_j/\sqrt{N_j}$ represents the uncertainty in the worst case of fully developed speckle where each contribution is independent from the others (a collection of random variables that are independent and identically distributed).

The signatures of the eight ships under test are highlighted by red rectangles in Fig. 14. The measured RCSs ($\hat{\sigma}_j$), instead, are reported in Table VI and compared to the RCSs deriving from the electromagnetic model ($\sigma_j$). The angle $\varphi$ is computed from the ship bearing provided with the AIS data. The freeboard height $h$, instead, is evaluated from the ship length according to the 1974 SOLAS Convention [37] because AIS data provide only ship length, width, and draught. The values of $\varphi$ and $h$ are shown in Table VI for each ship signature analyzed. All the other parameters involved in the electromagnetic model are either retrieved from the ancillary data of the SAR sensor ($k$ and $N$) or set equal to the mean value of the distribution function shown in Table II. For each ship, the absolute ($E_j$) and the relative ($e_j$) errors of measurement are computed according to the following formulas and reported in Table VI:

$$E_j = \sigma_j - \hat{\sigma}_j \quad j = 1, 2, \ldots, 8$$

$$e_j = \frac{\sigma_j - \hat{\sigma}_j}{\sigma_j} \quad j = 1, 2, \ldots, 8.$$  \hspace{1cm} (18)

Results highlight that the electromagnetic model always underestimates the measured RCS on real SAR images. In particular, the average absolute error of measurement is $0.0646 \text{ m}^2$, while the average relative error of measurement is $-0.4137$ so, in other words, the model underestimates the measured RCS of $1.5 \text{ dB}$ on average. The discrepancy between the model and the measured RCSs may be caused by the simplified geometry of the canonical ship where no superimposed structure is taken into account.

Outcomes also show that all the measured RCSs are included in the FR identified in Section IV ($\sigma_{10} = 9.10 \cdot 10^{-2} \text{ m}^2$ and $\sigma_1 - \sigma_{10} = 1.26 \text{ m}^2$). Therefore, the matching between the measured RCSs and the proposed ship model with the Gamma distribution for the HH polarization can be considered suitable. As a counter-example, a region of interest, highlighted with a green rectangle [Fig. 14(b)], is selected in the second SAR image. It represents the signature of a ship whose RCS is greater than the upper bound of the FR chosen in the proposed model. A zoom of the ship signature is shown in Fig. 15. Unfortunately, AIS signal of this ship is not available and, therefore, it is not possible to retrieve any information about the shape and the size of the ship. However, from the analysis of Fig. 14, a big mast (at the back) and some superimposed structures are clearly identified. As already underlined in Section V-II, in this particular scenario, the electromagnetic model introduced leads to an underestimation of the RCS because it is not able to describe all the scattering mechanisms. The measured RCS is $3.21 \text{ m}^2$ but, excluding the mast contribution from the evaluation of the double-reflection contribution, the RCS is reduced to $1.07 \text{ m}^2$, thus falling in the selected FR of the model.

VII. CONCLUSION AND FUTURE EXTENSIONS

In this paper, a novel model-based approach for the RCS evaluation of a canonical ship has been presented. The best pdfs have been identified for each polarization at X band...
within the hypotheses introduced on the input parameters of the model: the Gamma and the Weibull distribution are the pdfs which best approximate the simulated RCS data for the co- and cross-polarized channels, respectively (see Table V). The same analysis may also be performed at C and S band.

The influence of an imperfect knowledge of the input parameters on the retrieval of the RCS of the canonical ship has been evaluated through an error budget analysis: the proposed model is affected by the uncertainties on the freeboard height, the orientation angle (see Fig. 10), and the ratio of the roughness parameters (see Fig. 13), while it is robust with respect to the uncertainty on the dielectric material composing the hull of the ship (see Figs. 11 and 12).

In general, when a better knowledge on the input parameters is available, different distributions could be considered for them, leading to a different shape and distribution of the RCS values. For example, in specific areas characterized by high maritime traffic and/or geographical straits, ship routes may be more bounded. In these cases, the orientation angle can be more easily evaluated.

Preliminary results are promising as a good match between the measured RCS on real SAR images and the theoretical RCS has been found on a good number of different ships. The hypotheses made, in order to work with a simplified model of the ship, may lead to an underestimation of the real RCS due to superimposed structures and evaluated to be 1.5 dB on average (see Section VI-II). However, this underestimation of the RCS is a minor issue in the SAR ship-detection algorithms meaning that such targets can only be more easily detectable in real scenarios.

The model introduced is interesting especially in consideration of its applicability scenarios. The authors are already working at its inclusion in an SAR-based tool for ship detection. A likelihood-ratio test can indeed be performed at the detector stage leading to an improvement of the overall performance (lower false alarm rate and higher probability of detection) of the algorithms.

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REFERENCES


Pasquale Iervolino (S’12) was born in Naples, Italy, on August 16, 1985. He received the B.S. (cum laude) and the M.S. (cum laude) degrees in telecommunications engineering from the University of Naples Federico II, Naples, Italy, in 2008 and 2010, respectively. He is currently pursuing the Ph.D. degree at the Surrey Space Centre (SSC), University of Surrey, Guildford, U.K., in electronics engineering (remote sensing application group).

His research interests include microwave remote sensing, the development and inversion of scattering models from natural and man-made surfaces, and SAR ship-detection algorithms.

Raffaella Guida (S’04–M’08) was born in Naples, Italy, on October 24, 1975. She received the Laurea degree (cum laude) in telecommunications engineering and the Ph.D. degree in electronic and telecommunications engineering from the University of Naples Federico II, Naples, Italy, in 2003 and 2007, respectively.

In 2006, she received a 2-year research grant from the University of Naples Federico II to be spent at the Department of Electronic and Telecommunication Engineering on the topic of electromagnetic field propagation in urban environment. In 2008, she was also a Guest Scientist with the Department of Photogrammetry and Remote Sensing, Technische Universität München, Munich, Germany. In 2008, she joined the Surrey Space Centre (SSC), University of Surrey, Guildford, U.K., as a Lecturer of Satellite Remote Sensing, where she is currently a Senior Lecturer and leads the Remote Sensing Applications Research Group. Her research interests include electromagnetics and microwave remote sensing, particularly in simulation and modeling of synthetic aperture radar signals relevant to natural surfaces and urban scenes, new remote sensing mission concepts and applications. Dr. Raffaella is one of the recipients of the IEEE J-STARS Best Paper Award 2013.

Philip Whittaker received the B.Eng. degree in electronic engineering from the University of Sussex, Brighton, U.K., in 1993, the M.Sc. degree in telecommunications system engineering from the University of Kent, Canterbury, U.K., in 1997, and the Ph.D. degree in spectrum monitoring from low Earth orbit satellites from the University of Surrey, Guildford, U.K., in 2001. Since 2001, he has been working with Surrey Satellite Technology Ltd, Guildford, U.K., where he has worked on platform RF systems, navigation payloads, and microwave remote sensing systems and is currently a Product Manager for synthetic aperture radar systems.