Game Theory Based Radio Resource Allocation for Full-Duplex Systems

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***Abstract*—Full-duplex transceivers enable transmission and reception at the same time on the same frequency, and have the potential to double the wireless system spectral efficiency. Recent studies have shown the feasibility of full-duplex transceivers. In this paper, we address the radio resource allocation problem for full-duplex system. Due to the self-interference and inter-user interference, the problem is coupled between uplink and downlink channels, and can be formulated as joint uplink and downlink sum-rate maximization. As the problem is non-convex, an iterative algorithm is proposed based on game theory by modeling the problem as a noncooperative game between the uplink and downlink channels. The algorithm iteratively carries out optimal uplink and downlink resource allocation until a Nash equilibrium is achieved. Simulation results show that the algorithm achieves fast convergence, and can significantly improve the full-duplex performance comparing to the equal resource allocation approach. Furthermore, the full-duplex system with the proposed algorithm can achieve considerable gains in spectral efficiency, that reach up to 40%, comparing to half-duplex system.**

***Keywords*—Full-duplex, radio resource allocation, game theory.**

I. INTRODUCTION

Full-duplex communications is an emerging technique and is theoretically capable of doubling the link capacity, or equivalently halving the spectral resource usage. The main idea behind wireless full-duplex is to enable radios to transmit and receive simultaneously on the same frequency band at the same time slot [1]. This contrasts with conventional half-duplex operation where transmission and reception either differ in time, or in frequency. Full-duplex transmission has the potential to be deployed in different scenarios to improve the overall system spectral efficiency. A full-duplex central node, such as a base-station in the cellular network, can communicate simultaneously in uplink and downlink with a full-duplex terminal or with two half-duplex terminals using the same spectral resources. It is assumed that by doubling each single-link capacity, full-duplexing can significantly increase system level throughput in diverse applications in wireless communication networks. This is because the full-duplex approach provides additional degrees of freedom for protocol design, by removing constraints of conventional half-duplex system protocol, e.g. the use of strictly framed structured signals for bidirectional transmission.

Full-duplex wireless communications has recently attracted a considerable attention [2]–[6]. However, most related works deal with modelling and cancellation of the self-interference which is the coupling of transmit signal to the collocated receiver. Deployment of full-duplex systems in a network however needs to address issues beyond link level, e.g. intra-cell and inter-cell in-band interference management. To verify the performance of any large scale deployment of full-duplex nodes in wireless network, system-level evaluation of such systems is necessary. However such an analysis has received little attention in the literature so far.

This paper presents system-level analysis of full-duplex deployment in a single-cell scenario by evaluating the overall spectral efficiency improvement. It is noticed that self-interference cancellation problem in the full-duplex receiver is less stringent in the small-cell scenario due to relatively short physical distance between link ends (and thus lower required transmit power). Moreover, in base-stations because of the larger physical dimensions compared to user equipment, better isolation between transmitter and receiver circuits can be achieved by antenna separation, which results in better isolation and lower self-interference level. For those reasons we focus on the small-cell scenario in which full-duplex base-station communicates to a number of half-duplex nodes.

Due to the effect of interference, radio resource allocation plays an important role in optimizing the full-duplex system performance. Current radio resource allocation algorithms are designed for half-duplex systems [7], [8], where the uplink and downlink channels are orthogonal to each other, hence, can be optimized independently. On the contrary, the uplink and downlink resource allocation problem is coupled in full-duplex, and has to be optimized jointly. Thus, it is not possible to apply the conventional half-duplex resource allocation algorithms to full-duplex systems in a straightforward manner. In this paper, we address the joint radio resource allocation problem for uplink and downlink channels in a single-cell full-duplex system, with the objective of sum-rate maximization. As the problem is non-convex, we model the problem as a non-cooperative game between the uplink and downlink channels, and propose an iterative algorithm to achieve the Nash equilibrium. The simulation results show that the proposed algorithm achieves fast convergence rate and the full-duplex significantly outperforms half-duplex performance. The remainder of the paper is organized as follows: Sec. II presents the system model and the resource allocation problem formulation. In Sec. III, we propose a novel full-duplex resource allocation algorithm for the system under study based on the game theory. In Sec. IV, we evaluate the convergence and the performance of the proposed algorithm. Finally, concluding remarks are drawn in Sec. V.

II. System Model and Problem Formulation

We consider a single-cell with a full-duplex base-station that communicates with half-duplex users' terminals through multicarrier orthogonal channels, such as Orthogonal Frequency-Division Multiplexing (OFDM). A simplified diagram of the considered full-duplex system is depicted in Fig. 1. The available bandwidth is divided into a set of subcarriers $\mathcal{N} = \{1, \ldots, N\}$. The base-station uses the available subcarriers to transmit to the *downlink* users. In the same time, the *uplink* users can use the same subcarriers to transmit to the base-station. At the base-station, the downlink transmitted signal will leak into its own receiver RF chain (which is referred to as self-interference) and mixed with the received signals from the uplink channel. Advanced analog and digital self-interference cancellation techniques are able to suppress significant amount of this interference [3]. Similarly, at the user terminal, the downlink channel will suffer from the inter-user interference due to the uplink transmission from other users. However, unlike the self-interference, there is no cancellation implemented for the inter-user interference. Let $\mathcal{K} = \{1, \ldots, K\}$ be the set of users that transmit in the uplink, and $\mathcal{J} = \{1, \ldots, J\}$ be the set of users that receive in the downlink. Using Shannon's formula for Gaussian channel, the user's rate on a subcarrier in the downlink channel is given by

$$R_{j,n} = x_{j,n} \log_2 \left(1 + \frac{p_n h_{j,n}}{\sigma_e^2 + I_{j,n}}\right), \quad \forall j \in \mathcal{J}, n \in \mathcal{N}, \quad (1)$$

where $I_{j,n}$ is the inter-user interference from the uplink users to the jth downlink user on the nth subcarrier, and it's given by

$$I_{j,n} = \sum_{k \in \mathcal{K}} p_{k,n} g_{k,j}. \quad (2)$$

Here, p_n is the power transmitted by the base-station on the nth subcarrier, and $h_{j,n}$ is the downlink channel gain between the jth user and the base-station on the nth subcarrier. $p_{k,n}$ is the power transmitted by the kth user on the nth subcarrier in uplink. $g_{k,j}$ is the channel gain between the kth and jth users, and σ_e^2 is the additive white Gaussian noise (AWGN) power per subcarrier at the user equipment. $x_{j,n}$ is the subcarrier allocation binary indicator where $x_{j,n}$ equals to 1 if subcarrier n is allocated to user j, and 0 otherwise. In the downlink, each subcarrier is allocated to one user only. For uplink channel, the users' rate on each subcarrier will be

$$R_{k,n} = \log_2 \left(1 + \frac{p_{k,n} h_{k,n}}{\sigma_b^2 + I_{k,n} + p_n \beta}\right), \quad \forall k \in \mathcal{K}, n \in \mathcal{N}, \quad (3)$$

where β represents the self-interference cancellation factor at the base-station, i.e., $p_n \beta$ will be the residual self-interference on the nth subcarrier. $h_{k,n}$ is the uplink channel gain between the kth user and the base-station on the nth subcarrier, and σ_b^2 is the AWGN power per subcarrier at the base-station. $I_{k,n}$ is the interference the kth user sees from other uplink users on the nth subcarrier. Assuming that the users are decoded in an increasing order of their indices, the first user to be decoded, $k = 1$, will see interference from all the other users $k = 2, \cdots, K$, and the second user to be decoded will see interference from the users $k = 3, \cdots, K$, and so on. Thus, the interference $(I_{k,n})$ each user experience on each subcarrier with this decoding order will be

$$I_{k,n} = \sum_{j=k+1}^{K} p_{j,n} h_{j,n}, \quad k = 1, \cdots, K-1. \quad (4)$$

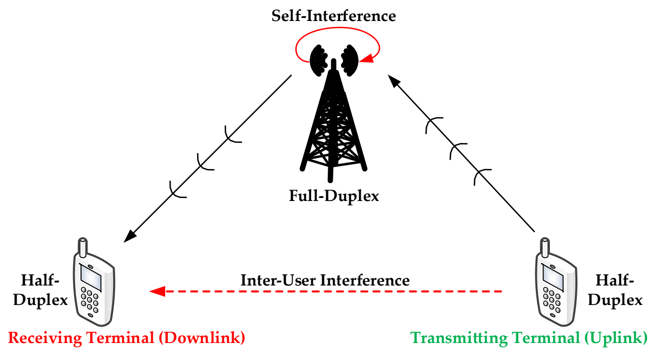

Fig. 1. Full-duplex system model.

It is worth mentioning that the decoding order does not affect the sum-rate, and any arbitrary decoding order can be assumed. It is clear from (1) and (3) that the power allocation is coupled between downlink and uplink channels due to the self-interference and inter-user interference. Thus, a joint downlink and uplink power allocation need to be implemented to optimize the system performance in both transmission directions. With the knowledge of channel state information, the downlink subcarrier and power allocation problem that maximizes the system spectral efficiency can be formulated as follows

$$\max_{x_{j,n}, p_n, p_{k,n}} \sum_{j \in \mathcal{J}} \sum_{n \in \mathcal{N}} R_{j,n} + \sum_{k \in \mathcal{K}} \sum_{n \in \mathcal{N}} R_{k,n} \quad (5)$$

subject to

$$\sum_{j \in \mathcal{J}} x_{j,n} \leq 1, \quad x_{j,n} \in \{0, 1\}, \quad \forall n \in \mathcal{N}, \forall j \in \mathcal{J}, \quad (6)$$

$$\sum_{n \in \mathcal{N}} p_n \leq P_T, \quad (7)$$

$$\sum_{n \in \mathcal{N}} p_{k,n} \leq P_k, \quad \forall k \in \mathcal{K}, \quad (8)$$

where P_k and P_T are the maximum transmit powers of the kth user and the base-station, respectively. Unfortunately, the problem (5-8) cannot be expressed as a convex optimization problem because the objective functions (1) and (3) are non-convex functions due to the interference terms. Finding the optimal solution is computationally difficult and intractable for systems with large number of users $(J$ and $K)$ and subcarriers (N). Consequently, instead of seeking the global optimal, we will solve the problem for competitively optimal power allocation by modeling the problem as a noncooperative game.

III. Proposed Algorithm

Initially, we consider the two users case, i.e. one uplink user (u) and one downlink (d) user, and extend the solution to more generic scenario later. A utility function (or payoff function) need to be defined to construct a game based on it.

The users' data rates can be considered as the reward obtained by transmitting power. The rates of the two users will be

$$R^u = \sum_{n\in\mathcal{N}} \log_2\left(1+\frac{p_{u,n}h_{u,n}}{\sigma_b^2+p_n\beta}\right), \tag{9}$$

$$R^d = \sum_{n\in\mathcal{N}} \log_2\left(1+\frac{p_n h_{d,n}}{\sigma_e^2+p_{u,n}g_{u,d}}\right). \tag{10}$$

It can be noticed that when each transmitter (uplink or downlink) increases its power, it will increase its data rate, however, at the same time it will increase the interference on the other channel direction. This can be modeled as noncooperative game between the uplink and downlink channels. In this sitting, each user attempts to maximise its performance regardless of the other user performance. This process can be done continuously, and if there is an equilibrium point it will converge. This equilibrium is referred to as Nash equilibrium, which is defined as a strategy set in which each user strategy is an optimal response to the other user's strategy. The following theorem shows the condition under which a Nash equilibrium will exists for the full-duplex two users noncooperative game.

Theorem 1. *At least one Nash equilibrium strategy exists in the two users full-duplex game, if the following condition is satisfied*

$$\beta < \min_{n\in\mathcal{N}}\left(\frac{h_{u,n}h_{d,n}}{g_{u,d}}\right). \tag{11}$$

Proof: The two users' rate can be reformulated as follows

$$R^u = \sum_{n\in\mathcal{N}} \log_2\left(1+\frac{p_{u,n}}{z_{u,n}+\alpha_{u,n}p_n}\right), \tag{12}$$

$$R^d = \sum_{n\in\mathcal{N}} \log_2\left(1+\frac{p_n}{z_{d,n}+\alpha_{d,n}p_{u,n}}\right), \tag{13}$$

where

$$z_{u,n} = \frac{\sigma_b^2}{h_{u,n}}, \quad \alpha_{u,n} = \frac{\beta}{h_{u,n}}, \tag{14}$$

$$z_{d,n} = \frac{\sigma_u^2}{h_{d,n}}, \quad \alpha_{d,n} = \frac{g_{u,d}}{h_{d,n}}. \tag{15}$$

It has been shown that for (12) and (13), if $\alpha_{u,n}\alpha_{d,n} < 1$ $\forall n \in \mathcal{N}$, there exists at least one Nash equilibrium strategy in the two users scenario [9]. With some algebraic manipulation, it can be shown that this condition is satisfied when $\beta < \min_{n\in\mathcal{N}}\left(\frac{h_{u,n}h_{d,n}}{g_{u,d}}\right)$. ■

Considering the large-scale channel effect, i.e. the pathloss which is function of the users' distance from the base-station, the required self-interference cancellation (β) will be proportional to the users' pathloss and inversely proportional to the distance between the two users. The farther the users from the base-station, the more self-interference cancellation is required (i.e. lower β), also, the more distance between the users, the less self-interference cancellation needed (i.e. higher β). For practical scenarios, it can be shown that this condition is generally satisfied. The largest pathloss values will occur when both users are at the cell-edge, i.e. for a base-station communication range up to 200m [10], this means at the worst case scenario both users are 200m from the base-station.

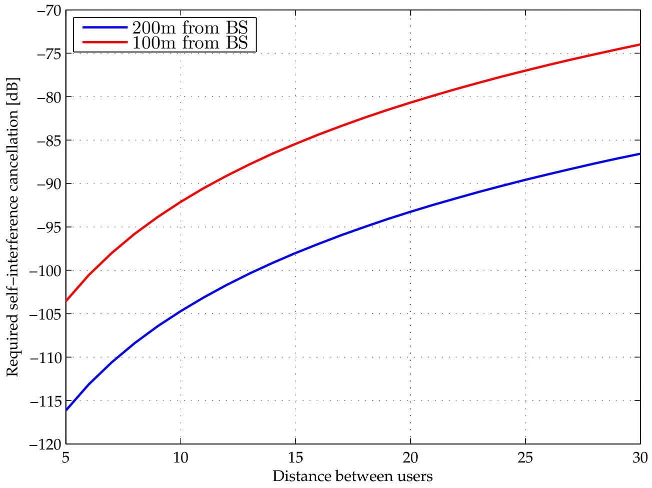

Fig. 2. Required self-interference cancellation for different users' distances.

Fig. 2 shows an example of the required self-interference cancellation, β, to satisfy the condition (11) versus the distance between the two users, for the worst case scenario (200m) and more moderate case (100m). It can be noticed that even for the worst case scenario, the Nash equilibrium existence condition (11) can be satisfied with the current reported self-interference cancellation values [4]. Note that a base-station's communication range is usually less than 100m in a small cell. Clearly, considering the interference from the other user as noise, the optimal power allocation strategy for each user is water-filling. Consequently, the Nash equilibrium can be reached by iteratively performing water-filling considering the interference from the other user. When the condition (11) satisfied, the iterative water-filling is guaranteed to reach to an equilibrium from any starting point.

We now extend the noncooperative game to the multiuser scenario. For a given uplink power allocation, the downlink subcarrier and power allocation can be formulated as follows

$$\max_{x_{j,n},p_n} \sum_{j\in\mathcal{J}}\sum_{n\in\mathcal{N}} x_{j,n}\log_2\left(1+\frac{p_n h_{j,n}}{\sigma_e^2+I_{j,n}}\right), \quad \text{subject to (6) and (7).} \tag{16}$$

The optimal solution for this problem can be found by allocating each subcarrier to the user that has the maximum unit power Signal to Interference-plus-Noise Ratio (SINR) [11], i.e.

$$j_n^\star = \arg\max_{j\in\mathcal{J}}\left(\frac{h_{j,n}}{\sigma_e^2+I_{j,n}}\right), \quad \forall \in \mathcal{N}. \tag{17}$$

Then, the base-station power is distributed over the subcarriers through water-filling policy. Thus, the downlink rate will be

$$R_{DL} = \sum_{n\in\mathcal{N}} \log_2\left(1+p_n\tilde{h}_n^{dl}\right), \tag{18}$$

where

$$\tilde{h}_n^{dl} = \max_{j\in\mathcal{J}}\left(\frac{h_{j,n}}{\sigma_e^2+\sum_{k\in\mathcal{K}} p_{k,n}g_{k,j}}\right). \tag{19}$$

The downlink power allocation will be

$$p_n = \left[\frac{1}{\lambda}-\frac{1}{\tilde{h}_n^{dl}}\right]^+ \tag{20}$$

Algorithm 1 Iterative radio resource allocation

Initialization: Set $p_n = 0, \ \forall n \in \mathcal{N}$ and $p_{k,n} = 0, \ \forall k \in \mathcal{K}, n \in \mathcal{N}$.

repeat

1) ***Optimal uplink power allocation:***

for $k = 1$ to K **do**

Perform water-filling over $(\sigma_b^2 + p_n\beta + \sum_{i \in \mathcal{K}\setminus k} p_{i,n}h_{i,n})$

end for

2) ***Optimal downlink subcarrier and power allocation:***

Find the maximum SINR for each subcarrier:

$\tilde{h}_n^{dl} = \max_{j\in\mathcal{J}}\left(\frac{h_{j,n}}{\sigma_e^2 + \sum_{k\in\mathcal{K}} p_{k,n}g_{k,j}}\right)$

Perform water-filling for the downlink over $\tilde{g}_n^{dl}$

until Stopping criterion is reached.

where $[x]^+ = \max(0, x)$ and λ is known as the water-level that should satisfy the power constraint in (7). It can be noticed that the multiuser downlink channel after optimal subcarrier allocation (17) can be represented by an *effective* single user with channel gains given by (19). Now, for a given downlink power allocation, the uplink power allocation is the solution for the following problem

$$\max_{p_{k,n}} \sum_{k\in\mathcal{K}}\sum_{n\in\mathcal{N}} \log_2\left(1 + \frac{p_{k,n}h_{k,n}}{\sigma_b^2 + I_{k,n} + p_n\beta}\right), \quad \text{subject to (8)}, \tag{21}$$

where the objective function (21) can be reformulated as

$$\max_{p_{k,n}} \sum_{n\in\mathcal{N}} \log_2\left(1 + \frac{\sum_{k\in\mathcal{K}} p_{k,n}h_{k,n}}{\sigma_b^2 + p_n\beta}\right). \tag{22}$$

This problem represents a classical multicarrier multiple access channel channel, and the global optimal power allocation can be found by iterative water-filling [12]. Hence, there is optimal power allocation strategy for the uplink users given the downlink subcarrier and power allocation, and there is optimal subcarrier and power allocation strategy for the downlink users given the uplink power allocation. Therefore, the multiuser resource allocation can be formulated as noncooperative game between the uplink and the downlink channels, where the downlink channel represented by an *effective* user with channel gains given (19) and water-filling as the optimal power allocation in response to the uplink power allocation. The uplink channel is also represented by *effective* user with iterative water-filling as the optimal power allocation in response to the downlink power allocation. Consequently, by iteratively implementing the optimal resource allocation in the uplink and downlink, an equilibrium can be reached. The detailed algorithm is listed in Algorithm 1.

IV. Numerical Results

In this section, the system-level performance of the proposed radio resource allocation is evaluated through Monte Carlo simulation. A single-cell with 200 m radius is considered, where the users' locations are randomly generated and uniformly distributed within the cell. The full-duplex base-station has 30 dBm maximum transmit power, and the half-duplex users each has 23 dBm maximum transmit power.

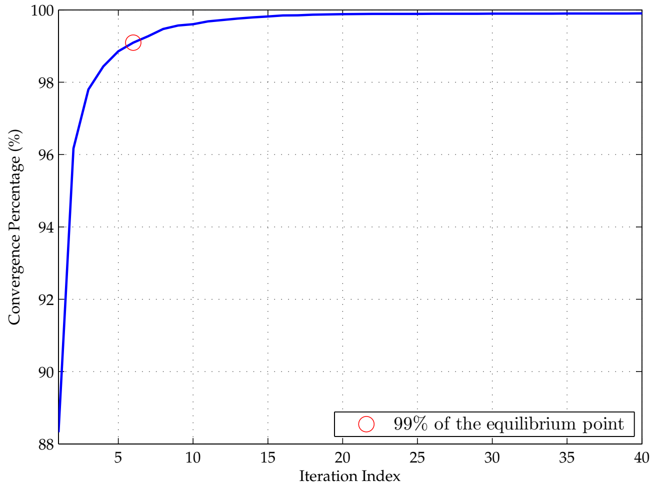

Fig. 3. Average convergence of the proposed iterative algorithm..

The total number of users is 20, with 10 downlink users and 10 uplink users. In the simulation, the iterative Algorithm 1 stops if the gain in spectral efficiency between two successive iterations is less than 10^{-5} bit/s/Hz. The system bandwidth is 10 MHz consisting of 50 resource blocks, and the noise power spectral density is -173 dBm/Hz. Noise figures of 5 dB and 9 dB are considered for the base-station and user equipment, respectively. The ITU pedestrian B fast fading model and the line-of-sight (LoS) pathloss model for pico-cell environment are used [10]. With 2 GHz frequency, the pathloss will be

$$PL_{\text{LoS}} = 41.1 + 20.9\log_{10}(d), \quad \text{(dB)}, \tag{23}$$

where d is the distance in meters between the users and the base-station. COST231 Hata propagation model [13] is used to obtain the large scale fading between two users, which is given by

$$PL_{\text{UE2UE}} = 35.68 + 38\log_{10}(d), \quad \text{(dB)}. \tag{24}$$

First, we examine the convergence behaviour of Algorithm 1. Fig. 3 shows the average rate of convergence of the algorithm in terms of percentage of the equilibrium, where the results are averaged over 100 channel realizations. The results show that, on average the algorithm reaches about 99% of the convergence point with 6 iterations only, which is significantly faster than algorithms seeking the optimal solution of non-convex problem [14]. Based on this observation, for further simulations we have used 20 as the maximum number of iterations for Algorithm 1 to keep the complexity low.

Fig. 4 shows the cumulative distribution function (CDF) of the sum-rate for full-duplex system, with different self-interference cancellation values. As benchmarks for comparison, we used full-duplex with equal power allocation and time-division duplex (TDD), i.e. half-duplex system, with optimal subcarrier and power allocation. For the TDD downlink, the resource allocation from [11] is used, while for uplink we use the iterative water-filling [12]. It is worth mentioning that these two algorithms give an upper bound on the TDD system sum-rate. The equal power allocation is simulated with -100 dB self-interference cancellation. The figure shows that the proposed resource allocation algorithm can significantly improves the full-duplex performance comparing to equal power allocation. For the same cancellation factors, the proposed algorithm

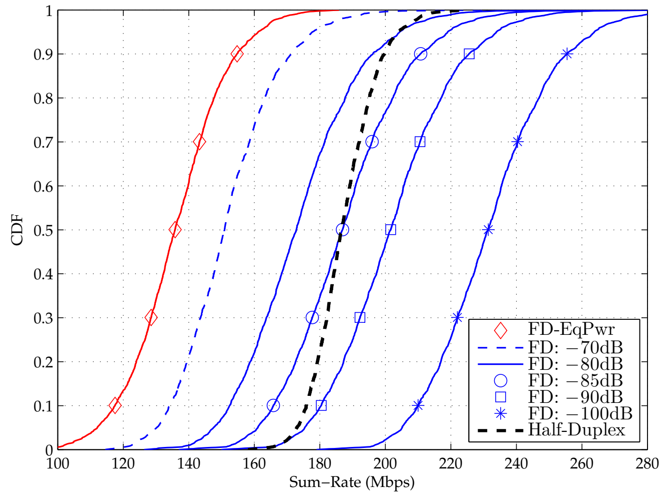

Fig. 4. Sum-rate comparison for full-duplex and half-duplex systems.

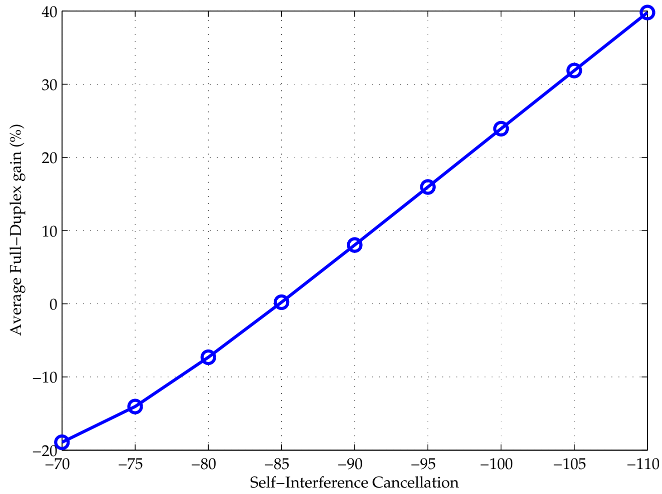

Fig. 5. Sum-rate gain of full-duplex comparing to half-duplex.

achieves about 70% gain in average sum-rate comparing to equal power allocation. Also, the figure shows that for self-interference cancellation more than -85 dB, the full-duplex significantly improves the system spectral efficiency comparing to the half-duplex system. Fig 5 shows the percentage of full-duplex gain in average sum-rate over half-duplex system. Up to 40% gain can be achieved with the currently reported self-interference cancellation capabilities [4].

Overall, it can be concluded that the proposed algorithm has fast convergence rate, and greatly outperforms the equal power allocation approach. Also, with proper radio resources allocation, considerable gain in sum-rate can be achieved by using the full-duplex technique with the currently feasible self-interference cancellation capabilities. We believe these results give insights into the potential gains and provide practical guidance for implementing the full-duplex technique.

V. Conclusion

In this paper, we have addressed the problem of joint radio resource allocation for uplink and downlink in the full-duplex system. The problem is modeled as a noncooperative game between the uplink and downlink channels. An algorithm is proposed that iteratively implements optimal downlink and uplink resource allocation to reach Nash equilibrium point. Simulation results show that the algorithm has a fast convergence rate, while on average reaches about 99% of the equilibrium point with few number of iterations (6 iterations for the simulated scenario). Although the proposed algorithm does not achieve the global optimal, it achieves significant improvement in the full-duplex system performance comparing to equal power allocation. The results suggest that despite the self-interference and inter-user interference, the full-duplex with the proposed algorithm achieves considerable spectral efficiency gains comparing to half-duplexing.

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