Efficient Spectrum Utilization Using Statistical Modeling of Channel Availability

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Abstract

Cognitive radio systems have been suggested as a method to improve spectrum utilization by detecting and accessing vacant spectrum. In such a network, sub-bands of a spectrum are shared by licensed (primary) and unlicensed (secondary) users in that preferential order. It is generally recognized that the spectral occupancy by primary users exhibit dynamical spatial and temporal properties and hence the fundamental issue is to characterize the sub-band spectrum occupancy in terms of probabilities. Given statistical analysis of the frequency band of interest are available, it has been shown that adaptive searching for white spaces could improve by 70% when compared to random searching. In the open literature, there exist no accurate/efficient time-varying model representing the spectrum occupancy that the wireless researchers could employ for evaluating new algorithms and techniques designed for dynamic spectrum access (DSA). Therefore, the objective is to propose an accurate and efficient analytic model that can be used to enhance the sensing operations.

Using real-time measurements conducted in different geographic locations, existing research has validated that subchannel availability is suitably modeled as independent but non identical (i.n.i.d.) Bernoulli variables characterized by $p_i$, the probability of availability of the $i^{th}$ subchannel. The magnitude of $p_i$’s could be extracted from sensed measurements or a geolocation database. Based on the i.n.i.d. paradigm, we develop a predictive model by probabilistically computing the distribution of the number of available subchannels over a wide-band at a given time. However, the combinatorial complexity behind the exact distribution computation alludes the need for accurate and efficient alternative approaches that can support frequency bands with a large number of non-overlapping subchannels. We propose 3 different techniques based on convolution, recursive, and hybrid convolution-recursive methods to resolve this complexity. We assess their efficiency by analyzing each algorithm’s time complexity and further compare their performance against existing models in the literature.

Moreover, knowing the availability of the channel’s immediate neighbors can allow efficient power management as well as prioritize channels allocation to secondary users. Therefore, we categorize available channels into three different types based on the oc-
cupancy of its two adjacent channels then model their availability. Additionally, from a network performance analysis perspective, predicting the count of available channels has to be evaluated against the probability of detecting these channels within the same i.n.i.d. framework. Respectively, we propose a novel approach to calculate the probability of detecting multi-channels simultaneously. Finally, we validate the effectiveness of the proposed models using several real-time measurements and further present 2 associated applications where one features novel 2-Dimensional (time, freq) availability prediction.

**Key words:** Cognitive Radio, Energy Efficiency, Modeling Techniques, Modeling Algorithms, Spectrum Modeling

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<td>Dynamic Spectrum Access</td>
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<tr>
<td>CR</td>
<td>Cognitive Radio</td>
</tr>
<tr>
<td>PDF</td>
<td>Probability Distribution Function</td>
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<tr>
<td>CDF</td>
<td>Cumulative Distribution Function</td>
</tr>
<tr>
<td>PU</td>
<td>Primary User</td>
</tr>
<tr>
<td>SU</td>
<td>Secondary User</td>
</tr>
<tr>
<td>i.i.d.</td>
<td>independent and identically distributed</td>
</tr>
<tr>
<td>i.n.i.d.</td>
<td>independent not identically distributed</td>
</tr>
<tr>
<td>n-SS</td>
<td>Spectrum of interest composed of n non-overlapping Subchannels</td>
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<tr>
<td>BS</td>
<td>Base Station</td>
</tr>
<tr>
<td>QoS</td>
<td>Quality of Service</td>
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<tr>
<td>FFT</td>
<td>Fast Fourier Transform</td>
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<td>RSSI</td>
<td>Received Signal Strength Indication</td>
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<td>AWGN</td>
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<td>CCDF</td>
<td>Complementary Cumulative Distribution Function</td>
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<td>Channel State Information</td>
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<tr>
<td>AP</td>
<td>Access Point</td>
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<tr>
<td>IEEE</td>
<td>Institute of Electrical and Electronics Engineers</td>
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<tr>
<td>GPS</td>
<td>Global Positioning System</td>
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<tr>
<td>EE</td>
<td>Energy Efficiency</td>
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<tr>
<td>SNR</td>
<td>Signal to Noise Ratio</td>
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<tr>
<td>SNIR</td>
<td>Signal to Noise and Interference Ratio</td>
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<tr>
<td>UT</td>
<td>User Terminal</td>
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Notation

\( n \) Number of subchannels in operating spectrum
\( p_i \) i-th subchannel free probability
\( \bar{p}_i \) i-th subchannel occupancy probability \((1 - p_i)\)
\( N_f(n) \) Discrete random variable that represents the total number of available subchannels in the n-SS
\( N_i \) Discrete random variable that represents the total number of available type-i subbands
\( x_i \) Binary random variable to indicate occupancy of the i-th subchannel
\( X_j \) Binary random variable to indicate occupancy of the j-th type-i subband
\( P(N_f = r) \) Exact probability of having r free subchannels
\( T(n) \) Time complexity of an algorithm
\( \alpha, \beta \) Beta distribution parameters
\( \Omega \) Lower bound of time complexity function growth rate
\( c_i \) steps or operation taken to execute an algorithm
\( k \) Number of subbands in a spectrum of interest
\( m \) Number of subchannels in a subband
\( P_d \) Probability of detection
\( P_{fa} \) Probability of false alarm
\( \mathcal{H}_{(0,1),j} \) Subchannel availability hypothesis
\( R_j \) Secondary received signal
\( S_j \) Primary transmitted signal
\( W_j \) Received noise
\( H_{ij} \) Channel gain between primary transmitter and secondary receiver
\( \gamma_j \) Decision threshold of subchannel \( j \)
\( M \) Samples of interest
\( \mathcal{CN}(\mu, \sigma^2) \) Complex Gaussian distribution with mean \( \mu \) and variance \( \sigma^2 \)
\( N_d \) Discrete random variable to represent the number of detected available subchannels
\( O_d \) Discrete random variable to represent the number of detected occupied subchannels
\( R \) Aggregate opportunistic throughput
\( C \) Aggregate interference to the primary users
\( \rho \) Decision threshold of meeting a request
<table>
<thead>
<tr>
<th>dB</th>
<th>Decibel</th>
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<tr>
<td>(\mathbb{E}[x])</td>
<td>Expectation of (x)</td>
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Chapter 1

Introduction

Wireless communication systems have been exploited since the early days of radio communications under a fixed spectrum management policy. Portions of the spectrum separated by guard bands have been allocated to particular licensees over large geographical regions, on a long term basis, and under protected status. Under this static regulatory regime, the overwhelming proliferation of new operators, services and wireless technologies has resulted in the reduction of spectrum bands with commercially attractive radio propagation characteristics. As a result, the Dynamic Spectrum Access (DSA) paradigm based on the Cognitive Radio (CR) technology [1] has gained popularity, motivated by the currently inefficient utilization of spectrum already demonstrated by many spectrum measurement campaigns performed all around the world [2–6]. The basic underlying principle of DSA/CR is to allow unlicensed users to access in an opportunistic and non-interfering manner some licensed bands temporarily unoccupied by licensed users. Unlicensed (secondary) DSA/CR terminals monitor the spectrum in order to detect spectrum gaps left unused by licensed (primary) users and opportunistically transmit. Secondary unlicensed transmissions are allowed according to this operating principle as long as they do not result in harmful interference to the licensees. As a result of the opportunistic nature of the DSA/CR principle, the behavior and performance of a network of DSA/CR nodes depends on the primary spectrum occupancy pattern therefore realistic and accurate modeling of such patterns becomes essential and extremely useful in the domain of DSA/CR research. Models of spectrum use can find applications in a wide variety of fields, rang-
Chapter 1. Introduction

ing from analytical studies to the design, dimensioning and performance evaluation of secondary networks, including the development of innovative simulation tools as well as novel DSA/CR techniques.

Moreover, by properly exploiting spectrum opportunity models, performance of spectrum sensing can be significantly enhanced. Given a wide-band system, the primary user occupancy in the entire spectrum is very difficult to determine due to its time complexity, as the larger the bandwidth to be scanned the higher will be the time needed for sensing. Therefore, instead of sensing the frequency band of interest, one can exploit knowledge of historical data to develop prediction models that can either select the most appropriate portions of spectrum or advise whether it is feasible to perform sensing at a given time and suggest alternative time. Hence, this reduces the need for exhaustive and extensive sensing [3] [4].

1.1 Objectives and Motivation

There are several benefits associated with estimating the likelihood of availability of a certain number of free channels, i.e. the probability that a count of channels are available: \( Pr(N_f = r) \), and their configuration/location over a spectrum. It can provide a trackable representation of the statistical properties of spectrum usage that can adequately be employed in the analytical studies or implemented in simulation tools for the performance evaluation of CR technologies. Moreover, this information can act as a pre-requisite for performing further processing to establish the communication link. In this manner, we can avoid unnecessary overhead calculations and processing in the scenarios where the chances of establishing a link can satisfy the expected QoS. Modeling the number of available channels and their configuration also provides useful information to establish if the constraints in terms of communication strategy can or cannot be met (e.g. FFT size that the transceiver can process, number of pilots used for channel estimation, carrier aggregation strategies/algorithms available to transceiver, etc.). If any of these constraints in a transceiver imply that the expected number and configuration of the channels is not suitable, we can avoid doing the unnecessary further processing involving sensing, channel
1.2 Fundamental Assumptions

To date, most developed spectrum occupancy models, used in cognitive radio research, rely on assumptions and oversimplifications. For instance, most studies within spectrum modeling assume that channel availability over any spectrum follows an independent and identically distributed (i.i.d.) Bernoulli sequence [7] [10]. Yet, real-time measurements indicate that channel availability depends on the channel frequency/location [4], which suggests that an i.n.i.d. model should be employed instead. This motivated us to consider building accurate predictive models on spectrum availability. With the aid of real-time energy-sensed measurements conducted over several bands in different geographic locations, we use detection threshold to extract the availability probability for all subchannels of interest. We assume that these subchannels probability distributions follow the i.n.i.d. model which has been extensively validated in [11] [12]. We employ convolution, recursive, and hybrid convolution-recursive methods to develop accurate and efficient techniques that facilitate the computation of the availability probability of r subchannels, \( P(N_f = r) \), where \( N_f \) denotes the number of available subchannels at any given time instant. Furthermore, we discuss the impact of spectrum availability in network performance in terms of detection probability and aggregate throughput. Additionally, as an attempt to minimize interference to primary users, we propose to categorize available subchannels into three different types, based on the occupancy of its two adjacent subchannels, then, probabilistically model their availability using the proposed hybrid model. Finally, through computer simulations, we validate the effectiveness of our models using the real time measurements, assess their efficiency using algorithm time complexity analysis, and further demonstrate an application for spectrum availability modeling.

1.2 Fundamental Assumptions

The following assumptions were made in the context of channel probability modeling as well as network performance analysis:

- In the entire spectrum, the channel occupancy is statistically independent, i.e., the arrival of a primary user in one channel does not depend on the arrival of another
primary user in any other channel at the same time instant. Therefore, channels availability random variables are modeled as independent Bernoulli random variables. However, these availabilities are not necessarily identically distributed, i.e., the probability that the $i$-th channel is available is not necessarily the same as the probability that the $j$-th channel being available. Moreover, although most studies within modeling available spectrum assume that channel availability over any spectrum follows an independent and identically distributed (i.i.d.) Bernoulli sequence, extensive real-time spectrum measurement campaigns indicate that channel availability depends on the channel frequency/location and further suggests that an i.n.i.d. model should be employed instead. This has motivated us to consider the i.n.i.d. model in our research. This assumption is validated in Chapter 2.

- As an input for the modeling stage, we assume that channel availability probabilities are known. These probabilities could be determined at a given location using two schemes: 1) A data-driven approach where spectrum measurements for that location are applied. It requires widespread measurements at low sensitivity thresholds and may take a long time to complete. Furthermore, these measurements will have to be repeated whenever the primary user’s transmission characteristics, such as transmit power, antenna height, license terms, etc., change. 2) A model-driven approach where RF propagation models are employed to compute spectrum availability. Given a transmitter’s location, transmit power, antenna directionality, etc., these models are used to estimate the received signal strength indication (RSSI) at any given location [13–16].

- Reliable channel estimation relies on some kind of exchange of information between transmitter and receiver (feedback channel, pilot symbols, etc.). This might be particularly difficult to establish in a cognitive radio environment since primary users are not supposed to modify their transmission due to the existence of secondary users. Therefore, a more realistic assessment is to take into account imperfect channel knowledge, where the statistics of channel coefficients are estimated by the secondary users without the presence of information exchange with the primary transmitters [17–19].
Every white space device primarily relies on the database to determine the white space availability. Hence, this results in a delay in the device learning about changes in spectrum availability. Either the device will have to poll the database, or the database will have to push updates to the device. This problem becomes worse when the devices are mobile. If mobile, the device could have traveled some distance between the time it receives two subsequent spectrum updates. Throughout our work, we focus mainly on fixed channels, i.e., TV channels. Future work can accommodate the mobility factor and consider adding protection range of the distance to learn further about determining white spaces.

1.3 Thesis Structure and Outline

The remainder of the thesis is structured as follows:

- **Chapter 2: ”State-of-the-art Spectrum Modeling”**
  This chapter focuses on providing an overview of existing spectrum availability models as well as white spaces sensing techniques. Firstly, we illustrate some of the existing modeling approaches categorized as time-, frequency-, and space-dimension, each of which describe the statistical properties of spectrum usage in the corresponding domain. Secondly, we validate the i.n.i.d. model for channel availability probabilities over two sets of real time measurements at two disparate geographic locations using McNemar’s and Pearson’s Chi-square tests.

- **Chapter 3: ”Probabilistic Modeling of Spectrum Occupancy”**
  In a cognitive radio network, sub-bands of a spectrum are shared by licensed (primary) and unlicensed (secondary) users in that preferential order. It is generally recognized that the spectral occupancy by primary users exhibit dynamic spatial and temporal properties and hence it is a fundamental issue to characterize the spectrum occupancy in terms of probability. With the sub-band free probabilities being available, an analytical model is proposed for spectrum occupancy in a cognitive network. To reduce the computational complexity of the actual distribution
of total number of free sub-bands, we employ convolution, recursive and hybrid convolution-recursive techniques and develop efficient algorithms. Furthermore, we characterize certain types of free channels based on the occupancy of its adjacent channels. Then, the probability distribution of each type subband is computed.

- **Chapter 4: ”Network Performance Analysis and Potential Applications”**
  In this chapter, we consider a multi-channel joint detection framework in which a technique takes into account the detection of primary users across multiple channels. In addition, we present the impact of the modeling on network performance and propose an efficient approach to determine the probability of detecting multiple channels simultaneously. Furthermore, we evaluate the accuracy and efficiency of our proposed models and compare them to the ones proposed in the state-of-art. Finally, we illustrate 2 applications associated with the use of the modeling.

- **Chapter 5: ”Conclusions”**
  This chapter provides a conclusive summary and puts into perspectives the numerous insights and findings that have been obtained from the previous chapters. Furthermore, deeper focus on QoS aspects of the proposed applications as well as open issues on propagated error are discussed and future work directions on this topic are proposed.

### 1.4 Overview of Contributions

The main contributions of this thesis can be summarized as follows:

- **Modeling of spectrum occupancy**: 3 models that enable to reduce the computational complexity of the actual distribution of total number of available channels based on the i.n.i.d paradigm.

- **Categorization of certain types of free channels** based on the occupancy of their neighbor channels.

- **A multi-channel joint detection framework** is presented with respect to the network performance where the probability of detecting multi-channels simultaneously has
been derived.

- Performance of the modeling: The proposed models show Furthermore, 2 applications associated with the use of the modeling are illustrated. One features novel 2-Dimensional (time, freq) availability modeling.

1.5 Publications

The research carried out during the course of this PhD has resulted in the following publications:
State-of-the-art Spectrum Modeling

The basic idea behind development of opportunistic spectrum access technologies such as cognitive radio is to increase the spectral utilization. This goal can be achieved by identifying and utilizing the spectrum holes; given the, the conventional definition of spectrum hole: a band of frequencies that are not being used by the licensed user of that band at a particular time in a particular geographical area [20]. Hence, based on this definition, identifying spectrum holes requires exploitation of three dimensions of frequency, time and space by the spectrum sensing device. In other words, the main task of any spectrum sensing device is to determine if the frequency band of interest is occupied by the licensed user during a time slot within a certain geographical area [21]. Spectrum modeling depends solely on spectrum sensing. For example, in order to evaluate the performance of statistically modeling spectrum holes, one has to refer to spectrum sensing. Therefore, in this chapter we provide an overview of existing spectrum sensing techniques as well as spectrum availability modeling approaches.

2.1 Spectrum Sensing Techniques

In what follows, firstly, we will consider the original problem of local narrow-band spectrum sensing and provide a comprehensive study of existing solutions and secondly, the concept of local sub-band sensing is considered along with the state-of-the-art sub-band spectrum sensing algorithms.
2.1. Spectrum Sensing Techniques

2.1.1 Narrow-Band Spectrum Sensing

The general problem of spectrum sensing can be modelled as the binary hypothesis testing with hypotheses: $\mathcal{H}_0$, when the frequency band of interest is vacant and no other users is utilising this frequency band; and $\mathcal{H}_1$, when the frequency band of interest is occupied by other users and not available for opportunist usage. Therefore, this problem mathematically reads as:

$$
\mathcal{H}_0 : y(t) = v(t), \\
\mathcal{H}_1 : y(t) = s(t) + v(t),
$$

(2.1)

where $y(t)$ denotes the received signal at the spectrum sensing device, $v(t)$ is the Additive White Gaussian Noise (AWGN) with zero mean, and $s(t)$ represents the signal transmitted by the existing active users. Hence, based on 2.1 spectrum sensing techniques can be evaluated through two classical metrics, namely probability of detection (PD) and probability of the false alarm (PFA).

Mathematically, the probability of false alarm is defined by [22]

$$
PFA \triangleq Pr\left(\mathcal{T} \geq \lambda | \mathcal{H}_0\right) \tag{2.2}
$$

where $\mathcal{T}$ denotes the test statistics and $\lambda$ is the detection threshold. As it can be observed from 2.2, PFA reflects the probability of an event where the spectrum sensing device reports an alarm when the signal is actually not being transmitted.

While, probability of detection is defined by

$$
PD \triangleq Pr\left(\mathcal{T} \geq \lambda | \mathcal{H}_1\right) \tag{2.3}
$$

From 2.3 it can be concluded that PD reflects the probability of an event where the spectrum sensing device reports an alarm when the signal is indeed there. Spectrum sensing problem explained in 2.1 has been quite well investigated in the literature, for which there are many approaches reported. In general, the existing spectrum sensing approaches can be divided into three main categories:

- Exploiting the energy difference.
- Exploiting the stationarity difference.
• Exploiting the distribution difference.

In the following sections each of the above categories is explained and compared in detail.

2.1.2 Exploiting Energy Difference

The spectrum sensing approaches which fall into this category make a decision based on the estimated energy of the received signal at the spectrum sensing device. The most well-known spectrum sensing approach developed under this category is the energy detection [22].

Energy Detection

Energy detection (radiometer) is the simplest spectrum sensing approach introduced in the literature [22–25]. Due to its low computational complexity it is also the most common technique used in practice. This approach can be thought of as a blind spectrum sensing approach since it does not require a priori knowledge of the signal being detected. Therefore, it is robust to variation of the signal being detected and thus it is known as the optimal detector in the case where we assume absolutely no deterministic knowledge about the signals being detected. Figure 2.1 and 2.2 show block diagrams of the energy detection algorithm. As it can be observed this technique can be implemented in either frequency or time domain. It is worth mentioning that the conventional energy
detection is implemented in time domain, while the frequency domain version was later introduced for sub-band [26] purposes. Frequency domain energy detection is usually used to simultaneously detect the vacancy of several sub-bands, i.e., filter bank based energy detection [26].

The test statistics $\mathcal{T}_{ED}$ for time domain energy detection is given by

$$\mathcal{T}_{ED} = \frac{1}{N} \sum_{s=0}^{N-1} |y(n)|^2$$

(2.4)

where $y(n)$ and $N$ represents the sampled received signal and the observation length respectively. While the test statistics in frequency domain energy detection is also calculated as 2.4, with a minor difference that it is a function of $\tilde{y} = \mathcal{F}_{N \times N} y$, given that $\mathcal{F}_{N \times N}$ denotes an $N \times N$ Discrete Fourier Transform (DFT) matrix and $y = [y(0), y(1), ..., y(N-1)]^T$.

Using central limit theorem [27] it can be observed that the alternative hypothesis testing in 2.1 for energy detection can be expressed as [28]

$$\lim_{N \to \infty} \sim \begin{cases} 
\sigma_v^2, & \mathcal{H}_0 \\
(\sigma_s^2 + \sigma_v^2), & \mathcal{H}_1
\end{cases}$$

(2.5)

where $\sigma_v^2$ and $\sigma_s^2$ denote the noise and signal variance respectively. As it can be observed from 2.5 the energy difference which exists between the two hypotheses can be used as a metric for performing spectrum sensing. Thus, the availability of the frequency band of interest using energy detection approach can be determined using

$$\mathcal{T}_{ED} \xrightarrow{\mathcal{H}_1} \frac{\mathcal{H}_1}{\mathcal{H}_0} \lambda_{ED}$$

(2.6)

where $\lambda_{ED}$ denotes the energy detection threshold. It is clear from 2.5 and also from the derivations in [22] and [28] that the threshold value is directly proportional to the noise power. Thus, it can be concluded that the energy detection approach requires accurate knowledge of the noise power, specifically in low SNR values, in order to deliver a reliable performance.

However, noise power estimation error is unavoidable in practical systems, hence, this will give rise to a phenomenon called noise uncertainty [22] [28].
Figure 2.3: The SNR wall phenomenon.

Energy Detection Under Noise Uncertainty

In most communication systems noise is an aggregation of various independent sources, i.e. thermal noise, interference due to nearby unintended emissions, etc. Thanks to the central limit theorem [27], one can assume that noise at the receiver is a Gaussian random variable. We should bear in mind that the error due to this assumption will tend to zero as \( \frac{1}{\sqrt{N}} \), where \( N \) is the number of independent random variables being summed up. In a practical scenario \( N \) is usually moderate therefore this error can not be neglected, especially in low SNR environments. For some constant \( K \) the error due to this assumption can be modeled as [27]:

\[
|F_v(x) - \mathcal{N}(x)| \leq \frac{k}{\sqrt{N}}
\]  

(2.7)

where \( F_v(x) \) denotes the actual noise distribution and \( \mathcal{N}(x) \) denotes a zero mean Gaussian distribution. Nevertheless, most detectors operate under the assumption of the received noise being Gaussian. The other main factors causing noise uncertainty is the temperature variations at the receiver which leads to inaccurate noise power measurements. Hence, it can be concluded that if the SNR at the spectrum sensing device is sufficiently low, there would be enough uncertainty in the noise to render the energy detection useless. Consider the case with noise uncertainty factor \( U \), since the energy detector only sees the energy, distribution for the noise uncertainty can be summarized in a single interval.
\( \sigma_v^2 = [(1/U)\sigma_n^2, U\sigma_n^2] \).

It is further proved in [9] that as the observation length is increased, i.e., \( N \to \infty \), the minimum operating SNR in which the energy detection can operate desirably will converge to \( \text{SNR} \downarrow (U - (1/U)). \) This introduces a phenomenon called "SNR wall". SNR wall of a spectrum sensing device is defined as the maximal SNR such that for any \( \text{SNR} \leq \text{SNR}_{\text{wall}} \) detection is impossible for that particular detector.

The SNR wall for energy detector can be expressed as:

\[
\text{SNR}_{\text{wall}} = \frac{U^2 - 1}{U} \tag{2.8}
\]

In the last four decades since the publication of [22], many solutions have been developed to make energy detection more robust in terms of SNR wall (e.g. [29–31]), yet the noise uncertainty problem in spectrum sensing approaches based on the energy difference still exists. Hence, small noise power estimation error can result in significant performance loss [32]. Consequently the noise power has to be estimated dynamically. This is done by separating the noise and signal subspaces using multiple signal classification algorithm [33]. Noise variance estimated by using the value of the smallest eigenvalue of the incoming signals autocorrelation. In [34] an iterative algorithm is proposed to find the near optimum threshold value. The performance of energy detection technique over various fading channel models has been investigated in [25], where it is shown that fading channels can have a detrimental effect on the performance of the energy detection based spectrum sensing.

### 2.1.3 Exploiting Stationarity Difference

Stationarity is defined as a quality of a process in which the statistical parameters of the process do not change with time [35]. The spectrum sensing approaches which fall into this category exploit the various stationarity difference which exists between the noise and signal. In what follows the state-of-the-art spectrum sensing approaches which fall into this category are explained in detail.
Figure 2.4: Flow chart of second order cyclostationary based detection technique.

Second Order Cyclostationarity Detection

The initial works of spectrum sensing through stationarity difference can be traced back to work of Dandawate et. al. in [36], where second order cyclostationarity is employed. Cyclostationarity detection is based on exploiting the cyclostationarity feature of the received signal [37, 38]. This feature is caused by periodicity in the statistics of the transmitted signal which could be a result of modulation, coding or intentionally to assist spectrum sensing [39–41]. Process \( y(n) \) is considered sub sense cyclostationary process if

\[
\mu_y = E[y(n)] = E[y(n) + \mathcal{P}]
\]

and

\[
R_y(n) = E[y(n)y^*(n + \eta)] \\
= E[y(n + \mathcal{P})y^*(n + \mathcal{P} + \eta)]
\]

\( \forall n, \eta \in \mathbb{Z} \) The smallest value of \( \mathcal{P} \) for which 2.9 and 2.10 hold is called the period. Being periodic, \( R_y(n) \) follows Fourier Series expansions over cyclic frequencies with the set of cycles \( \mathcal{A} := \{ \kappa = k/\mathcal{P}, k = 0, 1, ..., \mathcal{P} - 1 \} \). Hence, the Fourier coefficients also called cyclic auto correlations are related to \( R_y(\eta) \) using

\[
\hat{R}_y^\kappa(\eta) = 1/\mathcal{P} \sum_{n=0}^{\mathcal{P}-1} R_y(\eta) \exp(-j2\pi \kappa n)
\]

As it can be observed from 2.11 the cyclic autocorrelation function at a given cyclic frequency determines the correlation between spectral components of the signal separated in frequency by an amount of \( 1/\mathcal{P} \), hence, given sufficient observation length the cyclic autocorrelation function of cyclostationary signals is nonzero only for set of cycles which fall in the set \( \mathcal{A} \). Thus, one can determine the vacancy of the frequency band of interest by analyzing the cyclic autocorrelation function of the received signal [36], i.e. using the second order cyclostationary feature. More interestingly, cyclostationarity based detection
can be employed to differentiate different types of signals [42]. Figure 2.4 illustrates the block diagram of a second order cyclostationarity based detection technique.

In the above procedures it has been assumed that the cyclic frequency is known at the receiver. However, this assumption may not be reasonable when the spectrum sensing device is required to perform sensing in a sub spectrum band. Since, the sensing device may not have knowledge of the period of all the users operating and storing these information covering the whole geographical area in which the sensing device (specifically for non stationary users) requires connection to an accurate location aided database. Thus, an exhaustive search is required to determine the operating cyclic frequencies. This will increase the complexity of the detector significantly and furthermore the detector will loss the ability of distinguishing between the signal and the interference which also have cyclic characteristics. It is shown in [36, 40] that the cyclostationarity based scheme can trade latency with high sensing reliability. Furthermore, It is less sensitive to the noise uncertainty provided the knowledge of signals cyclic frequency [43].

**Covariance Based Detection**

![Figure 2.5: Flow chart of covariance based detection.](image)

To overcome the requirement of cyclic frequency information in cyclostationary based detection while not suffering from the noise uncertainty problem which exists with energy detection, the covariance based detection was introduced [44]. This spectrum sensing method, as the name implies, is based on the estimated covariance matrix of the received signal, and utilizes the correlation which exists in the transmitted signal to determine the vacancy/occupancy of the frequency band of interest. Hence, it is proved to be very
effective when the transmitted signal is highly correlated \[44,45\].

The correlation of the received signal samples can be due to many factors, e.g., modulation, multi-path fading, multiple receivers or can be intentionally introduced by oversampling at the spectrum sensing device \[46\], while the noise samples are independent. Moreover, since the covariance matrix of noise is determined by the receiving filter at the receiver its structure is known to the spectrum sensing device, allowing us to differentiate the two hypothesis in 2.1 \[47\].

Consider \( L \) to be the number of consecutive samples used for estimation the covariance matrix, i.e.,

\[
y(n) = [y(n)y(n - 1)...y(n - L + 1)]^T,
\]

hence, the estimated covariance matrices can be expressed as

\[
\lim_{L \to \infty} C_y = C_s + \sigma_v^2 I_L
\]

where

\[
C_s = \mathbb{E}[s(n)s^*(n)] \quad (2.14)
\]

\[
C_y = \mathbb{E}[y(n)y^*(n)] \quad (2.15)
\]

\( I_L \) denotes an \( L \times L \) identity matrix and \( s(n) = [s(n)s(n - 1)...s(n - L + 1)]^T \). Given that the noise samples are uncorrelated based on 2.13, it can be concluded that considering the \( \mathcal{H}_0 \) scenario in 2.1, if the signal \( s(t) \) is not present, i.e., when \( C_s = 0 \), the off-diagonal elements of \( C_y \) are all zero. On the other hand, if the signal \( s(n) \) is present, i.e., \( \mathcal{H}_1 \) hypothesis in 2.1, \( C_y \) will no longer be a diagonal matrix due to the correlation between the \( s(n) \) samples, resulting in off diagonal elements. Hence, the vacancy of the frequency band of interest can be determined based on the sum value of the off-diagonal elements.

Based on \[44\], an effective test statistic for this purpose would be \( \mathcal{R}_{CD} = \frac{T_1}{T_2} \) where

\[
T_1 = \frac{1}{L} \lim_{n=0} L-1 \lim_{m=0} L-1 |c_{m,n}|
\]

\[
T_2 = \frac{1}{L} \lim_{n=0} L-1 |c_{n,n}|
\]

given that \( c_{m,n} \) denote the element of the matrix \( C_y \) at the \( m \)th row and \( n \)th column.

Considering the \( \mathcal{H}_0 \) scenario, \( \mathcal{R}_{CD} = 1 \) while given\( \mathcal{H}_1 \) hypothesis \( \mathcal{R}_{CD} > 1 \). However, this is based on the assumption that \( L \to \infty \), which is not a practical assumption,
hence in order to make a reliable decision $\mathcal{H}_{CD}$ should be compared to a threshold value which is function of observation length and the required PFA. Details of the threshold setting can be found in [44]. It should be noted that the performance of the covariance based detection is highly dependent on the correlation of the received signal hence, in the extreme case where the received sample are Independent and Identically Distributed (i.i.d.) this spectrum sensing approach will fail.

**Eigenvalue Based Detection**

![Figure 2.6: Block diagram of Eigenvalue based detection technique.](image)

Following the development of the covariance based spectrum sensing approach the eigenvalue based detection was introduced in [46–49]. The eigenvalue based detection scheme exploits orthogonality between the signal subspace and noise subspace using covariance matrix, i.e., second order stationarity features, to offer highly reliable spectrum sensing [46].

Hence, based on this approach the vacancy of the frequency band of interest is determined based on the fluctuation of the eigenvalues [35] of the covariance matrix, and hence many test statistics have been proposed to efficiently utilize this fluctuation [46–49].

Amongst the well known test statistics for eigenvalue based detection is the ratio of the maximum to minimum eigenvalues [46]. Let $\varrho_{\text{min}}$ and $\varrho_{\text{max}}$ denote the minimum and the maximum eigenvalues of the covariance matrix $C_y$, respectively, hence, the availability of the spectrum can be determined using

$$\mathcal{T}_{EV} = \frac{\varrho_{\text{max}}}{\varrho_{\text{min}}} \frac{\mathcal{H}_1}{\mathcal{H}_0} \lambda_{EV}$$

(2.18)

In [46], the asymptotic statistical characteristics of $\varrho_{\text{min}}$ and $\varrho_{\text{max}}$ under $\mathcal{H}_0$ have been investigated and furthermore, closed from expression of the decision threshold $\lambda_{ED}$ for a
given probability of false alarm has been provided.

An other popular test statistic used in eigenvalue based detection is the ratio of the maximum eigenvalue to the average eigenvalue, i.e, $\mathcal{J}_{EV} = \frac{\rho_{\text{max}}}{\rho_{\text{min}}}$, where

$$\rho = \frac{1}{L} \lim_{l \to 0} \rho_l$$

(2.19)

The asymptotic threshold value of the above test statistic is provided in [46]. However, like covariance based detection, the eigenvalue based detection will fail if and only if $C_s = \sigma_s I_L$ i.e., the received signal samples are i.i.d. . However, the correlation between the received signal can be forced with employment of multiple receive antennas or oversampling.

**Matched Filtering**

Matched filtering is known to be the optimum method for detecting signals when the transmitted signals air interface is completely known to the sensing device [50]. The main advantage of matched filtering is low latency and computational complexity. Match filtering requires a very short time to achieve the desired probability of false alarm or probability of detection as compared to all the existing detection methods while introducing a linear complexity even in a very low SNR environment since it maximize the received SNR at the sensing device [50]. On the other hand the shortcoming of the match filtering technique is that the spectrum sensing device needs to demodulate the received signal prior to determine the vacancy of the frequency band of interest. Hence it will introduce to main concerns 1) requiring a perfect knowledge of the transmitted signals signaling feature, 2) security issue, since this allows the spectrum sensing device to have access to the transmitted message.

Since the spectrum sensing device should be able to detect all the possible signals transmitted within the bandwidth of interest, the implementation complexity of such detection technique is impractically large, due to perfect knowledge of all the available signaling information of users [51]. One of the other disadvantages of match filtering is large power consumption as all the possible receiver algorithms needs to be executed for spectrum availability decision making.
Pilot Based Match Filtering Detection

![Figure 2.7: Block diagram of the pilot based detection technique.](image)

In practical communication systems, pilots are usually transmitted periodically for time or frequency synchronization applications, channel estimation, etc. These pilots, if known to the spectrum sensing device, can be utilized for coherent detection of the transmitted signal with the aid of match filtering. Therefore, it works even under a very low SNR region. Furthermore, it has lower complexity and latency than statistics based cyclostationary and covariance based detection while overcoming the noise uncertainty problem. Moreover, it does not require demodulation of the transmitted signal as the conventional match filtering since orthogonal to the data and can be considered independently. Therefore, pilot based coherent detection is always one of the preferred spectrum sensing schemes in practice.

In the scenarios where the pilot structure is known to the spectrum sensing device the optimum detection technique would be match filtering. Therefore, the test metric can be expressed as:

\[
T_{PM} = \mathbb{E}\{y(n)s_p^*(n)\} \tag{2.20}
\]

where \(s_p^*(n)\) denotes the known pilot signal. Hence, a decision can be made using,

\[
\mathcal{H}_1 \quad \frac{T_{PM}}{\mathcal{H}_0} \quad \lambda_{PM},
\]

where \(\lambda_{PM}\) denotes the threshold value to satisfy the required probability of false alarm.

In 2.20 perfect synchronization between the sensing device and the transmitter is assumed, while this condition is not feasible in practice. Hence, the sensing device has to perform an exhaustive search to find the timing offset which maximizes the \(T_{PM}\) value. Recently, various robust pilot-based coherent detection schemes have been proposed for spectrum sensing applications [52–54].
2.1.4 Exploiting The Distribution Difference

Given that in almost all communication system models, noise is assumed to be additive white and Gaussian, one can determine the vacancy of a particular frequency band by observing the difference of the received signals distribution and that of the AWGN. Based on this feature a number of well known spectrum sensing algorithms have been proposed, which are explained in this section.

Entropy Based Detection

In information theory, entropy is a measure of the uncertainty associated with a discrete random variable. The term by itself usually refers to the Shannon entropy, which quantifies the information conveyed in a message [55]. Recently, entropy based detection approaches have been employed for spectrum sensing applications [56,57]. Entropy based spectrum sensing can be thought of an approach which exploits the distribution difference in order to determine the vacancy of the frequency band of interest.

In order to allow robustness to noise uncertainty issue, the entropy based detection makes a decision based on the estimated entropy of the measured signal in the frequency domain with the probability space partitioned into fixed dimensions. This is due the fact that the entropy of the received signal in the time domain is related to the signal power and is sensitive to noise uncertainty [56]. Hence, the test statistic for this spectrum sensing approach can be expressed as [57]

$$ T_{EB} = -\lim_{i=0}^{L-1} \log \frac{K_i}{N} \frac{H_i}{H_0} \lambda_{EB} $$

(2.21)

where $L$ denotes the dimension of probability space, $N$ is the number of Discrete Fourier Transform (DFT) points, $K_i$ is the total number of occurrences at the $i^{th}$ probability state,
and $\lambda_{EB}$ is the threshold value used for decision making. Assuming that the estimated noise entropy follows a Gaussian distribution, the value of $\lambda_{EB}$ can be easily calculated based on the desired PFA and value of $L$ [57]. However, the entropy based detection will fail to deliver accurate results, when the transmitted signals also follow a Gaussian distribution and since the convergence to normality could be extremely slow, this approach will require relatively high observation length. However, this would is in contradiction to achieving channel capacity, which benefit from Gaussian noise like transmit signals.

**Kurtosis Based Detection**

![Flow chart of kurtosis based detection.](image)

Figure 2.9: Flow chart of kurtosis based detection.

In statistics, kurtosis is a descriptor of the shape of a probability distribution, i.e., it is a measure of the peakedness of the probability. Hence, the kurtosis based detection was introduced to exploit the non-Gaussianity of communication signals in order to determine the availability of the frequency band of interest [58, 59]. For example for randomly occurring signals that produce non Gaussian distributions, the kurtosis estimate can be less than 3 or it can have a value much greater.

This scheme features excellent accuracy at the price of large latency due to higher order statistics. A critical point is that the sensing performance degrades significantly when signals are approximately Gaussian. Kurtosis is defined by the ratio of the expected value of the fourth-order central moment and the square of the expected value of the second-order central moment. Hence, the test statistic of the kurtosis based detection can be formulated as

$$J_{KB} = \frac{\mathbb{E}\left[(y(n) - \mu_y)^4\right]}{\mathbb{E}\left[(y(n) - \mu_y)^2\right]^2} \xrightarrow{\mathcal{H}_1} \frac{\mathcal{H}_1}{\mathcal{H}_0} \lambda_{KB} \quad (2.22)$$

This scheme features excellent accuracy at the price of high computational complexity due to higher-order statistics. Furthermore, the convergence to normality could be extremely slow.
slow, and the sample estimate of the kurtosis can deviate substantially from its true value even with a large number of observations. Thus, for moderate sample sizes which is a prerequisite for any spectrum sensing device, the kurtosis test cannot be expected to be accurate.

2.1.5 Summary of The Narrow-Band Spectrum Sensing Approaches

In the above discussion, we have introduced various state-of-the-art narrow-band spectrum sensing techniques. As explained, various spectrum sensing techniques have different advantages/disadvantages and hence are applicable in different sensing scenarios. In general, the existing spectrum sensing approaches can be divided into three main categories:

<table>
<thead>
<tr>
<th>Approach</th>
<th>Air-Interface</th>
<th>Synchronisation</th>
<th>Computational Complexity</th>
<th>Latency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Energy Detection</td>
<td>×</td>
<td>×</td>
<td>Low</td>
<td>Low</td>
</tr>
<tr>
<td>Cyclostationarity Detection</td>
<td>✓</td>
<td>✓</td>
<td>High</td>
<td>High</td>
</tr>
<tr>
<td>Covariance Based Detection</td>
<td>×</td>
<td>×</td>
<td>Medium</td>
<td>Low</td>
</tr>
<tr>
<td>Eigenvalue Based Detection</td>
<td>×</td>
<td>×</td>
<td>Medium</td>
<td>Medium</td>
</tr>
<tr>
<td>Matched Filtering</td>
<td>✓</td>
<td>✓</td>
<td>Low</td>
<td>Low</td>
</tr>
<tr>
<td>Pilot Based Matched Filtering</td>
<td>✓</td>
<td>✓</td>
<td>Low</td>
<td>Low</td>
</tr>
<tr>
<td>Entropy Based Detection</td>
<td>✓</td>
<td>×</td>
<td>Medium</td>
<td>Medium</td>
</tr>
<tr>
<td>Kurtosis Based Detection</td>
<td>✓</td>
<td>×</td>
<td>High</td>
<td>High</td>
</tr>
</tbody>
</table>

Figure 2.10: Summary of the state-of-the-art local narrow-band spectrum sensing approaches.

- Exploiting the energy difference. The most well-known spectrum sensing approach developed under this category is the energy detection [22]. The energy detection is recognized as a blind sensing scheme with advantages such as low complexity and low latency. However, it is very sensitive to the noise uncertainty such that its performance is limited by the SNR wall [28]. In the last four decades since the publication of [22], many solutions have been developed to make energy detection more robust in terms of SNR wall (e.g. [29–31]), yet the noise uncertainty problem in spectrum sensing approaches based on the energy difference still exists.
• Exploiting the stationarity difference. The initial works of spectrum sensing through stationarity difference can be traced back to work of Dandawate et. al. in [36], where second order cyclostationarity is employed. The cyclostationarity based scheme can trade latency with high sensing reliability. It is less sensitive to the noise uncertainty provided the knowledge of signals cyclic frequency [43]. To overcome the requirement of cyclic frequency in cyclostationary based detection while not suffering from the noise uncertainty problem which exists with energy detection, the covariance based detection was introduced [44]. This spectrum sensing method, utilizes the correlation which exists in the transmitted signal to determine the vacancy/occupancy of the frequency band of interest. However, the performance of this approach degrades dramatically as the correlation of the transmitted signal decreases. Matched filtering is known to be the optimum method for detecting signals when the transmitted signals air interface is completely known to the sensing device [50]. The main advantage of matched filtering is low latency and computational complexity. However, this approach requires perfect synchronizations between the transmitter and the spectrum sensing device. Matched-filtering pilot based detection, given the knowledge of pilot symbols and reasonably good timing and frequency synchronizations, exploits the cyclostationary property of the pilot symbols, to deliver fast and reliable sensing. The eigenvalue-based detection scheme exploits orthogonality between the signal subspace and noise subspace using second order stationarity features to offer highly reliable spectrum sensing [46]. However, it often needs the support of multiple antennas, and the subspace decomposition costs cubic complexity.

• Exploiting the distribution difference. Given that in almost all communication system models, noise is assumed to be additive white and Gaussian, one can determine the vacancy of a particular frequency band by observing the difference of the received signals distribution and that of the AWGN. An example of such approaches would be the kurtosis-type scheme, which exploits the non-Gaussianity of communication signals [58, 59]. This scheme features excellent accuracy at the price of large latency due to higher-order statistics. A critical point is that the sensing performance degrades significantly when signals are approximately Gaussian. Entropy
based spectrum sensing can be thought of an approach which also benefits from this property [56], where the probability space is partitioned into fixed dimensions and the Shannon entropy is employed as the information measure of the received signal as the test statistic.

2.1.6 Sub-Band Level Spectrum Spectrum Sensing

With increase in the spectrum utilization, spectrum scarcity increases. This would call for spectrum sensing techniques that adopt an architecture to simultaneously search over multiple frequency sub-bands at a time, while meeting the mandatory requirements of spectrum sensing, i.e., 1) low latency, 2) high reliability and 3) low complexity. However, the literature of sub-band spectrum sensing is rather limited at this time. In this section we will provide a system model for sub-band level spectrum sensing and further explain the state-of-the-art sub-band level spectrum sensing techniques in detail.

Sub-Band Level Spectrum Sensing System Model

Consider a communication system operating over a sub-band channel that is divided into $K$ non-overlapping sub-bands, e.g., multi-carrier systems. However, in a particular geographical region within a certain time frame only $l$ number of the sub-bands are utilized by the users, where $l < K$. Thus, $(K - l)$ sub-bands are available for opportunistic access. The essential task of the spectrum sensing device is to determine the availability of these $(K - l)$ sub-bands.

Let $y(n)$ denote the received sub-band signal at the spectrum sensing device. Hence:

$$y(n) = \lim_{k=0}^{K-1} \Lambda_q s_k(n) + v(n) \quad (2.23)$$

where $\Lambda_q$ is the indicator function which denotes the presence of the transmitted signal $q$. The opportunistic user needs to determine which of the spectrum bands are unoccupied, in order to utilize them efficiently. Based on 2.23 a number of solutions were proposed in the literature which are fully explained in the following sections.
2.1. Spectrum Sensing Techniques

Filter-Bank Based Spectrum Sensing

The Filter-bank architecture allows sub-band sensing with the aid of multiple narrow band, band-pass filters [26,60]. Filter banks are often implemented based on a prototype filter. The prototype filter is a lowpass filter that is also used to realize the first sub-band of the filter bank. Other bands are realized through repetition of the prototype filter. Hence, all the N sub-bands of interest share the same structure.

The implementation of an spectral estimator that uses a filter bank for signal analysis is as follows: 1) the input process is passed through a bank of filters and 2) the output power of each filter is measured as an estimate of the spectral power over the associated sub-bands and finally the vacancy of each sub-band is determined based on the estimated power of that particular sub-band. However, like conventional energy detection this approach will face the noise uncertainty problem.

It is shown in [60] that the filter bank based spectrum sensing performs significantly better in filter bank-based multi-carrier communication systems, since the same filters can be utilized for sensing purposes. Hence, in such systems, channel sensing is done at virtually no cost. This is only possible given that all the users within the geographical area of interest share the same air-interface and furthermore the opportunistic user also employs the same air interface. However, in a more general case where users may have different signaling format the filter-bank approach will result in increased number of components and energy consumption.

Joint Multi-Band Detection

In order to improve the performance of the filter bank detection the joint multiband detection, was proposed in [61]. This approach jointly optimizes a bank of multiple narrow band detectors to improve the aggregate opportunistic throughput of the opportunistic users while limiting the interference to the existing users. In particular, the joint multi-band detection reformulates the original problem of subband spectrum sensing into a class of optimization problems, where the objective is to maximize the aggregate opportunistic throughput in an interference-limited network given the opportunistic rate and interference penalty on each sub-band are known to the spectrum sensing device. Hence, the
optimization problem can be summarized by [61]

\[
\max r^T \left[ 1 - PFA(\gamma) \right] \\
K-1 \sum_{i=0} \left[ 1 - PD_i(\gamma_i) \right] \leq \varepsilon_j, j = 1, 2, ..., J
\]

\[
PFA(\gamma) \leq \beta \left[ 1 - PD(\gamma) \right]
\]

where \( r = [r_1, r_2, ..., r_K]^T \) is a vector with the throughput achievable over all \( K \) sub-bands, \( \gamma = [\gamma_1, \gamma_2, ..., \gamma_K]^T \) is the vector denoting the threshold value for all sub-bands, \( 1 - \alpha = [1 - \alpha_1, 1 - \alpha_2, ..., 1 - \alpha_K]^T \) and \( \beta = [\beta_1, \beta_2, ..., \beta_K]^T \) are the minimum limit for opportunistic spectral utilization required from the spectrum sensing device and the upper limit for the interference introduced by the opportunistic users, respectively.

Hence, the threshold setting in this approach is in such a way to firstly assure that the sub-band with a higher opportunistic rate has a higher threshold. In other words reduce PFA for the corresponding sub-band to ensure best possible use by the opportunistic users. Secondly, the higher priority sub-band, i.e., sub-bands carrying important messages, have a lower threshold resulting in smaller PD in order to prevent opportunistic users interference. Finally, a little compromise on the sub-bands carrying less important information which might boost the opportunistic rate considerably. Thus, in the determination of the optimal threshold for each sub-band, it is necessary to balance the channel conditions, the opportunistic throughput, and the relative priority of each sub-band. It has been shown in [61] that the joint multi-band detection can improve the performance of the filter-bank spectrum sensing significantly, and that the performance of this approach can be improved considerably by further exploiting the spatial diversity, i.e., cooperation between the spectrum sensing devices.

However, this technique requires the knowledge of noise power and the squared values of the channel frequency responses, which makes this approach only practical in fixed wireless networks, i.e., TV broadcast bands.
Sequential Multi-Band Detection

In [62], a sequential detection scheme has been developed for multi-band spectrum sensing. This approach employs a bank of sequential probability ratio tests [63], i.e., one per sub-band. The sequential probability ratio test has a very simple structure where the likelihood ratio of the observed samples is tested against two thresholds. The sequential probability is known to minimize the average sample number amongst all detectors given the PD and PFA requirements of the system. Hence, this algorithm can be particularly useful in delay sensitive applications. The hypothesis testing for this approach for the $k^{th}$ sub-band can be expressed as

$$
\begin{align*}
\mathcal{S}_k^k &\geq \log A_k, \quad \mathcal{H}_0^k \\
\mathcal{S}_k^k &\leq \log B_k, \quad \mathcal{H}_1^k \\
\log B_k &\leq \mathcal{S}_k^k \leq \log A_k, \quad \text{take next sample}
\end{align*}
$$

(2.25)

where the test statistic $\mathcal{S}_k^*$ is the likelihood ratio estimated from the received signal and the threshold values are related to the false alarm probability and the miss detection probability, i.e., [63]

$$A_k \approx \frac{PD_k}{PFA_k} \quad B_k \approx \frac{1 - PD_k}{1 - PFA_k}$$

(2.26)

However, the key challenge associated with this detector is that the parallel sequential probability tests do not yield the same sample sizes. This is due to the fact that the observation length is variable which depends on the random received signal. Thus, the overall sensing delay will be considered as the largest detection delay among those of the parallel detectors, until the set of bands that can support the requested rate is discovered.

Wavelet Based Detection

![Figure 2.11: Block diagram of the wavelet based detection technique.](image)

The wavelet based spectrum sensing is able to perform sub-band sensing with the aid of edge detection [64,65]. Assuming that the power spectral characteristic is smooth within
each sub-band but exhibits a discontinuous change between adjacent sub-bands, wavelet
based detection has been proposed to identify and locate the spectrum holes by exploiting
the irregularities within the estimated Power Spectral Density (PSD) [35] with the aid
of the wavelet transform, an attractive mathematical tool for analysing singularities and
irregular structures of signals. Wavelet based detection has proved useful for fast coarse
spectrum sensing based on a number of non-stationary samples, by making use of the
signals non-stationarity features.

The wavelet based detection has been developed under four main assumptions: 1) The
total bandwidth for detection is known to the spectrum sensing device, 2) The number
of licensed users are unknown to the spectrum sensing device, 3) The PSD of all occupied
sub-band is smooth and almost flat, 4) The noise is AWGN, i.e., noise process has a flat
PSD within the whole observed bandwidth.

Hence, once the region of support is determined, wavelet-based approach will firstly es-
timate the PSD of the received signal and determine the number of sub-bands and the
corresponding frequency boundaries. Later, the PSD for each sub-band will be employed
to determine the vacancy of the estimated sub-bands. As a result, the wavelet based
detection is also known as the wavelet based edge detection. Hence, the availability of
the $k^{th}$ sub-band can be determined using the

$$\mathcal{G}_{WB}^k = \frac{1}{\cap f_k - \cap f_{k-1}} \int_{\cap f_{k-1}}^{\cap f_k} \mathcal{H}_j \lesssim \mathcal{H}_0 \lambda_{WB}$$ (2.27)

where $\cap f_k - \cap f_{k-1}$ denotes the estimated frequency boundaries of the $k^{th}$ sub-band ob-
tained using the wavelet transform [64, 65]. Based on the above, one of the main advan-
tages of the wavelet based detection is that it does not require any prior knowledge about
the signals features. However, the most important limitation of this spectrum sensing
approach is determining the correct smoothing function (mother wavelet) for the wavelet
transformation. Even though some common features are shared by most mother func-
tions, some can perform better than others in a given environment. Hence, in order to
obtain the best possible results in the wavelet based detection, the specific wavelet family
should be designed based on the characteristics of the transmitted signal.
2.1. Spectrum Sensing Techniques

Wigner-Ville Based Detection

The Wigner-Ville based spectrum sensing [66] derives a greyscale image of the time frequency description of the received signal through the Wigner-Ville transform, and similar to wavelet based detection with the aid edge detection is able to detect occupied frequency bands.

With the aid of the Wigner-Ville transform, it is possible to show the spectral components of a signal with respect to the time variable and therefore have a bi-dimensional description of the perceived signal [67]. The resulting image from the Wigner-Ville based detection shows the spectrum occupancy in both time and frequency, marking the occupied zones with higher brightness. Hence, such zones are to be avoided by the opportunistic user, who, thanks to an edge detection, is able to detect the vacant sub-bands.

Such two dimensional strategies such as Wigner-Ville and wavelet based detection, tend to improve the performance of the spectrum sensing device with respect to single dimensional approaches due to the phenomenon known as uncertainty relationship which describes the trade off between the spectral and temporal resolution.

At the final stage, the measured energy level is employed as the slot availability criterion, entailing that slots are considered occupied even when they present high energy even in a narrow spectral and time components. However, these approaches may suffer from noise uncertainty problem, due to use of energy detection.

2.1.7 Summary of The Sub-Band Spectrum Sensing Approaches

<table>
<thead>
<tr>
<th>Approach</th>
<th>Air-Interface Information</th>
<th>Synchronisation</th>
<th>Computational Complexity</th>
<th>Latency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Filter Bank</td>
<td>×</td>
<td>×</td>
<td>Low</td>
<td>Low</td>
</tr>
<tr>
<td>Joint Multi-Band Detection</td>
<td>✓</td>
<td>✓</td>
<td>High</td>
<td>High</td>
</tr>
<tr>
<td>Sequential Multi-Band Detection</td>
<td>✓</td>
<td>×</td>
<td>High</td>
<td>Low</td>
</tr>
<tr>
<td>Wavelet Based Detection</td>
<td>×</td>
<td>×</td>
<td>Medium</td>
<td>Low</td>
</tr>
<tr>
<td>Wigner-Ville Based Detection</td>
<td>×</td>
<td>✓</td>
<td>High</td>
<td>High</td>
</tr>
</tbody>
</table>

Figure 2.12: Summary of the state-of-the-art local sub-band spectrum sensing approaches.

As explained in this section, different spectrum sensing scenarios demand for different sub-band spectrum sensing approaches based on their requirements. Amongst the exist-
ing practical solutions, there are filter-bank [60] and wavelet [65] based spectrum sensing techniques. The Filter-bank architecture allows sub-band sensing with the aid of multiple narrow bands, which results in increased number of components and energy consumption. However, the filter-bank approach is one of the preferred solutions when the spectrum sensing device does not have any a-priori knowledge about the signaling information. Wavelet based spectrum sensing exploits the multi-resolution features of the wavelet transform to estimate the power spectral density. With the aid of edge detection, the spectrum band of interest is divided into a number of sub-bands. This technique is particularly useful, when there are limited number of non-stationary samples. Fine spectrum sensing is further required, in order to determine the vacancy of specified frequency subbands. The Wigner-Ville based spectrum sensing [66] derives a greyscale image of the time-frequency description of the received signal through the Wigner-Ville transform. Similar to the wavelet based detection, with the aid of edge detection, it is able to detect occupied frequency bands.

Recently, a multi-band joint detection for spectrum sensing has been introduced in [61], where spectrum sensing is performed through a class of optimization problem with the objective of improving the aggregate opportunistic throughput of the opportunistic spectrum access user while limiting the interference to the other users in the system. However, this technique requires the knowledge of noise power and the squared values of the channel frequency responses, which makes this approach only practical in fixed wireless networks, i.e., TV broadcast bands. While [62] investigates multi-band spectrum sensing algorithm, which supports quality-of-service traffic. In particular, [62] proposes a sequential sensing, where a bank of sequential probability ratio tests are run in parallel to detect the availability of sub-bands, while ensuring a fixed minimum rate for the opportunistic user. This approach is based on the assumption that the propagation channel between the transmitter and the spectrum sensing device is fixed and deterministically known to the opportunistic users, making this multi-band spectrum sensing approach also only suitable for fixed networks.
2.1.8 Summary

Spectrum sensing device needs to continuously monitor the spectrum for possible presence of the vacant frequency bands. In this section, we have discussed various local spectrum sensing techniques that exploit null, minimal, or full knowledge of the transmitted signal characteristics. We have also addressed the state-of-the-art local sub-band spectrum sensing approaches, which can effectively improve the overall spectrum utilization by simultaneously search over multiple frequency subbands at a time. Furthermore, it is realized that existing schemes can hardly meet the requirements of a fast and accurate spectrum sensing, particularly, in low SNR range, (considering that the target SNR for a reliable spectrum sensing sensitivity is about -20 dB [68]) without introducing high complexity to the system. This observation motivates us to develop a spectrum availability models, which can significantly improve the state-of- the-arts and provides a practical solution. Throughout our research, we focus on the energy detection scheme since the only real-data publicaly available is energy sensed data.
Chapter 2. State-of-the-art Spectrum Modeling

2.2 Spectrum Availability Modeling

Wireless communication systems have been exploited since the early days of radio communications under a fixed spectrum management policy. Portions of the spectrum separated by guard bands have been allocated to particular licensees over large geographical regions, on a long term basis, and under exclusive exploitation licenses. Under this static regulatory regime, the overwhelming proliferation of new operators, services and wireless technologies has resulted in the depletion of spectrum bands with commercially attractive radio propagation characteristics. As a result, the Dynamic Spectrum Access (DSA) paradigm based on the Cognitive Radio (CR) technology [1] has gained popularity, motivated by the currently inefficient utilization of spectrum already demonstrated by many spectrum measurement campaigns performed all around the world [2-6]. The basic underlying principle of DSA/CR is to allow unlicensed users to access in an opportunistic and non-interfering manner some licensed bands temporarily unoccupied by licensed users. Unlicensed (secondary) DSA/CR terminals monitor the spectrum in order to detect spectrum gaps left unused by licensed (primary) users and opportunistically transmit. Secondary unlicensed transmissions are allowed according to this operating principle as long as they do not result in harmful interference to the licensees. As a result of the opportunistic nature of the DSA/CR principle, the behavior and performance of a network of DSA/CR nodes depends on the primary spectrum occupancy pattern. Realistically and accurately modeling such patterns becomes therefore essential and extremely useful in the domain of DSA/CR research. Models of spectrum use can find applications in a wide variety of fields, ranging from analytical studies to the design, dimensioning and performance evaluation of secondary networks, including the development of innovative simulation tools as well as novel DSA/CR techniques.

The existing models can be categorized into time-, frequency- and space-dimension models, each of which describe the statistical properties of spectrum usage in the corresponding domain. Based on this classification, we review some existing modeling approaches and analyzes their merits and limitations.
2.2. Spectrum Availability Modeling

2.2.1 Time-Dimension Models

From the point of view of DSA/CR, spectrum usage can adequately be modeled by means of a two-state Markov chain, with one state indicating that the channel is busy and therefore not available for opportunistic access and the other one indicating that the channel is idle and thus available for secondary use. Although some alternative modeling approaches have been proposed in the literature [12], the two-state Markov chain is the most widely employed time-dimension model in DSA/CR research. This binary channel model can be employed to describe the occupancy pattern of a licensed channel in discrete and continuous time.

Discrete-Time Models

In the two-state Discrete-Time Markov Chain (DTMC) model the time index set is discrete. According to this, the channel remains in a given state at each step, with the state changing randomly between steps. The behavior of the DTMC channel model can be described by means of the set of transition probabilities between states (see Figure 2.13), which can be expressed in matrix form as:

\[
P = \begin{bmatrix}
p_{00} & p_{01} \\
p_{10} & p_{11}
\end{bmatrix}
\] (2.28)

where \( p_{ij} \) represents the probability that the system transitions from state \( s_i \) to state \( s_j \).

Note that the DTMC channel model in 2.28, commonly used in the literature, assumes a stationary (time-homogeneous) DTMC, where the transition matrix \( P \) is constant and independent of the time instant.

The mean occupancy level of a channel is certainly a straightforward metric and an accurate reproduction is a minimum requisite for any time-dimension spectrum occupancy model. The average occupancy of a channel can be expressed in terms of the Duty Cycle (DC), henceforth denoted as \( \Psi \). The DC of a channel can be defined as the probability (or fraction of time) that the channel is busy (i.e., resides in the busy state in the long
term), which for the DTMC model of Figure 2.13 is given by $\Psi = p_{01}/(p_{01} + p_{10})$. The stationary DTMC model of 2.28 can therefore be configured to reproduce any arbitrary $\Psi$ by selecting the transition probabilities as $p_{01} = p_{11} = \Psi$ and $p_{10} = p_{00} = 1 - \Psi$, which yields:

$$P = \begin{bmatrix} 1 - \Psi & \Psi \\ 1 - \Psi & \Psi \end{bmatrix}$$

(2.29)

Nevertheless, reproducing not only the mean DC but also the lengths of the busy and idle periods is an important feature of a realistic time-domain model for spectrum use. The stationary DTMC model has been proven to not be able to reproduce the statistical properties of busy and idle period lengths of real channels [69]. This limitation, however, can be overcome by means of a non-stationary (time-inhomogeneous) DTMC channel model with a time-dependent transition matrix:

$$P(t) = \begin{bmatrix} 1 - \Psi(t)\Psi(t) \\ 1 - \Psi(t)\Psi(t) \end{bmatrix}$$

(2.30)

In the stationary case of 2.29, $\Psi$ represents a constant parameter. However, in the non-stationary case of 2.30, $\Psi(t)$ represents a time-dependent function that needs to be characterized in order to characterize the DTMC channel model in the time domain. The development of mathematical models for $\Psi(t)$ may be relatively simple for primary systems where occupancy patterns are characterized by a strong deterministic component. This is for example the case of cellular mobile communication systems, which exhibit a

---

Figure 2.13: Discrete-Time Markov Chain (DTMC) model
2.2. Spectrum Availability Modeling

Figure 2.14: Empirical DC time evolution and corresponding deterministic model for: (a) DCs 1800 low/medium-load channel, (b) E-GSM 900 medium/high-load channel

periodic load variation pattern on a daily basis (see Figure 2.14). Adequate models for such deterministic patterns have been developed in [69]. In general, however, the traffic load supported by a radio channel is normally the consequence of a significant number of random factors such as the number of incoming and outgoing users and the resource management policies employed in the system. As a result, the channel usage level is itself a random variable that may more appropriately be characterized from a stochastic modeling perspective, which still constitutes an open issue.

Continuous-Time Models

Another popular model in the DSA/CR literature is the two-state Continuous Time Markov Chain (CTMC) model, where the channel remains in one state for a random time period before switching to the other state. The state holding time or sojourn time is modeled as an exponentially distributed random variable. Some works based on empirical measurements [70–73] have demonstrated, however, that state holding times do not follow exponential distributions in practice. In particular, it has been found that state holding times are more adequately described by means of generalized Pareto [74], a mixture of uniform and generalized Pareto [70,71], hyper Erlang [70,71], generalized Pareto and hy-
per exponential [72], as well as geometric and log-normal distributions. Based on these results, a more convenient model is therefore the Continuous Time Semi Markov Chain (CTSMC) model, where the state holding times can follow any arbitrary distributions. Appropriate parameters for the aforementioned distributions have been derived from field measurements performed with high time resolution [70–72] and low time resolution [73] measurement equipment, and for various radio technologies of practical relevance. It is worth noting that the CTSMC channel model provides an explicit means to characterize and reproduce the lengths of the busy and idle periods, which implicitly offers the possibility to reproduce any arbitrary DC by appropriately selecting the parameters of the sojourn time distributions in order to provide mean values $E_{T_i}$ such that:

$$\Psi = \frac{E_{T_i}}{E_{T_0} + E_{T_1}}$$

(2.31)

where $E_{T_0}$ and $E_{T_1}$ respectively represent the mean sojourn times in the idle and busy states (i.e. mean idle and busy period duration).

### 2.2.2 Frequency-Dimension Models

The previous section has reviewed existing models to describe the statistical properties of spectrum occupancy patterns on individual channels. Such modeling approaches can be extended by considering as a whole the channels belonging to the same allocated band and introducing additional models to describe the statistical properties of spectrum usage in the frequency dimension.

#### Beta Distribution Model

In the spectral domain, one of the simplest but probably most relevant aspects to be captured and reproduced is the statistical distribution of the channel occupancy levels. The DCs of individual channels belonging to the same band have been shown to follow a beta distribution [4], whose density function is as follows:

$$f_x(x; \alpha, \beta) = \frac{1}{B(\alpha, \beta)} x^{\alpha-1} (1 - x)^{\beta-1}, \quad 0 < x < 1$$

$$B(\alpha, \beta) = \int_0^1 t^{\alpha-1} (1 - t)^{\beta-1} dt$$

(2.32)
2.2. Spectrum Availability Modeling

Figure 2.15: Empirical DC distributions and corresponding beta and Kumaraswamy fits

where $\alpha > 0$ and $\beta > 0$ are shape parameters and $B(\alpha, \beta)$ is the beta function.

Kumaraswamy Distribution Model

Alternatively, the Kumaraswamy distribution can be used as well:

$$f_{K}(x; a, b) = abx^{a-1}(1-x^{a})^{b-1}, \quad 0 < x < 1$$

where $a > 0$ and $b > 0$ are shape parameters. The Kumaraswamy distribution is similar to the beta distribution, but easier to use in analytical studies due to the simpler form of its density function. Figure 2.15 shows some examples of empirical DC distributions and their corresponding beta and Kumaraswamy fits.

Both Beta and Kumaraswamy distributions can be configured to reproduce any arbitrary mean DC over the whole band, $\bar{\Psi}$, by properly selecting the shape parameters:

$$\bar{\Psi} = \frac{\alpha}{\alpha + \beta} = bB \left( 1 + \frac{1}{a}, b \right)$$

2.2.3 Space-Dimension Models

While time- and frequency-dimension models are intended to reproduce the statistical properties of real spectrum occupancy patterns of primary transmitters, space-dimension
models, in general, deal with the characterization of spectrum occupancy patterns as perceived by the DSA/CR users at various locations. Spectrum occupancy in the space domain is analyzed and characterized in [75] in terms of the spatial distribution of the Power Spectral Density (PSD) by means of spatial statistics and random fields. Concretely, a semivariogram analytical model is fitted to average PSD values obtained at various locations by means of field measurements (empirical model) and simulation tools (deterministic model). The resulting PSD values can be mapped to binary busy/idle perceptions at various locations by means of a thresholding technique.

An alternative probabilistic modeling approach has been developed in [76, 77], which is illustrated in Figure 2.16. In the first step, a radio propagation model is used to estimate, based on a set of input parameters \( p = (p_1, p_2, \ldots, p_M) \) such as operating frequency, distance, etc., the radio propagation loss \( L \) between the primary transmitter and the DSA/CR user. Based on the primary transmission power \( P_T \), the computed losses \( L \) are then employed to compute the primary power \( P_S \) that would be observed by DSA/CR nodes at various locations, which are translated to SNR values \( \gamma \) based on the receiver’s noise power \( P_N \). The resulting SNR values are then fed, along with an additional set of input parameters \( p' = (p'_1, p'_2, \ldots, p'_N) \), to a DC model, which outputs an estimation \( \Psi \) of the DC that would be perceived at the considered geographical locations [77] as follows:

\[
\Psi = \left( \frac{1}{K} \sum_{k=1}^{K} \alpha_k \right) P_{fa} + \sum_{k=1}^{K} \alpha_k Q \left( \frac{Q^{-1}(P_{fa}) - \gamma_k}{\sigma S_k} \right)
\]  

(2.35)

where \( K > 0 \) represents the number of transmission power levels that may be present in the channel, \( 0 < \alpha_k \leq 1 \) is the activity factor of the k-th power level, \( P_{fa} \) is the target probability of false alarm of the DSA/CR network, \( \gamma_k (dB) = P_{S_k} (dBm) - P_N (dBm) \) is
2.2. Spectrum Availability Modeling

Figure 2.17: Characterization of spatial spectrum occupancy perception at various locations in a realistic urban environment: (a) in terms of binary busy (white)/idle (gray) observations, (b) in terms of the probability to observe spectrum as busy

the SNR resulting from the $k^{th}$ average transmission power level, and $\sigma_{S_k}$, $\sigma_N$ are the standard deviation of the $k$-th signal and noise power levels respectively. These parameters constitute the input vector $p'$. Figure 2.17 shows an example of the spectrum occupancy perceived by DSA/CR users at various locations in a realistic urban environment at the ground level between buildings, inside buildings and at rooftops [76]. While PSD/power modeling methods combined with thresholding techniques provide a simple binary characterization of the spatial spectrum occupancy perception (figure 2.17(a)), the approach depicted in 2.16 provides a more sophisticated characterization (figure 2.17(b)) by means of the probability that the spectrum is observed as busy depending on the specific DSA/CR user location, the considered scenario and the surrounding radio propagation environment.

2.2.4 Summary

The development of the DSA/CR technology can significantly benefit from accurate and practical spectrum usage models. The purpose of such models is to provide a tractable,
yet realistic representation of the statistical properties of spectrum usage in real systems that can adequately be employed in analytical studies or implemented in simulation tools for the performance evaluation of DSA/CR techniques. In this context, this section has provided an overview of spectrum occupancy models recently proposed in the literature in the context of DSA/CR. The existing models can be broadly categorized into time-, frequency and space-dimension models. Based on this classification, this section has reviewed various modeling approaches, pointing out their advantages and shortcomings.
2.3 System Assumptions Validation

In this section, we validate the i.n.i.d. observation for channels idleness (availability) probabilities (CIPs) using the Pearson's chi-square statistic and McNemars test statistic. Furthermore, we validate the Bernoulli sequence assumption.

2.3.1 Independence Validation

Our intention is to validate that the availability probabilities of two adjacent channels are statistically independent. Pearson’s chi-square statistic, \( \chi^2_{\text{Ind}} \) [78], is used to assess independence. We compute \( \chi^2_{\text{Ind}} \) from the experimental results of Measurement Set 1 for random two adjacent channels 2500 (equivalently 520 MHz) and 2501 (520.2 MHz) during the 7-8 am interval using the 2 x 2 contingency table for them provided in Table 2.1. Similarly, Table 2.1 illustrates reported frequencies of two adjacent channels 1700 (1840 MHz) and 1701 (1840.2 MHz) during the morning (4-5 am) time interval. Each observation \( O_{i,j} \) consists of the values of two outcomes in the \( (i, j) \) cell and the null hypothesis is that the occurrence of these outcomes is statistically independent. Each observation is allocated to one cell of the contingency table, according to the values of the two outcomes. For instance, out of 2000 observations, \( O_{i,j} = 68(180) \) whereas \( i \) represents channel 2500 (1700) being busy and \( j \) represents channel 2501 (1701) being simultaneously busy. The theoretical frequency \( E_{i,j} \) for a cell, given the hypothesis of independence, is expressed as follows:

\[
E_{i,j} = \frac{\sum_{k=1}^{c} O_{i,k} \sum_{k=1}^{r} O_{k,j}}{N} \tag{2.36}
\]

where \( r \) is the number of rows, \( c \) is the number of columns in the table, and degrees of freedom is \( (r_1)(c_1) \). The value of the test statistic is given as:

\[
\chi^2_{\text{Ind}} = \sum_{i=1}^{r} \sum_{j=1}^{c} \frac{(O_{i,j} - E_{i,j})^2}{E_{i,j}} \tag{2.37}
\]

For the test of independence, a p-value of less than or equal to 0.05 is commonly interpreted as justification for rejecting the null hypothesis.

Based on the results presented in Table 2.1 obtained from Measurement Set 1, the value of \( \chi^2_{\text{Ind}} \) from Eq. 2.37 is 1.8125. The chi-squared statistic \( \chi^2_{\text{Ind}} \) can then be used to
Table 2.1: Contingency table for channels 2500 (1700) and 2501 (1701)

<table>
<thead>
<tr>
<th>Observations</th>
<th>Ch. 2501 (1701) Busy</th>
<th>Ch. 2501 (1701) Idle</th>
<th>Row total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ch. 2500 (1700) Busy</td>
<td>68 (180)</td>
<td>994 (852)</td>
<td>1062 (1032)</td>
</tr>
<tr>
<td>Ch. 2500 (1700) Idle</td>
<td>46 (167)</td>
<td>892 (801)</td>
<td>938 (968)</td>
</tr>
<tr>
<td>Column total</td>
<td>114 (347)</td>
<td>1886 (1653)</td>
<td>2000</td>
</tr>
</tbody>
</table>

calculate a p-value by comparing its value to a chi-squared distribution with specific $D_f$. The corresponding p-value is 0.1782 for Spectrum 1 data set. Since the p-value is greater than 0.05, we accept the hypothesis of statistical independence of idleness between channels 2500 and 2501. The value of $\chi^2_{Ind}$ for the data set in Spectrum 2 during the 4-5 am and 4-5 pm intervals are 0.0028 and 0.0014, respectively. The corresponding p-values are 0.9578 and 0.9698. Since both these values are higher than 0.05, we infer that the independence assumption of channel idleness between channels 1700 and 1701 is valid for the Maastricht measurements.

Similarly, we perform the same analysis with randomly selected adjacent channels 562 and 563, 1819 and 1820, 3155 and 3156, 3519 and 3520, and 6435 and 6436 from both the spectra. We need to emphasize here that independence test proved to be successful in majority of the adjacent channels we have randomly chosen from the Maastricht data set, especially in the Global System for Mobile Communication (GSM) 1800 MHz cellular band. However, we must admit that there are a few examples of adjacent channels like 3519 and 3520 and 1000 and 1001, which proved to have dependence in terms of channel availability implying occupancy by the same primary user.

2.3.2 Non-Identical Distribution Validation

For the validation of the claim for non-identical distribution of idleness between adjacent channels, we again refer to the experimental measurements and apply a non-parametric method called the McNemars test [78]. The test is applied to the same contingency table as in Table 2.1, which tabulates the outcomes of frequencies on adjacent channels 2500 (1700)
2.3. System Assumptions Validation

and 2501 (1701) for the Aachen (Maastricht) measurements. Here, the null hypothesis is the marginal homogeneity between these two adjacent channels, i.e., probability of channel 2500 (1700) being occupied and channel 2501 (1701) being idle \((p_{ib})\) subsequently is identical to the probability of channel 2500 (1700) being idle and channel 2501 (1701) being occupied \((p_{bi})\) subsequently. In the sequel, \((p_{bi})\), etc. are the theoretical probability of occurrences in cells with the corresponding label.

The McNemar test statistic with Yates correction for continuity is given by:

\[
\chi^2_{iden} = \frac{(|b - c| - 0.5)^2}{b + c} \tag{2.38}
\]

where \(b\) corresponds to the frequency (994 (852) from Table 2.1) in the cell of channel 2500 (1700) being busy and channel 2501 (1701) being idle subsequently and \(c\) corresponds to the frequency (46 (167) from Table 2.1) in the cell of channel 2500 (1700) being idle and channel 2501 (1701) being busy subsequently. Based on the computations, the statistic \(\chi^2_{iden}\) is equal to 862.3163 (452.89). The corresponding McNemars test probabilities are less than \(2.2e^{-16}\). For the test of marginal homogeneity, the McNemars test probability of less than or equal to 0.05 is commonly interpreted as justification for rejecting the null hypothesis. Therefore, the hypothesis of marginal homogeneity is rejected. Hence, the adjacent channels are non-identical in terms of occupancy or idleness. Similarly, we have performed the same analysis with randomly selected adjacent channels 1819 and 1820, 3155 and 3156, 1000 and 1001, and 3519 and 3520 and the hypothesis of non-identical distribution of occupancy or idleness were valid for both the spectra.

The Beta distribution model ?? for non-identically distributed channels availability probabilities in a spectrum is validated here by using the above mentioned two sets of measurements. In other words, we have partitioned the measurements in Spectrum 1 into four intervals of interest: (i) 7 to 8 am, (ii) 12 to 1 pm, (iii) 3 to 4 pm, and (iv) 11 pm to 12 am. Similarly, the measurements in Spectrum 2 are grouped into four intervals: (i) 4 - 5 am, (ii) 1 - 2 pm, (iii) 4 - 5 pm, and (iv) 9 pm - 10 pm.

A total of 100 random channels are considered in each spectrum and the corresponding channels availability probabilities are evaluated by considering the following detection thresholds: -114 dBm for channels in Spectrum 1 and -107 dBm for channels in Spectrum 2 and normalizing the results over the one hour periods. Idleness of a channel is decided based on whether the received power is higher or lower than the detection
threshold. At any given time instant, if the received power is greater than the detection threshold, the channel is said to be occupied and idle if received power is lower than the threshold. The mean availability $\mu_{av}$ and standard deviation $\sigma_{av}$ are then computed over the obtained channels availability probabilities. The four sets of $\mu_{av}$ and $\sigma_{av}$ are as follows:

**Spectrum 1:** 7 - 8 am: $\mu_{av} = 0.4176$, $\sigma_{av} = 0.1703$, 12 - 1 pm: $\mu_{av} = 0.3756$, $\sigma_{av} = 0.1377$,

3 - 4 pm: $\mu_{av} = 0.4654$, $\sigma_{av} = 0.1617$, and 11 pm - 12 am: 43, $\mu_{av} = 0.4571$, $\sigma_{av} = 0.1396$.

**Spectrum 2:**

4 - 5 am: $\mu_{av} = 0.3643$, $\sigma_{av} = 0.0953$, 1 - 2 pm: $\mu_{av} = 0.3521$, $\sigma_{av} = 0.0961$,

4 - 5 pm: $\mu_{av} = 0.3202$, $\sigma_{av} = 0.0962$, and 9 - 10 pm: $\mu_{av} = 0.3462$, $\sigma_{av} = 0.0907$.

Estimated $\hat{\alpha}$ and $\hat{\beta}$ parameters for the Beta distribution are computed using the following expressions [78]:

<table>
<thead>
<tr>
<th>Time of the day</th>
<th>Frequencies</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.0-0.2</td>
</tr>
<tr>
<td>Observed (7-8am)</td>
<td>21</td>
</tr>
<tr>
<td>Expected (7-8am)</td>
<td>22.27</td>
</tr>
<tr>
<td>Observed (12-1pm)</td>
<td>23</td>
</tr>
<tr>
<td>Expected (12-1pm)</td>
<td>23.3</td>
</tr>
<tr>
<td>Observed (3-4pm)</td>
<td>16</td>
</tr>
<tr>
<td>Expected (3-4pm)</td>
<td>16.86</td>
</tr>
<tr>
<td>Observed (11p-12am)</td>
<td>15</td>
</tr>
<tr>
<td>Expected (11p-12am)</td>
<td>15.6</td>
</tr>
</tbody>
</table>
2.3. System Assumptions Validation

Table 2.3: Observed and expected frequencies of spectrum availability for Maastricht, Netherlands data

<table>
<thead>
<tr>
<th>Time of the day</th>
<th>Frequencies</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.0-0.2</td>
<td>0.2-0.4</td>
<td>0.4-0.6</td>
<td>0.6-0.8</td>
<td>0.8-1.0</td>
</tr>
<tr>
<td>Observed (4-5am)</td>
<td>45</td>
<td>19</td>
<td>13</td>
<td>9</td>
<td>14</td>
</tr>
<tr>
<td>Expected (4-5am)</td>
<td>40.97</td>
<td>18.27</td>
<td>14.62</td>
<td>13.02</td>
<td>13.12</td>
</tr>
<tr>
<td>Observed (1-2pm)</td>
<td>46</td>
<td>19</td>
<td>15</td>
<td>6</td>
<td>14</td>
</tr>
<tr>
<td>Expected (1-2pm)</td>
<td>43.11</td>
<td>17.73</td>
<td>14.0</td>
<td>12.43</td>
<td>12.47</td>
</tr>
<tr>
<td>Observed (4-5pm)</td>
<td>47</td>
<td>25</td>
<td>10</td>
<td>5</td>
<td>13</td>
</tr>
<tr>
<td>Expected (4-5pm)</td>
<td>48.54</td>
<td>16.37</td>
<td>12.52</td>
<td>10.99</td>
<td>11.58</td>
</tr>
<tr>
<td>Observed (9-10pm)</td>
<td>44</td>
<td>24</td>
<td>13</td>
<td>5</td>
<td>14</td>
</tr>
<tr>
<td>Expected (9-10pm)</td>
<td>42.91</td>
<td>18.64</td>
<td>14.56</td>
<td>12.49</td>
<td>11.39</td>
</tr>
</tbody>
</table>

\[
\hat{\alpha} = \mu_{av} \left( \frac{\mu_{av}(1 - \mu_{av})}{\sigma_{av}} - 1 \right)
\]

\[
\hat{\beta} = (1 - \mu_{av}) \left( \frac{\mu_{av}(1 - \mu_{av})}{\sigma_{av}} - 1 \right)
\]  

(2.39)

For Spectrum 1 we have:
7 - 8 am: $\hat{\alpha} = 0.1786$, $\hat{\beta} = 0.2492$, 12 - 1 pm: $\hat{\alpha} = 0.2642$, $\hat{\beta} = 0.4392$,
3 - 4 pm: $\hat{\alpha} = 0.2506$, $\hat{\beta} = 0.2879$, and 11 pm - 12 am: $\hat{\alpha} = 0.3554$, $\hat{\beta} = 0.422$.

Similarly, for Spectrum 2 the estimated parameters are as follows:
4 - 5 am: $\hat{\alpha} = 0.5206$, $\hat{\beta} = 0.9086$, 1 - 2 pm: $\hat{\alpha} = 0.4842$, $\hat{\beta} = 0.8909$,
4 - 5 pm: $\hat{\alpha} = 0.4046$, $\hat{\beta} = 0.8590$, and 9 pm - 10 pm: $\hat{\alpha} = 0.5179$, $\hat{\beta} = 0.9780$.

Based on computed channel availability probabilities, we have classified them into five intervals of equal widths namely, 0.0-0.2, 0.2-0.4, 0.4-0.6, 0.6-0.8, and 0.8-1.0. The frequency of idle channels in each interval is specified as observed in Tables 2.2 and 2.3 for Spectrum 1 and Spectrum 2, respectively. The expected frequencies are obtained using the Beta distribution in Eq. 2.32 with the corresponding $\hat{\alpha}$ and $\hat{\beta}$ parameters and specific intervals. The observed frequencies and the Beta distribution with the estimated
parameters are shown in Figure 3.1 for both the spectra. The figure indicates that the Beta distribution is a good fit to the spectrum availability patterns in both Spectrum 1 and Spectrum 2 under investigation.

2.3.3 Bernoulli Sequence Validation

The Bernoulli trials process, named after Jacob Bernoulli, is one of the simplest yet most important random processes in probability [27]. Essentially, the process is the mathematical abstraction of coin tossing, but because of its wide applicability, it is usually stated in terms of a sequence of generic trials that satisfy the following assumptions:

- Each trial has two possible outcomes, in the language of reliability called success and failure.
- The trials are independent. Intuitively, the outcome of one trial has no influence over the outcome of another trial.
- On each trial, the probability of success is $p$ and the probability of failure is $1 - p$, where $p \in [0, 1]$ is the success parameter of the process.

Mathematically, we can describe the Bernoulli trials process with a sequence of indicator random variables:

$$X = (X_1, X_2, ..., X_n)$$ (2.40)

An indicator variable is a random variable that takes only the values 1 and 0, which in this setting denote success and failure, respectively. Indicator variable $X_i$ simply records the outcome of trial $i$. Thus, the indicator variables are independent and have the same probability density function:

$$P(X_i = 1) = p$$
$$P(X_i = 0) = 1 - p$$ (2.41)

The distribution defined by this probability density function is known as the Bernoulli distribution. In statistical terms, the Bernoulli trials process corresponds to sampling from the Bernoulli distribution. In particular, the first $n$ trials $(X_1, X_2, ..., X_n)$ form a random sample of size $n$ from the Bernoulli distribution. Note again that the Bernoulli
trials process is characterized by a single parameter $p$.

The system considered in this work satisfies the above 3 conditions as follows:

- Each trial in this system has 2 and only possible outcome. In detail, detecting a whitespace channel can result in either success or failure.

- Validated in 2.3.1 that the channels follows an independent framework.

- On each trial, the probability of detecting a whitespace channel is $p$ and the probability of miss detection is $1-p$, where $p \in [0,1]$ is the success parameter of the process.

Therefore, we have validated that the system follows an i.i.d Bernoulli sequence.
Chapter 3

Probabilistic Modeling of Spectrum Occupancy

3.1 Introduction

Recently, dynamic spectrum access (DSA) technologies have attracted significant interest in the research community [70, 79–83]. Mainly three facts have been the motivation of this work. First, the scarcity of radio spectrum complicates the process of allocating the spectrum to new wireless systems. Second, the success of wireless communication and the trend towards broadband systems further increases the need for radio spectrum. Third, researchers have found that a significant amount of officially licensed spectrum is unused. This has been reported on several extensive spectrum occupancy measurement campaigns [2–6].

Both, the DSA and cognitive radio research, require realistic models and good understanding of the current network as basis for performance evaluation. The occupancy of the channels exhibits dynamic spatial and temporal property due to randomness of access by the PUs. For example, the occupancy in a single channel at any instant in time can be determined by the SUs in a cooperative manner [7, 8, 10] as opposed to being determined individually by each user [84–87]. However, the instantaneous statistics of channel occupancy in the entire spectrum over time is very difficult to determine due to its time complexity. An instantaneous resource map is only available after the CR network has
the knowledge of the utilization of every channel in the entire spectrum. For this purpose, the CR network has to constantly monitor and collect the spectrum occupancy data for all channels. Additionally, the larger the bandwidth monitored or scanned, the higher will be the energy consumption for the SUs. McHenry and Taher et. al. [88, 89] have monitored the spectrum occupancy for different channels in Chicago as well as in Vienna, Virginia. The results show that the utilization of a channel can be small, moderate, or large, depending on the traffic load at a given time. Hence, instead of monitoring or sensing the entire spectrum, it is advisable to exploit the historical data [90] to select a set of preferable channels for further sensing. Instead of a direct map of occupancy, we analyze the channel occupancy in terms of the probability of capturing the dynamic spatial and temporal property. The probability of each channel being free, can be determined individually, without simultaneously monitoring all other channels, thereby significantly reducing the complexity. In particular, the following two aspects are evaluated to reflect the channel availability for SUs:

- **Total number of free channels** ($N_f$): At any given time instant, let $N_f$ be the total number of channels that are not used by the PUs. The entity $N_f$ indicates how many SUs can simultaneously access the network. $N_f$ also represents the probability that a SU can transmit packets, given the total number of SUs. Finally, the average and variance of $N_f$ indicates spectrum utilization by PUs over a period of time.

- **Occupancy of a free subband** ($N_i$): Given a free channel, the information about adjacent channels’ occupancy is critical in mitigating interference to its adjacent PUs. If two neighboring channels, i.e., $(i-1)^{th}$ and $(i+1)^{th}$ channels of an $i^{th}$ free channel are occupied by PUs, the transmission power from the $i^{th}$ channel interferes with both its adjacent neighbors. Hence, to reduce inadmissible interference, the transmission power in the $i^{th}$ channel has to be restricted. If two networks have the same number of free channels, the network with higher number of free subbands can support a larger number of transmissions with high power if we ignore other factors that affect the transmission power.

By taking advantage of the probability of a channel being free, we provide a statistical estimation of the number of available channels as well as the number of free subbands based
on the occupancy of immediate neighbors. The probability distribution of $N_f$ allows selection of preferable sections of the spectrum for further sensing so that spectrum sensing is performed accordingly on those selective sub-bands with high probabilities of being free. The importance of our research is further emphasized by Wellens et. al. [90], which stated that if the corresponding statistical analysis is available, the adaptive spectrum sensing could improve the probability of sensing success by 70% over random spectrum sensing. In addition, given a free channel, knowledge of occupancy of the adjacent channels can enable efficient power management [86] among requesting SUs. Thus, the allowable transmission power on a free sub-band can be adjusted based on the occupancy of its adjacent channels. Furthermore, the prioritized allocation of sub-bands to SUs [91, 92] can be implemented in such a way that a high priority SU is allocated a channel with higher transmission power when compared to a low priority SU.

Considering $N_f$ as a sum of independent but not identically distributed Bernoulli random variables, the traditional approach to calculate the exact probability distribution of $N_f$ is computationally complex. Therefore, we use convolution, recursive, and hybrid convolution-recursive techniques to compute the distribution of $N_f$. In order to analyze the occupancy of the neighborhood of free channels, we define three types of free subband (i.e., Type I-III subband) based on the occupancy of the first and third channels of the subband (having zero, one, or two free neighbors). The main contributions of this chapter are:

- Given the probability of the channels being available from a local database or using a sensing scheme, we derive the probability distribution of $N_f$ in any spectrum range.

- We derive the probability distribution of the total number of free subbands of a specific type $N_i$ (i.e., type I-III subband).

The rest of this chapter is organized as follows: Section 3.2 discusses the related work. Section 3.3 illustrates the network model and formulates the problem. In Section 3.4, we present the analytical models for probability distribution of $N_f$. In Section 3.5, we
characterize a free subband based on the occupancy of contiguous channels. Section 3.6 concludes the chapter.

### 3.2 Related Work

![Beta density histogram of idleness probabilities in two different geolocations](image)

**Figure 3.1:** Beta density histogram of idleness probabilities in two different geolocations [11]

In a CR network, study of spectrum occupancy involves three critical aspects: spectrum measurement, spectrum sensing, and channel allocation. The first two aspects, in turn, facilitates the channel allocation for SUs. The current research efforts mainly focus on spectrum sensing [7–10, 91] and subband allocation [84–87, 92]. For example, Ganesan et. al. [7] proposed a cooperative spectrum sensing approach in a two-user cognitive network. This approach has been further improved for multi-user cognitive networks [8] enhancing the detection capability of cognitive radio users by exploiting the spatial diversity. Ganesan et. al. [9] then discussed a spectrum sensing technique for a base station-controlled centralized cognitive network. It allows some cognitive users to act as relays for occupancy sensing of sub-bands while others transmit data in order to reduce average detection time. For efficient spectrum sensing, Tu et. al. [91] exploit physical layer attributes of PU transmissions like existence of cyclic prefix or fundamental symbol rate of signals.
Allocation of free channels involves many issues such as routing, traffic, and power constraints. For example, based on spectrum stability, Deng et. al. [85] present a method of selecting a set of sub-bands for a predetermined path between a source and a destination. Demestichas et. al. [86] describe a joint allocation of spectrum and radio access technologies using a learning and adaptation approach. The joint allocation models user requirements like traffic intensity, mobility characteristics, and quality of service guarantees. Wang et. al. [87] discussed a graph theoretic approach for joint route and spectrum selection. A time schedule improves the channel usage to ensure quality of service guarantees among the real-time applications. Chu et. al. [92] jointly optimize the power, time slots, and sub-carriers among the SUs in Orthogonal Frequency Division Multiple Access (OFDMA) cognitive radio systems. Spectrum measurement is critical to assess spectrum occupancy in the network. McHenry et. al. [88,89] have experimentally monitored spectrum occupancy for different sub-bands at multiple locations. They deployed a high dynamic range spectrum measurement system for spectrum monitoring ranging from hours to days. Sanders et al. [93] use the Radio Spectrum Measurement System to collect observations periodically on sub-bands in the 108 MHz to 10 GHz spectrum providing a wide selection of occupancy data. Roberson et al. [94] used passive monitoring over
the range of frequencies (30 MHz to 3 GHz) in order to categorize the degree of utilization of the sub-bands into four different classes: (i) subbands seldom used, (ii) subbands used during specific intervals of time, (iii) subbands infrequently used, and (iv) subbands heavily used. In contrast to these spectrum measurement approaches, our target is to perform probabilistic analysis of sub-band occupancy. Luo et. al. [84] assume an independent and identically distributed (i.i.d) model for the spectrum occupancy. This means that the occupancy of subbands are statistically independent with constant probability of occupancy over the entire spectrum. With this model, they developed a detection search strategy for free sub-band based on an average search time. However, this model using a constant probability of occupancy does not reflect practical temporal and spatial variations in the spectrum. Additionally, authors in [12] captured the characteristics of

![Figure 3.3: Probability Mass Function of 20 channels. On right, the channel probabilities are small and have low variance whereas subfigure on left, probabilities are high with very low variance [95, 96]](image)

primary user occupancies based on variations in transmission powers, selection of center frequencies and time durations of operations. They proposed a validated probabilistic model for channel occupancy using real time measurements in the paging band (928-948 MHz). Furthermore, a Beta distribution 2.2.2 [97] model for channel occupancy was proposed by characterizing inherent parameters ($\alpha$ and $\beta$) (refer to Figure 3.1) substantiated with real-time measurements [98]. Ghosh et. al. [99] extensively validated this model
for better comprehension of channel occupancy patterns of licensed users using real-time measurements on the 1500 MHz band conducted in Aachen, Germany.

The same aforementioned authors stated that it is imperative to find an efficient way to significantly reduce the computation complexity in calculating the exact distribution, therefore, they proposed a Normal [100] and Normal-Poisson [97] approximation method for spectrum availability modeling based on the independent but not identically distributed (i.n.i.d.) model [11] [101]. A snapshot of Ghosh et. al.’s simulation results is depicted in Figure 3.2.

On the other hand, Arshad et. al. [95, 96] presented a statistical model of spectrum opportunities in terms of the probability of each channel being free. They also developed a model called Camp-Paulson to characterize the number of spectrum opportunities available using the probability and approximation theory. Figure 3.3 illustrates some of their numerical results showing that the proposed approximation models of spectrum opportunities achieve good accuracy at significantly lower computational cost. Similarly, in [95, 96], Arshad et. al. presented a statistical model of spectrum opportunities in terms of the probability of each channel being free. An approximation model, namely Camp-Paulson, has been developed to characterize the number of available spectrum opportunities. Authors claimed that their proposed Camp-Paulson Model achieve good accuracy at significantly lower computational cost when compared to the exact conventional distribution as well as the Poisson-normal model proposed in [11].

To summarize, most relevant research within the area of channel availability modeling managed to develop approximations to facilitate the computation of the exact distribution of the number of available channels at a given time. Not only that these approximations can show unacceptable margin of errors, but also, they were only validated with a limited number of channels (a maximum of 20 and 50 channels for the Camp-Paulson and Poisson-normal model respectively). Moreover, the running time of the Poisson-normal algorithm is high due to the combinatorial complexity of the model (4.8 seconds compared to 0.035 second with respect of the Camp-Paulson model for an input of 50 channels). This has motivated us to consider developing accurate and more efficient models. The work
reported in this chapter has been published in [102].

The main novel contributions of our research are summarized as follows:

- We employ convolution, recursive, and hybrid convolution-recursive methods to develop accurate and efficient techniques which tackles the combinatorial complexity burden of the actual distribution computation of the number of available channels based on the i.n.i.d paradigm. The outcome is 3 effective algorithms that are able to compute the distribution efficiently in a negligible time as compared to others proposed in the State-of-the-art.

- Given a free channel, the information about adjacent channels’ occupancy is critical in mitigating interference to its adjacent PUs. If two neighboring channels, i.e., $(i - 1)^{st}$ and $(i + 1)^{st}$ channels of an $i^{th}$ free channel are occupied by PUs, the transmission power from the $i^{th}$ channel interferes with both its adjacent neighbors. Hence, to reduce inadmissible interference, the transmission power in the $i^{th}$ channel has to be restricted. We categorize available channels based on the occupancy of its two adjacent channels into three different types, then, probabilistically model their availability using the proposed hybrid model. In this context, we propose a model that is able to efficiently predict the channels’ availability while taking in consideration the the status of the adjacent neighbor channels.

- In order to assess the performance of the proposed models in practical scenarios, predicting the count of available channels has to be evaluated against the probability of detecting these channels. In this context, a multi-channel joint detection framework is presented with respect to the network performance where we propose a novel approach that enables to compute the probability of detecting multi-channels simultaneously in an efficient manner based on the i.n.i.d paradigm.

- We validate the effectiveness of proposed models using several real time measurements then assess their efficiency using analysis metrics such as algorithm time complexity and running time. Furthermore, we illustrate 2 applications associated with the use of the modeling where one features novel 2-Dimensional (time, freq) availability modeling.
3.3 System Model and Problem Formulation

An example of a band composed of \( n \) subchannels in which some are occupied by primary users is depicted in Figure 3.4. In a particular geolocation and time interval, we consider a primary communication system operating over a spectrum (wideband) that is composed of \( n \) non-overlapping subchannels. Some of the \( n \) subchannels are not occupied by the primary users and therefore available for opportunistic spectrum access. The number of occupied subchannels vary as a function of time depending on primary users patterns. Let \( \{x_1, x_2, ..., x_n\} \) be the set of \( n \) Bernoulli variables where \( x_i = 0 \) reflects the \( i^{th} \) subchannel being occupied by a primary user and \( x_i = 1 \) denotes it is a vacant subchannel. For example, the third subchannel in Figure 3.4 is occupied and its status is expressed as \( x_3 = 0 \). Similarly, the status of the \( (i + 5)^{th} \) subchannel is depicted as \( x_{(i+5)} = 1 \). The occupancy of the \( i^{th} \) subchannel at any given time is characterized by the probability \( p_i \) of it being available and hereafter referred to as the subchannel availability probability. This probability is extracted using a threshold by observing the detected signal power of the \( i^{th} \) subchannel over a desired period of time, then average it. We assume that \( x_i \)'s are modeled as independent, non identical (i.n.i.d) Bernoulli random variables (Validated in 2.3) [12]. In fact, in the frequency range of interest, the subchannel occupancy is statistically independent, i.e., the arrival of a primary user in one subchannel does not depend on the arrival of another primary user in any other subchannel at the same time.
instance. Moreover, the variables \( x_1, x_2, \ldots, x_n \) are not necessarily identically distributed, i.e., \( P(x_i = 1) \neq P(x_j = 1) \). We further define a random variable \( N_f \) to represent the total number of free sub-bands in the spectrum of interest. In other words, \( N_f \) is the sum of all \( N \) random variables \( x_i \) s, i.e., \( N_f = \sum_{n=1}^{N} x_i \). The possible values of \( N_f \) are \( 0, 1, \ldots, N \).

### 3.4 Computation of \( P(N_f = r) \)

There are several benefits associated with estimating the likelihood of availability of a certain number of free channels, i.e. \( P(N_f = r) \), and their configuration/location over a spectrum. It can provide a track-able representation of the statistical properties of spectrum usage that can easily be employed in the analytical studies or implemented in simulation tools for the performance evaluation of CR technologies. Moreover, this information can act as a pre-requisite for performing further processing to establish the communication link. In this manner, we can avoid some unnecessary overhead calculations and processing in the scenarios where the chances of establishing a link can satisfy the expected QoS. Modeling the number of available channels and their configuration also provides useful information to establish if the constraints in terms of communication strategy can be met or not (e.g. FFT size that the transceiver can process, number of pilots used for channel estimation, carrier aggregation strategies/algorithms available to transceiver, etc.). If any of these constraints in a transceiver imply that the expected number and configuration of the channels is not suitable, we can avoid doing the unnecessary further processing involving sensing, channel estimation, spectrum aggregation, etc.

In this section, we highlight the combinatorial complexity arising from the traditional (conventional) computation of the exact probability distribution and further introduce the proposed model for alternative computations. We define \( \bar{p}_i \) such as \( \bar{p}_i = 1 - p_i \) and the generic term \( n\text{-SS} \) to represent the frequency range (spectrum) of interest composed of \( n \) non-overlapping subchannels.

The computation of the exact probability distribution of \( N_f \) becomes complex when \( p_i \)’s
are not equal. Each case of \(P(N_f = r)\) generates \(\binom{n}{r}\) possible scenarios to compute, where \(n\) probabilities of each scenario are multiplied. These products are then added together involving \(\binom{n}{r}\) summands resulting in the overall probability of the case. Thus, the computational complexity of each case is \(n\binom{n}{r} - 1\). For example, the probability of having exactly 16 available subchannels in a 32-SS, \(P(N_f = 16)\), incurs \(\binom{32}{16}\approx 601\) million possible scenarios, where in each scenario 32 numbers are multiplied. As \(n\) increases, the enormous amount of computation becomes a memory constraint for the secondary user device and is very time consuming.

**Algorithm 1** Conventional Computation of \(N_f\) Probability Distribution

1. Input \(p_i = [p_1, p_2, ..., p_n]\)
2. \(\bar{p}_i = (1 - p_i)\)
3. \(N_f(0) \leftarrow \) element products of \(\bar{p}_i\)
4. for \(r = 1\) to \(n\) do
5. Initialize matrix of Zeros \(C\) of order \(\binom{n}{r}, r\)
6. Initialize matrix of Zeros \(\text{REST}\) of order \(\binom{n}{r}, n-r\)
7. Initialize matrix of Zeros \(\text{SCENARIOS}\) of order \(\binom{n}{r}, n\)
8. \(C \leftarrow \) combinations of \(k\) of the elements in the vector \(P\)
9. for \(j = 1\) to \(\text{size}(C, 1)\) do
10. \(\text{Temp} \leftarrow \) set of values in \(P\) that are not in \(C(j,: )\)
11. \(\text{REST} \leftarrow [\text{REST}; 1 - \text{Temp}]\)
12. end for
13. \(\text{SCENARIOS} \leftarrow [C, \text{REST}]\)
14. \(M \leftarrow \) product of elements of each row in \(\text{SCENARIOS}\)
15. \(N_f(r) \leftarrow \) summation of elements in \(M\)
16. end for
17. Output \(N_f\)

In order to determine the computation complexity, we define the entity \(T(n)\) which denotes the time complexity of an algorithm on a particular input of \(n\) values. \(T(n)\) represents the sum of running times for each statement executed, for example a statement that takes \(c_i\) operations or steps to execute and executes \(n\) times will contribute \(c_in\) to the total
running time. We use the big $O$ notation to provide a lower bound on the growth rate of the time complexity function.

The conventional Algorithm 1 is implemented to compute the probability distribution of $N_f$ as there exist no closed-form formula for this purpose. The main approach behind this algorithm is to determine all possible scenarios and list them in a matrix, then, multiply each row’s elements together resulting on a 1 dimensional vector/array. Adding the elements in this vector will result in: $P_r = P(N_f = r)$ i.e. $r = 0, 1, ..., n$. In details, the basic input of this algorithm is a given set of subchannel free probabilities $p_1, p_2, ..., p_n$ (Line 1) whereas the output is a set of discrete probabilities $P_1, P_2, ..., P_n$ stored in vector (array) named $N_f$, where $P_r = P(N_f = r)$ (Line 17). These discrete probabilities $P_i$’s form the actual distribution of $N_f$. All different combinations for each case $N_f = r$, i.e. $r = 0, 1, ..., n$ are determined in Line 9-12 then stored in a matrix named SCENARIOS (Line 13). Next, the product of the elements of each row in SCENARIOS is computed and stored it in a vector $M$ (Line 14). Adding all the elements in vector $M$ (Line 15) results on the $i$-th $P$. Given these steps are repeated $n$ times through the loop in Line 4, the actual distribution of $N_f$ is determined.

In order to determine the computation complexity, we define the entity $T(n)$ which denotes the time complexity of an algorithm on a particular input of $n$ values. $T(n)$ represents the sum of running times for each statement executed, for example a statement that takes $c_i$ operations or steps to execute and executes $n$ times will contribute $c_i n$ to the total running time. We use the big $O$ notation to provide a lower bound on the growth rate of the time complexity function. In the conventional Algorithm 1, the leading order term arises in line 8 inside the second loop of size $\binom{n}{r}$ which itself is inside a first loop in line 3 therefore the resultant complexity in determining the conventional exact distribution of $N_f$ is given by:

$$T(n) = \sum_{r=0}^{n} \left[ n \binom{n}{r} - 1 \right] \simeq n \sum_{r=0}^{n} \left( \binom{n}{r} \right) \simeq O(n.2^n)$$ (3.1)

In what follows, we introduce the proposed approaches to compute the exact probability distribution of $N_f$. Since most multi-carrier transmission schemes employ FFT operation,
we configure the proposed algorithms to run with a number of subchannels \( n \) to the power of 2 i.e. \( n = 64, 128, 256, 512, \text{ etc.} \)

### 3.4.1 Multi-Convolved Exact Probability Model

Consider a 32-SS where its \( P(N_f = r) \) generates a total of \( \sum_{r=0}^{32} \binom{n}{r} \approx 4.29 \text{ Billion scenarios} \) to compute whereas an 8-SS, its \( P(N_f = r) \) will generate a total of \( \sum_{r=0}^{8} \binom{n}{r} \approx 255 \) scenarios to compute. It is evident that the 8-SS has negligible scenarios when compared to the 32-SS. Hence, dividing this 32-SS into 4 subbands each of 8 subchannels, computing each subband availability distribution separately, and then convolving them will clearly reduce the computational complexity. Convolution is done in a tree manner, 2 adjacent subbands at a time. For better understanding refer to example in Figure 3.5.

Let \( X \) and \( Y \) represent two adjacent subbands with equal number of subchannels \( n/2 \). Consider the sum of two independent discrete random variables \( N_{fx} \) of the \( X \)-subband and \( N_{fy} \) of the \( Y \)-subband whose values are restricted to positive integers. The \( N_f \) probability distribution of \( X \cup Y \), \( P(N_f = r) \), is given by the discrete circular convolution formula [103] as follows:

\[
P(N_f = r) = \begin{cases} 
\sum_{j=0}^{r} P(N_{fx} = j) \cdot P(N_{fy} = r - j), & 0 \leq r \leq n/2 \\
\sum_{j=r-n/2}^{n/2} P(N_{fx} = j) \cdot P(N_{fy} = r - j), & n/2 < r \leq n 
\end{cases}
\]  

Equation (3.2)

Based on this model, the implemented Algorithm 2 will divide the \( n \)-SS into \( k \) subbands each composed of \( m \) subchannels. The \( N_f \) distribution of each subband is determined using the conventional algorithm. In detail, the basic input of this algorithm is a given set of subchannel free probabilities \( p_1, p_2, \ldots, p_n \) (Line 1) as well as parameters \( m \) and \( k \) (Line 2) whereas the output is a set of discrete probabilities \( P_1, P_2, \ldots, P_n \) stored in a vector named \( N_f \), where \( P_r = P(N_f = r) \) (Line 23). The input probabilities are divided into equal sets of smaller sizes depending on parameters \( k \) and \( m \) (Line 3). The exact distribution of these sets are computed using Algorithm 1 (Line 6-8) and stored in matrix \( D \) in Line 9. Following equations 3.2, these distributions are then convolved together 2
Figure 3.5: Computation of $N_f$ using the convolution technique in a subband composed of 32 subchannels. Subband decomposed into 4 portion of 8 subchannels where the distribution of each portion is calculated. After that, the distributions are convoloved.

at a time in a tree manner (refer to figure 3.5) using the while loop in Line 11-21. The output of the convolution stage is the exact distribution of the spectrum of interest (Line 23).

Since each subband’s distribution has been computed using the conventional algorithm, a complexity of $T(n) = k \cdot O(m \cdot 2^m) = O(n \cdot 2^m)$ is generated. These distributions are then
Algorithm 2 Multi-Convolution computation of the $N_f(n)$ probability distribution

1: Input $p_1, p_2, \ldots, p_n$
2: Input parameters $m$ and $k$
3: Divide the spectrum into $k$ subbands each of $m$ subchannels
4: Initialize matrix of Zeros $D$ of order $(m+1,k)$
5: for $i = 1$ to $k$ do
6:   for $j = 0$ to $m$ do
7:     compute $N_f(j)$ using Algorithm 1
8:   end for
9:   $D(:, i) \leftarrow N_f(:,)$
10: end for
11: Initialize matrix of zeros Temp
12: of order $\left(2(size(D, 1) - 1) + 1, size(D, 2)/2\right)$
13: $l \leftarrow size(D, 2)$
14: while $l > 1$ do
15:   Initialize $h = 1$
16:   for $t = 1$ to $size(D, 2)/2$ do
17:     $Temp(:, t) \leftarrow$ convolution of $D(:, h)$
18:     with $D(:, h + 1)$
19:     $h \leftarrow h + 2$
20:   end for
21:   $D \leftarrow Temp$
22:   $l \leftarrow size(D, 2)$
23: end while
24: $N_f \leftarrow D$
25: Output $N_f$

convolved to produce the $N_f(n)$ distribution of the whole spectrum. The complexity arising from the convolution is $n^2$ operations. Therefore, the total time complexity of Algorithm 2 is given by $T(n) = \Theta(n^2 m + n^2)$. 
3.4. Computation of $P(N_f = r)$

3.4.2 Recursive Model

This model provides a more efficient approach to compute the exact distribution without having to resort to the intensive conventional algorithm. The objective is to employ a recursive method in such a way we compute the distribution of the n-SS based on the distribution of its (n - 1)-SS. Similarly, the distribution of the (n-1)-SS is computed based on the distribution of (n-2)-SS, etc.

Figure 3.6 illustrates an example that demonstrates how to compute the distributions from $N_f(1)$ up to $N_f(8)$. For instance, the derivations of the distribution of $N_f(6)$ based on the distribution of $N_f(5)$ are as follows:

\[
\begin{align*}
P(N_f(6) = 0) &= p_6. P(N_f(5) = 0) \\
P(N_f(6) = 1) &= p_6. P(N_f(5) = 1) + p_6. P(N_f(5) = 0) \\
P(N_f(6) = 2) &= p_6. P(N_f(5) = 2) + p_6. P(N_f(5) = 1) \\
&\vdots \\
P(N_f(6) = 6) &= p_6. P(N_f(5) = 5)
\end{align*}
\] (3.3)

Let $N_f(n)$ be the random variable that represents the number of available subchannels in the n-SS and similarly, let $N_f(n - 1)$ be the random variable that represents the number of available subchannels in the (n-1)-SS. Then,

\[ N_f(n) = N_f(n - 1) + x_n \] (3.4)

Equation (3.4) reflects a hidden relationship between the exact probability distribution of the (n - 1) and the n-SS. In fact, given a possible value for $N_f(n)$ is represented by $r$, the case of $N_f(n) = r$ suggests that the (n-1)-SS could have already had $r$ available subchannels and that the $n^{th}$ subchannel was occupied ($x_n = 0$) which implies that $N_f(n - 1) = r$. On the other hand, the event that the $n^{th}$ subchannel was available, i.e. ($x_n = 1$), suggests that the (n - 1)-SS had ($r - 1$) available subchannels hence $N_f(n - 1) = r - 1$. Therefore, we develop the following mathematical model for the
Figure 3.6: Channel availability and distributions of $N_f(3)$ through $N_f(8)$

computation of $P(N_f(n) = r)$:

$$
\begin{align*}
P(N_f(n) = 0) &= \tilde{p}_n.P(N_f(n-1) = 0) \\
P(N_f(n) = r) &= \tilde{p}_n.P(N_f(n-1) = r) \\
&\quad + p_n.P(N_f(n-1) = r-1), \quad 1 \leq r < n \\
P(N_f(n) = n) &= p_n.P(N_f(n-1) = n-1)
\end{align*}
$$

(3.5)

where $p_n$ probabilistically reflects the arising of an available channel while adding the $n^{th}$ subchannel to the (n-1)-SS, whereas $(1 - p_n)$ indicates the opposite.

Algorithm 3 is implemented to compute the distribution of $N_f(n)$ using the recursive model. It starts by compute the distribution of $N_f(2)$ which contributes in finding the distribution of $N_f(3)$. Similarly, $N_f(3)$ contributes in finding the distribution of $N_f(4)$. This process is repeated until $N_f(n)$ is reached. In detail, the basic input of this algorithm is a given set of subchannel free probabilities $p_1, p_2, ..., p_n$ (Line 1) whereas the output is a set of discrete probabilities $P_1, P_2, ..., P_n$ stored in a vector named $N_f$, where $P_r = P(N_f = r) \quad r = 0, 1, ..., n$ (Line 18). Throughout the nested loop (Line 6-16), $P(N_f(n) = r)$ is being computed recursively according to equations 3.5 and stored in the matrix $Pr$. The n-th row of this matrix is the array $N_f$ which forms the exact availability distribution of
3.4. Computation of $P(N_f = r)$

**Algorithm 3** Recursive computation of $N_f(n)$ probability distribution

1: Input $p_1, p_2, ..., p_n$
2: $\bar{p}_i \leftarrow (1 - p_i)$
3: $Pr \leftarrow$ matrix of zeros of order $(n, n+1)$
4: $Pr(1, 1) \leftarrow p_{(1)}$
5: $Pr(1, 0) \leftarrow \bar{p}_{(1)}$
6: for $i = 2$ to $n$ do
7: for $r = 0$ to $i$ do
8: if $r = 0$ then
9: $Pr(i, r) = \bar{p}_{(i)} \cdot Pr(i - 1, r)$
else if $0 < r < n$ then
11: $Pr(i, r) = p_{(i)} \cdot Pr(i - 1, r - 1) + \bar{p}_{(i)} \cdot Pr(i - 1, r)$
else if $r = n$ then
13: $Pr(i, r) = p_{(i)} \cdot Pr(i - 1, r - 1)$
end if
14: end for
15: end for
16: $N_f \leftarrow Pr(n,:)$
18: Output $N_f$

the spectrum (Line 17).

The leading order term of the running time of this algorithm arises in line 7 where a nested loop is formed, i.e. outer loop in Line 6 and inner loop in Line 7, of sizes $n - 1$ and $n + 1$ respectively generating $\sum_{r=2}^{n} r = O(n^2)$ steps to be executed. Hence, the time complexity of this algorithm is given by $T(n) = O(n^2)$.

### 3.4.3 Hybrid Model

This technique is a merge between the Multi-Convolved and the recursive models for more efficiency. With the aid of Algorithm 4, this model consists of dividing the n-SS into $k$ subbands each composed of $m$ subchannels. The $N_f(m)$ distribution of each subband is computed using the recursive technique. In details, the basic input of this algorithm is
a given set of subchannel free probabilities \( p_1, p_2, ..., p_n \) (Line 1) as well as parameters \( m \) and \( k \) (Line 2) whereas the output is a set of discrete probabilities \( P_1, P_2, ..., P_n \) stored in a vector named \( N_f \), where \( P_r = P(N_f = r) \) (Line 19). The input probabilities are divided into equal sets of smaller sizes depending on parameters \( k \) and \( m \) (Line 3). The exact distribution of these sets are computed using Algorithm 3 (Line 5-7). Following equations 3.2, these distributions are then convolved together 2 at a time in a tree manner (refer to figure 3.5) using the while loop in Line 9-18. The output of the convolution stage is the exact distribution of the spectrum of interest (Line 19).

Since the time complexity of Algorithm 3 is \( O(n^2) \) for an input of \( n \) values, then using it \( k \) times for an input of \( m \) values will generate a time complexity of \( T(n) = k \cdot O(m^2) = O(k.m^2) \). The complexity arising from the convolution is \( n^2 \) operations. Therefore, the total time complexity of Algorithm 4 is given by \( T(n) = O(k.m^2 + n^2) \).

### 3.5 Probabilistic Modeling of Channel Types Availability

Interference management is a challenging task in cognitive radio. Broadcasters in TV bands are concerned that secondary users will cause excessive harmful interference to TV reception, particularly in areas of weak signal reception. Measurements in TV frequency bands reveal that the interference on a channel is mostly accumulated from co-channel and adjacent channel interference [18]. To reduce the introduced adjacent interference to the primary users, we propose a categorizing criteria to model subchannel availability based on the occupancy of its adjacent subchannels. This approach can predict availability of such subbands and hence allow transmissions with high power where less restrictions on the power has to be applied.

Consider an interior available subchannel \( i \) within a n-SS i.e. \( 1 < i < n \). This subchannel has two neighbors \( i - 1 \) and \( i + 1 \). As illustrated in Figure 3.7, each adjacent subchannel is either occupied or available giving rise to three types of subbands:

**Type I subband:** A subband whose central subchannel \( i \) is available and having an occupied \((i - 1)^{st}\) neighbor and an available subchannel as the \((i + 1)^{st}\) neighbor, or vice versa.

**Type II subband:** A subband whose central subchannel \( i \) is available and having both
3.5. Probabilistic Modeling of Channel Types Availability

Algorithm 4 Hybrid Algorithm of $N_f(n)$ Probability Distribution

1: Input $p_1, p_2, ..., p_n$
2: Input parameters $m$ and $k$
3: Divide the n-SS into $k$ subbands each of $m$ subchannels
4: Initialize matrix of Zeros $N_f$ of order $(m+1,k)$
5: for $j = 1$ to $k$ do
6:   Compute $N_f(:,j)$ using Algorithm 3
7: end for
8: $l \leftarrow \text{size}(N_f, 2)$
9: while $l > 1$ do
10:   Initialize matrix of zeros Temp of order $\left( 2\text{size}(N_f, 1) - 1, \text{size}(N_f, 2)/2 \right)$
11:   Initialize $h = 1$
12:   for $t = 1$ to $\text{size}(N_f, 2)/2$ do
13:     $\text{Temp}(,:t) \leftarrow \text{convolution of } N_f(:,h) \text{ with } N_f(:,h+1)$
14:     $h \leftarrow h + 2$
15:   end for
16:   $N_f \leftarrow \text{Temp}$
17:   $l \leftarrow \text{size}(N_f, 2)$
18: end while
19: Output $N_f$

the $(i-1)^{st}$ and the $(i+1)^{st}$ neighbors occupied.

Type III subband: A subband whose central subchannel $i$ is available and having both the $(i-1)^{st}$ and the $(i+1)^{st}$ neighbors available.

Following the above definitions, we employ the Hybrid model to compute the probability distribution of the number of available type-$i$ subbands, $P\left( N_i(n) = r \right)$ where $i = I, II, III$. Since the availability probability of a type-$i$ subband will depend on the probability of three adjacent subchannels, changes will apply to the analytical model and algorithm. In detail, the n-SS is divided into $K$ subbands each composed of $m$ subchannels. We determine the $N_i(m)$ distribution of each subband using the recursive technique, then, these distributions are convolved to produce the $N_i(n)$ distribution of the whole spectrum. Let $N_i(m)$ be the random variable that denotes the number of available type-
Chapter 3. Probabilistic Modeling of Spectrum Occupancy

Figure 3.7: Types of available subbands: (a) Type I, (b) Type II, (c) Type III

$i$ subbands in the $m$-subband whose subchannel probabilities are $p_1, p_2, ..., p_m$, possible values of $N_i(m)$ are $0, 1, ..., m - 2$. Similarly, let $N_i(m - 1)$ be the random variable which denotes the number of available type-$i$ subbands in the $(m - 1)$-subband whose subchannel probabilities are $p_1, p_2, ..., p_{m-1}$, possible values of $N_i(m - 1)$ are $0, 1, ..., m - 3$. Also, let the variable $X_j$ denotes the status of the $j^{\text{th}}$ type-$i$ subband i.e. available ($X_j = 1$) with probability $p_i(m)$ or being occupied ($X_j = 0$) where:

$$
\begin{align*}
 p_I(m) &= \tilde{p}_{(m-2)}p_{(m-1)}p_m + p_{(m-2)}p_{(m-1)}\tilde{p}_m \\
 p_{II}(m) &= \tilde{p}_{(m-2)}p_{(m-1)}\tilde{p}_m \\
 p_{III}(m) &= p_{(m-2)}p_{(m-1)}p_m
\end{align*}
$$

(3.6)

Given a possible value where $N_i(m)$ is represented by $r$, the case of $N_i(m) = r$ where $0 < r < m - 2$ suggests two possible events. Either the $(m - 1)$-SS had $r$ available type-$i$ subbands ($N_i(m - 1) = r$). Otherwise, the event that type-$i$ subband occurred while adding the $m^{\text{th}}$ subchannel suggests that the (m-1)-SS had $r - 1$ available type-$i$ subbands implying ($N_i(m - 1) = r - 1$). Moreover, the event $N_i(m) = 0$ can occur if and only if $N_i(m - 1) = 0$. Finally, the event $N_i(m) = m - 2$ can only arise from $N_i(m - 1) = m - 3$. Therefore, we develop the following mathematical equations for the
3.5. Probabilistic Modeling of Channel Types Availability

computation of \( P(N_i(m) = r) \):

\[
\begin{align*}
P(N_i(m) = 0) &= \bar{p}_i(m).P(N_i(m - 1) = 0) \\
P(N_i(m) = r) &= \bar{p}_i(m).P(N_i(m - 1) = r) \\
&+ p_i(m).P(N_i(m - 1) = r - 1), \quad 0 < r < m - 2 \\
P(N_i(m) = m - 2) &= p_i(m).P(N_i(m - 1) = m - 3)
\end{align*}
\]

(3.7)

**Proof by Induction: Binomial Case Study for Type I:**

In the m-SS of interest, we focus on the status of occupancy in the subchannels (m-2), (m-1), and m. Each of these subchannels are either free or occupied, giving rise to \( 2^3 = 8 \) possibilities depicted in Figure 3.7.

Consider the case when all subchannels have equal availability probabilities, i.e. \( p_i = p \), then \( N_I(m) \) follows a Binomial \( \sim \left( (m - 2), 2p^2(1 - p) \right) \). We start by computing the distribution of \( N_I(3) \) which is given by:

\[
\begin{align*}
P(N_I(3) = 0) &= 1 - 2p^2(1 - p) \\
P(N_I(3) = 1) &= 2p^2(1 - p)
\end{align*}
\]

(3.8)

Therefore, \( N_I(3) \) follows Binomial \( \sim \left( 1, 2p^2(1 - p) \right) \). \( N_I(4) \) can be determined in terms of \( N_I(3) \) using the recursive relationship \( N_I(4) = N_I(3) + X_4 \) as follows:

\[
\begin{align*}
P(N_I(4) = 0) &= \left[ 1 - 2p^2(1 - p) \right] \\
P(N_I(4) = 1) &= 2\left[ 1 - 2p^2(1 - p) \right] 2p^2(1 - p) \\
P(N_I(4) = 2) &= \left[ 2p^2(1 - p) \right]^2
\end{align*}
\]

(3.9)

It is evident that \( N_I(4) \) follows the Binomial \( \left( 2, 2p^2(1 - p) \right) \).

Suppose that \( 5 \leq n \leq m - 1 \) and the inductive hypothesis that \( N_I(n) \sim \text{Binomial} \left( (n - 2), 2p^2(1 - p) \right) \) is true. We prove that \( N_I(n + 1) \sim \text{Binomial} \left( (n - 1), 2p^2(1 - p) \right) \).

The Binomial distribution of \( N_I(n) \) is given by:

\[
\begin{align*}
P(N_I(n) = 0) &= \left[ 1 - 2p^2(1 - p) \right]^{(n-2)} \\
P(N_I(n) = 1) &= \binom{n-2}{1} 2p^2(1 - p) \left[ 1 - 2p^2(1 - p) \right]^{(n-3)} \\
&\vdots \\
P(N_I(n) = n - 2) &= \left[ 2p^2(1 - p) \right]^{(n-2)}
\end{align*}
\]

(3.10)
Knowing that \( N_1(n+1) = N_1(n)+X_m \) where possible values of \( X_1(n+1) = 0, 1, 2, \ldots, (n-2), (n-1) \). The probabilities of each of these values are computed separately as follows:

\[
P(N_1(n+1) = 0) = P(N_1(n) = 0) \left[ 1 - 2p^2(1-p) \right] = \left[ 1 - 2p^2(1-p) \right] \left[ 1 - 2p^2(1-p) \right]^{n-2} \quad (3.11)
\]

The case when \( 1 \leq r \leq n - 2 \), we have the following:

\[
P(N_1(n+1) = r) = P(N_1(n) = r) \left[ 1 - 2p^2(1-p) \right] + P(N_1(n) = r-1) \left[ 2p^2(1-p) \right]
\]
\[
= \binom{n-2}{r} \left[ 2p^2(1-p) \right]^r \left[ 1 - 2p^2(1-p) \right]^{n-2-r} \left[ 2p^2(1-p) \right]
\]
\[
+ \binom{n-2}{r-1} \left[ 2p^2(1-p) \right]^{r-1} \left[ 1 - 2p^2(1-p) \right]^{n-2-r+1} \left[ 2p^2(1-p) \right]
\]
\[
= \binom{n-2}{r} + \binom{n-2}{r-1} \left[ 2p^2(1-p) \right]^r \left[ 1 - 2p^2(1-p) \right]^{n-2-r+1} \left[ 2p^2(1-p) \right]
\]
\[
= \binom{n-1}{r} \left[ 2p^2(1-p) \right]^r \left[ 1 - 2p^2(1-p) \right]^{n-1-r} \quad (3.12)
\]

Now the probability of having \( (n-1) \) Type I subbands is expressed as:

\[
P(N_1(n+1) = n - 1) = P(N_1(n) = n - 2) \left[ 2p^2(1-p) \right]
\]
\[
= \left[ 2p^2(1-p) \right]^{n-1} \quad (3.13)
\]

From these probabilities, it is evident that \( N_1(n+1) \sim \text{Binomial} \left( (n-1), 2p^2(1-p) \right) \).

The ultimate goal of this model is to find the probability distribution of \( N_i(m) \) of each \( m \)-subband. To do so, Algorithm 5 computes the distribution of \( N_i(3) \) which contributes to finding the distribution of \( N_i(4) \), similarly \( N_i(4) \) contributes to finding the distribution of \( N_i(5) \), we continue this process until we reach the \( N_i(m) \) distribution of each subband, then, all these distributions are convolved to produce \( N_i(n) \) of the whole spectrum. In details, the basic input of Algorithm 5 is a given set of subchannel free probabilities \( p_1, p_2, \ldots, p_n \) (Line 1) as well as parameters \( m \) and \( k \) (Line 2) whereas the output is a set of discrete probabilities \( P_1, P_2, \ldots, P(n-2) \) stored in a vector named \( N_i \), where \( P_r = P(N_i = r) \) (Line 19). The input probabilities are divided into equal sets of smaller
3.5. Probabilistic Modeling of Channel Types Availability

Algorithm 5 Hybrid computation of $N_i(n)$

1: Input $p_1, p_2, \ldots, p_n$
2: Input parameters $m$ and $k$
3: Divide the n-SS into $k$ subbands of $m$ subchannels
4: Initialize matrix of Zeros $N_i$ of order $(m-1, k)$
5: for $j = 1$ to $k$ do
6: Compute $N_i(:, j)$ using Algorithm 3
7: end for
8: $l \leftarrow$ size($N_i, 2$)
9: while $l > 1$ do
10: Initialize matrix of zeros Temp of order $(2 \cdot \text{size}(N_i, 1) - 1, \text{size}(N_i, 2)/2)$
11: Initialize $h = 1$
12: for $t = 1$ to $\text{size}(N_i, 2)/2$ do
13: $\text{Temp}( :, t) \leftarrow$ convolution of $N_i(:, h)$ with $N_i(:, h + 1)$
14: $h \leftarrow h + 2$
15: end for
16: $N_i \leftarrow$ Temp
17: $l \leftarrow$ size($N_i, 2$)
18: end while
19: Output $N_i$

sizes depending on parameters $k$ and $m$ (Line 3). The exact distribution of these sets are computed according to equations 3.12 using Algorithm 3 (Line 5-7). Following equations 3.2, these distributions are then convolved together 2 at a time in a tree manner (refer to figure 3.5) using the while loop in Line 9-18. The output of the convolution stage is the exact distribution of $N_i$ in the spectrum of interest (Line 19).
3.6 Summary

With the advent of high bandwidth multimedia applications and the growing demand for ubiquitous information network access for mobile wireless devices, enhancing the efficiency of wireless spectrum utilization is essential for addressing the scarcity of available transmission bandwidth. Given all analysis in this chapter, following an independent and non-identical distribution (i.n.i.d) paradigm, we have illustrated accurate probabilistic modeling of free and contiguous subchannels in a cognitive radio network. The computation of the exact distribution of the total number of free subchannels (i.e., $N_f$) is prohibitively energy-intense and time-consuming. Hence, techniques and algorithms were implemented for efficient computations. We label these approaches as multi-convolution model, recursive model, and hybrid multi convolution-recursive model. In addition, we present the analysis of contiguous sub-bands in characterizing three different types of free subbands.
Network Performance Analysis and Potential Applications

4.1 Network Performance Analysis

Conventionally, the metrics of interests for performance evaluation of spectrum sensing are mainly the \( P_{fa} \), \( P_{d} \), and computational complexity. The \( P_{fa} \) is often formulated for the AWGN case since it would not be affected by the channel fading. The higher the probability of detection, the better the primary users are protected. However, the lower the probability of false alarm, the more chances the channel can be reused when it is available and thus the higher the achievable throughput for the secondary network. In this section, we evaluate these cognitive radio based sensing parameters and demonstrate an approach to calculate the probability of detecting multi-subchannels simultaneously.

4.1.1 Detection Probabilities

We consider a multi-subchannel joint detection framework in which a technique takes into account the detection of primary users across multiple subchannels. The detection problem on the \( j^{th} \) subchannel is illustrated as choosing between two hypotheses such as: \( \mathcal{H}_{0,n} \) ("0"), denotes the absence of primary signals (spectrum hole) while \( \mathcal{H}_{1,n} \) ("1"),
reflects the presence of primary signals (occupied subchannel). In order to decide whether the \( j \)^th subchannel is available or not, the following hypotheses are tested:

\[
\mathcal{H}_{0,j}: R_j = W_j \\
\mathcal{H}_{1,j}: R_j = H_j S_j + W_j,
\]

where \( R_j \) is the secondary received signal, \( S_j \) is the primary transmitted signal, \( W_j \) is the received noise (AWGN normally distributed in the time domain with an average time domain value of zero), and \( H_j \) is the channel gain between primary transmitter and secondary receiver. Given \( \gamma_j \) is the decision threshold of subchannel \( j \), the subchannel energies are computed over \( M \) samples of interest and the decision is made as follows:

\[
\begin{pmatrix}
Y_j \\
\end{pmatrix} \triangleq \sum_{s=1}^{M} |R_j(s)|^2 \xrightarrow{\mathcal{H}_{1,j}} \frac{\mathcal{H}_{1,j}}{\mathcal{H}_{0,j}} \gamma_j, \quad j = 1, 2, ..., n \tag{4.2}
\]

For simplicity and without loss of generality, we assume the following [104]:

- \( S_j, H_j, \) and \( W_j \) are independent of each other.
- The random variables \( W_j \) are independent and normally distributed such as \( W_j \sim \mathcal{CN}(0, \sigma_w^2) \) where \( \sigma_w^2 \) is the noise power.
- Reliable channel estimation relies on some kind of exchange of information between transmitter and receiver (feedback channel, pilot symbols, etc.). This might be particularly difficult to establish in a cognitive radio environment since primary users are not supposed to modify their transmission due to the existence of secondary users. Therefore, a more realistic assessment is to take into account imperfect channel knowledge, where the statistics of channel coefficients are estimated by the secondary users without the presence of information exchange with the primary transmitters.

For a large number of samples \( M \), central limit theorem [105] dictates that the statistics are approximately normally distributed [106] under each hypothesis and therefore \( Y_j \sim \mathcal{N}(\mathbb{E}[Y_j], \text{Var}(Y_j)) \) where:

\[
\mathbb{E}[Y_j] = \begin{cases} 
M \sigma_w^2, & \mathcal{H}_{0,j} \\
M (\sigma_w^2 + |H_j|^2), & \mathcal{H}_{1,j}
\end{cases}
\]

\tag{4.3}
and

\[ \text{Var}(Y_j) = \begin{cases} 
2M\sigma_w^4, & H_{0,j} \\
2M(\sigma_w^2 + 2|H_j|^2)\sigma_w^2, & H_{1,j}
\end{cases} \quad (4.4) \]

Therefore, the probabilities of detection and false alarm in the \(j\)th subchannel associated with (4.2) can be expressed as:

\[ P^{(j)}_{fa}(\gamma_j) = P(Y_j > \gamma_j | H_{0,j}) = Q\left( \frac{\gamma_j - M\sigma_w^2}{\sigma_w^2\sqrt{2M}} \right) \quad (4.5) \]

Similarly

\[ P^{(j)}_{d}(\gamma_j) = P(Y_j > \gamma_j | H_{1,j}) = Q\left[ \frac{\gamma_j - M(\sigma_w^2 + |H_j|^2)}{\sigma_w^2\sqrt{2M(\sigma_w^2 + 2|H_j|^2)}} \right] \quad (4.6) \]

Where \(Q(.)\) is the complementary distribution function of the standard Gaussian, i.e.,
\[ Q(x) = \int_x^\infty \exp \left( -\frac{t^2}{2} \right) dt. \]

For the multi-subchannel joint detection, we define the following vectors using (4.6) and (4.5) as follows:

\[ \gamma = [\gamma_1, \gamma_2, ..., \gamma_n]^T \]
\[ P_{fa}(\gamma) = [P^{(1)}_{fa}(\gamma_1), P^{(2)}_{fa}(\gamma_2), ..., P^{(n)}_{fa}(\gamma_n)]^T \]
\[ P_{d}(\gamma) = [P^{(1)}_{d}(\gamma_1), P^{(2)}_{d}(\gamma_2), ..., P^{(n)}_{d}(\gamma_n)]^T \]

(4.7)

One of the metrics to assess the performance of the proposed modeling is to compute the probability of detecting \(r\) available subchannels simultaneously i.e. \(P(N_d = r)\) where \(N_d\), a discrete random variable, that denotes the number of detected available channels. In order to do this, we first compute \(P(O_d = z)\) i.e. the probability of detecting \(z\) primary signals (or \(z\) occupied subchannels) where \(O_d\) is also a discrete random variable that represents the number of detected occupied channels. Since we are following an independent and non-identical distribution (i.n.i.d) paradigm which introduces combinatorial complexity, we use the recursive computations to compute \(P(N_d = z)\) as follows:

\[ \begin{aligned}
P(O_d(n) = 0) &= P^{(n)}_{d}(\gamma_n)P(O_d(n - 1) = 0) \\
P(O_d(n) = z) &= P^{(n)}_{d}(\gamma_n)P(O_d(n - 1) = z) \\
&\quad + P^{(n)}_{d}(\gamma_n)P(O_d(n - 1) = z - 1), \quad 1 \leq z < n \\
P(O_d(n) = n) &= P^{(n)}_{d}(\gamma_n)P(O_d(n - 1) = n - 1)
\end{aligned} \]

(4.8)
Chapter 4. Network Performance Analysis and Potential Applications

Detecting $z$ occupied subchannels implies that $r = n - z$ available subchannels are detected. Hence, $P(N_d = z) = 1 - P(O_d = r)$.

4.1.2 Aggregate Throughput and Interference

The aggregate opportunistic throughput of the CR system as well as the aggregate interference to the primary users can be defined respectively as [107]:

$$R(\gamma) \triangleq r^T[1 - P_{fa}(\gamma)]$$ (4.9)

$$C(\gamma) \triangleq c^T[1 - P_{d}(\gamma)]$$ (4.10)

In (4.9), vector $r = [r_1, r_2, ..., r_n]^T$, where $r_j \leq 0$ denote the throughput achievable over the $j^{th}$ subchannel which can be estimated using the Shannon capacity formula [108] given $H_j$ and $S_j$ are known. In (4.10), vector $c = [c_1, c_2, ..., c_n]^T$, where $c_j \leq 0$ represents the associated interference cost with the primary user in the $j^{th}$ subchannel.

4.2 Simulation Results

In this section, we provide a comparative study of the proposed models for spectrum availability using existing real-time energy-sensed data obtained from extensive experimental measurements as follows:

- **Set 1**: 20 – 1500MHz band with center frequency 770MHz inside an office building at Aachen (Latitude: 50°47′24.01″ North and Longitude: 6°3′47.42″ East).

- **Set 2**: 1500 – 3000MHz band with center frequency 2250MHz on a rooftop location in a residential area at Maastricht (Latitude: 50°50′23.34″ North and Longitude: 5°43′14.93″ East).

- **Set 3**: 1240 – 1850MHz band on a rooftop of the 22 story IITRI Tower locater at the 35th South State Street roughly three miles south of the Chicago Loop, the business center of the city.
4.2. Simulation Results

- **Set 4**: Microsoft spectrum data measured on the 6th of August 2013 over the band 108Mhz to 174Mhz on a roof top of a 6 story building at Seattle, USA (GPS position: 47.621571,-122.33808)\(^1\).

In detail, Set 1 and Set 2 of these measurements were carried over 8192 subchannels each with a resolution bandwidth of 200 kHz and an average sweep time of 1.8 sec. For more details about the set-up refer to [4]. More details about the experiment set up of Set 3 are found in [88].

Additionally, we propose 2 potential applications associated with the use of the modeling. Application 2 features novel 2-Dimensional (time, frequency) modeling.

### 4.2.1 Experiment 1: Comparison with the State-of-the-Art

![Comparison of distribution results with existing models for Aachen, Germany measurements during 7:00-8:00 am](http://spectrum-observatory.cloudapp.net/Station/StationIndex?activeTab=stations)

Figure 4.1: Comparison of distribution results with existing models for Aachen, Germany measurements during 7:00-8:00 am

\(^{1}\text{http://spectrum-observatory.cloudapp.net/Station/StationIndex?activeTab=stations}\)
In this scenario, we compare our proposed models against the existing ones in the State-of-the-Art that follow the same i.n.i.d. classification. These models include the Poisson-normal approximation [11] as well as the Camp-Paulson approximation in [95]. We consider 50 subchannels \((n = 50)\) from Set 1 which availability probabilities \(p_i\)'s, \(i \in 1, 2, 3, ..., 50\), are computed and normalized over a period of 30 minutes. It is evident from figure 4.1 that the proposed recursive model follows the conventional exact distribution with a running time of 0.0004 second. Using the Chi-Square error criteria, the normal, Camp-Paulson, and Poisson-normal approximation models achieve an accuracy of 0.2, 0.17, and 0.15 maximum error along with an algorithm running time of 0.006, 0.035 and 4.8 second(s) respectively. Authors in [11] claimed the effectiveness of the Poisson-normal model for spectrum availability when compared to the inaccuracy of the normal approximation approach. However, due to the combinatorial complexity, this model is not only inefficient, but also, can only support small number of subchannels and therefore not an effective model for wide bands where \(n\) is a large number. On the other hand, although the Camp-Paulson model is more accurate model, yet it is less efficient than the Normal approximation since it requires more parameters to calculate. Table 4.1 illustrates a comparison of the algorithms running time given the input \(n\) is varied. Unlike these approximation methods, we have provided accurate modeling. Furthermore, supported by the Central Limit Theorem [105], our results prove that as the number of subchannels \(n\) increases, the accuracy error of the normal approximation tend to decrease (refer to experiment 2).
Table 4.1: Comparison of Algorithms running time (second) for an input \((n)\)

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>(n = 32)</th>
<th>(n = 64)</th>
<th>(n = 256)</th>
<th>(n = 512)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Conventional</td>
<td>482</td>
<td>2866</td>
<td>*</td>
<td>*</td>
</tr>
<tr>
<td>M-conv ((m = 4))</td>
<td>0.015</td>
<td>0.046</td>
<td>0.14</td>
<td>0.52</td>
</tr>
<tr>
<td>Recursive</td>
<td>0.0001</td>
<td>0.0005</td>
<td>0.002</td>
<td>0.015</td>
</tr>
<tr>
<td>Hybrid ((m = 8))</td>
<td>0.0003</td>
<td>0.0006</td>
<td>0.001</td>
<td>0.006</td>
</tr>
<tr>
<td>Poisson-normal</td>
<td>3.18</td>
<td>9.7</td>
<td>3028</td>
<td>*</td>
</tr>
<tr>
<td>Camp-Paulson</td>
<td>0.012</td>
<td>0.098</td>
<td>0.81</td>
<td>1.93</td>
</tr>
</tbody>
</table>

* Not applicable due to the memory constraint

Similarly, we compare the proposed models with the work provided in [95,96]. Authors had also proposed an approximation model based on Camp-Paulson approach. We consider 20 subchannels \((n = 20)\) from Set 3 which availability probabilities \(p_i\)'s, \(i \in 1, 2, 3, \ldots, 20\), are computed and normalized over a period of 30 minutes. It is evident from Figure 4.2 that the Camp-Paulson model achieves an accuracy of 0.15 compared to the exact distribution. However, the accuracy tend to decrease as the number of subchannels increases whereas the complexity order is much higher compared to the proposed poisson-normal approximation in [11].
Table 4.2 illustrates a comparison of the proposed models against the State-of-the-Art models in terms of running time matrix as well as computation complexity. In practical systems, some solutions may not be exact but may be able to provide a reasonably accurate estimation in a much more time-efficient way. In this context, it is evident that the proposed recursive and hybrid models are much quicker to run when compared against the poisson-normal as well as the Camp-paulson model. However, the multi-convolved’s running time is similar to the aforementioned models. Note that all results presented in this section are produced using a 32-bit system type machine that features Dual-core 2.10 GHz processor and 4GB of RAM. Moreover, the models proposed in the State-of-the-Art have been reproduced by the author.
## 4.2 Simulation Results

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Algorithm</th>
<th>Comp Time (sec)</th>
<th>Comp Complexity (add&amp;multi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>64-spectrum</td>
<td>conventional</td>
<td>*</td>
<td>$1.1 \times 10^{21}$</td>
</tr>
<tr>
<td></td>
<td>multi-conv</td>
<td>0.34</td>
<td>$2.46 \times 10^{3}$</td>
</tr>
<tr>
<td></td>
<td>recursive</td>
<td>$3 \times 10^{-4}$</td>
<td>$6.2 \times 10^{3}$</td>
</tr>
<tr>
<td></td>
<td>hybrid</td>
<td>$1 \times 10^{-5}$</td>
<td>$4.8 \times 10^{3}$</td>
</tr>
<tr>
<td></td>
<td>poisson-normal</td>
<td>0.22</td>
<td>$16.3 \times 10^{3}$</td>
</tr>
<tr>
<td></td>
<td>camp-paulson</td>
<td>0.19</td>
<td>$12.9 \times 10^{3}$</td>
</tr>
<tr>
<td>256-spectrum</td>
<td>conventional</td>
<td>*</td>
<td>$2.9 \times 10^{79}$</td>
</tr>
<tr>
<td></td>
<td>multi-conv</td>
<td>1.56</td>
<td>$98.6 \times 10^{3}$</td>
</tr>
<tr>
<td></td>
<td>recursive</td>
<td>0.03</td>
<td>$72.7 \times 10^{3}$</td>
</tr>
<tr>
<td></td>
<td>hybrid</td>
<td>$2 \times 10^{-3}$</td>
<td>$69.5 \times 10^{3}$</td>
</tr>
<tr>
<td></td>
<td>poisson-normal</td>
<td>1.73</td>
<td>$108.2 \times 10^{3}$</td>
</tr>
<tr>
<td></td>
<td>camp-paulson</td>
<td>2.31</td>
<td>$123.5 \times 10^{3}$</td>
</tr>
<tr>
<td>512-spectrum</td>
<td>conventional</td>
<td>*</td>
<td>$6.8 \times 10^{156}$</td>
</tr>
<tr>
<td></td>
<td>multi-conv</td>
<td>3.24</td>
<td>$402.3 \times 10^{3}$</td>
</tr>
<tr>
<td></td>
<td>recursive</td>
<td>0.06</td>
<td>$393.8 \times 10^{3}$</td>
</tr>
<tr>
<td></td>
<td>hybrid</td>
<td>0.01</td>
<td>$271.1 \times 10^{3}$</td>
</tr>
<tr>
<td></td>
<td>poisson-normal</td>
<td>3.55</td>
<td>$456.9 \times 10^{3}$</td>
</tr>
<tr>
<td></td>
<td>camp-paulson</td>
<td>5.02</td>
<td>$570 \times 10^{3}$</td>
</tr>
</tbody>
</table>

* Not applicable due to the memory constraint
4.2.2 Experiment 2: Comparison of the Proposed Models

Beta 1: 64-Channel Spectrum Modeling

Consider a spectrum of 64 channels \( n = 64 \) where channel availability probabilities are generated using \( \text{Beta}(0.5, 0.5) \) distribution (defined in 2.2.2). Figure 4.3 shows the beta output and further include the distribution of \( N_f(64) \) using the proposed techniques, its normal approximation as well as the distributions of \( N_I(64), N_{II}(64), \) and \( N_{III}(64) \). The multi-convolution and hybrid models were configured to run with the parameters \( k = 8 \) and \( m = 8 \) to keep the time complexity low and target ultimate efficiency. The overlapping of the \( N_f \) distributions using the proposed models confirm accuracy and further suggest a mean of 27 available channels compared to an approximate mean of 13, 8 and 5 for the distribution of \( N_I, N_{II}, \) and \( N_{III} \) available type channels respectively which validates the accuracy of channel type distribution as \( N_f \geq N_I + N_{II} + N_{III} \) considering...
the availability of the two channels in the spectrum edges. The normal approximation achieves an accuracy of 0.16 maximum error when compared to the exact distribution.

**Beta 2: 256-Channel Spectrum Modeling**

![Figure 4.4: 256-spectrum channel availability modeling](image)

Consider a spectrum of 256 channels \((n = 256)\) where channel availability probabilities are generated using \(\text{Beta}(0.5, 0.7)\) distribution (defined in 2.2.2). Figure 4.4 illustrates the distribution of \(N_f(256)\) using the proposed techniques, its normal approximation as well as the distributions of \(N_I(256)\), \(N_{II}(256)\), and, \(N_{III}(256)\). The multi-convolution and hybrid models were configured to run with the parameters \(k = 32\) and \(m = 8\) to keep the time complexity low and target ultimate efficiency. The normal approximation achieves an accuracy of 0.11 maximum error when compared to the exact distribution.
Real data 1: 512-Channel Spectrum Modeling

Next, we compare our proposed models in terms of accuracy and efficiency using real measured data. We consider a set of 512 subchannels \( n = 512 \) from Set 1. The \( p_i \)'s, \( i \in \{1, 2, ..., n\} \), are computed and normalized over a period of 30 minutes (i.e., 1000 time sweeps). Figure 4.5 illustrates the dynamical spectral occupancy of the spectrum as well as the distributions of \( N_f, N_I, N_{II}, N_{III} \), and the normal approximation. In order to keep the time complexity low and target ultimate efficiency, the multi-convolution and hybrid models were configured to run with parameters \( m = 8 \) and \( k = 64 \). It is evident that the proposed models achieve 100% accuracy when compared to the conventional exact distribution. On the other hand, the normal approximation achieves an accuracy of \( 8 \times 10^{-2} \) maximum error.

Real data 2: 1024-Channel Spectrum Modeling

Similarly, we consider a set of 1024 subchannels \( n = 1024 \) from Set 2. The \( p_i \)'s, \( i \in \{1, 2, ..., n\} \), are computed and normalized over a period of 30 minutes (i.e., 1000 time sweeps). Figure 4.6 illustrates the dynamical spectral occupancy of the spectrum as well as the distributions of \( N_f, N_I, N_{II}, N_{III} \), and the normal approximation. In order to keep the time complexity low and target ultimate efficiency, the multi-convolution and hybrid models were configured to run with parameters \( m = 8 \) and \( k = 128 \). It is evident that the proposed models achieve 100% accuracy when compared to the conventional exact distribution. On the other hand, the normal approximation achieves an accuracy of \( 1 \times 10^{-2} \) maximum error.

4.2.3 Performance Analysis

In order to assess the efficiency of the proposed algorithms, we refer to algorithm’s time complexity and running time as evaluation metrics. Figure 4.8 illustrates the time complexity whereas figure 4.7 depicts the running time of the proposed algorithms (parameters \( k \) and \( m \) are varied). It is evident how the computational complexity of the proposed models is significantly lower than the conventional one. It is also clear that the hybrid model is
the most efficient especially when running with parameters \((m = 8, k = n/m)\). It has been observed for both Multi-convolved and the Recursive algorithms that as we try to lower the \(m\) parameter, i.e., \(m = 2\), the convolution complexity dominates as the number of PMFs to be convolved increases. Vice versa, when \(m\) is a large number, i.e., \(m = 16\) with respect of the Multi-convolved algorithm, the combinatorial complexity tends to dominate and, consequently, the efficiency decreases tremendously. Yet, this case has no effects on the Hybrid algorithm as both recursive and convolution complexity functions share similar growth rate. Targeting ultimate efficiency, the ideal selection of the \(m\) parameter for both Multi-convolved and Hybrid model is either 4 or 8. Furthermore, as demonstrated in figure 4.7, this optimal selection is independent of the number of subchannels \(n\).

In order to evaluate the effect of the threshold uncertainty factor on the modeling approach proposed in this experiment, we have obtained the receiver operating characteristic (ROC) curves through computer simulations (refer to Figure 4.9). In details, the communication channel is generated according to the WINNER channel model under B2 outdoor scenario. The threshold settings are based on the assumed/estimated noise power while the real noise power varies with each Monte Carlo realization (10,000 realizations in total) by a certain degree depending on the uncertainty factor [109]. Furthermore, we have presented the effect of the sample size (M) constraint given energy detection was used in this experiment.

It can be observed that the performance of the availability modeling is considerably dependent on the noise uncertainty factor. This is due to the fact that the performance of the energy detection used prior to the modeling is a function of the uncertainty factor. Regardless of any employed technique to monitor the primary users activity, any error made through inaccurate decision-making at the sensing device will have a direct effect on the modeling algorithm. Further limitations of this data have been analyzed in [4] [110] [75]. Moreover, it has been proven in [111] that increasing the observation length \(M\) does not affect the performance of the energy detection scheme when the exact noise power is not known.

Additionally, we investigate the error propagated to the modeling stage through inaccurate threshold level decision at the sensing device. Figure 4.10 depicts 4 different evaluations of this criteria on 32 channels from Set 3. Each channel has a bandwidth
of 6 MHz. The actual distribution depicts sensed measurements at a threshold level, i.e. $\gamma_a = -60 dB$, while ensuring meeting a requirement of 10% $P_{fa}$. This threshold setting has then been shifted up and down by 10 dB and led to new sets of energy measurements across the 32 channels. The channels probability availabilities are normalized and computed based on the sensed measurements. Similarly, following the multi-channel detection scheme and the i.n.i.d. paradigm, 2 scenarios are considered where a random error has been introduced to each channel’s threshold level. Simulations show a considerable error when threshold sensing level is simply translated up and down. The number of available channels has changed dramatically. However, a marginal level of accuracy is secured when the error is scattered gently up and down across the channels.

4.3 Applications

4.3.1 Application 1: Assessment of Service Success

As a simplified application, we consider the framework where an upper layer CR application is inquiring the physical layer about current spectrum availability and making a request to access a defined number of available channels. In a pragmatic scenario, this request is also associated with an expectation of a certain level of QoS to be ensured by the physical layer. However, even if we assume a best case scenario of excellent channel state (no fading) and no interference on all available channels, we can have a first coarse estimation about the possibility of providing the service to the CR application. Hence, prior to establishing other credentials that will ensure desired QoS, as a first step this application will rely on a binary decision from our proposed modeling stage to determine whether the request can be met or not. In detail, the modeling stage refers to the database of historical availability of the spectrum and investigates the success in finding the requested number of channels (initially assumed to be ideal channels), then determines if the probability of meeting the request is above a threshold $\rho$. If $P(N_f \geq r) > \rho$, modeling will return 1 (Yes) to the application and only then: more energy-intensive processing for spectrum sensing, evaluation of channel state information, estimation of the interference
levels to be experienced on each subchannel, etc., are performed to determine if the QoS on these channels will be acceptable or not. Otherwise, it will return 0 (No).

As an experiment, we consider a set of 64 TV channels ($n = 64$) from Set 4. We examine a request of 2 LTE carriers (appr. 6 TV channels) at 6:05pm made by the upper layer CR application and investigate the success of meeting this request. We collect and compute the set of $p_i$’s, $i \in \{1, 2, ..., n\}$, based on a threshold of -40dBm over 4 successive days at the same given time and average them. Figure 4.11 displays the complimentary cumulative distribution (CCDF) of $N_f$ (the probability of finding $r$ or more available channels) in the modeling stage as well as the probability of detecting these channels in the sensing stage at 3 different times. Assuming $\rho = 0.6$, the modeling stage suggests that the request cannot be met at 6:05 pm since $P(N_f \geq 6) = 0.3 < \rho$. This prediction is evaluated against a detection probability of $P(N_d \geq 6) = 0.1$. Alternatively, based on 80% chance that 6 or more channels are available, the modeling stage advise the application to wait and recommend 6:10 pm as a substitute time to proceed for spectrum sensing. This prediction is evaluated against 70% chance of detecting 6 or more available channels at the sensing stage. It is evident in this case that a significant amount of energy is saved by avoiding unnecessary sensing.

4.3.2 Application 2: Database-Assisted Distributed Spectrum Sharing

We consider an infrastructure-based white-space network where multiple access points (APs) are operating on white spaces. Such infrastructure-based architecture has been adopted in IEEE 802.22 standard [112] and Microsoft Redmond campus white-space networking experiment [13]. Each AP is managing a set of TV channels at a specific geolocation and updates the database of its location and transmission power via wire-line connections. The database in return sends back the occupancy status of the requested TV channels set. An AP must choose one feasible channel to serve the secondary users within its transmission range. Similarly, each SU is expected to associate with one AP. Challenges in such infrastructure include:

1) When APs are non-cooperative (belong to different network operators), an SU should decide on the best AP/network profile to associate with.
2) In a cooperative model, when an AP is overloaded, SU may improve its throughput by moving and associating with a less congested AP owned by the same network. Therefore, modeling the channels availability at each AP will optimize the selection profile for the SU and further maximize the system-wide throughput. Moreover, by applying time-series modeling, SU will be able to make a decision based on the availability at the time of request as well as on future availability predictions.

Similarly, we consider a simplified scenario where a white-space wireless system consists of 4 APs scattered across a 2500 $m^2$ area. We assume that each AP has high processing capability and large energy resources and that all APs are transmitting at the same power where AP1/AP4 and AP2/AP3 belong to two different network operators. Each AP is managing a set of $n = 16$ TV channels where the occupancy is determined by consulting the geolocation database. The database can establish whether a channel is idle or not by performing spectrum sensing measurements or otherwise by computing the availability using RF propagation models [13]. To assess this scenario, we use real time occupancy data\(^2\) conducted by Microsoft on the 6\(^{th}\) of August 2013 over the band 108Mhz to 174Mhz on a roof top of a 6 story building at Seattle, USA (GPS position: 47.621571,-122.33808). Figure 4.12 illustrates the modeling of the channels availability when requested by a secondary user at time $t$. It also show the future prediction of the availability at $t + 1$ by applying Fuzzy time series modeling [113] based on the historical 10 time slots. The $t + 1$ prediction is compared against the actual occupancy featuring 10% maximum error. Hence, by applying 2D modeling, the SU can decide wisely on which AP to associate with. The predictions could also be extended to feature the availability at time $t + 2, t + 3$, etc., (refer to next scenario). It is evident from the figure that the best AP profile to be selected by the SU will be AP2 and AP3 with a peak of 58% and 39% chance of 12 and 10 available channels respectively. However, if the SU is already connected with AP4, he can increase his data rates by moving and associating with another cooperative AP within the same network provider which in this case is AP1 (less congested).

Additionally, we consider another scenario where 2 APs are scattered across an area of a length of 400 m. Each AP is managing a set of 64 TV channels, and operates with a specific transmission power based on its coverage and primary user protection requirements. Each

\(^2\)http://spectrum-observatory.cloudapp.net/Station/StationIndex?activeTab=stations
4.3. Applications

AP can acquire information about vacant channels (channels occupancy) at its location from the geolocation database. The bandwidth of each channel is 6MHz, the noise power is $W_n = -60$ dBm. The average data rate of a secondary user decreases when the number of contending users increase. To improve the data rate, the secondary user can choose to move to another AP with less users. When working cooperatively, for example, the case of APs deployed in a university campus. APs can coordinate together to maximize the entire campus network throughput where they adopt a random back-off mechanism to resolve channel contention. For instance, in this scenario, we predict the availability of type-III subband based on 10 historical observations (Type-III is a subband whose central channel $i$ is available and having both the $(i-1)^{st}$ and the $(i+1)^{st}$ neighbors available). Each observation represents 1 minute of computed and normalized probabilities of 60 time sweeps over the 64 channels. We also use the Fuzzy model [113] to predict the time series at time $t$, $t+1$ and $t+2$ which corresponds to the $11^{th}$, $12^{th}$ and $13^{th}$ observation respectively then compare them against the actual occupancy. Each $i^{th}$ prediction is estimated based on the $(i-1)^{th}$ prediction rather than the actual. It is evident from figure 4.13 that AP1 has more available channels as well as type-III subbands, i.e. A mode of 6 subbands with 42% availability at AP1 compared to a mode of 3 subbands with 48% availability predicted at $t+2$. When both APs are cooperative, SUs can associate directly with AP1 to maximize the network throughput. Additionally, the allowable transmission power on a free channel can be adjusted based on the occupancy of its adjacent channel. Therefore, since both adjacent channels are free in this case, AP1 and AP2 can support a larger number of transmissions with higher power if we ignore other factors that affect the transmission power. The prioritized allocation of these subband types to SUs can be implemented in such a way that a high priority SU is allocated a subband with higher transmission power when compared to a low priority SU. For instance, in the case of a high priority SU requesting one LTE carrier (approximately 3 TV channels where 1 broadcast channel is 6Mhz in the USA), AP1 is more suitable to assist and allocate 1 type-III subband to the SU than AP2 based on future availability of these subbands. On the other hand, when APs are owned by different network operators, here each AP is generally selfish and only concerns about its own throughput maximization. Formally, the SU need to determine the optimal AP to associate with in such a way its data rate
is maximized. As demonstrated, modeling of channel availability at each AP as well as future availability prediction can facilitate this task for SUs.

![Actual vs. Predicted Availability at AP1 on left and AP2 on right](image)

Figure 4.13: A square area of a length 400m with 2 scattered APs. Each AP is managing a set 8 TV channels. Actual vs. Predicted future availability at time $t = SU$ request, $t + 1$, $t + 2$ are presented
4.4 Summary

Comparison of results using approximation error as the evaluation criterion as well as time complexity and running time prove the efficacy of our proposed models for spectrum availability. Furthermore, it exhibits the inefficiency of the conventional computation and other approaches such as the Normal, the Poisson-Normal, and Camp-Paulson approximations presented in the existing research work. Moreover, probabilistic analysis of network performance has been presented. A novel approach to calculate the probability of detecting multiple channels simultaneously is derived. In addition, the accuracy and efficiency of our proposed models are evaluated and compared against the ones proposed in the state-of-art. 2 associated applications are presented, one of which features novel 2 dimensional (time, frequency) future availability predictions.
Figure 4.5: Dynamic spectral occupancy in a spectrum of 512 subchannels for 30 minutes accompanied by the availability distributions
4.4. Summary

Figure 4.6: Dynamic spectral occupancy in a spectrum of 1024 subchannels for 30 minutes accompanied by the availability distributions
Figure 4.7: Performance comparison in terms of algorithm running time
Figure 4.8: Performance comparison in terms of time complexity
Figure 4.9: The effect of energy sensing impairment on the primary user activity modeling performance in terms of receiver operating characteristics [109]
4.4. Summary

Figure 4.10: The effects of threshold level decision at the modeling stage

Figure 4.11: Success probability in finding \( r \) or more available subchannels
Figure 4.12: A square area of a length 500m with 4 scattered APs. Each AP is managing a set 16 TV channels. PDFs of the number of available channels at time $t = SU$ request as well as $t+1$ prediction are presented.
Chapter 5

Conclusions

5.1 Summary of Insights

With the advent of high bandwidth multimedia applications and the growing demand for ubiquitous information network access for mobile wireless devices, enhancing the efficiency of wireless spectrum utilization is essential for addressing the scarcity of available transmission bandwidth. The development of the DSA/CR technology can significantly benefit from accurate and practical spectrum usage models. The purpose of such models is to provide a tractable, yet realistic representation of the statistical properties of spectrum usage in real systems that can adequately be employed in analytical studies or implemented in simulation tools for the performance evaluation of DSA/CR techniques. In this context, we presented the following:

In Chapter 2, we have conducted probabilistic analysis of free and contiguous subchannels in the cognitive radio network. The critical entity in our analysis is the distribution of total number of free subchannels. As we have shown, the computation of the exact distribution of \(N_f\) is prohibitively time-consuming and thus accurate and efficient techniques were presented. We labeled these novel approaches as multi-convolution, recursive, and hybrid convolution-recursive models. In addition, we focused on the analysis of a free subband, characterizing three different types. We developed and illustrated algorithms...
for the computation of the fore-mentioned distributions.

Subsequently, in Chapter 3, a multi-subchannel joint detection framework in which a technique takes into account the detection of primary users across multiple subchannels is considered. A novel approach to compute the probability of detecting simultaneously multi-subchannels is demonstrated. Furthermore, the accuracy and efficiency of our proposed models are evaluated and compared against the ones proposed in the state-of-art. Finally, 2 associated applications are presented, one of which features novel 2-dimensional (time, frequency) future availability prediction.

5.2 Future Work

This thesis has presented novel availability models under the i.n.i.d. paradigm. Some points to broaden the scope as well as make the work more practical are discussed in this section.

5.2.1 Limitations of the Energy Measurements

The thesis address partially the issue of the limitations of energy measurements as an input for the modeling stage, concerns such as propagated error and impact of the number of sensing samples $M$ were covered. For further experimental verification, one has to understand and report the role of energy threshold level and selection. This is a very tricky issue as it is not only the level of selected threshold, but the whole pattern that affects the measurements. For instance, the noise at receiver chain needs to be considered, and one would need to understand if the measurement data is biased. The threshold itself thus is not a free parameter as setting it too low would affect the statistics through the noise sensitivity of the equipment. Equally, setting it too high leads to known problems in the primary detection statistics. Moreover the spectral and time resolution itself has possible effects that should be discussed.
5.2. Future Work

5.2.2 Utility of the RF Propagation Models

While the ruling of the FCC provides broad guidelines for the database, some specifics of its design, features, implementation, and use are yet to be determined. Published work claim that a database-driven white space network can determine white spaces more effectively using sophisticated signal propagation modeling as opposed to spectrum sensing. Given parameters such as transmitter’s location, transmit power, antenna directionality, etc., these RF propagation models, such as Free Space, Egli, L-R with/without terrain, can estimate the received signal strength indication (RSSI) at any given location [13] and references therein. The proposed availability models could benefit from this approach and not having to rely solely on spectrum measurement. Unfortunately, we were not able to find some of this data throughout this work but it would be interesting to investigate its use for the modeling and compare the performance against the usage of measurement data. Also, take into consideration the propagated error from false positive rate (Loss of white spaces) and the computational complexity of the RF models.

5.2.3 Mobility

Every white space device primarily relies on the database to determine the white space availability. Hence, this results in a delay in the device learning about changes in spectrum availability. Either the device will have to poll the database, or the database will have to push updates to the device. This problem becomes worse when the devices are mobile. If mobile, the device could have traveled some distance between the time it receives two subsequent spectrum updates. Future work can accommodate this issue and consider adding protection range of the distance to learn further about determining white spaces.

5.2.4 Applications QoS

Although the proposed applications give an insight of the utility of availability modeling, QoS factors such as coverage area as well as transmit power should be considered. For example, the worse case down-link throughput at the boundary of the APs coverage area
can be computed according to the physical interference model [114,115] as follows:

$$U_n(a) = B \log_2 \left(1 + \frac{P_n/d_{in}^\theta}{w_{a_n} + \sum_{i=1}^n P_i/d_{in}^\theta} \right)$$

(5.1)

Where $\theta$ is the path loss factor, $B$ is the channel bandwidth, $d_n$ denotes the radius of the coverage area of the AP of interest, and $d_{in}$ denotes the distance between the AP and the benchmark location at the boundary of the coverage area of AP. Furthermore, $w_{a_n}$ depicts the background noise power including the interference from incumbent users on the channel $a_n$, and $\sum_{i=1}^n P_i/d_{in}^\theta$ represents the accumulated interference from other APs that choose the same channel $a_n$.

On the other hand, following the conventions in IEEE 802.22 standard [112] and Microsoft Redmond campus white-space networking experiment [13,115], each AP can select one vacant channel to operate on in order to avoid interference with other APs. How does this feature affect the modeling stage and what happens when each AP can select multiple channels to operate on?
Bibliography


Two user networks,” *IEEE Transactions on Wireless Communications*, vol. 6, no. 6, pp. 2204–2213, June 2007.


[44] Y. Zeng and Y.-C. Liang, “Covariance based signal detections for cognitive radio,”
Bibliography


Bibliography


[68] *IEEE draft standard for information technology telecommunications and information exchange between systems local and metropolitan area networks specific requirements part 22.1: Standard to enhance harmful interference protection for low power licensed devices operating in TV broadcast bands*. IEEE 802.22.1, 2010.


