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Abstract: This paper presents the results of a series of tests on short span reinforced concrete beams which were strengthened in shear with various arrangements of externally bonded Carbon Fibre Reinforced Polymer (CFRP) sheets. The objective of the tests was to determine the effect of changing the area and location of the CFRP sheet within the shear span. A total of fifteen 150 mm x 300 mm x 1,675 mm concrete beams were tested of which four were un-strengthened control specimens. The remaining eleven beams were strengthened with varying configurations of CFRP sheets. Parameters varied in the tests included the area of CFRP sheet, its anchorage length and the distance of the CFRP sheet from the support. The experimental results revealed that the CFRP is more effective when it is placed close to the supports and even small areas of CFRP can give significant increases in shear strength. The experimental results were compared with the three different existing shear prediction models for estimating shear contribution of CFRP sheets. A simple strut-and-tie model (STM) is presented which gives reasonable predictions of shear strength for the beam specimens, which were strengthened with CFRP over the full depth of the beam. The superposition method of design is replaced in EC2 by the variable angle truss model in which all the shear is assumed to be resisted by the truss mechanism. A simple regression equation is proposed for the calculation of effective stress in FRP to be used in EC2.

Shear strengthening of short span reinforced concrete beams with CFRP sheets

ABSTRACT

This paper presents the results of a series of tests on short span reinforced concrete beams which were strengthened in shear with various arrangements of externally bonded Carbon Fibre Reinforced Polymer (CFRP) sheets. The objective of the tests was to determine the effect of changing the area and location of the CFRP sheet within the shear span. A total of fifteen 150 mm x 300 mm x 1,675 mm concrete beams were tested of which four were un-strengthened control specimens. The remaining eleven beams were strengthened with varying configurations of CFRP sheets. Parameters varied in the tests included the area of CFRP sheet, its anchorage length and the distance of the CFRP sheet from the support. The experimental results revealed that the CFRP is more effective when it is placed close to the supports and even small areas of CFRP can give significant increases in shear strength. The experimental results were compared with the three different existing shear prediction models for estimating shear contribution of CFRP sheets. A simple strut-and-tie model (STM) is presented which gives reasonable predictions of shear strength for the beam specimens, which were strengthened with CFRP over the full depth of the beam. The superposition method of design is replaced in EC2 by the variable angle truss model in which all the shear is assumed to be resisted by the truss mechanism. A simple regression equation is proposed for the calculation of effective stress in FRP to be used in EC2.

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INTRODUCTION

Many existing structures designed to then current codes are unsafe according to current design codes. Other concrete structures have become structurally unsound due to deterioration over time. These structures can either be rebuilt or retrofitted. Strengthening is often the most viable choice since rebuilding it usually more costly and time consuming. Structures can be strengthened with a variety of conventional techniques such as steel plate bonding, ferro-cement and increasing the cross-section but experimental studies[1] have shown that the use of Fibre Reinforced Polymers (FRP) has many advantages over conventional methods. CFRP composites are the most commonly used of the various types of FRP since they offer many benefits including ease of handling, light weight, durability, strength, corrosion resistance and field-workability.

The shear strength of reinforced concrete beams can be increased by externally bonding CFRP sheets to the sides of the beam cross-section. The CFRP transfers loads across diagonal tension cracks in the concrete in a similar way to steel stirrups. Three different wrapping schemes are commonly used to strengthen reinforced concrete beams in shear with CFRP. Firstly, the CFRP is bonded to the sides of the beam, secondly, it is used to wrap the sides and bottom of the beam and thirdly, the complete section is wrapped.

The first research on shear strengthening of RC beams with composite materials was conducted by Berset in 1992. He conducted experiments on several reinforced concrete beams strengthened with externally bonded glass FRP (GFRP) laminates and proposed a

1 simple analytical model to estimate the shear strength contribution of the GFRP composites.
2 After Berset, Uji[4] studied the shear behaviour of eight RC beams strengthened in shear
3 using externally bonded Carbon Fibre Reinforced Polymers (CFRP) sheets. He found that
4 the application of CFRP improves the shear capacity of reinforced concrete beams. Chajes et
5 al[5] conducted experiments on T-beams strengthened in shear using different types of FRP
6 fabrics named aramid, E-glass, and carbon. They found an average increase in ultimate
7 strength of 83 to 125 percent. The FRP contribution was modelled in analogy with steel
8 stirrups contribution and limiting FRP strain of 0.005 mm/mm, determined from the tests,
9 was assumed. The method is applied and experimentally verified in the case of wrapped
10 beams without stirrups. Sato et al[6] also conducted research on shear strengthening using
11 CFRP strips and continuous laminates. They described the observed failure mode (debonding
12 of CFRP) through a simple model to account for partial shear transfer by CFRP debonding.
13 Umezu et al[7] also studied the effectiveness of totally wrapped Aramid and CFRP sheets in
14 improving shear strength of simply supported beams. Araki et al[8] conducted experiments
15 on RC beams strengthened with various amount of totally wrapped AFRP and CFRP sheets.
16 The conclusion drawn was that the shear capacity of RC members increased in proportion to
17 the amount of FRP sheets. The contribution of FRP to the shear capacity was evaluated
18 similar to calculation of stirrups. They proposed strength reduction factors of 0.6 and 0.45 for
19 tensile strength of CFRP and AFRP sheets respectively. Norris et al[9] discussed the results
20 of a series of experimental investigations on uncracked and cracked concrete beams
21 strengthened in shear and flexure with CFRP sheets. The experimental results show
22 dependence of the strength, stiffness and failure modes on the fibre orientation. Malek
23 and Saadatmanesh [11] studied shear behaviour using FRP bonded plates using Compression
24 Field Theory and truss analogy. They proposed a method for calculating the inclination angle
25 of the shear cracks and ultimate shear capacity of RC beams externally bonded FRP plates.
26 Malek and Saadatmanesh also presented analytical models to calculate stresses in the
27 strengthened beam and the shear force resisted by the composite plate. It was shown that
28 shear failure of the strengthened beams was controlled by either FRP fracture at a stress level
29 below its ultimate due to stress concentration or by debonding of FRP from the concrete
30 surface.
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Triantafillou [12] presented a design model for computing the shear capacity of RC
beams strengthened with FRP composites. He treated external FRP shear reinforcement
similar to the internal reinforcement and assumed that at the ultimate limit state, the FRP
develops an effective strain, ϵ_{fe} , which is less than the ultimate tensile strain, ϵ_{fu} , of FRP.
Khalifa et al[13] presented a modified model to calculate ϵ_{fe} on the basis of few more test
results. In ACI Committee 440 report, shear design guidelines for FRP construction were
based on the equations proposed by Khalifa et al[13]. In 2000, Triantafillou and
Antonopoulos presented three equations for ϵ_{fe} which were derived from a regression analysis
of data from seventy five beam tests. In July 2001, Technical Report on the "Design and use
of externally bonded fibre reinforced polymer reinforcement (FRP EBR) for reinforced
concrete structures" was published by working party of fib Task Group 9.3. The shear
prediction guidelines in the report are based on the model proposed by Triantafillou and
Antonopoulos.

Adhikary and Mutsuyoshi [2] conducted experiments on eight concrete beams using
different configurations of CFRP sheets to evaluate shear strength. They found significant
increase in ultimate shear strength of strengthened beams. In another research, they
conducted an experimental investigation for enhancing the shear capacity of reinforced
concrete beams using different techniques. In 2004, the Concrete Society published revised
guidelines for strengthening beams in shear with FRP in the second edition of TR55. Zhang
et al[17] carried out research work on shear strengthened concrete beams with CFRP and

1 observed that the failure mechanism is different for CFRP strips and woven fabric and
2 concluded that strips are more efficient.

3 It is observed from the above review that there are few studies on shear strengthening
4 of RC beams. Mostly, the researchers have focused on improvement in shear capacity by
5 externally bonded CFRP composites using arrangements like complete wrapping, U shaped
6 wrapping and complete side wrapping of the FRP to the beams surface. These arrangements
7 of CFRP do not address the issue of shear enhancement by the external application of CFRP
8 in various configurations and anchorage lengths along the different areas of the shear span. In
9 practice, mostly the beam elements are built integrally with the slab and are not of rectangular
10 cross section as considered in most of the researches. In addition, the situations may arise
11 when the beams are required to be strengthened in some specific locations instead of CFRP
12 application along the entire shear span. The effect of varying the configuration and wrapping
13 scheme of the CFRP has not yet thoroughly assessed. This experimental program was
14 designed to investigate the effect of the CFRP configuration and wrapping scheme on the
15 shear strength of short-span reinforced concrete beams deficient in shear. The major limit of
16 the research work is that only one test has been performed for each strengthening scheme due
17 to which some unexpected results may be difficult to identify. This limitation has been
18 observed in the already published literature on the subject therefore it is suggested to increase
19 the database by conducting more experiments in future. The strength of the tested beams is
20 compared with the predictions of the models proposed by i) Khalifa et al[13], ii)
21 Triantafillou and Antonopoulos[14] and iii) Zhang and Hsu[17]. The strength of the beams
22 has also been assessed with a simple strut-and-tie model which was originally formulated for
23 the design of beams with steel shear reinforcement. The strut and tie model is shown to give
24 good predictions of the shear strength of beams strengthened with CFRP. A simple regression
25 equation is also proposed to be used in EC2, for the calculation of effective stress in CFRP. It
26 is shown that the variable truss angle in EC2 can be used for beams strengthened with CFRP.
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35 EXPERIMENTAL PROGRAMME

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37 **Test Specimens:** Fifteen high-strength concrete beams were tested. All the beams measured
38 150mm wide by 300mm deep in cross-section and 1,675 mm in length. Two 19mm diameter
39 bars were used as flexural reinforcement in each beam and no internal shear reinforcement
40 was used. Four beams were used as control specimens and eleven were strengthened using
41 CFRP sheets. The beam details and CFRP configurations are illustrated in Figures 1 and 2
42 respectively. CFRP sheet was applied only to the sides of the beams and no flexural
43 strengthening was done. The strengthened beams were divided into two groups A and B,
44 depending upon the depth of CFRP sheet. Group A was composed of six rectangular concrete
45 beams strengthened up to the full depth, whereas the remaining five beams, with reduced
46 anchorage length of CFRP sheets, were placed in Group B. The beams in Group B were
47 strengthened over half the beam depth to simulate the case of a T or down-stand beam where
48 it is not possible to apply the CFRP over the full beam depth.
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53 **Material properties:** The mean compressive cylinder strength of the concrete used in the
54 beams was 49.2MPa at 28 days. Limestone aggregate was used with a maximum aggregate
55 size of 19mm. The longitudinal reinforcement consisted of deformed bars with yield strength
56 of 494MPa. The relevant material properties of the CFRP sheet are given in Table 1.
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1 **Fabrication of Beam Specimens:** The beams were cast in steel forms and were cured at
2 room temperature for 28 days alongside 300mm long by 150mm diameter concrete cylinders.
3 After grinding, the surfaces of the beam were cleaned and a two part epoxy was applied in
4 accordance with the manufacturer's recommendations. After curing the epoxy, the beam
5 surfaces were again ground and cleaned to remove any loose dust particles. The CFRP sheet
6 was cut to the proper length and infused with two part epoxy before being applied to the
7 beam. The sheets were pressed firmly in place with a plastic roller to remove air bubbles and
8 excess epoxy. The sheets were placed on the sides of the beam with the main fibres vertical in
9 the configurations shown in Figure 2.
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11 **Test Procedure:** Each beam was simply supported over a span of 1200mm and tested under
12 three point loading as shown in Figure 1. The ratio between the clear shear span and the
13 effective depth (a_v/d) was 2. The beams were loaded with hydraulic jacks at a constant rate in
14 an internal reaction load frame. Deflections were recorded at mid-span and at the supports(to
15 observe any settlement of supports). The cracks and crack pattern were recorded at each
16 increment in load.
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19 **EXPERIMENTAL RESULTS:** Cracks were marked on the beams throughout the tests to
20 enable the cracking patterns and failure mechanisms in the CFRP strengthened beams to be
21 compared with the control beams. The shear strength of the beams was compared with the
22 predictions of three different models available in the literature. The beams in Group A were
23 also analysed with a strut-and-tie model (STM) developed by the Sagaseta and Vollum[19],
24 which is consistent with the recommendations for STM in EC2. Experimental results are
25 shown in Tables 1 and 2.
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29 **Strength:** Table 2 shows that that the CFRP sheet was effective in strengthening the beams
30 but the contribution of the CFRP varied depending on its area and configuration. Beams C-3
31 and C-9 in group A were strengthened with the same area of CFRP sheet (300 x 300mm) but
32 the position of the sheets in the shear span was different. The CFRP sheet was applied
33 adjacent to the supports in beam C9, whereas it was placed 150mm from the supports in
34 beam C3. The increase in shear strength in beam C9 was 47.15kN whereas it was only
35 32.7kN in beam C3. Beams C6 and C8 were also strengthened with the same sized sheets of
36 CFRP (150 x 300mm) but the distances of the sheets from the supports were 225mm and
37 75mm respectively. The increase in strength in of C8 was 18.35kN compared with an
38 increase of 13.55kN in beam C6. The shear strength of C5 with complete side wrap was
39 greatest at 66.45kN whilst the increase in strength in C11 was only 3.90kN.
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42 The increase in strength was 32.7kN in C2 which was strengthened with CFRP
43 throughout its shear span over the lower half of the beam depth within the flexural tension
44 zone. Beams C4 and C7 were similarly strengthened over half the beam depth with CFRP
45 sheets measuring (300 x 150mm) placed at the centre of the shear span and adjacent to the
46 support respectively. The increase in shear strength was 18.35kN in beam C7 and 13.55kN in
47 beam C4. Beams C10 and C12 were strengthened similarly with CFRP sheets measuring 150
48 x150mm placed at varying distances from the support. The shear strength of both beams was
49 increased by 8.75kN.
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52 The increase in beams shear strength is given in Table 1 which shows that beam C9
53 was the most efficient in terms of its combined increase in strength and cost effectiveness.
54 Consideration of Table 2 in conjunction with Figure 2 shows that it is beneficial to apply
55 CFRP sheets close to the support. Moreover, the area of CFRP sheet can be minimized with
56 considerable increase in strength if the sheet is applied near the support. It is also shown that
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1 the shear strength of the beams in Group B was reduced significantly compared with the
2 beams in Group A by reducing the anchorage length of the CFRP.

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4 **Ductility:** Figure 4 shows that the stiffness and ultimate deflection of the strengthened beams
5 were greater stiffness than in the control beams. The deflection of the strengthened beams was
6 found to depend on the position of the CFRP sheet and its anchorage length. Increasing the
7 distance of sheet from the support and reducing the anchorage length decreased the deflection
8 at failure. Zhang and Hsu[17] also found that CFRP strengthened beams give not only an
9 increase in shear strength but also an increase in ductility. It is concluded that strengthening
10 beams in shear with CFRP increases ductility in addition to strength.
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13 **Failure Mechanism:** All the control beams failed in shear with mean shear strength of 121.1
14 kN. The CFRP sheets resisted the crack propagation in the shear span and changed the mode
15 of failure to flexure shear rather than shear failure in the control beams.
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17 Beams C-2, C-3 and C4 failed as a result of flexure shear cracking along with
18 delamination of the CFRP sheet. Beam C5 failed due to de-lamination of the CFRP sheet
19 from the concrete surface with the concrete failing in tension underneath the epoxy. Splitting
20 of concrete at the top face was also observed at failure. The bonding between the CFRP sheet
21 and the epoxy was good, except at few spots where small pieces of epoxy were pulled away
22 from the surface of the CFRP sheet. The beam failed due to the formation of a flexural shear
23 crack. Most of the beams failed due to de-lamination of the CFRP sheet from the concrete
24 surface. Complete de-bonding of the CFRP sheet occurred due to diagonal cracking in one
25 shear span of beams C6 and C10 whereas the CFRP sheet resisted crack propagation in the
26 other shear span. Flexure shear failure was observed in all the strengthened beams except
27 beam C11 where only one 75mm wide CFRP strip was provided at each end. The crack
28 pattern at failure is shown for all the beams in Figure 3.
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33 **Shear Strength Prediction Models:** The nominal shear strength (V_n) of FRP strengthened
34 concrete beams is conventionally calculated by adding the individual contributions of
35 concrete (V_c), steel stirrups (V_s) and FRP (V_f) as follows:
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$$37 V_n = V_c + V_s + V_f \quad (1)$$

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39 In ACI-318, the design shear strength is obtained by multiplying the nominal shear strength
40 by a strength reduction factor, ϕ for which Khalifa et al[13] suggested a value of 0.70 for V_f .
41 The contribution of the CFRP sheet to shear strength can be evaluated with the following
42 equation which is similar to that used to determine the shear contribution of steel stirrups.
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$$46 V_f = \rho_f E_f \varepsilon_{fe} b_w d_f (1 + \cot \beta) \sin \beta \quad (2)$$

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48 where ρ_f is the CFRP shear reinforcement ratio ($2t_f w_f / b_w s_f$), E_f is the elastic modulus
49 of CFRP, ε_{fe} is the effective tensile strain of CFRP, b_w is the beam width, t_f is the thickness of
50 CFRP reinforcement and w_f is the width, s_f is the spacing of CFRP which becomes equal to w_f
51 for a continuous vertical CFRP reinforcement. The angle β describes the fibre orientation
52 with respect to the longitudinal axis of the beam. d_f is the effective depth of CFRP
53 reinforcement measured from the centre of the tensile flexural reinforcement towards the
54 flexural compressive zone in the beam.
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58 Triantafillou[12], observed that the effective strain (ε_{fe}) is a function of the axial
59 rigidity ($\rho_f E_f$) of the externally bonded CFRP strips or sheet. Triantafillou[12] determined the
60 effective strain in the CFRP by back calculation from experimentally derived values of V_f .
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An empirical relationship was developed between strain and axial rigidity by plotting effective strain versus axial rigidity for test data from 40 beams published by various researchers. Khalifa et al[13] modified Triantafillou's[12] method for calculating ε_{fe} on the basis of a slightly enlarged data base of 48 beams. The experimental data used by Khalifa et al[13] included two types of FRP materials (Carbon and Aramid), three different wrapping configurations (sides only, U-shaped wrapping and complete wrapping), with both continuous sheets and strips of FRP. Khalifa et al[13] presented three equations for calculating the reduction factor (R) of which the lowest value is used to calculate the effective strain. The resulting effective strain is used in Equation (2) to calculate the contribution of the CFRP to the shear strength of the RC beam. Although Equation (3) was developed from regression analysis of test data including both rupture and de-bonding failure modes of CFRP, Khalifa et al[13] suggested using it for CFRP rupture only.

$$R = 0.5622(\rho_f E_f)^2 - 1.2188(\rho_f E_f) + 0.778 \quad (3)$$

The reduction factor for CFRP de-bonding is given by:

$$R = \frac{0.0042(f'_c)^{2/3} w_{fe}}{(t_f E_f)^{0.58} \varepsilon_{fu} d_f} \quad (4)$$

where w_{fe} is the effective width of the CFRP sheet which is taken as

$$w_{fe} = d_f - 2e^{\{6.134 - 0.58 \ln(t_f E_f)\}} \quad (5)$$

Khalifa et al[12] also suggested an upper limit of 0.5 to R to control the shear crack width and loss of aggregate interlock.

In 2002, Triantafillou and Antonopoulos[14] presented three different equations using regression analysis of seventy five experimental data, two for CFRP sheets and one for fully wrapped Aramid FRP sheets. The equation for fully wrapped CFRP sheet is given by:

$$\varepsilon_{fe} = 0.17(f'_c)^{2/3} / \rho_f E_f)^{0.30} \varepsilon_{fu} \quad (6)$$

and for U-shaped or side wrapped CFRP is:

$$\varepsilon_{fe} = \min[0.65(f'_c)^{2/3} / \rho_f E_f)^{0.56} * 10^{-3}; 0.17(f'_c)^{2/3} / \rho_f E_f)^{0.30} \varepsilon_{fu}] \quad (7)$$

The contribution of the CFRP sheet to the shear carrying capacity is calculated by substituting ε_{fe} from equation (7) into Equation (2).

In 2005, Zhang et.al[17] presented two alternative equations for calculating the R-value. They considered the effect of concrete strength in the following equation which was derived from a regression analysis of test data:

$$R = 1.4871(\rho_f E_f / f'_c)^{-0.7488} \quad (8)$$

They also developed the following analytical equation for calculating R from an analysis of the bonding mechanism:

$$R = \tau_{\max} L_e / 2 f_{ft} t_f \leq 1 \quad (9)$$

where L_e is assumed to be 75mm (but further research is needed), f_{fu} is the ultimate tensile stress of CFRP and τ_{max} is to be calculated from the equation proposed by Hsu et.al[10] as follows:

$$\tau_{max} = (7.64 * 10^{-4} f_c^2) - (7.64 * 10^{-2} f_c) + 6.38 \quad (10)$$

where τ_{max} is the ultimate direct shear strength in MPa.

The lowest of the values of R from Equations (8) and Equation (9) is used to calculate the effective tensile strain in the CFRP. Zhang et al[17] also recommended a maximum value of R equal to 0.4. Zhang et al[17] presented an equation equivalent to Equation (2) for calculating V_f . They[17] took the contribution of continuous CFRP sheet to shear strength as:

$$V_f = w_{fe} t_f f_{fe} \sin^2 \beta \leq (2\sqrt{f_c} b_w d / 3 - V_s) \quad (11)$$

where w_{fe} is defined in Equation (5).

Comparison of measured and predicted shear strengths: The measured and predicted contributions of the CFRP to shear strength, V_f are compared in Table 2 and Fig. 5. The experimental values of V_f were calculated by subtracting the mean shear strength of the control beams from the shear strength of the beams with CFRP. The predicted values of V_f were calculated in accordance with the recommendations of Khalifa et al[13], Triantafillou and Antonopoulos[14] and Zhang et al[17]. V_f was calculated with Equation (2) with $\rho_f = 2t_f w_f / (b_w s_f)$. The spacing s_f of the discrete strips of CFRP was taken as the clear shear span $a_v = 525$ mm. The efficiency of the truss action is reduced when the CFRP only extends over half the beam depth as in some of the authors tests. Equation (1) is based on the truss analogy in which stirrups are assumed to extend over the full height of the beam. The efficiency of the CFRP also decreases due to the reduction in its anchorage length when it only extends over half the beam depth. This loss of efficiency in the CFRP was included in Equations (2) and (5) by measuring its effective depth d_f to the top of the CFRP. Table 2 includes a comparison of the ratio V_{fmeas}/V_{fpred} for each design method. It seems likely that the shear strength was increased in the beams in which the CFRP extended over half the beam depth as a result of the angle of the critical shear plane being increased by the presence of the CFRP.

The comparison is presented in Table 2 for all the specimens, the specimens with CFRP over the full beam depth and over half the beam depth. The method of Khalifa et al[13] gives the most consistent predictions for V_f for all the authors beams and that of Triantafillou and Antonopoulos[14] the least. The underestimate of V_f for beams C3 and C6 may be due to the early de-lamination of the CFRP sheet in beam C3 and de-bonding of the CFRP sheet on one of the side of beam in test C6 in which the shear crack crossed the CFRP sheet and propagated towards support, causing premature failure of the beam.

Equation (2) is based on the truss analogy and is theoretically applicable to beams in which the CFRP strips are evenly distributed within the shear span. Equation (2) seems less applicable for short span beams reinforced with a single CFRP strips in the shear span as in many of the tested beams. Eurocode 2 (EC2) and BS8110 state that shear reinforcement is only effective in short span beams with $a/d < 2$ if placed within the central three quarters of the shear span. The tests suggest that CFRP strips may be more effective, possibly due to enhancement of dowel action, in short span beams when positioned close to the support rather in the central three quarters of the shear span. Therefore, V_f was recalculated in terms of the total area of CFRP within the shear span as follows:

$$V_f = 2t_f w_f E_f \varepsilon_{fe} \quad (12)$$

The resulting values of V_f are given in Table 2 which shows that the shear strength contribution calculated using the total area of CFRP overestimates the shear carrying capacity.

Strut-and-tie model: The authors have analysed the beams in Group A, which were strengthened over their full height, with a strut-and-tie model (STM) which was developed by Sagaseta and Vollum[19] for short span beams with steel shear reinforcement. The STM model is consistent with the design recommendations in EC2 for strut and tie modelling. It is assumed that the shear force is transferred to the supports via, firstly, a direct strut and, secondly, a truss system consisting of two indirect struts equilibrated by stirrups as shown in Fig. 6. The proportion of the shear force taken by the direct strut (λ) and its angle of inclination to the horizontal (θ) are found iteratively by solving equations (12) to (16). Equations (13) and (14) are derived from considerations of geometry whilst Equations (15) and (16) are derived from consideration of horizontal equilibrium at the bottom node.

$$\cot \phi'_i = \frac{a_v + l_b - \frac{2(n-i)+1}{2n} \cdot (1-\lambda)l_b - S_i + \frac{l_t}{2} \cdot n_{lp}}{h - 2c + \frac{2(n-i)+1}{2n} \cdot (1-\lambda)2c - C'_i} \quad (13)$$

$$\cot \theta = \frac{a_v + \frac{l_b \lambda}{2} + \frac{l_t \lambda}{4} \cdot n_{lp}}{h - c\lambda - \left(\frac{T'_i + T_d / 2}{bf_{cnt}} \right)} \quad (14)$$

$$T'_i = T_{Si} \cdot \sum_1^n \cot \phi'_i \quad (15)$$

$$T_d = \frac{\lambda}{1-\lambda} \cdot \cot \theta \cdot \sum_1^n T_{Si} \quad (16)$$

$$\frac{\lambda}{1-\lambda} \cdot \sum_1^n T_{Si} = (\lambda l_b \sin^2 \theta + c\lambda \sin 2\theta) b \cdot 0.6\nu f_{cd} \quad (17)$$

All the terms in equations (13) to (17) are defined in Fig. 6. The tensile forces T'_i and T_d in equation (14) and (15) are the horizontal components of force in the indirect strut III and direct strut I respectively.

The critical failure mode is assumed to be crushing of the direct strut, and is implicit in equation (17). The width of the direct strut was calculated in terms of the geometry of the bottom node. The effective concrete strength of the direct strut was assumed to be $0.6\nu f_{cd}$ where $\nu = (1 - f_{ck}/250)$ as defined in EC2. The bearing stress under the plates was assumed to be uniform and was limited to $0.85\nu f_{cd}$ at the bottom nodes and νf_{cd} at the top nodes, as recommended in EC2. The top boundary of strut III is assumed, for simplicity, to be linear so that the distance C'_i can be easily estimated from horizontal equilibrium at the top node.

The strut and tie model is statically determinate if the stress in the shear reinforcement is known at failure. Sagaseta and Vollum[19] found that steel stirrups yield at failure ($T_{Si} = A_{sw} f_y$) for stirrup indices (SI) less than 0.1, where $SI = n A_{sw} f_y / (b_w h f_c)$. The STM can be

1 applied to beams with CFRP shear reinforcement if the effective tensile stress is known in the
2 CFRP at failure.

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4 The CFRP sheets were assumed to be located at the centre of the shear span in the
5 STM as shown in Fig. 6. The tensile force in the CFRP was calculated as the product of the
6 effective area of each strip (see Fig. 6) and the effective tensile stress in the CFRP. CFRP was
7 only assumed to be effective if positioned within the central three quarters of the clear shear
8 span as stated in EC2 for beams with steel stirrups. This assumption was found to be
9 reasonable for specimens C5, C8 and C9 in which the area of CFRP outside the central three
10 quarters of the shear span was neglected. Several assumptions needed to be made regarding
11 the geometry of the bottom node since the specimens were supported on rollers (see Fig. 6).
12 These assumptions were based on a previous analysis of a series of beams supported on
13 rollers tested by Shin et al. [24] which were reinforced with steel shear reinforcement. The
14 beams had a_v/d ratios of 1.5 and 2.0. Beams which failed due to local crushing of the concrete
15 at the support were not considered in the analysis. The bottom node was modelled assuming
16 an equivalent bearing plate length $l_{b,eff} = 2c \cdot \cot \alpha$ (see Fig. 6), where α is the dispersion angle
17 measured from roller centreline to the flexural reinforcement to the horizontal. An optimal
18 value of 47.8° was obtained for the dispersion angle α from a back analysis of Shin's[24] test
19 results with the STM. It is suggested that α is conservatively taken as 50° in practice.
20 Shin's[24] beams were reanalysed with $\alpha = 50^\circ$ obtaining a mean and standard deviations of
21 P_{test}/P_{calc} of 1.07 and 0.23 respectively for the 16 beams.

22 Table 2 shows that the strut-and-tie model described in this paper gives good
23 predictions of the shear strength for beams in group A. The mean value of P_{test}/P_{calc} was 0.98
24 for the six beams analysed and the standard deviation was 0.16. The worst predictions were
25 obtained for beams C6 and C11, which appear to have failed prematurely due to de-bonding
26 of the CFRP sheets without concrete failure. Nevertheless, Table 2 shows that the STM
27 provided safe estimates of the ultimate strength when standard material factors of safety were
28 applied ($\gamma_c=1.5$ and $\gamma_f=1.35$, according to *fib* report[15]) as shown in Table 2.

29 The STM predictions are relatively accurate even though the specimens had a clear
30 shear span to effective depth ratio of 2 which is at the upper limit of the range for which the
31 model is applicable[23].

32 The STM tends to give better estimates of the shear strength of the beams in series A
33 strengthened with CFRP than the empirical design equations described in this paper. It is
34 interesting to note that there are substantial conceptual differences between the STM and
35 empirical design approaches. The design formulas, which are based on a classical truss
36 superposition concept (V_c+V_f), were derived assuming a constant concrete contribution which
37 was estimated from the shear strength of the control beams. On the other hand, the shear
38 component of the direct strut ($V_c=\lambda V$) reduces with increasing stirrup index in the STM. The
39 test data were investigated to determine which of these assumptions is most realistic. V_c was
40 estimated by subtracting the calculated value of V_f for each method from the ultimate shear
41 strength obtained in the experiments. Figure 7 shows that the values of V_c obtained from this
42 analysis were closer to the predictions of the STM than the constant value assumed in the
43 remaining design methods. Even though the concrete component seemed to be overestimated
44 in the superimposition methods, the ultimate loads predicted were similar to the STM
45 predictions. This suggested that the reduction factor R derived in superposition methods must
46 compensate for this overestimation of the concrete component.

47 The existing design empirical formulas described in this paper do not take into
48 account the relative position of the shear reinforcement relative to the clear shear span.
49 Although the strut-and-tie model makes allowance for changing the position of the stirrups
50 (S_i) the effect of changing this variable has a minor influence on the predicted ultimate
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strength of the beam. The increase in strength observed in beam C3 compared with C9, and in lesser extent in beams C8 and C6, due to changing the position of the CFRP closer to the support is not captured by the STM. This increase in strength could be due to enhancement of the contribution of the dowel action, which is not considered in the STM.

APPLICATION OF EUROCODE 2

The draft ENV version of EC2 included the “Standard” design method for beams in shear which was similar to Equation (1). The “Standard” method was removed during the final development of EC2[28] which now only gives the variable strut inclination method for the design of shear reinforcement in beams. It is assumed in the variable strut inclination method that the shear force is resisted by a truss consisting of the concrete struts acting in compression and the shear reinforcement acting in tension. The angle of the concrete struts varies from 21.8 to 45 degrees to the longitudinal axis of the beam depending upon the applied shear force. For members with inclined shear reinforcement, the design value of the shear force is given by:

$$V_{Rd,s} = A_{sw} (0.9d) f_{ywd} (\cot\theta + \cot\beta) \sin\beta / s \quad (18)$$

where A_{sw} is the area of steel shear reinforcement; f_{ywd} is the yield strength of the shear reinforcement; ‘ s ’ is the spacing of the stirrups; θ is the angle in degrees of concrete strut to the longitudinal axis of the beam; β is the inclination angle of shear reinforcement. The value of $\cot\theta$ is limited to $1 \leq \cot\theta \leq 2.5$. EC2 also imposes a maximum limit on $\cot\theta$ which is governed by the crushing of concrete struts.

$$V_{Rd,max} = 0.9 b_w d v f_{cd} / (\cot\theta + \tan\theta) \quad (19)$$

where v is a strength reduction factor for concrete cracked in shear and f_{cd} is the design value of the concrete compression force in the direction of the longitudinal member axis.

A simple regression equation is proposed for the calculation of effective stress in CFRP to be used in EC2. Experimental data from 35 beams strengthened with CFRP, in which all the required test data was available, has been analysed to determine whether the VSI method in EC2 is suitable for the design of beams. All the beams were U-wrapped and details of the 35 beams considered are given in Table 3. It was assumed that the external CFRP reinforcement can be treated in the same way as internal steel stirrups if the stress in the CFRP is calculated in terms of the effective strain which is lower than the ultimate value for the naked CFRP as previously discussed. Equation 18 can be rewritten as:

$$V_{Rd,f} = 0.9 \rho_f b_w d \varepsilon_{fe} E_f (\cot\theta + \cot\beta) \sin\beta \quad (20)$$

where ρ_f is the CFRP reinforcement ratio which is given by $\rho_f = 2t_f w_f / b_w a_v$; ε_{fe} is the effective tensile strain in the sheet, a_v is the clear shear span and β is the angle of inclination of FRP to the longitudinal axis of the beam.

The effective stress in the CFRP was calculated by back substitution into Equation 20 using the experimental values of shear strength. The reduction factor R was calculated from the ratio of the effective stress (f_{fe}) to the ultimate strength (f_{fu}) of the FRP. The resulting reduction factors are plotted against axial rigidity in Figure 8. A power relationship was derived between the reduction factor (R_{EC2}) and the axial rigidity ($\rho_f E_f$) in a regression analysis. Figure 8 shows that the r-squared value is relatively high indicating that a simple power expression gives a reasonable representation of the relationship between axial rigidity

and R_{EC2} . The corresponding proposed power equation for calculating the effective strain in FRP is given by:

$$\varepsilon_{fe} = \varepsilon_{fu} \{0.0812(\rho_f E_f)^{-0.9434}\} \quad (21)$$

The value of effective strain from Equation (21) is used in Equation (20) to calculate the shear strength of the concrete beam strengthened in shear using CFRP sheets or strips. The experimental and predicted shear strengths are compared in Figure 9. The design datum was obtained by multiplying the effective strain given by Equation (21) by a reduction factor of 0.87 to achieve a lower bound to the test data. Figure 9 also shows the shear strengths predicted with Equation (1) with V_c calculated in accordance with EC2 using a material factor of safety of 1.5 for concrete. It is concluded that the variable truss model can be used to calculate the design shear strength of beams strengthened with CFRP.

CONCLUSIONS

Following conclusions have been drawn from the research work presented in this paper:

1. Experimental results revealed that significant increase in shear strength and ductility can be achieved by proper application of CFRP sheets to shear deficient concrete beams. The presence of CFRP sheet resists the crack propagation and alters the brittle failure mode to ductile.
2. For short beams, the application of CFRP sheet closer to the supports was found beneficial as the area of CFRP sheet can be minimized with considerable increase in shear strength.
3. It was observed that all the strengthened beams showed relatively greater stiffness than the control beams however the ultimate deflection was found higher. The deflection of the strengthened beams was found dependent upon the placement of CFRP sheet and its anchorage length, as increasing the distance of sheet from the support and reducing the anchorage length resulted in the corresponding decrease in deflection.
4. Comparison of experimental results with three different prediction models revealed that the model proposed by Khalifa et al. predicted the experimental results with good accuracy and safety margin. Although the model was proposed for complete side wrap, it can also be applied effectively to different arrangements of CFRP sheet and anchorage lengths along the shear span of the beam.
5. The ultimate strength of the short span beams strengthened with CFRP sheets up to the full depth can be well predicted using the simple strut-and-tie model suggested by the authors. The STM predictions were reasonable despite that the clear shear span to effective depth ratio was 2, which is near the limit of validity of the strut-and-tie model. Although the STM model allows for changing the position of the vertical reinforcement along the clear shear span, the influence of these variations into the ultimate strength are negligible. The strut-and-tie model agreed with predictions from empirical approaches, although the concrete contribution was not constant in the STM, as assumed in the empirical methods. This conceptual difference between both approaches raises the question of whether the reduction factor R , which is obtained empirically assuming a classic truss concept (V_c+V_s), should be applied to other methods such as STM.
6. The proposed equation for calculation of effective stress in CFRP can be effectively used in EC2 and it is shown that the variable angle truss model in EC2 can be used to calculate the shear strength of CFRP strengthened beams.

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Table 1: Specimen Details and CFRP Properties

Beam Ref	f'_c (MPa)	a/d	ρ_l (%)	Section Details			CFRP properties and wrapping schemes				
				b_w (mm)	d (mm)	d_s (mm)	t_f (mm)	E_f (GPa)	f_{fu} (MPa)	β	ρ_f ($\times 10^{-3}$)
Control	49.13 (avg)	2.00	1.43	150	300	-	-	-	-	-	-
C-2	49.1	2.00	1.43	150	300	150	0.34	234.5	3,450	90	4.46
C-3	48.28	2.00	1.43	150	300	300	0.34	234.5	3,450	90	2.23
C-4	49.1	2.00	1.43	150	300	150	0.34	234.5	3,450	90	2.23
C-5	48.62	2.00	1.43	150	300	300	0.34	234.5	3,450	90	4.46
C-6	49.79	2.00	1.43	150	300	300	0.34	234.5	3,450	90	1.11
C-7	48.97	2.00	1.43	150	300	150	0.34	234.5	3,450	90	2.23
C-8	47.93	2.00	1.43	150	300	300	0.34	234.5	3,450	90	1.11
C-9	50.35	2.00	1.43	150	300	300	0.34	234.5	3,450	90	2.23
C-10	51.38	2.00	1.43	150	300	150	0.34	234.5	3,450	90	1.11
C-11	49.38	2.00	1.43	150	300	300	0.34	234.5	3,450	90	0.56
C-12	48.41	2.00	1.43	150	300	150	0.34	234.5	3,450	90	1.11

Table 2: Comparison of Experimental Results

Beam No	Experimental Results			Triantafillou et al.		Khalifa et al		Zhang et al		$V_{fcal} = \frac{2t_f w_f E_f \epsilon_{fe}}{V_{fcal}}$		STM*		EC2		Failure mode
	P_u (kN)	V_{exp} (kN)	V_f (kN)	V_{fT} (kN)	V_f/V_{fT}	V_{fK} (kN)	V_f/V_{fK}	V_{fZ} (kN)	V_f/V_{fZ}	V_{fcal}	V_f/V_{fcal}	P_{calc} (kN)	P_{calc}/P_u	V_{EC2}	V_{exp}/V_{EC2}	
Control	242.3 (avg)	121.15	-	-	-	-	-	-	-	-	-	260.3	1.07	-	-	Shear
C-2	307.7	153.85	32.7	46.46	0.7	29.58	1.11	9.84	3.32	128.8	0.25	-	-	-	-	Sheet delamination
C-3	307.7	153.85	32.7	68.06	0.48	56.94	0.57	41.67	0.78	64.4	0.51	319.7	1.04	116.6	1.32	Sheet delamination
C-4	269.3	134.65	13.55	34.24	0.4	19.76	0.69	14.38	0.94	53.37	0.25	-	-	-	-	Sheet delamination
C-5	375.1	187.55	66.45	92.57	0.72	59.16	1.12	28.47	2.33	130.8	0.51	371.4	0.99	121.3	1.55	Sheet delamination
C-6	269.3	134.65	13.55	50.65	0.27	18.7	0.72	27.23	0.5	26.81	0.51	296.8	1.1	112.1	1.2	Debonding
C-7	279	139.5	18.35	34.21	0.54	19.72	0.93	14.37	1.28	72.28	0.25	-	-	-	-	Sheet delamination
C-8	278.9	139.45	18.35	49.93	0.37	18.24	1.01	26.88	0.68	36.3	0.51	292	1.05	112.1	1.25	Sheet delamination
C-9	336.5	168.25	47.15	69.14	0.68	58.56	0.81	42.28	1.12	92.86	0.51	317.8	0.94	116.6	1.44	Sheet delamination
C-10	259.7	129.85	8.75	25.63	0.34	10.14	0.86	14.62	0.6	34.62	0.25	-	-	-	-	Debonding
C-11	250	129.85	3.9	37.37	0.1	4.69	0.83	13.57	0.29	7.65	0.51	283.1	1.13	107.8	1.16	Debonding
C-12	259.7	125	8.75	25.06	0.35	9.74	0.9	14.31	0.61	34.62	0.25	-	-	-	-	Debonding

Notes: STM* values obtained with a material safety factors of 1.0.

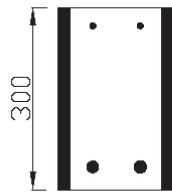
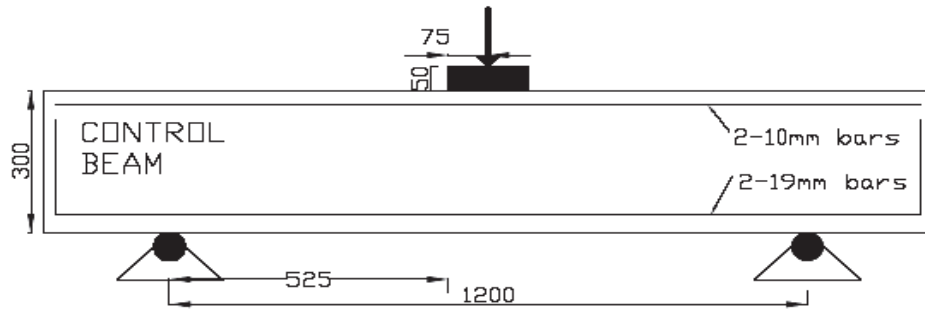
Table 3: Comparison of Results according to EC2

Sr No	Beam No	Section Details		FRP Properties		ρ_f ($\times 10^{-3}$)	$\rho_f E_f$	V_{exp} [kN]	R_{EC2} (f_{re}/f_{fu})	V_{EC2} [kN]	V_{exp}/V_{EC2}
		b_w (mm)	d (mm)	E_f (GPa)	β (Deg)						
1	T(S1a)	70	100	235	90	2.2	0.52	21.75	0.19	15.37	1.41
2	T(S1b)	70	100	235	90	2.2	0.52	19.45	0.17	15.37	1.27
3	T(S2a)	70	100	235	90	3.3	0.78	24.05	0.14	15.73	1.53
4	T(S2b)	70	100	235	90	3.3	0.78	21.1	0.12	15.73	1.34
5	T(S3a)	70	100	235	90	4.4	1.03	21.4	0.09	15.99	1.34
6	T(S3b)	70	100	235	90	4.4	1.03	18.75	0.08	15.99	1.17
7	T(S1-45)	70	100	235	45	2.2	0.52	22.25	0.20	15.22	1.46
8	T(S2-45)	70	100	235	45	3.3	0.78	23.65	0.14	15.57	1.52
9	T(S3-45)	70	100	235	45	4.4	1.03	20.35	0.09	15.83	1.29
10	K(B-CO2)	150	255	228	90	0.88	0.20	88	0.33	87.05	1.01
11	K(B-CO3)	150	255	228	90	2.2	0.50	113	0.17	91.68	1.23
12	K(C-BT2)	150	355	228	90	2.2	0.50	155	0.16	138.21	1.12
13	K(C-BT3)	150	355	228	90	2.2	0.50	157.5	0.16	138.21	1.14
14	K(C-BT4)	150	355	228	90	0.88	0.20	162.5	0.41	131.23	1.24
15	K(C-BT5)	150	355	228	90	0.88	0.20	121.5	0.30	131.23	0.93
16	K(A-SO3-2)	150	255	228	90	0.88	0.20	131	0.46	94.26	1.39
17	K(A-SO3-3)	150	255	228	90	1.32	0.30	133.5	0.31	96.45	1.38
18	K(A-SO3-4)	150	255	228	90	2.2	0.50	144.5	0.20	99.28	1.46
19	K(A-SO4-2)	150	255	228	90	0.88	0.20	127.5	0.44	94.26	1.35
20	Z(Z4 90)	152.4	228.6	238	45	4.96	1.18	73.65	0.07	62.37	1.18
21	Z(Z4 45)	152.4	228.6	238	90	4.96	1.18	82.77	0.08	63.00	1.31
22	CH(RS90-1)	150	250	150	90	6.67	1.00	87.5	0.06	97.42	0.90
23	CH(RS90-2)	150	250	150	90	6.67	1.00	95	0.07	97.42	0.98
24	CH(RS135-1)	150	250	150	135	4.44	0.67	94	0.25	40.39	2.33
25	CH(RS135-2)	150	250	150	135	4.44	0.67	99.5	0.26	40.39	2.46
26	BC(C2)	152.4	304.8	234.5	90	1.8	0.42	115.40	0.18	105.67	1.09
27	BC(C3)	152.4	304.8	234.5	90	4.46	1.05	126.95	0.08	111.24	1.14
28	BC(C5)	152.4	304.8	234.5	90	1.8	0.42	135.60	0.21	105.67	1.28
29	BC(C6)	152.4	304.8	234.5	45	1.8	0.42	144.27	0.22	104.61	1.38
30	BC(D6)	152.4	304.8	234.5	45	1.8	0.42	138.50	0.22	104.61	1.32
31	AD (B-4)	150	200	230	90	2.23	0.51	58.6	0.11	69.33	0.85
32	AD (B-5)	150	200	230	90	4.45	1.02	60.3	0.06	72.10	0.84
33	AD (B-6)	150	200	230	90	4.45	1.02	80.8	0.08	72.10	1.12
34	AD (B-7)	150	200	230	90	2.23	0.51	68.5	0.13	69.33	0.99
35	AD (B-8)	150	200	230	90	2.23	0.51	85.8	0.17	69.33	1.24

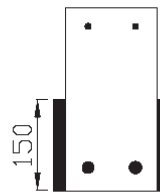
T=Triantafillou[12,14]; K=Khalifa et al [13,18], Z= Zhang et al[17,21]; CH = Challal et al[25]; BC=

Bukhari et al[26], AD = Adhikary et al[2]

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3 **Figure 1: Beam Cross-section, reinforcement details and Anchorage length of CFRP**
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(1) Group A



(2) Group B

Figure 2: Beam Configuration Details

(1) GROUP A

(2) GROUP B

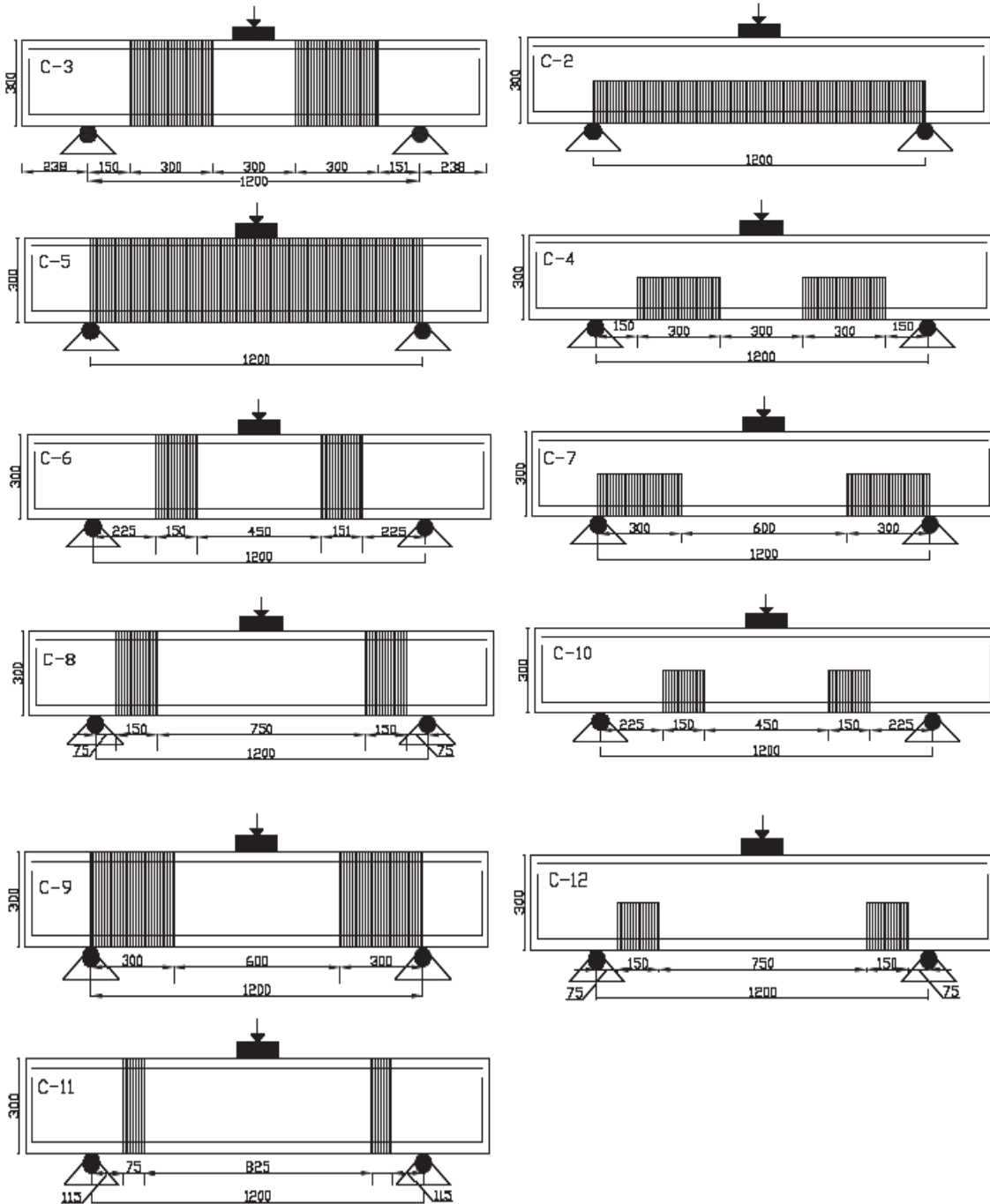
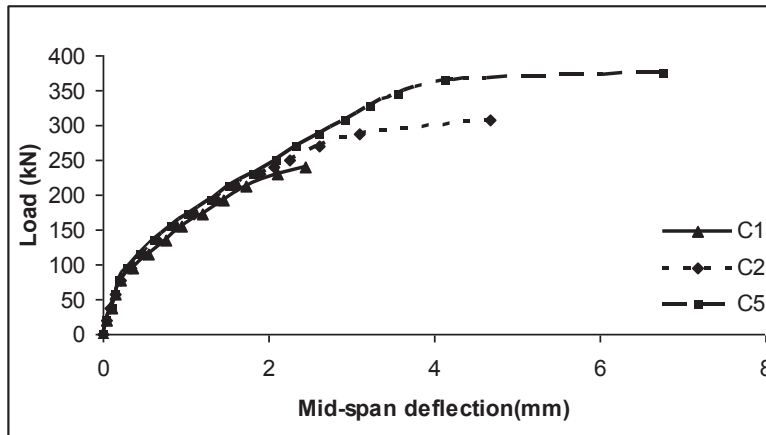


Figure 3: Crack pattern in tested beams

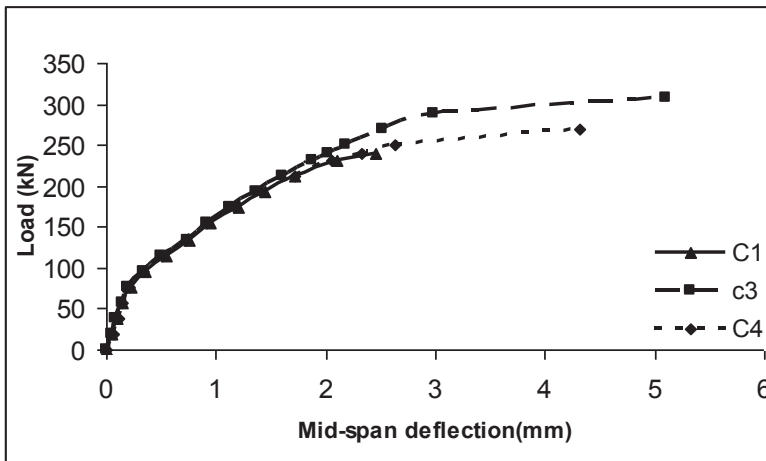
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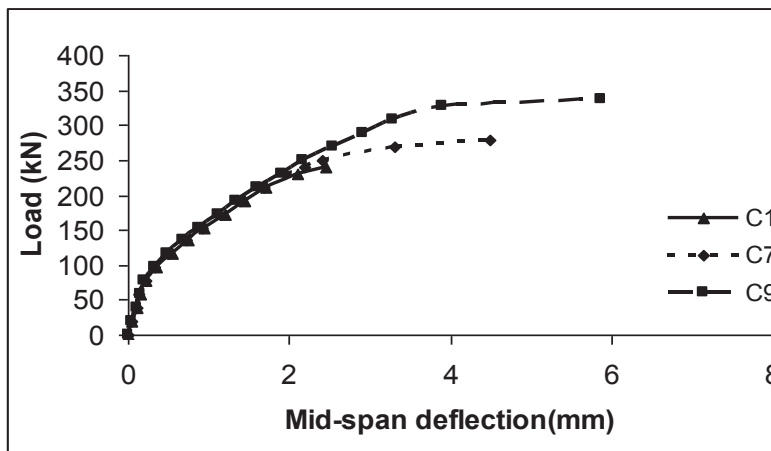
Figure 4: Load Deflection Curves



(a)



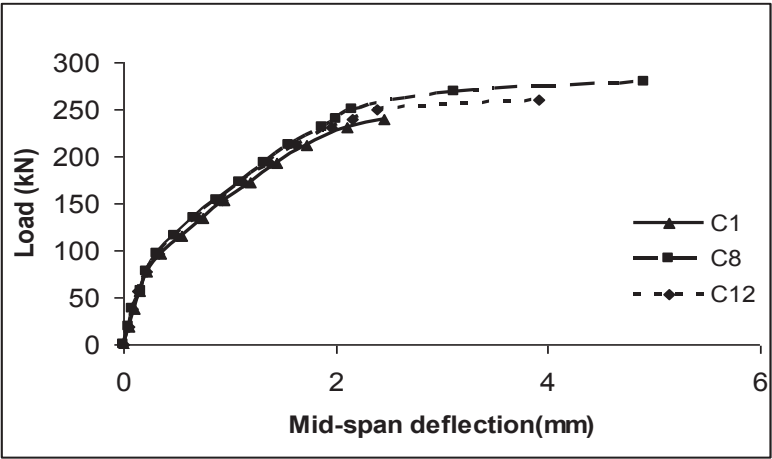
(b)



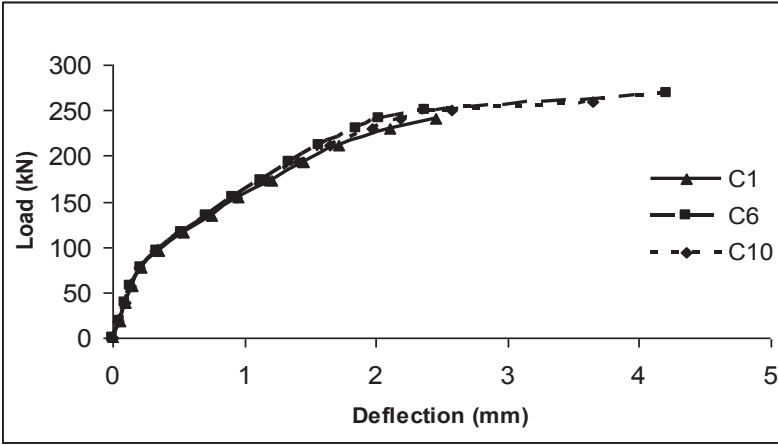
(c)

Figure 4 -contd

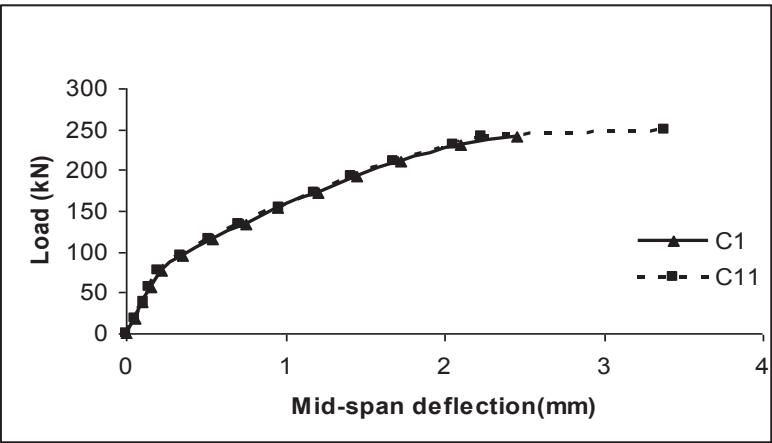
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(d)



(e)



(f)

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Figure 5: Prediction of shear capacity of CFRP sheets

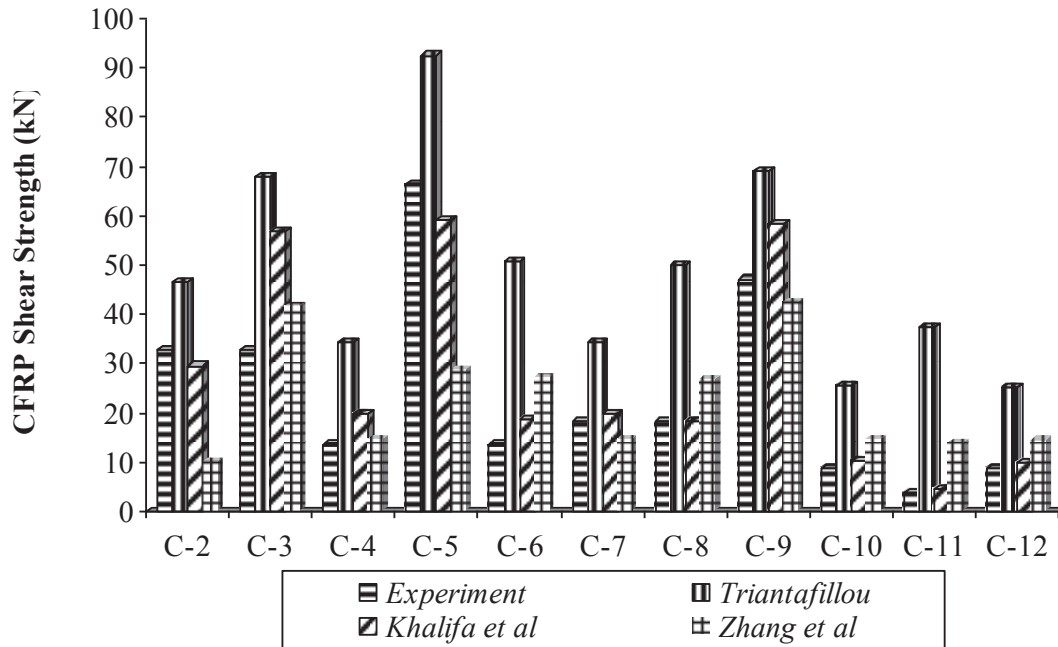


Figure 6: Strut-and-tie model for beams A

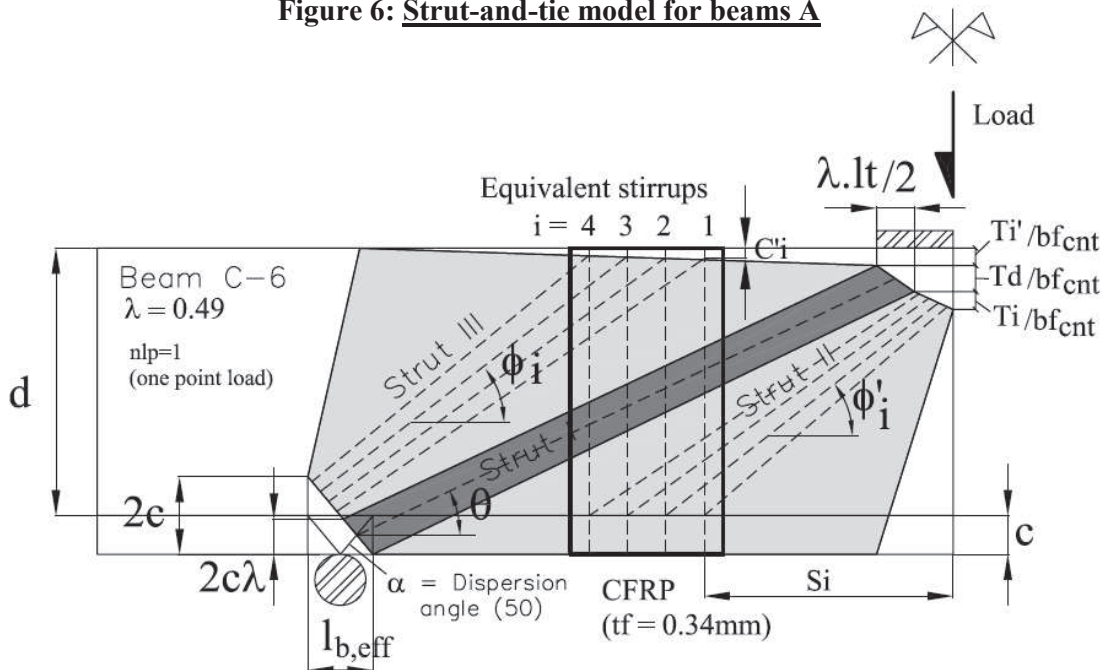


Figure 7: Concrete shear component in beams in Group A

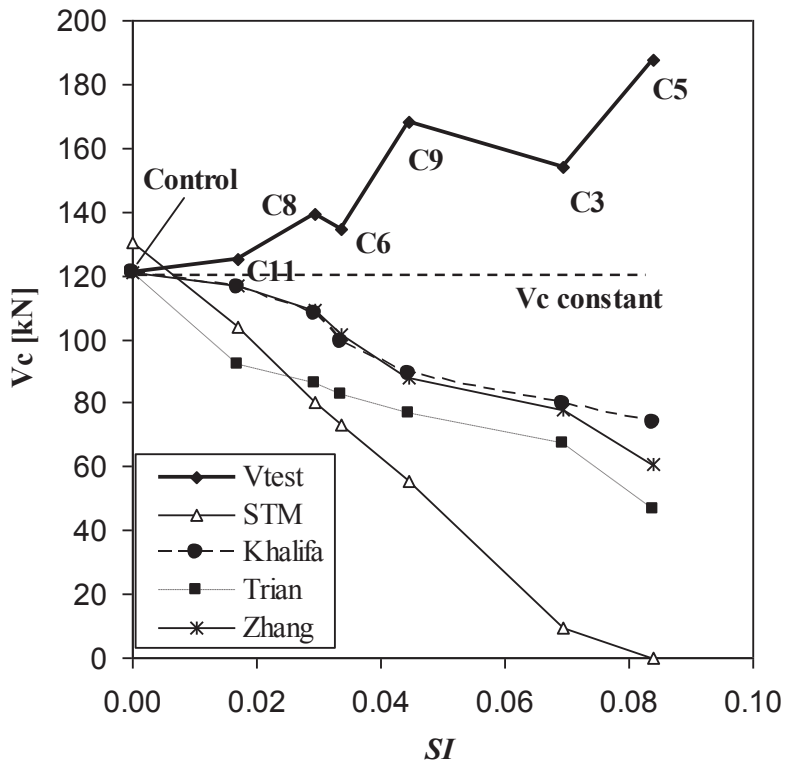


Figure 8: Relationship between R_{EC2} and Axial Rigidity

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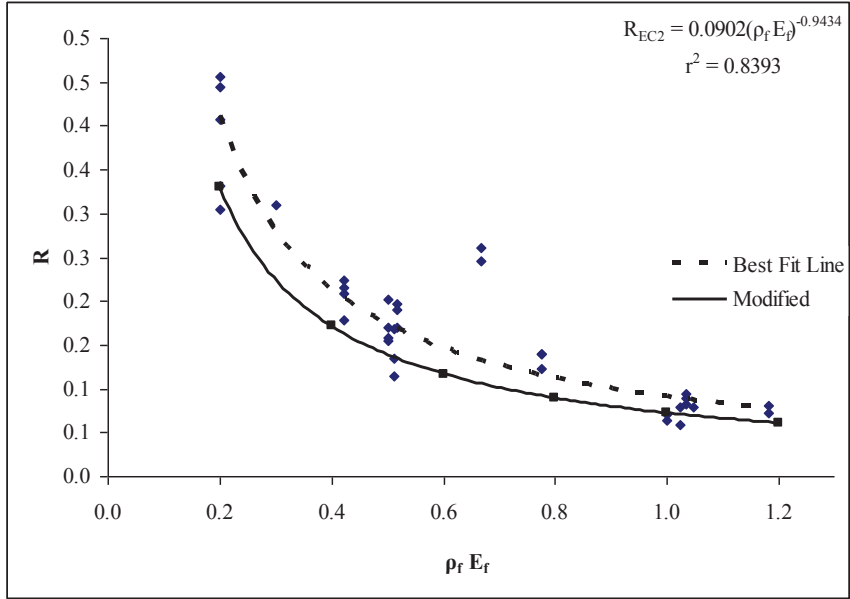


Figure 9: Curve between V_{exp} and V_{EC2} (calculated), (35 tests)

