Calculations of Compound Nucleus Spin-parity Distributions Populated via the (p,t) Reaction in Support of Surrogate Reaction Measurements

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The (p,t) transfer reaction is being studied for its potential use in surrogate reaction analyses. A theoretical model has been developed to predict spin-parity distributions of final states excited in the reaction. The model, after comparisons with experimental data, may provide a predictive capability to identify candidate isotopes for measurement. Preliminary results are presented for the ^{92}Zr(p,t)^{90}Zr reaction at incident proton energy $E_p = 28.5$ MeV. New experimental data for this reaction at a similar energy, and for several other stable Zr isotopes, will soon be available.

I. INTRODUCTION

The surrogate reaction method can determine the cross section for neutron-induced reactions that are not directly accessible through standard experimental techniques. This is achieved by creating the same compound nucleus as for the desired reaction but through a different entrance channel, e.g. populated by a direct transfer reaction.

To date, the surrogate technique has been applied with reasonable success to determine the fission cross section for a number of actinides [1], but has been less successful when applied to other reactions, e.g. (n,γ), due to a spin-parity mismatch [2]. This mismatch, between the distributions of the excited levels of the compound system populated by the desired and the surrogate channels, leads to different decay probabilities and hence reduces the validity and reliability of this surrogate reaction to infer the cross sections in the desired channel.

A better theoretical understanding of the distribution of states populated in the desired and the surrogate channels is required to attempt to address this mismatch quantitatively and allow the method to be utilised with greater confidence.

We discuss the (p,t) two-neutron transfer reaction which allows the technique to be used for isotopes furtherremoved from the line of stability. We outline the theoretical model developed to predict the spin-parity distributions following (p,t) reactions. The first results of calculations are presented for the case of ^{92}Zr(p,t)^{90}Zr for which new experimental data, as well as for other stable Zr isotopes, will soon be available for direct comparisons with predictions.

II. THEORETICAL MODEL

Calculations of cross sections for the direct (p,t) transfer reaction on a mass $A + 2$ target nucleus, populating specific J$^\pi$, A-body final states, involve a number of components, see e.g. Glendenning [3]. The expression used for the cross section is

$$\frac{d\sigma}{d\Omega}(0^+ \to J^\pi) = |\sum_{NLS} G_{NLSJ} B_{NLSJ}(\vec{k}_p, \vec{k}_t)|^2 \ , \quad (1)$$

where B_{NLSJ} is the (p,t) transition amplitude calculated via the distorted wave Born approximation (DWBA) method and G_{NLSJ}, comprised of nuclear *structural* factors, is broadly analogous to the spectroscopic amplitude of single-nucleon transfer reactions. The labels $NLSJ$ refer to the principal, orbital, spin and total angular momentum quantum numbers of the wave function of the pair of transferred neutrons. G is given, more specifically, by the product of terms

$$G_{NLSJ} = \sum_{\gamma} g_\gamma \beta_{\gamma LSJ} \Omega_n \langle n0, NL; L | n_1 l_1, n_2 l_2; L \rangle \ , \quad (2)$$

where $g_\gamma = \sqrt{2/(1 + \delta_{ij})}$ is a symmetry factor dependent on the (like or unlike) pair of orbitals i, j occupied by the two transferred neutrons, $\langle n0, NL; L | n_1 l_1, n_2 l_2; L \rangle$ is a *Moshinsky bracket* [4], $\beta_{\gamma LSJ}$ is the two-particle *parentage coefficient*, and Ω_n reflects the overlap of the two-nucleon relative motion wave functions between the initial and final states. Ω_n is assumed to be unity in the calculations presented here.

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At present we consider only even-even target nuclei and thus transfer from 0^+ ground states. The major assumption made is that the two neutrons are transferred simultaneously, in a single step, as a spin-0^+ *di-neutron* cluster. Thus, for even-even targets, the total and orbital angular momentum transferred are equal, $L = J$, and only *natural parity* final states with $\pi = (-1)^J$ are populated.

III. MODEL CALCULATIONS

A. Structural Factors

For the (p,t) pickup reaction the parentage coefficient, denoted by $\beta_{\gamma LSJ}$, measures the component in the (0^+) mass $A+2$ target nucleus ground state of a specific J^π A-body residual nucleus final state plus two neutrons with quantum numbers $\gamma \equiv ([n_1 l_1 j_1], [n_2 l_2 j_2])$ coupled to $[L, S]J$. Explicit forms relevant to our cases are detailed by e.g. Glendenning [3].

The value of the Moshinsky bracket gives the amplitude (within the harmonic oscillator approximation) for the overlap between the wave functions of the two neutron single-particle orbitals, i.e. γ, in the target nucleus and the 0^+ di-neutron configuration, $[L, S = 0]J$. Here the $S = 0$ and relative s-wave restrictions are dictated, in the one-step approximation, by the $\langle \mathrm{p}|\mathrm{t}\rangle$ structure vertex with the outgoing triton.

B. Energy Levels

The energies of the populated final states and the Q values for the individual transitions required for the DWBA transfer calculations are a necessary input. For these we used the experimental two-neutron separation energy S_{2n} of the target nucleus combined with Hartree-Fock (HF) calculations of the energies of the bound neutron single-particle states. The different degenerate J^π final states arising from the removal of neutrons from each pair of occupied neutron orbitals were split based on the phenomenological expectations, see e.g. Casten [5], from an attractive two-nucleon residual interaction.

The spherical HF [6] was used for these calculations. A variety of different Skyrme mean-field interactions were tested, and the SkX model [7] was used for the current zirconium isotopes study.

C. DWBA Calculations

The Surrey-version of the DWBA transfer code TWOFNR [8] was used for the (p,t) reaction cross sections calculations. These cross sections showed little dependence on the choice of the proton optical model potential (OMP) selected. However, there was more significant sensitivity to the triton OMP used. Thus, calculations were performed using two available global triton OMPs [9, 10] to provide a first assessment of the uncertainty due to this physical input. In the results presented below, the proton OMP of Bechetti and Greenlees [11] was used with the triton OMP of Pang *et al.* [9].

D. Final-state Energy Spreading

There will be physical spreading of the strengths of the final states about the estimated energies (from the HF plus two-neutron residual interaction) of the final states. We take this into account in the present calculations by the introduction of a parameterised spreading width, $\Gamma(E)$, for each state, dependent on its excitation energy above the Fermi energy of the residual nucleus E_F. We assume [12]

$$\Gamma(E) = \frac{\epsilon_0 (E - E_F)^2}{(E - E_F)^2 + E_0^2} + \frac{\epsilon_1 (E - E_F)^2}{(E - E_F)^2 + E_1^2}\ , \quad (3)$$

where $\epsilon_0, \epsilon_1, E_0, E_1$ are chosen constants; taken here from Ref. [13] where they were used for ^{60}Ni .

This approach follows that of Brown and Rho [12]. The spreading of the strength of the transfer yield with excitation energy is distributed with a Breit-Wigner shaped form factor with a FWHM of $\Gamma(E)$. This method has been employed for related analyses of yield distributions following single-nucleon transfer reactions.

IV. RESULTS

A. Example of ^{92}Zr(p,t)^{90}Zr

Preliminary results are presented in Fig. 1, which show the J^π and excitation energy of the predicted excited levels for the (p,t) reaction and an incident proton energy of 28.5 MeV.

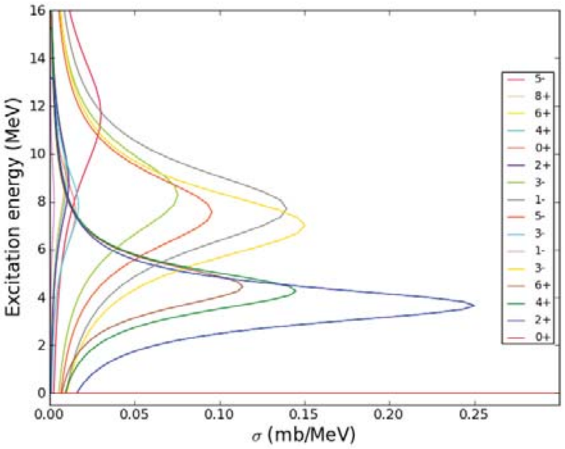

FIG. 1. Calculated excited levels for the ^{92}Zr(p,t)^{90}Zr reaction for an incident proton energy of 28.5 MeV.

The calculated level energies compare reasonably with those reported in the literature [14], and given the number of measured levels which are not assigned a definite J^π value, as is shown in Fig. 2.

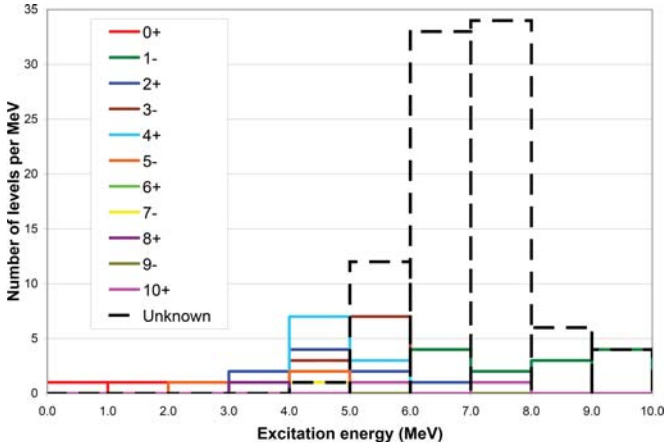

FIG. 2. Densities of previously measured levels of the ^{90}Zr nucleus.

B. Future Comparisons with Measurements

Experiments detect the γ-ray cascade from the decay of the final nucleus following the (p,t) reaction. The angular distributions of the outgoing tritons are also determined, yielding information on the J^π of the states. To enable a full comparison with such experimental data for the Zr isotopes, which are expected to be available soon, the TALYS code [15] will be used to generate the γ-ray cascade based on the predicted (J^π, E^*) populations of the excited compound nuclei calculated via this model.

These calculated γ-ray cascades will be complemented by the angular distributions already generated during the TWOFNR DWBA calculations allowing for even more detailed comparison with the measured data.

V. CONCLUDING COMMENTS

A theoretical model has been developed to calculate J^π distributions of levels excited via the (p,t) transfer reaction. Comparisons with new experimental data for reactions on the stable Zr isotopes will be made in the near future.

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[1] J.T. Burke *et al.*, PHYS. REV. C **73**, 054604 (2006).
[2] N.D. Scielzo *et al.*, PHYS. REV. C **81**, 034608 (2010).
[3] N.K. Glendenning, PHYS. REV. **137**, B102 (1965).
[4] M. Moshinsky, NUCL. PHYS. **A13**, 104 (1959).
[5] R.F. Casten, NUCLEAR STRUCTURE FROM A SIMPLE PERSPECTIVE, OUP, Chapter II.4.
[6] Code DENS, provided as part of OXBASH. B.A. Brown *et al.*, *Oxbash for Windows*, MSU-NSCL Report 1289 (2004).
[7] B.A. Brown, PHYS. REV. C **58**, 220 (1998).
[8] J.A. Tostevin, http:/www.nucleartheory.net/NPG/code.htm.
[9] D.Y. Pang *et al.*, PHYS. REV. C **79**, 024615 (2009).
[10] X. Li *et al.*, NUCL. PHYS. **A789**, 103 (2007).
[11] F.D. Bechetti, G.W. Greenlees, PHYS. REV. **182**, 1190 (1969).
[12] G.E. Brown, M. Rho, NUCL. PHYS. **A372**, 397 (1981).
[13] M. Matoba *et al.*, NUCL. PHYS. **A581**, 21 (1995).
[14] R. Capote *et al.*, NUCL. DATA SHEETS **110**, 3107 (2009).
[15] A.J. Koning *et al.*, PROC. OF ND2007, 211 (2008).