On the use of SEREP for satellite FEM validation

G.S. Aglietti and S.J.I. Walker
School of Engineering Sciences, University of Southampton, Southampton, UK, and
A. Kiley
EADS-Astrium Ltd, Stevenage, UK

Abstract

Purpose – The purpose of this paper is to assess the suitability of various methods for the reduction of a large finite element model (FEM) of satellites to produce models to be used for correlation of the FEM with test results. The robustness of the cross-orthogonality checks (COC) for the correlation process carried out utilizing the reduced model is investigated, showing its dependence on the number of mode shapes used in the reduction process. Finally the paper investigates the improvement in the robustness of the COC that can be achieved utilizing optimality criteria for the selection of the degrees of freedom (DOF) used for the correlation process.

Design/methodology/approach – A Monte Carlo approach has been used to simulate inaccuracies in the mode shapes (analysis and experimental) of a satellite FEM that are compared during the COC. The sensitivity of the COC to the parameters utilized during the reduction process, i.e. mode shapes and DOFs, is then assessed for different levels of inaccuracy in the mode shapes.

Findings – The System Equivalent Expansion Reduction Process (SEREP) has been identified as a particularly suitable method, with the advantage that a SEREP reduced model has the same eigenvalues and eigenvector of the whole system therefore automatically meeting the criteria on the quality of the reduced model. The inclusion of a high number of mode shapes in the reduction process makes the check very sensitive to minor experimental or modelling inaccuracies. Finally it was shown that utilizing optimality criteria in the selection of the DOFs to carry out the correlation can significantly improve the probability of meeting the COC criteria.

Research limitations/implications – This work is based on the FEM of the satellite *Aeolus*, and therefore the numerical values obtained in this study are specific for this application. However, this model represents a typical satellite FEM and therefore the trends identified in this work are expected to be generally valid for this type of structure.

Practical implications – The correlation of satellite FEM with test results involves a substantial effort, and it is crucial to avoid failures of the COC due to numerical issues rather than real model inaccuracies. This work shows also how an inappropriate choice of reduction parameters can lead to failure of the COC in cases when there are only very minor differences (e.g. due to minor amount of noise in the results) between analytical and test results. Vice versa, the work also shows how the robustness of the reduced model can be improved.

Originality/value – The paper shows how the robustness of the correlation process for a satellite FEM carried out utilising a SEREP reduced model needed to be investigated, to demonstrate the suitability of this method to reduce large FEM of satellites.

Keywords Artificial satellites, Finite element analysis, Modelling, Finite element model, System equivalent expansion reduction process, SEREP, FEM reduction, FEM correlation, Validation

Paper type Research paper

1. Introduction

The validation of finite element models (FEM) is particularly important in the space industry (Friswell and Mottershead, 1995; Girard and Roy, 2008) because the loads that will be experienced by a satellite during launch can only be accurately predicted by performing a coupled analysis, where the FEM of the satellite is joined with that of the launch vehicle (LV). For practical reasons, mostly related to the size of the LV, the complete system cannot be dynamically tested on the ground to verify its dynamic behaviour. Hence it is necessary to have accurate FEM of the subsystems (i.e. satellite and LV) that can then be integrated to obtain a reliable estimate of the loads exchanged at their interface.

The issue of the accuracy of FEMs has been tackled by several researchers in academia and in industry (Avitabile, 2006; Hasselman *et al.*, 2000) but limiting our attention to linear dynamic systems and classical modal analysis, there are relatively few criteria that are used to quantify the quality of the correlation between a FEM and the physical structure/system that this represents (Randall, 2003).

In the space industry, the quality of the correlation is mainly assessed by matching the FEM eigenvalues and eigenvectors with experimentally retrieved natural frequencies and mode shapes (ECSS-E-ST-32-11C, 2008).

According to the European Space Agency's requirements, the natural frequencies of the main bending modes of the spacecraft from FEM predictions are required to match experimental values within 3 percent, whereas for the other modes (generally up to 100 Hz) the requirement can be relaxed to 5 percent and then down to 10 percent according to the importance of the modes (e.g. their effective mass, or impact on relevant responses).

Although there are criteria to compare the frequency response functions (FRF) in the whole frequency range (e.g. frequency response assurance criteria (FRAC) or response vector assurance criteria (RVAC)), and that could be used to quantify the quality of match between FRF predicted using the FEM with those retrieved during the physical tests, these criteria are not frequently used in the space industry.

Comparing resonance frequencies (or FRF) is only meaningful if the associated mode shapes are the same, hence matching mode shapes is a crucial issue.

The quality of the match can be expressed in terms of the modal assurance criteria (MAC) (Allemang, 2003) or performing cross orthogonality checks (COC) (ECSS-E-ST-32-11C, 2008) between the analysis mode shapes (i.e. FEM eigenvectors) and experimental mode shapes with respect to the FEM mass matrix. (Please note that some authors call COC the test that checks the orthogonality of the experimental mode shapes and they call pseudo orthogonality check (POC) the check that "compares" the orthogonality of experimental vs computational mode shapes).

As there will always be some differences between computational model predictions and experimental data, the criteria specify some thresholds for the relevant parameters. If MAC is used, it is common practice to consider that $MAC > 0.9$ corresponds to a good match between vectors ($MAC = 1$ is obtained when comparing two identical vectors). However, a higher or lower threshold can be applied according to the importance of the modes. An important issue concerning the use of MAC is that the degrees of freedom (DOF) are not weighted to reflect different importance, as in reality some DOFs (typically the ones associated with higher inertia) are more significant in determining the dynamic behaviour of the structure and its loads during launch. In the COC, the mass matrix is used in the mathematical expression to compare the two set of mode shapes

(FEM eigenvectors and experimental mode shapes) and this has the effect of adding a mass weighting to the DOFs, so that the result is more influenced by the most significant DOFs. If the two sets of mode shape that are compared were exactly the same, as the mode shapes are orthogonal to the mass matrix, the result of the COC would be a diagonal matrix (that when the modes are mass normalised is equal to the identity matrix). However, as the two sets will have some differences the quality threshold associated with the COC is that the elements on the diagonal of the matrix should be > 0.9 and those off-diagonal < 0.1 (this assumes mass normalised mode shapes).

Typically the FEM of a satellite will have several hundred thousand DOFs and therefore this will be the number of elements in its eigenvectors; however experimental tests rarely capture the acceleration of more than a few hundred locations (which will correspond to some specific DOFs of the FEM) and therefore the experimental mode shapes will only have a maximum of few hundred elements. The MAC or COC algorithms require the vectors (or matrices) to be compared to have the same size and therefore it is necessary either to expand the experimental results or to reduce the analytical ones (Avitabile, 2005). Modal expansion or reduction can be carried out using the FEM and if the same FEM is used to reduce or to expand mode shapes, then the test at reduced or expanded level would give the same results. However, as often the initial FEM does not meet the specified accuracy criteria (in terms of matching the physical test results) and needs to be updated/modified to improve correlation with the experimental results (Friswell and Mottershead, 1995), its use to expand experimental mode shapes would lead to corrupted expanded mode shapes, unsuitable to be used as targets to modify the FEM. Hence it is preferable to keep the experimental mode shapes to their size and reduce the analysis (FEM) mode shapes. This can be accomplished by simply eliminating from the analytical eigenvectors all the elements corresponding to DOFs that were not monitored during the tests. A potential issue here is that even a MAC close to one only confirms that there is a very good match between the monitored DOFs, but it is possible that parts of the model and structure which were not monitored behaved quite differently.

To select the DOFs to be monitored during a structures modal survey test, or the dual problem of selecting the DOFs that have to be preserved in the FEM reduction process it is ultimately necessary to decide which DOFs are "most representative" of the behaviour of the structure. There are some reasonably established general criteria that can be followed (Van Langenhove and Brughmans, 2010; Papadimitriou, 2004; Kammer and Tinker, 2004): the DOFs should meet some observability criteria and allow the modes to be distinguished, they should be associated with large masses, etc. however optimality criteria on DOFs selection also depend on the method that has been used to carry out the reduction.

There are various procedures available to reduce the size of a FEM. The selection of the most suitable procedure to obtain the reduced FEM (usually called test analysis model (TAM)) is usually based on:

(1) practical computational consideration and company heritage;
(2) the quality with which the TAM can actually reproduce the full model behaviour; and
(3) finally the robustness of the TAM for the purpose of carrying out COC.

In fact it has been demonstrated that certain algorithms can produce reduced models which are so sensitive to noise that even minor differences in the mode shapes (analysis vs experiment) may prevent the model from meeting the COC specification (Bergman *et al.*, 2010). In this case the following phase of FEM updating becomes a worthless exercise, simply correcting the model to produce mode shapes that match the experimental results distorted by their "noise".

In this article it is shown that, compared to other methods (e.g. Guyan reduction, dynamic reduction, improved reduction system (IRS), Craig Bampton) whose suitability for FEM reduction have been investigated by various authors (Koutsovasilis and Beitelschmidt, 2008 or Bergman *et al.*, 2010), the system equivalent expansion reduction process (SEREP) is a method that is particularly suitable for spacecraft FEMs. The issue of the robustness of the TAM is then investigated in detail to verify its sensitivity to the relevant parameters (e.g. number and location of the accelerometers, number of modes preserved in the TAM) and identify ranges of these parameters which deliver appropriate robustness for the correlation of typical spacecraft FEM.

Finally the use of an optimality criterion for accelerometer location (e.g. through the use of the Effective Independence (EfI) matrix) is investigated as a vehicle to improve robustness of the COC.

2. Model reduction techniques

The standard equation that governs the dynamics of a system and that is implemented using the FEM is:

$$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{K}\mathbf{u} = \mathbf{f} \tag{1}$$

where $\mathbf{M}$ and $\mathbf{K}$ are the mass and stiffness matrices, respectively, $\mathbf{u}$ and $\mathbf{f}$ are the vectors of the physical displacements and applied forces, and where, for simplicity, the damping has been neglected. There are various methods that can be employed to reduce the size of a FEM and that can be used to produce the TAM mass matrix.

2.1 Static reduction

Static reduction, also known as Guyan reduction, is the oldest method, and here the full FEM in equation (1) is partitioned as illustrated in equation (2):

$$\begin{bmatrix} \mathbf{M_{MM}} & \mathbf{M_{MS}} \\ \mathbf{M_{SM}} & \mathbf{M_{SS}} \end{bmatrix} \cdot \begin{Bmatrix} \ddot{\mathbf{u}}_\mathbf{M} \\ \ddot{\mathbf{u}}_\mathbf{S} \end{Bmatrix} + \begin{bmatrix} \mathbf{K_{MM}} & \mathbf{K_{MS}} \\ \mathbf{K_{SM}} & \mathbf{K_{SS}} \end{bmatrix} \cdot \begin{Bmatrix} \mathbf{u_M} \\ \mathbf{u_S} \end{Bmatrix} = \begin{Bmatrix} \mathbf{f_M} \\ \mathbf{0} \end{Bmatrix} \tag{2}$$

where the subscript S denotes DOFs that can be eliminated (slaves) whereas the subscript M denotes DOFs that have to be retained (masters), e.g. for the process of correlation with test results. If the inertia associated with the slave DOFs is small, then the associated partitions of the mass matrix can be neglected and the second line of equation (2) can be written as:

$$\mathbf{K_{SM}}\mathbf{u_M} + \mathbf{K_{SS}}\mathbf{u_S} = 0 \tag{3}$$

Equation (3) can be utilized to obtain the slave displacements as a function of the master displacements and then substitute back in equation (2) as a transformation matrix to eliminate the slave DOFs from the equation:

$$\begin{bmatrix} \mathbf{M_{MM}} & \mathbf{M_{MS}} \\ \mathbf{M_{SM}} & \mathbf{M_{SS}} \end{bmatrix} \begin{bmatrix} \mathbf{I} \\ -\mathbf{K_{SS}^{-1}K_{SM}} \end{bmatrix} \cdot \{\ddot{\mathbf{u}}_\mathbf{M}\} + \begin{bmatrix} \mathbf{K_{MM}} & \mathbf{K_{MS}} \\ \mathbf{K_{SM}} & \mathbf{K_{SS}} \end{bmatrix} \begin{bmatrix} \mathbf{I} \\ -\mathbf{K_{SS}^{-1}K_{SM}} \end{bmatrix} \cdot \{\mathbf{u_M}\} = \{\mathbf{F_M}\} \quad (4)$$

Now, the TAM mass matrix, to be used in the COC can be obtained pre-multiplying equation (4) by the transpose of the transformation matrix:

$$\begin{bmatrix} \mathbf{I} \\ -\mathbf{K_{SS}^{-1}K_{SM}} \end{bmatrix}^\mathbf{T} \begin{bmatrix} \mathbf{M_{MM}} & \mathbf{M_{MS}} \\ \mathbf{M_{SM}} & \mathbf{M_{SS}} \end{bmatrix} \begin{bmatrix} \mathbf{I} \\ -\mathbf{K_{SS}^{-1}K_{SM}} \end{bmatrix} = \mathbf{M}_{TAM\_Stat} \quad (5)$$

Concerning the "quality" of the reduced model with respect to the full FEM, it must be highlighted that equation (4) will give the exact static solutions, however as the effect of the inertia associated with the slave DOFs has been lost, natural frequencies and mode shapes will be different from those of the original FEM. The magnitude of the differences depends on how much inertia has been neglected, and will generally increase with the frequency of the modes. Therefore, even if the original FEM was a perfect representation of the physical system (with exact match of natural frequencies and mode shapes), the static reduction of the model introduces errors that can easily lead to non-compliance with the COC requirements.

2.2 Dynamic reduction

The effect of the inertia of the slave DOFs, which was completely ignored in the static reduction, is considered here but its representation is only exact at a particular reference frequency. At this specific frequency, assuming harmonic response at an angular frequency ω, the second line of equation (2) can be written as:

$$(-\boldsymbol{\omega}^2\mathbf{M_{SM}} + \mathbf{K_{SM}})\mathbf{u_M} + (-\boldsymbol{\omega}^2\mathbf{M_{SS}} + \mathbf{K_{SS}})\mathbf{u_S} = 0 \quad (6)$$

and from equation (6) it is possible to calculate the slave displacements as functions of the masters and hence obtain the transformation matrix:

$$\begin{Bmatrix} \mathbf{u_M} \\ \mathbf{u_S} \end{Bmatrix} = \begin{bmatrix} \mathbf{I} \\ -(\mathbf{K_{SS}} - \boldsymbol{\omega}^2\mathbf{M_{SS}})^{-1}(\mathbf{K_{SM}} - \boldsymbol{\omega}^2\mathbf{M_{SM}}) \end{bmatrix} \cdot \{\mathbf{u_M}\} \quad (7)$$

Similarly, as shown in the static reduction, equation (7) can be substituted into equation (2) and then this can be pre-multiplied by the transformation matrix to obtain the mass TAM matrix in a format similar to that in equation (5).

This reduction is exact at the reference frequency, and good accuracy is still preserved in its neighbourhood, but natural frequencies and mode shapes of the reduced model decrease in accuracy as we move away from the reference frequency.

2.3 Improved reduction system methods

The standard IRS (O'Callahan, 1989) method, like the dynamic reduction, improves a static reduction including extra terms that consider the inertia neglected in the static reduction. However, it has been demonstrated that the mass matrix produced by the basic version of the IRS is less suitable for use in COC than a mass matrix obtained with a static reduction. Friswell *et al.* (1995) proposed an iterated IRS technique which improves the standard IRS method utilizing a dynamically reduced model to obtain the basic transformation that is then iterated to converge to the eigenvalues and eigenvectors of the full model. Friswell *et al.* (1995) also show that this method converges to the results obtained by SEREP transformation (if the modes are observable using the selected master DOFs).

2.4 Craig-Bampton

The Craig-Bampton (CB) method (Craig and Bampton, 1968) has been used for years and it is still the main method used to reduce the size of the FEM of a satellite in order to allow integration with the LV model and maintain the number of DOFs of the coupled model within a manageable size. This is particularly useful during long transient analyses (in the time domain) with a small time step (to capture "high frequency" behaviour), such as a typical coupled loads analysis (CLA).

The CB reduction utilizes the concept of mode superposition, where the dynamics of a system with a large number of DOFs (and related physical displacements contained in the vector $\mathbf{u}$) can be described using a combination of a relatively small number of mode shapes (contained in the modal matrix $\mathbf{\Phi}$) multiplied by appropriate modal coordinates (contained in the vector $\boldsymbol{\eta}$):

$$\mathbf{u} = \mathbf{\Phi}\boldsymbol{\eta} \tag{8}$$

This transformation can be substituted in equation (1), which can then be pre-multiplied by the transpose of the modal matrix to obtain a much smaller set of equations which can still capture the dynamic behaviour of the system.

In practice, in the space sector, as the model of the satellite has to be connected with that of the LV, the interface DOFs are preserved and the transformation in equation (8) typically takes the form:

$$\begin{Bmatrix} \mathbf{u_R} \\ \mathbf{u_L} \end{Bmatrix} = \begin{bmatrix} \mathbf{I} & \mathbf{0} \\ \boldsymbol{\phi}_\mathbf{R} & \boldsymbol{\phi}_\mathbf{L} \end{bmatrix} \cdot \begin{Bmatrix} \mathbf{u_R} \\ \boldsymbol{\eta}_\mathbf{m} \end{Bmatrix} \tag{9}$$

where $\mathbf{u}_\mathbf{R}$ contains the displacements of the interface $\boldsymbol{\eta}_\mathbf{m}$ are the modal displacement $\boldsymbol{\phi}_\mathbf{R}$ contains interface static mode shapes (i.e. the static shapes obtained for unitary displacements of the interface DOFs) and $\boldsymbol{\phi}_\mathbf{L}$ are the elastic mode shapes of the structure (with the interface nodes constrained).

The main issue here in terms of verification of the C-B reduced model against test results, is that the model resulting from this reduction uses a mixture of physical ($\mathbf{u}_\mathbf{R}$) and generalized (modal) displacements ($\boldsymbol{\eta}_\mathbf{m}$) that cannot be immediately compared with physical displacements (e.g. at the DOFs monitored with accelerometers during a modal survey test). In practice to obtain physical quantities like node displacements or accelerations, or stresses in the elements, the modal displacements output of the simulations have to be expanded using appropriate output transformation matrices (OTM). A modal reduction (i.e. CB) followed by an expansion using appropriate OTM is what is in practice carried out using the SEREP which is discussed in the next section.

2.5 System equivalent expansion reduction process

SEREP consists essentially in building a transformation matrix obtained from a modal reduction followed by an expansion. Equation (8) can be partitioned to separate DOFs that have to be retained (masters) for modal correlation purposes, from those that will be “eliminated” (slaves). Eliminating the rows corresponding to the slave DOFs, equation (8) can be rewritten as:

$$\mathbf{u}_M = \mathbf{\Phi}_M \boldsymbol{\eta} \tag{10}$$

where $\mathbf{\Phi}_{\mathrm{M}}$ is a reduced modal matrix which contains only the rows corresponding to the master DOFs. At this point, if the number of modal coordinates “m” is equal to the number of master DOFs “n”, then the reduced modal matrix $\mathbf{\Phi}_{MM}$ is square and equation (10) can be inverted to obtain:

$$\boldsymbol{\eta} = \mathbf{\Phi}_M^{-1} \mathbf{u}_M. \tag{11}$$

This can then be substituted into equation (8) to obtain all the displacements $\mathbf{u}$ as a functions of the displacements of the master DOFs $\mathbf{u}_{\mathrm{M}}$. This procedure is commonly known as SEREPa.

However, as the most common case is that only a relatively few number of modes are important to determine the dynamic behaviour of the system (i.e. the reduced modal matrix will only have a few columns) there will be more master DOFs then modal coordinates, and therefore the inversion of equation (11) has to be performed using the pseudo inverse:

$$\boldsymbol{\eta} = \left(\mathbf{\Phi}_M^T \mathbf{\Phi}_M\right)^{-1} \mathbf{\Phi}_M^T \mathbf{u}_M = \mathbf{\Phi}_M^P \mathbf{u}_M \tag{12}$$

where the superscript P indicates the pseudo inverse of the reduced modal matrix. Therefore, in the most general case the reduction of the initial FEM can be carried out substituting:

$$\mathbf{u} = \mathbf{\Phi} \;\; \mathbf{\Phi}_M^P \mathbf{u}_M \tag{13}$$

In equation (1) and pre-multiplying the equation by the transpose of the modal matrix and by the transpose of the pseudo inverse of the modal matrix to obtain:

$$\mathbf{\Phi}_M^{P^T} \mathbf{\Phi}^T \mathbf{M} \mathbf{\Phi} \;\; \mathbf{\Phi}_M^P \ddot{\mathbf{u}}_M + \mathbf{\Phi}_M^{P^T} \mathbf{\Phi}^T \mathbf{K} \mathbf{\Phi} \;\; \mathbf{\Phi}_M^P \mathbf{u}_M = \mathbf{\Phi}_M^{P^T} \mathbf{\Phi}^T \mathbf{f} \tag{14}$$

Now, it must be noted that if the modes were mass normalised, which in most software is standard practice, then:

$$\mathbf{\Phi}^T \mathbf{M} \mathbf{\Phi} = \mathbf{I} \tag{15}$$

and therefore the reduced mass matrix in equation (14), which is the TAM mass matrix will be:

$$\mathbf{M}_{TAM\_SEREP} = \mathbf{\Phi}_M^{P^T} \mathbf{\Phi}_M^P. \tag{16}$$

As it can be seen from equations (16) and (12) the $\mathbf{M}_{\mathrm{TAM\_SEREP}}$ can be simply assembled from the mode shapes, without requiring manipulation of the original FEM mass matrix. A very similar procedure can be applied to obtain the reduced stiffness matrix, however to do so it is also necessary to have the eigenvalues (or the natural frequencies) of the system.

2.6 Application to satellite FEM reduction

In comparison with classical static and dynamic reductions a SEREP reduced model will exactly reproduce the natural frequencies and mode shape of the original FEM. This is an important advantage as most launch agencies specify a maximum value for the discrepancy (for example in terms of natural frequencies percentage difference) between reduced models and complete FEM predictions. A statically or dynamically reduced model might not meet this requirement, whereas a SEREP model will, by definition, exactly reproduce natural frequencies and mode shapes that were used during the reduction process.

In comparison with the IRS, from a computational and practical perspective SEREP is much easier to implement. In addition, as the first part of the SEREP procedure is a modal reduction like the CB, and because CB will most likely be carried out anyway to produce the model for the CLA, the use of the SEREP procedure seems very appropriate.

3. Sensitivity of the COC to SEREP parameters

As mentioned in the introduction, the purpose of the COC is to measure the level of correlation between the set of mode shapes calculated using the FEM and mode shapes determined experimentally, for example from a modal survey test. NASA and ESA state that the FEM is sufficiently accurate if the correlation meets the quality criteria expressed in equation (17):

$$\mathbf{\Phi}_{Exp}^{T}\mathbf{M}_{FEM}\mathbf{\Phi}_{FEM} = \mathbf{XOR}\begin{cases} XOR_{i,j} > 0.9 & \forall i = j \\ XOR_{i,j} < 0.1 & \forall i \neq j \end{cases} \tag{17}$$

As the relative importance of the various DOFs is weighted using the mass matrix ($\mathbf{M}_{\mathrm{TAM}}$), the results are likely to be influenced by the specifics of the procedure that has been implemented to produce this matrix.

As SEREP has been identified as the most efficient procedure to carry out the reduction, this is the method selected for further investigations.

The two most important factors in the implementation of this method are the modes considered in the reduction (first step of the SEREP process) and the DOFs that will be preserved (masters) in the reduced modal matrix and in $\mathbf{\Phi}_{\mathrm{FEM}}$. The purpose here is therefore to investigate the sensitivity of the orthogonality check to the number of modes considered in the reduction and the logic used to select the DOFs to be included in the reduced mode shape vectors.

It is recognized that both set of mode shapes are likely to be affected by "errors" (i.e. differences between the analytical or experimental mode shape and the true mode shape of the physical structure) that can be attributed to various phenomena, and difficult to quantify exactly. For example, for the experimental mode shapes, typical sources of errors can be: inaccuracies in the accelerometers sensitivities, non-ideal boundary conditions, issues related with the method/algorithm used to extract the mode shapes from the experimental data, etc. For the analytical mode shapes, approximations of the geometry and properties of the structure and possibly minor errors/inaccuracies in production of the FEM are the most likely causes of errors.

In order to assess the sensitivity of SEREP to these errors it is necessary to know and control the magnitude of such errors/inaccuracies. Therefore, for this investigation it is more appropriate to use some synthetic experimental mode shapes as "experimental" mode shapes. These are obtained by distorting the analytical (FEM) shapes using an appropriately controlled level of errors (or noise) that will be described in the next section. This method also allows the production of a large number of test sets with the same level of "inaccuracy" to carry out Monte Carlo simulations. This procedure can then be used to assess the statistical probability of meeting COC criteria as function of the parameters utilized to carry out the SEREP reduction (selected mode shapes and DOFs) for particular levels of distortion of the mode shapes (i.e. difference between test mode shapes and the "true" mode shape).

Along with the synthetic experimental mode shapes, the analysis (FEM) modes will also be distorted with known levels of errors to simulate typical inaccuracies, and these inaccuracies of the FEM will also have effects on the $\mathbf{M}_{TAM}$ (as this is produced from the analytical mode shapes (equation (16)).

The procedure followed in this study is to consider the likelihood of meeting the check in equation (17) for different levels of distortion of the mode shapes and in three different situations:

(1) exact $\mathbf{\Phi}_{FEM}$ and distortions of $\mathbf{\Phi}_{Exp}$, to simulate experimental errors;
(2) distortions of $\mathbf{\Phi}_{FEM}$ and exact $\mathbf{\Phi}_{Exp}$, to simulate modelling errors; and
(3) distortions of both $\mathbf{\Phi}_{FEM}$ and $\mathbf{\Phi}_{Exp}$, to simulate simultaneous errors in modelling and experimental results simultaneously.

In the latter situation the same level of errors is used in both sets of mode shapes. The whole process is then repeated using different numbers of mode shapes in the reduction process.

3.1 Modelling of the mode shapes inaccuracies

The distortion of the mode shapes is produced using random coefficients uniformly distributed in the neighbourhood of 1 (i.e. open interval $1 - \delta, 1 + \delta$ where δ represent the level of distortion) and used to multiply the elements of the vectors representing the modes shapes.

The size of the intervals considered ranged from $\delta = 0.5$ (i.e. a level of distortion of ± 50 percent) to $\delta = 0.00001$ (i.e. level of distortion ± 0.001 percent) and for each interval size (level of distortion) 1,000 sets of mode shapes were generated.

Two cases of distortion have been tested, uncorrelated noise (where different, uncorrelated coefficients where used to multiply the elements of the vectors – this to simulate the most general case) and correlated noise (where the same "noise" coefficients where used across the mode shapes – e.g. to simulate an error in an accelerometer sensitivity which affects the same elements across the set of mode shapes).

Both types of noise gave the very similar results in the COC described in the next section, and therefore in this paper the most general case (uncorrelated noise) is used.

3.2 Model for sensitivity study

To ensure that the findings of this study are applicable to real spacecraft FEM developed in industry, this work has been performed using "real" data from the FEM of the Aeolus satellite (Figure 1), developed by Astrium UK.

The frequency range of interest for the satellite modal correlation is generally up to 100 Hz although modes up to say 120-140 Hz should be considered, as their effect could easily extend down to 100 Hz and just below. In the range up to 100 Hz the modal analysis of the Aeolus FEM carried out with NASTRAN showed 142 modes, with a further 127 modes in the range between 100 and 140 Hz. This situation is fairly typical, and it would be an incredibly daunting task to try to correlate all the modes, also because a typical base driven modal survey would only be able to identify at best some tens of modes. There are various criteria to select which modes to correlate (target modes), and in

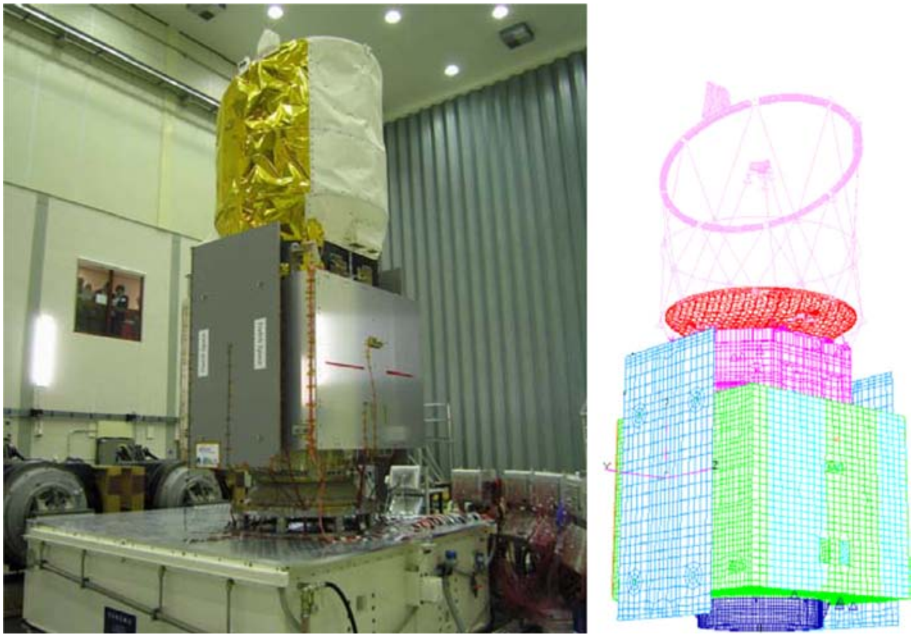

Figure 1.
Aeolus structural model during the lateral sine test at ESA-ESTEC (Holland) and FEM

the space sector the relevant criteria is the effective mass percentage of the total mass associated with the mode. The modes with the highest effective masses are those that will generally produce the highest loads at the interface with the LV and therefore they are very important. However, from the satellite manufacturer's perspective, all the modes that contribute significantly to responses where the margins of safety are relatively low should be correlated. A more detailed discussion on the selection of target modes can be found for example in Chung (1999). Figure 2 shows the main modes along the three directions X, Y and Z and their percentage fraction of the total mass of the model, and in addition another seven modes have been selected on the basis of relatively high effective mass or because they were the main contributor to the response of a relevant subsystem. In the context of this study ten target modes to correlate is a reasonable number, although sometime the correlation exercise can cover up to say 15-20 modes.

Hundreds of accelerometers are generally used to monitor a modal survey test, but for various reasons not all the accelerometers are used for the correlation. To correlate ten modes it was decided to use 99 DOFs that correspond to accelerometers that were placed according some sound engineering judgment.

3.3 SEREP sensitivity to the number of mode shapes

The first task undertaken was the assessment of the effect of number of modes utilized in the SEREP reduction process.

The two extreme situations are:

(1) only use the target modes and carry out the transformation using the pseudo inverse in equation (12); and

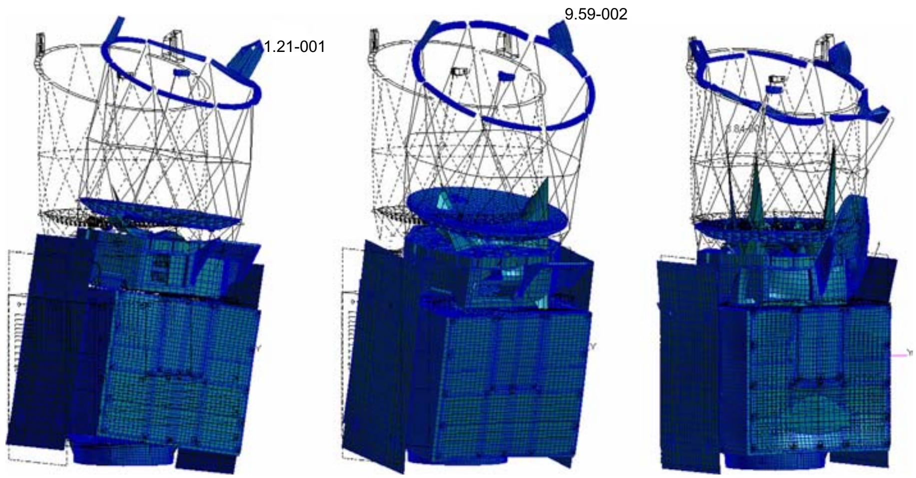

Figure 2.
Modes with highest effective masses along X, Y and Z

Notes: First mode 15.96 Hz, 42 percent effective mass X-direction, second mode 16.62 Hz, 45 percent effective mass Y-direction, 56th mode 64.36 Hz, 22 percent effective mass Z-direction

(2) extend the set of modes choosing another 89 modes in addition to the ten target modes in order to obtain a square reduced modal matrix that can be simply inverted as shown in equation (11) (as its determinant should be different from zero if all the modes are different from each other); this approach is called SEREPa.

Between these two extremes it is indeed possible to choose any number of shapes and here the cases of 20 and 50 mode shapes have been investigated.

Table I shows the results of the Monte Carlo simulations first for the case of exact FEM mode shapes to be compared with experimental mode shape affected by different levels of errors $\delta_{i,j}$, and Figure 3 shows two examples of the XOR matrix for a correlation that met the criteria and one that did not:

$$\mathbf{\Phi}^{T}_{Exp}(\delta_j)\mathbf{M}_{TAM}\mathbf{\Phi}_{FEM} = \mathbf{XOR} \tag{18}$$

then considering errors in the analytical mode shapes to be compared with an exact set of experimental mode shapes:

$$\mathbf{\Phi}^{T}_{Exp}\mathbf{M}_{TAM}(\delta_I)\mathbf{\Phi}_{FEM}(\delta_i) = \mathbf{XOR} \tag{19}$$

and finally the inaccuracies of analytical mode shapes and experimental mode shapes were considered simultaneously:

$$\mathbf{\Phi}^{T}_{Exp}(\delta_j)\mathbf{M}_{TAM}(\delta_i)\mathbf{\Phi}_{FEM}(\delta_i) = \mathbf{XOR} \tag{20}$$

It has to be noticed that:

- When the distortions δ are applied to the FEM mode shapes (equations (19) and (20)), these also naturally propagated in $\mathbf{M}_{TAM}$ as this is assembled from the FEM mode shapes which are affected by distortions (equation (16)).

Number of mode shapes used in SEREP	50%	25%	10%	5%	2.50%	1%	0.10%	0.01%	0.001%	
	Percentage probability of meeting XOR criteria (XORi,i > 0.9; XORi,j < 0.1) — Level of noise (%)									
10	0	0	27.6	74.2	99.6	100				Exact FEM
20		0	1.5	46.4	93.5	100				inaccurate
50					0	28.2	100			experimental
99 SEREPa								0	65.5	results
10	0	0.1	5.5	77.1	99.8	100				Exact
20			0	30	81.4	100				experimental
50				0	0.1	29.7	100			results,
99 SEREPa								0	66.7	inaccurate FEM
10	0	0	5.3	60.4	93.6	100				Inaccurate
20			0	11.3	72.8	99.8	100			experimental
50					0	5.5	100			results and FEM
99 SEREPa								0	29.3	

Table I.
Monte Carlo simulation results – sensitivity to mode number and noise

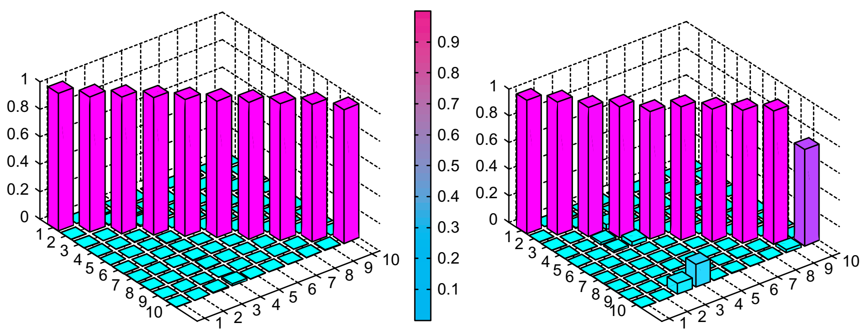

Figure 3.
XOR matrix representation showing a successful and unsuccessful correlation

- In equation (20) two different sets or uncorrelated random coefficients δ have been applied to the experimental and FEM mode shapes as the errors in the two sets are assumed to be uncorrelated.

For each different level of distortion δ_i and for each number of mode shapes utilized to generate the mass TAM matrix (i.e. 10, 20, 50 and 99 modes), 1,000 sets of mode shapes were generated in order to have statistically representative results.

The 99-modes rows (in bold in the table), correspond to the SEREPa condition, and although this situation has the computational advantage of generating a square reduction matrix (easy to invert) it is clear that even a very small level of error (0.001 percent) gives a one in three chance of failing the European Space Agency COC criteria (ECSS-E-ST-32-11C, 2008). This very high sensitivity makes SEREPa not particularly suitable for the validation of FEM against test results as differences of mode shape coefficients in the region of 1 percent would generally be expected and acceptable. However, they would lead to a failure in meeting the correlation criterion.

To carry out the SEREP reduction using only the target mode shapes that will need to be ultimately correlated seems indeed a more reasonable option.

From the results in Table I it appears that a level of error in the region of 2.5 percent would still allow meeting the cross orthogonality check criteria in over 99 percent of the cases with errors in the FEM or in the experimental results, which is probably in line with the current expectations. The probability of failing the COC then increases as the level of errors increase and if the errors are in the region of 10 percent then the probability of failing the XOC is around 95 percent, which again seems reasonable as a level of discrepancy in the mode shapes the region of 10 percent would generally be regarded as excessive.

Perhaps it is worth noting that the COC appears to be more sensitive to errors in the FEM than to errors in the experimental mode shapes. The reason for this is likely to be the fact that in using the SEREP procedure the errors in the FEM shapes are also affecting the TAM mass matrix. The errors are effectively being multiplied as the XOC (equation (19)) $\boldsymbol{\phi}$ is multiplied by $\mathbf{M}_{\mathrm{TAM}}$. Conversely the errors in the experimental mode shape (equation (18)) are not multiplied by any other error.

As the COC is actually composed of two tests on the XOR matrix:

(1) one the elements of the diagonal that have to be >0.9; and

(2) one on the elements off-diagonal that have to be <0.1 an investigation has been carried out to assess the probability of meeting these two criteria independently.

The graph in Figure 4 shows the minimum elements of the diagonal and the maximum off-diagonal for each XOR matrix generated by each of the 1,000 sets of mode shapes produced for the specific case of 2.5 percent noise. This graph shows that the diagonal members of XOR are smaller than 0.9 in 64 out of the 1,000 cases considered, whereas there are only 12 off-diagonal elements that are greater than 0.1. Further investigation showed that in each of these 12 cases the element on the diagonal was smaller than 0.9, hence the check would have failed anyway. In summary, the data shows that just checking the diagonal members of XOR would have been sufficient, as there are no cases where the

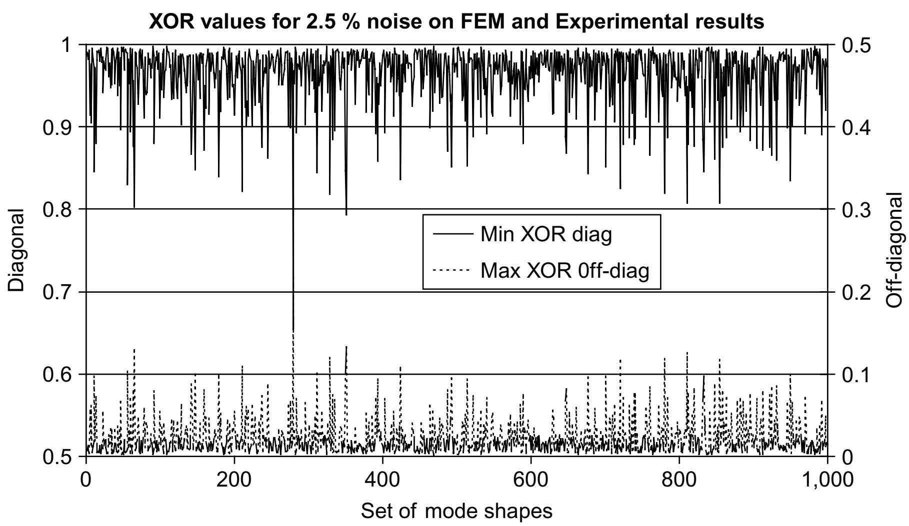

Figure 4.
Minimum value on the XOR diagonal and maximum value off-diagonal per each simulation with 2.5 percent noise on both FEM and experimental results, 99 DOFs monitored, and SEREP reduction algorithm considering ten modes

off-diagonal terms are greater then 0.1, but the element on the diagonal are greater than 0.9. This investigation has been repeated for different levels of noise and different numbers of mode shapes in the SEREP reduction, and in each case we found that the ensemble of cases that failed the COC because of the diagonal terms in XOR included the ensemble of fails due to the off-diagonal members. Therefore, in this specific case, the check on the diagonal members would have been sufficient to detect all cases of missed correlation.

4. Sensitivity to measurement DOFs optimization

Assuming that 50 accelerometer outputs are enough to correlate the ten target modes, it is possible to carry out the COC as in the previous section, but where the mode shape vectors will now contain only 50 elements each. The 50 DOFs corresponding to these 50 elements are initially randomly selected out of the previous group of 99.

As the trade-off in the previous section has indicated that it is appropriate to use only the target mode shapes in the SEREP process, this approach has been used consistently in this analysis.

Two situations are considered in this section:

(1) errors in the FEM; and

(2) errors in the FEM and in the tests.

The probability of meeting the COC criteria for these two situations and various levels of errors are reported in Table II. For each combination and level of error, 1,000 runs were carried out.

As it can be seen from the table, a level of noise of 5 percent is enough to give a very low probability of meeting the COC.

Now, instead of randomly selecting the 50 DOFs used in the analysis, an optimality criterion is applied to select the most appropriate DOFs. The criterion utilized consists of selecting the DOFs that maximize the independence of the target modes. In practice here this is implemented by computing the effective independence matrix:

$$\mathbf{EfI} = \mathbf{\Phi}_M \left(\mathbf{\Phi}_M^T \mathbf{\Phi}_M\right)^{-1} \mathbf{\Phi}_M^T$$

and eliminating the DOF corresponding to the smallest element in the diagonal of the EfI matrix (Friswell and Mottershead, 1995). Eliminating this DOF from the modal vectors allows the re-computation of the EfI matrix. The procedure is repeated until we are left only with 50 DOFs.

This has been implemented to select the DOFs in both situations (errors in the FEM, and errors in the FEM and in the tests) and the results are reported in Table II.

Considering for example the column corresponding to the 5 percent level of noise, it is possible to see that the use of this optimality criteria has improved by just over

Table II. Monte Carlo simulation results – sensitivity to noise and DOFs optimization

Percentage meeting XOR criteria (XORi,i > 0.9; XORi,j < 0.1) Level of noise (%)	10%	5%	2.50%	1%	
50 accel outputs random DOFs	0	13.3	83.9	100	Inaccurate experimental results and inaccurate FEM
50 accel outputs optimised DOFs	4.1	55.4	94.2	100	
50 accel outputs random DOFs	0	28.9	98.8	100	Exact FEM, inaccurate experimental results
50 accel outputs optimised DOFs	7.5	82.8	99.6	100	

four folds the probability of meeting the COC requirement when noise was present both in the experiment and FEM and nearly three times the probability of meeting the criteria when the noise was only in the FEM data. When the level of errors is small (1 percent), then it is not necessary to optimise the selection of DOFs, and vice-versa, when the errors are quite big 10 percent or more, although optimising the DOFs improves the situation, the chances of meeting the COC is still quite remote.

To summarise, it seems that the greatest impact in using an optimality criterion to select the DOFs is when the noise (errors) is between 2.5 and 5 percent. In this region it can quite dramatically change the probability of meeting the COC. Indeed, in practice, other methods, such as the coordinate orthogonality check (CORTHOG) (Avitabile and Pechinsky, 1998) could be used to select the DOFs to correlate.

5. Conclusions

In this work, the various techniques which are available to reduce a FEM to produce a mass matrix of an appropriate size to carry out COC have been briefly reviewed. SEREP has been identified as a particularly suitable method for two main reasons:

(1) because the first part of the SEREP algorithm is a modal reduction, which will be carried out anyway to provide the C-B model to the launch agency; and

(2) because SEREP guarantees that the reduced matrix has the same eigenvalues and eigenvector of the whole system therefore automatically meeting the criteria on the quality of the reduced model (which are given in terms of capability to reproduce mode shapes and natural frequency similar to those generated by the full model).

The robustness of a COC carried out utilizing a SEREP reduced model has been investigated, and the results show that to include a high number of mode shapes in the reduction process makes the check very sensitive to minor experimental or modelling inaccuracy. Finally it was shown that utilizing optimality criteria in the selection of the DOFs to carry out the correlation can significantly improve the probability of meeting the COC criteria.

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Corresponding author

G.S. Aglietti can be contacted at: gsa@soton.ac.uk