Reducing the Peak to Average Power Ratio of LDS-OFDM Signals

Mohammed AL-Imari and Reza Hoshyar
Centre for Communication Systems Research
University of Surrey
Guildford GU2 7XH, Surrey, U.K.
Email:{M.Al-imari, R.Hoshyar}@surrey.ac.uk

***Abstract*—Low Density Signature-Orthogonal Frequency Division Multiplexing (LDS-OFDM) has been introduced recently as an efficient multiple access technique. High Peak to Average Power Ratio (PAPR) is an important obstacle to multicarrier communication systems. This paper concentrates on the PAPR investigation and its reduction for LDS-OFDM signals. Specifically, we will investigate the impact of subcarrier allocation schemes and the phases of the signatures on the PAPR of LDS-OFDM signals. Firstly, the PAPR of LDS-OFDM with conventional signatures is investigated. Then we propose two methods for PAPR reduction; Newman phases and DFT pre-coding. The former method is simple and doesn't imply changes in the system structure while the DFT pre-coding implies a modification in the system. Simulation results show that using Newman phases considerably reduces the PAPR of LDS-OFDM. Further PAPR reduction is achieved using DFT pre-coded LDS-OFDM on the cost of higher complexity.**

***Index Terms*—LDS-OFDM, peak-to-average power ratio, Newman phases, DFT pre-coding.**

I. Introduction

Multicarrier transmission has been considered as a promising technique for high data rate transmission in wireless and wired applications. Orthogonal Frequency Division Multiplexing (OFDM) is one of the prominent multicarrier transmission techniques. For wireless applications, an OFDM-based system offers greater immunity to multipath fading and impulsive noise with low complexity per subcarrier equalization [1].

Orthogonal Frequency Division Multiple Access (OFDMA) is an efficient extension of OFDM transmission to a multiuser communication scenario. In OFDMA systems, the set of subcarriers is divided into several mutually exclusive subsets that are assigned to different users for simultaneous transmission. As highlighted in [2] OFDMA requires an efficient error control coding to harvest the channel frequency diversity.

Low Density Signature OFDM (LDS-OFDM) technique is introduced in [3] as an efficient uplink multicarrier multiple access scheme. The technique combines the benefits of OFDM based multicarrier transmission with a recent idea on LDS based spreading proposed for Code Division Multiple Access (CDMA) systems in [4]. LDS-OFDM combines the two efficient concepts of LDS and OFDM to establish an efficient multiple access over wideband channels. In LDS-OFDM, due to low density signature structure, every data symbol will only be spread on a small subset of subcarriers (effective processing gain) and also every subcarrier will only be used by a small subset of data symbols that could belong to different users.

The LDS-OFDM technique is able to exploit both the channel and the multiple access interference diversities over the frequency domain. In [3] it is shown that LDS-OFDM outperforms the performance of OFDMA over a typical multipath fading channel and under different spectral efficiency conditions. On the other hand, LDS-OFDM detector has larger complexity than conventional single-user receivers. This is due to employment of the iterative message passing algorithm over the LDS graph. The incurred computational complexity is justified thanks to improved performance and also due to the less complexity restrictions of the network side where the LDS-OFDM receiver is employed.

A major drawback of multicarrier communication systems is their high Peak to Average Power Ratio (PAPR) in comparison to single carrier systems. This high PAPR will reduce the efficiency of high-power amplifiers as more back-off will be required to avoid non-linear effects on multi-carrier signal. Non-linearity of the power amplifier will cause signal distortion and as a result increase in Bit Error Rate (BER) and out-of-band radiation. In this regard, reduction of the signal's PAPR is more crucial for multicarrier systems.

LDS-OFDM technique inherits the PAPR drawback due to its multicarrier nature. In this paper, we evaluate the PAPR of LDS-OFDM signal and will consider efficient methods for its PAPR reduction. Specifically, we will investigate the impact of the subcarrier allocation and the tuning of LDS signature phases on the signal PAPR. We show that with proper subcarrier allocation and phase adjustment, the PAPR can be significantly reduced. Moreover, we propose Discrete Fourier Transform (DFT) pre-coding for further reduction of the PAPR.

This paper is organized as follows: Section II presents the LDS-OFDM system model. Issues related to subcarrier allocation and chip phases of LDS-OFDM are discussed in section III. The PAPR analysis is presented in section IV. In section V, two effective methods for PAPR reduction are proposed and evaluated. Finally, section VI is devoted to concluding remarks.

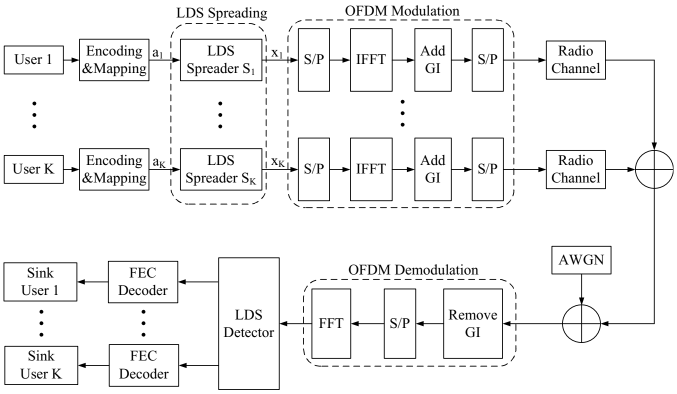

Fig. 1. LDS-OFDM block diagram.

II. LDS-OFDM System Model

In an LDS-OFDM system, the original data streams are first multiplied with the low density spreading sequences and then transmitted over different subcarriers. The conceptual block diagram of an uplink LDS-OFDM system is shown in Fig. 1. We consider LDS-OFDM system with N subcarriers and K users. Let $\mathbf{a}_k = [a_{k,1}, a_{k,2}, \cdots, a_{k,M}]^T, k = 1, ..., K$, be a data vector of user k consisting of M modulated data symbols. Without loss of generality all users are assumed to take their symbol from the same constellation alphabet $\mathbb{X}$. Also, all users are assumed to have the same number of data symbols M and allocated the same number of subcarriers N_u.

Each data symbol $a_{k,m}, m = 1, ..., M$, is spread using a low density signature $\mathbf{s}_{k,m} \in \mathbb{C}^{N\times 1}$. The LDS is a sparse vector consisting of N chips. Among the N chips only d_v chips have non-zero values, where d_v is the effective spreading factor. The positions (indices) of the non-zero chips represent the subcarrier mapping. We define the signature matrix of user k as $\mathbf{S}_k = [\mathbf{s}_{k,1}, \mathbf{s}_{k,2}, \cdots, \mathbf{s}_{k,M}]$. All signatures are assumed to have the same effective spreading factor d_v. So, the maximum number of subcarriers occupied by each user is $N_u = Md_v$. The chips vector belonging to user k after the spreading process is given by;

$$\mathbf{x}_k = \mathbf{S}_k \; \mathbf{a}_k. \tag{1}$$

Each user's generated chip will be transmitted over a subcarrier of the OFDM system. As users are not bound to exclusively use the subcarriers, at the receiver side users' signals that are using the same subcarrier will be superimposed. The LDS structure can be captured by a low density graph, thus similar to application of LDS for CDMA system the detection of LDS-OFDM could be based on the Message Passing Algorithm (MPA) presented in [4]. More details regarding the LDS receiver can be found in [4].

III. Subcarrier Allocation and Chips' Phases

Signature construction in LDS-OFDM system includes the subcarrier allocation scheme and the phases of the chips. It has been shown that the subcarrier allocation and the signature's phases have considerable effect on the PAPR of the multicarrier signals [5], [6]. In this regard, it is necessary to analyze the effect of signature design schemes on the PAPR of LDS-OFDM signals. In this section we will explain the possible options of the signature design.

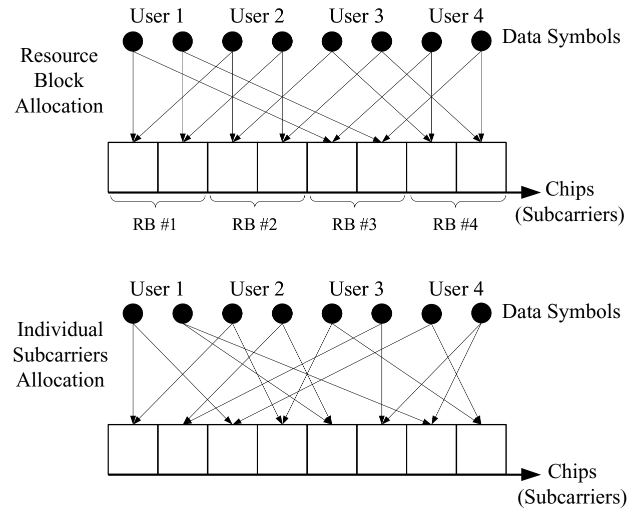

Fig. 2. Subcarrier allocation schemes for LDS-OFDM.

A. Subcarrier Allocation

In LDS-OFDM system, the generated chips at the output of the LDS spreader are mapped to the subcarriers of the employed OFDM signal. Here we assume that a static subcarrier allocation is to be employed. Individual subcarriers can be allocated to the users for transmission. However, to facilitate the channel estimation a common practice is to group a number of adjacent subcarriers in both time and frequency domain in a form of a chunk. The time and frequency (N_t and N_z) sizes of the chunks are ensured to be less than the coherence time and bandwidth of the channel, respectively. As a result the channel will be almost flat over a chunk and the channel can be efficiently estimated by included pilots within each chunk. The chunk concept is adopted in 3GPP Long Term Evolution (LTE), where a chunk is denoted as Resource Block (RB). So, instead of individual subcarriers, resource blocks are allocated to the users. In addition to that, resource block allocation will enable the application of PAPR reduction techniques for LDS-OFDM as we will explain in the next sections. Here we will consider that the time size of the resource block N_t is equal one. Fig. 2 depicts two types of subcarrier allocation schemes, recourse block based and individual subcarriers based. It shows an example of 4 users with two symbols ($M = 2$) per user and effective spreading factor equal to two ($d_v = 2$). In resource block based allocation each user is allocated two resource blocks with size two ($N_z = 2$).

B. Chips' Phases

Several phasing schemes, describing the chips' phases in a closed form, are given in the literature [7], [8]. Their advantage is that the closed-form construction rule allows easy

TABLE I
SIMULATION PARAMETERS.

Parameter		Value
Data streams per user		10
Number of data subcarriers		60
FFT size		64
Effective spreading factor		3
Oversampling		4
Modulation		QPSK
Number of subcarriers per user (N_u)	OFDMA	10
	LDS-OFDM	30

generation of signals with a varying number of subcarriers. In [7], Newman constructed a set of phases for N equally spaced (in frequency) sinusoids so that the PAPR of their sum is minimized. In [9] Newman phases have been studied in details and showed that a 2.6 dB PAPR is always obtained for large N. So, we suggest for LDS-OFDM, instead of random phases, Newman phases to be used for the generation of the signatures to reduce the PAPR. The Newman phases are given by;

$$\theta_n = \frac{\pi(n-1)^2}{N}, \qquad n = 1, 2, ..., N. \tag{2}$$

As in LDS-OFDM the signatures are not orthogonal, applying Newman phases will not affect the system's performance. Another advantage is that applying these phases will not add complexity to the system. The evaluation of the PAPR for Newman phases comparing with conventional random phases will be given in next sections.

IV. PAPR OF LDS-OFDM

In this section, the PAPR of LDS-OFDM signals is analyzed and compared with that of OFDMA. The complex baseband representation of the time-domain LDS-OFDM signal for the kth user is given by;

$$y_k(t) = \frac{1}{\sqrt{N}} \sum_{n=1}^{N} x_{k,n} e^{j2\pi f_n t}, \qquad 0 \leq t \leq T_s, \tag{3}$$

where $f_n = n/T_s$ and T_s is the time duration of the LDS-OFDM symbol. The PAPR of the transmitted signal of the kth user, $y_k(t)$, is defined as;

$$PAPR = \frac{\max\limits_{0 \leq t \leq T_s} |y_k(t)|^2}{\frac{1}{T_s} \displaystyle\int_0^{T_s} |y_k(t)|^2 dt}. \tag{4}$$

The numerator in (4) denotes the peak power and the denominator denotes the average power. By sampling $y_k(t)$ defined in (3) at frequency $f_s = LN/T_s$, where L is the oversampling factor, the discrete-time LDS-OFDM symbol can be written as;

$$y_{k,l} = \frac{1}{\sqrt{N}} \sum_{n=1}^{N} x_{k,n} e^{j2\pi \frac{nl}{LN}}, \qquad l = 1, 2, \cdots, NL. \tag{5}$$

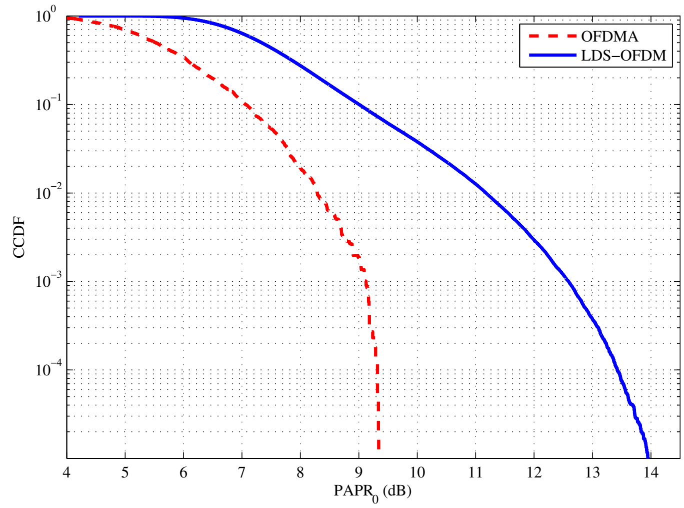

Fig. 3. CCDF of PAPR for OFDMA and LDS-OFDM with conventional signatures.

It is shown in [10] that $L = 4$ can provide adequately precise PAPR results. The PAPR calculated from the L times oversampled time domain signal's samples is given by;

$$PAPR = \frac{\max\limits_{l \in [1, NL]} |y_{k,l}|^2}{E\{|y_{k,l}|^2\}}, \tag{6}$$

where $E[.]$ denotes expectation. In the remaining part of this paper, we will numerically compare the PAPR characteristics using the Complementary Cumulative Distribution Function (CCDF) of PAPR. The CCDF of PAPR denotes the probability that PAPR exceeds a certain PAPR value, $Pr(PAPR > PAPR_0)$. In addition to the CCDF curves, we will compare the PAPR values that exceed with probability 0.1%, and will refer to it as the 99.9-percentile PAPR. The 99.9-percentile PAPR is defined as $Pr(PAPR > PAPR_{99.9}) = 10^{-3}$. The CCDF of PAPR is calculated by Monte Carlo simulation. In the simulations, 10^5 uniformly random symbols (OFDMA and LDS-OFDM symbols) per user were generated and 4 times oversampling was used to acquire the CCDF of PAPR. We will consider contiguous subcarrier allocation for OFDMA. The same spectral efficiency per user is considered for both systems. The simulation parameters are listed in Table I.

Here we will investigate the PAPR of LDS-OFDM with conventional signatures, which use random phases and individual subcarrier allocation scheme. Fig. 3 shows the PAPR of LDS-OFDM and OFDMA. As it can be observed from the figure, LDS-OFDM with conventional signatures has high PAPR. High PAPR for LDS-OFDM can be justified by considering that random phases are assigned to the signatures without

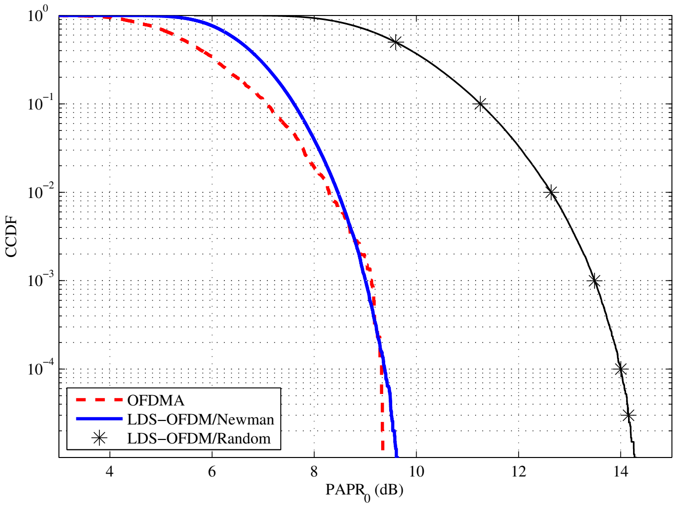

Fig. 4. CCDF of PAPR for OFDMA and LDS-OFDM with recourse block allocation.

taking into account that the chips belonging to each symbol are correlated. Table II lists the 99.9-percentile PAPR values that each signal experiences for all the simulation carried out in this paper.

V. PAPR Reduction for LDS-OFDM

As shown earlier, LDS-OFDM suffers from high PAPR for randomly generated phases. In this section, we will propose and evaluate two methods for reducing the PAPR. The first method is using Newman phases that benefits from low PAPR feature. Also, this method does not require modification in the LDS-OFDM system structure. The second method is applying DFT pre-coding for LDS-OFDM system.

A. Newman Phases

At this section, the effect of applying Newman phases on the PAPR of LDS-OFDM is evaluated using numerical results. Newman phases require the subcarriers allocated to the user to be equally spaced. Restricting all the users to have equally spaced subcarrier allocation in LDS-OFDM will result in a fully connected graph, which reduce the receiver efficiency. So, we propose applying Newman phases for LDS-OFDM combined with resource block based allocation.

The PAPR of LDS-OFDM with resource block based allocation is shown in Fig. 4 for Newman phases and random phases. Each resource block consists of 10 adjacent subcarriers ($N_z = 10$). We can see from the figure, there is a significant reduction in the PAPR of LDS-OFDM when Newman phases are used. It can be observed that the PAPR of LDS-OFDM with Newman phases is close to OFDMA, and it's 3.6 dB less than conventional signatures. Also, random phases still suffer high PAPR regardless of the allocation scheme used.

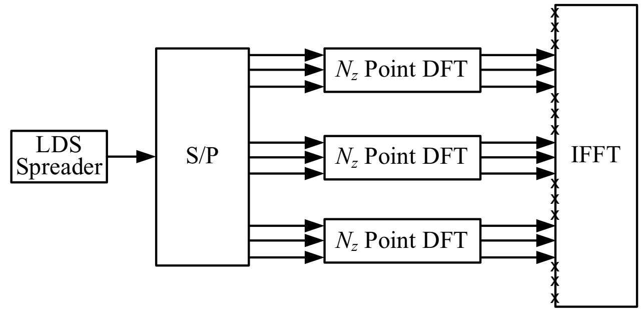

Fig. 5. DFT pre-coding for LDS-OFDM.

B. DFT Pre-Coding

Discrete Fourier Transform (DFT) is used in many systems (MC-CDMA and MC/DS-CDMA) as spreading or pre-coding techniques to reduce the PAPR [11], [12]. For further PAPR reduction, we propose DFT pre-coding for LDS-OFDM system. Using of LDS spreading combined with DFT based spreading will increase the density of the resulted signature matrix. To benefit fro both LDS structure and DFT property in PAPR reduction, we propose using DFT per resource block. Doing this and assuming negligible channel variation, at the receiver side, we can perform a simple IDFT to remove the effect of DFT spreading. Assume the total number of subcarriers is divided into B resource blocks, and each resource block consists of N_z contiguous subcarriers, where $B = N/N_z$. So, the chips vector of the kth user in (1) can be rewritten as $\mathbf{x}_k = [\mathbf{x}_{k,1}^T, \mathbf{x}_{k,2}^T, ..., \mathbf{x}_{k,B}^T]^T$ with $\mathbf{x}_{k,b} \in \mathbb{C}^{N_z \times 1}$, consists of B groups of chips, $\mathbf{x}_{k,b} \neq \mathbf{0}$ if the user k uses the bth resource block and $\mathbf{x}_{k,b} = \mathbf{0}$ otherwise.

Fig. 5 depicts the DFT pre-coding for LDS-OFDM. DFT pre-coding is applied for each non-zero group of chips as follows;

$$\hat{\mathbf{x}}_{k,b} = \mathbf{F}_{N_z}\,\mathbf{x}_{k,b}, \tag{7}$$

where $\mathbf{F}_{N_z}$ is N_z x N_z DFT matrix. The received signal per resource block $\mathbf{r}_b \in \mathbb{C}^{N_z \times 1}$ can be defined as follows;

$$\mathbf{r}_b = \sum_{k \in \mathcal{J}_b} \mathbf{H}_{k,b}\,\mathbf{F}_{N_z}\,\mathbf{x}_{k,b} + \mathbf{v}_b, \tag{8}$$

where $\mathcal{J}_b = \{k : \mathbf{x}_{k,b} \neq \mathbf{0}\}$ is the set of users that use the bth resource block, $\mathbf{v}_b$ is the Additive White Gaussian Noise (AWGN) and $\mathbf{H}_{k,b}$ is the frequency domain channel transfer function of user k on the bth resource block. The number of subcarriers per resource block (N_z) is chosen such that the resource block bandwidth is smaller than coherence bandwidth of the channel. In this way, highly correlated fading is experienced over all subcarriers in a resource block resulting in ($\mathbf{H}_{k,b} = h_{k,b}\,\mathbf{I}_{N_z}$). Therefore, the transmitted chips can be

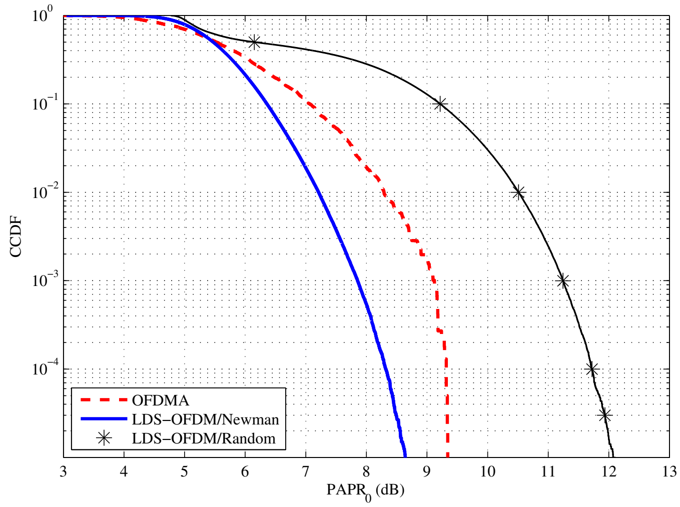

Fig. 6. CCDF of PAPR for OFDMA and DFT pre-coded LDS-OFDM

simply retrieved by applying IDFT as follows;

$$\begin{aligned}\tilde{\mathbf{r}}_b &= \mathbf{F}_{N_z}^{-1}\left(\sum_{k\in\mathcal{J}_b} h_{k,b}\,\mathbf{F}_{N_z}\,\mathbf{x}_{k,b}+\mathbf{v}_b\right)\\ &= \sum_{k\in\mathcal{J}_b}\mathbf{F}_{N_z}^{-1}\,h_{k,b}\,\mathbf{F}_{N_z}\,\mathbf{x}_{k,b}+\mathbf{F}_{N_z}^{-1}\,\mathbf{v}_b\\ &= \sum_{k\in\mathcal{J}_b} h_{k,b}\,\mathbf{x}_{k,b}+\mathbf{F}_{N_z}^{-1}\,\mathbf{v}_b, \qquad (9)\end{aligned}$$

where $\mathbf{F}_{N_z}^{-1}$ is N_z x N_z IDFT matrix.

Fig. 6 shows the PAPR of the DFT pre-coded LDS-OFDM with Newman phases and random phases. The results show that DFT pre-coding combined with Newman phases reduces the PAPR of LDS-OFDM by 4.75 dB comparing to conventional signatures. Furthermore, DFT pre-coding has small PAPR reduction when random phases are used.

VI. Conclusions

Increasing interest in high data rate services demands high spectral efficiency techniques. In this regard LDS-OFDM was introduced as an efficient multiple access technique. To benefit from this effective technique, it is necessary to consider its practical issues such as PAPR. The impact of subcarriers' allocation schemes and the phases of the signatures on the PAPR of LDS-OFDM signals was investigated. The PAPR problem for LDS-OFDM is tackled using Newman phases and DFT pre-coding.

Applying Newman phases to LDS-OFDM reduces its PAPR significantly. A major advantage for this method is that it doesn't affect the performance while maintaining the same level of complexity as the original LDS-OFDM system. DFT pre-coding is able to further reduce the system PAPR, but it has some added complexity compared to Newman phases technique. On the other hand, the decoding of DFT is applicable when the channel is not highly selective and the resource block size is small in comparison to the coherence bandwidth of the channel. This could be achieved by carefully selecting the resource block size.

TABLE II
Comparison of 99.9-percentile PAPR.

		99.9-percentile PAPR
OFDMA		9.1
LDS-OFDM/conventional signatures		12.6
LDS-OFDM/Resource Block	Newman	9
	Random	13.48
DFT pre-coded LDS-OFDM	Newman	7.85
	Random	11.25

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