Modeling Leakage of Ephemeral Secrets in Tripartite/Group Key Exchange*

SUMMARY We propose a security model, referred as g-eCK model, for group key exchange that captures essentially all non-trivial leakage of static and ephemeral secret keys of participants, i.e., group key exchange version of extended Canetti-Krawczyk (eCK) model. Moreover, we propose the first one-round tripartite key exchange (3KE) protocol secure in the g-eCK model under the gap Bilinear Diffie-Hellman (gap BDH) assumption and in the random oracle model.

key words: group key exchange, group-oriented extended Canetti-Krawczyk model, tripartite key exchange, gap Bilinear Diffie-Hellman assumption, random oracle model

1. Introduction

Design and analysis of Key Exchange (KE) protocols is amongst the oldest research topics in cryptography that has found its direct way into practice. Although KE was introduced back in 1976 [23], it was not until 1993 when Bellare and Rogaway [9] made the first step towards capturing the security requirements for these protocols in a formal way, by designing a security model, which influenced later developments in the design and analysis of KE protocols. Research efforts on provable security in KE protocols, in the public key setting, can roughly be classified along two-party KE (2KE), e.g., [7], [11], [19], [20], [22], [34], [35], [41], [46], and group KE (GKE), e.g., [14], [17], [26], [31], [39], [40], [47], reaching out to other KE flavors such as password-based solutions [1], [4], [8], [10], [12], multi-factor KE protocols [44], and to flexible combinations of GKE and 2KE [2]. The most standard security notion, common to all KE flavors, takes its roots in [9] and is called authenticated key exchange security, or AKE-security for short. Although AKE-security has been modeled in different flavors, for different types of adversarial capabilities and corruptions, the common idea behind this notion is to protect the secrecy of exchanged keys by means of indistinguishability of some test session key from a randomly chosen one.

In the field of two-party KE (2KE), the extended Canetti-Krawczyk (eCK) model, which captures essentially all non-trivial leakage of static and ephemeral secret keys of participants, has been proposed [35].

In the field of group KE (GKE), the models [17], [25], which capture (restricted) leakage of static and ephemeral secret keys of participants, incorporate key compromise impersonation (KCI) attacks and forward security (FS). However, no security definition, that captures essentially all non-trivial leakage of static and ephemeral secret keys of participants, is known yet.

From practical point of view, since there are many participants in the case of GKE, the possibility of leakage of static and ephemeral secret keys increases. So, such leakage is even more important to take care of in the case of group key exchange. From theoretical point of view, it is natural to consider all non-trivial leakage of static and ephemeral secret keys of participants by removing unnecessary restrictions from the models of [17], [25]. Thus, it is worthy goal to model extended Canetti-Krawczyk (eCK) security in the group KE (GKE).

In this paper, we propose a security model, referred as g-eCK model, for group key exchange that captures essentially all non-trivial leakage of static and ephemeral secret keys of participants, i.e., group key exchange version of extended Canetti-Krawczyk (eCK) model [35].

Moreover, we propose the first one-round authenticated tripartite key exchange (3KE) protocol that is secure in the g-eCK model and is based on the Joux’s one-round (unauthenticated) 3KE [28], [29]. There have been several authenticated 3KE protocols [5], [36]–[38], [45], however, none of these protocols are secure in the g-eCK model. So, we provide the proposed protocol to demonstrate that the g-eCK model can be satisfied. The proposed protocol is secure in the g-eCK model under the gap Bilinear Diffie-Hellman (gap BDH) assumption and in the random oracle model. Although the gap BDH assumption is strong, we adopt the assumption to make the protocol efficient, i.e., we can use the BDH assumption by applying the twin Diffie-Hellman technique, but it requires twice number of keys and more pairing operations than the proposed protocol.

The proposed protocol is efficient, i.e., is one-round and requires 4 shared secrets, 4 pairing operations, and 4 exponential operations (including the exponentiation for the ephemeral public key).
1.1 Related Works

Ephemeral Key Leakage. The security model proposed in this paper (and [40]), which was stated in a more general GKE setting, considers a very strong attacker, who may adaptively compromise static and ephemeral secret keys used in the protocol session (with an obvious restriction that at least one of these keys per participant must remain secret). Leakage of ephemeral secrets, typically of exponents used in the computation of ephemeral public keys (Diffie-Hellman) keys, is especially damaging for implicitly authenticated protocols, where for better efficiency one may desire to pre-compute ephemeral public keys off-line, in which case used secrets must be stored temporarily, resulting in a higher risk for the security of the protocol. Even if ephemeral secret keys are chosen (and erased) within the protocol session, attacks exploiting side-channels of the implementation may threaten their secrecy. In general, motivation for considering leakage of ephemeral secrets in KE protocols stems from 2KE domain, e.g. as first mentioned in [19], [34] and explicitly modeled in AKE-security definitions from [35], [46]. Various efforts towards construction of 2KE leakage-resilient protocols have been taken, e.g. [24], [33], [35], [42], [43], [46]. In general, modeling and designing ephemeral key-leakage resilient KE protocols should not be taken for granted — Cremers [21], [22] demonstrated how various technical elements of 2KE models such as the notions of session ids and partnering as well as conditions for freshness of the test session may affect the strength of AKE-security definition with ephemeral key-leakage resilience, when it comes to comparability of models and 2KE protocols. The model proposed in this paper (and [40]) is so far the only GKE security model that focuses on ephemeral key-leakage in test sessions and has recently been applied in [47], for the analysis of a two-round implicitly authenticated ephemeral key-leakage resilient GKE protocol.

GKE models. Group key exchange (GKE) protocols are essentially the generalization of 2KE protocols to the group case. However, this generalization brings additional problems both in the design and the analysis of the protocols. The first formal model for GKE protocol was described by Bresson et al. [14] inspired by the two-party approach in [9]. Many modifications and improvements appeared thereafter, see the survey in [39]. GKE models mainly focus on the outsider security which is modeled through the requirement of AKE-security, e.g. [13], [14], [18], [31], as this requirement deals explicitly with the secrecy of the established keys, which becomes meaningless if the adversary is an insider. Several models, e.g. [15], [17], [25], [26], [30], also consider the optional insider security aiming to prevent attacks by which insiders force parties to complete either with different keys (usually modeled as MA-security) or with keys that have some biased distribution (usually modeled as contributiveness). Protection against insider attacks, however, can often be achieved through generic compilers e.g. [15], [16], [30]. Another key difference amongst GKE models is in the treatment of corruptions: earlier models, e.g. [14], [31], considered weak corruptions allowing the adversary to obtain users’ static keys, but not their ephemeral session secrets. Later models, e.g. [17], [18], [25] assumed strong corruptions allowing the adversary to learn both static private keys and session specific secrets through a single query. Manulis and Bresson [17], inspired by the two-party approach in [19] refined the notion of strong corruptions in GKE allowing the adversary to obtain static keys independently from ephemeral session secrets; yet, restricting the leakage of ephemeral secrets to sessions for which the adversary does not need to distinguish the key. The reason is that GKE protocols known today become insecure if ephemeral secrets used to compute a group key leak, in other words leaking ephemeral secrets of one session affects the security of other non-partnered sessions. As a result many GKE protocols are insecure if parties for better performance pre-compute their ephemeral secrets off-line. The model from [17] was subsequently strengthened by Gorantla et. al. [25] (also more recent journal version [27]) to address key compromise impersonation attacks.

Tripartite Key Exchange. In 2000, a powerful GKE subclass of tripartite KE (3KE) emerged with the introduction of pairings to KE by Joux [28], [29]. Thanks to the bilinear property only one communication round amongst three parties is necessary in [28], [29] to compute the session key, whereby each party has to communicate only a constant amount of bits (one group element) and perform a constant number of operations (one exponentiation and pairing evaluation). The original protocol in [28] was unauthenticated and so efforts were taken to achieve protection against active attacks, yet without sacrificing the unique efficiency properties of the protocol. This goal turned out to be non-trivial. In fact, adopting traditional authentication techniques such as digital signatures, as previously applied to unauthenticated 2KE Diffie-Hellman in [19], would inherently result in at least two rounds of communication amongst 3KE participants due to the necessary use of nonces for preventing replay attacks on the signed ephemeral public keys; as is also obvious from the possible application of signature-based GKE authentication compilers from [18], [30]–32. 3KE protocols with at least two communication rounds have also been known in other authentication settings, e.g. using passwords [3]. Intuitively, the only way to preserve one communication round with constant bit communication complexity from [28], [29] is to resort to an implicitly authenticated solution, in which session key is derived through a binding of static (long-term) and ephemeral (session-dependent) secrets. Multiple attempts to achieve this form of authentication for 3KE, e.g. [5], [36]–[38], [45] failed (as partly demonstrated in subsequent attempts against the previous ones and also summarized and extended in [40]) and the so-far only implicitly authenticated 3KE protocol that provably fulfills this goal is the protocol proposed by Manulis, Suzuki, and Ustaoglu in this paper (and [40]).
2. Proposed g-eCK Model

We propose a security model, referred as g-eCK model, for group key exchange that captures all non-trivial patterns of leakage of static and ephemeral secret keys of participants, i.e., group key exchange version of extended Canetti-Krawczyk (eCK) model [35]. All non-trivial patterns of leakage of static and ephemeral secret keys is described by Condition 3 and Condition 4 in the definition of freshness (Definition 1) using StaticKeyReveal query and StateReveal query. This model can be seen as an extension of the strong authenticated key exchange model for two-party protocols from [41] to the group setting and it is described using the classical notations and terminology of existing models for GKE protocols, e.g., [17],[25],[27],[31].

(1) Protocol Participants and Initialization.

Let \( \mathcal{U} := \{U_1,\ldots,U_N\} \) be a set of potential protocol participants and each user \( U_i \in \mathcal{U} \) is assumed to hold a static private/public key pair \((s_i,S_i)\) generated by some algorithm \( \text{Gen}(1^k) \) on a security parameter \( 1^k \) during the initialization phase.

(2) Protocol Sessions and Instances.

Any subset of \( \mathcal{U} \) can decide at any time to execute a new protocol session and establish a common group key. Participation of some \( U \in \mathcal{U} \) in multiple sessions is modeled through an number of instances \( \{\Pi^s_U \mid s \in [1\ldots n], U \in \mathcal{U} \} \), i.e., the \( \Pi^s_U \) is the \( s \)-th session of \( U \). Each instance is invoked via a message to \( U \) with a partner id \( \text{pid}^s_U \in \mathcal{U} \), which encompasses the identities of all the intended session participants (note that \( \text{pid}^s_U \) also includes \( U \)). Then, we say that \( U \) owns the instance \( \Pi^s_U \). In the invoked session, \( \Pi^s_U \) accepts if the protocol execution was successful, in particular \( \Pi^s_U \) holds then the computed group key \( K^s_U \).

(3) Session State.

During the session execution, each participating \( \Pi^s_U \) creates and maintains a session id \( \text{sid}^s_U \) and an associated internal state \( \text{state}^s_U \) which in particular is used to maintain ephemeral secrets used by \( \Pi^s_U \) during the protocol execution. Concretely, \( \text{sid}^s_U = \{(m_{1,1},\ldots,m_{n,1}),\ldots,(m_{1,j},\ldots,m_{n,j})\} \), where \( m_{i,j} \) is the \( j \)-th outgoing message from participant \( U_i \) in the session \( \Pi^s_U \) and \( \text{pid}^s_U = \{U_1,\ldots,U_n\} \).

We say that \( U \) owns session \( \text{sid}^s_U \) if the instance \( \Pi^s_U \) was invoked at \( U \). Furthermore, we assume that instances that accepted or aborted delete all information in their respective states.

(4) Partnering.

Two instances \( \Pi^s_U \) and \( \Pi^t_U \), are called partnered or matching if 1) \( \Pi^s_U \) and \( \Pi^t_U \) have accepted, 2) \( \text{sid}^s_U = \text{sid}^t_U \), and 3) \( \text{pid}^s_U = \text{pid}^t_U \).

Note also that the notion of partnering is self-inclusive in the sense that any \( \Pi^s_U \) is partnered with itself. If the protocol allows a user \( U \) to initiate sessions with \( U \), then the equality \( \text{pid}^s_U = \text{pid}^t_U \) is a multi-set equality.

(5) Adversarial Model.

The adversary \( \mathcal{A} \), modeled as a PPT machine, can schedule the protocol execution and mount own attacks via the following queries:

- **AddUser** \((U,S_U)\): This query allows \( \mathcal{A} \) to introduce new users. In response, if \( U \notin \mathcal{U} \) (due to the uniqueness of identities) then \( U \) with the static public key \( S_U \) is added to \( \mathcal{U} \). Note that \( \mathcal{A} \) is not required to prove the possession of the corresponding secret key \( s_U \).

- **Send** \((\Pi^s_U,m)\): With this query, \( \mathcal{A} \) can deliver a message \( m \) to \( \Pi^s_U \) whereby \( U \) denotes the identity of its sender. \( \mathcal{A} \) is then given the protocol message generated by \( \Pi^s_U \) in response to \( m \) (the output may also be empty if \( m \) is not required or if \( \Pi^s_U \) accepts). A special invocation query of the form \( \text{Send}(U,\text{'start'},U_1,\ldots,U_n) \) with \( U \in \{U_1,\ldots,U_n\} \) creates a new instance \( \Pi^s_U \) with \( \text{pid}^s_U = \{U_1,\ldots,U_n\} \) and provides \( \mathcal{A} \) with the first protocol message.

- **SessionKeyReveal** \((\Pi^s_U)\): This query models the leakage of session group keys and provides \( \mathcal{A} \) with \( K^s_U \). It is answered only if \( \Pi^s_U \) has accepted.

- **StaticKeyReveal** \((U)\): This query provides \( \mathcal{A} \) with the static private key \( s_U \).

- **StateReveal** \((\Pi^s_U)\): \( \mathcal{A} \) is given the ephemeral secret information contained in \( \text{state}^s_U \) at the moment the query is asked. Note that the protocol specifies what the state contains.

- **Test** \((\Pi^s_U)\): This query models the indistinguishability of the session group key according to the privately flipped bit \( \tau \). If \( \tau = 0 \) then \( \mathcal{A} \) is given a random session group key, whereas if \( \tau = 1 \) the real \( K^s_U \). The query can be queried only once and requires that \( \Pi^s_U \) has accepted.

Observe that via the \( \text{Send}(\Pi^s_U,m) \) query the adversary can control delivery of messages between instances, i.e., adversary can drop or modify messages. Therefore, the model captures active attacks such as replay and man in the middle attacks.

(6) Correctness.

A GKE protocol is said to be correct if in the presence of a benign adversary all instances invoked for the same protocol session accept with the same session group key.

(7) Freshness.

The classical notion of freshness of some instance \( \Pi^s_U \) is transcribed...
ditionally used to define the goal of AKE-security by specifying the conditions for the Test(\(\Pi_U^t\)) query. For example, the model in \([31]\) defines an instance \(\Pi_U^t\) that has accepted as fresh if none of the following is true: (1) at some point, \(\mathcal{A}\) asked SessionKeyReveal to \(\Pi_U^t\) or to any of its partnered instances; or (2) a query StaticKeyReveal(\(U_s\)) with \(U_s \in \text{pid}^t_U\) was asked before a Send query to \(\Pi_U^t\) or any of its partnered instances.

Unfortunately, these restrictions are not sufficient for our purpose since \(\Pi_U^t\) becomes immediately unfresh if the adversary gets involved into the protocol execution via a Send query after having learned the static key \(s_U\), of some user \(U_s\), those instance participates in the same session as \(\Pi_U^t\).

The recent model in \([17]\) defines freshness using the additional AddUser and StateReveal queries as follows. According to \([17]\), an instance \(\Pi_U^t\) that has accepted is fresh if none of the following is true: (1) \(\mathcal{A}\) queried AddUser(\(U_s, S_U\)) with some \(U_s \in \text{pid}^t_U\); or (2) at some point, \(\mathcal{A}\) asked SessionKeyReveal to \(\Pi_U^t\) or any of its partnered instances; or (3) a query StaticKeyReveal(\(U_s\)) with \(U_s \in \text{pid}^t_U\) was asked before a Send query to \(\Pi_U^t\) or any of its partnered instances; or (4) \(\mathcal{A}\) queried StateReveal to \(\Pi_U^t\) or any of its partnered instances at some point after their invocation but before their acceptance.

Although this definition is already stronger than the one in \([31]\) it is still insufficient for the main reason that it excludes the leakage of ephemeral secrets in instances in the period between the protocol invocation and acceptance. Also, this definition of freshness does not model key compromise impersonation attacks.

The recent update of the freshness notion in \([25]\), \([27]\) addressed the lack of key compromise impersonation resilience. In particular, it modifies the above condition (3) by requiring that if there exists an instance \(\Pi_U^t\), which is partnered with \(\Pi_U^t\) and \(\mathcal{A}\) asked StaticKeyReveal(\(U_s\)) then all messages sent by \(\mathcal{A}\) to \(\Pi_U^t\) on behalf of \(\Pi_U^t\) must come from \(\Pi_U^t\) intended for \(\Pi_U^t\). This condition should allow the adversary to obtain static private keys of users prior to the execution of the attacked session while requiring its benign behavior with respect to the corrupted user during the attack.

Yet, this freshness requirement still prevents the adversary from obtaining ephemeral secrets of participants during the attacked session. What is needed is a freshness condition that would allow the adversary to corrupt users and reveal the ephemeral secrets used by their instances in the attacked session at will for the only exception that it does not obtain both the static key \(s_U\), and the ephemeral secrets used by the corresponding instance of \(U_s\); otherwise security can no longer be guaranteed. In the following we define freshness taking into account all the previously mentioned problems.

Definition 1: An accepted instance \(\Pi_U^t\) is fresh if none of the following is true:

1. \(\mathcal{A}\) queried AddUser(\(U_s, S_U\)) with some \(U_s \in \text{pid}^t_U\); or
2. \(\mathcal{A}\) queried SessionKeyReveal to \(\Pi_U^t\) or any of its accepted partnered instances; or
3. \(\mathcal{A}\) queried both StaticKeyReveal(\(U_s\)) with \(U_s \in \text{pid}^t_U\) and StateReveal(\(\Pi_U^t\)) for some instance \(\Pi_U^t\), partnered with \(\Pi_U^t\); or
4. \(\mathcal{A}\) queried StaticKeyReveal(\(U_s\)) with \(U_s \in \text{pid}^t_U\) and there exists no instance \(\Pi_U^t\) partnered with \(\Pi_U^t\).

Note that since \(U \in \text{pid}^t_U\), and since the notion of partnering is self-inclusive, Condition 3 prevents the simultaneous corruption of static and ephemeral secrets for the corresponding instance \(\Pi_U^t\) as well. In case when users are allowed to own two partnering instances i.e., they can initiate protocols with themselves, the last condition should be modified to say that the number of instances \(U\) equals the number of times \(U\) appears in \(\text{pid}^t_U\). Note also that the above definition captures key compromise impersonation resilience through Condition 4: \(\mathcal{A}\) is allowed to corrupt participants of the test session in advance but then must ensure that instances of such participants have been honestly participating in the test session. In this way we exclude the trivial break of security where \(\mathcal{A}\) reveals static keys of users prior to the test session and then actively impersonates those users during the session. On the other hand, as long as \(\mathcal{A}\) remains benign with respect to such users their instances will still be considered as fresh.

(8) g-eCK Security.

We are ready to generalize the strong AKE-security definition from \([35]\), \([41]\) to a group setting.

Definition 2: Let \(P\) be a correct GKE protocol and \(\tau\) be a uniformly chosen bit. We define the adversarial game \(\text{Game}^{\text{ake}, \tau}_{\mathcal{A}, P}(\kappa)\) as follows: after initialization, \(\mathcal{A}\) interacts with instances via queries. At some point, \(\mathcal{A}\) queries Test(\(\Pi_U^t\)), and continues own interaction with the instances until it outputs a bit \(\tau'\). If \(\Pi_U^t\) to which the Test query was asked is fresh at the end of the experiment then we set \(\text{Game}^{\text{ake}, \tau}_{\mathcal{A}, P}(\kappa) = \tau'\). We define

\[
\text{Adv}^{\text{ake}}_{\mathcal{A}, P}(\kappa) = 2\Pr[\tau = \tau'] - 1
\]

and denote with \(\text{Adv}^{\text{ake}}_{\mathcal{A}, P}(\kappa)\) the maximum of the advantage over all PPT adversaries \(\mathcal{A}\). We say that a GKE protocol \(P\) provides g-eCK security if advantage \(\text{Adv}^{\text{ake}}_{\mathcal{A}, P}(\kappa)\) is negligible.

3. Proposed g-eCK Secure Tripartite Protocol

In this section, we propose a one-round tripartite key exchange (3KE) protocol secure in the g-eCK model under the gap Bilinear Diffie-Hellman (gap BDH) assumption and in the random oracle model.

3.1 Proposed 3KE Protocol

We propose a one-round 3KE protocol \(\Pi\) and prove in Theorem 4 that the proposed 3KE protocol \(\Pi\) is g-eCK secure under the gap BDH assumption in the random oracle model.
Let $\kappa$ be the security parameter. Let $\mathbb{G}$ and $\mathbb{G}_T$ be cyclic groups of prime order $q$. Let $e: \mathbb{G} \times \mathbb{G} \to \mathbb{G}_T$ be a non-degenerate bilinear map, called pairing, from group $\mathbb{G} \times \mathbb{G}$ to group $\mathbb{G}_T$, respectively. Let $H: \{0, 1\}^* \to \{0, 1\}^k$ be cryptographic hash function modeled as a random oracle. Let $P$ be the protocol identifier of the 3KE protocol $\Pi$.

Let $D, E, F \in \mathbb{Z}_q$ be constants s.t. $D, E, F \neq 0, 1$ and published as the system parameter.

For a user $U_A$, we set $U_A$’s static and ephemeral keys $A_0 = g^{a_0}$ and $A_1 = g^{a_1}$, respectively, and the lower-case letters are the private keys.

In the description, users $U_A$ with static key $A_0$, $A_B$ with static key $B_0$, and $U_C$ with static key $C_0$ communicate with each other, and compute the session key by the following one-round 3KE protocol.

1. $U_A$ selects a random ephemeral private key $a_1 \in \mathbb{Z}_q$, computes the ephemeral public key $A_1 = g^{a_1}$, stores ephemeral private key $a_1$ as state information, and broadcasts $(P, (U_A, A_B, U_C), (A_1))$ to $U_B$ and $U_C$.

2. $U_B$ selects a random ephemeral private key $b_1 \in \mathbb{Z}_q$, computes the ephemeral public key $B_1 = g^{b_1}$, stores ephemeral private key $b_1$ as state information, and broadcasts $(P, (U_A, A_B, U_C), (B_1))$ to $U_B$ and $U_C$.

3. $U_C$ selects a random ephemeral private key $c_1 \in \mathbb{Z}_q$, computes the ephemeral public key $C_1 = g^{c_1}$, stores ephemeral private key $c_1$ as state information, and broadcasts $(P, (U_A, A_B, U_C), (C_1))$ to $U_A$ and $U_B$.

4. Upon receiving $(P, (U_A, A_B, U_C), (B_1), (U_A, A_B, U_C), (C_1))$, $U_A$ verifies $B_1, C_1 \in \mathbb{G}$, computes $m$ shared secrets

$$\sigma_1 = e(B_0B_1, C_0C_1)^{a_0+b_1},$$

$$\sigma_2 = e(B_0B_1^{e_k}, C_0C_1^{a_0+a_1}),$$

$$\sigma_3 = e(B_0B_1, C_0C_1^{a_0+b_1}),$$

$$\sigma_4 = e(B_0B_1^{e_k}, C_0C_1^{a_0+a_1}),$$

obtains the session key $K = H(\sigma_1, \sigma_2, \sigma_3, \sigma_4, P, U_A, A_0, A_1, U_B, B_0, B_1, U_C, C_0, C_1)$, and completes the session.

5. Upon receiving $(P, (U_A, A_B, U_C), (C_1), (U_A, A_B, U_C), (A_1))$, $U_B$ verifies $C_1, A_1 \in \mathbb{G}$, computes $m$ shared secrets

$$\sigma_1 = e(C_0C_1, A_0A_1^{a_0+b_1}),$$

$$\sigma_2 = e(C_0C_1, A_0A_1^{b_0+b_1}),$$

$$\sigma_3 = e(C_0C_1, A_0A_1^{g_0+b_1}),$$

$$\sigma_4 = e(C_0C_1^{e_k}, A_0A_1^{b_0+b_1}),$$

obtains the session key $K = H(\sigma_1, \sigma_2, \sigma_3, \sigma_4, P, U_A, A_0, A_1, U_B, B_0, B_1, U_C, C_0, C_1)$, and completes the session.

6. Upon receiving $(P, (U_A, A_B, U_C), (A_1), (U_A, A_B, U_C), (B_1))$, $U_C$ verifies $A_1, B_1 \in \mathbb{G}$, computes $m$ shared secrets

$$\sigma_1 = e(A_0A_1^{D_0}, B_0B_1)^{e_k^{g_0+c_1}},$$

$$\sigma_2 = e(A_0A_1, B_0B_1^{e_k^{g_0+c_1}}),$$

$$\sigma_3 = e(A_0A_1, B_0B_1^{g_0+F_1}),$$

$$\sigma_4 = e(A_0A_1^{D_0}, B_0B_1^{e_k^{g_0+F_1}},$$

obtains the session key $K = H(\sigma_1, \sigma_2, \sigma_3, \sigma_4, P, U_A, A_0, A_1, U_B, B_0, B_1, U_C, C_0, C_1)$, and completes the session.

All users $U_A, U_B$, and $U_C$ compute the same shared secrets

$$\sigma_1 = d_T^{(a_0+b_1)(b_0+b_1)(k_0+c_1)},$$

$$\sigma_2 = d_T^{(a_0+a_1)(b_0+b_1)(e_0+c_1)},$$

$$\sigma_3 = d_T^{(a_0+a_1)(b_0+b_1)(e_0+C_1)},$$

$$\sigma_4 = d_T^{(a_0+b_1)(b_0+b_1)(e_0+C_1)},$$

and so compute the same session key $K$.

The 3KE protocol corresponding to Example 3 requires 4 shared secrets, 4 pairing operations, and 4 exponential operations (including the exponentiation for the ephemeral public key).

Instead of that constants $D, E, F \in \mathbb{Z}_q$ is provided as a part of the system parameter, constants $D, E, F \in \mathbb{Z}_q$ can be generated on the fly by $D = H'(A_1), E = H'(B_1), F = H'(C_1)$, where $H'$ is a hash function and $A_1, B_1, C_1$ are the ephemeral keys of parties $U_A, U_B, U_C$ and $D, E, F \neq 0, 1$ holds with overwhelming probability.

3.2 Security

For the security of the proposed protocol, we need the gap Bilinear Diffie-Hellman (gap BDH) assumption\(^1\) [6] described below. Let $BDDH : \mathbb{G}^3 \to \mathbb{G}_T$ be a BDH function $BDDH(g', g', g'' \in \mathbb{G})$ and $BDDH : \mathbb{G}^3 \times \mathbb{G}_T \to \{0, 1\}$ be a predicate which takes an input $(g'^a, g'^b, g'^c, e(g, g'))$ and returns bit 1 if $x = 0$ and 0 otherwise.

An adversary $\mathcal{A}$ is given input $g'^a, g'^b, g'^c \in \mathbb{G}$ selected uniformly random and oracle access to $BDDH(\cdot, \cdot, \cdot, \cdot)$ oracle, and tries to compute $BDDH(g'^a, g'^b, g'^c)$. For adversary $\mathcal{A}$, we define advantage

$$Adv_{gapBDH}(\mathcal{A}) = Pr[g'^a, g'^b, g'^c \in \mathbb{G},$$

$$\mathcal{A}^{BDDH(\cdot, \cdot, \cdot)}(g'^a, g'^b, g'^c) = BDDH(g'^a, g'^b, g'^c)],$$

where the probability is taken over the choices of $g'^a, g'^b, g'^c$ and $\mathcal{A}$’s random tape.

**Definition 3** (gap BDH assumption): We say that $\mathbb{G}$ and $\mathbb{G}_T$ satisfy the gap BDH assumption if, for all polynomial-time adversaries $\mathcal{A}$, advantage $Adv_{gapBDH}(\mathcal{A})$ is negligible in security parameter $\kappa$.

**Theorem:** We now prove security of the proposed 3KE protocol

\(^1\)In pairing group, we can use the pairing as DDH oracle, but we do not have BDDH oracle. So we need gap BDH assumption.
in the g-eCK model.

**Theorem 4:** If $\mathcal{G}$ and $\mathcal{G}_T$ are groups where the gap BDH assumption holds and $H$ is a random oracle, the proposed 3KE protocol $\Pi$ is secure in the g-eCK model.

The proof of Theorem 4 is provided in Appendix, we provide a intuitive discussion here.

*Proof:* (Sketch) All users $U_A$, $U_B$, and $U_C$ can compute the same shared secrets as shown above and so can compute the same session key $K$.

The gap BDH solver $S$ extracts the answer $g_T^{\sigma_{user}}$ of an instance $(U = g'^x, V = g'^y, W = g'^z)$ of the gap BDH problem using adversary $A$. For instance, we assume the case that test session $\id$, owner of which is user $U_A$, has no partnered sessions $\id$, owners of which are users $U_B$ and $U_C$, adversary $A$ is given $a_0$, and adversary $A$ does not obtain $a_1$, $b_0$, and $c_0$ from the condition of the freshness. In this case, solver $S$ can perfectly simulate StaticKeyReveal query by selecting random $a_0$ and setting $A_0 = g'^a$, and solver $S$ embeds the instance as $A_1 = U (= g'^a)$, $B_0 = V (= g'^b)$ and $C_0 = W (= g'^c)$ to extract $g_T^{\sigma_{user}}$ from the shared secrets

$$\sigma_1 = g_T^{(a_0+b_0+c_0 + 1)},$$
$$\sigma_2 = g_T^{(a_0+b_0+1+c_0)},$$
$$\sigma_3 = g_T^{(a_0+b_0+c_0 + 1)},$$
$$\sigma_4 = g_T^{(a_0+b_0+c_0 + 1)}.$$

The solver $S$ can extract the answer of the gap BDH instance as follows. By eliminating the terms including $a_0$ using the knowledge of $a_0$, solver $S$ can obtain

$$\sigma_1' = (\sigma_1 e(b_0 b_1, C_0 C_1)^{-a_0} b_0 b_1 + c_0 + 1) = g_T^{(b_0 b_1 + c_0 + 1)},$$
$$\sigma_2' = (\sigma_2 e(b_0 b_1^2, C_0 C_1)^{-a_0} b_0 b_1 + c_0 + 1) = g_T^{(b_0 b_1 + c_0 + 1)},$$
$$\sigma_3' = (\sigma_3 e(b_0 b_1, C_0 C_1)^{-a_0} b_0 b_1 + c_0 + 1) = g_T^{b_0 b_1 + c_0 + 1},$$
$$\sigma_4' = (\sigma_4 e(b_0 b_1^2, C_0 C_1)^{-a_0} b_0 b_1 + c_0 + 1) = g_T^{b_0 b_1 + c_0 + 1}.$$

By using these 4 linearly independent terms, solver $S$ can extract the answer $g_T^{\sigma_{user}}$ as

$$((\sigma_1 e(b_0 b_1, C_0 C_1)^{-a_0} b_0 b_1 + c_0 + 1)/(\sigma_2 e(b_0 b_1^2, C_0 C_1)^{-a_0} b_0 b_1 + c_0 + 1)) = g_T^{b_0 b_1 + c_0 + 1}.$$

Notice that, in other cases, the solver $S$ can eliminate the terms including the specific secret key using the knowledge of the secret key, can obtain 4 linearly independent terms, and can extract the answer $g_T^{\sigma_{user}}$ by solving the linear equation.

The solver $S$ can check whether the shared secrets are correctly formed w.r.t. static and ephemeral public keys, and can simulate $H$ and SessionKeyReveal queries consistently. More precisely, in the simulation of the $H(\sigma_1, \sigma_2, \sigma_3, \sigma_4, P, U_A, A_0, A_1, U_B, B_0, B_1, U_C, C_0, C_1)$ query, solver $S$ needs to check that the shared secrets $\sigma_1, \sigma_2, \sigma_3, \sigma_4$ are correctly formed, and if so return session key $K$ being consistent with the previously answered

**SessionKeyReveal:**

For instance, we assume the case that

$$\text{BDDH}(A_0 A_1^D, b_0 b_1, C_0 C_1, \sigma_1) = 1,$$
$$\text{BDDH}(A_0 A_1, B_0 B_1^F, C_0 C_1, \sigma_2) = 1,$$
$$\text{BDDH}(A_0 A_1, B_0 B_1^F, C_0 C_1, \sigma_3) = 1,$$
$$\text{BDDH}(A_0 A_1^D, B_0 B_1^F, C_0 C_1, \sigma_4) = 1,$$

and this implies $\sigma_1, \sigma_2, \sigma_3, \sigma_4$ are correctly formed.

Notice that, in other cases, the solver $S$ can check if shared secrets $\sigma_1, \sigma_2, \sigma_3, \sigma_4$ are correctly formed or not by the same procedure.

\[\square\]

### 4. Conclusion

We proposed a security model, referred as g-eCK model, for group key exchange that captures essentially all non-trivial leakage of static and ephemeral secret keys of participants, i.e., group key exchange version of extended Canetti-Krawczyk (eCK) model [35]. Moreover, we proposed the first one-round tripartite key exchange (3KE) protocol secure in the g-eCK model under the gap Bilinear Diffie-Hellman (gap BDH) assumption and in the random oracle model. The proposed protocol is efficient, i.e., is one-round and requires 4 shared secrets, 4 pairing operations, and 4 exponential operations (including the exponentiation for the ephemeral public key).

### References


In this section, we provide the proof of Theorem 4. We need
the gap BDH (Bilinear Diffie-Hellman) assumption, where one tries to compute BCDH(U, V, W) accessing the BDDH oracle. Here, we denote BCDH(g^a, g^b, g^{ab}) = e(P, P)^{ab}, and the BDDH oracle on input (g^a, g^b, g^{ab}, e(g, g)^{xy}) returns the bit 1 if \( xy = x \) and the bit 0 otherwise. We also need two variants of gap BDH assumption where one tries to compute BCDH(U, U, U) or BCDH(U, U, W) instead of BCDH(U, V, W). We call the first/second variant as the cubic/square gap BDH assumption, respectively. These two variants are equivalent to the gap BDDH assumption as well.

Given a challenge \( U \) of the cubic gap BDH assumption, one sets \( V = U', W = U'' \) for random integers \( s, t \in \mathbb{R} [1, p - 1] \), and then can compute BCDH(U, V, W)^{1/4} = BCDH(U, U, U). Given a challenge \( U, V, W \) of the gap BDH assumption, one sets \( U_1 = UVW = g^{s+tw}, U_2 = UVW^{-1} = g^{s-tw}, U_3 = UV^{-1}W = g^{n+tw}, U_4 = UV^{-1}W^{-1} = g^{n-tw} \) and then can compute BCDH(U, V, W) from BCDH(\( U_i, U_j, U_k \)) \( i = 1, \ldots, 4 \). One can show the equivalence of the square gap BDH assumption similarly.

Let \( k \) denote the security parameter, and let \( \mathcal{A} \) be a polynomially (in \( k \)) bounded adversary. We assume that \( \mathcal{A} \) succeeds in an environment with \( n \) users \( \{U_i\} \), at most \( s \) instances \( \{\Pi^l_{U_i}\} \) within a user \( U_i \). We use \( \mathcal{A} \) to construct a gap BDH solver \( \mathcal{S} \) that succeeds with non-negligible probability. The adversary \( \mathcal{A} \) is said to be successful with non-negligible probability if \( \mathcal{A} \) wins the distinguishing game with probability \( 1/2 + \rho(k) \), where \( \rho(k) \) is non-negligible, and the event \( M \) denotes a successful \( \mathcal{A} \).

Let \( \Pi^l_{U_i} \) be the test instance owned by user \( U_A \) with session id \( \text{sid}' \) = \( (P, U_A, A_0, A_1, U_B, B_0, B_1, U_C, C_0, C_1) \). Let \( \Pi \) be any completed instance owned by an honest user with session id \( \text{sid}' \) such that \( \text{sid} \neq \text{sid}' \). Let \( H^* \) be the event that \( \mathcal{A} \) queries \( (Z_1, \ldots, Z_m, \text{sid}'') \) to \( H \), where \( Z_1, \ldots, Z_m \) are correctly formed. Let \( \overline{H}^* \) be the complement of event \( H^* \). Since \( \text{sid} \) and \( \text{sid}' \) are distinct, the inputs to the key derivation function \( H \) are different for \( \text{sid} \) and \( \text{sid}' \). Since \( H \) is a random oracle, \( \mathcal{A} \) cannot obtain any information about the session key of test instance \( \Pi^l_{U_i} \) from the session key of instance \( \Pi \). Hence \( \Pr(M \land \overline{H}^*) \leq 1/2 \) and \( \Pr(M) = \Pr(M \land H^*) + \Pr(M \land \overline{H}^*) \leq \Pr(M \land H^*) + 1/2 \), and we have \( \Pr(M \land H^*) \geq \rho(k) \). Henceforth the event \( M \land H^* \) is denoted by \( M' \).

We will consider the non-exclusive classification of all possible events in the following tables. In the tables and hereafter, we denote by \( (A_0 = A, A_1 = X), (B_0 = B, B_1 = Y), (C_0 = C, C_1 = Z) \) the static and ephemeral public keys of users \( U_A, U_B, U_C \) in the test session \( \text{sid}' \). Events can be classified not exclusively as in Table A.1 when \( A, B, C \) are distinct, as in Table A.2 when \( A = B \neq C \), as in Table A.3 when \( A = C \neq B \), as in Table A.4 when \( A \neq B = C \), and as in Table A.5 when \( A = B = C \). Since the classification covers all possible events, at least one event \( E_{xy} \land M' \) in the tables occurs with non-negligible probability if event \( M' \) occurs with non-negligible probability. We will investigate each of these events in the following subsections, we provide detailed description for event \( E_{1a} \land M' \), which is the most difficult case, and outline for other events.

A.1 Event \( E_{1a} \land M' \)

A.1.1 Setup

The algorithm \( \mathcal{S} \) begins by establishing \( n \) honest users that are assigned random static key pairs. \( \mathcal{S} \) embed instance \( (U, V, W) \) of gap BDH as follows. \( \mathcal{S} \) randomly selects three users \( U_A, U_B, U_C \) and integer \( j \in [1, s] \). \( \mathcal{S} \) selects static and ephemeral key pairs on behalf of honest users with the following exceptions. The \( j \)-th ephemeral public key \( X \) is chosen to be \( U \), the static public key \( B \) is selected on behalf of \( U_B \) is chosen to be \( V \), and the static public key \( C \) is chosen to be \( W \), \( S \) does not possess the corresponding static and ephemeral private keys. We denote static and ephemeral public keys of user \( U_i \) by \( S_i \) and \( X_i \).

A.1.2 Simulation

\( \mathcal{S} \) activates \( \mathcal{A} \) on this set of users and simulates oracle queries as follows.

1. Send \( U_{\alpha}(`\text{start}') \), \( U_{\beta} \), \( U_{\gamma} \)): \( \mathcal{S} \) selects ephemeral private key \( X \) randomly, computes ephemeral public key \( X_i = g^{X_i} \), returns \( (P, U_i, U_i, X_i) \), and records it.

2. Send \( \Pi_{U_i}^l \), \( (P, U_i, U_j, U_k, X_i, X_j, X_k) \): if \( (P, U_i, U_j, U_k, X_i) \) is recorded, \( \mathcal{S} \) records instance \( \Pi_{U_i}^l \) is completed. Otherwise, \( \mathcal{S} \) records instance \( \Pi_{U_i}^l \) is not completed.

3. SessionKeyReveal(\( \Pi_{U_i}^l \), \( (P, U_i, S_i, X_i, U_j, X_j, X_k, X_{s}, X_{e}) \)): \( \mathcal{S} \) maintains list \( L_S \) of query \( \Pi_{U_i}^l \) and answered session key \( K \).

   a. If instance \( \Pi_{U_i}^l \) is not completed, \( \mathcal{S} \) returns error.

   b. Else if instance \( \Pi_{U_i}^l \) is recorded in \( L_S \), \( \mathcal{S} \) returns recorded session key \( K \).

   c. Else if \( (Z_1, \ldots, Z_m, P, U_i, S_i, X_i, U_j, S_j, X_j, U_k, S_k, X_k) \) is recorded in \( L_H \), \( \mathcal{S} \) verifies whether \( Z_1, \ldots, Z_m \) are correctly formed w.r.t. \( S_i, S_j, X_j, S_k, X_k \) or not by the procedure Check described below. If \( Z_1, \ldots, Z_m \) are correctly formed, \( \mathcal{S} \) returns session key \( K \) and records it in \( L_S \).

   d. Otherwise, \( \mathcal{S} \) returns random session key \( K \), and records it in \( L_S \).

4. \( H(Z_1, \ldots, Z_m, P, U_i, S_i, X_i, U_j, S_j, X_j, U_k, S_k, X_k) \): \( \mathcal{S} \) maintains list \( L_H \) of \( H \) query and answered hash value \( K \).

   a. If \( (Z_1, \ldots, Z_m, P, U_i, S_i, X_i, U_j, S_j, X_j, U_k, S_k, X_k) \) is recorded in \( L_H \), \( \mathcal{S} \) returns recorded hash value \( K \).

   b. Else if instance \( \Pi_{U_i}^l = (P, U_i, S_i, X_i, U_j, S_j, X_j, U_k, S_k, X_k) \) is recorded in \( L_S \) and \( \mathcal{S} \) records session key \( K \) and records it in \( L_S \).
described below. If $Z_1, \ldots, Z_m$ are correctly formed, $S$ returns recorded session key $K$ and records it in $L_H$.

c. Else if $(P, U_i, S_i, X_i, U_j, S_j, X_j, U_k, S_k, X_k)$ is corresponding to the test instance $(P, U_A, A, X = U, U_B, B = V, Y, U_C, C = W, Z)$, $S$ verifies whether $Z_1, \ldots, Z_m$ are correctly formed w.r.t. $S_i, S_j, S_k, X_i, X_j, S_k, X_k$ or not by the procedure Check described below using the knowledge of $a_0$. If $Z_1, \ldots, Z_m$ are correctly formed, $S$ computes the answer of the gap BDH problem by the procedure Extract described below using the knowledge of $a_0$. Then $S$ stops and is successful by outputting answer of gap BDH problem.

d. Otherwise, $S$ returns random hash value $K$, and records it in $L_H$.

5. $H_s(X_i)$: $S$ simulates random oracle in the usual way.
6. $\text{StateReveal}(\Pi_i^a)$: If ephemeral public key of instance $\Pi_i^a$ is $U$, then $S$ aborts with failure, otherwise responds to the query faithfully.
7. $\text{StaticKeyReveal}(U)$: If static public key of user $U_i$ is $V$ or $W$, then $S$ aborts with failure, otherwise responds to the query faithfully.
8. $\text{AddUser}(U_i, S)$: $S$ responds to the query faithfully.
9. $\text{Test}(\Pi_i^a)$: If ephemeral public key of the owner is $U$ and static public keys of the other users are $V, W$ in instance $\Pi_i^a$, then $S$ responds to the query faithfully, otherwise $S$ aborts with failure.
10. If $A$ outputs a guess $y$, $S$ aborts with failure.

(1) Extract.

The solver $S$ can extract the answer of the gap BDH instance as follows. By eliminating the terms including $a_0$ using the knowledge of $a_0$, solver $S$ can obtain

$$
\begin{align*}
\sigma_1' &= (\sigma_1/e(B_0B_1, C_0C_1)^{a_0})^{1/D} = g_T^a(b_b+b_t)(c_u+c_i), \\
\sigma_2' &= (\sigma_2/e(B_0B_1^E, C_0C_1)^{a_0}) = g_T^a(b_b+b_t)(c_u+c_i), \\
\sigma_3' &= (\sigma_3/e(B_0B_1, C_0C_1^F)^{a_0}) = g_T^a(b_b+b_t)(c_u+c_i), \\
\sigma_4' &= (\sigma_4/e(B_0B_1^E, C_0C_1^F)^{a_0})^{1/D} = g_T^a(b_b+b_t)(c_u+c_i).
\end{align*}
$$

By using these 4 linearly independent terms, solver $S$ can extract the answer $g_T^{a(b(u+c))}$ as

$$
((\sigma_1' \sigma_2' \sigma_3' \sigma_4')^{-1} (\sigma_1' \sigma_2' \sigma_3' \sigma_4')^{-1})^{1/(E-1)(F-1)} = g_T^{a(b_u+c_j)}.
$$

(2) Check.

The solver $S$ can check if shared secrets $\sigma_1, \sigma_2, \sigma_3, \sigma_4$ are correctly formed w.r.t. the static and ephemeral public keys by asking BDH oracle

$$
\begin{align*}
\text{BDDH}(A_0A_1^D, B_0B_1^D, C_0C_1^D, \sigma_1) &= 1, \\
\text{BDDH}(A_0A_1, B_0B_1^E, C_0C_1, \sigma_2) &= 1, \\
\text{BDDH}(A_0A_1, B_0B_1, C_0C_1^F, \sigma_3) &= 1.
\end{align*}
$$

$$
\text{BDDH}(A_0A_1^D, B_0B_1^E, C_0C_1^F, \sigma_4) = 1,
$$

and this implies $\sigma_1, \sigma_2, \sigma_3, \sigma_4$ are correctly formed.

A.1.3 Analysis

The simulation of $A$ environment is perfect except with negligible probability. The probability that $A$ selects the instance, where ephemeral public key of the owner is $U$ and static public keys of the other users are $V, W$, as the test instance $\Pi'$ is at least $1/(n^3 s)$. Suppose this is indeed the case, $S$ does not abort as in Step 9, and suppose event $E_{1a} \land M^*$ occurs, $S$ does not abort in Step 7 and Step 6.

Under event $M^*$ except with negligible probability, $A$ queries $H$ with $(P, U_A, U_A, A, X = U, U_B, B = V, Y, U_C, C = W, Z)$, Therefore $S$ is successful as described in Step 4c and does not abort as in Step 10.

Hence, $S$ is successful with probability $Pr(S) \geq p_{1a}/(n^3 s)$, where $p_{1a}$ is probability that $E_{1a} \land M^*$ occurs.

A.2 Other Events

Event $E_{1b} \land M^*$ can be handled same as the event $E_{1a} \land M^*$ in Sect. A.1, except that $S$ embeds gap BDH instance $(U, V, W)$ as $A = U, B = V, C = W$.

Event $E_{2a} \land M^*$ can be handled same as the event $E_{1a} \land M^*$ in Sect. A.1, except that $S$ embeds gap BDH instance $(U, V, W)$ as $X = U, Y = V, Z = W$.

Event $E_{2b} \land M^*$ can be handled same as the event $E_{1a} \land M^*$ in Sect. A.1, except that $S$ embeds gap BDH instance $(U, V, W)$ as $A = U, B = V, Z = W$.

Event $E_{3a} \land M^*$ can be handled same as the event $E_{1a} \land M^*$ in Sect. A.1, except that $S$ embeds gap BDH instance $(U, V, W)$ as $A = U, B = V, Z = W$.

Event $E_{3b} \land M^*/E_{3a} \land M^*$ can be handled same as the event $E_{3a}/E_{3b} \land M^*$, because of symmetry of $B$ and $C$.

A.3 Cases of Reflection Attack

In the case of $A = B \neq C$, events $E_{1a}^{1}, E_{2a}^{1}, E_{3a}^{1}, E_{3a}^{1}$ in Table A-2 can be handled same as events $E_{1b}, E_{2a}, E_{3b}, E_{3a}$ in Table A-1, with condition $A = B \neq C$ and square gap BDH assumption.

In the case of $A = C \neq B$, events $E_{1a}^{1}, E_{2a}^{1}, E_{3a}^{1}, E_{3a}^{1}$ in Table A-3 can be handled same as events $E_{1b}, E_{2a}, E_{3b}, E_{3a}$ in Table A-1, with condition $A = C \neq B$ and square gap BDH assumption.

In the case of $A \neq B = C$, events $E_{1a}^{2}, E_{2a}^{2}, E_{2a}^{2}, E_{2a}^{2}$ in Table A-4 can be handled same as events $E_{1b}, E_{2a}, E_{2b}, E_{2a}$ in Table A-1, with condition $A \neq B = C$ and square gap BDH assumption.

In the case of $A = B = C$, events $E_{1a}^{3}, E_{2a}^{3}$ in Table A-5 can be handled same as events $E_{1b}, E_{2a}$ in Table A-1, with condition $A = B = C$ and cubic gap BDH assumption.
Table A.1 Classification of events, when A, B, C are distinct. “ok” means the static key is not revealed, or a partnered instance exists and its ephemeral key is not revealed. “r” means the static or ephemeral key may be revealed. “r/n” means the ephemeral key may be revealed if the corresponding partnered instance exists, or no corresponding partnered instance exists.

<table>
<thead>
<tr>
<th>Event</th>
<th>A</th>
<th>X</th>
<th>B</th>
<th>Y</th>
<th>C</th>
<th>Z</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_{1a}$</td>
<td>r</td>
<td>ok</td>
<td>ok</td>
<td>r/n</td>
<td>ok</td>
<td>r/n</td>
</tr>
<tr>
<td>$E_{1b}$</td>
<td>ok</td>
<td>r</td>
<td>ok</td>
<td>r/n</td>
<td>ok</td>
<td>r/n</td>
</tr>
<tr>
<td>$E_{2a}$</td>
<td>r</td>
<td>ok</td>
<td>r</td>
<td>ok</td>
<td>r</td>
<td>ok</td>
</tr>
<tr>
<td>$E_{2b}$</td>
<td>ok</td>
<td>r</td>
<td>r</td>
<td>ok</td>
<td>r</td>
<td>ok</td>
</tr>
<tr>
<td>$E_{3a}$</td>
<td>r</td>
<td>ok</td>
<td>ok</td>
<td>r/n</td>
<td>r</td>
<td>ok</td>
</tr>
<tr>
<td>$E_{3b}$</td>
<td>ok</td>
<td>r</td>
<td>ok</td>
<td>r/n</td>
<td>r</td>
<td>ok</td>
</tr>
</tbody>
</table>

Table A.2 Classification of events, when $A = B \neq C$.

<table>
<thead>
<tr>
<th>Event</th>
<th>A</th>
<th>X</th>
<th>B</th>
<th>Y</th>
<th>C</th>
<th>Z</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_{1a}$</td>
<td>ok</td>
<td>r</td>
<td>ok</td>
<td>r/n</td>
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<td>r</td>
<td>ok</td>
<td>r</td>
<td>ok</td>
<td>r</td>
<td>ok</td>
</tr>
<tr>
<td>$E_{3a}$</td>
<td>ok</td>
<td>r</td>
<td>ok</td>
<td>r/n</td>
<td>r</td>
<td>ok</td>
</tr>
</tbody>
</table>

Table A.3 Classification of events, when $A = C \neq B$.

<table>
<thead>
<tr>
<th>Event</th>
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<th>X</th>
<th>B</th>
<th>Y</th>
<th>C</th>
<th>Z</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_{1a}$</td>
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<td>r</td>
<td>ok</td>
<td>r/n</td>
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<td>r/n</td>
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<td>r</td>
<td>ok</td>
<td>r</td>
<td>ok</td>
<td>r</td>
<td>ok</td>
</tr>
<tr>
<td>$E_{3a}$</td>
<td>ok</td>
<td>r</td>
<td>ok</td>
<td>r/n</td>
<td>r</td>
<td>ok</td>
</tr>
</tbody>
</table>

Table A.4 Classification of events, when $A \neq B = C$.

<table>
<thead>
<tr>
<th>Event</th>
<th>A</th>
<th>X</th>
<th>B</th>
<th>Y</th>
<th>C</th>
<th>Z</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_{1a}$</td>
<td>r</td>
<td>ok</td>
<td>ok</td>
<td>r/n</td>
<td>ok</td>
<td>r/n</td>
</tr>
<tr>
<td>$E_{2a}$</td>
<td>ok</td>
<td>r</td>
<td>ok</td>
<td>r/n</td>
<td>ok</td>
<td>r/n</td>
</tr>
<tr>
<td>$E_{3a}$</td>
<td>r</td>
<td>ok</td>
<td>r</td>
<td>ok</td>
<td>r</td>
<td>ok</td>
</tr>
</tbody>
</table>

Table A.5 Classification of events, when $A = B = C$.

<table>
<thead>
<tr>
<th>Event</th>
<th>A</th>
<th>X</th>
<th>B</th>
<th>Y</th>
<th>C</th>
<th>Z</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_{1a}$</td>
<td>ok</td>
<td>r</td>
<td>ok</td>
<td>r/n</td>
<td>ok</td>
<td>r/n</td>
</tr>
<tr>
<td>$E_{2a}$</td>
<td>r</td>
<td>ok</td>
<td>r</td>
<td>ok</td>
<td>r</td>
<td>ok</td>
</tr>
</tbody>
</table>

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