Elastoplastic Behaviour of Flat Grids

by

Nasrollah Dianat

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Abstract

The validity of any theoretical prediction can only be assessed in practice. The limitations are the range and precision of the observations.

Traditionally in the field of structural engineering, new mathematical idealizations or methods of analysis need to be evaluated or confirmed by experiments on test structures.

In the present work reported here, the elastoplastic behaviour of four grid structures has been recorded throughout an experimental study. The recorded data can be reliably used to evaluate the validity of the analytical techniques. The contents of this thesis are arranged in the following manner:

Part I, following the introduction, consists of Chapters 1-5 dealing with the experimental work. The grid structures were made to a high standard of precision from stress-free bars whose mechanical properties were obtained from a series of tension, torsion, and bending tests. Recent developments in experimental techniques and equipment were employed for accurate measuring and automatic data acquisition. Grids were gradually loaded under computer control far beyond the elastic limit. In addition to the electrical measuring systems, close-range photogrammetry was used to record the deformed shape of the grids.

In Chapter 6 the analytical technique employed to analyse the test grid is discussed, followed by the description of the computer program and results of analysis of a number of large square diagonal grids.

Chapter 7 is concerned with the results of experimental and theoretical studies on the test grids, every experimental curve being plotted along with the theoretically predicted curve. This comparison provides an example of how these experiments may be used to assess the validity of any relevant theory.

Chapter 8 deals with the conclusions and presents a practical analysis technique referred to as 'Structural Factoring'. This technique is a means for the elastic and elastoplastic analysis of large flat grids by the available computer, which is inevitably limited in capacity.
To the memory of my uncle Mohammad Dianat
The author wishes to express his gratitude and sincere thanks to Dr. H. Nooshin for his continuous help, advice and encouragement throughout this work.

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Introduction

Structural systems are studied with two aims. The first is to assist practical engineers, who have to design and build real structures in such a way that they are safe and economical. Such engineers need rapid, simple and self-checking procedures for predicting the stresses, deflections and collapse loads. The second objective is to develop a consistent and logical theoretical framework which will provide a general understanding of the behaviour of structures.

For structures whose behaviour under applied loads could reasonably be expected to lie in (or close to) the linear elastic range, mathematical methods of analysis are well proven and widely available. Current studies are therefore largely devoted to the other aspects of behaviour, plasticity is one of those aspects which has attracted the attention of many investigators.

Load-carrying capacity of a redundant structure of ductile material is rarely exhausted when the yield stress is attained. Excluding the possibility of buckling, collapse can only occur when plastic flow is taking place at two or more cross-sections simultaneously. By recognition of this fact and the possibility of utilizing ductility of materials to improve the design of engineering structures, the plastic methods were originated.

Today the methods of structural analysis concerned with plasticity are of two major classes: plastic analysis and elastoplastic analysis.

Plastic methods of analysis ignore all elastic deformations and assume that a structure does not distort at all until sufficient plastic regions have formed to convert it into a mechanism. Plastic methods are therefore concerned solely with structures at the point of collapse; they derive the
collapse load with no attempt to provide accurate data on displacements.

In elastoplastic methods of analysis the fact that the plastic hinges do not develop all at once is taken into account. Thus these methods trace the load-displacement behaviour of structures loaded beyond the elastic range up to the ultimate load. Such methods therefore do provide information on the displacements and internal stresses at every load level.

A grid may be defined as any plane network of interconnected beams with an external loading system composed of forces normal to the plane of the network and/or moments whose axes lie in this plane. Roof and floor systems, bridges and ship decks, elevated highways and raft foundations are examples of constructions in which grids are commonly used.

A considerable amount of work has been done on the study of elastic behaviour of grids (1-7) but little has appeared on the elastoplastic behaviour and collapse behaviour of this type of structure. A survey of the literature dealing with these aspects of behaviour of grids has been presented by the author in reference (8).

In the present work the experimental study of elastoplastic behaviour of four grid structures is reported, the aim being to establish facts which could be used to evaluate the validity of analytical techniques.

Analytical techniques deal with mathematical models. Simulation of a structure with a mathematical model is based on some idealizations and simplifying assumptions. Behaviour predicted by these techniques therefore deviates from the actual behaviour and we need to assess these deviations by the aid of observations on real structures.
Experimental data on elastoplastic behaviour of a structure could be obtained as it is loaded gradually beyond the elastic range up to the point of collapse. This is a costly process. Care should be taken therefore to obtain sufficient and accurate data on the initial structure as well as on its behaviour under load. In the present study the test grids were constructed with care; and recent developments in the experimental analysis techniques were employed for accurate measurement and automatic data acquisition.

Stress-free bars of rectangular solid sections were welded to each other to form the grids; great care was exercised to minimize probable residual stresses and to achieve the design geometry.

The loading system consisted of three servo-controlled hydraulic jacks matched to apply three equal loads, while tests were carried out under displacement control.

Measurement systems consisted of electric resistance strain gauges, and a system of LVDT-type displacement transducers. In addition close-range photogrammetry was employed to record the deformed shapes of the grids at some stages during the test.

The whole process of the test, including loading, measuring and recording, was controlled automatically, a mini-computer under program control being employed for the purpose.

To design and carry out the experimental study successfully, some knowledge of elastoplastic behaviour of grids proved necessary. A piece-wise linear analysis technique was therefore employed, and a computer program was written in FORTRAN.
During the test for each grid, a large number of measurements (17,000-22,000) were recorded and 6-12 pairs of stereo photographs taken. The recorded values were processed by a computer and a series of 'Load-Displacement' and 'Load-Moment' curves were plotted. Also, from the stereo photographs, 'Level Contours' were obtained.

These observations could be used reliably to assess deviation of any analytical results from actual behaviour. As an example, the analytical results from the above-mentioned program are compared with the experimental results.

For design purposes a new approximate technique referred to as 'Structural Factoring' is presented. This method may prove to be a general and economical means for elastic and elastoplastic analysis of large flat grids by the available computer, which is always limited in size. The technique is supported by the results of elastic and elastoplastic analysis of a number of large diagonal grids, carried out with the program.

The report on experimental work forms Part I of this Thesis. The analytical technique and the computer program are dealt with in Part II, where the results and conclusions are also presented. The data from tension, torsion and bending tests carried out to obtain the mechanical properties of grid members are given in Appendices B, C and D. Appendix A is concerned with the specifications of strain gauges and calibration of bridge outputs used throughout the experimental study.
Part I

Experimental Work
GENERAL REMARKS

The aim of the experimental study was to produce some facts which could be used to evaluate the validity of analytical techniques. Therefore, in the design of the set up the main considerations were:

1. To obtain information over a wide range of grid structures regarding the difficulties and the effort needed in experimental studies.
2. To have accurate information about the grid structures and the boundary conditions.
3. To obtain accurate measurements during the test.
4. To record sufficient information during the test.

The experimental set up of which Fig. I.1 is a picture, consisted of: grid structures; test frame; loading system; measurement systems.

Four grid structures of the same layout but of two different member cross-sections were tested under two different loading arrangements. The layout was determined by the number of the supports and by analytical studies. In order to eliminate any residual stresses which might be caused by imperfections in the supports, it was decided to have three supports and, by analytical studies on some triangulated three-way grids, the layout of Fig.I.2 was chosen. This layout had three axes of symmetry, and the reliability of measurements could be assured by comparison of measurements on symmetric joints and members.

The overall dimensions of the grids and the cross-sectional dimensions of the members were determined thus:

1. Structure to be small enough to be made and handled under laboratory conditions and large enough to get reliable measurements.
2. The load-carrying capacity of the grid to be well within the range of the available loading system.
Fig. I.1

The experimental set-up
3. To eliminate the possibility of lateral buckling.

4. The difference between the torsional rigidities of the two sections to be large.

The joints had to have very good rigidity (fixity) and their dimensions had to be relatively small. Studies on different types of joints proved that, regarding the dimensions of the grid and members cross-section, welded joints were the only answer.

Necessary information about the grid consisted of data on its geometry, its material and residual stresses throughout the grid. To obtain this information accurately was an important part of the experimental study,
because these data provided a sound base for the evaluation of the experimental results. There was no easy way of measuring residual stress. Stress-free bars were therefore welded, to construct the grids, and care taken to minimize the stresses which might have been induced in the process of welding. The following two Chapters will report on the grid structures, first on the material and mechanical properties of the members and, second, on the construction of the grids. Chapter 3 is concerned with the test frame and loading system, Chapter 4 deals with the measurement systems, and the test procedure is reported in Chapter 5.
Chapter 1

MATERIAL AND MEMBERS' PROPERTIES

1.1 Material

Bright mild steel bars were used as members of the grid structures. To study the elastoplastic behaviour of the grids, mild steel was a proper choice of material, and the bright finish was necessary to obtain the accuracy on the cross-sectional dimensions. Bright finish of steel bars is produced by cold rolling; this process increases the yield stress and hardness of the steel, and induces some residual stresses, which in this study were undesirable and therefore the material had to be annealed. All bars of the same cross-section came from the same batch.

1.2 Annealing

Stress relief annealing was carried out at the Department of Metallurgy and Materials Technology of the University of Surrey, in an air circulating electric furnace (WILD BARFIELD) Fig.1.1. Members and test specimens which had been cut and machined to size, were positioned in the furnace vertically, to avoid deformation at high temperatures; there was adequate spacing between adjacent bars, Fig. 1.2 and Fig. 1.3. For this purpose specially made cages were employed. Air circulation resulted in even temperature all over the furnace. The furnace was large enough to accommodate only one third of all the bars and necessary test specimens for a grid. However, check on the temperature variation of the loaded furnace with time proved the excellent repeatability of the process.
Fig. 1. Air circulating furnace

Fig. 1.2 Arrangement of bars in furnace

Fig. 1.3 Arrangement of bars in furnace
The annealing process was as follows:

1. Furnace was loaded at room temperature.
2. The temperature control was set to 650°C and the furnace was switched on.
3. When 650°C temperature was reached, it was held for one hour.
4. The temperature control was then set to 600°C for one hour.
5. The furnace was switched off and the bars were left in to cool down to room temperature.

The stress-strain curves derived from tension tests on one of the materials before and after annealing are given in page 20.

To obtain the necessary information about the mechanical properties of the materials the following tests: (a) tension test; (b) torsion test; (c) bending test, were carried out. However, before these tests are discussed the effect of straining rate on the yield stress of mild steel will be considered.

1.3 Straining Rate

As reported by Manjoine(9); Leblois and Massonnet (10), among others, the effect of increasing the strain rate is generally to increase the yield stress of mild steel. Thus to have compatible results from different tests on the same mild steel, this effect should be taken into account. In a test on a complex structure in which, obviously, not all parts will deform with the same strain rate, the strain rates should be kept within a range, in which the variation of yield stress is minimum. In the present study this matter was carefully considered.
STRESS - STRAIN curve derived from tensile test on a bar of rectangular solid section of nominal dimension of 30x25mm specimen G

This is a test on the material before being stress relieved

STRESS - STRAIN curve derived from tensile test on a bar of rectangular solid section of nominal dimension of 30x25mm specimen C

This is a test on the material after stress relieving
1.4 Tension Test

Tension tests were carried out to obtain the following information about the materials:

1. The value of modulus of elasticity, $E$.
2. The value of Poisson's ratio, $v$.
3. The upper and lower values of the yield stress, $\sigma_{yu}$ and $\sigma_{yf}$, respectively.
4. The stress-strain curve.

Fig. 1.4 shows the experimental set-up. It consisted of:
- test machine, test specimen, strain transducers, recording and control system.

1.4.1 Testing Machine - A servo controlled 500 kN hydraulic machine made by SATEC Systems Ltd., was employed. The load verification of this system was to grade A of BS1610-1967. But to enable automatic recording of the load signal, it was fed to a data logger and the final output was calibrated with a Mayes 500 kN load cell having 0.25% accuracy of applied load. There was excellent agreement between the values obtained from this load cell and the corresponding values displayed by the machine. The machine was equipped with wedge grips which were suitable for mild steel, and using proper liners care was exercised to achieve good alignment of the specimen.

1.4.2 Test Specimens - Specimens were full size and taken at random from the annealed bars for the grids, one specimen from the bars annealed in a batch, a total of six specimens for either of the sections. Fig. 1.5. End pieces were welded in a jig very similar to the members being welded in the grids, and straight specimens were obtained. The dimensions of the cross-sections were measured by a set of 0.001mm accuracy micrometers.
Fig. 1.4 Tension test set-up

Fig. 1.5 Tension specimens; A-F are stress relieved.
1.4.3 Strain Transducers - To obtain E and ν values, strain should be measured with high accuracy (with class A extensometer ASTM E83-67). In these tests precision electric resistance strain gauges produced by M-M (Micro-Measurements) were used. In four of the six specimens of each material two pairs of gauges were used, one pair for axial strain and the other for transverse strain. Gauges were installed on opposite sides of specimens. In the other two specimens an additional pair of strain gauges were used for axial strain, this pair of gauges being installed on the other sides of the specimens. Fig. 1.6. A full bridge arrangement for each pair of gauges was employed. The inactive gauges were of the same type as the active ones and were installed on similar specimens and located close to the specimen under test. Fig. 1.7. The specification of the gauges, the calibration of output signal and nonlinearity correction of the system are reported in Appendix A. In the specimens with three pairs of gauges the difference between the two values obtained for axial strain was at most 10 micro strains.

In addition to the strain gauges a dual extensometer using LVDT type displacement transducers, with a gauge length of 50mm, made by R.D.P. Howden was employed. Fig. 1.7. The calibration of this extensometer was carried out to an accuracy better than grade C (ASTM E 83-67).

The recording and control system is explained in Chapter 4.

1.4.4 Testing - When the specimen was set in the machine and instrumentation completed, the whole system was left for some time to reach a stable temperature. Then at zero load the signals of all transducers were measured and the values were recorded. Loading under a constant rate of extension was then started, and, every 20 seconds, a set of measurements was obtained and recorded under program control. Each set of measurements consisted of three readings from
Fig. 1.6 Strain gauge locations on a tension specimen.

Fig. 1.7 Dual extensometer on a specimen.
each transducer; if the difference between any two of these three values was found to be more than a given tolerance, another set of measurements was obtained immediately. The loading continued through the strain-hardening region up to 3% strain. All values were recorded on punched paper tape, and during the test the operator was informed of the test situation by approximate values of load, stress, strain, modulus of elasticity and Poisson's ratio, which were printed on teletype. The ambient temperature during the test was constant and was checked by a digital thermometer and recorded by the system, once for each set of measurements. Also to obtain the actual strain rate the time was recorded.

1.4.5 Results - The data obtained from the test were analysed by a computer program. The stress and strain values were calculated and systematic corrections carried out. The desired information about the materials was obtained as follows:

1. Modulus of elasticity $E$: In the stress-strain coordinate system a line was fitted by the least squares method to all points below a given stress limit. The points which had a strain deviation from this line greater than 5 $\mu$ strains were eliminated and another line was fitted to the remaining points. If the slope of this line differed from the previous one by more than 0.5%, the process was repeated until the difference between two successive slopes was found to be equal to or less than 0.5% (ASTM E111-61).

2. Poisson's ratio $\nu$: This was obtained according to ASTM E 132-61 but $\frac{d\varepsilon}{dp}$ and $\frac{d\sigma}{dp}$ were found as explained above. The $G$ value also was calculated from $G=E/2(1+\nu)$. 
STRESS - STRAIN curve derived from tensile test on a bar of rectangular solid section of nominal dimension of 30*25mm specimen B

Modulus of Elasticity $E = 201000$ N/mm$^2$
Poisson's Ratio $\nu = 0.283$
Shear Modulus $G = 78000$ N/mm$^2$
Lower Yield Stress $\sigma_y = 363.56$ N/mm$^2$
Upper Yield Stress $\sigma_u = 575.26$ N/mm$^2$
Rate of Strain in/on linear part $\dot{\varepsilon} = 1.67 \mu$ strain/sec
Cross Sectional Area $A = 739.30$ mm$^2$
3. The upper value of yield stress was obtained as the maximum stress at which the strain deviation from the line of slope E was not more than 10 μ strains. The stress values of those points which were on the yield plateau were averaged to give the lower yield stress.

4. Stress-Strain curve was plotted by the microfilm facilities of the U.L.C.C. (University of London Computing Centre).

Page 26 presents the above information for a typical specimen; for other specimens see Appendix B.

The average of E, ν, G, σ_{y,α}, σ_{y,β} values obtained from six specimens and their corresponding standard deviations for both materials are given in Table 1-1 at the end of this Chapter.

1.5 Torsion Test

Torsion tests were carried out to obtain the following information about the materials:

1. The value of shear modulus of rigidity, G.
2. The value of yield stress at shear, τ_y.
3. The Torque-Twist curve.

Fig. 1.8 shows the experimental set up, consisting of:
- testing machine; test specimen; twist gauges; recording and control system.

1.5.1 Testing Machine - A machine made by Additional Equipment Company was used to apply torque to the specimen. Rotation of end 'B' Fig.1.9, was prevented by a proving lever, one end of which was fixed to the shaft, the other end resting on a load cell, loading it axially.
Fig. 1.8 Torsion test set-up

Fig. 1.9 Torsion testing machine
This system was used to measure the resisting torque. The load cell was of type D012/C, of range 0-1000 lbf (0-4.5kN) made by Coutant Transducers. It was energized by a d.c. pulsing current and the output was calibrated to an accuracy better than 1% of the applied load. The leverage arm was measured to an accuracy of 0.06mm. The machine was equipped with fixed jaws and specimens were machined accordingly, Fig.1.10. A very good alignment being obtained; the end 'B' of the shaft was free to slide in its bearing.

1.5.2 Test specimens - Six specimens were machined for each of the two materials and each was annealed along with one annealing batch. Fig.1.11, shows these specimens. A fine finish was obtained for installation of strain gauges, and their diameters were measured with a micrometer of accuracy 0.001mm.

1.5.3 Twist Gauges - To obtain the G value; strain should be measured with high accuracy. In these tests precision electric resistance strain gauges were used. A full bridge arrangement of four active arms was employed, Fig.1.12, and special arrangements made for the accurate positioning of the gauges. The specification of the gauges and the calibration of the output signal is given in Appendix A.

In addition to the strain gauges the angle of twist over a gauge length of 75mm was measured by an accurately made twistmeter, employing LVDT type transducers, Fig.1.9 and Fig.1.10. The results of these two independent measurements were found to be in very good agreement.

The recording and control system is explained in Chapter 4.
Fig. 1.10 Machine Jaws, test specimen and twistmeter

Specimens from 25x30mm Section

Fig. 1.11 Tested Torsion Specimens

Specimens from 35x16mm Section
1.5.4 Testing - When the specimen had been set in the machine and instrumentation completed, the whole system was left for some time to reach a stable temperature. Then at zero torque, the signals of all transducers were measured and the values were recorded. Then the application of torque under a constant rate of rotation was started, and every 10 seconds a set of measurements was obtained and recorded under program control. Each set of measurements consisted of three readings from each transducer. If the differences between any of these three values was found to be more than a given tolerance, another set of measurements was obtained immediately. All values were recorded on punched paper tape, and during the test the operator was informed of the test situation by approximate values of torque, maximum stress, twist and G which were printed on teletype. The ambient temperature during the test was constant and was checked by a digital thermometer and recorded by the system, once for each set of measurements.

1.5.5 Results - The data obtained from the test were processed by a computer program. The torque and twist values were calculated and systematic errors corrected; the desired information about the materials was then obtained as follows:

1. Shear modulus of rigidity G was obtained in the same way as E value, (see sub-section 1.4.5) according to ASTM E143-61.
2. Yield stress at shear $\tau_y$ was obtained from $\tau_y = \frac{2T_{max}}{\pi r^3}$, where $r$ is the radius of specimen, $T_{max}$ is maximum torque for which the twist deviation from the line of slope G in torque twist coordinates is not more than $10^{-5}$ radian/mm.
3. Torque-twist curve, was plotted by the microfilm facilities of U.L.C.C.
TORQUE - TWIST curve for a specimen of circular section of 15mm diameter machined from a bar of 30*25mm section, specimen A
Fig. 1.12 Installation of twist gauges and bridge arrangement

Fig. 1.13 Bending test set-up
Page 32 presents the above information for a typical specimen; for other specimens see Appendix C.

The average of G, τ values obtained from six specimens and their corresponding standard deviations for both materials are given in Table 1-1 at the end of this Chapter.

1.6 Bending Test

Bending tests were carried out in order:
1. To obtain the Moment-Curvature curves for both materials.
2. To calibrate the strain gauges which were to be used for the measurement of bending moment in grid members.

The test set up consisted of: test frame; test specimen; loading system; transducers; recording and control system.

1.6.1 Test Frame - Fig 1.13 illustrates the test frame when a specimen was being tested. This shows a four-point load beam test. The system was designed to maintain symmetry about the plane A-A throughout the test.

Fig. 1.14 shows one of the supports. The support axis lay on the beam's neutral plane, parallel to the plane of symmetry (the axis of cylindrical surface of element A). Friction-free movement of this axis along the beam axis, and the beam's rotation about the support axis were both provided. The support reaction was always vertical.

Fig. 1.15 illustrates one of the loading heads, the load application axis lay on the beam's neutral plane, parallel to the plane of symmetry, (the axis of cylindrical surface of element A).
Fig. 1.14 Support arrangement in bending test

Fig. 1.15 Loading head in bending test
1.6.2 Test Specimens - Each specimen consisted of three parts. Fig. 1.16
The middle part which was to be studied was taken at random from the annealed bars for the grids. Lateral parts were of the same cross-section, which were welded to the middle part in a jig, rather as the members were welded in the grids and an almost straight specimen was obtained. For either type of section three specimens were made.

1.6.3 Loading System - Two servo-controlled hydraulic actuators were matched to apply equal loads while the test was carried out under displacement control. The accuracy of load control and measurement was better than ± 0.5% and the accuracy of displacement control and measurement was ± 1mm. This system is explained in Section 3.2. Unconstrained horizontal movements of loading points were provided by the slight pivoting of the actuators, which was made possible by the design of their bases, Fig. 1.17.

1.6.4 Transducers - Precision electrical strain-gauges of two types were installed on the specimens, Fig. 1.18. These gauges were to be calibrated for measurement of bending moment in grid members. A half-bridge arrangement having two active arms was employed. The bridge was completed by a highly stable half-bridge. The specification of the gauges and the calibration of the output signal are given in Appendix A.

In addition to the strain gauges the curvature of the specimen was measured at two places over a gauge length of 50mm using two accurately made curvature meters, employing LVDT type transducers, Fig. 1.19. The values of curvature obtained from the strain gauges were found to be in very good agreement with the values obtained from the curvature meters.

The recording and control system is explained in Chapter 4.
1.6.5 Testing - First, 'A' elements of Fig.1.14 and Fig.1.15 were fixed to the specimen; these elements also determined the load and reaction positions. So, to locate them accurately, specially made gauges of accuracy 0.01mm (at 20°C) were employed. When the specimen was set in the frame and the instrumentation completed, the whole system was left for some time to reach a stable temperature. At zero load, the signals of all transducers were measured, and the values recorded. Then, under a constant rate (1mm in 60 seconds) a vertical displacement was applied to the specimen at load points. After a few seconds a set of measurements was obtained and recorded under computer control. Each set of measurements consisted of three readings from each transducer; if the difference between any of these three values was found to be more than a given tolerance, another set of measurements was carried out immediately. Then another 1mm displacement was applied and a new set of measurements obtained and recorded as before. The process was continued as far as the limitations of the set-up allowed, Fig.1.17. All values were recorded on punched paper tape and during the test the operator was informed of the test situation by approximate values of loads, displacements, bending moments, curvatures, ..., which were printed on teletype. The ambient temperature during the test was constant and was checked by a digital thermometer and recorded by the system, once for each set of measurements.

1.6.6 Results - The data obtained from the test were processed by a computer program and curvature values at five stations on the specimen were calculated and systematic errors corrected. A graphical presentation of the results is shown on page 42. This graph, which is produced with the microfilm facilities of the U.L.C.C. is for a typical specimen. For other specimens
Fig. 1.16 Tested bending specimens

Fig. 1.17 Here, both actuators have tilted towards the plane of symmetry A-A; gaps in the bases are shown by arrows.
Fig. 1.18 The pairs of gauges marked 3 and 4 were of type EA-06-250BF-350 and the pair of gauges marked 5 were of type EP-08-250BF-350. (see Appendix A).

Fig. 1.19 Curvature-meters located on specimen.
see Appendix D. The graph contains the experimental moment-curvature curves at five stations along the beam and also the curve obtained by the classical theory of bending. According to this theory the variation of curvature with bending moment is given by: 

$$K = \frac{M}{EI}$$

(where: $K$ is the curvature, $M$ is the applied bending moment, $E$ is the modulus of elasticity, and $I$ is the second moment of area). The relation is applicable until stress at top and bottom fibres reach the value of the lower yield stress or $M$ becomes equal to $M_e$, the maximum elastic moment. Then the curvature is obtained by:

$$K = \frac{2\sigma_y l}{EH} \cdot \frac{M_e}{\sqrt{3M_e - 2M}}$$

where $\sigma_y$ is the lower yield stress and $H$ is the height of the section.

The bending moment at the limit of proportionality found to be higher than $M_e$, this is as explained by Leblois and Massonnet (10), because of the upper yield stress.

1.7 Mechanical Properties

The mechanical properties of the materials obtained from tension and torsion tests on six specimens of each material are presented in Table 1-1.

### Table 1-1 Mechanical Properties of the Materials

<table>
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<tr>
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<th>TENSION TESTS</th>
<th></th>
<th></th>
<th></th>
<th>TORSION TESTS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\sigma_y$ N/mm²</td>
<td>$\sigma_{yu}$ N/mm²</td>
<td>E N/mm²</td>
<td>$\nu$</td>
<td>G N/mm²</td>
</tr>
<tr>
<td>Mean</td>
<td>196.6</td>
<td>206.5</td>
<td>199500</td>
<td>0.274</td>
<td>78330</td>
</tr>
<tr>
<td>S. D.</td>
<td>3.85(2.0%)</td>
<td>3.37(1.8%)</td>
<td>164.00(0.8%)</td>
<td>0.007(2.5%)</td>
<td>516.00(0.7%)</td>
</tr>
<tr>
<td>Mean</td>
<td>365.9</td>
<td>337.5</td>
<td>201000</td>
<td>0.282</td>
<td>78670</td>
</tr>
<tr>
<td>S. D.</td>
<td>7.0 (1.9%)</td>
<td>8.6 (2.3%)</td>
<td>632.00(0.3%)</td>
<td>0.001(0.4%)</td>
<td>516.00(0.7%)</td>
</tr>
</tbody>
</table>
As it may be seen, the difference between the two values of G for each material one obtained from tension tests and the other from torsion tests, is less than one percent.

Each of the graphs on page 42 and page 43 may be taken as representative of bending tests results on one material.
MOMENT CURVATURE curve for a beam of rectangular solid section of nominal dimension of 30x25 mm specimen B

B.M. at proportional limit = 1209.59 Kn.mm
The Maximum Rate of Straining = \( \epsilon' = 2.84 \ \mu \text{strain/sec} \)
for linear part

- Width of Beam \( B = 50.05 \ \text{mm} \)
- Height of Beam \( H = 24.83 \ \text{mm} \)
MOMENT CURVATURE curve for a beam of rectangular solid section of nominal dimension of 16.35 mm specimen B

Bending Moment in 10^6 KN.mm

Curvature in 10^-5 mm

1. Curvaturemeter
2. Curvaturemeter
3. Strain Gauge
4. Strain Gauge
5. Strain Gauge

Classical Bending Theory

B.M. at proportional limit = 781.92 KN.mm

The Maximum Rate of Straining ε' = 5.45 μ strain/sec

(on linear part)

Width of Beam B = 15.93 mm
Height of Beam H = 34.84 mm
Chapter 2

CONSTRUCTION OF THE GRIDS

2.1 General Remarks

Stress free bars were to be welded to each other to form the grids such that:

1. The resulting geometry to match the design geometry within the close tolerances.
2. The residual stresses throughout the members and joints to be negligible.

Considerations involved in satisfying the above conditions are as follows:

1. The volume of the weld metal to be minimized and balanced symmetrically about the neutral plane of the grids.
2. Weld passes on both sides of the neutral plane to be balanced.
3. Work to be positioned for downhand and horizontal welding.
4. Grids to be divided into smaller units and proper jigs (fixtures) to be used.
5. A welding sequence to be followed.

In this Chapter the joint design, welding procedure, jigs, assembling of the units and assembling of the grids are described.

2.2 Joint Design and Welding Procedure.

Joint designs for both types of grids are given in drawings on pages 45 and 46. Clearly for given bars these joints were the smallest possible, and regarding the shape (double-V groove weld),
Drawing of a test grid with member X-section 30x25 mm
Drawing of a test grid with member X-section 16x35 mm
included angle and root opening, the volume of the weld metal was minimum and was symmetrically balanced about the centre of the thickness. The weld at either side was deposited in three or five layers (depending on grid type), each layer annealing the previous one, but before applying a further layer to one side the other side was welded up to the same stage. The weld stresses caused by the last layers of welds were largely eliminated when the excess weld was machined off.

The joint element was cut and machined to size from bright mild steel bars of hexagonal cross-section. Its centre hole was drilled and reamed. Fig. 2.1. Then its 45° sides were machined. Fig. 2.2.

Semi-automatic Gas Metal-Arc Welding Process was employed using mild steel electrode of BOC Bostrand LW1 of 1mm diameter and shielding gas of BOC Argo shield 20 (Argon + 20% CO₂ + 2% Oxygen). The speed of the electrode wire was 7-8 m/min, and the rate of gas flow was 2 m³/h. The amperage and voltage as shown on the welder were respectively 200-220 A. and 21-20V.

The joint design and welding procedure were studied and improved by experiments on a good number of sample joints. The inspection of sample joints was mainly visual and carried out on each weld pass, finished joint and its cut surfaces at different sections, Fig. 2.3. In addition the x-ray radiographs from one sample was studied. When sound joints were repeatedly obtainable the construction of grids was started.

The weld shrinkage at a joint in the direction of any bar was measured to be 0.3 to 0.4mm, and was taken into account in bar length and root opening measurements.
2.3 Jigs

As is shown on the drawings on pages 45 and 46, either type of the grids could be divided into ten small units of four types. If a jig was made for assembling the unit I of either type, it could be employed for assembling of the other units of that type. But the geometrical difference between the two types of grid was in the cross-sectional dimensions of the members. A jig was therefore designed for assembling the units of both types of grids. This jig will be denoted as Jig U. To assemble these units to form the grids another jig was necessary, which will be denoted as Jig G.

2.3.1 Jig U - Fig. 2.4 is a general view of this jig with one member and one joint element of either type placed in its bottom half.

In the design of Jig U the main considerations were:
1. Bars and joint elements to be positioned accurately.
2. Elongation and shrinkage of the bars and their limited rotation about their axes to be allowed.
3. Measurements to be adjusted for the weld shrinkage.
4. Easy rotation of the jig about a horizontal axis to be provided. Downhand and horizontal welding could then be carried out for every layer at either side of the midplane.

Page 52 gives a drawing of this jig. To make the jig, seven holes shown in Fig. 2.5 were first drilled in the two plates forming the main body of the jig which had been previously spot-welded to each other. The centre of these holes were used as basic geometrical points for measurements in making and assembling the jig. The accuracy obtained in the centre-to-centre distance of any two corresponding holes was better than 0.1mm.
Fig. 2.1 Joint element as cut from a hexagon bar and drilled.

Fig. 2.2 Joint element in final form.

Fig. 2.3 Some of the sample joints
These plates were too big for the available jig boring machines, so a triangular jig shown in Fig. 2.6 was employed. This jig consisted of three bushes having centre-to-centre distances of 400.3 ± 0.02mm.

Finally the positions of these reference points were transferred to seven detachable plates C, Fig. 2.4 (these plates were used to position the joint element, see section 2.4). Then the semi-circular and circular pieces at the corners and centre of the jig's plates were removed to allow full access to each side of the joint for welding.

In making this jig a high degree of symmetry and a large number of similar parts helped to achieve high accuracy. This was also aided by the use of additional jigs for the production of individual parts.

2.3.2 Jig.G. - In the design of this jig the main considerations were:
1. Accurate positioning and alignment of the units.
2. Measurements to be adjusted for weld shrinkage.
3. Easy rotation of the jig about a horizontal axis to be provided. Downhand and horizontal welding could then be done for every layer at either side of the grid's midplane.

Fig. 2.7 is a general view of this jig while units of a grid are positioned to be welded, and Fig. 2.8 illustrates the rotational facility.

When the frame of the jig was made, holes 1-10 of Fig. 2.9 were drilled in it, for positioning of the units. To drill these holes, the triangular jig of Fig. 2.10 was used. But to do so, 6 auxiliary holes in 6 detachable pieces 'A' had to be drilled. Fig 2.9 The accuracy of the centre-to-centre distances of any corresponding two of these ten holes was found to be better than 0.1mm.
Fig. 2.4 A general view of jig U with one member and one joint element of either type placed in its bottom half.
Fig. 2.5 The centres of holes 1-7 are the geometrical reference points for constructing the jig U.

Fig. 2.6 The triangular jig used when drilling the holes shown in Fig.2.5
2.4 Assembling the Units

Jig U was employed to assemble the units, as follows:

1. Plates C Fig. 2-4 giving the positions of joints were attached to the jig.

2. The stress-relieved bars with polished ends were positioned through the slots of the jig, and the joints elements were set at their positions on plates C.

3. At one or more joints the root openings were set to a predetermined gap with a feeler gauge, and the first pass weld on side A; Fig. 2.4 was made for all bars meeting at these joints in a sequence to minimize the distortions and stresses caused by welding.

4. The jig was rotated half a turn and plates C at the welded joint were detached and the first pass weld on side B was deposited.

5. Then second, third, ..., welds at either side of the mid-plane were deposited in turn (first on side A).

Fig. 2-12 and Fig. 2-13 show a joint of each type at different stages of welding.

The welding process was carried out under continuous supervision, each pass being inspected followed by any necessary repairs and improvements before the next weld was deposited.

In the case of grids of 16 x 35mm member's cross-section, in units I & II, the welding of corner joints was carried out to the level of the joint element, Fig 2.12. In all other joints after any weld was deposited over the joint element obscuring the centre hole the continuation of the hole in the weld layer was drilled from the other side.
Fig. 2.7 A general view of jig G with grid units positioned for welding

Fig. 2.8 Rotational facility of jig G
Fig. 2.9  PLAN OF JIG 'G'
Fig. 2.10 The jig used to set the grid geometry on jig G.

Fig. 2.11 Welding stages in Unit I
2.4.1 Welding Sequence - In the case of units III & IV (single jointed units), when the welding was completed, the unit was left in the jig to cool down. During this time in which weld shrinkage was taking place, all bars were free to move and no axial stresses were expected.

In the case of Unit I the welding was carried out in three stages Fig. 2.11. First the welding of the centre joint was completed and the weld was left to cool down; at this stage the movement of bars caused by weld shrinkage was not restricted. Then the welding of three alternate corner joints was completed and the welds were left to cool. At this stage the jig did not allow any change of centre-to-centre distances between these three corner joints and the centre joint. Weld shrinkage at these joints therefore caused axial stresses in the radial bars welded to the joints. Then the welding of the other three joints was completed and the welds were left to cool. At this stage all three members meeting at each of these joints had to bear the axial stress caused by weld shrinkage, as happened with the radial bars at the previous stage. Therefore the welding condition for all bars was almost the same and, regarding the geometry, the stresses in the bars must have been nearly the same; when the unit was removed from the jig, elastic deformation took place relieving the stresses. However, because of the imperfections in the jig and any small differences between the welding conditions at different joints some stresses must have been left in the bars; these were not expected to be large, especially after the excess weld had been machined off.

In the case of unit II the welding sequence was the same as unit I and it could be shown that the stress caused by welding was not large.
Fig. 2.12 This figure and Fig. 2.13 illustrate a joint of each type at different stages of welding, each showing the stages on one side of the joints' mid-plane.
Fig. 2.13 This figure and Fig. 2.12 illustrate a joint of each type at different stages of welding, each showing the stages on one side of the joints' mid-plane.
2.5 Assembling the Grids

The prepared units for each grid were assembled in the following way:

1. The units were positioned on the jig $G$, each by a pin fitted through the relevant joint into the corresponding hole of the jig.

2. The free ends of members of each unit were clamped to the adjacent units as shown in Fig. 2.14 and Fig. 2.15. The clamps were designed to allow reasonable access for ease of welding.

3. Shimming was carried out under the units until the desired flatness for the grid was achieved. The flatness was checked in all directions by a 2m straight edge and a feeler gauge.

4. By a length of cotton on the three corner joints, three lines were set giving the edges of the grid. With the aid of these lines the alignment of the outer units was checked and adjustments were made to achieve the best compromise. Then by a straight edge the centre unit was aligned. The units were then clamped down to the jig. Fig. 2.7.

5. To minimize the distortion and any stress build up, first, all units of type III were welded to Unit I, and each of the units of type IV was welded to its adjacent unit of type II. Then three parts $b$ were welded to the part $a$. Fig. 2.16 At both stages the movement of parts was effectively prevented by clamps, and any pair of symmetric welds of each layer was deposited in sequence. At stage two, the first three pairs were the nearest to the axes of symmetry and the last three pairs were the farthest. Obviously, before any further layer of weld was applied to one side of the joints, the jig was
Fig. 2.14 Clamp at an edge joint used for assembling grids.

Fig. 2.15 Clamps at an inner joint used for assembling to the grids.
rotated half a turn and the other side was welded to the same stage. At the end of each stage the axial stresses, which were almost equal for all welded bars, were relieved by removing the clamps.

The tolerances in the diameter of the positioning pins and root opening were chosen to give acceptable tolerances in the lengths of members, the coordinates of joints and the overall dimensions of grids, allowing for weld shrinkage. The final check showed ±0.3mm accuracy on member lengths, ±0.6mm on the x, y coordinates of joints and ±1mm accuracy on overall dimensions had been achieved.

In the case of grids of 16 x 35mm member's cross-section, after any weld was deposited over the joint element obscuring the centre hole, the continuation of the hole in the weld layer was drilled from the other side.

The excess weld was machined off from the joint and a flatness check with a 2m straight edge after machining showed closer tolerances (maximum -0.4mm). Regarding the size of the grid, special machining arrangements proved necessary.
Fig. 2.16 The joining sequence for units: they were welded to form parts a and b first, and then by joining these parts the grid was constructed. (see page 61).
Chapter 3

TEST FRAME AND LOADING SYSTEM

3.1 Test Frame

The test frame consisted of a base, three columns, three supports and fixtures for the displacement transducers, Fig. 3.1.

3.1.1 Base - The base was designed to be very stiff, as it was used as the origin for displacement measurements. An analysis was carried out having the base supported at three corners for different loading cases, and the deflections were found to be undetectable with the measuring instruments used. A drawing of the base is given on page 66.

The geometry of the grid structure was transferred to the base by the centres of 28 holes of $\frac{1}{2}$" diameter, the centre of each corresponding to the centre of a joint on the grid. These holes were drilled using the triangular jig of Fig. 2.10. The accuracy of centre-to-centre distances between any two adjacent holes was better than 0.05mm and the accuracy of the x and y coordinates of each hole's centre was better than 0.1mm. These holes were used for positioning the fixtures for the displacement transducers, the loading actuators and the columns.

3.1.2 Columns - The three columns supporting the grids were positioned at the corners of the base, and were fixed perpendicular to the base, while it was set horizontal. The height of these columns was determined by considering the loading equipment, and the system of displacement transducers. Their stiffness was high enough to prevent any deformations measurable by the measuring instruments used.
A drawing of the test frame.
Fig. 3.1 The test frame and a tested grid.
3.1.3 Supports - The three supports were identical, Fig. 3.2 and Fig. 3.3 show one of them. A support consisted of two hardened plates with fine ground surfaces, with a bearing race between them, and part 1 Fig. 3.2 attached to the grid. It rested on the column on 4 adjusting screws (1\" B.S.F.) with spherically machined heads. It may be seen that z translation of the joint was fully constrained while its x, y translations and all rotations were almost frictionless. The centre of the spherical surface of part 1 which was machined and hardened for each type of the grids, lay on the joint's centre (on the midplane of the grid); the support reaction during the test therefore passed through the joint's centre. Four vertical bars were fixed to the four sides of the column around the support to prevent any accidental movement of the grid whilst the test was being set up.

3.1.4 Fixtures for Displacement Transducers - Twenty two fixtures of three different heights were used to hold twenty two transducers which were of three lengths. Fig. 3.1. A fixture consisted of three plates and two bars of circular cross-section Fig. 3.4. The transducer sat on the recess of plate 2 passing through the reamed hole of plate 1, and it could be positioned by sliding plate 2 over the bars Fig. 3.5. A 1/4" diameter reamed hole at the centre of plate 3 was used to position the fixture on the base. As may be seen in Fig. 3.4. A 1/4" diameter pointed pin was fitted through this hole into the corresponding hole in the base, and the fixture was attached to the base by two 1/4" screws. The four jacking screws were used to adjust the fixture, and the transducer was set vertical by the use of a specially made plumb bob, Fig. 3.5.
Fig. 3.2 General view of support arrangement.

Fig. 3.3 A closer view of support arrangement.
Fig. 3.4 Fixture for displacement transducer.

Fig. 3.5 Alignment of a displacement transducer with plumb.

Fig. 3.6 Positioning of transducer armature.
3.2 Loading System

Regarding the physical limitations and the results of analytical studies, the load consisted of three forces with the following characteristics:

1. Having constant proportion to each other.
2. Controllable to produce strain rates in the desired range.
3. To be controlled by computer.

The equipment available was supplied by R.D.P. Howden Ltd., and comprised three hydraulic actuators 'A', together with a hydraulic power pack 'P', and a console 'C' housing, separate closed-loop control systems for each hydraulic actuator. Facilities for controlling and monitoring the load and displacement of each actuator also were housed in this console Fig. 3-7.

The specification of the hydraulic actuators was as follows:

- Maximum capacity of static testing: 25kN
- Total working stroke: 250mm
- Accuracy of load control and measurement: ±0.5%
- Accuracy of displacement control and measurement: ±1mm

To have loads of constant proportion two of the actuators were set to load control and both made to follow the output load signal of the third, the proportional relationship of the three loads being adjustable.

To meet the other two conditions the third actuator was set to displacement control and was fed by a signal controlled by the computer (Alpha 16 mini-computer). This signal was produced by a ramp generator which was built utilizing two digital-to-analogue converters, a real-time event controller and relevant interfaces. The ramp generator could be programmed to produce a signal variation with time as shown in Fig. 3-8.
Fig. 3.7 This is to illustrate the loading equipment comprising three hydraulic actuators A, a power pack P, and console C housing control systems for actuators and the power pack.
Fig. 3.8 A type of signal variation which the ramp generator could produce.

Fig. 3.9 A section through the loading head. The arrangement provides the facility for applying a concentrated load passing through the joint centre.
Fig. 3.10
Arrangement of actuators.

Fig. 3.11
Base for an actuator. The arrangement provides the slight pivoting facility when the jack is working in tension.

Fig. 3.12
Loading head. The arrangement provides the facility for applying a concentrated load passing through the joint centre.
The curve consists of straight segments of zero or positive and negative slopes, however, for safety reasons sharp changes were not allowed.

Fig. 3.10 shows how the actuators were used to apply the load. Each actuator was attached to the test frame by an arrangement called the 'actuator's base', and was attached to the grid structure by an arrangement called 'the loading head'. The load was applied by pulling the grid downwards.

3.2.1 Actuator's Base - Fig. 3.11 is an illustration of the base. It was positioned and fixed to the test frame and adjusted to align the actuator. The base was designed to allow a limited pivoting of the actuator (about 0.7°), when it was working in tension. By this means a horizontal displacement of the loaded joint up to about 15mm was allowed.

3.2.2 Loading Head - Fig. 3.12 shows the loading head during a test and Fig. 3.9 is a section of it. As may be seen in this section, the centres of the spherical surfaces of part 1 and part 3 lay on the joint's centre (on the grid's midplane), when part 1 was being pushed by part 3 through the bearing race 2. Thus, applied force was enabled to pass through the joint's centre. For each type of grid structure a different type of part 1 was necessary to ensure that the load passed through the centre of the joint. Part 1, part 3 and all bearings were of high hardness.
Chapter 4

MEASUREMENT SYSTEMS

4.1 General Remarks

Quantities to be measured during the test were as follows:
1. The load values at loaded joints.
2. The displacements of a number of points on the grid (sufficient to give the grid's deformed shape).
3. The internal moments at some sections of certain members.

Because of the number of quantities to be measured and the speed of the test, electronic measuring systems were mainly employed; in addition close-range photogrammetry was used to record the deformed shapes of grids in some stages during the test. Fig. 4.1 is a general view of the set-up when a grid was being tested.

The electronic measuring systems comprise three basic parts; the transducer, the signal-conditioning equipment and the recording equipment. In all the measuring systems used for the different quantities in this work, the same recording system was employed, while each system had different transducers and signal conditioning equipment. The recording system is therefore described first, followed by the report on the transducers and signal conditioning equipment of each system and its other specifications.

4.2 Recording System

Parts forming this system may be seen in Fig. 4.2. This system was part of a COMPULOG data logging system made by Intercol Systems Ltd., and consisted of 100 input channels with the necessary switching, a precision d.c. amplifier (all housed in cabinet 'A'), a digital voltmeter 'B', fast paper tape punch 'C' and page printer 'D'. The analogue signal input from a channel was scaled by the amplifier, measured by the digital voltmeter and
Fig. 4.1 A general view of the experimental set-up when a grid was being tested.
Fig. 4.2 Data-logging system

Fig. 4.3 Load cell fixed to the actuator
recorded on punched paper tape for use in subsequent analysis, and/or printed.

The channel switching, scaling, measurement and recording were controlled by an Alpha 16 mini-computer 'E'. If the values were stored in the computer's memory, about 15-17 measurements per second could be acquired by this arrangement. The system was set such that signals up to 5V could be measured. The output value of this system was an integer of 0 to +9999, which could be interpreted as a voltage by the Table 4-1 if the scale value of an integer of 0-8 was known.

Table 4-1 Voltage ranges and conversion factors of output for different scales

<table>
<thead>
<tr>
<th>Scale</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Max. value.</td>
<td>10mV</td>
<td>20mV</td>
<td>50mV</td>
<td>100mV</td>
<td>200mV</td>
<td>500mV</td>
<td>1V</td>
<td>2V</td>
<td>5V</td>
</tr>
<tr>
<td>Bit sensitivity.</td>
<td>1μV</td>
<td>2μV</td>
<td>5μV</td>
<td>10μV</td>
<td>20μV</td>
<td>50μV</td>
<td>100μV</td>
<td>200μV</td>
<td>500μV</td>
</tr>
</tbody>
</table>

The scale value had to be determined by the program before measurement.

4.3 Load Measurement

A load cell (STRAINSERT FLU 2.5 precision grade), Fig.4.3 was used to convert the value of the load applied by each actuator to an electrical signal which was amplified to give 1 volt for 1 kN by the signal conditioning equipment of the system. This output was fed to the recording system via a system of buffer amplifier, filter and divider. The system was calibrated so that in scale 8, one increment in the integer value of the output was due to one newton increment in the load values. Loads larger than 9999 newtons could not therefore be recorded. Larger loads values during one of the tests were recorded manually from the single digital indicator of the loading system, the load signal of each actuator being switched to the indicator manually.
4.4 The Displacement Measurement at Loaded Joints

The value of the vertical displacement of each loaded joint was converted to a 5kHz signal by the LVDT type displacement transducer of the actuator which, through the signal conditioning equipment of the system, was converted to a d.c. signal of 1V output for 10mm displacement. This signal was fed to the recording system via a system of buffer amplifier, filter and divider. The system was calibrated so that in scale 8, one increment in the integer value of the output was due to 0.01mm increment in displacement. Thus, displacements larger than 99.99mm in either direction from the actuator's middle position could not be recorded.

4.5 Displacement Measurement at Other Joints

4.5.1 Transducers - L.V.D.T. (Linear Variable Differential Transformer) type displacement transducers made by R.D.P. Electronics were used for the measurement of vertical displacement of the grid joints. 22 transducers having two or three different stroke ranges were used, Fig.3.1. Stroke length being determined by the expected displacements. The transducers were D5/1000, D5/2000 and D5/4000, with working ranges ± 1", ±2" and ± 4" and accuracy grades A1, A1 and A (B.S.I.), respectively. Energizing supply was 5V rms @ 5kHz (50 mA max).

4.5.2 Signal Conditioners - For each group of transducers of the same stroke range an amplifier (2027 R.D.P. Electronics) was employed to enable their output signals to be independently scaled. The 5kHz output signal of a transducer was converted to a d.c. voltage of the appropriate sign and proportional in magnitude to the transducer's signal. To use one signal conditioner for a number of transducers, a scanner was necessary;
Utilizing the available facilities of the logging system a compatible scanning system was made. With this system each transducer of a range could be switched to the respective amplifier; when a transducer was not selected it was fed, with a d.c. warming current to eliminate self-heating errors (1 V @ 16, 20 or 24 mA, according to the stroke range). The output signal of each signal conditioner was fed to the recording system.

4.5.3 Calibration - In this system there is no independent scaling facility for the output of each transducer. Therefore, each signal conditioner was scaled to produce an output of nearly ±2 volts (the absolute value always to be less than 2 volts) for any of the transducers at the limits of its working range. Then the final output for each transducer was calibrated as follows:

1. The transducer was selected by the scanner, the output of the corresponding signal conditioner was switched to the recording system and the scale was set to 7.

2. Using the set up shown in Fig. 4.4, by fine adjustment the armature was positioned to give zero output. At this time the slip gauge 'A' was of height of 25, 50 or 100 mm according to the working stroke range of the transducer.

3. Then the slip gauge was changed by 5mm intervals between 0-50, 0-100 or 0-200 according to the working stroke range, and the output for each height was recorded.

4. The slope of the line fitted to these points by the least square method was used as the calibration factor to convert the output data into the displacement in mm.

The ambient temperature during the calibration was 20°C, the slip gauges were ground to an accuracy of ±0.02mm, and the repeatability of the set up was good.
Fig. 4.4 Arrangement for calibration of displacement transducers

Fig. 4.5 Arrangement for positioning the armature of a displacement transducer.
4.5.4 Arrangement to Position the Armature - The body of the transducer mounted in the fixture was positioned and set vertical under a joint of the grid as described in subsection 3.1.4. Its armature had to be hung from the joint, the joint's centre hole being used for this purpose, and a plastic coated stranded flexible high tensile steel wire with ultimate load 20 kgs (as used in fishing) was employed. The arrangement shown in Fig.4.5 was used for both types of grid. The 5mm diameter screw of end 'B' passed through the joint's hole and the strand was centred on the joint by end 'A' while its head was sitting (directly or through the ring 'C') on the joint's surface. The armature was fixed to end 'B' through an adaptor 'D'. This adaptor also served for fine adjustments of the position of the armature up to 12mm. The strand was soldered into a brass tube of 3.1mm diameter and 20mm length at either end. The arrangement showed zero creep when checked over a long period.(72 hours).

4.6 Measurement of Bending Moments

4.6.1 Transducer - Precision electric resistance strain gauges produced by Micro-Measurements were used. The specification of these gauges is given in Appendix A. One pair of gauges was installed (according to the manufacturer's instructions) on the top and bottom surfaces of the member at the section where the bending moment was to be measured, which in most cases was at the middle of the member. The position and direction of the gauges were scribed on the member by the arrangement of Fig.4.6. A half-bridge was used having two active gauges as shown in Fig.4.7. The bridge was completed by a highly stable half-bridge within the logging system (100 such half bridges were available).
Fig. 4.6 The arrangement for marking the gauge location.

Fig. 4.7 Strain gauges installed on grid members. A half bridge arrangement was used comprising two active gauges, one stuck on the top surface and the other on the bottom surface.
The bridge was energized by a constant current, while the measurement was taking place and the output signal was fed to the recording system. The scale was programmed to change to higher range during the test when the signal could not be resolved by the existing range and this provided higher accuracy in the measurement of small values.

4.6.2 Calibration - The main reason for carrying out the bending tests described in Section 1.6 was to calibrate the output of this system, to the bending moment, for each type of section. The results of these tests given in pages 42 and 43 were used in the analysis to convert curvature (or strain) into bending moment.

4.7 Temperature Measurement

Although the ambient temperature during the test was almost constant \(20^\circ C\) a digital electrical thermometer utilizing a thermocouple was used to measure this temperature. The output signal of the device (1 V for 1000\(^\circ\)C) was fed to the recording system. The integer output on scale 4 was calibrated as five increments for 0.1\(^\circ\)C. This thermomenter was made by Digitron Instrumentation Ltd.

4.8 Photogrammetry

Close-range photogrammetry was used to record the deformed shape of grids in certain stages during the test.

4.8.1 Photographing - A pair of Zeiss stereometric cameras (SMK-40) was employed. The cameras were fixed to a gantry and centred on the grid structure at a height of 2.15m above it, Fig. 4.1, giving stereo coverage of the entire area of the grid structure in one overlap, with a base-to-depth ratio of 1/5.3. The object distance was chosen to give the
Fig. 4.8 Zeiss stereometric cameras SMK-40 fixed to the gantry beam

Fig. 4.9 The mobile platform used for access to the cameras for loading and unloading plates and cocking the shutter releases
largest possible scale of photography, the format of the photographic plates being 80 x 100 mm. The wide-angle lenses of these cameras were focused so that with their fixed aperture of f/11, they would give sharp pictures over a range from 2.5m to 8m. Thus, the use of close-up lenses was necessary, one of the pairs supplied with the camera being suitable. The result was ±50cm depth of field at an object distance of 181cm, and the focal lengths of 60.95mm and 60.97mm were changed by a factor of 0.9971.

The cameras were to be loaded with glass plates 90 x 120mm, the types of plates used being:

60 off silver-eosine plates orthochromatic, fine grain ASA12 Agfa.
36 off fine grain, high-speed panchromatic ASA 160, Ilford FP4.

The cameras were fixed rigidly to the bottom of the gantry beam Fig. 4.8, by a specially made taper and fixture, and when they were positioned, every moving part of the gantry was fixed. A lighting arrangement was employed to illuminate the grids and reduce the exposure time, and the structure's top surface was painted white, the resulting exposure time being 1 second for 12ASA plates and 1/15 second for 160ASA plates.

A mobile platform was necessary for access to the cameras for loading and unloading the plates and cocking the shutter. Fig. 4.9. The shutters of the two cameras were triggered in synchronism by a remote shutter release unit. 'R'. Fig. 4.2.

The fiducial marks and a number for identification of the photo pairs appeared on the photographs, and a label on the test frame identified the grid and the stage of test. The separation between fiducial marks at the middle of the negative sides (small crosses) were 76.00 and 96.00 mm. Fig. 4.10 and Fig. 4.11 show a typical positive print of a pair of plates with a scale of 2.07.
Fig. 4.10 This figure and Fig. 4.11 are positive prints from a typical stereo pair of photographs taken from the grids during test; the scale of enlargement is 2.07.
Fig. 4.11 This figure and Fig. 4.10 are positive prints from a typical stereo-pair of photographs taken from the grids during test; the scale of enlargement is 2.07.
4.8.2 Marked Points - In order to form a digital model of the grid structure at any stage, the coordinates of a sufficient number of points must be known. Thus the following points were marked:
1. Centre of the joints.
2. Four points on top of each loading head.
3. 18 points on each member at six sections, each section comprising three points one at the centre and two others near to the edges.

These points could be identified by reference to the joint numbering system, Fig. 4.12 and Fig. 4.13. To mark the points, circles of self-adhesive paper were first stuck on the members using a jig; another jig was then used to draw the crosses with a line thickness of 0.7mm. Jigs for both types of grid are shown in Fig. 4.14.

4.8.3 Control points - Control was obtained from 11 control points attached to the test frame, Fig. 4.10. Coordinates of the control points were determined from a trilateration survey and precise levelling.

4.8.4. Photocoordinate Measurement - The photocoordinate measurements of a number of marked points on the stereopairs of photographs was carried out by Hunting Surveys Ltd. utilizing the Zeiss Stecometer C. This stereocomparator was capable of readings to the nearest micrometre (micron) and its mean error was reported to be not more than ± 2μm. Also the accuracy by which the reference floating mark could be set on the three-dimensional image of the point produced in the stereocomparator was an important factor in the accuracy of the photocoordinate results.

From each stereo pair of photographs the photocoordinates of the following points were obtained:
1. Centres of joints; at each joint the 3.2mm diameter hole of element 'A' which was fitted into the joint's centre hole Fig. 4.12 was used to find the x, y coordinate of the joint.
Fig. 4.12
Marked points on grid members and the joint numbering.

Fig. 4.13
Marked points on grid members and the joint numbering.

Fig. 4.13
These jigs were used to stick the circular labels and draw the crosses on grid members of both types.
2. A point on the surface of each joint near to the centre which was used to obtain the z coordinates of the joint.

3. Six points on each member; these were the points marking the centre line of the member. Initially the photocoordinates of some points marked on the member edges were measured too, but when their corresponding ground coordinates were calculated, they did not provide much useful information regarding the available accuracy.

4. Eleven control points and the fiducial marks.

4.8.5 Ground Coordinates - The photocoordinates which were in the arbitrary comparator x, y axes system were transformed to the fiducial x, y axes system and scaled in the x and y directions by the accurately known distances of the fiducial marks.

Then, having the photocoordinates and ground coordinates of the control points, the coordinates of each exposure station and its camera axes directions were obtained by solving the system of nonlinear equations of collinearity conditions.

Then, from the photocoordinate of each point its ground coordinates were found by the least-squares solution of the equations of collinearity conditions.

4.8.6 Accuracy - The accuracy of the ground coordinates could be judged in the following ways:

1. Comparing the ground coordinates of control points obtained by trilatration surveying and levelling with the values obtained from the photogrammetry. The root mean square error on all points was about 1mm in x, y and z coordinates.

2. Comparing the vertical displacement of each joint obtained by photogrammetry with the value obtained by electrical transducers.
Table 4.1 is a comparison of this type. It comprises results for grid 4, at two different stages of loading. At this first stage the load value at each loaded joint was 2.8 kN and at the second stage it was 4 kN. At each stage the vertical displacement of each joint obtained from photogrammetry is listed alongside the value obtained by the electrical transducer. This table contains only 20 joints, those where electrical transducers were employed to measure their vertical displacements. Concerning the values obtained from the photogrammetry, it should be mentioned:

(a) Each value is the difference of two z coordinates of a point, one obtained from a pair of photographs taken before load application and the other from the pair of photographs taken when the load of known value was applied; thus the error in this difference may be twice as large as the error in the z coordinates.

(b) As mentioned before, the z coordinate of each joint was taken as the z coordinate of a point on the joint's surface near to its centre, which itself is another cause of error, especially in large deformations.

Table 4-1 Vertical displacements at joints of grid 4. at two stages of loading obtained by photogrammetry and electrical transducers.

<table>
<thead>
<tr>
<th>Joint No.</th>
<th>First stage</th>
<th>Second stage</th>
<th>Joint No.</th>
<th>First stage</th>
<th>Second stage</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>photogrammetry</td>
<td>electrical transducer</td>
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<td>1</td>
<td>21.7</td>
<td>22.95</td>
<td>46.6</td>
<td>47.49</td>
<td>15</td>
</tr>
<tr>
<td>2</td>
<td>40.4</td>
<td>39.8</td>
<td>86.4</td>
<td>86.4</td>
<td>17</td>
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<td>106.18</td>
<td>18</td>
</tr>
<tr>
<td>4</td>
<td>40.7</td>
<td>40.16</td>
<td>86.57</td>
<td>86.57</td>
<td>19</td>
</tr>
<tr>
<td>5</td>
<td>25.2</td>
<td>23.13</td>
<td>49.</td>
<td>47.77</td>
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<tr>
<td>6</td>
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<td>34.09</td>
<td>34.09</td>
<td>21</td>
</tr>
<tr>
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<td>23</td>
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<tr>
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<td>17.9</td>
<td>17.42</td>
<td>31.1</td>
<td>31.29</td>
<td>24</td>
</tr>
<tr>
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<td>14</td>
<td></td>
<td></td>
<td></td>
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<td></td>
</tr>
</tbody>
</table>

All values are in mm.
THE TEST

5.1 General Remarks

The grid structures were tested under computer control. An Alpha 16 mini-computer under program control served this purpose. The program was written in BASIC and called machine code routines to control the data logger and the loading system. Setting up and testing procedures are described below.

5.2 Setting

Setting up the test rig was carried out as follows:

1. The actuators and displacement transducers were positioned according to the loading arrangement for the grid to be tested.

2. The grid structure was positioned on the supports, the movement of the supports being prevented by 'T' blocks as shown in Fig. 5.1. To each of the support joints a bearing (Part 1 of Fig. 3.3) was attached, and also a bearing (Part 1 of Fig. 3.12) was attached to each of the joints to be loaded. At this stage the strain gauges were already installed and the photogrammetric points had been marked.

3. The wiring of the strain gauges was completed and checked.

4. The armatures of the displacement transducers were positioned.

5. The thermocouple was positioned in the centre of the test frame.
6. The whole system was left for some time to reach a stable temperature.

The lab temperature could be controlled only by heating. Tests were therefore started at 5 am when the outside weather was cooler. Tests were carried out at a temperature of 20°C., and during the 3 hours of each test temperature variations were negligible (less than 1°C).

7. The power pack of the hydraulic system was switched on. The calibration voltage for each load and displacement output was checked. The loading heads were attached and the system left running until the oil reached a stable temperature.

8. The positions of the photogrammetric cameras were checked, and plates were loaded and the shutter releases were cocked (using the mobile platform).

5.3 Testing

Testing was started and carried out by running the computer program as follows:

1. Under zero load the signals of all transducers were measured and their values were recorded (as explained in the following item 4), and the computer real-time clock was set to zero.

2. The first stereo-pair of photographs were taken and the time was recorded, in order to relate the photogrammetric results with the other measurements.

3. A period of loading took place by the application of controlled displacements as defined in 3.2.

4. After a short interval (3 seconds) a set of measurements were taken and the results were stored in an array.
A set of measurements consisted of three measurements from each transducer. The contents of the array, the time, the temperature's reading, and the scale of the voltmeter for certain types of transducer were punched on paper tape.

5. If the difference between the values of three measurements from each transducer were found to be more than a given tolerance, a reject message and identification number of the transducer (or transducers) was printed out on the teletype and another set of measurements was carried out immediately. Also the scale of the voltmeter was changed for the relevant transducers when necessary.

The operator was informed of the test situation by approximate values of load and displacement at the loaded joints, displacement values of a pair of symmetric joints, and the strain values at four sections of two symmetric pairs, which were printed on the teletype.

6. The items 3, 4 and 5 were repeated 'n' times; then another stereo-pair of photographs were taken. Obviously by the time the exposed plates had been replaced by unexposed plates, the shutter releases had been cocked.

7. Item 6 was repeated 'm' times and the grid structure was taken well beyond the elastic limit. Grids 3 and 4 were deformed as far as the limitations of the test set-up allowed.

8. Gradual unloading was carried out.

9. A set of measurements was obtained and the last stereo-pair of photographs was taken.
The number 'm' was decided by studies on the Load-Displacement curves obtained from the theoretical analysis, and also the limitations of the photogrammetry.

The number 'n' in item 6 was determined by dividing the displacement to be applied by the actuator (See 3.2 Loading System) between the two successive stereo-pairs of photographs by the displacement applied at a period of loading. This number could of course be adjusted by the operator's interaction, during the test.

To obtain sufficient information it was decided that the loading period be 60 seconds while 15-17 seconds was needed for each set of measurements. The displacement applied at each period of loading was 1.5mm. This gave a range of straining rate compatible with the strain rates present during the tests on the materials (See 1.2 Straining Rate).

Two movie cameras, one loaded with 16mm black and white film and the other loaded with colour super 8mm film, were used to record the tests of grid 3 and grid 4 from two different angles, taking a single shot every few seconds (approximately 3 seconds on grid 3 and 4 seconds on grid 4).

The test procedure, instrumentation and the program were checked and developed by a series of tests on a dummy model.
Fig. 5.1 Here the movement of support is prevented by T blocks for locating the grid.
Part II

Theoretical Work;
Results;
Conclusions
THEORETICAL WORK

6.1 General Remarks

In linear analysis the first assumption is that the individual components of a structure behave in a linear way. Thus it is assumed that the equations

\[ P_m = K_m e_m \] (6.1)

which relate the end forces \( P_m \) to the deformations \( e_m \) for the disconnected members are linear. (\( K_m \) is a diagonal matrix representing the elastic properties of the structure with the completely unconnected members without containing any information about their orientation).

The second assumption is that the equations of joint equilibrium:

\[ P = C P_m \] (6.2)

(where \( P \) is the load vector, at the joints and \( C \) is the connection matrix) may be written for the undistorted structure, rather than for the distorted structure in which they must actually be satisfied. By this assumption the matrix \( C \) is made a constant matrix independent of the loading on the structure. Strictly speaking this procedure is always an approximation, and it is justified by the fact that, in most structures, the displacements under working loads are small compared with the dimensions of the structure. This assumption is repeated when the equations of displacement compatibility, \n \[ e_m = C^t d \] (6.3)

are deduced (\( d \) is the joint displacement vector).

Obviously when either or both of the above assumptions cannot be made, then the load-deformation equations of the structure are nonlinear and it is usually necessary to employ an iterative type of solution.
While most practical grid structures behave in an approximately linear manner in the elastic range, their load-deflection characteristics become markedly nonlinear with the initiation of unrestricted plastic flow at one or more member cross-sections. Thus the analysis methods concerning the elastoplastic behaviour of grid structures comprise both linear and nonlinear analysis, linear analysis being carried out until plastic flow is detected. Through the nonlinear analysis the displacements are usually assumed to be small compared with the dimensions of the structure, and nonlinearity is considered to be due to the load-deformation characteristics of members in which plastic flow has occurred in one or more of their sections (members having plastic hinges).

In this chapter the analytical technique employed to analyse the test grid structures is described, with a brief reference to the fundamentals of the theory of plasticity. Sections 6.2 to 6.4 deal with this theory; sections 6.5 to 6.9 discuss the analytical technique; Section 6.10 describes the computer program developed on the basis of the technique; and in Section 6.11 some analytical results are presented.

6.2 The Yield Surface

When several member force components interact in a cross-section the critical combination of these which produces a yield condition is defined by the yield surface. When perfect plasticity is assumed, a yield surface is defined for each stage of the plastification of the member cross-section. The region between the initial yield surface and the ultimate yield surface, reflecting the contained plastic flow, is neglected. The members of the structure are therefore assumed to retain their properties unchanged until all fibres in the cross-section yield. The ultimate yield surface defines this condition, and this surface will be referred to as the yield surface.
Fig. 6.1 Characteristics of yield surface

Fig. 6.2 Evaluation of load factor $\lambda$
For a stable elastic-plastic material as defined by Drucker\(^{(11)}\), the yield surface defined by:

\[ \phi(R_1, R_2, \ldots, R_n) = \text{constant} \quad (6.4) \]

(where \(R_1, R_2, \ldots, R_n\) are force parameters) has the following characteristics:

1. While the yield surface does not necessarily have continuous normals, it is a convex, closed surface.

2. At a point where the yield surface is smooth, the incremental plastic deformation vector has the direction of the outward normal. At a sharp ridge or corner, this vector lies between the outward normals to the intersecting surface elements.

The significance of these characteristics is illustrated for the yield curve shown in Fig. 6.1. A space, defined in terms of plastic deformation components \(e_1^p\) and \(e_2^p\), is superimposed on the force space in such a way that the \(e_1^p\) and \(R_1\) axes coincide, as do the \(e_2^p\) and \(R_2\) axes.

At every point where the yield surface is smooth, the second characteristics just defined may be stated as

\[ de^p = \mu N \quad (6.5) \]

in which

- \(de^p\) = The plastic deformation increment vector at the yield hinge.
- \(\mu\) = A positive scalar, called the flow constant, which defines the magnitude of the plastic deformation at the hinge, and has units of deformation.
- \(N\) = The outwardly directed normal vector at the point where the force vector \(R\) meets the yield surface.

The situation at a sharp corner is pictured at point A in Fig. 6.1. While the plastic deformation increment vector \(de^p\) is indeterminate at the corner, it is known to lie between the normals to
As the force vector cannot extend beyond the yield surface, any force increment vector \( dR \) corresponding to a plastic deformation at the cross-section must lie in the surface. This requirement is expressed by the normality condition

\[ N^t dR = 0 \]  

(6.6)

As indicated by equations 6.5 and 6.6 the force increment vector and the plastic deformation increment vector are always tangential and normal respectively, to the yield surface.

6.3 Theorems of Limit Analysis

Statically Admissible Stress Field - A stress field is said to be statically admissible for a given load if it satisfies the equilibrium conditions and nowhere exceeds the yield limit.

Kinematically Admissible Velocity Field - A velocity field is said to be kinematically admissible if it satisfies the velocity (or displacement) constraints and the rate of work done by the actual loads on this velocity field is positive.

Lower Bound Theorem - Under loads for which a statically admissible stress field can be found, the structure will not collapse or will just be at the point of collapse.

Upper Bound Theorem - The structure must collapse under loads for which a kinematically admissible velocity field can be found.

Although in certain cases the correct collapse load could be obtained without much difficulty, in complex problems it may be difficult or impossible to obtain the exact collapse load. In such cases, by the help of upper and lower bound theorems, the true collapse load could be bounded between two values from above and below.
These theorems were first presented by Gvozdev\textsuperscript{(12)} and independently proved by Hill\textsuperscript{(13,14)} for rigid-perfectly plastic material, by Drucker et al \textsuperscript{(15,16)} for the elastic-perfectly plastic material.

6.4 Yield Surface Equation

The derivation of the exact yield surface equation is generally difficult. For the yield curve of a beam under combined bending and torsion, Hendelman\textsuperscript{(17)} and Hill\textsuperscript{(18)} have derived a second order, nonlinear, partial differential equation, using the variational technique. However, with the exception of a numerical solution for a square section by Steele\textsuperscript{(19)} no solution of this equation has been obtained.

Lacking an exact solution, the theorems of limit analysis have been used to obtain upper and lower bounds for the yield curve. A general approach to this problem has been given by Hill and Siebel\textsuperscript{(20,21)}. In what follows a lower bound solution is given.

6.4.1 Lower Bound Yield Curve - consider a shearing stress distribution similar to that of the plastic section in pure torsion, but of reduced magnitude. Thus

\[ \tau^2 = (\tau_x^2 + \tau_y^2) < (\sigma_0 / \alpha)^2 \]

and \[ T / T_0 = \alpha \tau / \sigma_0 . \] (6.7)

Simultaneously, consider a normal stress distribution similar to that in pure plastic bending, but of reduced magnitude. Thus

\[ \sigma < \sigma_0 \]

and \[ M / M_0 = \sigma / \sigma_0 . \] (6.8)

Since the artificial state of stresses must not violate the yield criterion, the best approximation is obtained by taking

\[ c^2 + \alpha^2 \tau^2 = \sigma_0^2 . \] (6.9)

Substitution from equations 6.7 and 6.8 into equation 6.9 then leads
to the approximate lower bound interaction curve:

\[(M/M_0)^2 + (T/T_0)^2 = 1 \]  \hspace{1cm} (6.10)

The accuracy of this equation for different sections, with or without considering the effect of warping restraint on the value of \(T_0\), has been studied by different investigators (21-24), by comparing it with the experimental results as well as with the upper bound solution. This equation also has been used by a number of authors in elastoplastic analysis of grids (8, 25, 26), as well as in the present work.

6.4.2 Yield Surface Equation in Terms of Load Factor - It is a usual practice in plastic analysis to find the collapse load as a multiple of the applied load. Thus the multiplier, called the load factor, is to be obtained. In incremental procedures, to calculate the load factor at any cross-section for a particular loading increment, the yield surface equation and the member force parameters must be expressed in terms of load factor. The situation at a particular cross-section is shown in Fig. 6.2.

At the start of the increment, the force vector is \(R\), and the force increment vector corresponding to the application of the current external load increment has a known direction, \(dR\). It is then necessary to determine the load factor, \(\lambda\), for which the force increment vector \(dR\) just reaches the yield surface. The coordinates of the point where vector \(dR\) pierces the yield surface are

\[ R^P_i = R_i + \lambda dR_i, \]  \hspace{1cm} (6.11)

in which \(R^P_i\) is the \(i\)th coordinate of the piercing point, and \(R_i\) and \(dR_i\) are components of vectors \(R\) and \(dR\), respectively.

The equation 6.10, in terms of load factor becomes:

\[ \left( \frac{M + \lambda dM}{M_0} \right)^2 + \left( \frac{T + \lambda dT}{T_0} \right)^2 = 1. \]  \hspace{1cm} (6.12)

This is the yield curve for a beam under combined bending and torsion.
6.5 General Remarks on Analytical Technique

The technique employed here to analyse the test grids, was presented by Morris and Fenves (27). This is a piecewise linear analysis for tracing the load-displacement behaviour of frameworks loaded beyond the elastic range up to the ultimate load, and all structural forces, displacements and reactions are determined at various load level. Approximate lower bound yield surface equations are used to determine the load level at which a plastic hinge will form at a given cross-section, and to define the load-deformation characteristics of a member having plastic hinges.

In order to linearize the structural behaviour piecewise, the yield surface was linearized and the elastic stiffness matrix, and fixed end force vector for a member were modified by a procedure to take account of plastic hinges.

6.6 Assumptions and Limitations

The imposed assumptions and limitations of the technique concerning the analysis of grid structures are:

1. The stress-strain curve is assumed to be linearly elastic-perfectly plastic, and all stress-strain characteristics are assumed to be time-independent.

2. The material is assumed to be stable according to the definition given by Drucker (11).

3. A shape factor of 1.0 is assumed for all cross-sections, i.e. the cross-section is assumed to make an abrupt transition from a completely elastic state to a state where all fibres are stressed to the yield level and unrestricted plastic flow can occur.
4. Plastic yielding is confined to individual cross-sections with no spread length along the member.

5. All members are prismatic and straight.

6. Strengthening effect due to warping restraint is neglected.

7. Direct shear distortions are ignored.

8. Changes in structural geometry and loading due to deflections are neglected.

9. The structure is loaded by consecutive loading systems, each consisting of concentrated joint loads and any number of concentrated loads applied anywhere along the members.

6.7 Yield Curve and its Linearization

In the grid structures, at a member cross-section, bending moment and torque are the main force components. Hence the yield curve of equation 6.12 was used. As indicated by equations 6.5 and 6.6, the force increment vector and plastic deformation vector are always tangential and normal, respectively, to the yield surface. Since the surface is generally curved, the two vectors constantly change direction as the cross-section deforms plastically, which is considered to be the cause of the nonlinear behaviour of the structure. In order to linearize the load-deformation characteristics of the structure picewise, the infinitesimal force and plastic deformation vectors that would accumulate at a plastic hinge during a loading interval must be replaced by a finite force increment vector and a finite plastic deformation increment vector.

The linearization is accomplished by drawing the force vector \( \mathbf{R} \) back a specified distance, \( h \), from the tangent plane at the point of contact with the yield surface as shown in Fig. 6.3. If vector \( \mathbf{AB} \) is designated by \( d\mathbf{R} \) the required force reduction is specified by

\[
N^{(A)} \mathbf{dR} = -h
\]

(6.13)

where \( N^{(A)} \) is the normal vector to the yield surface at \( A \).
The force increment vector at the cross-section is then constrained to remain parallel to the yield surface tangent plane at A and to trace out line BC until the yield surface is again encountered at some point C. If vector BC is designated dR, the subject condition is expressed by

\[ N(A)^T dR = 0 \]  

(6.14)

when the force vector reaches point C, a new normal vector \( N(C) \) and a new tangent plane are established, and the process is repeated.

6.8 Member Force - Deformation Relation

At the start of elastoplastic analysis a typical member of the structure is continuously elastic, and throughout the analysis it may contain plastic hinges. In this case, elastic return may occur, in any of the hinges. So member force-deformation for each of these cases is given in this section. When in a member cross-section, the force vector reaches the yield surface, the member's force-deformation relation will be changed. At this stage the incremental force-deformation relation of the member is to be approximated to behave linearly by linearization of the yield surface as described in section 6.7. For this purpose, member forces and deformations should first be modified to draw the force vector at a given hinge a specified distance back from the yield surface tangent plane, Fig. 6.3. This is described in sub-section 6.8.2, and in sub-section 6.8.3 the incremental force-deformation relation is obtained.

6.8.1 Continuous Elastic Member - Force-displacement relations for a continuous elastic member of a structure loaded along its length are:

\[ P_i = K_{11} d_i + K_{12} d_j + R_i \]  

(6.15)

\[ P_j = K_{21} d_i + K_{22} d_j + R_j \]  

(6.16)

where \( K_{11}, K_{12}, K_{21} \) and \( K_{22} \) are the stiffness submatrices of the member,
\( P_i \) and \( P_j \) are the end force vectors, \( d_i \) and \( d_j \) are the end displacement vectors and \( R_i \) and \( R_j \) are reactive forces applied at end \( i \) and end \( j \) of the member respectively, if the member ends were completely fixed. \( R_i \) and \( R_j \) are referred to as fixed end force vectors.

The equilibrium condition for a member could be written as:

\[
P_i + H_{ij} P_j = 0 ,
\]

\( H_{ij} \), which is called the equilibrium matrix, also relates the displacement of \( j \) end of the member to the displacement of its other end \( i \) with equation

\[
e_j^E = d_j - H_{ij} d_i ,
\]

where \( e_j^E \) is called the elastic deformation vector of the member.

Also with the equilibrium matrix all member stiffness submatrices could be expressed in terms of \( K_{22} \) as follows

\[
K_{11} = H_{ij} K_{22} H_{ij}^t ,
\]

\[
K_{12} = -H_{ij} K_{22} ,
\]

\[
K_{21} = -K_{22} H_{ij}^t .
\]

Therefore the force deformation relationship for a typical member could be adequately represented if \( K_{22} \) and \( R_j \) are known.

\( R_j \) could be expressed as:

\[
R_j = -K_{22} e_j^E,
\]

whence,

\[
P_j = K_{22} e_j^E - K_{22} e_j^E
\]

where \( e_j^E \) is the cantilever displacement of end \( j \) under member loads, which for \( n \alpha \), concentrated loads \( P_\alpha \) applied at points \( \alpha \), is found from:

\[
e_j^E = \sum_{\ell=1}^{n \alpha} H_{\ell j}^t F_{22 \ell} P_\alpha ,
\]

where \( F_{22} \) is the flexibility submatrix of segment \( i \ell \) at end \( \ell \).
When member loading and \( P_j \) are known, the force at any other cross-section could be calculated.

6.8.2 Reduction of Force Vector at Plastic Hinge - Consider member \( ij \) in Fig. 6.4 loaded with concentrated loads \( P_k \) and end forces \( P_i \) and \( P_j \). Under the action of this loading it is assumed that the member has suffered a distortion \( e_j \) and that the plastic hinges have formed at cross-section \( k \). An additional plastic hinge is about to form at \( s' \). Before the subsequent external loading increment is applied to the member, it is necessary to draw the force vector at hinge \( s' \) a distance \( h \) back from the tangent plane where the force vector meets the yield surface. If the ends of the member are fixed, the force reduction can be accomplished by artificially imposing a pattern of plastic deformations at all hinges on the member. The plastic deformations modify the end forces which, in turn, produce the desired force reduction at hinge \( s' \).

Assuming that the increment of member distortion corresponding to the imposed plastic deformation is \( \delta e_j^P \), compatibility requires that

\[
\delta e_j^E + \delta e_j^P = 0 \tag{6.24}
\]

where \( \delta e_j^E \) is the elastic distortion increment. The plastic deformation increment vector at any yield hinge \( k \) was shown to be \( \delta e_k^P = N_k \theta_k^P \). Hence, the plastic member distortion increment is

\[
\delta e_j^P = \sum_{k=1}^{nh} H_{kj}^t N_k \theta_k^P = GM \tag{6.25}
\]

where:

- \( nh \) is the number of plastic hinges, including the new hinge \( s' \).
- \( G \) is an \( nc \times nh \) matrix, the \( k^{th} \) column of which is \( H_{kj}^t N_k \).
- \( nc \) is the number of force components acting on member cross-section.
- \( M \) is a vector of length \( nh \) containing the flow constants \( \theta_k \) at the plastic hinge \( s' \).
Fig. 6.3 Force increment vectors at plastic hinge

Fig. 6.4 Member with plastic hinges
The force increment vector at j end is
\[ dP_j = K_{22} \, dE_j. \]  \hfill (6.26)

Hence, substitution of Eqs. 6.24 and 6.25 into Eq. 6.26 yields
\[ dP_j = -K_{22} \, GM. \]  \hfill (6.27)

From statics, the force increment vector at any hinge k can then be written as
\[ dP_k = -H_{k,j} K_{22} \, GM. \]  \hfill (6.28)

To solve for \( M \), Eq. 6.13 is applied at the new hinge's, and Eq. 6.14 at all other existing hinges \( k \). Then, substituting Eq. 6.28, the set of simultaneous equations
\[ N_i^t H_{ij} K_{22} \, GM = 0, \]
\[ N_k^t H_{kj} K_{22} \, GM = 0, \]  \hfill (6.29)
\[ N_s^t H_{sj} K_{22} \, GM = h, \]
is obtained.

Then defining a vector \( T \) of \( nh \) elements as:
\[ T^t = [0, 0, ..., 0, 0, h] \]  \hfill (6.30)

it can be seen from Eq. 6.25 that Eqs 6.29 can be written as
\[ (G^t K_{22} \, G) \, M = T, \]  \hfill (6.31)
or
\[ M = (G^t K_{22} \, G)^{-1} \, T = ET \]  \hfill (6.32)
where \( E = (G^t K_{22} \, G)^{-1} \).  \hfill (6.33)

Finally, substituting Eq. 6.33 into Eqs. 6.27 and 6.28 the force increment vectors at j and at a typical yield hinge k, can be written as
\[ dP_j = -K_{22} \, G \, E \, T, \]  \hfill (6.34)
\[ dP_k = -H_{k,j} K_{22} \, G \, E \, T. \]  \hfill (6.35)
The force increment vector given by Eq. 6.34 produces a force increment vector $dP_s$ at hinge 's' which has a projection of magnitude $h$ in the direction of the inwardly directed yield surface normal vector. In addition to the normal component, vector $dP_s$ has in general a component parallel to the yield surface tangent plane. Simultaneously, vector $dP_j$ produces force increment vector $dP_k$ at all other yield hinges which are parallel to their appropriate yield surface tangent plane.

If, instead of a fixed-ended member, a typical member of a structure is considered, the force reduction is achieved by first applying appropriate forces to the joints at the ends of the member. The joint force applied at end $j$ is the negative of the force increment vector given by Eq. 6.34, while that applied at the $i$ end of the member is the static effect there of the latter force increment. The critical hinge is then inserted, the member stiffness matrix is modified as presented in subsection 6.8.3 and a new linear analysis of the structure is performed to obtain the final force increment at the critical hinge.

This increment vector has two components, one corresponding to the fixed end-force vector given by Eq. 6.34, and a second resulting from the displacements of the joint at the end of the member due to the applied joint forces. The first component draws the force vector at the critical hinge the required distance back from the yield surface tangent plane. As is shown in 6.8.3, the second component is parallel to the yield surface tangent plane. Hence the required force reduction at the critical hinge is achieved.

6.8.3 Incremental Force-Deformation Relationships - Having suitably modified the force vector at the new plastic hinge's in member $ij$, it is necessary to determine the force-deformation relationships for the member as it is subjected to subsequent incremental loads $dP_j$ and incremental end forces $dP_j$ and $dP_i$. 
During the subsequent loading increment, the member undergoes a distortion \( \delta e_j \) which includes an elastic component \( \delta e_j^E \) and a component \( \delta e_j^P \) caused by plastic deformation at the yield hinges. Compatibility requires that
\[
\delta e_j^E = \delta e_j - \delta e_j^P. \quad (6.36)
\]

Analogous to Eq. 6.22, the incremental force vector at \( j \) is
\[
dP_j = K22 \delta e_j^E - K22 \delta e_j^P, \quad (6.37)
\]
where \( \delta e_j^j \) is the cantilever deflection due to incremental loads on the member. Substituting Eq. 6.36 into Eq. 6.37 yields
\[
dP_j = K22 \delta e_j - K22 \delta e_j^P - K22 \delta e_j^P. \quad (6.38)
\]
Substituting Eq. 6.25, Eq. 6.38 can be written
\[
dP_j = K22 \delta e_j - K22 GM - K22 \delta e_j^P. \quad (6.39)
\]

From statics, the incremental force vector at any hinge \( k \) is
\[
dP_k = H_{kj} dP_j + \sum_{f=1}^{n_f} H_{kf} dP_f, \quad (6.40)
\]
where \( n_f \) is number of incremental loads \( dP_f \) on member segment \( kj \).

Substituting Eq. 6.39 into Eq. 6.25 gives
\[
dP_k = H_{kj} K22 \delta e_j - H_{kj} K22 GM - H_{kj} K22 \delta e_j^P + \sum_{f=1}^{n_f} H_{kf} dP_f. \quad (6.41)
\]

If, now, Eq. 6.41 is substituted into the normality equation (Eq. 6.14) and all \( n_k \) yield hinges on the member are considered, the simultaneous equations
\[
N_k^t H_{kj} K22 GM = N_k^t H_{kj} K22 \delta e_j - N_k^t H_{kj} K22 \delta e_j^P + \sum_{f=1}^{n_f} H_{kf} dP_f, \quad (6.42)
\]
are obtained.
Vectors $N^t_{kH_kj}$ are again combined into a matrix $G^t_t$, and a vector $dQ$ of $n_h$ elements is defined, the $k^{th}$ element of which is given by

$$dQ_k = N^t_{kH_kf} \sum_{f=1}^{n_f} H_{kf} dP_f , \quad (6.43)$$

Eq. 6.42 now can be rewritten as

$$(G^t_t K_{22} G^t_t) M = G^t_t K_{22} \, d e_j^t - G^t_t K_{22} \, d e_j^t + dQ . \quad (6.44)$$

Solving Eq. 6.44 for $M$, and employing Eq. 6.33, yields

$$M = E G^t_t K_{22} \, d e_j^t - E G^t_t K_{22} \, d e_j^t + E \, dQ \quad (6.45)$$

Finally, substituting Eq. 6.45 into Eq. 6.39, the incremental force-deformation equations for the member are

$$dP_j = (K_{22} - K_{22} G E G^t_t K_{22}) \, d e_j^t - (K_{22} - K_{22} G E G^t_t K_{22}) \, d e_j^t - K_{22} G E dQ . \quad (6.46)$$

Then, by definition, the incremental stiffness submatrix at $j$ is

$$K_{22}^M = K_{22}(I - G E G^t_t K_{22}) , \quad (6.47)$$

where $I$ is a unit matrix. Similarly, the incremental fixed-end force vector is given

$$dR_j = -K_{22}^M \, d e_j^t - K_{22} G E dQ . \quad (6.48)$$

6.8.4 Elastic Return - As a structure is being loaded, a change in load distribution or the formation of a plastic hinge at a particular location may cause one or more previously formed plastic hinges to unload and again become elastic. This phenomenon, referred to as elastic return, occurs whenever there is a reversal in the direction of the plastic deformation increment vector at a yield hinge. It can be seen from Eq. 6.5 that elastic return at any yield hinge during a given load increment is signalled by a negative flow constant $\mu$ at the hinge.

Thus, to test for elastic return at any yield hinge on a member, it is necessary to calculate $M$ from Eq. 6.45, and then test the sign of each element of $M$ in turn.
If elastic return is detected at any plastic hinge, the hinge must be removed and the member stiffness matrix recalculated to reflect its absence.

6.9 Analysis Procedure
The analysis procedure consists of repeated cycles of the following steps:
1. Linear analysis.
2. Calculation of minimum load factor.
3. Test for elastic return.
5. Test of termination criteria.

The structural characteristics and loading information required are similar to those needed for typical elastic analysis. Each load system may consist of any combination of concentrated joint loads and concentrated loads on the members.

Linear analysis - In a given cycle, the linear analysis is performed either for a single set of joint force vectors, or simultaneously for two separate sets: the current external loading increment, and the auxiliary set of joint loads required to reduce the force vector at a newly formed or modified plastic hinge as previously stated.

Calculation of Minimum Load Factor - The load factor for the current external load increment is determined for each potential plastic hinge location in the structure, and the minimum load factor retained along with the identifiers of the critical hinge and the member on which it is located.

Test for Elastic Return - To test whether the current load increment has caused elastic return at any previously formed plastic hinge, the flow constant vector $M$ is calculated for each member containing plastic hinges. If elastic return is detected at any plastic hinge, the hinge is removed
and the member stiffness matrix is recalculated. In addition, the joint forces at the ends of the member are recalculated using the corrected stiffness matrix, the structure is reanalysed for the same external loading as previously, a new minimum load factor is calculated and a new test made for elastic return.

Modification - when no elastic return is detected, the modification phase is initiated. If the minimum load factor for the current cycle is greater than 1.0, the entire loading increment is applied without any modification of the stiffness characteristics of the structure. Accordingly, the cumulative member forces and loads, joint forces, loads, displacement, and reactions and the plastic hinge distortions are incremented by the appropriate values obtained from the linear analysis. Then the termination criteria are checked and, if the analysis is to be continued, the next load system is applied and a new analysis cycle initiated.

If the minimum load factor is 1.0 or less, the cumulative loads, forces, reactions, displacements and plastic hinge distortions are incremented by the product of the minimum load factor and the corresponding values from the linear analysis. The critical plastic hinge is then inserted or modified by evaluating the yield surface normal vector N at that location, and the appropriate member stiffness and joint force vectors are modified. Next the auxiliary joint force vectors necessary to reduce the force vector at the newly formed hinge are generated. The termination criteria are then applied, and if a continuation of the analysis is indicated, all external incremental loads and the corresponding joint forces are multiplied by the factor (1.0 - minimum load factor), and the next linear analysis is begun.

The modification procedure is completed at the end of the linear analysis step of the subsequent cycle. The force reduction at the critical hinge is effected by incrementing the cumulative member forces, joint
displacements and reactions, and plastic hinge distortions by the correspond-
ing values from the analysis for the auxiliary joint loads.

Test of termination criteria - in each analysis cycle, the absolute magnitude of the linear displacement of each joint is calculated and, if it exceeds a defined limit, the analysis is terminated. Also if, a zero diagonal appears in the stiffness matrix of the structure or the number of analysis cycles exceeds a given value, the analysis is terminated.

6.10 The Computer Program
6.10.1 General Remarks - To design and carry out a successful experimental study of elastoplastic behaviour of grid structures, some knowledge of their behaviour was necessary regarding the laboratory limitations, and efforts needed in such a study. The analytical technique described in the previous sections was therefore employed and a computer program was developed to implement this technique for elastoplastic analysis of flat grids.

In addition, in developing the program it was intended to produce an economical means of studying the elastoplastic behaviour of large grids. For this reason a piecewise linear analysis technique was chosen rather than a nonlinear analysis (26).

The program is written in standard FORTRAN IV and can be divided into three parts: automatic data preparation; analysis; and results, which are presented graphically as well as numerically. To describe the program these parts are discussed in what follows:

6.10.2 Automatic Data Preparation - Data which are to provide a complete description of the structure to the program arranged as follows:

1. Mechanical properties of the material.
2. Interconnection pattern of the structural system.
3. Geometrical properties of member, i.e., cross-sectional
1. Carry out the elastic solution.
2. Compute values of joints deflections, member-distortions and member-end forces.
3. Set all cumulative totals to zero and cycle \( K=0 \)

1. Find the smallest positive real load factor \( \lambda_i \), which makes an elastic member-end become plastic, or the force vector at formerly formed hinge intersects the yield curve.
   \[ \lambda_i = \text{Minimum} \left( \lambda_i^{(K)} \right) i = 1, N \]
2. Multiply the unit values of the joint deflections, member end forces by the minimum \( \lambda_i \), these represent the increment values.
3. Add to the incremental values the previous cumulative values to obtain the new cumulative values.

Remove the plastic hinge

Has elastic return occurred?

Yes

No

1. Find an auxiliary set of joint loads required to reduce the force vector at a newly formed or modified plastic hinge, (needed to linearize the behaviour of the structure).
2. Add this load to the remaining portion of previous set, if there is any.
3. Modify the stiffness matrix of members with plastic hinge.
4. Modify the stiffness matrix of the structure 'SB'

Is 'SB' singular?

Yes

Write the results

No

1. Solve the system of equations for both sets of external and auxiliary loads.
2. Find joint displacements and member end forces for each set of loads.
3. Find the smallest positive real load factor, \( \lambda_i \), to the auxiliary loads which makes an elastic member end become plastic or the corresponding force vector at formerly formed hinge intersects the yield curve.
   \[ \lambda_i = \text{Minimum} \left( \lambda_i^{(K)} \right) i = 1, N \]

Is this factor equal to or greater than 1?

Yes

No

1. Multiply the corresponding joint deflections and member end forces by the current load factor and add them to the cumulative values.
2. Store the remaining portion of the auxiliary loads in the corresponding load vector.

Add the incremental values due to auxiliary loads to the cumulative values.
properties, length and orientation of each member.

4. Kinematical characteristics of the joints, i.e., degrees of freedom and constraints at each joint.

5. Loading system, i.e., the load specification at each joint.

Data preparation is straightforward process involving merely a systematic recording of known facts. For large and complex systems, however, the sheer bulk of information that has to be handled makes data preparation a tedious, error-prone and time-consuming task. Thus it was most desirable to adopt a method of automatic data preparation. This was achieved by the use of formex algebra (28), a mathematical system which provides a convenient means for algebraic representation and processing of configurations. Formex algebra consists of a set of abstract entities known as formices and a set of rules, through which these entities are manipulated.

The idea is that one starts with representing, by a few formices of small order, the basic information related to the set-up of a structural system and then generates further information by operating on these formices.

Based on this algebra the part of the program dealing with data preparation was written calling FORTRAN SUBROUTINES (29), for numerical implementation of formex operations. By execution of this part data needed in the analysis was obtained.

6.10.3 Analysis - The second part of the program provides a means for elastoplastic analysis of flat grids loaded by a system of point loads applied at joints. The analysis is of piecewise linear type based on the technique described in the previous sections. The procedure employed is illustrated by the flow diagram given in page 120. It consists of repeated cycles of linear analysis, calculation of minimum load factor, test for elastic return, modification and test of termination criteria.
The stiffness matrix is generated, being stored in a one-dimensional array. The method of Gaussian eliminations is employed to solve the system of linear equations, at each cycle of linear analysis, taking advantage of the properties of the matrix which is symmetric, positive definite and banded.

If the structural system is symmetric the program is capable of taking full advantage of symmetry i.e., if there are \( n \) planes of symmetry in the grid structure, information about its elastoplastic behaviour could be obtained by analysis of a portion containing \((1/2n)^{th}\) of the structure. Consider the grid of Fig. 6.5 which has four planes of symmetry; the portion between planes I and II with numbered joints is to be analysed. The joint number 3 is a fictitious joint whose rotation about the x axis is constrained. However, if a plastic hinge is formed at end 5 of member 3-5, the stiffness matrix of the structure will become singular, which is contrary to the real structure. This can be overcome by applying proper constraint at this stage; the constraint is to be removed if elastic return occurred in the plastic hinge.

6.10.4 Results - During the analysis, results, including data on displacements, member forces and history of plastic hinge formation in the grid structure, are available at certain load levels. The results could be obtained in different arrangements on different media (printed, punched paper tape, magnetic tape and graphs), according to requirements. The facility was made available by the third part of the program. The section dealing with graphics was made as a separate program using the intermediate results obtained on punched paper tape and/or magnetic tape.
Fig. 6.5 A grid structure having four planes of symmetry

Fig. 6.6 A group of square diagonal grids supported at four corners, carrying a system of U.D.L.
6.11 Analytical Results

As mentioned earlier, this program was used to analyze the test grids, details of which are discussed in Chapter 7. However, this was not the only use of the program and a number of large square diagonal grids were also analyzed. A group of these grids and the results of their analysis are presented below.

Fig 6.6 represents this group of grids, each grid being supported at four-corners and loaded with a system of U.D.L. The inner beams in all of the grids have the same cross-sectional properties. The edge beams in a grid are the same; while their cross-sectional properties vary from one grid to another by a scalar multiplier. This scalar, which is a function of the number of inner beams, also applies to the load system.

Each grid was analyzed to obtain the load factor at the elastic limit $\lambda_e$ and the load factor at collapse $\lambda_c$, it being assumed, for inner beams,

$$EI_i=1.0 \text{ kN.mm}^2 \quad GJ_i=0.05 \text{ kN.mm}^2 \quad \text{and} \quad M_0_i=0.01 \text{ kN.mm}$$

and, for all beams,

$$\alpha = \sqrt{\frac{M_o^2}{T_o^2}} = 25.0$$

The specific data on each grid and the values of $\lambda_e$ and $\lambda_c$ are listed in Table 6-1. $EI$ and $GJ$ are bending and torsional rigidities of member cross-section, respectively. $M_o$ being its full moment in pure bending; $W$ is the total value of uniformly distributed load (U.D.L.). Subscript $i$ is for inner beams and subscript $e$ being for edge beams.
Table 6.1 List of specific data on each grid from the group of grids shown in Fig.6.6 (see section 6.11)

<table>
<thead>
<tr>
<th>n</th>
<th>(a) mm</th>
<th>(w) kN</th>
<th>(EI_{e_{mm}^2})</th>
<th>(GJ_{e_{mm}^2})</th>
<th>(M_{\theta_{mm}})</th>
<th>(\lambda_e)</th>
<th>(\lambda_c)</th>
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<tr>
<td>20</td>
<td>0.500</td>
<td>1.000</td>
<td>140.00</td>
<td>7.00</td>
<td>1.40</td>
<td>0.7738</td>
<td>0.7745</td>
</tr>
<tr>
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<td>0.556</td>
<td>0.895</td>
<td>125.26</td>
<td>6.26</td>
<td>1.25</td>
<td>0.7726</td>
<td>0.7768</td>
</tr>
<tr>
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<td>0.625</td>
<td>0.789</td>
<td>110.53</td>
<td>5.53</td>
<td>1.10</td>
<td>0.7709</td>
<td>0.7739</td>
</tr>
<tr>
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<td>0.684</td>
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<td>0.7700</td>
</tr>
<tr>
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<td>0.579</td>
<td>81.05</td>
<td>4.05</td>
<td>0.81</td>
<td>0.7627</td>
<td>0.7709</td>
</tr>
<tr>
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<td>66.31</td>
<td>3.32</td>
<td>0.66</td>
<td>0.7466</td>
<td>0.7546</td>
</tr>
</tbody>
</table>
RESULTS

7.1 General Remarks

During tests, for each of the grids 1, 2 and 3 some 17,000 measurements and for grid 4 some 21,000 measurements were obtained and the values recorded on punched paper tape. Also the deformed shapes of each grid at certain stages were recorded by stereo pairs of photographs. (6-12 pairs of photographs were taken from each grid). The data recorded on punched paper tape were processed by a computer, the values of loads, displacements and internal moments calculated and a series of 'Load-Displacement' and 'Load-Moment' curves plotted.

These results could be used reliably to assess deviation of any analytical results from actual behaviour. As an example, the analytical results obtained by the technique explained in Chapter 6 are compared with the experimental results.

From each pair of stereo photographs the coordinates of a number of points on the grid were obtained and then by a computer program the 'Level-Contours' for the deformed grids at the corresponding stages of loading were plotted.

This chapter describes the processing of the experimental data, details of the theoretical analysis and the content of the final results.

7.2 Processing of Experimental Data

The experimental data for each test which were punched on paper tape was processed by a computer to obtain the values of: loads, vertical displacements of joints and internal bending moments at certain member sections. As mentioned in Chapter 5 the data consisted of sets of
measurements, each set comprising three measurements from each transducer. It should be added that after the first measurements from all transducers had been obtained, the second round of measuring was carried out in the same order as the first round, and repeated for the third round.

The procedure employed for data analysis was as follows:

1. The three measurements from each transducer at zero load were averaged, and stored in the first row of a two-dimensional array.

2. In the next set of measurements, if the differences between every three corresponding values were found to be less than a given tolerance, then the three values from each transducer were averaged and stored in the next row of the array in the same order as the measuring rounds. Otherwise, first, second third measurements of all transducers were taken independently and stored in the next three rows of the array. This was carried out for all sets of measurements.

3. From the relevant data stored in each row of the array, the load value at each of the three loaded joints was calculated by subtraction of the zero load reading and application of the relevant calibration factor. Then the three load values were averaged, standard deviation always being less than 0.5%.

4. The vertical displacement of each joint was calculated in the same way as the loads. But any change of scale during the measurement was taken into account.

5. From the output of each strain gauge bridge, the strain value at the corresponding member section was calculated in the same way as the displacement value (item 4). From the strain value at the top or bottom fibre of the member at a section, its curvature value at that section was calculated. Having the curvature value,
the corresponding value of the bending moment could be found from the relevant Moment-Curvature curve. (graphs of pages 42 and 43) This was automated as follows: each of the graphs on pages 42 and 43 include five experimental moment-curvature curves. Two of the curves, which are due to the strain gauges marked 3 and 4 (the most accurate measuring devices of the five), were replaced by a curve comprising 

number of straight segments, and the coordinates of the vertices were supplied as input data to the program. Then, having the value of the curvature, the moment value was found by linear interpolation between the two nearest points. The number of straight segments and the length of each segment were obtained by the shape of the curves and the acceptable tolerances. Tolerances were dictated by the accuracy of the measuring devices.

6. The data concerning the vertical displacements of each group of symmetric joints of the loaded grid structure were punched on paper tape in a sequence with an identifier, following the load data. The same action was taken for the data concerning the bending moments of each group of symmetric member sections.

7. The joints vertical displacements at a time at which a stereo-pair of photographs was taken were printed out, for comparison with the photogrammetric results.

7.3 Details of Theoretical Analysis

The computer program explained in Chapter 6 was employed to analyse the grid structures which were tested. The input data for this program were as follows:

1. The modulus of elasticity E, and shear modulus of rigidity G,

The values obtained from tension and torsion tests were supplied, Table 1-1.
2. Member List - In this list each member was identified by three integer numbers. The first two were the numbers identifying the two joints which the member was connected to at its ends, the first number being the smaller. Fig. 7.1 shows the joint-numbering system which was used throughout this study (experimental as well as theoretical). The third number identified the type of member.

3. Typical Members - The member type was defined by its components on the x and y axes of the global coordinate system, and its sectional properties including the second moment of area I, the torsional constant J, the value of the fully plastic moment in pure bending $M_0$, and the ratio $\alpha = \sqrt{\frac{M^2}{T^2}}$, where $T_0$ was the value of fully plastic torque in pure torsion. The value of $M_0$ was found from $M_0 = BH^2\sigma_y/4$ where H and B were the height and width of the section, respectively, and $\sigma_y$ is the value of the lower yield stress of the material obtained from tension tests, Table 1.1. The value of $T_0$ was found by Sand Hill analogy $T_0 = B^2(3H-B)\tau/12$ where $\tau$ is the value of shear yield stress. All members of each grid were of the same material having the same length and the same cross-section but were orientated in three different directions.

4. The list of loaded joints; each loaded joint was identified by two integer numbers, the first being the joint number and the second the type of load.

5. Typical loads - The loading consisted of the grid’s self-weight plus three increasing forces normal to the grid, each applied at a joint. The initial value of each of these three forces was taken as 1 kN. The self-weight of the grid was considered as a system of point loads, each applied at a joint; the value of this load at each joint was the weight of the joint plus half
the weight of all members joined to it. (Obviously in calculating of the member weight, its length was taken as the distance between the two sections adjacent to the joints to which member was connected). Excluding the supported joints, these loads were of two different values, so there were three typical loads.

6. The list of constrained joints; including the three corner joints which their vertical movements were constrained.

7. The list of joints whose vertical displacements at different load levels were required. This list consisted of one joint from each group of symmetric joints.

8. The list of members for which the values of internal bending moment at certain sections at different load levels were required. The list consisted of one member section from each group of symmetric sections.

The output data from this analysis included:

1. The vertical displacements at different load levels for the joint which had the maximum displacements.

2. The joints' displacements at ultimate load.

3. The members' forces at ultimate load.

4. The history of plastic hinge formation.

5. The variation of bending moment with load at certain joints, as required.

6. The variation of bending moment with load at certain member sections, as required.

The values of these last two items were punched on paper tape for subsequent plottings.
Fig. 7.1 The joint numbering for test grids used throughout the study.

THE SEQUENCE OF PLASTIC HINGE FORMATION IN GRID No. 3
This is the point of collapse according to the theory, load at each loaded joint being 9.051 KN.
7.4 Load-Displacement and Load-Moment Curves

The results obtained in Sections 7.2 and 7.3 were used to produce the graphs of two types which are described in what follows.

7.4.1 Load-Displacement Curves - A graph of this type is shown on page 142. Such graphs consist of one or more (up to six) experimental curves and one theoretical curve plotted as a full line. Each of the experimental curves shows the variation of vertical displacement of a joint of the grid structure with load during the test. All joints for which load-displacement curves are given on this graph are marked in the sketch at the bottom right corner of the graph. Also the type of line identifying each curve is specified. As may be seen, these joints are symmetric and the theoretical curve is common to all of them. The data for this curve were obtained from theoretical analysis and, to make it comparable with the experimental curves, the first value of displacement, which was due to self-weight, was subtracted from all theoretical displacement values.

In some cases the vertical displacement of certain joints happened to be larger than the working range of the displacement transducers used, so the corresponding load-displacement curve of each was plotted up to the range limit.

7.4.2 Load-Moment-Curves - A graph of this type is shown on page 148. Such graphs consist of one or more (up to six) experimental curves and one theoretical curve plotted as a full line. Each of the experimental curves shows the variation of bending moment at a member section of the grid structure with load during the test. All members for which the load
moment-curve at a section is given on this graph are marked on the sketch at bottom right corner of the graph. Also, the type of line identifying each curve is specified. As may be seen, these member sections are symmetric and the theoretical curve is common to all of them. The data of this curve were obtained from theoretical analysis, and to make it comparable with the experimental curves, the first value of moment, which was due to self-weight, was subtracted from all theoretical moment values.

7.5 History of Plastic Hinge Formation

The position of plastic hinges and the sequence of their formation in one of the grids are given on the sketch of Fig.7.2. This is obtained from theoretical analysis, where it was assumed that plastic hinges were formed in the vicinity of joints.

During the test, visual inspection was carried out to check the position of plastic hinges and the sequence of their formation; this was helped by the oxide scale produced on bars as a result of annealing, Fig.7.3, and Fig.7.4. The sequence was the same as predicted by theory, but it was difficult to determine the exact position of the plastic hinges. However, at each section which was predicted as a plastic hinge, large plastic deformation was detected when the strain gauges were present.

7.6 Level-Contours

A typical graph of this type is given on page 152. These graphs were produced from the results of close-range photogrammetry. As mentioned in Section 4.8, the coordinates of certain points on the grid structure at certain stages of loading were obtained by close-range photogrammetry.
It should be added that at loaded joints the loading heads had hidden a number of marked points, Fig. 4.11, so direct calculation of their coordinates was not possible. The coordinates of each of these points were obtained by interpolation between the coordinates of the relevant joint and of the nearest point on the same member. The coordinates of these joints were obtained from the coordinates of the marked points on the loading heads.

Having these coordinates, the contour lines were produced on microfilm by the computer. The structure of the program was as follows:

1. Basic Routine. Given four points A, B, C, and D, not all on one line, a routine checked if a horizontal plane of level z (z=constant) cut the plane ABC and/or ACD. This was done by examining if the plane z cut or passed through any of the lines AB, BC, AC, CD, and AD. If this was the case, the x, y coordinates of 2 or 4 points were obtained giving one or two segments of the contour line of level z. The x and y coordinates were stored in two one-dimensional arrays, where the x and y coordinates of any pair of points of level z obtained in the subsequent calls to this routine followed. Thus each segment of a contour line could be plotted independently.

2. The points shown on Fig. 7.5 were organized in groups of four as follows: three members of the grid forming a triangle were taken. Then the three joints and 3 x 6 points marking the centre lines of these members were grouped in eleven groups as shown in Fig. 7.6, where I<J<K. I, J, K were joint numbers, the joint numbering system being the same as shown on Fig. 7.1. As may be seen, the joints were taken as repeated points (A and D of item 1), and in each quadrilateral the dashed line joined
Fig. 7.3 Formation of a plastic hinge

Fig. 7.4 Formation of two plastic hinges
Fig. 7.5 Marked points on a test grid. Coordinates were obtained by photogrammetry.

Fig. 7.6
LEVEL CONTOURS FOR GRID NO 1
The deformed shape of originally flat grid is caused by its own weight and 100kN forces applied at joints marked by Q.

Fig. 7.7 The Z difference between supports and the nearest level contour is 2.5 mm

Fig. 7.8 The Z difference between supports and the nearest level contour is 2.4 mm
point A to point C. There were thirty-six such triangles in each grid and, obviously the same arrangement was followed for all of them.

3. The basic routine was called for all of these groups of points with the same horizontal plane of level \( z \). Then the contour line of level \( z \) was plotted.

4. Item 3, was carried out for different \( z \) values and all required level contours were plotted.

As may be seen in a typical graph on page 152, no smoothing was done. The \( x, y \) coordinates were rounded to 0.001mm, while the accuracy of \( z \) coordinates of the points were 1mm (rms), see Section 4.8. As a result in the rather flat parts of the grid, a change of 0.1mm in the \( z \) value gave quite a noticeable difference in the shape of the level contours. Fig. 7.7 and Fig. 7.8 show this difference. Both were obtained from the same set of coordinates which was for grid 1 when under its own weight with only 100N at each of the three loaded joints. In Fig 7.7 the \( z \) difference of the supports with the nearest level contour is 2.5mm, and in Fig 7.8 this difference is 2.4mm. Thus, from the first stereo pair of photographs taken from each grid when only under its own weight, correct contour lines were not obtainable.
7.7 Grid No.1

The first grid structure tested was of 35 x 16mm members' cross-section. It was supported and loaded as shown in Fig.7.9, having three axes of symmetry. Fig.7.9 also shows the position of strain gauges and displacement transducers used respectively, to measure the bending moments at member sections and vertical displacement at joints.

During the test on this grid a total of 130.5mm of vertical displacement was applied to joint number 9 with the actuator which was under displacement control (see Section 3.2). This displacement after unloading was reduced to 88mm, due to the plastic deformation. The vertical displacements at joints numbered 3, 5, 14, 18 and 23 were larger than the working range of the displacement transducers used.

The section properties supplied as data for the theoretical analysis were as follows: second moment of area $I = 56465 \text{mm}^4$; torsional constant $J = 33621 \text{mm}^4$. Fully plastic moment in pure bending $M_o = 955 \text{kN.mm}$, and $\alpha = \sqrt{\frac{M_o^2}{J^2}} = 5.005$. These values were obtained taking the cross-sectional dimensions as: height $H = 34.91 \text{mm}$ and width $B = 15.94 \text{mm}$, each being the average of eighteen measurements by micrometers of accuracy of 0.001mm; standard deviations were under 0.03mm.

To consider the grid's self-weight a vertical force of 0.052 kN was applied at each of the inner joints and 0.035 kN vertically at each of the side joints.

The Load-Moment and Load-Displacement curves obtained from the results of study on this grid as explained in Section 7.4 are presented on the following pages, Table 7.1 is an index for these graphs, listing the joints or members whose curves are presented in each graph, together with the type of transducers used.
During the test on this grid, eight pairs of stereo-photographs were taken at different load levels, the second pair being unfortunately lost because of photographic fault. From the first, seventh and eighth pairs the coordinates of points, shown in Fig 7.5 at corresponding load levels were calculated. The first pair was taken when the load value at each of the loaded joints was 0.1kN; the seventh pair was taken when the load had its maximum value of 5.293kN; and the eighth pair was taken when the grid was unloaded. The Level Contours for these two last stages are obtained as explained in Section 7.6 and are presented on pages 152 and 153. The deformed shape of the grid and the approximate position of the plastic hinges (or yield lines) may be seen in these graphs.

The position of plastic hinges and sequence of their formation obtained from the theoretical analysis of this grid are given graphically on page 154. According to this analysis the collapse load of the grid must have been 4.563 kN.
Fig. 7.9  A sketch of grid No. 1 showing: supported joints, loaded joints, position of displacement transducers and position of strain gauges. (See Notes.)

Table 7.1  Index for graphs on elastoplastic behaviour of grid No. 1

<table>
<thead>
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<th>LOAD-DISPLACEMENT CURVES</th>
<th>LOAD-MOMENT CURVES</th>
</tr>
</thead>
<tbody>
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<td><strong>Groups of symmetric Joints</strong></td>
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<tr>
<td>7.1.1</td>
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<tr>
<td>7.1.2</td>
<td>3, 5, 14</td>
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<td>7.1.3</td>
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<td>7.1.4</td>
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<td>7.1.5</td>
<td>13 &amp; 27</td>
</tr>
<tr>
<td>7.1.6</td>
<td>4</td>
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</tbody>
</table>

**Notes:**

* A1 and A are the grade of displacement transducers (B.S.I.); ±350 and ±100 being their working range in millimetres. Each joint where a transducer was used for measuring vertical displacement is marked by .

+ Displacement transducers are those of the actuators.

** A and B are the type of strain gauges. A for EA-06-250 BF-350 gauges which are shown by — , and B for EP-08-250 B-350 gauges being shown by —— . (See Appendix A).

Supported joints are shown by .

Loaded joints are shown by .
LOAD-DISPLACEMENT Curves for the grid no.1.
These curves are for vertical displacements of
the joints marked by ○.

Fig. 7.10
LOAD-DISPLACEMENT Curves for the grid no. 1.
These curves are for vertical displacements of
the joints marked by .

* Displacement larger than the working range of transducers.

Fig. 7.11
LOAD-DISPLACEMENT Curves for the grid no.1.
These curves are for vertical displacements of the joints marked by $\bigcirc$.

Fig.7.12
LOAD-DISPLACEMENT Curves for the grid no. 1.
These curves are for vertical displacements of
the joints marked by O.
LOAD-DISPLACEMENT Curves for the grid no. 1. These curves are for vertical displacements of the joints marked by ○.
LOAD-DISPLACEMENT Curves for the grid no.1. These curves are for vertical displacements of the joints marked by 0. * Displacement transducer of the actuator initially was out of its working range.

![Graph showing load-displacement curves with legend for joints and supported joints.](image)

Fig.7.15
LOAD - MOMENT Curves for the grid no. 1.
These curves are for the bending moments at the midpoint of the members marked by •.

<table>
<thead>
<tr>
<th>Member</th>
<th>24 - 27</th>
</tr>
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<tbody>
<tr>
<td>Member</td>
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<td>Member</td>
<td>12 - 13</td>
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<td>Member</td>
<td>6 - 12</td>
</tr>
<tr>
<td>Member</td>
<td>8 - 9</td>
</tr>
<tr>
<td>Member</td>
<td>2 - 9</td>
</tr>
</tbody>
</table>

Elastoplastic Analysis
Members for L-M curves O→O
Loaded joints O
Simply supported joints O

Fig.7.16
LOAD - MOMENT  Curves for the grid no. 1.
These curves are for the bending moments at the
midpoint of the members marked by .

Member 21 - 24
Member 20 - 24
Member 12 - 17
Member 11 - 12
Member 9 - 15
Member 9 - 10
Elastoplastic Analysis
Members for L-M curves O - O
Loaded joints O
Simply supported joints O

Fig. 7.17
LOAD - MOMENT Curves for the grid no. 1.
These curves are for the bending moments at the
midpoint of the members marked by .

Member 4 - 5
Member 3 - 4
Member 22 - 25
Member 18 - 22
Member 19 - 23
Member 14 - 19

Elastoplastic Analysis
Members for L-M curves O•O
Loaded joints O
Simply supported joints O

Fig. 7.18
LOAD - MOMENT Curves for the grid no. 1.
These curves are for the bending moments at the plastic hinge end of the members marked by • •.

Fig. 7.19
LEVEL CONTOURS FOR GRID NO 1

The deformed shape of originally flat grid is caused by its own weight and 5.293kN forces applied at joints marked by ▼.

Fig.7.20

Supports have zero level.
Contours for 25mm increments
Contours for 2.5mm increments
Simply supported joints
Loaded joints
All members are of the same rectangular section of:
Height H=34.91mm
Width B=15.95mm
LEVEL CONTOURS FOR GRID NO 1

The deformed shape of originally flat grid is caused by its own weight and 5.293KN forces applied at joints marked by \( \nabla \). Forces are removed.

Fig. 7.21

Supports have zero level.
Contours for 25mm increments
Contours for 2.5mm increments
Simply supported joints
Loaded joints
All members are of the same rectangular section of:
Height \( H = 34.91 \text{mm} \)
Width \( B = 15.95 \text{mm} \)
THE SEQUENCE OF PLASTIC HINGE FORMATION IN GRID No. 1
This is the point of collapse according to the theory, load at each loaded joint being 4.563 KN.

Fig. 7.22

Plastic hinges
Numbers 1, 2,... give the sequence of hinge formation
Simply supported joints
Loaded joints
All members are of the same rectangular section of:

Height $H=34.91$ mm
Width $B=15.95$ mm
7.8 Grid No.2

The second grid structure tested was of 30 x 25mm members' cross-section. It was supported and loaded as shown in Fig.7.23, having three axes of symmetry. Fig. 7.23 also shows the position of strain gauges and displacement transducers used, respectively, to measure the bending moment at member sections and vertical displacement at joints.

During the test on this grid a total of 162.6mm of vertical displacement was applied to joint number 9 with the actuator which was under displacement control (see Section 3.2). This displacement after unloading was reduced to 61.1mm, due to the plastic deformation. The vertical displacements at joints numbered 3, 5, 14, 18 and 23 were larger than the working range of the displacement transducers.

The section properties supplied as data for the theoretical analysis were as follows. Second moment of area $I=39034 \text{ mm}^4$, torsional constant $J=33621\text{ mm}^4$, fully plastic moment in pure bending $M_o=1715\text{kN mm}$ and $\alpha=\sqrt{\frac{M_o}{T_o^2}}=1.435$. These values were obtained taking the cross-sectional dimensions as: height $H=24.98$ and width $B=30.05\text{mm}$, each being the average of eighteen measurements by micrometers of accuracy of 0.001mm; standard deviations were under 0.03mm.

To consider the grid's self weight a vertical force of 0.067 kN was applied at each of the inner joints and a 0.046kN vertically at each of the side joints.

The Load-Moment and Load-Displacement curves obtained from the results of studies on this grid as explained in Section 7.4 are presented on the following pages. Table 7-2 is an index for these graphs, listing the joints or members whose curves are presented in each graph, together with the type of transducers used.
During the test on this grid, seven pairs of stereo-photographs were taken at different load levels, six pairs of them unfortunately being lost because of photographic fault. Remaining pair fortunately was the pair which was taken when the load had its maximum value of 10.853kN at each loaded joint. The Level-Contours for this stage are obtained as explained in Section 7.6 and is presented on page 169. The deformed shape of the grid and the approximate position of the plastic hinges (or yield lines) may be seen in this graph.

The position of plastic hinges and their formation sequence obtained from the theoretical analysis of this grid are given graphically on page 170. According to this analysis the collapse load of the grid must have been 9.051 kN (see Section 8.3).
Fig 7.23 A sketch of grid No.2 showing: supported joints, loaded joints, position of displacement transducers and position of strain gauges. (See Notes)

Table 7.2 Index for graphs on elastoplastic behaviour of grid No.2

<table>
<thead>
<tr>
<th>LOAD-DISPLACEMENT CURVES</th>
<th>LOAD-MOMENT CURVES</th>
</tr>
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<tbody>
<tr>
<td><strong>Fig. No.</strong></td>
<td><strong>Groups of symmetric joints</strong></td>
</tr>
<tr>
<td>7.24</td>
<td>10, 11, 15, 17, 20 &amp; 21</td>
</tr>
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<td>7.25</td>
<td>3, 5, 14, 18 &amp; 23</td>
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<td>7.26</td>
<td>2, 6, 8, 13 &amp; 27</td>
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<td>7.27</td>
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</tr>
<tr>
<td>7.28</td>
<td>4, 19 &amp; 22</td>
</tr>
<tr>
<td>7.29</td>
<td>9, 12 &amp; 24</td>
</tr>
</tbody>
</table>

Notes:

* A1 and A are the grade of displacement transducers (B.S.I.); 450 and 1100 being their working range in millimetre. Each joint where a transducer was used for measuring vertical displacement is marked by .

+ Displacement transducers are those of the actuators.

** A and B are the type of strain gauges. A for EA-10-250 BF-350 gauges which are shown by --- , and B for EP-08-250 BF-350 gauges being shown by --- . (See Appendix A).

Supported joints are shown by .

Loaded joints are shown by .
LOAD-DISPLACEMENT Curves for the grid no. 2.
These curves are for vertical displacements of
the joints marked by O.

Fig. 7.24
LOAD-DISPLACEMENT Curves for the grid no. 2.
These curves are for vertical displacements of the joints marked by O.
* Displacement larger than the working range of transducers.

Fig. 7.25
LOAD-DISPLACEMENT Curves for the grid no. 2.
These curves are for vertical displacements of
the joints marked by O.

Fig. 7.26
LOAD-DISPLACEMENT Curves for the grid no. 2.

These curves are for vertical displacements of the joints marked by ⊗.
LOAD-DISPLACEMENT Curves for the grid no. 2.
These curves are for vertical displacements of the joints marked by ○.
LOAD-DISPLACEMENT
Curves for the grid no. 2.
These curves are for vertical displacements of
the joints marked by ○.
LOAD - MOMENT Curves for the grid no. 2. These curves are for the bending moments at the midpoint of the members marked by .

Fig. 7.30
LOAD - MOMENT Curves for the grid no. 2.
These curves are for the bending moments at the
midpoint of the members marked by .

Member 21 - 24
Member 20 - 24
Member 11 - 12
Member 9 - 10
Member 12 - 17
Member 9 - 15

Elastoplastic Analysis
Members for L-M curves O - O
Loaded joints O
Simply supported joints O

Load in K. Newton

Bending Moment in Kn.mm /10

Fig.7.31
LOAD - MOMENT Curves for the grid no. 2.
These curves are for the bending moments at the
midpoint of the members marked by •.

Member 10 - 11
Member 17 - 21
Member 15 - 20
Elastoplastic Analysis
Members for L-M curves
Loaded joints
Simply supported joints

Fig. 7.32
LOAD - MOMENT Curves for the grid no. 2.
These curves are for the bending moments at the midpoint of the members marked by .

Fig. 7.33
LOAD - MOMENT Curves for the grid no. 2.
These curves are for the bending moments at the plastic hinge end of the members marked by .

Fig.7.34
LEVEL CONTOURS FOR GRID NO 2

The deformed shape of originally flat grid is caused by its own weight and 10,860 KN forces applied at joints marked by ▼.

Fig. 7.35

Supports have zero level.
Contours for 25mm increments
Contours for 2.5mm increments
Simply supported joints
Loaded joints
All members are of the same rectangular section of:
Height H=24.98mm
Width B=30.05mm
THE SEQUENCE OF PLASTIC HINGE FORMATION IN GRID No. 2

This is the point of collapse according to the theory, load at each loaded joint being 7.675 KN.

Plastic hinges
Numbers 1, 2, ..., give the sequence of hinge formation
Simply supported joints
Loaded joints
All members are of the same rectangular section of:
  Height  H=24.98 mm
  Width   B=30.05 mm

Fig.7.36
7.9 Grid No.3

The third grid structure tested was of 30 x 25mm members' cross-section. It was supported and loaded as shown in Fig.7.37, having one axis of symmetry. Fig.7.37 also shows the position of strain gauges and displacements transducers used respectively, to measure the bending moment at member sections and vertical displacement at joints.

During the test on this grid a total of 160.0mm of vertical displacement was applied to joint number 16 with the actuator which was under displacement control (see Section 3.2). This displacement after unloading was reduced to 44.3mm due to the plastic deformation. The vertical displacement at joints numbered 3 and 5 were larger than the working range of the displacement transducers used.

The data on section properties and grid's self-weight supplied for the theoretical analysis were the same as given in Section 7.8 for grid No.2.

The Load-Moment and Load-Displacement curves obtained from the results of studies on this grid as explained in Section 7.4 are presented on the following pages. Table 7-3 is an index for these graphs. Listing the joints or members whose curves are presented in each graph, together with the type of transducers used.

During the test on this grid, six pairs of stereo-photographs were taken at different load levels, the third pair unfortunately being lost because of photographic fault. From the first, fifth and sixth pairs, the coordinates of points shown in Fig.7.5 at the corresponding load levels were calculated. The first pair was taken when the grid was only under its own weight; the fifth pair taken when the load had its maximum value of
8.536kN at each loaded joint; and the sixth pair was taken when the grid was unloaded. The level-contours for these two last stages are obtained as explained in Section 7.6 and are presented on page 195 and page 196. The deformed shape of the grid and the approximate position of the plastic hinges (or yield lines) may be seen in these graphs.

The position of plastic hinges and their formation sequence obtained from the theoretical analysis of this grid are given graphically on page 197. According to this analysis the collapse load of the grid must have been 7.675 kN.
Table 7.3 Index for graphs on elastoplastic behaviour of grid No.3

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<td>27</td>
</tr>
<tr>
<td>7.51</td>
<td>10 &amp; 11</td>
</tr>
</tbody>
</table>

Fig. 7.37 A sketch of grid No.3 showing: supported joints, loaded joints, position of displacement transducers and position of strain gauges. (See Notes)

Notes:
* $A_l$ and $A$ are the grade of displacement transducers (B.S.I.); $250$ and $1100$ being their working range in millimetre. Each joint where a transducer was used for measuring vertical displacement is marked by $\bigcirc$.
† Displacement transducers are those of the actuators.
** $A$ and $B$ are the type of strain gauges. $A$ for EA-06-250 BF-350 gauges which are shown by $\bigcirc$, and $B$ for EP-08-250 B-350 gauges being shown by $\Box$. (See Appendix A).
Supported joints are shown by $\bigcirc$.
Loaded joints are shown by $\Box$. 
LOAD-DISPLACEMENT Curves for the grid no. 3.
These curves are for vertical displacements of
the joints marked by O.

Fig. 7.38
LOAD-DISPLACEMENT Curves for the grid no. 3.
These curves are for vertical displacements of
the joints marked by O.
* Displacements larger than the working range of transducers.

Fig. 7.39
LOAD-DISPLACEMENT Curves for the grid no. 3.
These curves are for vertical displacements of
the joints marked by O.

Fig. 7.40
LOAD-DISPLACEMENT Curves for the grid no. 3.
These curves are for vertical displacements of
the joints marked by ○.
LOAD-DISPLACEMENT Curves for the grid no. 3.
These curves are for vertical displacements of the joints marked by ○.

Fig.7.42
LOAD-DISPLACEMENT Curves for the grid no. 3.
These curves are for vertical displacements of the joints marked by ○.
LOAD-DISPLACEMENT Curves for the grid no. 3.
These curves are for vertical displacements of the joints marked by O.

Fig. 7.44
LOAD-DISPLACEMENT Curves for the grid no. 3. These curves are for vertical displacements of the joints marked by O.
LOAD-DISPLACEMENT Curves for the grid no. 3.
These curves are for vertical displacements of
the joints marked by ○.

Fig.7.46
LOAD-DISPLACEMENT Curves for the grid no. 3.
These curves are for vertical displacements of the joints marked by O.

Fig.7.47
LOAD-DISPLACEMENT Curves for the grid no. 3.
These curves are for vertical displacements of the joints marked by ○.

Fig. 7.48
LOAD-DISPLACEMENT Curves for the grid no. 3.
These curves are for vertical displacements of
the joints marked by O.
LOAD-DISPLACEMENT Curves for the grid no. 3. These curves are for vertical displacements of the joints marked by O.
LOAD-DISPLACEMENT Curves for the grid no. 3.
These curves are for vertical displacements of
the joints marked by O.
LOAD - MOMENT Curves for the grid no. 3.
These curves are for the bending moments at the
midpoint of the members marked by .

Fig. 7.52
LOAD - MOMENT Curves for the grid no. 3.
These curves are for the bending moments at the
midpoint of the members marked by .

Member 4 - 11
Member 4 - 10
Elastoplastic Analysis
Members for L-M curves O - O
Loaded joints O
Simply supported joints O

Fig. 7.53
LOAD - MOMENT Curves for the grid no. 3.
These curves are for the bending moments at the midpoint of the members marked by .

Member 12 - 17
Member 9 - 15
Elastoplastic Analysis
Members for L-M curves O - O
Loaded joints
Simply supported joints

Bending Moment in Kn.mm /10

Fig. 7.54
LOAD - MOMENT Curves for the grid no. 3. These curves are for the bending moments at the midpoint of the members marked by . . .

Fig.7.55
LOAD - MOMENT Curves for the grid no. 3. These curves are for the bending moments at the midpoint of the members marked by •.

Fig. 7.56
LOAD - MOMENT Curves for the grid no. 3.
These curves are for the bending moments at the plastic hinge end of the members marked by .

Fig. 7.57
LOAD - MOMENT Curves for the grid no. 3.
These curves are for the bending moments at two sections at the middle of the members marked by . .

Fig. 7.58
LEVEL CONTOURS FOR GRID NO. 3
The deformed shape of originally flat grid is caused by its own weight and 8.530KN forces applied at joints marked by ▼.

Fig.7.59

Supports have zero level.
Contours for 25mm increments
Contours for 2.5mm increments
Simply supported joints
Loaded joints
All members are of the same rectangular section of:
Height: H=24.98mm
Width: B=30.05mm
LEVEL CONTOURS FOR GRID NO 3

The deformed shape of originally flat grid is caused by its own weight and 8.530kN forces applied at joints marked by ▼. Forces are removed.

Supports have zero level.
Contours for 25mm increments
Contours for 2.5mm increments
Simply supported joints
Loaded joints
All members are of the same rectangular section of:
Height     H=24.98mm
Width      B=30.05mm

Fig.7.60
THE SEQUENCE OF PLASTIC HINGE FORMATION IN GRID No. 3

This is the point of collapse according to the theory, load at each loaded joint being 9.051 KN.

Plastic hinges
Numbers 1, 2, ..., give the sequence of hinge formation
Simply supported joints
Loaded joints
All members are of the same rectangular section of:
  Height  H=24.98 mm
  Width   B=30.05 mm

Fig. 7.61
7.10 Grid No. 4

The fourth and last grid structure tested was of 35 x 16mm members' cross-section. It was supported and loaded as shown in Fig. 7.62, having one axis of symmetry. Fig. 7.62 also shows the position of strain gauges and displacement transducers used respectively, to measure the bending moments at member sections and vertical displacement at joints.

During the test on this grid a total of 189 mm of vertical displacement was applied to joint number 16 with the actuator which was under displacement control (see Section 3.2). This displacement after unloading was reduced to 138.88mm due to plastic deformation.

The data on section properties and grid's self-weight supplied for the theoretical analysis were the same as given in Section 7.7 for grid No.1.

The Load-Moment and Load-Displacement curves obtained from the results of studies on this grid as explained in Section 7.4 are presented on the following pages. Table 7-4 is an index for these graphs, listing the joints or members whose curves are presented in each graph, together with the type of transducers used.

During the test on this grid, twelve pairs of stereo-photographs were taken at different load levels. From the first, eighth, eleventh and twelfth pair, the coordinates of points shown in Fig. 7.5 at corresponding load levels were calculated. From the fourth and seventh pairs, coordinates of a number of points (some 60 points) including the joints, also were obtained. The first pair was taken when the grid was only under self-weight; the eighth pair was taken when the load at each loaded joint was 4.00kN; the eleventh pair was taken when the load had its maximum value of 4.71 kN; and the twelfth pair was taken when the grid was unloaded. The level-contours for these three last stages are obtained as explained
in Section 7.6 and are presented on pages 224, 225 and 226. The deformed shape of the grid and the approximate position of the plastic hinges (or yield lines) may be seen in these graphs.

The position of plastic hinges and their formation sequence obtained from the theoretical analysis of this grid are given graphically on page 227. According to this analysis the collapse load of the grid must have been 3.889kN (see Section 8.3).
Table 7-4  Index for graphs on elastoplastic behaviour of grid No.4

<table>
<thead>
<tr>
<th>LOAD DISPLACEMENT CURVES</th>
<th>LOAD MOMENT CURVES</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fig. No.</td>
<td>Groups of symmetric Joins</td>
</tr>
<tr>
<td>7.63</td>
<td>2 &amp; 6</td>
</tr>
<tr>
<td>7.64</td>
<td>3 &amp; 5</td>
</tr>
<tr>
<td>7.65</td>
<td>4</td>
</tr>
<tr>
<td>7.66</td>
<td>6 &amp; 13</td>
</tr>
<tr>
<td>7.67</td>
<td>9 &amp; 12</td>
</tr>
<tr>
<td>7.68</td>
<td>14 &amp; 18</td>
</tr>
<tr>
<td>7.69</td>
<td>15 &amp; 17</td>
</tr>
<tr>
<td>7.70</td>
<td>19 &amp; 22</td>
</tr>
<tr>
<td>7.71</td>
<td>20 &amp; 21</td>
</tr>
<tr>
<td>7.72</td>
<td>23</td>
</tr>
<tr>
<td>7.74</td>
<td>27</td>
</tr>
<tr>
<td>7.76</td>
<td>10 &amp; 11</td>
</tr>
</tbody>
</table>

Notes:

* A1 and A are the grade of displacement transducers (B.S.I.); &50 and &100 being their working range in millimetre. Each joint where a transducer was used for measuring vertical displacement is marked by .

† Displacement transducers are those of the actuators.

** A and B are the type of strain gauges. A for EA-06-250 BF-350 gauges which are shown by , and B for EP-08-250 B-350 gauges being shown by  . (See Appendix A).

Supported joints are shown by .

Loaded joints are shown by .
LOAD-DISPLACEMENT Curves for the grid no. 4. These curves are for vertical displacements of the joints marked by ⊙.
LOAD-DISPLACEMENT Curves for the grid no. 4.

These curves are for vertical displacements of the joints marked by O.

* There are steps in the curve corresponding to joint 3, where the armature was released after being stopped by friction.

---

Fig. 7.64
LOAD-DISPLACEMENT Curves for the grid no. 4.
These curves are for vertical displacements of
the joints marked by O.

Fig. 7.65
LOAD-DISPLACEMENT

Curves for the grid no. 4.

These curves are for vertical displacements of the joints marked by O.

Fig. 7.66

Vertical Displacement in mm

Load in K. Newton
LOAD-DISPLACEMENT Curves for the grid no. 4. These curves are for vertical displacements of the joints marked by ○.
LOAD-DISPLACEMENT Curves for the grid no. 4.
These curves are for vertical displacements of
the joints marked by O.

Fig. 7.68
LOAD-DISPLACEMENT Curves for the grid no. 4.
These curves are for vertical displacements of the joints marked by O.
LOAD-DISPLACEMENT Curves for the grid no. 4. These curves are for vertical displacements of the joints marked by ○.
LOAD-DISPLACEMENT Curves for the grid no. 4.
These curves are for vertical displacements of
the joints marked by O.

Fig. 7.71
LOAD-DISPLACEMENT Curves for the grid no. 4.
These curves are for vertical displacements of the joints marked by O.
LOAD-DISPLACEMENT Curves for the grid no. 4.

These curves are for vertical displacements of the joints marked by ○.
LOAD-DISPLACEMENT Curves for the grid no. 4.
These curves are for vertical displacements of
the joints marked by O.

Fig. 7.74
LOAD-DISPLACEMENT Curves for the grid no. 4.
These curves are for vertical displacements of the joints marked by ○.

Fig. 7.75
LOAD-DISPLACEMENT Curves for the grid no. 4.

These curves are for vertical displacements of the joints marked by O.

* Displacement transducers of the actuators initially were out of their working range.

---

Fig. 7.76
LOAD - MOMENT Curves for the grid no. 4.
These curves are for the bending moments at the midpoint of the members marked by .

Fig. 7.77
LOAD - MOMENT Curves for the grid no. 4.

These curves are for the bending moments at the midpoint of the members marked by .

Member 6 - 12
Member 2 - 9
Elastoplastic Analysis
Members for L-M curves O - O
Loaded joints O
Simply supported joints O

Bending Moment in Kn.mm

Fig.7.78
LOAD - MOMENT Curves for the grid no. 4. These curves are for the bending moments at the midpoint of the members marked by . .

Fig.7.79
LOAD – MOMENT Curves for the grid no. 4.
These curves are for the bending moments at the
midpoint of the members marked by •.

Member 12 – 17
Member 9 – 15
Elastoplastic Analysis
Members for L-M curves O•O
Loaded joints O
Simply supported joints O

Bending Moment in Kn.mm

Fig.7.80
LOAD - MOMENT Curves for the grid no. 4.
These curves are for the bending moments at the
midpoint of the members marked by . .

Member 21 - 22
Member 19 - 20
Elastoplastic Analysis
Members for L-M curves O O
Loaded joints O
Simply supported joints O

Fig. 7.81
LOAD - MOMENT Curves for the grid no. 4. These curves are for the bending moments at the midpoint of the members marked by .

Fig.7.82
LOAD - MOMENT Curves for the grid no. 4.
These curves are for the bending moments at the midpoint of the members marked by **.

Member 20 - 21
Elastoplastic Analysis
Members for L-M curves
Loaded joints
Simply supported joints

Bending Moment in Kn.mm

Fig.7.83
LOAD - MOMENT
Curves for the grid no. 4.
These curves are for the bending moments at the midpoint of the members marked by .

Fig. 7.84
LOAD - MOMENT Curves for the grid no. 4. These curves are for the bending moments at the midpoint of the members marked by •.

Fig. 7.85
LEVEL CONTOURS FOR GRID NO 4

The deformed shape of originally flat grid is caused by its own weight and 4.000kN forces applied at joints marked by ⬤.

Fig.7.86

Supports have zero level.
Contours for 25mm increments
Contours for 2.5mm increments
Simply supported joints
Loaded joints
All members are of the same rectangular section of:
Height H=34.91mm
Width B=15.95mm
LEVEL CONTOURS FOR GRID NO 4

The deformed shape of originally flat grid is caused by its own weight and 4.710KN forces applied at joints marked by ▼.

Supports have zero level.
Contours for 25mm increments
Contours for 2.5mm increments
Simply supported joints
Loaded joints
All members are of the same rectangular section of:

Height H=34.91mm
Width B=15.95mm

Fig.7.87
LEVEL CONTOURS FOR GRID NO. 4
The deformed shape of originally flat grid is caused by its own weight and 4.710KN forces applied at joints marked by . forces are removed.

Supports have zero level.
Contours for 25mm increments
Contours for 2.5mm increments
Simply supported joints
Loaded joints
All members are of the same rectangular section of,

Height H=34.91mm
Width B=15.95mm

Fig. 7.88
The sequence of plastic hinge formation in grid No. 4

This is the point of collapse according to the theory, load at each loaded joint being 3.889 KN.

Plastic hinges
Numbers 1, 2, ..., give the sequence of hinge formation
Simply supported joints
Loaded joints
All members are of the same rectangular section of:

Height \( H = 34.91 \) mm
Width \( B = 15.95 \) mm

Fig. 7.09
CONCLUSIONS

8.1 General Remarks

Four grid structures made of stress-free bars to a high precision, were deformed plastically under gradually increasing loads. Their elastoplastic behaviour was recorded automatically to a high accuracy; by employing recent developments in experimental techniques. The results presented in Chapter 7 could be reliably used to evaluate the validity of analytical techniques.

In addition to the electrical measuring systems, close-range photogrammetry was used to record the deformed shapes of grids at certain load levels. This technique proved to be a reliable means of displacement measurement, especially for large displacements.

A piecewise linear analysis technique was employed and a computer program developed for elastoplastic analysis of grids. The test grids were analysed by this program and results compared with the experimental data. The technique proved to be a reliable and economical one for predicting the load-displacement behaviour and the collapse load of the flat grids.

Also, with this program a number of large square diagonal grids were analysed. The results (see Section 6.11) do support a new approximate method of analysis called "Structural Factoring" presented in this Chapter.

This method may prove to be a general and economical means for elastic and elastoplastic analysis of large flat grids by the available computer which is always limited in size.
The above-mentioned points are further discussed in what follows.

8.2 Notes on Experimental Results

Comments on any particular load-displacement or load-moment curve are given on the relevant graph, the general notes on the groups of these curves being as follows:

1. The curves not only illustrate the elastoplastic behaviour of the grids. In some cases they also show the effect of strain hardening.

2. Vertical displacements at all the symmetric joints in the grids at any load level are almost the same until plastic flow begins at a member cross-section. The reason for small differences after this stage is that not all expected symmetric plastic hinges form simultaneously.

3. Bending moments at all the symmetric sections in the grids at any load level are almost the same until plastic flow begins at a member cross-section. The differences after this stage may be quite large but they disappear as the result of subsequent deformations.

4. The variations of displacements and moments with load in the elastic range are almost linear as may be seen in the majority of graphs. However, there are some cases in which the load-moment relation in the elastic range is markedly nonlinear.

5. As a result of further load increase, beyond a certain load level, moment decreases were observed at some sections, Fig. 7.82.
8.3 Analytical Results compared with Experimental Results

In comparing the analytical and experimental results, points to be mentioned are discussed below.

8.3.1 Load-Displacement Curves

1. The ratio of two load values, one corresponding to the limit of the first cycle of linear analysis and the other corresponding to the limit of proportionality in the experimental curves, is nearly equal to the ratio of the full plastic moment of the member in pure bending and the moment corresponding to the proportionality limit observed in bending tests on the member. This is the direct effect of the unity-shape factor for all cross-sections, as assumed in the theory. However, it should be added that, as an effect of upper yield stress in the mild steel, this ratio was much less than 1.5, which is the shape factor of rectangular cross-sections under pure bending.

2. At any load level the maximum percentage differences between displacement values in linear parts of the curves are as follows:
   - 4\% for grid No.1,
   - 10\% for grid No.2,
   - 12.5\% for grid No.3,
   - 0.5\% for grid No.4,
theoretical values always being the larger.

Large differences for grids No.2 and No.3 are thought to be mainly due to the effect of joint stiffness which is not taken into account in the theory.
8.3.2 Load-Moment Curves - In most cases the deviations of theoretical values from experimental values were within ±15%. However, in some sections quite large differences were observed, these being mainly sections under low stresses.

In reference (26) Dinno and Saffarini have presented the results of elastoplastic analysis of three grids. These grids were analysed by both a nonlinear and a piecewise linear method; results were compared and significant differences found on the moment and torque values. Predicted collapse loads and the sequences of hinge formations were found to be almost the same.

8.3.3 Sequence of Plastic Hinge Formation and Collapse Load. As reported in Section 7.5 the sequence of plastic hinge formation predicted by theory was the same as observed in experiments, except for small delays between the formation of hinges in a group of symmetric sections. These hinges were predicted to form simultaneously.

For each grid the maximum load value applied to it during the test, and the collapse load predicted by theory, are listed below in kN:

<table>
<thead>
<tr>
<th>Grid No.</th>
<th>Maximum experimentally applied load</th>
<th>Collapse load predicted by theory</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5.257</td>
<td>4.563</td>
</tr>
<tr>
<td>2</td>
<td>10.853</td>
<td>9.051</td>
</tr>
<tr>
<td>3</td>
<td>8.528</td>
<td>7.675</td>
</tr>
<tr>
<td>4</td>
<td>4.653</td>
<td>3.889</td>
</tr>
</tbody>
</table>

Obviously, if the test on each grid were continued by application of further displacement increments, larger loads would be needed compared with the maximum applied load. As mentioned earlier, the test for each grid was concluded for a different reason in each instance. Thus to draw
consistent conclusions from the comparison of experimental load values with theoretical collapse loads, for the grids No.2 and No.4 the maximum experimentally applied load was replaced by the load for which, in each Load-Displacement curve, there is an increase in slope. This is thought to be the result of an increase in the grid stiffness caused by strain-hardening. Table 8-1 compares experimental and theoretical collapse loads for the four grids.

Table 8-1 Comparison of experimental and theoretical collapse loads (kN).

<table>
<thead>
<tr>
<th>Grid No.</th>
<th>Experimental load</th>
<th>Theoretical load</th>
<th>Percentage difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5.257</td>
<td>4.563</td>
<td>13.25</td>
</tr>
<tr>
<td>2</td>
<td>10.2</td>
<td>9.051</td>
<td>11.3</td>
</tr>
<tr>
<td>3</td>
<td>8.528</td>
<td>7.675</td>
<td>10.0</td>
</tr>
<tr>
<td>4</td>
<td>4.25</td>
<td>3.889</td>
<td>8.5</td>
</tr>
</tbody>
</table>

Theoretical loads were always smaller.

8.4 Photogrammetry

The deformed shape of grids at different load levels were recorded with close-range photogrammetry. As a result the Level-Contours were presented in Chapter 7.

The accuracy of measurements obtained was discussed in Section 4.8 and it was shown that the displacements calculated from photogrammetric results are comparable with the results obtained from electrical transducers, Table 4-1.
Thus the method was proved to be a reliable means of displacement measurement. However, if a larger photographic format were used, and cameras were properly orientated at the ends of a wider base, instead of the available stereometric cameras, better accuracy could be achieved.

Also it was checked whether the bending moment at a member cross-section could be estimated from the photogrammetric results. Using the general equation of beam deflection for a grid member, the end forces (M and R) could be calculated if the deflections of four points on the member are known. In this case the deflections at eight points on each grid member were available. Thus M and R were obtained by the method of least squares. At sections at which strain gauges were present, the moment values measured by the gauge system were used to judge the accuracy of the values obtained from the photogrammetric results; the latter were found to be unacceptable. The reason is that the deflections lack the accuracy needed in such calculation.

8.5 Structural Factoring

Concurrently with the research being carried out to develop more general and more accurate theories for better understanding of grid behaviour, simple and rapid procedures should be developed for the analysis of large grids with the available computer. Structural factoring is an attempt in this direction. This method, which is presented below may prove to be a general and economical means for elastic and elastoplastic analysis of large flat grids.

Suppose that two identical grid structures with identical loads have hypothetically occupied the same space. Since every two corresponding points have the same displacements, no interaction appears, Fig.8.1a.
So the overall structure, whose load and stiffness are twice that of either of the structures, will behave like either of them. If one of the grids is shifted in their common plane, a small distance, Fig. 8.1.b, then some interaction appears and the behaviour of either of the grids is only approximately the behaviour of the overall grid. In Fig. 8.2, c and d are two identical grids and are combined to construct the grid e. Analysis of c or d is an approximation to the analysis of e.

Now the 'Structural Factoring' may be explained by its application to the grid f of Fig. 8-3, which is loaded by a system of U.D.L. The grid f is divided into grids g and h as follows:

1. The boundary condition for g and h is the same as for f.
2. The positions of the beams AA and BB appear to have an important effect on the behaviour of grid f. Thus the grids g and h have beams at the same positions. The stiffness of each of these beams is half that of the corresponding beam at f.
3. One of every two adjacent beams in f is given to g and the other to h, their positions being the same as in f.
4. The load of g or h is half the load of f.
5. From the analysis of grid g or grid h, the load factor at elastic limit $\lambda_e$ and the load factor at collapse $\lambda_c$ for grid f could be obtained. These are approximate values, because the interaction of g and h is neglected.

In the above example the grid was factored by 2.0, but we may deduce that the factor may be any reasonable real value. For example grid k, Fig. 8.4 is divided into three grids l, m and n; it may be seen that if either grid l or grid m is factored by 2.0, the resulting grid will be almost like grid n. Thus either l or m is obtained from k by a factor 2.5.
In Section 6.11 the results of elastic and elastoplastic analysis of six square diagonal grids were presented. It may be seen that each structural system is factored from the first one; the factors being 1.0, 1.114, 1.258, 1.444, 1.696 and 2.053, respectively.

If the $\lambda_e$ and $\lambda_c$ for each grid are compared with their corresponding values for the first grid, the percentage differences for the first five grids are less than 1.5% and, for the sixth grid, less than 3.6%. The numbers of equations involved for each grid were 663, 543, 435, 339, 255 and 185, respectively; when taking advantage of symmetry only a quarter of each grid was analysed. Generally, in elastoplastic analysis the number of cycles of linear analysis decreases by the decrease in the number of equations involved.

The example shows that this method cuts sharply the cost of analysis by reducing the computation time and the need to use the computer memory, without much loss in accuracy.

However, to establish the accuracy limit and the range of structures to which this method could be reliably applied, needs further work to be done.

The idea was originally used in experimental study of a cable roof (37).
Fig. 8.2 c and d are two identical grids and are combined to construct grid e (see section 8.5)
Fig. 8.3 Grid f is divided into grids g and h (see section 8.5).

Fig. 8.4 Grid k is divided into grids l, m and n (see section 8.5).
References


<table>
<thead>
<tr>
<th>No.</th>
<th>Author(s)</th>
<th>Title</th>
<th>Details</th>
</tr>
</thead>
</table>
Appendix A

STRAIN GAUGES;

BRIDGE ARRANGEMENTS;

CALIBRATION
A.1 Strain Gauges

Throughout the experimental study, precision electrical resistance strain gauges were used for strain measurements. According to the application and level of expected strains, three different types of gauges were employed. All gauges used were produced by MICRO-MEASUREMENT and were of the types:

1. EA-06-250BF-350. This type was used for longitudinal and transverse strains in tension tests, for curvature measurement in bending tests and on grid members.

2. EP-08-250BF-350. This type was used for curvature measurement in bending tests and on grid members where plastic flow was expected.

3. CEA-06-187 UV-120. This was used in torsion tests.

Every five gauges were supplied in a box with a data sheet. A typical data sheet for the first type of gauge is given here as the next page.

The necessary data on the other types are as follows:

<table>
<thead>
<tr>
<th>Gauge type</th>
<th>EF-08-250BF-350</th>
<th>CEA-06-187 UV-120</th>
</tr>
</thead>
<tbody>
<tr>
<td>Resistance</td>
<td>350.0 Ω ± 0.15%</td>
<td>120.0 Ω ± 0.4%</td>
</tr>
<tr>
<td>Gauge factor</td>
<td>2.095 ± 0.15%</td>
<td>1.99 ± 0.5% or</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2.035 ± 1.0%</td>
</tr>
<tr>
<td>Transverse sensitivity</td>
<td>+ 0.3%</td>
<td>+ 1.4% or 1.6%</td>
</tr>
<tr>
<td>Self-Temp compensation for steel</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Approx. strain limits</td>
<td>20%</td>
<td>5%</td>
</tr>
</tbody>
</table>

The cement used was M-Bond 200, and all gauges installed were coated with two layers of M-Coat D (supplied by the manufacturers of the corresponding gauges).
GENERAL INFORMATION: SERIES EA STRAIN GAGES

GENERAL DESCRIPTION: EA Series Gages are a general-purpose family of constant-strain gages widely used in experimental stress analysis. These gages are of open-faced construction with a 1 mil (0.025 mm) tough, flexible polymide film backing.

TEMPERATURE RANGE: -100°F (-75°C) to +350°F (+175°C) for continuous use in static measurements; -320°F (-160°C) to +400°F (+204°C) for special or short-term exposure.

SELF-TEMPERATURE COMPENSATION: See data curve below.

STRAIN LIMITS: Approximately 5% for gage lengths 1/8" (3.2 mm) and larger; and approximately 3% for gage lengths under 1/8" (3.2 mm).

FATIGUE LIFE: 10^8 cycles at ±1200 µin/µin; 10^9 cycles at ±1500 µin/µin; 10^10 cycles at ±1800 µin/µin) undirectional tension or compression only. Longer gage lengths and lower resistances result in lower endurance and lower fatigue life.

CEMENT: Compatible with M-M Certified M-Bond 200 but normally not recoatable, the great fatigue life. Micro-Measurements M-Bond AE-10/15, M-Bond GA-2, M-Bond 800, and M-Bond 810 are excellent. M-Bond 810 is the best choice over the entire operating range. Refer to M-M Bulletin A-142 for information on bonding agents, and Bulletins B-127, B-130, and B-137 for installation procedures.

SOLDER: If operating temperature will not exceed +300°F (+150°C), M-Line solder type 361 (63-37 tin-lead solder) may be used for lead attachment. M-Line solder type 450 (95.5 tin-antimony) is satisfactory to +400°F (+204°C). When solder turns (Option D) are supplied on these gages, they are formed with +370°F (+190°C) lead-tin silver solder alloy. Refer to M-M Bulletin A-132 for further information on solder, and Bulletin TT-127 and TT-129 for lead attachment techniques.

PROTECTIVE COATINGS: These EA open-faced gages should always be protected with a suitable coating that is applied as soon as possible after gage installation. Refer to M-M Bulletin A-134 for information on Strain Gage Protective Coatings.

BACKING: The backing of EA Series Gages has been specially treated for optimum bond formation with all appropriate strain gage adhesives. No further cleaning is necessary if contamination of the prepared surface is avoided during handling.

TEST PROCEDURES USED BY MICRO-MEASUREMENTS FOR STRAIN GAGE PERFORMANCE EVALUATION

- OPTICAL DEFECT ANALYSIS: M-M Procedures and Standards
- GAGE FACTOR AT 75°F: ASTM E251-67 (Constant Stress Calibrator Method)
- G.F. VARIATION WITH TEMPERATURE: ASTM E251-67 (Step Deflection Method)
- APPARENT STRAIN VERSUS TEMPERATURE: ASTM E251-67 (Slow Heating Rate, Continuously Recorded)
- TRANSVERSE SENSITIVITY: M-M Procedure, Direct NBS Traceability on Resistance Standards
- INITIAL RESISTANCE:
- FATIGUE LIFE: NAS 942 (Modified)
- STRAIN LIMITS: NAS 942 (Modified)
- GAGE THICKNESS:
- CREEP AND DRIFT: M-M Procedure (Similar to NAS 942 Method)

NOTE: This data is obtained in an uniaxial stress field with Poisson's ratio of approximately 0.25.
A.2 Bridge Arrangements

The bridge arrangements used were as follows:

1. Tension tests. The bridge arrangement shown in Fig. A.1 was employed. This comprises two active gauges of 350Ω registering equal strains of the same sign, installed on opposite sides of the specimen to cancel the effect of any bending, and two inactive gauges of 350Ω. The arrangement was used for axial strains as well as for transverse strains. Its output, as expected, was nonlinear.

2. Torsion tests. The bridge arrangement shown in Fig. A.2 was employed. This comprises four active gauges of 120Ω each pair being subjected to equal and opposite strains. The output of this arrangement was linear.

3. Bending tests and grids. The arrangement shown in Fig. A.3 was employed. This comprises two active gauges of 350Ω subjected to equal and opposite strains and two inactive highly stable arms of 24kΩ. The output of this arrangement was linear.

A.3 Calibration

During the tests, outputs of all bridge arrangements were fed into the recording system (see Section 4.2). Subsequently, they were scaled with respect to their values and measured. Thus the final output of each arrangement for all scales used had to be calibrated. The device and procedures employed are described below.

A.3.1 Calibrator - The device used was a Strain Indicator Calibrator Model 1550, made by Vishay Instruments. This calibrator is a Wheatstone bridge with the facility to generate the true change of resistance in one
or two arms of the bridge. The device simulates the actual behaviour of
the strain gauge in both positive and negative strains, the accuracy being
0.025% of setting ± 1μ strain, and repeatability ± 1μ strain. This calibrator
is for 120 Ω and 350 Ω resistance gauges.

A.3.2 Arrangement 1 - To calibrate the strain gauge outputs from tension
tests, the bridge of Fig.A.1 was simulated by a full bridge having one
active arm Fig.A.4. This bridge was energized and its output fed to the
recording system; the resistance of the active arm was changed and outputs
corresponding to strain changes up to 25000μ strain were calibrated as follows:

1. The scale of the recording system (see Section 4.2) was set to one
   of the ranges used during the tests.
2. The bridge was set to zero strain and the output checked to be
   zero.
3. A resistance increment then was applied to the active arm of the
   bridge with the push buttons of the device. The resistance
   increment caused by each push button was equivalent to the effect
   of its indicated strain increment for a gauge of factor 2.
4. The strain value and the output value were recorded.
5. Items 3 and 4 were repeated until the output value reached the
   range limit.
6. The procedure was carried out for all ranges used during the
   tests.

The strain increments within each range were chosen to give enough
points to justify assuming linear variation between any two successive
points. In the analysis of experimental results, these sets of data
were used to convert the output values to strain values by linear inter-
polation.
A.3.3 Arrangement 2. To calibrate the strain gauge outputs from torsion tests, the bridge of Fig.A.2 was simulated by a full bridge with two active arms Fig.A.5. The bridge was energized by the same constant pulsing current as the main bridge and the output fed to the recording system. Outputs corresponding to strain changes up to 48000 \( \mu \) strain were calibrated as explained in A.3.2.

In this case the variation of output value with strain within each range was found to be linear, so by the method of least squares a line was fitted to the points obtained and its slope was used as a factor to convert output values to strain values.

A.3.4 Arrangement 3. To calibrate the strain gauge outputs from bending tests and grid tests, the same bridge of Fig.A.3 was arranged, with the half bridge of the calibrator and a half bridge from the logging system. The bridge was energized by the same current as the main bridges, and the output was fed to the recording system. Outputs corresponding to strain changes up to 30000 \( \mu \) strain were calibrated as explained in A.3.2. In this case the variation of output values with strain within each range was found to be linear as in case A.3.3, and the conversion factor for each range was found by the same method.
Fig. A.1 Bridge arrangement 1, used in tension tests.

Fig. A.2 Bridge arrangement 2, used in torsion tests.

Fig. A.3 Bridge arrangement 3, used in bending tests and grids.

Fig. A.4 This bridge was used for calibration of arrangement 1.

Fig. A.5 This bridge was used for calibration of arrangement 2.
Appendix B

RESULTS OF TENSION TESTS
STRESS - STRAIN curve derived from tensile test on a bar of rectangular solid section of nominal dimension of 30x25mm specimen A

Modulus of Elasticity $E = 201000$ N/mm$^2$

Poisson's Ratio $\nu = 0.280$

Shear Modulus $G = 79000$ N/mm$^2$

Lower Yield Stress $\sigma_y = 371.84$ N/mm$^2$

Upper Yield Stress $\sigma_u = 583.62$ N/mm$^2$

Rate of Straining (on linear part) $\varepsilon' = 1.68 \mu$ strain/sec

Cross Sectional Area $A = 735.40$ mm$^2$
STRESS - STRAIN curve derived from tensile test on a bar of rectangular solid section of nominal dimension of 30*25mm specimen B

Modulus of Elasticity $E = 201000 \text{ N/mm}^2$
Poisson's Ratio $\nu = 0.283$
Shear Modulus $G = 78000 \text{ N/mm}^2$
Lower Yield Stress $\sigma_y = 363.36 \text{ N/mm}^2$
Upper Yield Stress $\sigma_u = 375.26 \text{ N/mm}^2$
Rate of Straining (linear part) $\dot{\varepsilon} = 1.67 \mu \text{ strain/sec}$
Cross Sectional Area $A = 739.30 \text{ mm}^2$
STRESS - STRAIN curve derived from tensile test on a bar of rectangular solid section of nominal dimension of 30.25mm specimen C

Modulus of Elasticity \( E = 202000 \) N/mm²
Poisson's Ratio \( \nu = 0.283 \)
Shear Modulus \( G = 79000 \) N/mm²
Lower Yield Stress \( \sigma_y = 353.78 \) N/mm²
Upper Yield Stress \( \sigma_u = 361.58 \) N/mm²
Rate of Straining (linear part) \( \varepsilon = 1.54 \) µ strain/sec
Cross Sectional Area \( A = 745.00 \) mm²

Strain in \( 10^4 \times \text{mm/mm} \)

Stress in Newton/mm²

80
160
320
480
584.0x845.0
STRESS - STRAIN curve derived from tensile test on a bar of rectangular solid section of nominal dimension of 30*25 mm specimen D

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Modulus of Elasticity $E = 200000 \text{ N/mm}^2$

Poisson's Ratio $\nu = 0.282$

Shear Modulus $G = 78000 \text{ N/mm}^2$

Lower Yield Stress $\sigma_y = 369.70 \text{ N/mm}^2$

Upper Yield Stress $\sigma_u = 380.58 \text{ N/mm}^2$

Rate of Straining in linear peri $\gamma = 1.29 \mu \text{ strain/sec}$

Cross Sectional Area $A = 737.00 \text{ mm}^2$

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Strain in $10^{-3} \text{ mm/mm}$

Stress in Newton/mm²

480

400

320

240

160

80

40

80

120

160

200

240

280
STRESS - STRAIN curve derived from tensile test on a bar of rectangular solid section of nominal dimension of 30.25mm specimen E

- Modulus of Elasticity: \( E = 201000 \, \text{N/mm}^2 \)
- Poisson's Ratio: \( \nu = 0.282 \)
- Shear Modulus: \( G = 79000 \, \text{N/mm}^2 \)
- Lower Yield Stress: \( \sigma_y = 364.74 \, \text{N/mm}^2 \)
- Upper Yield Stress: \( \sigma_u = 378.37 \, \text{N/mm}^2 \)
- Rate of Straining (on linear part): \( \varepsilon = 0.82 \, \mu \text{strain/sec} \)
- Cross Sectional Area: \( A = 740.00 \, \text{mm}^2 \)
STRESS - STRAIN curve derived from tensile test on a bar of rectangular solid section of nominal dimension of 30*25mm specimen F

Modulus of Elasticity  $E = 201000 \text{ N/mm}^2$

Poisson's Ratio  $\nu = 0.283$

Shear Modulus  $G = 79000 \text{ N/mm}^2$

Lower Yield Stress  $\sigma_y = 372.06 \text{ N/mm}^2$

Upper Yield Stress  $\sigma_u = 385.30 \text{ N/mm}^2$

Rate of Straining (on linear part)  $\varepsilon' = 1.33 \mu \text{ strain/sec}$

Cross Sectional Area  $A = 738.30 \text{ mm}^2$

Strain in $10^4 \text{ mm/mm}$
STRESS - STRAIN curve derived from tensile test on a bar of rectangular solid section of nominal dimension of 30x25mm specimen G

This is a test on the material before being stress relieved.

Modulus of Elasticity $E = 199000 \text{ N/mm}^2$

Poisson's Ratio $\nu = 0.284$

Shear Modulus $G = 78000 \text{ N/mm}^2$

0.01 Proof Stress $\sigma_{0.01} = 433.52 \text{ N/mm}^2$

0.02 Proof Stress $\sigma_{0.02} = 465.46 \text{ N/mm}^2$

Rate of Straining $\dot{e} = 1.62 \mu \text{ strain/sec}$

Cross Sectional Area $A = 745.56 \text{ mm}^2$
STRESS - STRAIN curve derived from tensile test on a bar of rectangular solid section of nominal dimension of 35*16mm specimen A

Modulus of Elasticity \( E = 200000 \text{ N/mm}^2 \)
Poisson's Ratio \( \nu = 0.277 \)
Shear Modulus \( G = 78000 \text{ N/mm}^2 \)
Lower Yield Stress \( \sigma_y = 196.39 \text{ N/mm}^2 \)
Upper Yield Stress \( \sigma_u = 206.00 \text{ N/mm}^2 \)
Rate of Straining (linear part) \( \varepsilon' = 1.43 \mu \text{ strain/sec} \)
Cross Sectional Area \( A = 549.64 \text{ mm}^2 \)
STRESS - STRAIN curve derived from tensile test on a bar of rectangular solid section of nominal dimension of 35\*16mm  specimen B

- Modulus of Elasticity: $E = 200000$ N/mm$^2$
- Poisson's Ratio: $\nu = 0.278$
- Shear Modulus: $G = 78000$ N/mm$^2$
- Lower Yield Stress: $\sigma_y = 193.79$ N/mm$^2$
- Upper Yield Stress: $\sigma_u = 209.96$ N/mm$^2$
- Rate of Straining (linear part): $\varepsilon_a = 1.42 \mu$ strain/sec
- Cross Sectional Area: $A = 548.80$ mm$^2$
STRESS - STRAIN curve derived from tensile test on a bar of rectangular solid section of nominal dimension of 35*16 mm specimen C

- Modulus of Elasticity: $E = 201000 \text{ N/mm}^2$
- Poisson's Ratio: $\nu = 0.277$
- Shear Modulus: $G = 79000 \text{ N/mm}^2$
- Lower Yield Stress: $\sigma_y = 196.50 \text{ N/mm}^2$
- Upper Yield Stress: $\sigma_u = 201.13 \text{ N/mm}^2$
- Rate of Strain (on linear part): $\dot{\varepsilon} = 1.10 \mu \text{ strain/sec}$
- Cross Sectional Area: $A = 547.00 \text{ mm}^2$

Strain in $10^4 \text{ mm/mm}$
STRESS - STRAIN curve derived from tensile test on a bar of rectangular solid section of nominal dimension of 35*16 mm specimen D

Modulus of Elasticity \( E = 197000 \text{ N/mm}^2 \)
Poisson's Ratio \( \nu = 0.260 \)
Shear Modulus \( G = 78000 \text{ N/mm}^2 \)
Lower Yield Stress \( \sigma_y = 203.90 \text{ N/mm}^2 \)
Upper Yield Stress \( \sigma_u = 210.17 \text{ N/mm}^2 \)
Rate of Straining (linear part) \( \dot{e} = 1.16 \mu \text{ strain/sec} \)
Cross Sectional Area \( A = 549.10 \text{ mm}^2 \)
STRESS - STRAIN curve derived from tensile test on a bar of rectangular solid section of nominal dimension of 35*16mm specimen E

Modulus of Elasticity \( E = 198000 \, \text{N/mm}^2 \)
Poisson's Ratio \( v = 0.274 \)
Shear Modulus \( G = 78000 \, \text{N/mm}^2 \)
Lower Yield Stress \( \sigma_y = 195.95 \, \text{N/mm}^2 \)
Upper Yield Stress \( \sigma_u = 205.34 \, \text{N/mm}^2 \)
Rate of Strain (linear part) \( \varepsilon^* = 1.04 \, \mu\text{strain/sec} \)
Cross Sectional Area \( A = 548.30 \, \text{mm}^2 \)
STRESS – STRAIN curve derived from tensile test on a bar of rectangular solid section of nominal dimension of 35*16mm specimen F

- Modulus of Elasticity: $E = 201000 \text{ N/mm}^2$
- Poisson's Ratio: $\nu = 0.276$
- Shear Modulus: $G = 79000 \text{ N/mm}^2$
- Lower Yield Stress: $\sigma_y = 193.08 \text{ N/mm}^2$
- Upper Yield Stress: $\sigma_u = \text{N/mm}^2$
- Rate of Straining (on linear part): $\varepsilon' = 1.34 \mu \text{ strain/sec}$
- Cross Sectional Area: $A = 543.85 \text{ mm}^2$
STRESS - STRAIN curve derived from tensile test on a bar of rectangular solid section of nominal dimension of 35*16mm specimen G

This is a test on the material before being stress relieved.

Modulus of Elasticity \( E = 201000 \text{ N/mm}^2 \)
Poisson's Ratio \( \nu = 0.291 \)
Shear Modulus \( G = 78000 \text{ N/mm}^2 \)
0.01 Proof Stress \( \sigma_{0.01} = 315.48 \text{ N/mm}^2 \)
0.02 Proof Stress \( \sigma_{0.02} = 360.80 \text{ N/mm}^2 \)
Rate of Straining \( \dot{\varepsilon} = 1.18 \mu \text{ strain/sec} \)
Cross Sectional Area \( A = 546.50 \text{ mm}^2 \)
Appendix C

RESULTS OF TORSION TESTS
TORQUE - TWIST curve for a specimen of circular section of 15mm diameter machined from a bar of 30*25mm section, specimen A

Strain gauge
Twistmeter

Shear Modulus $G = 79000$ N/mm$^2$
Rate of Twist (linear part) $\theta' = 9.24 \times 10^6$ Radian/sec
Diameter of Specimen $D = 14.99$ mm

$T_s = 195.01$ N/mm
TORQUE - TWIST curve for a specimen of circular section, specimen B of 15\( \text{mm} \) diameter machined from a bar of 30\( \times \)25\( \text{mm} \) section.

- Strain gauge
- Twistmeter

Shear Modulus: \( G = 78000 \text{ N/mm}^2 \)
- Rate of Twist: \( \frac{\text{rad}}{\text{mm}} \), \( \frac{\text{rad}}{\text{mm}} \)
- Diameter of Specimen: \( D = 14.97 \text{ mm} \)

- Torque in Kn.m
- Angle of Twist in 10\(^4 \) Radian/mm
TORQUE - TWIST curve for a specimen of circular section of 15mm diameter machined from a bar of 30*25mm section, specimen C

Shear Modulus \( G = 78000 \text{ N/mm}^2 \)
Rate of Twist (linear part) \( \dot{\theta} = 9.25 \times 10^5 \text{ radian/sec} \)
Diameter of Specimen \( D = 14.95 \text{ mm} \)

Shear Stress \( T_s = 204.68 \text{ N/mm}^2 \)
TORQUE - TWIST curve for a specimen of circular section of 15mm diameter machined from a bar of 30*25mm section. specimen D

Torque in Kn.mm

80
120
160
200

Angle of Twist 10° Radian/mm

Strain gauge
Twistmeter

Shear Modulus
T, = 199.05 N/mm²
G = 79000 N/mm²
Rate of Twist (linear part)
θ' = 11.11 10° Radian/sec
Diameter of Specimen
D = 15.03 mm
TORQUE - TWIST curve for a specimen of circular section of 15mm diameter machined from a bar of 30.25mm section, specimen E.

\[ \text{Strain gauge} \]
\[ \text{Twistmeter} \]

Shear Modulus \( G = 79000 \text{ N/mm}^2 \)
Rate of Twist \( \theta' = 8.86 \times 10^3 \text{ Radian/sec} \)
Diameter of Specimen \( D = 15.04 \text{ mm} \)
TORQUE - TWIST curve for a specimen of circular section of 15mm diameter machined from a bar of 30.25mm section. specimen F.

Strain gauge
Twistmeter

Shear Modulus
G = 80000 N/mm²

Rate of Twist (linear part)
θ' = 9.00 10⁴ Radian/sec

Diameter of Specimen
D = 14.99 mm
TORQUE - TWIST curve for a specimen of circular section of 15mm diameter machined from a bar of 35*16mm section. specimen A

Torque in Kn.mm

Angle of Twist \(10^5\) Radian/mm

Strain gauge
Twistmeter

Shear Modulus \(\tau = 107.98\) N/mm²
Rate of Twist (on linear part) \(\theta = 6.73 \times 10^5\) Radian/sec
Diameter of Specimen \(D = 14.94\) mm

G = 80000 N/mm²
TORQUE - TWIST curve for a specimen of circular section of 15mm diameter machined from a bar of 35*16mm section. specimen B.

Strain gauge
Twistmeter

Shear Modulus $G = 76000 \text{ N/mm}^2$
Rate of Twist (on linear part) $\theta' = 8.81 \times 10^4 \text{ Radian/sec}$
Diameter of Specimen $D = 14.96 \text{ mm}$

Angle of Twist $10^4 \times \text{ Radian/mm}$

Torque in Kn.mm

24

12

84

72

60

48

36

20

40

60

80

100

120

140

160

180

200

220
TORQUE - TWIST curve for a specimen of circular section of 15mm diameter machined from a bar of 35*16mm section, specimen C

Shear Modulus, \( G = 78000 \text{ N/mm}^2 \)
Rate of Twist (linear part), \( \dot{\theta} = 7.89 \times 10^4 \text{ Radian/sec} \)
Diameter of Specimen, \( D = 15.00 \text{ mm} \)
TORQUE - TWIST curve for a specimen of circular section of 15mm diameter machined from a bar of 35*16mm section, specimen D

Strain gauge
Twistmeter

Torque in Kn.mm
24
12
36
48
60
72
84
96

Angle of Twist $10^5 \times$ Radian/mm

100
200
300
400
500
600
700
800
900

Diameter of Specimen
$D = 14.97$ mm

Shear Modulus
$G = 77000$ N/mm$^2$

Rate of Twist (on linear part)
$\theta' = 7.22 \times 10^5$ Radian/sec

$T_r = 125.29$ N/mm$^2$
TORQUE - TWIST curve for a specimen of circular section of 15mm diameter machined from a bar of 35*16mm section, specimen E

Strain gauge
Twistmeter

Shear Modulus
G = 78000 N/mm²

Rate of Twist (line part)
6° = 3.19 10⁻⁴ Radian/sec

Diameter of Specimen
D = 15.02 mm

Torque in Kn.mm

12 24 36 48 60 72 84 96

Angle of Twist 10⁻⁴ Radian/mm

19 39 59 79 99 119 139 159 179
TORQUE - TWIST curve for a specimen of circular section of 15mm diameter machined from a bar of 35*16mm section, specimen F

Strain gauge
Twistmeter

Shear Modulus
G = 77000 N/mm²
Rate of Twisting (linear part)
\( \theta_f = 7.41 \times 10^4 \) Radian/sec
Diameter of Specimen
D = 15.00 mm
Appendix  D

RESULTS OF BENDING TESTS
MOMENT CURVATURE curve for a beam of rectangular solid section of nominal dimension of 30.25 mm, specimen A

B.M. at proportional limit = 1316.92 kN.mm
The Maximum Rate of Straining \( \epsilon' = 2.87 \) \( \mu \) strain/sec

Width of Beam \( B = 30.07 \) mm
Height of Beam \( H = 24.97 \) mm
MOMENT CURVATURE curve for a beam of rectangular solid section of nominal dimension of $30 \times 25 \text{ mm}$ specimen B

B.M. at proportional limit $= 1209.59 \text{ Kn.mm}$

The Maximum Rate of Straining $\varepsilon' = 2.84 \mu \text{ strain/sec}$

Width of Beam $B = 30.05 \text{ mm}$

Height of Beam $H = 24.83 \text{ mm}$
MOMENT CURVATURE curve for a beam of rectangular solid section of nominal dimension of 30×25 mm. Specimen C

B.M. at proportional limit = 1287.10 Kn.mm
The Maximum Rate of Straining \( \varepsilon' = 2.91 \mu \text{strain/sec} \)

Width of Beam \( B = 30.04 \text{ mm} \)
Height of Beam \( H = 24.83 \text{ mm} \)
MOMENT CURVATURE curve for a beam of rectangular solid section of nominal dimension of 16x35 mm specimen A

B.M. at proportional limit = 810.35 Kn.mm
The Maximum Rate of Straining e' = 3.81 μ strain/sec

Width of Beam
B = 15.94 mm

Height of Beam
H = 34.84 mm
MOMENT CURVATURE curve for a beam of rectangular solid section of nominal dimension of 16.35 mm specimen B.

B.M. at proportional limit = 781.92 kN.mm

Width of Beam
B = 15.93 mm

Height of Beam
H = 34.84 mm

The Maximum Rate of Straining e' = 5.43 μ strain/sec

Curvature in 10^5 mm⁻¹

Bending Moment in 10^1 KN-mm
MOMENT CURVATURE curve for a beam of rectangular solid section of nominal dimension of 16*35 mm specimen C

B.M. at proportional limit = 810.38 Kn.mm

The Maximum Rate of Straining $\epsilon^* = 4.27 \mu$ strain/sec (on linear part)

Width of Beam $B = 15.94$ mm

Height of Beam $H = 34.80$ mm