Application of WLF to OFDMA MU-MIMO Systems I: Frequency-Domain Equalization

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***Abstract*— This paper presents a novel iterative receiver strategy incorporating widely linear filtering for uplink Orthogonal Frequency Division Multiple Access (OFDMA) multiuser multiple-input, multiple-output (MIMO) systems. The proposed iterative receiver scheme achieves better performance without the loss of spectrum efficiency compared to the conventional iterative receivers; The superiority of the investigated scheduler coupled with the innovative iterative receiver scheme over conventional solutions is verified by both simulation and analytical results.**

I. Introduction

In 3GPP LTE, Single Carrier (SC) Frequency Division Multiple Access (FDMA) [1] used for uplink transmission, whereas the OFDMA signaling format is used for the downlink transmission [2]–[4]. There are also some proposals on using OFDMA for uplink transmission in LTE advanced (LTE-A) standard, in which both SC-FDMA and OFDMA can be considered as two options for uplink transmission.

This paper investigates receiver algorithms for the OFDMA based uplink multi-user MIMO system. Frequency domain equalization (FDE) is commonly used for OFDMA. This includes frequency domain linear equalization (FD-LE) [5], decision feedback equalization (DFE) [6], and the more recent turbo equalization (TE) [7]. FD-LE is analogous to time domain LE. A zero-forcing (ZF) LE [8] eliminates intersymbol interference (ISI) completely but introduces degradations in the system's performance due to noise enhancement. Superior performance can be achieved by using the minimum mean square error (MMSE) criterion [8], which accounts for additive noise in addition to ISI.

Recent studies have shown that the use of multiple antennas in a wireless communication system significantly improves the system's spectral efficiency, enables a growth in transmission rate which is linear in the minimum number of antennas at either end [9], [10], and improves link reliability and coverage [11]. However, the main problem for transmission over multiple-input, multiple-output (MIMO) channels is the separation or equalization of the parallel data streams. In order to exploit the capacity and performance gains promised by MIMO, we must deal with the co-antenna interference (CAI). It was shown in [10] that iterative (turbo) detection provides an effective means to combat CAI and to approach the capacity offered by the MIMO systems. In its original form, the iterative receiver employs the maximum a posteriori probability (MAP) algorithm [12], which has a high computational complexity that increases exponentially with the spatial diversity and modulation orders. To reduce the complexity, a MIMO turbo receiver based on soft interference cancelation was proposed in [13]. The basic idea is to iteratively cancel out the CAI with soft symbol estimates and suppress the residual interference with a ZF or MMSE filter. We call this method 'IC-ZF/MMSE' in the sequel. The main computational complexity of this approach is incurred by the matrix inverse in the filter coefficient computation, which is much simpler than the MAP algorithm.

The second-order properties of a complex random process are completely characterized by its autocorrelation function as well as the pseudo-autocorrelation function [14]. Most existing studies on receiver algorithms only exploit the information contained in the autocorrelation function of the observed signal. The pseudo-autocorrelation function is usually not considered and is implicitly assumed to be zero. While this is the optimal strategy when dealing with proper complex random processes [15], it turns out to be sub-optimal in situations where the transmitted signals and/or interference are improper complex random processes, for which the the pseudo-autocorrelation function is non-vanishing, and the performance of a linear receiver can be improved by the use of widely linear filtering (WLF) [16]. It was shown in [14] that the performance gain of WLF compared to conventional processing in terms of mean square error can be as large as a factor of 2. MIMO transceiver design was considered in [17], where it was shown that when channel information is available both at the transmitter and receiver, joint design of the precoder and decoder using WLF yields considerable performance gains at the expense of a limited increase in the computational complexity, compared to the conventional linear transceiver in the scenario where real-valued symbols are transmitted over complex channels. By using the same principle, a real-valued MMSE (RV-MMSE) beamformer was developed in [18] for a binary phase shift keying (BPSK) modulated system, and was shown to offer significant enhancements over the standard complex-valued MMSE (CV-MMSE) design in terms of bit error rate performance and the number of supported users.

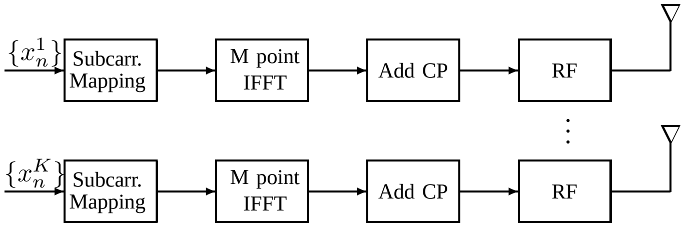

Fig. 1. OFDMA based MIMO Transmitter.

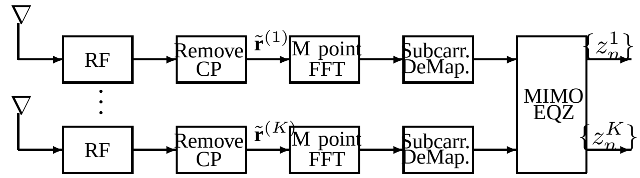

Fig. 2. OFDMA based MIMO Receiver.

In this paper, we show that the existing iterative receiver designs are sub-optimum and their performance can be improved by exploitation of the complete second-order statistics of the received signal. Throughout this paper, $(\cdot)^{\mathcal{T}}$ denotes matrix transpose, $(\cdot)^{\mathcal{H}}$ matrix conjugate transpose, $(\cdot)^*$ matrix conjugate, $\mathrm{E}[\cdot]$ expectation, $\|\cdot\|$ Euclidean norm, $\|\cdot\|_F$ Frobenius norm, $\mathrm{Tr}(\cdot)$ trace operation, and $\mathbf{I}_N$ an $N \times N$ identity matrix.

II. System Model

The cellular multiple access system under study has n_R receive antennas at the BS and a single transmit antenna at the ith user terminal, $i = 1, 2, \cdots, K_T$ where K_T is the total number of users in the system. We consider the multi-user MIMO case with K ($K < K_T$) users being served at each time slot and $K = n_R$. The system model for an OFDMA based MIMO transmitter and receiver is shown in Figs. 1 and 2, respectively. On the transmitter side, the user data block containing N symbols first goes through a subcarrier mapping block, these symbols are then mapped to M ($M > N$) orthogonal subcarriers followed by an M-point Inverse Fast Fourier Transform (IFFT) to convert to a time domain complex signal sequence.

There are two approaches to mapping subcarriers among Mobile Stations (MSs) [4]: localized mapping and distributed mapping. The former is usually referred to as localized FDMA transmission, while the latter is usually called distributed FDMA transmission scheme. With the localized FDMA transmission scheme, each user's data is transmitted by consecutive subcarriers, whereas with the distributed FDMA transmission scheme, the user's data is transmitted by distributed subcarriers [4]. Because of the spreading of the information symbol across the entire signal band, the distributed FDMA scheme is more robust against frequency selective fading and can thus achieve more frequency diversity gain. For the localized FDMA transmission, in the presence of frequency selective fading channel, the multiuser diversity and frequency selective diversity can also be achieved if assigning each user to subcarriers with favorable transmission characteristics.

In this work, we only consider the localized FDMA transmission. A Cyclic Prefix (CP) is inserted into the signal sequence before it is passed to the Radio Frequency (RF) module. On the receiver side, the opposite operating procedures are performed after the noisy signals are received by the receive antennas. A MIMO Frequency Domain Equalizer (FDE) is applied to the frequency domain signals after subcarrier demapping as shown in Fig. 2. For simplicity, we can employ a linear MMSE receiver, which provides a good tradeoff between the noise enhancement and CAI mitigation [19].

In the following, we let $\mathbf{D}_{F_M} = \mathbf{I}_K \otimes \mathbf{F}_M$ and denote by $\mathbf{F}_M$ the $M \times M$ Fourier matrix with the element $[\mathbf{F}_M]_{m,k} = \exp(-j\frac{2\pi}{M}(m-1)(k-1))$ where $k, m \in \{1, \cdots, M\}$ are the sample number and the frequency tone number, respectively. Here $\otimes$ is the Kronecker product, $\mathbf{I}_K$ is a $K \times K$ identity matrix. We denote by $\mathbf{D}_{F_M}^{-1}$ the $KM \times KM$ dimension inverse Fourier matrix defined as $\mathbf{I}_K \otimes \mathbf{F}_M^{-1}$, where $\mathbf{F}_M^{-1}$ is the $M \times M$ inverse Fourier matrix with the element $[\mathbf{F}_M^{-1}]_{m,k} = \frac{1}{M}\exp(j\frac{2\pi}{M}(m-1)(k-1))$. Furthermore, we let $\digamma_n$ represent the subcarrier mapping matrix of size $M \times N$ and $\digamma_n^{-1}$ is the subcarrier demapping matrix of size $N \times M$.

The received signal after the RF module and removing CP becomes $\tilde{\mathbf{r}} = \tilde{\mathbf{H}}\mathbf{D}_{F_M}^{-1}(\mathbf{I}_K \otimes \digamma_n)\mathbf{x} + \tilde{\mathbf{w}}$, where $\mathbf{x} = [\mathbf{x}_1^{\mathcal{T}}, \cdots, \mathbf{x}_K^{\mathcal{T}}]^T \in \mathbb{C}^{KN \times 1}$ is the data sequence of all K users, and $\mathbf{x}_i \in \mathbb{C}^{N \times 1}$, $i \in \{1, \cdots, K\}$, is the transmitted user data block for the ith user; $\tilde{\mathbf{w}} \in \mathbb{C}^{Mn_R \times 1}$ is a circularly symmetric complex Gaussian noise vector with zero mean and covariance matrix $N_0\mathbf{I} \in \mathbb{R}^{Mn_R \times Mn_R}$, i.e., $\tilde{\mathbf{w}} \sim \mathcal{CN}(\mathbf{0}, N_0\mathbf{I})$; $\tilde{\mathbf{H}}$ is an $n_R M \times KM$ channel matrix.

The signal after performing the FFT operation, subcarrier demapping and the MIMO FDE, is given by

$$\begin{aligned}\mathbf{z} &= \mathbf{G}^{\mathcal{H}}(\mathbf{I}_K \otimes \digamma_n^{-1})\mathbf{D}_{F_M}\tilde{\mathbf{r}} \\ &= \mathbf{G}^{\mathcal{H}}(\mathbf{I}_K \otimes \digamma_n^{-1})\mathbf{D}_{F_M}(\tilde{\mathbf{H}}\mathbf{D}_{F_M}^{-1}(\mathbf{I}_K \otimes \digamma_n)\mathbf{x} + \tilde{\mathbf{w}}) \\ &= \mathbf{G}^{\mathcal{H}}(\mathbf{H}\mathbf{x} + \mathbf{w}) = \mathbf{G}^{\mathcal{H}}(\mathbf{H}\mathbf{P}\mathbf{s} + \mathbf{w}) = \mathbf{G}^{\mathcal{H}}\mathbf{r}, \qquad (1)\end{aligned}$$

where

$$\mathbf{H} = (\mathbf{I}_K \otimes \digamma_n^{-1})\mathbf{D}_{F_M}\tilde{\mathbf{H}}\mathbf{D}_{F_M}^{-1}(\mathbf{I}_K \otimes \digamma_n) \in \mathbb{C}^{KN \times KN}$$

is a channel matrix in the frequency domain and $\mathbf{r} = \mathbf{HPs} + \mathbf{w}$; $\mathbf{G}$ is a $KN \times KN$ equalization matrix; $\mathbf{w} \in \mathbb{C}^{n_R N \times 1}$ is a circularly symmetric complex Gaussian noise vector with zero mean and covariance matrix $N_0\mathbf{I} \in \mathbb{R}^{n_R N \times n_R N}$, i.e., $\mathbf{w} \sim \mathcal{CN}(\mathbf{0}, N_0\mathbf{I})$. The vector $\mathbf{x}$ can be expressed as $\mathbf{x} = \mathbf{Ps}$, where $\mathbf{s} = [\mathbf{s}_1^{\mathcal{T}} \cdots \mathbf{s}_K^{\mathcal{T}}]^{\mathcal{T}}$ and $\mathbf{s}_i \in \mathbb{C}^{N \times 1}$, $i \in \{1, 2, \cdots, K\}$, is the user data block for the ith user, and $\mathrm{E}[\mathbf{s}_i\mathbf{s}_i^{\mathcal{H}}] = \mathbf{I}_N$. The power loading matrix $\mathbf{P} \in \mathbb{R}^{KN \times KN}$ is a block diagonal matrix with its ith sub-matrix expressed as $\mathbf{P}_i$ =diag$\{\sqrt{p_{i,1}}, \sqrt{p_{i,2}}, \cdots, \sqrt{p_{i,N}}\} \in \mathcal{R}^{N \times N}$ and $p_{i,n}$ ($i \in \{1, 2, \cdots, K\}$) is the transmitted power for the ith user at the nth subcarrier; $\mathbf{s} \in \mathbb{C}^{KN\times 1}$ represents the transmitted data symbol vector from different users with $E[\mathbf{s}\mathbf{s}^{\mathcal{H}}] = \mathbf{I}_{KN}$.

In the case when proper modulation schemes are employed, the conventional equalizer $\mathbf{G}$ can be derived from the cost function $e = \mathrm{E}[\|\mathbf{z}-\mathbf{s}\|^2] = \mathrm{E}[\|\mathbf{G}^{\mathcal{H}}\mathbf{r}-\mathbf{s}\|^2]$. Minimizing this cost function leads to the optimum solution

$$\mathbf{G} = (\mathbf{H}\mathbf{P}\mathbf{P}^{\mathcal{H}}\mathbf{H}^{\mathcal{H}} + N_0\mathbf{I})^{-1}\mathbf{H}\mathbf{P}. \qquad (2)$$

III. Iterative frequency domain equalizer

A. Conventional Iterative Equalizer

We denote the symbol vector $\mathbf{s} = \begin{bmatrix} s_1 & \ldots & s_{NK}\end{bmatrix}^{\mathcal{T}}$ which comprises the transmit symbol of NK parallel data streams. The data symbols are assumed to be uncorrelated and to have zero mean and identical energy σ_s^2, i.e., $\mathrm{E}[\mathbf{s}\mathbf{s}^{\mathcal{H}}] = \sigma_s^2\mathbf{I}_{NK}$. The received signal can be expressed as

$$\mathbf{r} = \mathbf{H}\mathbf{P}\mathbf{s} + \mathbf{v} = \sum_{i=1}^{NK}\mathbf{h}_i s_i + \mathbf{v}, \qquad (3)$$

where $\mathbf{r} = \begin{bmatrix} r_1 & r_2 & \ldots & r_{NK}\end{bmatrix}^{\mathcal{T}}$ is the received signal vector; $\mathbf{v} = \begin{bmatrix} v_1 & v_2 & \ldots & v_{NK}\end{bmatrix}^{\mathcal{T}}$ denotes the complex additive white Gaussian noise vector with zero mean and covariance matrix $\sigma_v^2\mathbf{I}_{NK}$, i.e., $\mathbf{v} \sim \mathcal{CN}(\mathbf{0}, \sigma_v^2\mathbf{I}_{NK})$. The channel matrix $\mathbf{H} \in \mathbb{C}^{NK\times NK}$ contains the complex channel gains, and $\mathbf{h}_i$ is the ith column of $\mathbf{HP}$. We assume uncorrelated Rayleigh fading channel model, and the channel coefficients are independent complex Gaussian random variables.

Suppose the symbol s_n is to be decoded. According to (3), the received vector after interference cancelation is given as

$$\mathbf{r}_n = \mathbf{r} - \mathbf{H}\mathbf{P}\bar{\mathbf{s}}_n = \mathbf{H}\mathbf{P}[\mathbf{s}-\bar{\mathbf{s}}_n] + \mathbf{v} \in \mathbb{C}^{NK\times 1}, \qquad (4)$$

where $\mathbf{r}_n$ is the interference canceled version of $\mathbf{r}$, and

$$\begin{aligned} \mathbf{s} &= \begin{bmatrix} s_1 & \ldots & s_{n-1} & s_n & s_{n+1} & \ldots & s_{NK}\end{bmatrix}^{\mathcal{T}}; \\ \bar{\mathbf{s}}_n &= \begin{bmatrix} \bar{s}_1 & \ldots & \bar{s}_{n-1} & 0 & \bar{s}_{n+1} & \ldots & \bar{s}_{NK}\end{bmatrix}^{\mathcal{T}}. \end{aligned} \qquad (5)$$

The vector $\bar{\mathbf{s}}_n$ contains the soft estimate of the interference symbols from the previous iteration. The derivation of $\bar{\mathbf{s}}_n$ will be given later on.

Note that (4) represents a decision-directed iterative scheme, where the detection procedure at the the p^{th} iteration uses the symbol estimates from the $(p-1)^{th}$ iteration. The performance is improved in an iterative manner due to the fact that the symbols are more accurately estimated (leading to better interference cancellation) as the iterative procedure goes on. For simplicity, the iteration index is omitted, whenever no ambiguity arises.

In order to further suppress the residual interference in $\mathbf{r}_n$, an instantaneous linear filter is applied to $\mathbf{r}_n$, to obtain $z_n = \mathbf{w}_n^{\mathcal{H}}\mathbf{r}_n$, where the filter coefficient vector $\mathbf{w}_n \in \mathbb{C}^{NK\times 1}$ is chosen by minimizing $e_n = \mathrm{E}\{|\mathbf{w}_n^{\mathcal{H}}\mathbf{r}_n - s_n|^2\}$ or $e_n = \mathrm{E}\{|\mathbf{w}_n^{\mathcal{H}}(\mathbf{r}_n - \mathbf{v}) - s_n|^2\}$, respectively, under the MMSE and ZF criteria. It can be derived as

$$\begin{aligned} \mathbf{w}_n &= \sigma_s^2[\mathbf{H}\mathbf{P}\mathbf{V}_n\mathbf{P}^{\mathcal{H}}\mathbf{H}^{\mathcal{H}} + N_0\mathbf{I}]^{-1}\mathbf{h}_n \quad \text{for MMSE;} \\ \mathbf{w}'_n &= \sigma_s^2[\mathbf{H}\mathbf{P}\mathbf{V}_n\mathbf{P}^{\mathcal{H}}\mathbf{H}^{\mathcal{H}}]^{-1}\mathbf{h}_n \quad \text{for ZF.} \end{aligned} \qquad (6)$$

The matrix $\mathbf{V}_n \in \mathbb{R}^{NK\times 1}$ is formed as

$$\mathbf{V}_n = \mathrm{diag}\{\mathrm{var}(s_1)\ldots\mathrm{var}(s_{n-1}) \quad \sigma_s^2 \quad \mathrm{var}(s_{n+1})\ldots\mathrm{var}(s_{NK})]\}, \qquad (7)$$

where $\mathrm{var}(s_j) = \mathrm{E}[|s_j - \bar{s}_j|^2]$. Refer to [7], [13] for a detailed description of this conventional algorithm.

B. Improved iterative solution for ASK system

The filter design shown in (6) is optimum for systems with proper modulations, such as M-QAM and M-PSK (for which $E[\mathbf{s}\mathbf{s}^{\mathcal{T}}] = \mathbf{0}$). However, for the improper modulation schemes, such as M-ary ASK (for which $E[\mathbf{s}\mathbf{s}^{\mathcal{T}}] \neq \mathbf{0}$), the above mentioned design criterion is sub-optimum. We propose a new scheme based on an error criterion defined by

$$\begin{aligned} e_n &= \mathrm{Re}\{\mathbf{g}_n^{\mathcal{H}}(\mathbf{H}\mathbf{P}[\mathbf{s}_n - \bar{\mathbf{s}}_n] + \mathbf{v})\} - s_n \\ &= 0.5\mathbf{g}_n^{\mathcal{H}}\mathbf{H}\mathbf{P}[\mathbf{s}_n - \bar{\mathbf{s}}_n] + 0.5(\mathbf{g}_n^{\mathcal{H}}\mathbf{H}\mathbf{P})^*[\mathbf{s}_n - \bar{\mathbf{s}}_n] \\ &\quad + 0.5[\mathbf{g}_n^{\mathcal{H}}\mathbf{v} + (\mathbf{g}_n^{\mathcal{H}}\mathbf{v})^*] - s_n. \end{aligned} \qquad (8)$$

Since only the real part of this output is relevant for the decision in a system with an improper constellation, minimization of the modified cost function in (8) will result in a better estimator [20].

The modified MSE function can be written as follows

$$\begin{aligned} \eta_n = \mathrm{E}[|e_n|^2] &= 0.25\sigma_s^2(\mathbf{g}_n^{\mathcal{H}}\mathbf{H}\mathbf{P}\mathbf{V}_n\mathbf{P}^{\mathcal{H}}\mathbf{H}^{\mathcal{H}}\mathbf{g}_n \\ &+ \mathbf{g}_n^{\mathcal{H}}\mathbf{H}\mathbf{P}\mathbf{V}_n\mathbf{P}^{\mathcal{T}}\mathbf{H}^{\mathcal{T}}\mathbf{g}_n^* + \mathbf{g}_n^{\mathcal{T}}\mathbf{H}^*\mathbf{P}^*\mathbf{V}_n\mathbf{P}^{\mathcal{H}}\mathbf{H}^{\mathcal{H}}\mathbf{g}_n \\ &+ \mathbf{g}_n^{\mathcal{T}}\mathbf{H}^*\mathbf{P}^*\mathbf{P}^{\mathcal{T}}\mathbf{V}_n\mathbf{H}^{\mathcal{T}}\mathbf{g}_n^*) \\ &- 0.5\sigma_s^2(\mathbf{g}_n^{\mathcal{H}}\mathbf{h}_n + \mathbf{g}_n^{\mathcal{T}}\mathbf{h}_n^* + \mathbf{h}_n^{\mathcal{H}}\mathbf{g}_n + \mathbf{h}_n^{\mathcal{T}}\mathbf{g}_n^*) \\ &+ \sigma_v^2(\mathbf{g}_n^{\mathcal{H}}\mathbf{g}_n + \mathbf{g}_n^{\mathcal{T}}\mathbf{g}_n^*) + \sigma_s^2. \end{aligned}$$

Setting the partial derivative of η_n with respect to $\mathbf{g}_n$ to zero results in the following equation

$$2\mathbf{h}_n = \mathbf{H}\mathbf{P}\mathbf{V}_n\mathbf{P}^{\mathcal{H}}\mathbf{H}^{\mathcal{H}}\mathbf{g}_n + \mathbf{H}\mathbf{P}\mathbf{V}_n\mathbf{P}^{\mathcal{T}}\mathbf{H}^{\mathcal{T}}\mathbf{g}_n^* + \frac{\sigma_v^2}{\sigma_s^2}\mathbf{g}_n. \qquad (9)$$

The above equation holds since

$$\begin{aligned} &\frac{\partial \mathbf{g}_n^{\mathcal{H}}\mathbf{g}_n}{\partial \mathbf{g}_n} = \frac{\partial \mathbf{g}_n^{\mathcal{T}}\mathbf{g}_n^*}{\partial \mathbf{g}_n} = \mathbf{g}_n^*; \quad \frac{\partial \mathbf{g}_n^{\mathcal{T}}\mathbf{h}_n^*}{\partial \mathbf{g}_n} = \frac{\partial \mathbf{h}_n^{\mathcal{H}}\mathbf{g}_n}{\partial \mathbf{g}_n} = \mathbf{h}_n^*; \\ &\frac{\partial \mathbf{g}_n^{\mathcal{H}}\mathbf{h}_n}{\partial \mathbf{g}_n} = \frac{\partial \mathbf{h}_n^{\mathcal{T}}\mathbf{g}_n^*}{\partial \mathbf{g}_n} = \mathbf{0}; \quad \frac{\partial \mathbf{g}_n^{\mathcal{H}}\mathbf{H}\mathbf{P}\mathbf{V}_n\mathbf{P}^{\mathcal{T}}\mathbf{H}^{\mathcal{T}}\mathbf{g}_n^*}{\partial \mathbf{g}_n} = \mathbf{0}; \\ &\frac{\partial \mathbf{g}_n^{\mathcal{T}}\mathbf{H}^*\mathbf{P}^*\mathbf{V}_n\mathbf{P}^{\mathcal{H}}\mathbf{H}^{\mathcal{H}}\mathbf{g}_n}{\partial \mathbf{g}_n} = 2\mathbf{H}^*\mathbf{P}^*\mathbf{V}_n\mathbf{P}^{\mathcal{H}}\mathbf{H}^{\mathcal{H}}\mathbf{g}_n; \\ &\frac{\partial \mathbf{g}_n^{\mathcal{H}}\mathbf{H}\mathbf{P}\mathbf{V}_n\mathbf{P}^{\mathcal{H}}\mathbf{H}^{\mathcal{H}}\mathbf{g}_n}{\partial \mathbf{g}_n} = (\mathbf{H}\mathbf{P}\mathbf{V}_n\mathbf{P}^{\mathcal{H}}\mathbf{H}^{\mathcal{H}}\mathbf{g}_n)^*; \\ &\frac{\partial \mathbf{g}_n^{\mathcal{T}}\mathbf{H}^*\mathbf{P}^*\mathbf{V}_n\mathbf{P}^{\mathcal{T}}\mathbf{H}^{\mathcal{T}}\mathbf{g}_n^*}{\partial \mathbf{g}_n} = \mathbf{H}^*\mathbf{P}^*\mathbf{V}_n\mathbf{P}^{\mathcal{T}}\mathbf{H}^{\mathcal{T}}\mathbf{g}_n^*; \end{aligned}$$

Denoting

$$\begin{aligned} \mathbf{H}\mathbf{P}\mathbf{V}_n\mathbf{P}^{\mathcal{H}}\mathbf{H}^{\mathcal{H}} &= \mathbf{A}_r + j\mathbf{A}_i; \qquad \mathbf{g}_n = \mathbf{g}_r + j\mathbf{g}_i; \\ \mathbf{H}\mathbf{P}\mathbf{V}_n\mathbf{P}^{\mathcal{T}}\mathbf{H}^{\mathcal{T}} &= \mathbf{B}_r + j\mathbf{B}_i; \qquad \mathbf{h}_n = \mathbf{h}_r + j\mathbf{h}_i, \end{aligned} \qquad (10)$$

then Eq. (9) can be reformed as

$$\begin{aligned} &2\mathbf{h}_r + 2j\mathbf{h}_i = \\ &(\mathbf{A}_r + j\mathbf{A}_i)(\mathbf{g}_r + j\mathbf{g}_i) + (\mathbf{B}_r + j\mathbf{B}_i)(\mathbf{g}_r - j\mathbf{g}_i) + \xi(\mathbf{g}_r + j\mathbf{g}_i) \\ &= (\mathbf{A}_r\mathbf{g}_r - \mathbf{A}_i\mathbf{g}_i) + j(\mathbf{A}_r\mathbf{g}_i + \mathbf{A}_i\mathbf{g}_r) \\ &+ (\mathbf{B}_r\mathbf{g}_r + \mathbf{B}_i\mathbf{g}_i) + j(\mathbf{B}_i\mathbf{g}_r - \mathbf{B}_r\mathbf{g}_i) + \xi\mathbf{g}_r + j\xi\mathbf{g}_i \\ &= (\mathbf{A}_r\mathbf{g}_r - \mathbf{A}_i\mathbf{g}_i + \mathbf{B}_r\mathbf{g}_r + \mathbf{B}_i\mathbf{g}_i + \xi\mathbf{g}_r) \\ &\quad + j(\mathbf{A}_r\mathbf{g}_i + \mathbf{A}_i\mathbf{g}_r + \mathbf{B}_i\mathbf{g}_r - \mathbf{B}_r\mathbf{g}_i + \xi\mathbf{g}_i), \end{aligned}$$

where $\xi = \frac{\sigma_v^2}{\sigma_s^2}$. The real and imaginary parts of the vector $2\mathbf{h}$ can be expressed in a vector form as

$$\begin{bmatrix} 2\mathbf{h}_r \\ 2\mathbf{h}_i \end{bmatrix} = \begin{bmatrix} \mathbf{A}_r + \mathbf{B}_r + \xi & \mathbf{B}_i - \mathbf{A}_i \\ \mathbf{A}_i + \mathbf{B}_i & \mathbf{A}_r - \mathbf{B}_r + \xi \end{bmatrix}\begin{bmatrix} \mathbf{g}_r \\ \mathbf{g}_i \end{bmatrix},$$

leading to the improved MMSE filter $\mathbf{g}_n = \mathbf{g}_r + j\mathbf{g}_i$, where $\mathbf{g}_r$ and $\mathbf{g}_i$ are derived as

$$\begin{bmatrix}\mathbf{g}_r\\ \mathbf{g}_i\end{bmatrix} = \begin{bmatrix}\mathbf{A}_r + \mathbf{B}_r + \xi & \mathbf{B}_i - \mathbf{A}_i\\ \mathbf{A}_i + \mathbf{B}_i & \mathbf{A}_r - \mathbf{B}_r + \xi\end{bmatrix}^{-1}\begin{bmatrix}2\mathbf{h}_r\\ 2\mathbf{h}_i\end{bmatrix}. \quad (11)$$

The ZF solution is derived by minimizing the following function with respect to $\mathbf{g}_n$

$$\begin{aligned}\eta_n' = {}& 0.25\sigma_s^2(\mathbf{g}_n^{\mathcal{H}}\mathbf{HPV}_n\mathbf{P}^{\mathcal{H}}\mathbf{H}^{\mathcal{H}}\mathbf{g}_n + \mathbf{g}_n^{\mathcal{H}}\mathbf{HPV}_n\mathbf{P}^{\mathcal{T}}\mathbf{H}^{\mathcal{T}}\mathbf{g}_n^*\\ &+ \mathbf{g}_n^{\mathcal{T}}\mathbf{H}^*\mathbf{P}^*\mathbf{V}_n\mathbf{P}^{\mathcal{H}}\mathbf{H}^{\mathcal{H}}\mathbf{g}_n + \mathbf{g}_n^{\mathcal{T}}\mathbf{H}^*\mathbf{P}^*\mathbf{V}_n\mathbf{P}^{\mathcal{T}}\mathbf{H}^{\mathcal{T}}\mathbf{g}_n^*) + \sigma_s^2\\ &- 0.5\sigma_s^2(\mathbf{g}_n^{\mathcal{H}}\mathbf{h}_n + \mathbf{g}_n^{\mathcal{T}}\mathbf{h}_n^* + \mathbf{h}_n^{\mathcal{H}}\mathbf{g}_n + \mathbf{h}_n^{\mathcal{T}}\mathbf{g}_n^*).\end{aligned}$$

Following the same procedure as shown previously, we can form the improved ZF filter as $\mathbf{g}_n' = \mathbf{g}_r' + j\mathbf{g}_i'$, where $\mathbf{g}_r'$ and $\mathbf{g}_i'$ are derived as

$$\begin{bmatrix}\mathbf{g}_r'\\ \mathbf{g}_i'\end{bmatrix} = \begin{bmatrix}\mathbf{A}_r + \mathbf{B}_r & \mathbf{B}_i - \mathbf{A}_i\\ \mathbf{A}_i + \mathbf{B}_i & \mathbf{A}_r - \mathbf{B}_r\end{bmatrix}^{-1}\begin{bmatrix}2\mathbf{h}_r\\ 2\mathbf{h}_i\end{bmatrix}. \quad (12)$$

Unlike the MMSE solution, the ZF filter does not need any knowledge of the noise as shown by (12).

C. Improved iterative solution for QPSK system

The performance of the iterative receiver for QPSK systems can be improved by applying WLF and exploiting the complete second-order statistics of the received signals. It was shown in [21] that for proper constellations, such as QPSK, the improperness of the observation occurs in the course of the iterations. To exploit this property, we not only process $\mathbf{r}_n$, but also its conjugated version $\mathbf{r}_n^*$ in order to derive the filter output [21], [22], i.e., $z_n = \mathbf{a}_n\mathbf{r}_n + \mathbf{b}_n\mathbf{r}_n^* = \mathbf{\Psi}_n^{\mathcal{H}}\mathbf{y}_n$, where $\mathbf{\Psi}_n = \begin{bmatrix}\mathbf{a}_n & \mathbf{b}_n\end{bmatrix}^{\mathcal{H}}$ and $\mathbf{y}_n = \begin{bmatrix}\mathbf{r}_n^{\mathcal{T}} & (\mathbf{r}_n^*)^{\mathcal{T}}\end{bmatrix}^{\mathcal{T}}$. The filter $\mathbf{\Psi}_n$ can be derived by minimizing the MSE $\mathrm{E}\{|e_n|^2\}$, where $e_n = z_n - s_n = \mathbf{\Psi}_n^{\mathcal{H}}\mathbf{y}_n - s_n$. According to the orthogonality principle,

$$\mathrm{E}[\mathbf{y}_n e_n^*] = \mathrm{E}[\mathbf{y}_n(\mathbf{\Psi}_n^{\mathcal{H}}\mathbf{y}_n - s_n)^{\mathcal{H}}] = \mathbf{0},$$

leading to the solution

$$\mathbf{\Psi}_n = (\mathrm{E}[\mathbf{y}_n\mathbf{y}_n^{\mathcal{H}}])^{-1}\,\mathrm{E}[\mathbf{y}_n s_n^*] = \mathbf{\Phi}_{\mathbf{yy}}^{-1}\mathbf{\Phi}_{\mathbf{y}s}, \quad (13)$$

where

$$\begin{aligned}\mathbf{\Phi}_{\mathbf{yy}} &= \mathrm{E}\{\mathbf{y}_n\mathbf{y}_n^{\mathcal{H}}\} = \mathrm{E}\left\{\begin{bmatrix}\mathbf{r}_n\\ \mathbf{r}_n^*\end{bmatrix}\begin{bmatrix}\mathbf{r}_n^{\mathcal{H}} & \mathbf{r}_n^{\mathcal{T}}\end{bmatrix}\right\}\\ &= \begin{bmatrix}\mathbf{HPV}_n\mathbf{P}^{\mathcal{H}}\mathbf{H}^{\mathcal{H}} + \sigma_n^2\mathbf{I} & \mathbf{HP}\tilde{\mathbf{V}}_n\mathbf{P}^{\mathcal{T}}\mathbf{H}^{\mathcal{T}}\\ \mathbf{H}^*\mathbf{P}^*\tilde{\mathbf{V}}_n^*\mathbf{P}^{\mathcal{H}}\mathbf{H}^{\mathcal{H}} & \mathbf{H}^*\mathbf{P}^*\mathbf{V}_n\mathbf{P}^{\mathcal{T}}\mathbf{H}^{\mathcal{T}} + \sigma_n^2\mathbf{I}\end{bmatrix};\end{aligned} \quad (14)$$

the matrix $\mathbf{V}_n$ is defined in (7) and

$$\mathbf{\Phi}_{\mathbf{y}s} = \mathrm{E}\{\mathbf{y}_n s_n^*\} = \mathrm{E}\left\{\begin{bmatrix}\mathbf{r}_n s_n^*\\ \mathbf{r}_n^* s_n^*\end{bmatrix}\right\} = \begin{bmatrix}\mathbf{h}_n\\ \mathbf{0}\end{bmatrix} \quad (15)$$

The matrix $\tilde{\mathbf{V}}_n$ is calculated as

$$\begin{aligned}\tilde{\mathbf{V}}_n &= \mathrm{E}\{[\mathbf{s}_n - \bar{\mathbf{s}}_n][\mathbf{s}_n - \bar{\mathbf{s}}_n]^{\mathcal{T}}\}\\ &= \mathrm{diag}\{\Lambda_1 \quad \ldots \quad \Lambda_{n-1} \quad 0 \quad \Lambda_{n+1} \quad \ldots \quad \Lambda_N\}.\end{aligned} \quad (16)$$

Denoting the complex QPSK symbol $s_p = s_{p,I} + js_{p,Q}$, and $\bar{s}_p = \bar{s}_{p,I} + j\bar{s}_{p,Q}$, where $\bar{s}_p = \mathrm{E}[s_p]$, the pth diagonal element of $\tilde{\mathbf{V}}_n$ is calculated as [21]

$$\begin{aligned}\Lambda_p &= \mathrm{E}[(s_p - \bar{s}_p)^2] = \mathrm{E}[s_p^2] - (\bar{s}_p)^2\\ &= \mathrm{E}[s_{p,I}^2 + 2js_{p,I}s_{p,Q} - s_{p,Q}^2] - (\bar{s}_{p,I})^2 - 2j\bar{s}_{p,I}\bar{s}_{p,Q} + (\bar{s}_{p,Q})^2\\ &= (\bar{s}_{p,Q})^2 - (\bar{s}_{p,I})^2.\end{aligned}$$

In what follows, we demonstrate how the vector $\bar{\mathbf{s}}_n$ in (5), the matrix $\mathbf{V}_n$ in (7) as well as $\tilde{\mathbf{V}}_n$ in (16) can be derived in order to carry out the iterative process for QPSK and 4-ASK systems. The filter output can be expressed as

$$z_n = \mathbf{g}_n^{\mathcal{H}}\mathbf{r}_n = \mu_n s_n + \nu_n, \quad (17)$$

where the combined noise and residual interference ν_n can be approximated as a Gaussian random variable [23], i.e., $\nu_n \sim \mathcal{CN}(0, N_\nu)$. The parameters μ_n, N_ν can be determined as [24]

$$\begin{aligned}\mu_n &= \mathrm{E}\{z_n s_n^*\} = \mathbf{g}_n^{\mathcal{H}}\,\mathrm{E}[\mathbf{r}_n s_n^*] = \mathbf{g}_n^{\mathcal{H}}\mathbf{C}_{\mathbf{r}s};\\ N_\nu &= \mu_n - \mu_n^2,\end{aligned} \quad (18)$$

where $\mathbf{C}_{\mathbf{r}s} = \mathrm{E}[\mathbf{r}_n s_n^*] = \mathbf{h}_n$.

After computing the values of μ_n and N_ν, the conditional probability density function (PDF) of the filter output can be obtained as

$$f(z_n|s_n = s_m) = \frac{1}{\pi N_\nu}\exp\left(-\frac{|z_n - \mu_n s_m|^2}{N_\nu}\right),$$

For QPSK and 4-ASK systems, each symbol s_n corresponds to two information bits, denoted as b_n^0 and b_n^1. For the 4-ASK system, the log-likelihood ratio (LLR) for the first information bit b_n^0 can be computed as

$$\begin{aligned}\lambda(b_n^0) &= \ln\frac{f(z_n|b_n^0 = 0)}{f(z_n|b_n^0 = 1)} = \ln\frac{f(z_n|s_n = s_0) + f(z_n|s_n = s_3)}{f(z_n|s_n = s_1) + f(z_n|s_n = s_2)}\\ &\approx \ln\frac{\exp(-|z_n - \mu_n s_n^+|^2/N_\nu)}{\exp(-|z_n - \mu_n s_n^-|^2/N_\nu)}\\ &= \frac{1}{N_\nu}\{|z_n - \mu_n s_n^-|^2 - |z_n - \mu_n s_n^+|^2\}\\ &= \frac{1}{1-\mu_n}\mathrm{Re}\{[2s_n^{+*}z_n - \mu_n|s_n^+|^2] - [2s_n^{-*}z_n - \mu_n|s_n^-|^2]\},\end{aligned} \quad (19)$$

where s_+ denotes the 4-ASK symbol corresponding to $\max\{f(z_n|s_0), f(z_n|s_3)\}$, and s_- denotes the 4-ASK symbol corresponding to $\max\{f(z_n|s_1), f(z_n|s_2)\}$ since the real part of the symbols s_0, s_3 corresponds to 0, and the real part of the symbols s_1, s_2 corresponds to 1 as shown in Fig. 3.

For the QPSK system, since all the signal candidates have the same energy, i.e., $|s_0|^2 = |s_1|^2 = |s_2|^2 = |s_3|^2$, the LLR value of b_n^0 can thus be simplified to

$$\lambda(b_n^0) \approx \frac{2}{1-\mu_n}\mathrm{Re}\{s_n^{+*}z_n - s_n^{-*}z_n\},$$

and the definition of s_+ and s_- is the same as described previously. The LLR value for the second information bit can be obtained in a similar manner as

$$\begin{aligned}\lambda(b_n^1) &= \ln\frac{f(z_n|s_0) + f(z_n|s_1)}{f(z_n|s_2) + f(z_n|s_3)}\\ &\approx \begin{cases}\frac{1}{1-\mu_n}\mathrm{Re}\{[2s_n^{+*}z_n - \mu_n|s_n^+|^2] - [2s_n^{-*}z_n - \mu_n|s_n^-|^2]\}; & \text{for 4-ASK}\\ \frac{2}{1-\mu_n}\mathrm{Re}\{s_n^{+*}z_n - s_n^{-*}z_n\}; & \text{for QPSK}\end{cases}\end{aligned}$$

where s_+ denotes the 4-ASK/QPSK symbol corresponding to $\max\{f(z_n|s_0), f(z_n|s_1)\}$, and s_- denotes the 4-ASK/QPSK symbol corresponding to $\max\{f(z_n|s_2), f(z_n|s_3)\}$ since the imaginary part of the symbols s_0, s_1 corresponds to 0, and the imaginary part of the symbols s_2, s_3 corresponds to 1 as shown in Figs. 3 and 4.

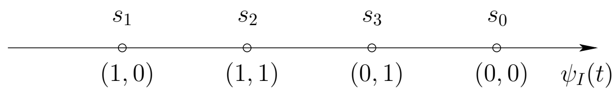

Fig. 3. 4-ASK constellation and bit-to-symbol mapping.

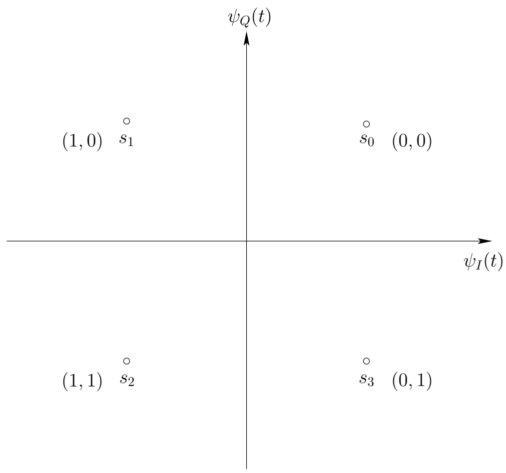

Fig. 4. QPSK constellation and bit-to-symbol mapping.

Since the a priori probability of each symbol $P_r(s_j) = P_r(b_j^0) \cdot P_r(b_j^1)$ in an uncoded system, according to the symbol-to-bit mapping shown in Figs. 3 and 4, we have

$$\begin{aligned}
P_r(s_j = s_0) &= P_r(b_j^0 = 0) \cdot P_r(b_j^1 = 0) \\
P_r(s_j = s_1) &= P_r(b_j^0 = 1) \cdot P_r(b_j^1 = 0) \\
P_r(s_j = s_2) &= P_r(b_j^0 = 1) \cdot P_r(b_j^1 = 1) \\
P_r(s_j = s_3) &= P_r(b_j^0 = 0) \cdot P_r(b_j^1 = 1)
\end{aligned}$$

where

$$\begin{aligned}
P_r(b_j^0 = 0) &= \frac{e^{\lambda(b_j^0)}}{1 + e^{\lambda(b_j^0)}}; \qquad P_r(b_j^0 = 1) = \frac{1}{1 + e^{\lambda(b_j^0)}} \\
P_r(b_j^1 = 0) &= \frac{e^{\lambda(b_j^1)}}{1 + e^{\lambda(b_j^1)}}; \qquad P_r(b_j^1 = 1) = \frac{1}{1 + e^{\lambda(b_j^1)}}
\end{aligned}$$

With the a priori probability of each symbol $P_r(s_j)$, the soft estimate $\bar{s}_j$ in the vector $\bar{\mathbf{s}}_n$ and the variance $\mathrm{var}(s_j)$ in the matrix $\mathbf{V}_n$, respectively, can be calculated as [24]

$$\begin{aligned}
\bar{s}_j &= \mathrm{E}\{s_j\} = \sum_{m=0}^{3} s_m P_r(s_j = s_m); \\
\mathrm{var}(s_j) &= \mathrm{E}[|s_j|^2] - |\bar{s}_j|^2,
\end{aligned}$$

where $\mathrm{E}[|s_j|^2] = \sum_{m=0}^{3} |s_m|^2 P_r(s_j = s_m)$.

Due to the space limit, the simulation results are shown and conclusions drawn in another paper submitted to ISWCS'13 [25], where one can see that for both QPSK and 4ASK systems, he improved iterative receiver scheme achieves better performance at medium to high SNR region.

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REFERENCES

[1] Z. Lin, P. Xiao and B. Vucetic. "Analysis of Receiver Algorithms for LTE SC-FDMA Based Uplink MIMO Systems", *IEEE Transactions on Wireless Communications*, Vol. 9, No. 1, Nov. 2010, pp. 60-65.

[2] Z. Lin, P. Xiao, T. B. Sorensen, B. Vucetic. "Spatial Frequency Scheduling for Uplink SC-FDMA based Linearly Precoded LTE Multi-user MIMO Systems", *Transactions on Emerging Telecommunications Technologies*, Vol. 21, No. 3, April. 2010, pp. 213-223.

[3] Z. Lin, T. B. Sorensen, P. E. Mogensen. "Downlink SINR distribution of linearly precoded multi- user MIMO systems", *IEEE Communications Letters*, Vol. 11, No. 11, Nov. 2007.

[4] 3GPP TR 25.814 V7.0.0, "Physical Layer Aspects for Evolved UTRA," Tech. Rep., June 2006.

[5] F. Pancaldi, G. M. Bitetta. "Block channel equalization in the frequency domain", *IEEE Trans. Commun.*, vol. 53, no. 1, pp. 110-121, 1995.

[6] N. Benvenuto and S. Tomasin. "Iterative design and detection of a DFE in the frequency domain", *IEEE Trans. Commun.*, vol. 53, no. 11, pp. 1867-1875, 2005.

[7] M. Tuchler, R. Koetter, A. Singer. "Turbo equalization: principles and new results", *IEEE Trans. Commun.*, vol. 50, pp. 754–767, May 2002.

[8] D. Tse and P. Viswanath. *Fundamentals of Wireless Communications*, Cambridge University Press, 2004.

[9] G. Foschini. "Layered space-time architecture for wireless communication in a fading environment when using multiple-element antennas", *Bells Labs Tech. J.*, vol. 1, pp. 41–59, 1996.

[10] G. Foschini, M. Gans. "On limits of wireless communications in a fading environment when using multiple antennas", *Wireless Personal Commun.*, vol. 6, no. 3, pp. 311-335, March 1998.

[11] A. Paulraj, R. Nabar, D. Gore. *Introduction to Space-Time Wireless Communications*, Cambridge, University Press, 2003.

[12] C. Douillard, M. Jezequel, C. Berrou. "Iterative correction of intersymbol interference: turbo-equalization". *European Trans. on Telecommun.*, pp. 507-511, Sept. 1995.

[13] J. Wang, S. Li. "Reliability based reduced-complexity MMSE soft interference cancellation MIMO turbo receiver", *Proc. PIMRC*, pp. 1–4, Sept. 2007.

[14] P. Schreier, L. Scharf, and C. Mullis. "Detection and estimation of improper complex random signals," *IEEE Trans. Inform. Theory*, vol. 51, no. 1, pp. 306–312, Jan. 2005.

[15] F. Neeser, J. Massey. "Proper complex random processes with applications to information theory", *IEEE Trans. Inform. Theory*, vol. 39, no. 4, pp. 1293–1302, July 1993.

[16] B. Picinbono and P. Chevalier. "Widely linear estimation with complex data," *Trans. Sig. Proc.*, vol. 43, no. 8, pp. 2030–2033, Aug. 1995.

[17] F. Sterle. "Widely Linear MMSE Transceivers for MIMO Channels", *IEEE Trans. Sig. Proc.*, vol. 55, no. 8, pp. 4258-4270, Aug. 2007.

[18] S. Chen, S. Tan, and L. Hanzo. "Linear Beamforming Assisted Receiver for Binary Phase Shift Keying Modulation Systems", *Proc. IEEE WCNC*, pp. 1741-1746, April 2006.

[19] A.J. Paulraj, R. Nabar & D. Gore. *Introduction to Space-Time Wireless Communications*, Cambridge University Press, 1st edition, September 2003.

[20] S. Buzzi, M. Lops, A. Tulino. "A new family of MMSE multiuser receivers for interference suppression in DS/CDMA systems employing BPSK modulation". *IEEE Trans. on Commun.*, vol. 49, no. 1, pp. 154–167, Jan. 2001.

[21] A. Lampe, R. Schober, W. Gerstacker, J. Huber. "A novel iterative multiuser detector for complex modulation schemes".

[22] M. Witzke, S. Baro, J. Haeauer. "Iterative detection of generalized coded MIMO signals using a widely linear detector". *Proc. IEEE Globecom'03*, vol. 4, pp. 1821–1825, Dec. 2003.
IEEE J. Sel. Area Comm., vol. 20, no. 2, pp. 339–350, Feb. 2002.

[23] V. Poor, S. Verdu. "Probability of error in MMSE multiuser detection", *IEEE Trans. Commun.*, vol. 43, pp. 858–971, May 1997.

[24] A. Dejonghe and L. Vandendorpe. "Turbo-equalization for multilevel modulation: an efficient low-complexity scheme", *Proc. IEEE ICC*, vol. 3, pp. 1863–1867, 2002.

[25] Z. Lin, P. Xiao, Y. Wu. "Application of WLF to OFDMA MU-MIMO Systems II: Frequency-Domain Packet Scheduling". *The Tenth International Symposium on Wireless Communication Systems, ISWCS 2013*.