The nuclear structure of neutron-rich \( N > 126 \) nuclei has been investigated following their production via relativistic projectile fragmentation of a \( E/A = 1 \) GeV \( ^{238}\text{U} \) beam. Metastable states in the \( N = 128 \) isotones \( ^{208}\text{Hg} \) and \( ^{208}\text{Tl} \) have been identified. Delayed \( \gamma \)-ray transitions are interpreted as arising from the decay of \( I^\pi = (8^+ \) and \( 17/2^+ \) ) isomers, respectively. The data allow for the so far most comprehensive verification of the shell-model approach in the region determined by magic numbers \( Z < 82 \) and \( N > 126 \).

The understanding of how shell structure arises and develops is a major goal in contemporary nuclear physics. To this end, it is of particular importance to measure the properties of nuclei in the vicinity of closed shells. Information on the single-particle energies, proton-neutron interactions, and two-body residual interactions can be derived from experimental observables such as masses, energies of excited states, and transition probabilities [1]. Furthermore, information on the global behavior of nuclei can be obtained from the energy spacing of the lowest lying states in even-even systems. Recently, it was shown that there is direct empirical correlation between the \( p-n \) interaction strength and the growth of collectivity determined from the energies of the first \( 2^+ \) and \( 4^+ \) excitations [2]. The \( p-n \) interaction [2–4], especially among valence nucleons, is an important factor in controlling the onset and development of collectivity and deformation in nuclei and in determining the structure of nuclear transition regions. However, there is no double-magic nucleus (above mass 48), around which spectroscopic data are available in all four quadrants beyond the one- and two-particle neighbors. Although many nuclei in the \( ^{208}\text{Pb} \) region have been studied, we have no information on the excited states of even-even nuclei in the “southeast” quadrant defined by \( Z < 82 \) and \( N > 126 \). Yet, such nuclei, representing the particle-hole sector surrounding \( ^{208}\text{Pb} \), are critical for understanding the effects of seniority, the onset of proton-neutron configuration mixing that drives collectivity and nuclear deformation. This study provides the first spectroscopic data on an even-even nucleus in this region, namely \( ^{208}\text{Hg} \), and it allows the first detailed verification of the shell-model approach and nucleon-nucleon interaction in this region away from the semi-magic nuclei and the particle-hole neighbor \( ^{208}\text{Tl} \) [5]. Recently, the mass of \( ^{208}\text{Hg} \) was measured [4], allowing the extraction of the average \( p-n \) interaction for \( ^{210}\text{Pb} \), the first value in the proton-hole-neutron-particle quadrant. The combination of mass and spectroscopic data is essential in understanding the evolution of structure near doubly magic nuclei. Indeed, it has recently been shown that the link between masses and structure is stronger and more sensitive than hitherto thought [6]. Furthermore, because the newly accessible region near \( ^{132}\text{Sn} \) [7] shares many similarities with the Pb region, studies...
in the latter may have broader implications for the former and for other doubly magic regions in exotic nuclei.

To date, our knowledge of the properties of heavy neutron-rich nuclei at or near the $N = 126$ shell is very limited. In the case of nuclei with $Z < 82$ and $N > 126$, excited states were reported only in $^{208}$Tl [8] and $^{209}$Tl [9,10]. The lack of information on nuclei in this region is mainly from the difficulties in creating and populating excited states in these neutron-rich nuclei. Fragmentation has proven to be an efficient tool for producing exotic nuclear species, and when combined with high sensitivity γ-detection arrays, structural information can be gained for otherwise inaccessible nuclei. The highest sensitivity is achieved with both isomeric and β-delayed γ-ray spectroscopy techniques; delayed γ rays are time-correlated with individually identified ions, thereby minimizing the associated background radiation [11–13]. Information on the excited states populated in this way can be obtained when producing only a few hundred nuclei of interest [12]. In this rapid communication, results on the structure of heavy neutron-rich nuclei with $N > 126$ are reported. Isomeric states in the $N = 128$ isotones $^{208}$Hg and $^{209}$Tl have been identified for the first time. Preliminary experimental results have been reported in conference proceedings [14,15].

Heavy neutron-rich nuclei were populated in relativistic energy projectile fragmentation. The primary $^{238}$U beam at an energy of $E/A = 1$ GeV was provided by the SIS-18 accelerator at GSI, Darmstadt, Germany. The maximum primary beam intensity was $\sim 10^{10}$ ions/spill. The $\sim 2$ s spills were separated by $\sim 2$ s periods without beam. The $^{238}$U ions impinged on a target composed of 2.5 g/cm$^2$ $^9$Be + 223 mg/cm$^2$ Nb, where the Nb foil serves for electron stripping of the reaction products. The nuclei of interest were selected and identified in flight on an event-by-event basis by the Fragment Separator (FRS) [16]. The FRS was optimized for the transmission of $^{205}$Pt ions. Details of the experiment and particle identification technique are given in Refs. [14, 15], and [17–19]. The transmitted (and identified) ions were slowed down in a variable-thickness aluminium degrader and finally implanted in an active stopper. The total number of implants included $\sim 700$ $^{208}$Hg and $\sim 620$ $^{209}$Tl nuclei.

The stopper covered an area of $15 \times 5$ cm$^2$ and had a thickness of 2 mm. It consisted of six double-sided silicon detectors, each of size $5 \times 5$ cm$^2$ and 1 mm thickness [20]. It was surrounded by the Rare Isotope Investigations at GSI (RISING) germanium array in the “Stopped Beam” configuration. The array consists of 15 Euroball cluster germanium detectors and has a photopeak efficiency of $\sim 15\%$ at 661 keV [18].

Correlated with the implanted ions, γ decays following both internal decay and β decay have been recorded. The particle identification is confirmed by the observation of the previously reported isomeric decays in $^{204}$Au [21], $^{205}$Au [21,22], and $^{206}$Hg [23]. The γ-ray spectrum as well as the decay curve associated with $^{206}$Hg are shown in Fig. 1(a). The half-life of the $I^+ = 10^+$ isomeric state obtained in our work, $T_{1/2} = 96(15)$ ns, is in good agreement with the previously measured value of $T_{1/2} = 92(8)$ ns [23].

Evidence of decays from isomeric states in the $N = 128$ isotones $^{208}$Hg and $^{209}$Tl is observed. Delayed γ rays associated with $^{208}$Hg nuclei are shown in Fig. 1(b). Three γ-ray transitions with energies 203, 425, and 669 keV, together with characteristic Hg Kα γ rays are identified. The three γ-ray transitions are in mutual coincidence, and they have similar half-lives within experimental uncertainties. The measured half-life is $T_{1/2} = 99(14)$ ns [see inset to Fig. 1(b)]. The measured relative γ-ray intensities are $I_{\gamma}(669.0) = 100(16)$, $I_{\gamma}(424.9) = 107(16)$, and $I_{\gamma}(203.0) = 77(11)$. Assuming that the three γ-ray transitions form a single cascade, the total transition intensities have to be equal, and the conversion coefficient of the 203 keV transition can be determined.

![FIG. 1. (Color online) Delayed γ-ray spectra and decay curves associated with (a) $^{208}$Hg [23], (b) $^{208}$Hg, and (c) $^{209}$Tl.](image-url)
TABLE I. Comparison between experimental and shell-model B(E2) transition strengths. Values are given in Weisskopf units. Effective charges of 1.5 e for protons and 1.0 e for neutrons were assumed.

<table>
<thead>
<tr>
<th>Nucleus</th>
<th>Transition</th>
<th>B(E2)_{exp}</th>
<th>B(E2)_{calc}</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{209}$Pb</td>
<td>$8^+ \rightarrow 6^+$</td>
<td>0.64(7) [28,29]</td>
<td>0.69</td>
</tr>
<tr>
<td>$^{208}$Hg</td>
<td>(10^+)$ \rightarrow (8^+)$</td>
<td>0.99(18) [23,30]</td>
<td>0.87</td>
</tr>
<tr>
<td>$^{208}$Hg</td>
<td>(8^+)$ \rightarrow (6^+)$</td>
<td>1.95(39)$ - 1.58(22)^a$</td>
<td>1.22</td>
</tr>
<tr>
<td>$^{209}$Tl</td>
<td>(17/2^+)$ \rightarrow (13/2^+)$</td>
<td>1.87(22)$ - 1.51(18)^a$</td>
<td>0.96</td>
</tr>
</tbody>
</table>

*a Assuming a transition energy between 20 and 80 keV.

The 8^+ transitions are considered to have E2 character: $\alpha = 0.36(6)$. This suggests that the 203 keV transition has E2 ($\alpha_{\text{obs}} = 0.37$) character. The amount of $K_\alpha$ X rays following the conversion electron emission is in agreement with the observed intensity.

In $^{209}$Tl, an isomeric decay in a similar time range is in evidence. $\gamma$ rays with energies of 137, 323, and 661 keV, together with the characteristic Tl $K_\alpha$ X ray are identified [see Fig. 1(c)]. The 137, 323, and 661 keV transitions are in mutual coincidence, and their half-lives agree within errors. Therefore, they form a cascade. No parallel branches are observed. From the $\gamma$-ray intensities, $I_\gamma(323,1) = 100(15)$, $I_\gamma(661,2) = 96(19)$, and $I_\gamma(136,8) = 41(10)$, the conversion coefficient of the 137 keV transition can be determined: $\alpha = 1.5(4)$, suggesting that it is an E2 ($\alpha_{\text{obs}} = 1.60$). The amount of $K_\alpha$ X rays following the conversion electron emission is in agreement with the observed intensity. The measured half-life is $T_{1/2} = 95(11)$ ns [see inset to Fig. 1(c)].

To obtain a quantitative understanding of the underlying single-particle structure of the excited states in the $N = 128$ nuclei $^{208}$Hg and $^{209}$Tl, shell-model calculations have been performed. The OXBASH code [24] was employed. The model space considered consisted of the proton orbitals 2d5/2, 2d3/2, 3s1/2, and 1h11/2 below the Z = 82 closed shell and the neutron orbitals 2g9/2, 1i11/2, and 1j15/2 above the closed $N = 126$ shell. Therefore, no core excitations across the $^{208}$Pb double-shell closure are allowed. The single-proton-hole and neutron-particle energies are taken from the experimental spectra of $^{207}$Tl and $^{209}$Pb, respectively. The two-body interaction matrix elements (TBMEs) are from Ref. [25]. Those are based on the Kuo-Herling realistic interaction [26] for proton-proton and neutron-neutron TBMEs derived from a free nucleon-nucleon potential with core polarization renormalization needed due to the finite model space. The proton-neutron interaction is the bare H7B $G$ matrix [27] without core polarization as justified in Ref. [25]. The only additional correction made in this work is a shift of +40 keV to the $(v_{g9/2})_i^2$ TBME to get the correct ordering of the 6^+ and 8^+ sequence in $^{208}$Hg.

The interaction reproduces very well binding energies, excited states, and B(E2) transition strengths (see Table I) in the two-proton-hole and two-neutron-particle nuclei $^{208}$Hg and $^{210}$Pb.

The $^{208}$Hg nucleus has two-proton holes and two-neutron particles outside the doubly magic $^{208}$Pb core. The results of the shell-model calculations are shown in Fig. 2. The comparison with the experimental information suggests an $8^+$ assignment for the observed isomer. The three observed transitions at 669, 425, and 203 keV correspond to the $6^+ \rightarrow 4^+ \rightarrow 2^+ \rightarrow 0^+$ sequence. The $8^+ \rightarrow 6^+$ transition is not observed because of the high conversion coefficient at low energies. All these states are of predominantly $v_{g9/2}$ character. The low intensity of the observed $K_\alpha$ X ray indicates that the energy of this missing transition is below the binding energy of the $K$ electron (i.e., below 83.1 keV). The transition strength extracted from the experiment is slightly larger than the calculated value of $B(E2) = 1.22$ W.u. (see Table I). The ground-state mass [4] is well reproduced by the calculations.

The $^{209}$Tl nucleus has one-proton-hole and two-neutron-hole particles outside the doubly magic $^{208}$Pb core. The calculation (see Fig. 2) suggests a 17/2^+ isomeric state that...
would decay via the $17/2^+ \rightarrow 13/2^+ \rightarrow 9/2^+ \rightarrow 7/2^+ \rightarrow 3/2^+ \rightarrow 1/2^+$ sequence. The isomer has predominantly a $v(g_{9/2})_s^2\pi s_{1/2}$ character and decays via an unobserved low-energy $E2$ transition into the mainly $\nu g_{9/2}$ $13/2^+$ state. The 137 and 661 keV lines are interpreted as the $\gamma$-decay of unobserved $M1$ transitions connecting states with a $(\nu g_{9/2})_s^2\pi s_{1/2}^{-1}$ configuration is not observed. The low intensity of the observed $K_a$ line indicates that the energies of both unobserved transitions are below the binding energy of the $K$ electron (i.e., below 85.5 keV). The $\gamma$-decay scenario is a consequence of the $(210\text{Po}; I) \times s_{1/2}$ weak-coupling structure of $^{209}\text{Tl}$, which forbids $\Delta I = 2 M1$ transitions as reflected in the shell-model wave functions.

The region around the doubly magic nucleus $^{208}\text{Pb}$ presents a unique testing ground of basic nuclear physics concepts. With the new results on $^{208}\text{Hg}$, this is the only region of the nuclide chart above $^{48}\text{Ca}$ where information is available on excited states on all four neighboring even-even nuclei (see Fig. 3). The general trend is that with an increasing number of valence protons and neutrons, the yrast $E(2^+)$ energy decreases and the $E(4^+)/E(2^+)$ ratio increases toward 3.3. By concentrating on two-proton, two-neutron nuclei outside the closed shell, one observes that the $2^+$ energies are similar, $\sim$700 keV, for three of these nuclei ($^{208}\text{Hg}$, $^{212}\text{Po}$, and $^{206}\text{Po}$) but much lower for $^{204}\text{Hg}$. Likewise, the $E(4^+)/E(2^+)$ ratio is around 1.6–2.0 for the former three and much larger for $^{204}\text{Hg}$. This is understood by considering the individual proton and neutron orbitals around the closed shell. For example, in $^{208}\text{Hg}$, the predominant character of the observed yrast states is $(\nu g_{9/2})_s^2\pi s_{1/2}^{-1}$. Therefore, their energies can be directly compared with the corresponding states in the two-neutron nucleus $^{210}\text{Pb}$. The energies of the $4^+$, $6^+$, and $8^+$ states are very similar, indicating that the proton admixture into these states in $^{208}\text{Hg}$ is small. However, the $2^+$ state in $^{208}\text{Hg}$ is 130 keV lower than in $^{210}\text{Pb}$. The lowering is primarily caused by mixing the $\pi s_{1/2}d_{3/2}$ configuration (which is predominant in the $2^+$ state of $^{206}\text{Hg}$) with the $(\nu g_{9/2})_s^2\pi s_{1/2}$ partition. A similar constellation preserves the dominant $\pi g_{9/2}$ configuration in $^{208,210,212,84}\text{Po}$. In contrast, $^{204}\text{Hg}$ is governed by $j = 1/2–5/2$ low-spin orbitals resembling the classical deformation driving the SU(3) structure in the $sd$ shell and the $f_{5/2}, p$ shell above $^{56}\text{Ni}$.

From mass measurement, it was shown [4] that the general rule that the average $p-n$ interaction is large if both protons and neutrons are above or below the shell closure, and small if one of them is above and the other below such a closure, applies for the nuclei around $^{208}\text{Pb}$ [4]. However, as we see in Fig. 3, this symmetry does not apply if we look into energies of the simplest excitations. To understand the excitation spectrum of nuclei, configuration-specific information about the nucleon-nucleon interaction is needed [31]. The information on the excited states of $^{208}\text{Hg}$ and $^{209}\text{Tl}$ obtained in our work tests this configuration-specific nucleon-nucleon interaction.

In summary, we have identified metastable states in the $N = 128$ isotones, $^{208}\text{Hg}$ and $^{209}\text{Tl}$. The data provide the first comprehensive experimental test of shell-model calculations and residual interactions for the model space $Z < 82, N > 126$. Our results and those of Ref. [4] represent the beginnings of nuclear structure studies in this entire, hitherto unknown, major shell quadrant to the “southeast” of $^{208}\text{Pb}$. Other experiments, extending this information further into the quadrant in the direction toward increasing the neutron number and/or decreasing the proton number, although very challenging, would be highly valuable.

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NUCLEAR STRUCTURE “SOUTHEAST” OF $^{208}$Pb: . . .