Microstructure and Mechanical Properties of Multiphase Materials

by

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Abstract

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A systematic method for quantitative characterisation of the topological properties of two-phase materials has been developed, which offers an effective way for the characterisation of two-phase materials. In particular, a topological transformation has been proposed, which allows a two-phase microstructure with any grain size, grain shape and phase distribution to be transformed into a three-microstructural-element body (3-E body). It has been shown that the transformed 3-E body is mechanically equivalent along the aligned direction with the original microstructure.

The Hall-Petch relation developed originally for single-phase metals and alloys has been successfully extended to two-ductile-phase alloys. It has been shown that the extended Hall-Petch relation can separate the individual contribution to the overall efficiency of different kinds of boundaries as obstacles to dislocation motion.

A new approach to deformation behaviour of two-ductile-phase alloys has been developed based on Eshelby's continuum transformation theory and the microstructural characterisation developed in this thesis. In contrast to the existing theories of plastic deformation, this approach can consider the effect of microstructural parameters, such as volume fraction, grain size, grain shape and phase distribution. In particular, the interactions between particles of the same phase have also been taken into account by the topological transformation. Consequently, the newly developed theory can be applied in principle to a composite with any volume fraction. This approach has been applied to various two-ductile-phase alloys to predict the true stress-true strain curves, the internal stresses and the in situ stress and plastic strain distribution in each microstructural element. It is found that the theoretical predictions are in very good agreement with the experimental results drawn from the literature.

A new approach has also been developed for the prediction of the Young's moduli of particulate two-phase composites. Applications of this approach to Al/SiC<sub>p</sub> and Co/WC<sub>p</sub> composite systems and polymeric matrix composites have shown that the present approach is
superior to both the Hashin and Shtrikman's bounds and the mean field theory in terms of the good agreement between the theoretical predictions and the experimental results from the literature. Furthermore, this approach can be extended to predict the Young's moduli of multiphase composites by iteration. This iteration approach has been tested on some Ti-6Al-4V-TiB composites.

An experimental investigation has being carried out to study the in situ Ti-6Al-4V-TiB (hereafter, Ti/TiB is used for convenience) metal matrix composites produced through a rapid solidification route. Production of in situ Ti/TiB metal matrix composites through rapid solidification route can completely exclude problems such as wetting and chemical reaction encountered by alternative production routes. The relevant microstructural phenomena in in situ Ti/TiB metal matrix composites, such as the growth habit of TiB phase and the ω-phase transformation, have also been investigated. The TiB phase in the consolidated composites exhibits two distinguished morphologies: needle-shaped TiB and nearly equiaxed TiB. The needle-shaped TiB phase formed mainly from the solidification process always grows along the [010] direction of the B27 unit cell, leaving the cross-section of the needles consistently enclosed either by (100) and {101} type planes or by (100) and {102} type planes. It is also found that the cross-sections of the nearly equiaxed TiB particles formed from the B supersaturated Ti solid solution are also bounded by the same planes as above, although the growth rate along the [010] direction has been considerably reduced.

Experiments have also been performed to investigate the effect of pre-hipping heat treatments on the microstructure of RS products. It is found that pre-hipping heat treatments at a temperature below 800°C can lead to the precipitation of fine equiaxed TiB particles from the B super-saturated Ti solid solution, which are uniformly distributed throughout the α+β matrix. The majority of those TiB precipitates do not grow up by Ostwald ripening process after long time exposure at higher temperature.

Microstructural examination has confirmed the existence of a β to ω transformation in RS Ti-6Al-4V alloys with and without B addition after consolidation. In addition, the β to ω transformation has also been observed in RS Ti-Mn-B alloys after consolidation. Systematic electron diffraction work on the β-phase offers a strong experimental evidence for the β to ω transformation mechanism proposed by Williams et al.
Preface

This dissertation is submitted to the University of Surrey for the degree of Doctor of Philosophy, and is an account of the work carried out in the Department of Materials Science and Technology between April 1989 and August 1992, under the supervision of Professor A. P. Miodownik. This dissertation is not being currently submitted for any other degree, diploma or other qualifications at any other universities.

I would like to express my most sincere thanks to Professor A. P. Miodownik for his constant support, encouragement and stimulating discussions during his supervision of this work, particularly his emphasis on checking all assumptions made by past workers as well as those made by myself in the course of this work. I have learned a lot from his methodology. It is believed that the people of the East have a macroscopic way of thinking, while the people in the West have a microscopic way of thinking. To my surprise, my supervisor, a Westerner, has consistently adopted a telescopic view, while I, a typical Chinese, have found myself adopted a microscopic view. The "telescope" once put Chinese in the most advanced position in ancient science and technology for centuries, and it is the "microscope" which helped the West to become the leader of the modern science and technology. Any individual society or person, who is well equipped with both "telescope" and "microscope", has proved to be very instructive and very powerful.

I am also greatly indebted to Dr. P. Tsakirisopoulos and Dr. P. A. Smith for their encouragement and helpful discussions throughout this work and for their patience during the reading of various parts of the manuscript and correcting my abuse of the English language. I am grateful to Dr. P. A. Smith also for his useful comments on Chapter 4 and 5. Thanks also go to Dr. N. Saunders whose comments on chapter 4 have stimulated me made a major improvement in this chapter. I wish to thank Dr. L. Chandrasekaran for his considerable help in processing of the materials used in Chapter 8 and for his great kindness showed during our collaboration on the DRA project.

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Zhongyun Fan
February 1993.
Various aspects of the work described in this thesis have been presented in the following publications and presentations:


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"We are like dwarfs on the shoulders of giants, so that we can see more than they, and things at a greater distance, not by virtue of any sharpness of sight on our part or any physical distinction, but because we are carried high and raised up by their giant size."

Bernard of Chartres 1130.
A principal aim of materials science is the correlation of microstructure with properties. An example of such a relationship is the grain size dependence of yield stress and flow stress of polycrystalline metals and alloys. The theories which describe these correlations are well established and generally successful for single-phase metals and alloys. However, the extension of theories concerning structure-property relationships to materials containing two or more phases, such as two-ductile-phase alloys and metal matrix composites (MMCs), has been hindered by the lack of an effective method for quantitative characterisation of multiphase materials. The main objective of this dissertation is to develop first a systematic method for quantitative characterisation of two-phase materials, which is then used to develop new correlations between microstructures and mechanical properties in two-ductile-phase alloys and metal matrix composites. Experimental work has also been carried out to apply the predictions which have emerged from the theoretical development and to check the validity of some of the newly developed correlations.

A significant difference between single-phase and multiphase structures is the co-existence of different grain and phase boundaries. In order to establish new microstructure/property correlations for multiphase materials, a prerequisite is to develop an effective method for quantitative description of the two-phase microstructures, which includes the quantification of both the geometrical and the topological properties of multi-phase microstructures. For this purpose, a systematic method for quantitative characterisation of the topological properties of two-phase microstructures has been developed in Chapter 3, which, with the well established methods for geometrical quantification [72Und], offers an effective way for characterisation of two-phase materials. In particular, a topological transformation has been proposed, which allows a two-phase microstructure with any grain size, grain shape and phase distribution to be transformed into a three-microstructural-element body (3-E body). The three microstructural elements in the 3-E body with specified microstructural parameters are aligned along a particular direction of interests, usually the direction for uniaxial tension. It has been proved in detail in Chapter 3 and the subsequent chapters that the transformed 3-E body is mechanically equivalent along the aligned direction with the original microstructure before the transformation. The microstructural characterisation and the mechanical equivalence described in Chapter 3 can therefore be used as a basic tool for the theoretical development in the later chapters.
Chapter 1: Introduction

Since the publication of their experimental discovery of the grain size dependence of yield stress of polycrystalline metals and alloys by Hall [51Hal] and Petch [53Pet] thirty years ago, many experimental and theoretical investigations have been carried out on this subject. Now, the well known Hall-Petch relation is widely used [83Arm]. However, the validity of application of the Hall-Petch relation originally developed for single-phase metals and alloys to two-ductile-phase alloys has remained uncertain. In Chapter 4, the Hall-Petch relation has been successfully extended to two-ductile-phase alloys. It has been shown that an extended Hall-Petch relation can separate the individual contribution to the overall efficiency of all kinds of boundaries as obstacles to dislocation motion from different grain and phase boundaries. The applications of the extended Hall-Petch relation to various two-ductile-phase alloys have shown that phase boundaries are not always the strongest obstacles to the dislocation motion in two-ductile-phase alloy.

A number of technologically important alloys consist of mixtures of a softer phase and a harder phase, both of which are plastically deformable. Those alloys have been called two-ductile-phase alloys [82Tom]. Examples of this type of composite alloys are α-β brass, dual-phase steels and α-β titanium alloys. As a group, these alloys offer useful combinations of high strength, good ductility and promising fracture toughness and have been widely used in aerospace and automobile industries. Therefore, it is desirable to have a clear understanding of the deformation behaviour of two-ductile-phase alloys in terms of the effects of microstructural parameters and the mechanical properties of the constituent phases. For this purpose, a new approach to the deformation behaviour of two-ductile-phase alloys has been developed in Chapter 5, based on the Eshelby’s continuum transformation theory [57Esh, 59Esh, 61Esh] and the microstructural characterisation method developed in Chapter 3. In contrast to the existing theories of plastic deformation, the newly developed approach can consider the effect of microstructural parameters, such as volume fraction, grain size, grain shape and phase distribution. In particular, the interactions between particles of the same phase have also been considered by a topological transformation. Consequently, the newly developed approach can be applied in principle to a homogeneous composite with any volume fraction. In contrast, both the independent inclusion approach [70Mor, 71Bro] and the mean field approach [78Ped, 83ped] have no provision for dealing with this kind of interaction and, consequently, can not be applied to a composite with high volume fraction of second phase.

The present approach can be used to predict the true stress-true strain curves, the internal stresses and the in situ stress and plastic strain distribution in each microstructural element, and has been applied to various two-ductile-phase alloys to predict the above mechanical properties in Chapter 6. The theoretical predictions are in very good agreement with the experimental results drawn from the literature. It is found that the phase distribution described by the topological parameters has a significant effect on the local deformation behaviour of two-ductile-phase alloys in terms of mean internal stresses and the in situ stress and plastic strain distribution in the three microstructural elements, although it does not markedly affect the
macroscopic deformation behaviour described by the true stress-strain curves. The theoretical calculations of the true stress-true strain curves for various two-ductile-phase alloys show that there always is a drop in flow stress after the onset of plastic deformation in EIII or EII. This stress drop has been explained in terms of the elastic energy release, which is supported by the experimental evidence in the literature. In addition, the theoretical calculations of the internal stresses indicate that the role of the mean internal stresses is to impede the further plastic deformation in the softer element and to aid the further plastic deformation in the harder elements and consequently to make the plastic deformation of two-ductile-phase alloys tend to be more homogeneous throughout microstructure.

The Main incentive for research into metal matrix composites has been the large scale improvement in properties, compared with those achieved through conventional refinement of alloys. For example, it is possible to improve the stiffness on a large scale over the unreinforced alloys, although there is a drastic fall of ductility. In recent years, low material and fabrication costs, combined with good properties in all the directions, have stimulated considerable interests in short fibre and particulate metal matrix composites. Therefore, it is desirable to understand and to be able to predict the elastic properties of particulate MMCs. In Chapter 7, a new approach has been developed for the prediction of the Young’s moduli of particulate two-phase composites, based on the mean field theory [78Ped, 83Ped] and the microstructural characterisation in Chapter 3. Applications of this approach to Al/SiCp and Co/WCp composites and polymeric matrix composites have shown that the present approach is superior to both the Hashin and Shtrikman’s bounds [62Has, 63Has] and the mean field theory [78Ped, 83Ped] in terms of the better agreement between the theoretical predictions and the experimental results. Furthermore, the newly developed approach for two-phase composites can be extended to predict the Young’s moduli of multi-phase composites by an iterative approach.

Although a number of fabrication routes has been developed for metal matrix composites, two concurrent problems, namely, the poor wetting and the severe chemical reaction between the reinforcing phase and the matrix alloy, still remain unsolved in nearly all those production routes [88Mor, 91Mor], especially in the reactive composite system such as Ti/TiB₂ composite [91Pra]. In addition, the heterogeneous distribution of the reinforcing phase in the matrix also causes concern in those production routes [91Mor]. The mechanical properties of the final products are largely controlled by the extent to which those problems are solved. On the other hand, the rapid solidification techniques can largely extend the solid solubilities of metalloid elements (such as B, C and Si, which usually exhibit very low equilibrium solid solubilities in metals such as Ti) in metals and their alloys (e.g., Ti and Al) [91Sur]. Upon the subsequent annealing, metalloid supersaturated solid solutions produced by the RS techniques will precipitate fine ceramic phases which are uniformly distributed within the matrix. Therefore, rapid solidification techniques can provide a novel production route for in situ composites, in which the above problems can be successfully solved. An experimental investigation has been carried out to study in situ Ti/TiB metal matrix composites produced through rapid
solidification route. The experimental results are presented in Chapter 8. The relevant microstructural phenomena in in situ Ti/TiB metal matrix composites, such as \( \omega \)-phase transformation in the consolidated Ti-6Al-6V-XB alloys and the growth habit of TiB both from liquid alloy and from the B supersaturated Ti solid solution, will be discussed in Chapter 9. In the final part of Chapter 9, the approach for prediction of the Young's moduli of particulate two-phase composites developed in Chapter 7 will be extended to multi-phase composites and will be applied to in situ Ti/TiB composites to predict Young's moduli. It is shown that the theoretical predictions are in good agreement with the experimental results.

The general conclusions drawn from all the work done for this thesis are presented in Chapter 10, followed by suggestions for further work.

The general background relevant to the contents of this dissertation is summarised in Chapter 2. The Hall-Petch relation [51Hal, 53Pet] for describing the grain size effect on the deformation behaviour of polycrystalline metals and alloys is first introduced, and current analytical approaches which try to understand and explain the physical meaning of the Hall-Petch constants are discussed. The mechanical theories of phase mixtures are briefly reviewed in the second part of this literature survey, which include the classical linear law of mixtures and its various modifications, the lower and upper bounds for the effective elastic moduli of two-phase composites and the shear lag model for the nearly continuous fibre composites. The micromechanical theories of deformation of two-phase materials are then discussed in detail because they are the basis for the theoretical developments in this dissertation. This part of the review includes Eshelby's continuum transformation theory, the independent inclusion approaches, the mean field approaches and their applications. In particular, attention is paid to the various interactions in the deformation process of two-phase composites. In the final part, previous achievements on the rapid solidification of titanium alloys are reviewed in terms of processing, microstructure development and the improvement on the mechanical properties. The literature survey presented in Chapter 2 covers only the general background relevant to the content of this dissertation.

Because there are many different strands to this thesis, each chapter is presented in a self-contained manner, with its own introduction, results, discussion and conclusions. The introduction in each chapter is designed to refresh the background of the research on this subject and to introduce the objectives and main contents of this chapter. Following the presentation of the theoretical development and/or experimental results, a discussion is made to explain the results or to compare with the existing theories and/or experimental results from the literature. Conclusions are also drawn from the results in each chapter.

ISO units are used throughout this dissertation. The alloy compositions are always in weight percent (wt.-%), unless otherwise stated.
CHAPTER 2

Literature Survey

2.1. Introduction

In this chapter, the previous work relevant to this dissertation will be reviewed. Firstly, the Hall-Petch relation for describing the grain size effect on the deformation behaviour of polycrystalline metals and alloys is introduced, and the proposed analytical approaches (or models) which try to understand and explain the physical meaning of the Hall-Petch constants are discussed. Secondly, the mechanical theories of phase mixtures are briefly reviewed, which include the classical linear law of mixtures and its various modifications, the lower and upper bounds for the effective elastic moduli of two-phase composites and the shear lag model for the nearly continuous fibre composites. Thirdly, the micromechanical theories of deformation of two-phase materials are discussed in some detail. This part of the review will include the Eshelby’s continuum transformation theory, the independent inclusion approaches, the mean field approaches and their applications. In particular, attentions are paid to the various interactions in the deformation process of two-phase composites. Finally, the previous achievements on the rapid solidification of the titanium alloys are reviewed in terms of processing, microstructure development and the improvement of the mechanical properties.

2.2. On the Hall-Petch Relation

2.2.1. Background

Hall [51Hal] and Petch [53Pet] measured at ambient and liquid nitrogen temperatures, respectively, the values of lower yield stress, \( \sigma_y \), of \( \alpha \)-iron polycrystal specimens with different average grain sizes. Their results showed that \( \sigma_y \) was proportional to the inverse square root of the average polycrystal grain diameter, \( d \), and \( \sigma_y \) was expressed in terms of \( d \) by the following relationship

\[
\sigma_y = \sigma^0 + k_y d^{-1/2}
\]  

(2.2.1)

where \( \sigma^0 \) and \( k_y \) were taken as experimental constants. Of equal importance to the experimental determination of eqn (2.2.1) was their dislocation pile-up interpretation for the relationship. The reciprocal square root dependence of \( \sigma_y \) on grain size was explained by the theoretical
results obtained by Eshelby et al [51Esh]. In eqn (2.2.1), \( \sigma^0 \) was identified as the friction stress required to move individual dislocation in the microyielded slip band pile-ups confined to isolated grains, whereas \( k_y \) was proposed as a measure of the local intensified stress needed to propagate general yielding across the polycrystalline grain boundaries. For \( \alpha \)-iron, a large value of \( k_y \) was expected for the requirement of unpinning dislocation sources from the carbon atmosphere, in line with the model description previously developed by Cottrell and Bilby [52Cot] for the yield point phenomenon.

A natural consequence of \( k_y \) being a stress intensity for plastic yielding was the further consideration that an even larger stress intensity, or \( k \) value, should be measured for the cleavage fracture stress dependence on \( d^{-1/2} \) in agreement with experimental measurements [53Pet] at liquid nitrogen temperature. Such reasoning, closely coupled with eqn (2.2.1) was employed in a number of further studies to determine the influence on \( \sigma^0 \) and \( k_y \) of the following factors: carbon and nitrogen quenched in solution [55Cra], the test temperature [56Hes] and the relative difference in carbon, nitrogen and boron locking of dislocation sources [60Cod]. Petch showed that hydrogen lowered the stress intensity required for cleavage fracture [56Pet1] and that the true ductile fracture stress depended on grain size in the same way as the cleavage stress [56Pet2].

Armstrong et al [62Arm] investigated the application of the Hall-Petch relation to describe the dependence on grain size of the flow stress obtained at constant strain values for mild steel and a number of common metals and their alloys, with or without a yield point. Their results for mild steel at ambient temperature are shown in Fig 2.1. The results of lower yielding point (lyp) in Fig 2.1 relate directly to the results obtained by Cracknell and Petch [55Cra]. The data of the true ductile fracture stress are those from Petch [56Pet2], and have been corrected for the triaxiality of stress due to necking.

The linear dependence for all the measurements indicates that the total plastic stress-strain behaviour was amenable to a Hall-Petch relation if the flow stress at a constant strain value \( (\sigma_\varepsilon) \) was written as

\[
\sigma_\varepsilon = \sigma^0(\varepsilon) + k_\varepsilon d^{-1/2}
\]

At a constant strain \( \varepsilon \), of 0.1 for example, a \( k_\varepsilon \) value less than for the initiation of yield is obtained because the continued straining of the fully plastic material is achieved both from the newly created dislocation sources and from those unpinned for the lyp strain. Once the lyp strain is exceeded, additional plastic straining does not affect \( k_\varepsilon \) very much until plastic instability occurs at the necking stress and then \( k_\varepsilon \) increases rapidly to the value for the ductile fracture stress value which is very near to that measured for cleavage fracture. The intercept, \( \sigma^0(\varepsilon) \), is substantially increased past the lyp strain due to strain hardening within the polycrystal grains. This connection of \( \sigma^0(\varepsilon) \) with single crystal deformation is important
because the long-standing problem of accounting theoretically for large difference in strength of single crystal and the fine-grained polycrystals in the absence of any grain size dependence is reconciled to a large extent when the polycrystal $\sigma^0(\epsilon)$, not $\sigma_y$, is employed in the comparison.

\begin{figure}[h]
\centering
\includegraphics[width=0.7\textwidth]{figure21.png}
\caption{The grain size dependence of the lower yield stress, flow stress and ductile fracture stress of mild steels. The experimental results are from Armstrong et al [62Arm] (open circles) and Petch [56Pet2] (filled circles).}
\end{figure}

2.2.2. The Theoretical Analysis of $\sigma^0(\epsilon)$ and $k_\epsilon$

Many analytical approaches (or models) have been elaborated in order to provide a semiquantitative explanation for the experimentally observed relationship between grain size and the resistance of a polycrystal to plastic deformation. In fact, these proposed models are based on the dislocation theory. They have been reviewed by Li and Chou [70Li] and a detailed account of the experimental results on various metals and their alloys was given in the review papers by Armstrong [83Arm].

The existing analytical approaches can be classified into three broad categories [86Las]: the dislocation pile-up models, the work hardening models and the composite models. In this section, these three types of models will be briefly introduced.
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(1) Dislocation Pile-Up Models

One main analytical approach is the dislocation pile-up model proposed initially for the interpretation of the grain size dependence of yield strength (eqn (2.2.1)) by a number of researchers [51Hal, 53Pet, 58Cot, 67Li] and extended further to describe the grain size dependence of flow stress [62Arm]. The idea in Armstrong and coworkers' model [62Arm] is that in a favourably oriented grain slip bands are formed which terminate at a grain boundary (to form the pile-ups). In order for the shear displacement associated with the impinging slip band to be accommodated plastically in the adjacent grain, the stress concentration caused by the pile-ups at or near the grain boundary must be sufficient to nucleate slip along the trace of the shear zone. The final expression for stress is

$$\sigma_e = m \left[ \tau_R + k_s d^{-1/2} \right]$$

(2.2.3)

where $m$ is described as an orientation factor connecting $\sigma_e$ to the resolved shear stress, $\tau_R$, for slip on the deformation systems required to operate within the polycrystal grain volume to maintain continuity of the material at grain boundaries, and $k_s$ is specified as the locally intensified shear stress needed in the average instance for propagation of plastic flow across the polycrystal grain boundaries.

From eqn (2.2.2) and (2.2.3)

$$\sigma^0(\varepsilon) = m \tau_R$$

(2.2.4)

which is the relationship connecting deformation on the systems within the polycrystal grain volume and in a single crystal.

In addition, the linear work hardening of the friction stress of polycrystal specimens of $\alpha$-brass is related to single crystal work hardening results in this Hall-Petch interpretation by

$$\frac{d[\sigma^0(\varepsilon)\tau_R]}{d\varepsilon} = m^2 \frac{d\tau_R}{d\gamma}$$

(2.2.5)

where $\gamma$ is the resolved shear strain.

Li and Chou [70Li] described a number of the analytical and numerical calculations involved in establishing the theoretical properties of essentially static dislocation pile-ups, following on the early work by Eshelby et al [51Esh]. A typical expression is

$$\tau_e = \tau_R + \left[ \frac{\pi m^* G b \tau_C}{2\bar{\alpha}} \right]^{1/2} d^{-1/2}$$

(2.2.6)

where $G$ is the shear modulus, $b$ is the Burger's vector of dislocation, $\bar{\alpha}$ depends on the average dislocation character, $\bar{\alpha} = 2(1 - \nu)/(2 - \nu)$, with $\nu$ is the Poisson's ratio, $\tau_C$ is the critical shear stress and $m^*$ is an orientation factor. From eqn (2.2.3) and (2.2.6), subject to the condition that $\sigma_e = m \tau_e$, the result is reached

$$k_e = m k_s = m \left[ \frac{\pi m^* G b \tau_C}{2\bar{\alpha}} \right]^{1/2}$$

(2.2.7)
The slip band stress intensities, $k_e$ and $k_s$ are shown by eqn (2.2.7) to depend on $C^{1/2}$ and on the orientation of the deformation systems which are available within the polycrystal structure-type being considered.

(2) The Work Hardening Theories

The other analytical approach to the Hall-Petch relation is the dislocation density approach [63Con, 63Li], in which the stress concentrations at grain boundaries are not considered explicitly and the assumption of the dislocation pile-ups is avoided. Instead, the relationship which describes the flow stress is expressed in terms of dislocation density $\rho$, i.e.,

$$\sigma = \sigma^0(\varepsilon) + \alpha G b \rho^{1/2}$$  \hspace{1cm} (2.2.8)

where $\alpha$ is a numerical constant. The principle feature of the explanation of the grain size dependence of flow stress, based on the dislocation density, is the experimental finding that dislocation density, $\rho$, is an inverse function of grain size [60Bra, 63Keh]. This inverse functional dependence may be explained in a number of ways. Li [63Li], for example, has suggested that grain boundary ledges are sources for dislocation emission, so that the number of dislocations generated per unit strain is proportional to grain boundary surface, from which it then follows that for a given small strain, $\rho$ is proportional to $1/d$, for which there is indeed some evidence [67Con]. Therefore, eqn (2.2.8) takes the form of Hall-Petch relation and $k_e$ is a function of $\alpha G b$. Furthermore, for any given value of $d$, $\rho$ will also depends on the plastic strain $\epsilon$. Accordingly, Conrad et al [67Con] have shown that

$$k_e = \alpha G b^{1/2} \epsilon^{1/2}$$  \hspace{1cm} (2.2.9)

where $\beta$ is an experimentally determined constant.

(3) The Model Based on the Composite Theory

In this group of analytical approaches, the polycrystal is described as a composite material. This composite idea was first proposed by Kocks [70Koc] and further developed by Thompson et al [73Tho].

Ashby [70Ash] made conceptually the distinction between two types of dislocations: one type is the statistically stored dislocations which would accumulate during the deformation of a single crystal. The density of statistically stored dislocations ($\rho^S$) can be expressed as $\rho^S = \varepsilon (b/\lambda)$, where $\lambda$ is the average slip length and $\varepsilon$ is the strain amplitude; the other type is the geometrically necessary dislocations which are generated in addition to the statistically stored ones in order to maintain the continuity of the polycrystal. The density of geometrically necessary dislocations ($\rho^G$) can be expressed as $\rho^G = \varepsilon/(4bd)$. These two types of dislocations, of course, are not physically distinguishable, but may appear in largely separated regions.
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Thompson et al [73Tho] proposed that $\rho^S$ is randomly distributed over the entire grain, whereas $\rho^G$ is more concentrated in the vicinity of the grain boundaries. Based on this heterogeneous distribution of dislocations the stresses due to $\rho^S$ and $\rho^G$ can then be expressed by the following two equations

$$\sigma^S = K_1(1/\lambda^S)$$  \hspace{1cm} (2.2.10)

$$\sigma^G = K_2(1/d^{1/2})$$  \hspace{1cm} (2.2.11)

where $\sigma^S$ and $\sigma^G$ are the flow stresses due to $\rho^S$ and $\rho^G$, respectively, $\lambda^S$ is the statistical slip length which is a function of strain and grain size $d$, $K_1$ and $K_2$ are constants. By averaging the above two stresses over the relative volume fraction on which each stress act on, Thompson et al [73Tho] obtained the following expression for flow stress of the polycrystal specimen

$$\sigma_0 = \sigma_0 + \left(1 - \frac{\lambda^S}{d}\right)\left(\frac{K_1}{\lambda^S}\right) + \frac{\lambda^S}{d}\left(\frac{K_2}{d^{1/2}}\right)$$  \hspace{1cm} (2.2.12)

It is clear from eqn (2.2.12) that the flow stress is a complex function of grain size and it can only be reduced to eqn (2.2.2) for $\lambda^S = d$.

2.2.3. Applications of the Hall-Petch Relation to Two-Phase Alloys

The application of Hall-Petch relation to two-phase alloys was made first to unidirectionally solidified lamellar eutectic systems by Shaw [67Sha] in Cd-Zn alloys and by Cline and Stein [69Cli] in Ag-Cu alloys. The inerlamellar spacing was taken as the effective grain size in eqn (2.2.2). The eutectic Hall-Petch parameters in these systems were compared by Armstrong et al [83Arm]. They have reached the following conclusions: (i) the eutectic Hall-Petch stress intensities are of the comparable magnitude to those $k_e$ values reported for the single phase materials; (ii) the appreciable contribution of the $k_e d^{-1/2}$ terms to the total eutectic strength stems from the small effective $d$ values achieved in the eutectic systems.

There have been some efforts to apply the Hall-Petch relation developed for single-phase alloys to two-phase alloys [74Kar, 77Fis, 80Ugg, 81Ugg, 82Ugg, 82Ugg2, 83Wer, 84Wer, 85Wer]. However, in nearly all cases, the grain size of only one phase, usually the matrix grain size, was used.

A significant difference between the two-phase alloys and single-phase alloys is the co-existence of different grain boundaries and phase boundaries, although the response of phase boundaries to dislocation motion is not necessarily the same as that of grain boundaries. The above applications make no consideration of this difference. Recently, Werner and Stuwe [84Wer, 85Wer] investigated experimentally the grain size effect on yield strength and flow stress of $\alpha$-$\beta$ Cu-Zn system with a wide range of volume fraction and grain size. To apply Hall-Petch relation, the matrix grain size was used for eqn (2.2.2). They tried to evaluate the Hall-Petch coefficients for $\alpha$-$\alpha$ grain boundaries, $\beta$-$\beta$ grain boundaries and $\alpha$-$\beta$ phase boundaries by assuming the following equation
where \( f_{\alpha c} \) and \( f_{\alpha c} \) are the continuous volume fractions of the \( \alpha \)- and \( \beta \)-phases, respectively, \( k_y^c \) is the Hall-Petch slope for the whole \( \alpha \)-\( \beta \) phase mixture, \( k_y^\alpha \) and \( k_y^\beta \) are the Hall-Petch coefficients for single \( \alpha \)- and \( \beta \)-phases, respectively, and \( k_y^{\alpha \beta} \) is the Hall-Petch coefficient for \( \alpha \)-\( \beta \) phase boundaries. In eqn (2.2.13) \( k_y^c \), \( k_y^\alpha \) and \( k_y^\beta \) can be experimentally determined from \( \alpha \)-\( \beta \) phase mixture, single \( \alpha \)-phase alloys and single \( \beta \)-phase alloys, respectively. Thus \( k_y^{\alpha \beta} \) can be calculated from eqn (2.2.13).

At a given volume fraction of the \( \beta \)-phase \( (f_{\beta}) \) and plastic strain \( (\varepsilon) \), \( \sigma_e \) is a function of \( \alpha \)-grain size \( (d_{\alpha}) \) and \( \beta \)-grain size \( (d_{\beta}) \). In Werner and Stuwe's work, only the matrix grain size was considered for the Hall-Petch analysis, ignoring the grain size of the other phase. This is misleading since in dual-phase alloys the grain size of one phase can vary when the grain size of the other phase is kept constant. In other words, the friction stress and the Hall-Petch coefficient of a dual-phase alloy determined by only using the grain size of one phase are functions of the grain size of the other phase and are not independent of grain size, as required by eqns (2.2.1) and (2.2.2).

2.2.4. Summary

(1) Although the Hall-Petch relation has been widely accepted and frequently applied for analysis of the grain size effect on the strength of polycrystalline materials, it is not conclusive whether the exponent \( n=1/2 \) in the expression \( \sigma_e = \sigma_0(\varepsilon) + k_e d^{-n} \) is most suitable for describing the flow stress, especially at high strain. Other values of exponent \( n \) have often been reported, for example, Hutchson and Pascoe [72Hut] studied the dependence of flow stress on the grain size for copper-base alloys with the nonlinear regression analysis method, and found that an exponent of unity \( (n = 1) \) is most probable. Kocks [59Koc] has pointed out that when the flow stress is plotted against \( d^{-n} \), the deviation from a straight line is small, irrespective of any choice of \( n \) ranging from unity to 1/3 in the range of grain size usually investigated.

(2) The effect of the spread in the distribution of grain size is usually not considered in most studies. The shape of the grain size histogram can be an important parameter [79Kuh], either when the grains are flat or elongated or when there is a wide spread of size around the value of mean diameter.

(3) The application of Hall-Petch relation to two-phase alloys, such as \( \alpha \)-\( \beta \) brass, \( \alpha \)-\( \beta \) titanium alloys and dual-phase steels, is not justified. There are two problems which cause concern: one is how to define the grain size \( d \) in eqn (2.2.1) and (2.2.2), since there are two grain sizes, \( d_{\alpha} \) and \( d_{\beta} \); the other is what is the physical meaning of the obtained Hall-Petch slope, since there are different kinds of boundaries in a two-phase alloy.
namely, α-α grain boundaries, β-β grain boundaries and α-β phase boundaries and the
resistance to plastic deformation from different boundaries is not necessarily the same.

(4) In terms of the dislocation theory of plastic deformation, $k_y$ can be taken as a direct
measure of the efficiency of grain boundaries as obstacles to dislocation motion [77Fis,
65Kos, 74Mea]. If $k_y^C$ is taken as a measure of the overall efficiency of grain boundaries
and phase boundaries in dual-phase alloys, an interesting question which arises is
whether it is possible to separate the contributions to this overall efficiency from α-α
grain boundaries, β-β grain boundaries and the α-β phase boundaries.

2.3. The Mechanical Theories of Phase Mixtures

2.3.1. Law of Mixtures and Its Modifications

Voight [1889Voi] first proposed the linear law of mixtures, which expresses the mechanical
properties of a phase mixture in terms of the bulk properties and the relative amounts of the
constituent phases. Consider a composite which is a mixture of $n$ phases, the volume fraction
of the $i$th phase is $f_i$ and the yield strength of the $i$th phase is $\sigma_{yi}$, where $i = 1, 2, 3, ..., n$.

According to the law of mixtures, the yield strength of the composites is expressed by the
following equation

$$\sigma_y = \sum_{i=1}^{n} (f_i \sigma_{yi}) \quad (2.3.1)$$

A implicit assumption is that at yielding point, the total strains in all the constituent phases are
equal. This assumption can only be valid in certain condition. In fact, the linear law of mixtures
can be divided into two categories, the equal strain model and the equal stress model, according
to the different assumption made for the deformation of the phase mixtures. In what follows,
only the flow stress of the two-phase mixtures (α-β) is considered.

The equal strain model assumes that the strains in all the constituent phases are equal and equal
to the strain of the phase mixture, as illustrated in Fig 2.2(a), i.e.,

$$\varepsilon^\alpha = \varepsilon^\beta = \varepsilon^C \quad (2.3.2)$$

where the superscripts α, β and C refer to the α-phase, β-phase and the whole composite, and $\varepsilon$ is the strain. According to this equal strain assumption, the applied stress will be distributed
among the constituent phases according to their volume fractions. Thus, the flow stress of the
composite can be expressed as

$$\sigma^C_f = f_\alpha \sigma^\alpha_f + f_\beta \sigma^\beta_f \quad (2.3.3)$$

A realistic case in which the equal strain model is applicable is the one in which the α- and β-
phases are continuous and are aligned parallel along the tensile direction (see Fig 2.2(c)).

The equal stress model assumes that the flow stresses in all the constituent phases are equal and
equal to the flow stress of the phase mixture (see Fig 2.2(b)), i.e.,
\[ \sigma^\alpha = \sigma^\beta = \sigma^c \] (2.3.4)

As a direct consequence of the equal stress assumption, the total strain will be distributed among constituent phases according to their volume fractions, thus,

\[ \varepsilon^c = f_\alpha \varepsilon^\alpha + f_\beta \varepsilon^\beta \] (2.3.5)

The equal stress model is only applicable in the case where all the constituent phases are aligned parallel along the direction perpendicular to the tensile direction (see Fig 2.2(d)).

---

Fig 2.2 Schematic illustration of the equal strain and equal stress models. (a) The equal strain model \((\varepsilon^\alpha = \varepsilon^\beta = \varepsilon^c)\); (b) The equal stress model \((\sigma^\alpha = \sigma^\beta = \sigma^c)\); (c) the \(\alpha\)- and \(\beta\)-phases are continuous and are aligned parallelly along the tensile direction; (d) the \(\alpha\)- and \(\beta\)-phases are continuous and are aligned perpendicularly to the tensile direction.

The equal strain model has indeed been successfully applied to the Cu matrix reinforced by the parallelly aligned continuous W fibres, where Taya and Arsenault [89Tay] found that the predictions of Young's moduli and yield strengths were in excellent agreement with the
experimental results by McDanels et al [65McD]. However, the mechanical properties of most particulate systems of technical importance do not follow a linear law of mixture based on the bulk properties [88Cho].

A more general form of the law of mixtures relating to the average stresses and strains in the constituent phases was suggested for empirical use by Tamuran et al [73Tam], and was shown by Fischmeister and Karlson [77Fis] to describe adequately the distribution of stress and strain among the constituent phases. This form of law of mixture is often called in the literature the modified law of mixtures, because the relevant average stresses and strains of each phase are in situ values, and not in general, the bulk properties, nor necessarily the in situ flow properties. When an $\alpha$--$\beta$ phase mixture is under uniaxial loading, the stress and strain of the composite can be expressed by the following equations

$$\bar{\sigma}_f = \bar{\sigma}_f^\alpha f_\alpha + \bar{\sigma}_f^\beta f_\beta \quad (2.3.6a)$$

$$\bar{\varepsilon}_f = \bar{\varepsilon}_f^\alpha f_\alpha + \bar{\varepsilon}_f^\beta f_\beta \quad (2.3.6b)$$

Where $\bar{\sigma}$ and $\bar{\varepsilon}$ are the average values, respectively, of the directional components of stress and strain parallel to the direction of the applied load, averaged on the planes normal to the loading direction for stress or along the lines parallel to the load direction for strain [77Fis]. In the modified law of mixture, the interaction effects are incorporated into the stress and strain values associated with each phase. The empirical use of the modified law of mixture for stress and/or strain has shown good agreement with experiments [77Fis, 87Fun, 87Su, 82Ank1], theoretical modelling [85Goe, 78Lee] and finite element analysis [82Ank1, 82Ank2, 75Kar]. However, it has been shown recently by Cho and Gurland [88Cho] that this modified law of mixture can only be justified at small strains.

For accounting for the microstructural and interaction effects, Ankem and Margolin [86Ank] proposed an alternation of the linear law of mixture, which can be mathematically expressed by the following equation

$$\sigma_f = \sigma_{fa} f_\alpha + \sigma_{f\beta} f_\beta + I_{\alpha\beta}^f \quad (2.3.7)$$

where $\sigma_{fa}$ and $\sigma_{f\beta}$ are the bulk flow stress of the $\alpha$- and $\beta$-phases corrected for variations of grain size and texture, and $I_{\alpha\beta}^f$ is due to the interaction between the $\alpha$- and $\beta$-phases. The interaction term, $I_{\alpha\beta}^f$, may be additive or substractive depending on the microstructure, alloy system and composition.

Eqn (2.3.7) can feature the interaction for two phases explicitly, but it is very difficult to achieve a general approach for determination of $I_{\alpha\beta}^f$ for various alloy systems with different composition and microstructure.
2.3.2. The Upper and Lower Bounds for Stiffness

In contrast to continuously reinforced composites where the elastic modulus of the composite varies linearly with volume fraction, the elastic moduli of particulate composites have a non-linear dependence on the volume fraction of the reinforcement. This phenomenon arises from the interactions between the constituent phases which generally have different elastic constants, as well as from interactions between particles of the same phase. In order to predict the elastic constants of a composite body, extreme value theories have been applied. Paul [60Pau] derived a set of equations by examining the energy theorems of elasticity theory. In order to obtain the upper limit, the strains in both phases were assumed to be equal, and for the lower limit, the stresses in both phases were assumed to be equal, the result is commonly referred to the linear law of mixtures.

\[ E_U = f_\alpha E_\alpha + f_\beta E_\beta \] (2.3.8)

\[ E_L = \frac{E_\alpha E_\beta}{f_\alpha E_\beta + f_\beta E_\alpha} \] (2.3.9)

where \( E \) is the Young's modulus, and the subscripts U and L denote the upper bound and lower bound for the \( \alpha-\beta \) composite, respectively. Differences calculated by employing the above two equations are normally large, provided the difference in the two moduli is larger than a factor of three. An alternative formulation for the composite modulus, involving the variational principles in the linear theory of elasticity, was applied by Hashin and Shtrikman [62Has, 63Has] to derive the upper and lower bounds (denoted as HS lower and upper bounds). The calculated bounds provide a closer approximation, since the separation between the HS bounds is much less than that between Paul's bounds for a given volume fraction. Determination of the Young's modulus of a composite, using Hashin and Shtrikman's formulation, requires the calculation of the upper and lower limits for the bulk modulus, \( K \), and the shear modulus, \( G \). For an isotropic material, the bulk modulus and shear modulus can be defined in terms of the Young's modulus and Poisson's ratio, \( \nu \):

\[ K = \frac{E}{3(1 - 2\nu)} \] (2.3.10)

\[ G = \frac{E}{2(1 + \nu)} \] (2.3.11)

The upper and lower bounds are given by the following equations:

\[ K_L = K_\alpha + \frac{f_\beta}{1 + \frac{3f_\alpha}{K_\beta - K_\alpha (3K_\alpha + 4G_\alpha)}} \] (2.3.12)

\[ K_U = K_\beta + \frac{f_\alpha}{1 + \frac{3f_\beta}{K_\alpha - K_\beta (3K_\beta + 4G_\beta)}} \] (2.3.13)
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\[ G_L = G_\alpha + \frac{f_\beta}{1 + \frac{6(K_\alpha + 2G_\alpha)f_\alpha}{G_\beta - G_\alpha}} \]

\[ G_U = G_\beta + \frac{f_\alpha}{1 + \frac{6(K_\beta + 2G_\beta)f_\beta}{G_\alpha - G_\beta}} \]

Then the upper and lower bounds for the Young's modulus can be calculated from

\[ E = \frac{9KG}{3K + G} \]

where the upper limits of \( K \) and \( G \) are used to determine the upper bound and the lower limits of \( K \) and \( G \) are used to determine the lower bound. It follows that the separation between the HS lower and upper bounds is dependent upon the ratio \( E_\beta/E_\alpha \) of moduli of the constituent phases. When the moduli of the constituent phases are closely matched, the bounds predict values within 10% difference, while the bounds are remarkably separated and have limited predictive value when \( E_\beta/E_\alpha \) is about 20 [90Ahm]. The HS bounds, however, serve as a useful test of the approximate theories, since any theoretical solution outside these bounds can be regarded as invalid [90Ahm].

2.3.3. The Shear Lag Model

This method was originally derived by Cox [52Cox] in order to model the stiffness of fibrous paper, and has since been extensively used to determine the load transfer between the phases of the nearly continuous fibre composites. In this section of the literature survey, the theoretical treatment is adapted from Kelly and Macmillen [86Kel] and Piggott [80Pig].

The difference in stiffness between the matrix and fibres of an aligned fibre composite gives rise to different elastic strains in the two phases. The shear strain which develops parallel to the fibre axis as a result of this incompatibility of strain act to transfer stress to the included phase (see Fig 2.3a). By determining the form of these shear stresses it is possible to calculate the build-up of tensile stress along the fibre length. From this knowledge of the fibre loading it is a simple matter to derive the fibre modulus.

Cox [52Cox] considered a fibre of length \( l \) embedded in a matrix subject to a strain \( \epsilon \) in the fibre direction. If the fibre is displaced by \( u \) at a distance \( x \) from one end, then the stress within the fibre can be expressed in terms of the modulus of the fibre \( (E_F) \):

\[ \sigma_F = E_F \frac{du}{dx} \]

Cox [52Cox] then proposed that the rate of change of stress in the fibre was linearly related to the difference between the fibre displacement \( (u) \) and the displacement \( (v) \) of the same point were the fibre absent, i.e.,
![Diagram](image)

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\[
\frac{d\sigma_F}{dx} = f_t (u - v) \tag{2.3.18}
\]

where the constant \( f_t \) is a function of the composite geometry and the shear (\( G_M \)) and Young's (\( E_M \)) moduli of the matrix.

Since it is the matrix shear stress at the interface (\( \tau_M(r_0) \)) which maintains the fibre stress:

\[
\frac{d\sigma_F}{dx} = \frac{2\pi r_0 \tau_M(r_0)}{\pi r_0^2} \tag{2.3.19}
\]

so that

\[
f_t = \frac{2\pi \tau_M(r_0)}{\pi r_0 (u - v)} \tag{2.3.20}
\]

If \( w \) is the displacement of the matrix, then close (\( r=r_0 \)) to the fibre \( w=u \), and well away from the fibre \( w=v \) (this is taken to be at \( r=R \), the mean fibre separation).

For equilibrium of the matrix:

\[
2\pi r \tau_M(r) = 2\pi r_0 \tau_M(r_0) \tag{2.3.21}
\]
thus \[ \frac{\text{d}w}{\text{d}r} = \frac{\tau_M(r)}{G_M} = \frac{\tau_M(r_o)}{rG_M} \] (2.3.22)

Integrating: \[ \Delta w = \frac{\tau_M(r_o)}{G_M} r_o \ln(R/r_o) \] (2.3.23)

so that \[ t = \frac{2G_M}{r_o^2 \ln(R/r_o)} \] (2.3.24)

with \[ \ln(R/r_o) = 0.5 \ln\left(\frac{2\pi}{\sqrt{3f}}\right) \] for hexagonal packing

and \[ \ln(R/r_o) = 0.5 \ln\left(\frac{\pi}{f}\right) \] for square packing

\[ \ln(R/r_o) \] is not a strong function of the fibre arrangement.

At low volume fraction \( dv/dx = e \) so that differentiating and substituting:

\[ \frac{\text{d}^2\sigma_F}{\text{d}x^2} = \frac{t}{E_F} (\frac{\sigma_F}{E_F} - \nu) \] (2.3.25)

Solving the differential equation and using the boundary condition \( \sigma_F = 0 \) at \( x = 0 \), \( l \) (i.e., assuming there is no bond across the fibre end) the stress within the fibre can be calculated:

\[ \sigma_F = E_F e \left( 1 - \frac{\cosh(\beta (l/2 - x))}{\cosh(\beta l/2)} \right) \] with \( \beta = \sqrt{h/E_F} \) (2.3.26)

Hence the average stress within the fibre is given by:

\[ \overline{\sigma}_F = f E_F \left( 1 - \frac{\tanh(\beta l/2)}{\beta l/2} \right) \]

So that the applied stress \( \sigma^A \) is distributed between the two phases such that:

\[ \sigma^A = f \sigma_F + (1 - f) \overline{\sigma}_M \] (2.3.27)

thus the composite modulus \( (E_C) \) is given by:

\[ E_C = f E_F \left( 1 - \frac{\tanh(\beta l/2)}{\beta l/2} \right) + E_M (1 - f) \] (2.3.28)

and: \[ \tau_M(r_o) = E_F e \sqrt{\frac{G_M}{E_F 2\ln(R/r_o)}} \frac{\sinh(\beta (l/2 - x))}{\cosh(\beta l/2)} \] from eqn (2.3.19) (2.3.29)

It is clear from the above analysis that the matrix shear stress \( \tau_M \) is greatest at the fibre end, whereas the fibre stress \( \sigma_F \) rises to a maximum at the centre (see Fig 2.3b).

In summary, the shear lag theory has been widely adopted for the determination of internal stresses in fibrous composites because: (i) it is simple to calculate; (ii) all the relevant parameters are easily obtained; (iii) it is a continuum model thus requires minimum microstructural characterisation. However, it has a number of short comings: i) it is grossly oversimplistic, it assumes a radial distribution of matrix shear stress about the fibre axis within the matrix, which is reasonable for the aligned fibres of considerable length, but it is unsuitable for short fibres or particles; ii) it is not valid for high volume fractions, for example, the largest strain will occur at the distance \( R \) from the fibre (the largest distance from a fibre), but \( dv/dx=e \) at \( R \) so that throughout the composite the strains are everywhere less than the composite strain. This can only be true for the trivial case of infinite aspect ratio fibres when the strains is \( e \).
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everywhere; iii) it assumes that the fibres are not bonded to the matrix at the fibre ends; this is not important to the long fibres, but for the short ones a large proportion of the load transfer occurs not by the build-up of shear stresses in the matrix, but by the tensile stresses at the ends; iv) it does not limit correctly (e.g., when $E_F \Rightarrow E_M$; $E_C \neq E_M$; $t_M \neq 0$); this is because, under this model, the only means of loading a "fibre" is to allow shear strains to develop within the matrix, but for an elastically homogeneous medium there should be no shear stresses. In fact, the unrealistic boundary conditions used in solving eqn (2.3.26) lead to the non-zero shear stresses in the homogeneous case, so it is applicable only to aligned cylindrical fibre shapes.

These shortcomings are not serious for composites containing large aspect ratio fibres of very different elastic constants from the matrix [70Smi, 52Cox]. However for the short fibre and particulate systems, the errors (chiefly associated with the end effects) can be large. Nardone and Prewo [86Nar] have extended the model to include the transfer of load (but not the stress concentration effect) at fibre ends, but there are only limited improvement in the model's predictions [87Tay].

2.4. The Micromechanical Theory of Deformation of Two-phase Materials

2.4.1. Introduction

In recent years there has been a great deal of interests in short fibre metal matrix composites as materials of improved specific strength and elevated temperature creep properties compared with conventional alloys [89Wit]. Their potential derives from, and depends upon, an effective transfer of loads from the matrix to the stiffer reinforcing second phase. The other group of materials which interests materials scientists is the two-ductile-phase alloy, for instance, $\alpha$-Ti-alloys, $\alpha$-brass and dual-phase steels, for their excellent combination of strength, ductility and fracture toughness [86Ank, 88Cho]. It is therefore important to be able to predict their mechanical properties from the corresponding properties of the constituent phases and their microstructural parameters.

Applications of Eshelby's ideas [57Esh, 59Esh, 61Esh] in mean field models [70Tan, 71Bro] has illustrated the dispersion strengthening [79Bro] and the heterogeneous plastic flow in high cyclic fatigue [81Ped, 81Mug, 82Ped]. The mean field models were initially formulated in two non-equivalent ways: Tanaka-Mori's model [70Tan] minimises free energy under the simplifying assumption that the inclusions sample the external stress only; and the Brown-Stobbs model [71Bro] expresses stress equilibrium under the more general assumption that inclusions feel not only the external stress but also the image stress from the external surface. Later work on elastically homogeneous composites [73Bro, 74Tan, 73Mor] clarified the
models and the relationship between them. It was demonstrated that the models were equivalent if inclusions in both models were made to feel the mean matrix stress, which is necessarily derived from the stress equilibrium requirement. The models for elastically homogeneous composite, of course, need modification before they can hope to describe the effects of elastic heterogeneity, especially when the volume fraction is high [83Ped].

It was suggested, in a short account [78Ped, 79Ped], that a natural modification of the Brown-Stobbs model for random elastically heterogeneous composites is simply to express that equivalent inclusions sample the mean matrix stress. This modification has now been made to the Tanaka-Mori's model [81Tay1, 81Tay2].

In this section of the literature survey, only the stress equilibrium formulation will be emphasized. For energy conservation approaches, the paper by Benveniste [87Ben] and the book by Taya and Arsenault [89Tay] should be consulted. It should also be pointed out that, for convenience, the tensor formulation is used for general problems and the scalar formulation will be used for specific ones.

2.4.2. Eshelby's Continuum Transformation Theory

(1) The Transformed Homogeneous Inclusion

The problem to be solved here is what are the stress-strain fields inside and outside an ellipsoidal inclusion with a shape mismatch when it is embedded in an infinite isotropic matrix with elastic constants identical to those of the inclusion. By a series of cutting and welding exercises, Eshelby [57Esh] first solved the above problem.

A region (the inclusion) is cut from an unstressed elastically homogeneous material, and is then imagined to undergo a stress free shape change (strain $e^T$). Surface tractions are then applied to return it to its original shape before replacing it into the hole from which it was cut. This process has been schematically illustrated by Withers et al [89Wit], as shown in Fig 2.4. On the removal of the surface tractions, equilibrium is reached between the matrix and the inclusion at a constrained strain ($e^C$) of the inclusion relative to its initial shape before removal.

By Hooke's law, the stress in the inclusion can be expressed in terms of the elastic strain ($e^C - e^T$)

$$\sigma_i = C_M (e^C - e^T)$$  \hspace{1cm} (2.4.1)

where $\sigma_i$ is the stress within the inclusion and is uniform since the inclusion is ellipsoidal in shape [57Esh], $C_M$ is the elastic constants of the matrix, $e^C$ is the constrain strain of the inclusion, $e^T$ is the Eshelby's transformation strain, $e^C - e^T$ is the inclusion strain.
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Fig 2.4. Eshelby's cutting and welding exercises for misfitting ellipsoidal inhomogeneity and the corresponding equivalent homogeneous inclusion, which has been chosen so that the final stress states in the two composites are identical everywhere. After Ref [89Wit].

A necessary condition for the calculation of $\sigma_I$ is a knowledge of the final constrained strain $e^C$. Eshelby [57Esh] found that the constrain strain $e^C$ can be expressed in terms of the stress free transformation strain, $e^T$, and a tensor, termed as the Eshelby tensor, $S$, which depends solely on the inclusion geometry and Poisson's ratio of the matrix, i.e.,

$$e^C = S e^T$$  \hspace{1cm} (2.4.2)

$S$ thus relates the final constrained inclusion shape to the original shape mismatch between matrix and the inclusion. Although the derivation of the $S$ tensor is quite complicated, for a given major to minor axis ratio of the ellipsoidal inclusion, its final form is relatively simple to evaluate. Thus, without going into the details of the matrix stress field, the inclusion stress can be written in terms of the $S$ tensor and the shape mismatch $e^T$

$$\sigma_I = C_M (S - I) e^T$$  \hspace{1cm} (2.4.3)

where $I$ is the identity tensor. Eqn (2.4.3) is the expression for the Eshelby's continuum transformation theory in tensor form. It should be emphasised that the stress in an ellipsoidal inclusion is uniform while the stress outside the inclusion in the matrix is by no means uniform. Brown and Stobbs [71Bro] derived the stress field outside a spherical inclusion embedded in an infinitely extended homogeneous matrix, which is described by the following equations
\[ \sigma_{xx} = \frac{6GA}{r^3} \left( \frac{xz - 5x^3z}{r^2} \right) + \frac{6GAR_o^2}{r^5} \left( \frac{3xz - 7x^3z}{r^4} \right) \]  
\[ \sigma_{yy} = \frac{6GA}{r^3} \left( \frac{(1-2\nu)xz}{r^2} - \frac{5xy^2z}{r^4} \right) + \frac{6GAR_o^2}{r^5} \left( \frac{xz - 7xy^2z}{r^4} \right) \]  
\[ \sigma_{zz} = \frac{6GA}{r^3} \left( \frac{xz - 5xz^3}{r^2} \right) + \frac{6GAR_o^2}{r^5} \left( \frac{3xz - 7xz^3}{r^4} \right) \]  
\[ \sigma_{xz} = \frac{6GA}{r^3} \left( \frac{1+\nu}{3} - \frac{y^2}{r^2} - \frac{5x^2z^2}{r^4} \right) + \frac{6GAR_o^2}{r^5} \left( \frac{4}{5} - \frac{y^2}{r^2} - \frac{7x^2z^2}{r^4} \right) \]  
\[ \sigma_{yz} = \frac{6GA}{r^3} \left( \frac{xy}{r^2} - \frac{5xyz^3}{r^4} \right) + \frac{6GAR_o^2}{r^5} \left( \frac{xy - 7xyz^2}{r^4} \right) \]  
\[ \sigma_{xy} = \frac{6GA}{r^3} \left( \frac{yz}{r^2} - \frac{5x^2yz}{r^4} \right) + \frac{6GAR_o^2}{r^5} \left( \frac{yz - 7x^2yz}{r^4} \right) \]

where \( \Lambda = -\frac{EPr_0^2}{3(1-\nu)} \), \( \nu \) is Poisson's ratio, \( G \) is the shear modules and \( r \geq r_o \).

The stress field outside a spherical inclusion described by eqn (2.4.4) is quite complicated, but it does have certain simplifying features [71Bro]: (i) The stress outside the inclusion is decreasing dramatically by a power law with increasing distance \( r (r \geq r_0) \), and will have a zero value if \( r \) is large enough. Therefore, it has been called the local fluctuating stress; (ii) the mean value of each stress component outside the inclusion is zero [71Bro, 74Tan], and therefore these local fluctuation stresses do not contribute to the strengthening effect of the inclusion.

(2) The Transformed Inhomogeneous Inclusion

The question now is how to derive the stress field inside an inclusion which has elastic constants different from those of the infinite matrix. By a cutting and welding process similar to that used for the elastically homogeneous inclusion, the stress free strain mismatch can be considered as a stress free transformation strain \( \varepsilon_{T^*} \) of the ellipsoidal inhomogeneity with respect to the hole from which it came. On replacing the inclusion, it now takes up the constrained shape, strained \( \varepsilon_C \) relative to the original shape of the hole. Provided the inclusion is ellipsoidal in shape, the final constrained strain inside the inclusion is still uniform [57Esh], and can be expressed by the Hooke's law as

\[ \sigma_I = C_1 (\varepsilon_C - \varepsilon_{T^*}) \]  

But in this inhomogeneous case, there is no simple solution for the relationship between \( \varepsilon_C \) and \( \varepsilon_{T^*} \) unlike in the homogeneous case. In order to solve this problem, Eshelby [57Esh] proposed a powerful method called the equivalent homogeneous inclusion method (EHIM). As a result of this method, a second inclusion of the same elastic constants as the matrix can be imagined to undergo a stress free transformation, with the transformation strain \( \varepsilon_T \) (see Fig 2.4) chosen in such a way that when the surface traction is applied to give it the constrained shape of the inhomogeneity, it will contain the same uniform stress state. Thus the inhomogeneity and the equivalent homogeneous inclusion can be interchanged without
disturbing the matrix stress-strain field. This equivalent homogeneous problem has already been solved before; therefore, by calculating the equivalent stress free transformation strain $e^T$ required to initiate the constrained stress state in the inhomogeneity, the inhomogeneous problem is also solved.

The stress inside the inhomogeneous inclusion is expressed in terms of the elastic strain by eqn (2.4.5). Since the stress in the equivalent homogeneous inclusion must be identical, one has the following equation

$$\sigma_I = C_M (e^C - e^T) \quad (2.4.6)$$

The constrained strain for the elastically homogeneous problem is given in terms of the equivalent stress-free transformation strain ($e^T$) by eqn (2.4.2), so that equating eqns (2.4.5) and (2.4.6), one has

$$e^T = (C_I - C_M) + C_M^{-1} C_I e^T \quad (2.4.7)$$

The equivalent homogeneous transformation strain $e^T$ can therefore be expressed in terms of the stress-free transformation strain of the inhomogeneous inclusion $e^{T^*}$. So that the stress in the inhomogeneous inclusion can be calculated from eqn (2.4.6)

$$\sigma_I = C_M [S - I] (C_I - C_M) + C_M^{-1} C_I e^{T^*} \quad (2.4.9)$$

Thus by calculating the stress free shape of the equivalent inclusion required to initiate the shape and stress state of the inhomogeneous problem, the stress and strain common to both composites can be obtained.

(3) The Interaction between External Stress and the Inclusion

Until now, only the internal stress in the inclusion arising from a nature shape mismatch between the two regions has been calculated without any external applied stress. It has been demonstrated by Eshelby [57Esh, 59Esh, 61Esh] that the internal stress under the externally applied stress in an inclusion can also be derived by applying the cutting and welding process.

Suppose that a stress $\sigma^A$ is applied externally to the elastically homogeneous composite, which is a homogeneous inclusion embedded in an infinitely extended isotropic matrix. The stress within the composite is now necessarily the superposition of the applied stress and the internal stress which has already been calculated before, i.e., both the inclusion and the matrix undergo an extra uniform elastic distortion $e^A$. Accordingly

$$\sigma_I + \sigma^A = C_M (e^C - e^T) + C_M e^A \quad (2.4.10)$$

where $\sigma_I$ is the stress disturbance in the inclusion to the applied stress, $e^A$ is the elastic distortion a homogeneous inclusion would undergo ($\sigma^A = C_M e^A$), so that $\sigma_I$ is independent of the applied stress in the homogeneous case and is given by eqn (2.4.1).

In the case of inhomogeneity, upon loading, both the inhomogeneous and the equivalent homogeneous inclusions will extend by different amounts. Accordingly, in order to initiate a specific loaded state, the process of selecting the appropriate equivalent transformation strain
must be repeated. That this new equivalent homogeneous composite will have a shape and stress state different from the inhomogeneous composite (see Fig 2.5) when no load is applied is of no importance provided that, upon loading, it extends by \( \mathbf{e}^A \) such that the final stress states of the two composites are identical. The new equivalent homogeneous inclusion then has the constrained shape \( \mathbf{e}^C + \mathbf{e}^A \), and an identical shape and stress state to the inhomogeneous inclusion problem it initiates, so that the total stress in the inhomogeneous inclusion has the form calculated in the previous section. At the same time, it must also be possible to write the stress in terms of the elastic strain in the inhomogeneity as before, so that

\[
\sigma_1 + \sigma^A = C_I (\mathbf{e}^C + \mathbf{e}^A - \mathbf{e}^T) \quad \text{inhomogeneity} \tag{2.4.11}
\]

\[
= C_M (\mathbf{e}^C + \mathbf{e}^A - \mathbf{e}^T) \quad \text{equivalent inclusion} \tag{2.4.12}
\]

where \( \mathbf{e}^A \) is the elastic strain of the unreinforced matrix, i.e., \( \sigma^A = C_M \mathbf{e}^A \). However, the transformed shape of the equivalent homogeneous inclusion is now dependent upon the applied stress, and the elastically homogeneous transformation strain \( \mathbf{e}^T \) can be expressed in terms of the transformation strain of the inhomogeneity and the applied strain, i.e.,

\[
\mathbf{e}^T = [(C_1 - C_M) S + C_M]^{-1} [C_1 \mathbf{e}^T + (C_M - C_1) \mathbf{e}^A] \tag{2.4.13}
\]

Note that for the loading of a previously unstressed inhomogeneous composite \( \mathbf{e}^T = 0 \), but \( \mathbf{e}^T \neq 0 \) in the above equations.

![Fig 2.5](image.png)

**Fig 2.5.** The response of inhomogeneous composite to an applied load \( (\sigma^A) \), and the cutting and welding exercises required to select an equivalent transformation strain \( \mathbf{e}^T \) appropriate to the applied load \( (\sigma^A) \). After Ref. [88Wit].

As has already been pointed out by Withers et al [89Wit], it is always possible to imagine a parallel "experiment" involving an inclusion of the same elastic constants as the matrix, suitably
transformed in a stress free manner such that, under the relevant loading conditions, it is "equivalent" to the ellipsoidal inhomogeneity of interest, in that the two can be interchanged without changing the stress distribution anywhere. It is from this parallel "experiment" the values of stress and matrix strain common to both situations can be calculated. It does not matter that no single equivalent transformation strain will be suitable for all applied loads. Rather it is sufficient that, for any particular applied load and the inclusion/matrix strain mismatch, an equivalent stress free transformation strain $e^T$ can be found.

2.4.3. The Mean Field Model

(1) The Image Stress field

The foregoing analysis is mathematically rigorous for a single inclusion embedded in an infinitely extended isotropic matrix. But in order to utilise this powerful approach in more realistic situations, such as dispersion-hardened alloys, two-ductile-phase alloys and metal matrix composites, careful consideration is needed for effects caused by their finite nature. It was in order to satisfy the boundary conditions at the external surfaces of such finite composites that Eshelby [57Esh] introduced, without defining its precise form, the concept of "image" stress.

The problem arises as what is the precise form of the image stress field. The image stress can be thought of as arising from an external distribution of "image" inclusions, each cancelling appropriate stresses on the external surface of the cut specimen. It is clear that the image stress field is relatively uniform in case of homogeneous composites, and does not fall off with increasing distance from the inclusion particle according to any power law [11Bro]. The image stress probably tends to increase slowly with the increasing distance from the centre of the inclusion. However, when a random array of particles is present, it appears that the image stress and strain are to a good approximation uniform [57Esh], and therefore, in the case of homogeneous inclusion, it is reasonable to assume that the image stress is uniformly distributed throughout the whole composite [71Bro]. But the distribution of the image stress in matrix and inclusion in the case of inhomogeneous inclusions is far from clear [89Wit].

(2) The Mean Field Approximation for Elastically Homogeneous Composite

On finding that the local fluctuation stress field calculated by the Eshelby's method about a spherical inclusion averaged to zero throughout a spherical region of the matrix, Brown and Stobbs [71Bro] realised that, whatever the form of this image stress may be, its overall effect must balance the stress in the inclusion if the composite is free from surface traction, i.e.,

$$<\sigma>^{im} + V\sigma_I = 0$$

(2.4.14)

where $V$ is the volume proportion of the composite occupied by the inclusions and $<\sigma>^{im}$ is the mean image stress. Therefore, there should in general be a simple relation between the stress in the inclusion and the averaged value of the matrix. At this point it becomes clear that if an
appropriate assumption is made about how the image stress is distributed, its effect on the flow
behaviour of the matrix can be defined through the knowledge of the inclusion stress. For
homogeneous composites, it seems nature to assume that the image stress distributed uniformly
throughout the whole composite [89Wit], i.e.,

\[(1 - V) \langle \sigma >^\text{im} + V (\sigma_1 + \langle \sigma >^\text{im}) = 0 \] (2.4.15)

This is equivalent to state that on cutting the finite homogeneous composite from the infinite
one, the composite relaxes by distorting uniformly; the inclusion/matrix boundaries moving
proportionally to the new external surface. Under these circumstances, the fluctuating stress
field does not contribute to the average matrix stress, so that the mean image stress \(\langle \sigma >^\text{im}\)
is also the mean matrix stress \(\langle \sigma >_M\), and the mean inclusion stress fall from \(\sigma_1\) in the infinite
composite to \((\sigma_1 + \langle \sigma >_M)\) in the finite composite. For a finite number of inclusions constituting
a volume fraction of \(f\), the internal balance of the mean stresses can be rewritten as

\[(1 - f) \langle \sigma >_M + f \langle \sigma >_1 = 0 \] (2.4.16)

It is important to note that \(\langle \sigma >_M\) is the average stress (within the matrix), of the sum of
stresses of the inclusions when they are present in an infinite body, plus the image stress of all
the inclusions [73Mor]. The relative contributions of these two terms are not generally
determined. However, as shown by Tanaka and Mori [72Tan, 73Mor], the contribution from
each term can be calculated, if the specimen is of the similar shape to the inclusion, and in this
case \(\langle \sigma >_M\) is solely due to \(\langle \sigma >^\text{im}\). The derived mean strain in the matrix of a finite body by
Brown [73Bro] in a scalar form is

\[<\varepsilon_{ij}>_M = f(\varepsilon_{ij}^T - S_{ijkl} \varepsilon_k^T) \] (2.4.17)

The summation convention applies here. \(<\varepsilon_{ij}>_M\) is the mean strain in the matrix of a finite
composite, \(f_\beta\) is the volume fraction of \(\beta\)-phase, \(\varepsilon_{ij}^T\) and \(\varepsilon_k^T\) are the Eshelby’s transformation
strains, and \(S_{ijkl}\) is the Eshelby tensors.

Eqn (2.4.17) is valid subject to the following restrictions: (i) The inclusion has the same elastic
constants with the matrix; (ii) the inclusions are ellipsoidal in shape so that the internal strain
within the inclusion is uniform; (iii) the composite should contain sufficient number of
inclusions so that one thinks of a rather uniform distribution of them throughout the whole
composite; (iv) the volume fraction of inclusion is small enough to avoid the contacts between
them to any appreciable extent so that the interactions among the stress-strain fields of inclusion
particles can be neglected.

Brown and Clarke [75Bro], for convenience, defined the two rank strain accommodation
tensors \((\gamma_{ij})\) and stress accommodation tensors \((\Sigma_{ij})\) in the deformation coordinate frame to
connect the transformation strains and Eshelby’s tensors. By this treatment, the mean stresses
and strains in the matrix of a finite homogeneous composite arising from the plastic strain
incompatibility can be expressed by the following equations

\[<\varepsilon_{ij}>_M = \gamma_{ij} f_\beta \varepsilon_P^U \] (2.4.18)
2.4.19) where \(<\varepsilon_i^P>_M\) and \(<\sigma_i^P>_M\) are the mean strains and stresses in the matrix of a finite composite, respectively, \(G\) is the shear modulus of the composite, \(\gamma_{ij}\) and \(\Sigma_{ij}\) are the strain and stress accommodation tensors respectively, and \(\varepsilon_P^U\) is the unrelaxed plastic strain in the matrix. The \(\gamma_{ij}\) and \(\Sigma_{ij}\) for a sphere and infinitely long fibres have been computed by Brown and Clarke [75Bro]. It is important to point out that the use of eqns (2.4.18) and (2.4.19) are subject to the same restrictions to eqn (2.4.17) mentioned above.

(3) The Mean Field Approximation for Elastically Inhomogeneous Composite

Following the mean field approximation for homogeneous composite, Pedersen [78Ped, 79Ped, 83Ped] has proposed a mean field approach for the inhomogeneous composite, namely that, just as in the homogeneous case, the inhomogeneous inclusion will sample the mean matrix stress so that

\[ \langle \sigma >_I = \sigma_I + \langle \sigma >_M. \]

This is an approximation and is equivalent to state that the mean matrix stress is indiscriminately distributed throughout the inhomogeneous composite irrespective of the elastic constants of two phases. According to this approximation, the interaction between the inclusion and matrix is straightforward: the mean matrix stress further strains the equivalent homogeneous inclusion so that it is strained by \(e^C + e^T + \langle e >_M.\) Thus, the general expression for the stress within the inhomogeneous inclusion is given by

\[ \langle \sigma >_I + \sigma^A = C_I (e^C + e^A + \langle e >_M + e^T), \]

inhomogeneity

\[ = C_M (e^C + e^A + \langle e >_M + e^T), \]

equivalent inclusion

which gives

\[ e^T = \left[ (C_M - C_I) [S-f (S - I)] - C_M \right]^{-1} \left[ C_I e^T^* - (C_I - C_M) e^A \right], \]

where \(e = S e^T, \langle e >_M = C_M, \langle e >_M \) and \(\sigma^A = C_M e^A.\)

In order to calculate the mean matrix stress, a knowledge of the local fluctuating stress-strain field derived by Eshelby [57Esh] is unnecessary. Instead, the mean matrix stress can be calculated easily from the stress balance (eqn (2.4.16)), in terms of the mean inclusion stress. From eqn (2.4.12),

\[ \langle e >_M = -f (e^C - e^T) = -f (S - I) e^T, \]

\[ \langle e >_M = f (S - I) \left[ (C_M - C_I) [S-f (S - I)] - C_M \right]^{-1} \left[ C_I e^T^* - (C_M - C_I) e^A \right]. \]

This is the general form of the mean matrix strain derived by Pedersen [79Ped]. The mean matrix stress can then be expressed as

\[ \langle \sigma >_M = A e^T^* + B \sigma^A, \]

where

\[ A = f C_M (S - I) \left[ (C_M - C_I) [S-f (S - I)] - C_M \right]^{-1} C_I \]

\[ B = -f (S - I) \left[ (C_M - C_I) [S-f (S - I)] - C_M \right]^{-1} (C_I - C_M). \]

The tensors \(A\) and \(B\) are functions only of the elastic constants of the two phases. It is important to realise that the image stress is significant (being equal to the mean matrix stress in the homogeneous case) even at low volume fractions (first order in volume fraction, see eqn
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(2.4.14), however, only at high volume fraction of inclusion does the sampling the of the image stress by the inclusions become important (second order in volume fraction) [89Wit]. The mean field approach thus expresses both the equivalent transformation strain and the mean matrix stress as the sum of an "inelastic" term proportional to the stress free transformation strain \( e^T \) and an "elastic" term proportional to the applied stress \( \sigma^A \). The elastic term only becomes significant at high volume fraction and large inclusion aspect ratio [79Ped].

It must remember that although the Eshelby's analysis is exact when the volume fraction tends to zero, for finite \( f \) it is no longer so, and the mean field model is only an approximation which has to be justified in light of established theory. The model can be extended to describe the composite's overall elastic and inelastic strains and it is then found [78Ped 79Ped] that it reproduces both the Hashin-Shtrikman's [62Has, 63Has] bounds, and Levin's relation for thermal expansion [74Law]. With further extension and generalisation [83Ped], the model become equivalent to a combination of Walpole's [66Wal] and Law's [74Law] formulation for composites with two or more phases [89Wit]. The special advantage of the formulation based on the mean field model is that it expresses the equivalent transformation strain \( e^T \), and hence the local fluctuating stress and the average stress in the matrix, as a simple function of \( e^A \) and \( e^T \) [89Wit].

It is also important to note the fact that it is the mean stress which determines much of the mechanical behaviour of a well-bounded composite. Upon yielding, flow takes place in the ductile matrix by the movement of dislocations, which experience the local stress field they thread. The local fluctuation in stress will not, since they average to zero [71Bro, 72Tan], encourage movement along any single direction, and so will not give rise to net macroscopic flow. Instead it is the average stress field experienced by the dislocations which cause them to migrate through the composite and thus ultimately determines the flow behaviour. Local flow can however occur, resulting in microplasticity and this is often responsible for an early deviation from the initial linear elastic response to applied loading [89Wit]. It is also important to note that most experimental techniques evaluate the average stresses and strains. For example, the volumes analysed by conventional diffraction techniques are much larger than the "periodicity" of the local strain fluctuations [85Led, 87Wit]. Therefore mechanical testing will be governed by the average stresses.

2.4.4. Applications of the Mean Field Model

(1) The Continuum Approximation

The Eshelby method, described in the previous sections for determination of mean stresses and strains, is only applicable to a model system with an isotropic matrix (a continuum medium) and perfectly aligned ellipsoidal inclusions, randomly distributed within the matrix. It takes no account of the discrete nature of the real materials. Elastic and plastic deformation are
considered to be homogeneous. Whilst in practice, real materials are made up of atoms which flow on slip lines by the movement of dislocations. For wholly elastic behaviour, the representation of the medium by a continuum is a good approximation. However, if the model is applied to simulate plastic deformation, flow must be sufficiently finely distributed so that the displacement and the elastic stress arising from the individual dislocations can be averaged out [66Ash, 70Ash]. This occurs when a number of slip lines intersect each particle. For particles intersected by only a small number of slip lines, dislocations pile up as Orowan loops form. Errors in the application of the Eshelby model in such circumstances have been discussed by Brown [84Bro] and Hazzledine and Hirsch [74Haz], and the incremental slip line spacing has been derived by Pedersen and Brown [83Ped]. However, for the micron-sized inclusions typical of MMCs and tow-ductile-phase alloys, the slip line spacing is significantly less than the particle diameter, and thus one would not expect a significant discrepancy to arise from the continuum approximation [88Wit].

(2) The Elastic Deformation

The inclusions and matrix usually have markedly different elastic constants so that on loading they are stressed to different extent. Provided that no relaxation, decohesion of the inclusion/matrix interfaces, or fibre breakage occurs, the strain difference is accommodated completely elastically and the average disturbance to the uniform applied stress can be calculated [88With]. If the composite is unstressed before loading, the transformation strain (the strain mismatch between the hole and inclusion in unstressed state) is zero ($e^{T*} = 0$), so that from eqn (2.4.22) one gets the following equation

$$e^T = [ (C_M - C_I) [S - f (S - I)] - C_M ]^{-1} (C_I - C_M) e^A$$

(2.4.28)

and the mean matrix stress response to the applied load is

$$\langle \sigma \rangle_M = - f C_M (S - I) [ (C_M - C_I) [S - f (S - I)] - C_M ]^{-1} (C_I - C_M) e^A$$

$$= + B e^A$$

(2.4.29)

When the load is applied along the 3-axis (0, 0, $\sigma^A$, 0, 0, 0)

$$\langle \sigma_3 \rangle_M = + B_{33} \sigma^A$$

and

$$\langle \sigma_3 \rangle_I = - \frac{1 - f}{f} \langle \sigma_3 \rangle_M$$

(2.4.30)

As expected, for constituent phases obeying Hooke's law, the mean stress disturbance ($\langle \sigma \rangle$) in each phase is linearly related to the applied stress tensor $\sigma^A$. The sign of the mean stress is such that the average matrix stress is less than the applied stress, whilst the inclusion stress is greater. Consequently, the form of the $B$ tensor determines the extent of load transfer from the softer matrix to the stiffer fibres or particles, and therefore gives a measure of the "effectiveness" of the reinforcing phase as well as determining the increase in the stiffness and yield stress of the composite relative to the unreinforced alloy.
Prior to loading, a homogeneous composite is strained relative to its shape were there no transformed region by $f e^T$, but since $e^T$ is not a function of the applied load, on applying a load the strain becomes:

$$\text{total strain} = f e^T$$  \hspace{2cm} \text{before loading}  \hspace{2cm} (2.4.31)$$

$$\text{total strain} = e^A + f e^T$$  \hspace{2cm} \text{after loading}  \hspace{2cm} (2.4.32)$$

As one would expect, the composite modulus is unaffected by the presence of the homogeneous inclusion.

Since the stress and the displacement within the inhomogeneous composite and its inhomogeneous equivalent are the same, the total strain in the inhomogeneous case is also given by eqn (2.4.32), but now it is a function of both $e^T$ (the original misfit prior to loading) and the applied stress $\sigma^A$ (eqn (2.4.22)).

$$\text{total strain} = f e^T(e^T)$$  \hspace{2cm} \text{before loading}  \hspace{2cm} (2.4.33)$$

To calculate the extension of the composite under the load and hence the composite's stiffness, one subtracts the two terms of eqn (2.4.33) leaving only the component of eqn (2.4.22) dependent on the applied load ($e^T \text{ (Applied Stress)}$). The macroscopic elastic strain of the composite then is expressed by the following equations

$$e^C = e^A + f e^T (\text{Applied Stress})$$  \hspace{2cm} \text{(2.4.34)}$$

$$e^C = e^A - f[(C_M - C_I)[S-f(S-I)] - C_M]^{-1} (C_I - C_M) e^A$$  \hspace{2cm} \text{(2.4.35)}$$

Combining the above equation with the definition of the composite’s effective stiffness tensor, one has

$$\sigma^A = C_C e^C = C_C(e^A + f e^T (\text{Applied Stress}))$$  \hspace{2cm} \text{(2.4.36)}$$

The composite’s compliance can be written as:

$$C_C^{-1} = C_M^{-1} - f[(C_M - C_I)[S-f(S-I)] - C_M]^{-1} (C_I - C_M) C_M^{-1}$$  \hspace{2cm} \text{(2.4.37)}$$

or equivalently (from eqn (2.4.25)):

$$C_C^{-1} = C_M^{-1} - [S-I]^{-1} C_M^{-1} B$$  \hspace{2cm} \text{(2.4.38)}$$

From eqn. (2.4.36) it is a trivial matter to derive the Young’s modulus of the composite:

$$E_3 = \frac{1}{C_3^{-1}} = \frac{\sigma^A}{e^A + f e^T (\text{Applied Stress})}$$  \hspace{2cm} \text{(2.4.39)}$$

Based on the mean field model, Pedersen [83Ped] developed rigorous bounds for elastic modulus of two-phase composites, and found that the mean field model reproduces Hashin and Shtrikman’s [63Has, 63Has] upper and lower bounds for elastic modulus. Eqn (2.4.39) gives the lower bound, and the corresponding upper bound is obtained by simply interchanging the elastic constants of the matrix and fibres [83Ped]. The predicted lower and upper bounds for Young’s modulus of aligned Al/SiC composites by Withers [88Wit] are presented in Fig 2.6 as
functions of volume fraction and aspect ratio of SiC fibre. Not surprisingly, the margin between two bounds increases as the difference between the stiffness of the two phases increase. However for low volume fractions, the mean field approximation (namely that the inclusion samples the mean matrix stress) seems more reasonable than the converse [88Wit]. As a result, one would intuitively expect the lower bound to be more closely followed at low volume fractions, whilst the stiffness might approach the upper bound at higher volume fractions.

![Graph](image)

**Fig 2.6.** The Hashin and Shtrikman's bounds for the effective composite modulus, obtained using the mean field approach (lower bound) and by interchanging the elastic constants of the two phases (upper bound). The upper bound of \( l/d = 1 \) is coincidental with the lower bound of \( l/d = 5 \). After Ref. [88Wit].

(3) The Thermal Residual Stresses

In most composite materials the individual phases have different coefficients of thermal expansion (CTE). Fabrication at elevated temperature thus results in the development of the internal stresses on cooling [84Ars]. The evaluation of these stresses is of importance in determining the subsequent mechanical properties, especially for systems in which the CTE mismatch is large.

The objective of this section is to determine the elastic stresses in both phases arising from a temperature drop of \( \Delta T \) by applying the mean field theory. It is assumed that the relaxation does not occur. The mismatch between the stress-free changes in volume of the inclusion and of the same region, were it occupied by the matrix, can then be represented by the transformation strain \( e^{T*} \):
where \( \alpha_M \) and \( \alpha_I \) are the thermal expansion tensors for the matrix and inclusions respectively. If the applied stress is absent, the average and mean stresses are equal and given by eqn (2.4.26):

\[
\bar{\sigma}_M = \langle \sigma \rangle_M = A \mathbf{e}^{T*} = f C_M [S-I] [(C_M - C_I) [S - f(S-I)] - C_M]^{-1} C_I \mathbf{e}^{T*}
\]  

(2.4.41)

From eqn (2.4.41), it is clear that the elastic stresses and strains are linearly related both to the temperature drop (\( \Delta T \)) and to the difference between the CTEs of the two phases (\( \Delta \alpha \)). The form of \( A \) is similar, though not identical, to that of \( B \) and expresses the distribution of stress and elastic strain between the three principal directions as a result of the dilatation misfit strain.

Defining the composite's thermal expansion tensor \( \alpha_C \) as

\[
\bar{\varepsilon}_C = \alpha_C \Delta T
\]  

(2.4.42)

following the similar argument in the case of composite's stiffness in previous section, Takao and Taya [85Tak] derived the microscopic elastic strain, \( \varepsilon_C \), which is given by the following equation

\[
\bar{\varepsilon}_C = \alpha_M \Delta T + f \varepsilon^{T}(\text{Thermal Misfit})
\]  

(2.4.43)

Assuming there is no applied stress, the equivalent transformation strain for thermal expansion can be obtained from eqns (2.4.23) and (2.4.13) in the form:

\[
\varepsilon^{T}(\text{Thermal Misfit}) = - \frac{1}{f} [S-I]^{-1} C_M^{-1} A \mathbf{e}^{T*} = [(C_M - C_I) [S - f(S-I)] - C_M]^{-1} C_I \mathbf{e}^{T*}
\]  

(2.4.44)

Here \( \varepsilon^{T*} \) is given by eqn (2.4.40) and the equivalent homogeneous transformation strain is in a form closely related to the composite thermal expansion tensor \( \alpha_C \); using eqns (2.4.41-2.4.44), one gets the following equation

\[
\alpha_C = \alpha_M - [S-I]^{-1} C_M^{-1} A (\alpha_M - \alpha_I)
\]  

(2.4.45)

One can also write eqn (2.4.45) in terms of the composite compliance by eliminating the common factor \( [S-I]^{-1} \) in eqns (2.4.38) and (2.4.45) and using the identity:

\[
A = -B C_M [C_I - C_M]^{-1} C_I
\]  

(2.4.46)

i.e.,

\[
\alpha_C = \alpha_M + (C_C^{-1} - C_M^{-1}) C_M [C_I - C_M]^{-1} C_I (\alpha_M - \alpha_I)
\]  

(2.4.47)

The predicted longitudinal and transverse CTEs for aligned AVSiC composite by Withers [88Wit] is shown in Fig 2.7 as functions of volume fraction and aspect ratio of SiC phase.

Turning now to the bounds for the inelastic mean stress response of multiphase composites, Laws [74Law] has derived the following relation:

\[
\mathbf{e}_C = (C_C^{-1} - C_I^{-1}) [C_M^{-1} - C_I^{-1}]^{-1} \mathbf{e}_M + (C_C^{-1} - C_M^{-1}) [C_I^{-1} - C_M^{-1}]^{-1} \mathbf{e}_I
\]  

(2.4.48)

where \( \mathbf{e}_C, \mathbf{e}_M \) and \( \mathbf{e}_I \) are the composite, matrix and inclusion inelastic strains, respectively. For the case of thermal expansion \( (\alpha) \) eqn (2.4.48) becomes:
\[
\alpha_C = (C_C^{-1} - C_I^{-1})[C_M^{-1} - C_I^{-1}]^{-1} \alpha_M + (C_C^{-1} - C_M^{-1})[C_I^{-1} - C_M^{-1}]^{-1} \alpha_I
\]
\[
= (C_C^{-1} - C_M^{-1})[C_I^{-1} - C_M^{-1}]^{-1} (\alpha_I - \alpha_M) + (C_M^{-1} - C_I^{-1})[C_I^{-1} - C_M^{-1}]^{-1} \alpha_M
\]
\[
\alpha_C = \alpha_M + (C_C^{-1} - C_M^{-1})C_M[C_I^{-1} - C_M^{-1}]C_I(\alpha_M - \alpha_I)
\]
(2.4.49) (2.4.50)

which is the same as that derived before from the mean field theory (eqn (2.4.47)). It should be noted that eqn (2.4.48) is applicable not only to thermal expansion but to all forms of inelastic strain.

![Graph showing composite CTE variation with inclusion volume fraction and aspect ratio for an aligned Al/SiC composite after Ref [88Wit].](image)

**Fig 2.7.** The predicted variation in composite CTE with inclusion volume fraction and aspect ratio for an aligned Al/SiC composite after Ref [88Wit].

\[\text{Al: } \alpha_M = 23 \times 10^{-6} /K; \text{SiC: } \alpha_I = 4 \times 10^{-6} /K.\]

(4) **Derivation of the Flow Curves of Two-Ductile-Phase Alloys**

Tomotan et al [76Tom] divided the deformation process of two-ductile-phase alloys (an \(\alpha-\beta\) phase mixture, \(\beta\)-phase is relatively harder than \(\alpha\)-phase, but the elastic constants of the \(\alpha\)- and \(\beta\)-phases are the same) into three stages: Stage I, both phase deform elastically; Stage II, \(\alpha\)-phase deforms plastically while the \(\beta\)-phase deforms elastically; Stage III, both phase deform plastically. These three stages are delineated by two critical points. The first critical point is the yield strength of the \(\alpha\)-phase \((Y(i) = \sigma_y)\), when the applied stress reaches this point the \(\alpha\)-phase starts plastic deformation; the second critical point \((Y(ii))\) is defined by the condition in which the applied stress plus the internal stress in the \(\beta\)-phase equal to the yield strength of the \(\beta\)-
phase. They also assumed that the plastic strain is partitioned between the constituent phase according to the linear law of mixtures, i.e.,
\[ \varepsilon_p^C = f_\alpha \varepsilon_p^\alpha + f_\beta \varepsilon_p^\beta \]  
(2.4.51)

Then they applied the mean field approach (Tanaka-Mori's model [70Tan]) for homogeneous inclusion to each deformation stage and each critical point. Their results are summarised below:

Stage I:
\[ \sigma_{33}^A = E \varepsilon \]  
(\(\sigma_{33}^A < \sigma_y^A\))
(2.4.52)

First Critical point:
\[ \sigma_{33}^A = \sigma_y^A \]  
(\(\sigma_{33}^A = \sigma_y^A\))
(2.4.53)

Stage II:
\[ \sigma_{33}^A = \sigma_f^A (\varepsilon_p^A) + f_A \varepsilon_p^\alpha \]  
\[ (Y(ii) \geq \sigma_{33}^A \geq \sigma_y^A) \]  
(2.4.54)

Second Critical point:
\[ Y(ii) = \sigma_f^B (\varepsilon_p^B) - (1 - f_A) \varepsilon_p^\alpha(ii) \]  
\[ Y(ii) = \sigma_f^B (\varepsilon_p^B) + f_A \varepsilon_p^\alpha(ii) \]  
(\(\sigma_{33}^A = Y(ii)\))
(2.4.55)

Stage III:
\[ \sigma_f^\alpha(\varepsilon_p^A) = \sigma_{33}^A - f_A [\varepsilon_p^\alpha(ii) + (\delta \varepsilon_p^\alpha - \delta \varepsilon_p)] \]  
\[ \sigma_f^\beta(\varepsilon_p^B) = \sigma_{33}^A + f_A [\varepsilon_p^\alpha(ii) + (\delta \varepsilon_p^\alpha - \delta \varepsilon_p)] \]  
(\(\sigma_{33}^A \geq Y(ii)\))
(2.4.56)

Thus by choosing the empirical equations, such as the Hollomon equation, to represent \(\sigma_f^\alpha(\varepsilon_p)\) and \(\sigma_f^\beta(\varepsilon_p)\) and by choosing the strain increment \(\delta \varepsilon_p\) the stress-strain curve for the composite can be calculated.

2.4.5. Summary

Eshelby's continuum transformation theory and the subsequently developed mean field theory have been widely accepted for predictions and understanding of the physical and mechanical properties of composite materials. In comparison with the shear lag model described previously, the mean field theory has the following advantages: (i) it has been successfully applied to both continuous reinforced composites and dispersion hardened materials; (ii) it has a rigorous theoretical basis at low inclusion volume fraction; (iii) it facilitates the stress transfer across the whole inclusion/matrix interface; (iv) it can predict the stress field about different ellipsoidal inclusion shapes; (v) it limits sensibly; (vi) it can be used to predict many aspects of the composite behaviour in terms of the applied stress and the inclusion/matrix mismatch from a knowledge of the elastic properties; (vii) it can provide both the mean and fluctuating components of the matrix stress field; (viii) it can model misoriented and anisotropic
inclusions. However, despite its inherent shortcomings, such as the abstract nature of the concepts like image stress and equivalent inclusion construction, and difficult to derive, the mean field theory is also much less valid at high inclusion volume fraction, and the assumption that the inclusions and the matrix sample the image stress to the same extent does need further clarification, and it may cause a large discrepancy in the theoretical prediction of the properties of composite where the properties of inclusion phase are remarkably different from those of the matrix.

2.5. Rapid Solidification of Titanium Alloys

2.5.1. Introduction

Rapid solidification (RS) usually refers to the rapid cooling of molten metal through the solidification range. This is most readily achieved by ensuring that solidification occurs in a small cross-sectional dimension of the molten metal in good thermal contact with an effective heat sink. The method developed to achieve these requirements have been categorised in a number of ways, such as spray (atomisation), chill (ribbon), and weld (surface) methods [82Jon, 84Sav, 87Jon]. RS began from 1960 after the pioneer investigations of rapid quenching of metals and alloys from the liquid state by the late Professor Pol Duwez [60Duwl, 60Duw2, 60Kle], which heralded a new era in physical metallurgy by identifying the novel constitutional and microstructural effects achieved through RS. The development of RS techniques in the 1970s, to produce long and continuous tapes and ribbons of uniform cross-section gave further impetus to this fast developing field. The 1980s have achieved additional breakthroughs in the design and development of alloys with novel compositions, refined microstructures and improved mechanical and physical properties [91Sur]. Early work concentrated on the production and application of amorphous materials; more recently, the potential for new and improved crystalline materials has also received considerable attention, with many defined and potential applications under investigation [88Fro].

It is now well recognised that RS of liquid alloys at cooling rates \( >10^{-4} \text{ Ks}^{-1} \) results in significant undercooling of the melts and leads to several metastable effects which can be most conveniently divided into constitutional and microstructural categories:

(i) **Constitutional effects**: rapid solidification allows large departures from equilibrium constitution (the identity and compositional ranges of phases formed) resulting in large extension of solid solubility, formation of non-equilibrium or metastable crystalline phases and metallic glasses, and retention of disordered crystalline structures in normally ordered materials and intermetallic compounds.

(ii) **Microstructural effects**: the microstructural effects include changes in morphology and refinement of dimensions of microstructural features (size, shape, location of phases
present) with a large scale of reduction of segregation effects. The microstructural features that are refined include the grain size, dendrite arm spacing and the size of the constituent particles, precipitates and dispersoids.

Since the products of RS processing can be directly compacted into complex shapes, other benefits include improved fabricability, the production of net or near net shapes, and the elimination of highly textured products especially in hcp materials such as titanium [91Sur].

![Binary Titanium Alloys](image)

**Fig 2.8 Classification scheme of Ti-alloy phase diagrams (after Ref. [65Mol]).**

Titanium is one of the most important non-ferrous metals and finds extensive applications in aerospace, chemical and general engineering industries, because of its low density (4.51 g/cm³), excellent corrosion resistance, high strength level, attractive fracture behaviour, and high melting point (1678°C). Pure titanium exists in two allotropic forms: α (hcp, a = 0.29508 nm, c=0.46855 nm, and c/a = 1.588) up to 882°C and β (bcc, a = 0.33065 nm) up to the melting point, which gives an additional freedom is available for developing the microstructure and the mechanical properties, compared with elements such as aluminium or nickel which exhibit only one crystal structure. As a transition metal, titanium is characterised
by its ability to provide greater than 1 at.-% maximum equilibrium solid solubility for about 45 elements [86Mas] in contrast with aluminium where only the 8 elements surrounding it in the Periodic Table have greater than 1 at.-% maximum equilibrium solid solubility [82Jon]. The classes of elements for which the alloying potential of RS titanium appears greatest have been summarised as [91Sur]: (i) the rare earths which normally exhibit very low solid solubility; (ii) the metalloids (B, C, Ge and Si) which exhibit low solid solubility under equilibrium conditions; (iii) the eutectic formers which exhibit quite high solid solubility but are very segregation prone. This subject has been extensively covered by several reviews [83Sas, 84Wha, 86Fro1, 86Wha1 87Row1, 91Sur] and one conference proceeding [86Fro2].

2.5.2. Classification of Binary Titanium Alloys

A number of authors have attempted to categorise the titanium alloy phase diagram [87Mur, 60Mar, 65Mol, 84Col], all agreeing that there are two major divisions: α stabilised and β stabilised systems. The most convenient presentation is that developed by Molchanova [65Mol] and presented in Fig 2.8. Here the α stabilisers are divided into those having complete solubility, in which the α-phase can coexist with the liquid phase (e.g., Ti-N and Ti-O) and there is a simple peritectic reaction resulting in the formation of the β-phase. The other category is the one where the element has limited α stability, in which α forms by α peritectic reaction from β and a compound (β peritectoid). Examples of the later type of system are Ti-Al, Ti-B and Ti-C. Molchanova [65Mol] also divided the β-stabilisers into two categories: β isomorphous and β eutectoid. In the former system, an extended β solubility range exists with only a restricted α solubility range. Examples are Ti-Mo, Ti-Ta and Ti-V, with elements such as Hf and Zr occupying an intermediate position since they have a complete mutual solubility in the both the α and β phases. For the β eutectic systems, the β phase has a limited solubility range and decomposes into α plus a compound (e.g., Ti-Cr, Ti-Cu). This class can also be further subdivided depending on whether the β transformation is fast (e.g Ti-Cu, Ti-Ni and Ti-Sn) or sluggish (e.g., Ti-Cr, Ti-Fe and Ti-Mn).

2.5.3. Techniques for Rapid Solidification of Titanium Alloys

(1) Melting Processes

Titanium and its alloys are highly reactive and even the most chemically stable ceramic compounds generally dissolve in liquid titanium [65Gar, 83Hol]. The properties of titanium alloys are strongly influenced by even a small amount of interstitial elements (O, N, H, C) emphasising the containment problems in melting titanium alloys. Schemes for minimising the reaction of ceramic containers by external cooling and other means during melting have been developed [73Cli, 75Pri, 73Zan]. Containment problem for melting titanium alloys may also be
overcome through levitation melting, but since the quantity of alloy that can be melted at one time during levitation is limited, this technique cannot be used for bulk production [91Sur]. Thus contamination free melting techniques for titanium and its alloys are generally based on cold hearth or "skull" melting [68Noe, 86Chr]. These techniques rely on an intense heat source to melt the alloy, which is contained in a cooled high thermal conducting hearth, e.g., copper. This approach maintains a solid layer of titanium (skull) as the effective containment surface. The advantage of this process is that there is virtually no contamination during melting since no materials other than the solid alloy itself is in contact with the melt. The disadvantage is that the superheating is minimal and both solid and liquid coexist in the melt pool.

An alternative melting technique which has been used on a laboratory scale (and some commercial scale processes) is a localised melting with an intense localised heat source (electron beam, laser and arc) so that the melt is again only in contact with the remaining solid part of titanium metal. The characteristics of local melt based processes are the same as for skull melting, but there is a possibility of local compositional fluctuation resulting from heterogeneity in the starting material.

(2) RS Processes

(a) The Techniques

Gas atomisation (GA) is a well established process for the production of RS powders from various metals and their alloys. This technique, as shown schematically in Fig. 2.10(a), has been used to produce a wide range of Ti-base alloys [91Sur]. Ultrasonic and high pressure gas atomisation (USGA) is similar to standard GA except that the impinging gas is at a very high velocity and high frequency. USGA technique has been used to produce powders from Ti-Al alloys [83Sas] and TiAl [88Lon]. The laser spin atomisation (LSA) technique is schematically shown in Fig 2.9(b). This technique has been employed on a laboratory scale to produce powders from a number of Ti-alloys [91Sur]. Pendant drop melt extraction (PDME), as shown schematically in Fig. 2.10(c), has been very widely used for the production of RS titanium and other reactive and refractory metal alloy powders. The advantages of PDME include elimination of the crucible and the use of variety of feedstock. The disadvantages of PDME are chemical inhomogeneity of the product, including unmelted particles, and scale up difficulties. Electron beam splat quenching (EBSQ) technique produces splat foils using a focused electron beam which melt the button tip of a prealloyed titanium rod, as shown schematically in Fig. 2.10(d). This has remained a laboratory scale technique.
The melt spinning technique involves the injection of a molten stream of alloy from an upper chamber, by a pressure difference, through a nozzle (often made of W) onto a rapidly rotating high thermal conductivity metal wheel in a lower chamber. Whang et al [83Wha, 85Wha1, 86Wha2] constructed a melt spinner which is similar to the one used earlier for spinning lanthanum based alloys [80Tut]. In this apparatus, the conventional arrangement of melting and pressurised expulsion from a quartz nozzle was replaced by arc melting on the orifice of a water cooled copper anode (Fig 2.10). A new version of this apparatus incorporates an arrangement to feed solid granules of alloy automatically into the melt zone to convert larger quantities continuously into ribbon or flake. Masumoto et al [80Mas, 80Sur, 87Row2] used
leviation melting to melt titanium alloys in a chamber filled with argon gas. Other designs have also been reported in the literature [87Ana, 91Sur, 86Ray1]. The melt spun ribbon is somewhat thinner (15-50 μm) than that produced by melt extraction because of the high wheel speeds that can used for melt spinning [76Pon]. The cooling rates have been estimated to be of the order of $10^6$ Ks$^{-1}$. Melt spinning of titanium is a laboratory scale process with 1 kg quantities being currently produced.

Fig 2.10 Schematic drawing of arc melt spinning unit (after Ref. [86Whal]).

Fig 2.11 Schematic drawing for laser surface melting of Ti alloys (after Ref [81Pen]).

Laser and electron beam surface melting technique, shown in Fig 2.11, has been used to melt a thin surface layer of titanium alloys which solidified rapidly resulting from fast heat extraction.
by the bulk material substrate (e.g. [81Pen, 84Kon]). Surface melting processes have the advantage that the rate of solidification can be easily controlled by melt pool depth and scanning rate [77Cli] and the perfect thermal contact (ideal cooling) between the melt and the substrate results in the rapid quenching through both solidification and subsequent solid state cooling.

Other techniques have also been used to produce RS titanium alloys, which include melt overflow (e.g., [86Gas]), the hammer-and-anvil splat quenching technique (e.g., [85Lu]), the pulverisation sous vide (PSV) process [80Dev], the hydride process (Colt-Titanium, C-T process) [80Lau], electron beam rotating disc (EBRD) process [76Ste, 80Wah], the Hurd process [77Hur, 84Hay], The AIITi-OXY powder process [84Har, 86Meg] and the Goldschmidt calctiothermic process [89Egg].

Some characteristics of RS techniques used for titanium alloys are summarised in Table 2.1

(b) The cooling rate during rapid solidification

The rate of cooling of a metallic material before, during and after solidification can, in principle, be calculated and also measured experimentally [82Jon, 87Ana]. A knowledge of the cooling rate helps in evaluating the relative effectiveness of different quenching techniques. An understanding of the variables affecting the rate of cooling allows the optimization of the conditions to achieve the maximum possible cooling rates. Furthermore, the magnitude of cooling rate and its variation aids in both interpreting and optimising the microstructural features obtained during rapid solidification.

Table 2.1. Summary of processing techniques, cooling rate and product dimensions for RS Ti-alloys after Ref. [91Sur].

<table>
<thead>
<tr>
<th>RS technique</th>
<th>Cooling rate, K s⁻¹</th>
<th>Product form</th>
<th>Typical product dimension, μm</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gas atomisation</td>
<td>10² - 10³</td>
<td>Spherical</td>
<td>50-100 dia.</td>
<td>[86Yol, 86Mol]</td>
</tr>
<tr>
<td>Ultrasonic gas atomisation</td>
<td>10³ - 10⁵</td>
<td>Spherical</td>
<td>10-100 dia.</td>
<td>[83Sas]</td>
</tr>
<tr>
<td>Laser spin atomisation</td>
<td>10⁵</td>
<td>Spherical (±needles ?)</td>
<td>100-1000 dia.</td>
<td>[84Kon, 85Pen]</td>
</tr>
<tr>
<td>Plasma rotating electrode process</td>
<td>10⁴ - 10⁵</td>
<td>Spherical</td>
<td>50-200 dia.</td>
<td>[84Par]</td>
</tr>
<tr>
<td>Pendant drop melt extraction</td>
<td>10³ - 10⁵</td>
<td>Fibre, filament</td>
<td>20-100 dia.</td>
<td>[74Mar]</td>
</tr>
<tr>
<td>Electron beam splat quenching</td>
<td>10⁴ - 10⁶</td>
<td>Elongated splat</td>
<td>40-100 dia.</td>
<td>[84Sas, 88Sas]</td>
</tr>
<tr>
<td>Melt spinning</td>
<td>10⁵ - 10⁷</td>
<td>Ribbon</td>
<td>15-50 dia.</td>
<td>[86Wha, 85Wha]</td>
</tr>
<tr>
<td>Melt overflow</td>
<td>10⁶</td>
<td>Ribbon</td>
<td>20-50 dia.</td>
<td>[87Gas, 86Gas]</td>
</tr>
<tr>
<td>Hammer-and anvil quenching</td>
<td>10⁵ - 10⁷</td>
<td>Foil</td>
<td>10-50 dia.</td>
<td>[85Lu]</td>
</tr>
<tr>
<td>Laser surface melting</td>
<td>10⁵ - 10⁶</td>
<td>Surface</td>
<td>10-1000 dia.</td>
<td>[81Pen]</td>
</tr>
</tbody>
</table>

A correlation was established [81Pen, 83Pen] between secondary dendrite arm spacing d and cooling rate T in a laser processed Ti-15V-3Al-3Sn-3Cr alloy using an equation of the type d = B T⁻ⁿ with B = 80 μm (Ks⁻¹)ⁿ, n = 0.34, d is in μm, and T in Ks⁻¹. Broderick et al [85Bro] determined the effect of the section thickness on grain size since both these parameters can be measured directly. The section thickness, in turn, can be related to the cooling rate. The relationship between β grain size L and the estimated cooling rate T indicates a power relationship of the form L = A₁ T⁻ⁿ₁, where n₁ = 0.93 and A₁ = 3.1x10⁶ μm (Ks⁻¹)⁰.93, for Ti-
Chapter 2: Literature Survey

6Al-4V [85Bro]. By applying the classical theory of homogeneous nucleation and isotropic linear growth to grain formation during rapid solidification, Bosswell et al [77Bos] derived a relation relating grain size to solidification rate as \( L = A_2 \cdot T^{-n_2} \), where \( n_2 = 0.9 \) and \( A_2 = 1.8 \times 10^7 \mu m (Ks^{-1})^{0.9} \) for aluminium. The exponent \( n \) is almost exactly the same for aluminium and Ti-6Al-4V, but the absolute values of \( L \) for Ti-6Al-4V are an order of magnitude less than those predicted by Bosswell and Chadwick [77Bos] for aluminium. This work [85Bro] represented the first clear demonstration that the predicted power law [77Bos] can apply.

The above methods have been used to estimate the cooling rate using different techniques; Table 2.2 summarises these results. It should be emphasised, however, that the undercooling experienced by the melt and the subsequent solidification front velocity are more important than the cooling rate in determining the final constitutional and microstructural characteristics [88Per].

Table 2.2. A and n values in Equation \( L = A \cdot T^{-n} \) (after Ref [91Sur]).

<table>
<thead>
<tr>
<th>Alloy</th>
<th>Microstructural feature</th>
<th>( A, \mu m (Ks^{-1})^n )</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ti-6Al-3Al-3Sn-3Cr</td>
<td>Secondary dendrite arm spacing</td>
<td>131  80  0.3  0.34</td>
<td>[81Pen]  [83Pen]</td>
</tr>
<tr>
<td>Ti-6Al-4V</td>
<td>Grain size</td>
<td>3.1x10^6  0.93</td>
<td>[85Bro]</td>
</tr>
<tr>
<td>Ti eutectoid system</td>
<td>Grain size</td>
<td>3x10^4  3x10^5  0.62  0.79</td>
<td>[84Kri]  [84Bae]</td>
</tr>
<tr>
<td>Aluminium</td>
<td>Grain size</td>
<td>1.8x10^7  0.9</td>
<td>[77Bos]</td>
</tr>
</tbody>
</table>

(3) Compaction and Processing

For most technical applications, the RS product (powder, flake, ribbon, fibre etc.) needs to be consolidated into larger usable forms. Conventional consolidation techniques and processes used in powder metallurgy requires application of pressure at relatively high temperatures which may result in coarsening of the ultrafine RS structures and lead to reduced mechanical property levels. Thus, the challenge is to achieve the necessary integrity at a relatively low temperature so as to retain the refined microstructures and extended supersaturation. However, the particle deformation necessary for full densification and interparticle bonding is difficult to achieve at low temperature.

Thermomechanical processing (TMP) techniques have been used for consolidation of titanium alloy powders (including the powder pulverised from other forms of RS products), which include forging [88Lon], extrusion [85Mar], gun compacting [83Ray, 84Iye], explosive compaction [86Lin, 85Rom], hot isostatic pressing (hipping) [86Eyl], vacuum hot pressing [88Lon] and rapid omnidirectional compaction [87Eyl, 85Mah, 86Mah].

Thermochemical processing (TCP) is a term which has been coined to describe the use of hydrogen as a temporary alloying element in titanium alloys to enhance both the processability and final mechanical properties [85Fro]. Thermochemical processing has been successfully applied to a variety of powder metallurgy products [85Fro, 85Kol, 90Fro1, 90Fro2]. In an attempt to use TCP to control the microstructure of eutectoid forming elements, hydrogen was
introduced into PDME Ti-Cu alloy [87Tea], and also into Ti-B, Ti-C alloys [88Bha]. Similar results of microstructural refinement [89Yol, 89Eyl, 89Ste] and improved mechanical properties [89Eyl] have been obtained in α–β alloys and Ti3Al-Nb alloys using TCP.

It has been shown that hydrogen dramatically reduces the β transus [87Mur]. Thus, by adding the hydrogen and going above the lowered β transus temperature it was hoped that the desired α morphology (Widmanstatten for better creep behaviour) would be obtained while retaining the fine dispersoid. However, it was found that the increased diffusion rate in the more open bcc β lattice resulted in excessive coarsening of the dispersoid, despite the reduced temperature.

2.5.4. The Effects of RS on Microstructure

Solid solubility extension has been observed in many alloy systems [91Sur]. Titanium and its alloys usually solidify as a bcc β-phase which may be retained to room temperature or transformed by several different paths to the low temperature hcp α-phase, depending on alloy composition. Since the primary effect of RS is to shift the formation of equilibrium second phase from the solidification step to the solid state, the principal parameter for evaluating the advantage of a RS process over ingot solidification is the degree of extension of metastable solubility of solute elements in the high temperature β-phase. However, the situation is complicated by the decomposition occurred in both the high temperature β-phase and the low temperature α-phase during solid state cooling. Suryanarayanan et al [91Sur] have summarised the solid solubility extension in RS titanium alloys studied in the literature, and this is presented in Table 2.3.

RS techniques were successful in producing amorphous titanium alloys as early as 1968 [68Ray], but the techniques developed during 1970s to produce long and continuous tapes and ribbons of amorphous materials with possible superconducting behaviour revived the interest in 1970's [84Sur]. Binary and ternary titanium alloy systems that have been successfully produced in a glass state on rapid quenching from the melt have been reviewed by Suryanarayana and Froes [91Sur].

In addition to the extension of solid solubility limits and amorphous phase formation, RS can also lead to the formation of metastable crystalline phases and occasionally a quasicrystalline phase. These aspects have been investigated in great details for aluminium alloys [77Ana, 88Sur], but much less in titanium base alloys. For this reason, the number of metastable intermediate phases, and quasicrystalline phases, detected in RS titanium alloys to date is very limited [91Sur].

The metastable phases normally occurring in titanium alloys quenched from the solid state β include α', α'', (and other alternative forms of martensite) [71Fon], and the hexagonal ω-phase [86Fro, 72Sas]. Earlier studies indicated that the solid state martensitic transformation is independent of cooling rate, beyond a critical cooling rate [70Jep]. However, Whang [87Wha, 85Wha3] has shown that the martensitic start temperature Ms is lowered as the cooling rate
increases if rapid quenching takes place from the liquid state. For a given composition of the alloy studied no martensitic structure was observed at a cooling rate considerably above the critical cooling rate, while at a lower cooling rate (a result of the increased cross-section thickness) a clear martensitic structure was observed. Refinement of the \( \omega \)-phase particles was also noticed with increasing cooling rate and here again the \( \omega \)-phase transformation may be suppressed by sufficiently rapid quenching from the melt [91Sur].

**Table 2.3. Solid solubility limits of alloy additions to Titanium under equilibrium and RS condition.**

<table>
<thead>
<tr>
<th>Element</th>
<th>Matrix</th>
<th>Solu. in ( \alpha ), at-%</th>
<th>Solu. in ( \beta ), at-%</th>
<th>Technique</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>B</td>
<td>Binary</td>
<td>0.2</td>
<td>0.5</td>
<td>3.0</td>
<td>Splat [83Chi]</td>
</tr>
<tr>
<td>B</td>
<td>Binary</td>
<td>0.2</td>
<td>0.5</td>
<td>10.0</td>
<td>Splat [86Wha1]</td>
</tr>
<tr>
<td>Be</td>
<td>Binary</td>
<td>-0</td>
<td>7.5</td>
<td>15.0</td>
<td>Splat [79Tan]</td>
</tr>
<tr>
<td>C</td>
<td>Binary</td>
<td>1.6</td>
<td>0.6</td>
<td>0.6</td>
<td>Splat [86Wha]</td>
</tr>
<tr>
<td>Ce</td>
<td>Binary</td>
<td>1.2</td>
<td>0.6</td>
<td>0.6</td>
<td>Splat [83Wha, 85Wha2]</td>
</tr>
<tr>
<td>Co</td>
<td>...</td>
<td>0.8</td>
<td>2.5</td>
<td>14.5</td>
<td>...</td>
</tr>
<tr>
<td>Cr</td>
<td>...</td>
<td>0.2</td>
<td>&lt;5.0</td>
<td>100.0</td>
<td>...</td>
</tr>
<tr>
<td>Cu</td>
<td>...</td>
<td>1.6</td>
<td>&lt;7.5</td>
<td>13.5</td>
<td>13.5</td>
</tr>
<tr>
<td>Dy</td>
<td>Binary</td>
<td>...</td>
<td>...</td>
<td>&gt;0.6</td>
<td>EBSQ [84Sa1]</td>
</tr>
<tr>
<td>Er</td>
<td>Binary</td>
<td>0.3</td>
<td>...</td>
<td>0.9</td>
<td>0.7</td>
</tr>
<tr>
<td></td>
<td>6242*</td>
<td></td>
<td></td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td></td>
<td>Ti-5Al</td>
<td></td>
<td></td>
<td>1.5</td>
<td>H-A Splat [87Row1]</td>
</tr>
<tr>
<td>Fe</td>
<td>...</td>
<td>0.04</td>
<td>2.5</td>
<td>22.0</td>
<td>35.0</td>
</tr>
<tr>
<td>Gl</td>
<td>Binary</td>
<td>0.6</td>
<td>2.0</td>
<td>&lt;0.5</td>
<td>EBSQ [84Sa1]</td>
</tr>
<tr>
<td>Ge</td>
<td>Ti-5Al-2.5Sn</td>
<td>2.9</td>
<td>8.9</td>
<td>&gt;0.3</td>
<td>EBSQ [85Jac]</td>
</tr>
<tr>
<td>La</td>
<td>Ti-5Al-2.5Sn</td>
<td>-1</td>
<td>0.3</td>
<td>3</td>
<td>Splat [92Chi]</td>
</tr>
<tr>
<td>Mn</td>
<td>...</td>
<td>0.4</td>
<td>2.5</td>
<td>30.0</td>
<td>...</td>
</tr>
<tr>
<td>Mo</td>
<td>...</td>
<td>0.4</td>
<td>&lt;2.0</td>
<td>100.0</td>
<td>...</td>
</tr>
<tr>
<td>Ni</td>
<td>Binary</td>
<td>1</td>
<td>2</td>
<td>&gt;0.1</td>
<td>EBSQ [84Sa1]</td>
</tr>
<tr>
<td>Ni</td>
<td>...</td>
<td>0.2</td>
<td>5.0</td>
<td>10.0</td>
<td>...</td>
</tr>
<tr>
<td>Si</td>
<td>Ti-10 at-%Zr</td>
<td>0.5</td>
<td>6.0</td>
<td>3.5</td>
<td>...</td>
</tr>
<tr>
<td>W</td>
<td>...</td>
<td>0.2</td>
<td>2.5</td>
<td>100.0</td>
<td>...</td>
</tr>
<tr>
<td>Y</td>
<td>Ti-8Al</td>
<td>&lt;0.1</td>
<td>...</td>
<td>0.3</td>
<td>&gt;2.0</td>
</tr>
</tbody>
</table>

*6242: Ti-6Al-2Sn-4Zr-2Mo.

The matrix microstructure of as quenched RS titanium alloys can be significantly more complicated than aluminium alloys, for example, because of the allotropic change and the occurrence of the martensitic transformation. In order to document fully the microstructural development by RS care must be taken to distinguish clearly whether the matrix is \( \alpha \) or \( \beta \).

The solidification structure of RS titanium alloys has been studied primarily in substrate quenched alloys. In highly \( \beta \)-stabilised alloys, the alloys quenches to a relatively simple \( \beta \) matrix [84Bro]. In most alloys, however, the RS material solidifies to a \( \beta \) structure which then transforms to a hexagonal martensite on quenching to room temperature [85Bro]. On subsequent annealing, the matrix can evolve in two ways depending on the \( \beta \) grain size. If the \( \beta \) grain size is relatively large (\( \geq 5\mu m \)), lenticular \( \alpha \) forms from the martensitic laths [85Bro]. If, however, the \( \beta \) grain size is small (\( \leq 5\mu m \)), a quite different equiaxed \( \alpha \) morphology develops [83Jac].
In the substrate quenched material, solidification generally occurs by the growth of columnar β dendrites from the contact surface. Alloys that do not contain strongly segregating elements are columnar from the contact surface to the free surface in the melt extracted filaments or ribbons less than 75-100 μm in thickness. Alloys with rare earth additions exhibit a similar columnar microstructure, but with some evidence of rare earth oxide precipitation along dendrite boundaries in some regions.

Titanium alloys containing boron exhibit stronger segregation effects at higher solidification rate than alloys containing rare earth elements. The decoration of the solidification structure in boron containing alloys reveals a continuous variation of structure from the contact surface to the free surface of the ribbon, ranging from columnar β grains growing from the contact surface of the filament to an equiaxed dendritic microstructure near the top of the ribbon [86Row]. Within the columnar grains, the microstructure exhibits a transition from relatively segregation free solidification near the contact surface to cellular and dendritic segregation just before a transition to the equiaxed dendritic zone [86Row]. In melt spun and melt extracted ribbon, the zone of columnar solidification structure has been observed in different substrate quenching experiments, to extend from 1/4 to 3/4 of the distance to the top of the ribbon, apparently depending on the solidification conditions.

2.5.5. The Effects of Additional Elements

In the past few years, considerable efforts have been made to increase the elevated temperature strength and creep behaviour of titanium base alloys beyond those of conventional ingot metallurgy alloys by controlling the dispersion of hard second phase particles [86Fro, 87Row1]. Dispersion strengthened alloys generally display attractive elevated temperature strength with good creep resistance and stress rupture behaviour [91Sur]. Retention of these high temperature properties requires slow coarsening of the dispersoids at temperature necessitating low solute solubility and low diffusion rate. In conventional compositions, temperature improvement has not been easy because of several difficulties including (i) insufficient supersaturation in the as quenched alloys; (ii) heterogeneous distribution of dispersoids; (iii) rapid particle growth [91Sur]. The RS technique can potentially circumvent these problems. At high enough solidification rates supersaturated solid solution of dispersoid forming elements which have very low equilibrium solid solubilities and low diffusion can be obtained. On annealing at suitable temperatures, these supersaturated solid solutions decompose to produce very fine dispersoids which are generally uniformly distributed. Dispersions which have been introduced into the titanium alloy matrix are rare earth (RE) elements or their oxides, sulphides, borides, carbides and silicides [91Sur].

1) Rare Earth System

(a) The Effect on Microstructure

Vordahl [69Vor] first suggested RS of titanium alloys containing RE oxides and sulphides as a means of producing a fine stable dispersion of oxide and sulphide particles. The identification
of these fine stable dispersoids has stimulated considerable interests in RS of titanium alloys for elevated temperature applications. Until now, the RE elements added to titanium alloys include Ce, Dy, Er, Gd, La, Lu, Nd, Tb, Th, and Y by using numerous RS techniques [91Sur]. It has been realised that alloys in which the solid state decomposition reaction to produce the dispersoid occurred at low temperature generally have the finest dispersoid diameter [85Kon1]. The extension of solid solubility for RE elements is summarised in Table 2.3.

When RE elements are added to Ti in elemental form, they scavenge the interstitial oxygen from the titanium matrix decreasing the oxygen contamination in the solid solution. To maintain the oxygen concentration at the desired level and obtain oxide dispersion strengthening in Ti, additions of RE oxide to titanium alloys have also been investigated [86Sas]. Among all the potential alloying RE elements, Er additions have been studied extensively because they result in a maximum of 1 at.-% solid solubility and subsequent fine and uniform dispersion of second phase particles. An interesting observation is that alloys containing Er in excess of that required to form the oxide coarsen much more rapidly than alloys containing excess oxygen [84Kon1, 85Kon2, 87Lof]

(b) The Coarsening of Dispersoids

The resistance of dispersoids to coarsening at elevated temperatures is critical for the retention of properties both after consolidation and during service. The resistance to coarsening during consolidation is generally of more concern, since this operation is usually performed in the temperature range 800-1000°C, which is above the temperature range anticipated for applications.

Sastry et al [84Sas1] compared the dispersoid distribution in a series of binary Ti-RE alloys (Ce, Dy, Er, Gd, La, Nd and Y) after annealing for 1000 min. at 800°C, below the β transus temperature. Very fine dispersoids (<100 nm in diameter) were found in alloys containing Dy, Er, La and Nd, with particle spacing ranging from 0.3 to 0.8 μm. The dispersoid with greatest resistance to coarsening is Er₂O₃.

The mechanisms that account for the changes in dispersoid size are: (i) coarsening in the β-phase due to the higher diffusivity in β-phase than in the α-phase, (ii) coarsening resulting from the migration of internal boundaries and (iii) bulk Ostwald ripening in α-matrix [91Sur].

(c) Mechanical properties

The first study of the effect of the dispersoid additions on the mechanical properties of RS titanium alloys was on binary Ti-RE alloys [84Sas2]. It was found that dispersoid containing alloys were 60% stronger than high purity titanium, but had lower ductility. Dispersoids affect the strength in several ways. Firstly, grain refinement will increase the low temperature strength. However, the effect of RE element in getting oxygen from the matrix will reduce
oxygen interstitial strengthening, secondly, the maximum contribution to the strength of the alloy is derived from the Orowan strengthening by the dispersoid. This effect accounted for 205 MNm$^{-2}$ of the total 415 MNm$^{-2}$ flow stress increase in one study [84Sas2]. By applying the modified Orowan relation [64Ash] to analyse the dispersoid strengthening effect on dispersoid containing RS Ti-alloys, Snow and Giiameli [84Sno] concluded that the highest strengthening occurs for fine, closely spaced dispersoids. Whileas the finest dispersoids would be expected to form at low temperature, this implies that decomposition of the metastable solid solution to form these dispersoids should occur in the $\alpha$-Ti phase rather than from the higher diffusivity $\beta$-phase. With the objective of producing combined dispersion, precipitation and solid solution strengthening in titanium alloys, Sastry et al [86Sas] investigated the addition of different amount of Al to a Ti-2Er alloy. Their results of mechanical properties showed that a linear superposition of the various strengthening contributions account most satisfactorily for the measured yield strength. In comparison with the ingot metallurgy alloys, RS Ti-Al-Er alloys have 20-30% higher strength and greater ductility.

(2) Metalloid Systems

The properties of conventional titanium alloys can be enhanced with additions of metalloid elements. For example, additions of Si are used to increase the high temperature strength and creep resistance of commercial titanium alloys. The silicide Ti$_5$Si$_3$ in binary and pseudobinary Ti-Zr-Si [(Ti, Zr)$_5$Si$_3$] alloys provides good creep strength up to 600°C [82Ram, 71Flo]. However, silicide particles coarsen rapidly above 600°C resulting in a reduction of creep strength as well as decreased post-exposure ductility. The other metalloid elements B, C, and Ge generally offer little advantage when added to conventional ingot metallurgy titanium alloys, in large part because of their limited solid solubility and hence coarse morphology of second phase particles. Rapid solidification offers the potential to extend solubility limits and to refine the second phase particles which form on annealing. In this part of the literature survey, only addition of B will be reviewed in detail. For other metalloid additions see the review paper by Suryanarayana [91Sur].

Addition of B to titanium decreases the density and increases the specific modulus. The limited solid solubility of B in both $\alpha$- and $\beta$-Ti results in coarse needle like TiB particles in conventionally processed material. However, RS processing of Ti-B alloys results in non-equilibrium supersaturated solid solutions and uniformly distributed high aspect ratio TiB needles [86Row, 86Sas], B additions have been made to rapid solidified Ti [86Sas], Ti-6Al-4V [86Ray1], T-5Al-2.5Sn [84Chi], Ti-8Al-1Mo-1V [86Ray1], Ti-6Al-3Sn-3Zr [86Row], Ti-6Al-2Sn-4Zr-2Mo [86Row], Ti-6Al-6Zr [86Row] and Ti$_3$Al [86Ray1]. In some cases, B addition has been made to alloys containing other dispersoid formers like Er [86Row]. The precipitates in B containing Ti-alloys after annealing are reported to be TiB in most investigations, although the evidence is not very convincing (e.g., [85Row]).

Rapid solidification can increase the solid solubility of B in titanium up to 10 at.-% from the equilibrium value of 0.5 at.-% [86Whal, 86Wha3]. Boron contributes to solid solution
strengthening and the hardness can be increased further by age hardening; here, the increase can be as much as 20-40% from the as quenched state [83Chi].

In virtually all studies, with the possible exception of hammer-and-anvil splat quenching, boron was found to decorate the solidification interfaces [91Sur]. Boron containing alloys exhibit a two zone microstructure with columnar growth at the contact side changing to an "equiaxed dendritic" structure near the top of the melt extracted and the melt spun ribbons [86Row]. This is in contrast with the segregation free columnar solidification of alloys containing RE additions. Because of this segregation, boride containing alloys exhibit a stronger resistance to grain coarsening than alloys containing RE additions. Annealing of RS Ti-B alloys at temperatures above 500°C results in transformation of martensite to fine grained α and precipitation of fine needle like TiB precipitates. Large, well defined needles 1-10 μm in length develop as a result of coarsening after aging at 800-1000°C [86Row, 85Sas]. These precipitates are uniformly distributed throughout the matrix and higher boron concentration produces a correspondingly higher volume fraction of precipitate.

TiB dispersions in RS titanium alloys have the potential for significant strengthening by in situ fibre reinforcement. This results in substantial improvements in hardness, tensile strength, and stiffness compared with conventional material; the ductility is, however, lower (see Table 2.4) [86Ray1, 86Ray2]. For example, an addition of 0.5%B increases the Young's modulus by 30% and doubles the yield strength. This increased strength persists at high temperatures. Fig 2.12 shows the improvements in mechanical properties of titanium alloys resulting from B additions [86Sas]. Sastry et al [86Sas] also reported that boride dispersion modified Ti-5Al-2.5B showed rapid overaging at 600°C, indicating boride coarsening. Further investigation is needed to confirm the boride coarsening temperature.

Fig 2.12 Mechanical properties of RS Ti alloys containing boron (after Ref. [86Sas]).

The inherent instability of the metalloid compounds in titanium alloys limits their use to temperature of the order of 500-600°C. However, RS has been shown to be an effective means
of introducing a relatively high volume fraction of fine titanium-metalloid compounds without the formation of the large blocky inclusions, and advantages such as modulus improvement are apparent [91Sur].

The eutectoid forming systems (e.g., Ti-Ni, Cu, Co, Fe, Cr, Si and W systems) and the intermetallic systems (e.g., Ti₃Al (α₂) and TiAl (γ)) have also been extensively studied through the RS techniques, for a detailed review see [91Sur].

Table 2.4. Effect of boron on room and elevated temperature tensile properties of RS titanium alloys (after Refs. [86Ray1, 86Ray2].

<table>
<thead>
<tr>
<th>Alloy, wt-%</th>
<th>Temp., °C</th>
<th>0.2% YS, MNm⁻²</th>
<th>UTS, MNm⁻²</th>
<th>Elong., %</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ti-6Al-4V</td>
<td>RT</td>
<td>848</td>
<td>986</td>
<td>9.0</td>
</tr>
<tr>
<td></td>
<td>371</td>
<td>910</td>
<td>1048</td>
<td>3.1</td>
</tr>
<tr>
<td></td>
<td>537</td>
<td>620</td>
<td>862</td>
<td>8.8</td>
</tr>
<tr>
<td>Ti-6Al-4V-1B</td>
<td>RT</td>
<td>1090</td>
<td>1110</td>
<td>2.5</td>
</tr>
<tr>
<td></td>
<td>371</td>
<td>910</td>
<td>1048</td>
<td>3.1</td>
</tr>
<tr>
<td></td>
<td>537</td>
<td>620</td>
<td>862</td>
<td>8.8</td>
</tr>
<tr>
<td>Ti-8Al-1Mo-1V</td>
<td>RT</td>
<td>827</td>
<td>896</td>
<td>9.0</td>
</tr>
<tr>
<td>Ti-8Al-1Mo-1V-1B</td>
<td>RT</td>
<td>1138</td>
<td>1172</td>
<td>1.8</td>
</tr>
<tr>
<td></td>
<td>371</td>
<td>855</td>
<td>1000</td>
<td>8.2</td>
</tr>
<tr>
<td></td>
<td>537</td>
<td>696</td>
<td>821</td>
<td>8.2</td>
</tr>
<tr>
<td>Ti₃Al</td>
<td>RT</td>
<td>780</td>
<td>903</td>
<td>0.5</td>
</tr>
<tr>
<td>Ti₃Al-1B</td>
<td>RT</td>
<td>...</td>
<td>435</td>
<td>1.0</td>
</tr>
<tr>
<td></td>
<td>371</td>
<td>...</td>
<td>476</td>
<td>1.0</td>
</tr>
</tbody>
</table>

2.5.6. Summary

(1) Although extensive studies on the application of RS processing techniques to titanium alloys have started only recently, considerable progress has been made in design and development of novel compositions and improved properties. These improvements are based on increased solid solubility limits, the formation of microcrystalline and amorphous phases, and the refinement of microstructural features.

(2) The solid solubility of rare earth elements in titanium can be substantially increased by RS. On subsequent aging these alloys exhibit enhanced property combinations, because of the combined effects of solid solution, precipitation, and dispersion hardening in addition to the strengthening arising from the fine grain size.

(3) Rapid solidification of titanium-metalloid systems has proved to be an effective means of introducing a relatively high volume fraction of fine intermetallic compounds. Metalloid-containing RS titanium alloys show the improved stiffness and strength. However, further work is needed to enhance the improvement by further increasing the volume fraction of intermetallic compounds and to investigate their coarsening rate.

(4) The coarsening rate of dispersoids (RE oxides and Ti-metalloid compounds) can be controlled by the proper choice of the element addition based on the Solubility-diffusivity product.
3.1. Introduction

The quantitative characterisation of microstructure of two-phase materials involves the analysis of both geometrical and topological quantities. The geometrical characterisation of two-phase microstructure is well established and has been reviewed by Underwood [72Und]. This usually involves the analysis and measurements of grain size, volume fraction and particle spacing, but the topological characterization of two-phase microstructure is inherently more difficult. Efforts in this direction can be divided into three categories: (i) approaches based on the rigid topological theory, with the first attempt made by Rhines [58Rhi] in describing the sintering process, and further developed by Steele [67Ste] and Aigeltinger [70Aig]; (ii) an approach based on digital topology, introduced recently by Cerezo and coworkers [88Cer, 89Cer, 91Cer] to characterise the interconnected atomic scale microstructure in a number of Fe-Cr alloys; (iii) approaches based on the concept of contiguity proposed first by Gurland [58Gur] to describe the degree of particle contact of one phase in a two-phase microstructure. It has subsequently been found that many physical and mechanical properties, such as electric resistivity [66Gur], fracture toughness [76Che, 82Ugg1, 90Fan], Vicker's hardness [78Lee] and yield strength [80Ugg, 81Ugg, 82Ugg2, 82Ugg3, 83Wer, 84Wer] can be closely related to the contiguity parameter.

According to the different ways in which the second phase distributes in the matrix phase, two-phase microstructures can be divided into two broad categories: oriented structures and random structures. In terms of the quantitative description of two-phase microstructures, the former is just a specific case of the latter. Thus, it is easy to describe the oriented microstructure once a method for describing the random microstructure has been developed. Hombogen [83Hor, 84Hor, 86Hor] classified random two-phase microstructures into the following three elementary types: dispersion structures, net structures and duplex structures, as is schematically illustrated in Fig 3.1. In this chapter we only deal with net (Fig 3.1(b)) and duplex (Fig 3.1(c)) structures, which are combinations of a softer $\alpha$-phase and a harder $\beta$-phase, or so called coarse two-phase structure. The term "coarse" here refers to the fact that the two phases have grain sizes of the same order. The term "phase" does not have to be applied here in a strict sense; any microstructural region with unique physical and mechanical
properties, such as tempered martensite, can be treated as a phase, irrespective of the substructure of this region.

In this chapter a series of topological parameters will be developed based on the concept of contiguity [58Gurl], which can then be used for an effective description of phase distribution in coarse two-phase structures, and to facilitate the development of the microstructure/property correlationships in two-phase materials. The effect of volume fraction and relative grain size of the constituent phases on the various topological parameters of a random two-phase microstructure with equiaxed grains will be discussed. The concept of directional contiguity will be proposed to describe the oriented microstructure. In particular it will be shown in the last section of this chapter that considerable advantages can be obtained by introducing a topological transformation of two-phase microstructure. As far as the thesis itself is concerned, this chapter offers an effective method for characterisation of two-phase microstructures, which is a prerequisite of the analysis of mechanical properties in later chapters.

3.2. The Concept of Contiguity

Gurland [58Gur] first defined the contiguity of the α-phase in an α-β phase mixture as the fraction of the total internal surface area of the α-phase shared with particles of the same phase and designated as $C_\alpha$. The mathematical expression for $C_\alpha$ is

$$C_\alpha = \frac{2S_{V}^{\alpha\alpha}}{2S_{V}^{\alpha\alpha} + S_{V}^{\alpha\beta}}$$

(3.1)

where $S_{V}^{\alpha\alpha}$ is the interface area between α particles per unit volume, $S_{V}^{\alpha\beta}$ is the interface area between α and β particles per unit volume and $2S_{V}^{\alpha\alpha} + S_{V}^{\alpha\beta}$ is the total grain boundary areas of the α-phase in an α-β phase mixture, which is designated as $S_{V}^\alpha$, i.e.,

$$S_{V} = 2S_{V}^{\alpha\alpha} + S_{V}^{\alpha\beta}$$

(3.2)

The physical meaning of contiguity is the extent of particle contact of the same phase. The contiguity of the α-phase can then vary between 0 and 1 as the distribution of this phase changes from a completely dispersed to a completely agglomerated structure. The interface area $S_{V}^{\alpha\alpha}$ and $S_{V}^{\alpha\beta}$ can be obtained from a simple intercept measurement on a random plane of polish [72Und], and hence, the contiguity of the α-phase can also be expressed as

$$C_\alpha = \frac{2N_{L}^{\alpha\alpha}}{2N_{L}^{\alpha\alpha} + N_{L}^{\alpha\beta}}$$

(3.3)

where $N_{L}^{\alpha\alpha}$ and $N_{L}^{\alpha\beta}$ are, respectively, the numbers of intercepts of α-α interfaces and α-β interfaces within a random line of unit length on a plane of polish. Eqns (3.1) and (3.3) are valid for any particle size, shape and distribution.
Fig 3.1. Schematic illustration of three element types of random microstructures; (a) dispersion structure; (b) net structure; (c) duplex structure.

The relation between the phase contiguity and the mean free path, $\lambda$, can be derived by combining the previous equations with the definition of mean free path through the $\alpha$-phase ($\lambda_\alpha = 2f_\alpha/N L^\alpha$, where $f_\alpha$ is the volume fraction of the $\alpha$-phase) and with the number of intercepted particles ($f_\alpha/L_\alpha$) [66Gur].

$$\lambda_\alpha = \frac{L_\alpha}{1-C_\alpha}$$  \hspace{1cm} (3.4)
where $L_\alpha$ is the mean intercept length of $\alpha$-particles, and
\begin{equation}
L_\alpha = \frac{2f_\alpha}{2N_\alpha + N_\beta} \tag{3.5}
\end{equation}

Eqn (3.4) is valid for any grain size, shape and distribution.

Under the assumption of random distribution of equiaxed $\alpha$ and $\beta$ particles, Cremens [63Cre] has derived the following equation:
\begin{equation}
\frac{S_\alpha^{\alpha\beta}}{S_\alpha} = \frac{S_\beta}{S_\alpha + S_\beta} \tag{3.6}
\end{equation}

where $S_\alpha$ and $S_\beta$ are the total surface area of $\alpha$ and $\beta$ particles per unit volume, respectively, and
\begin{equation}
S_\beta = 2S_\alpha^{\beta\beta} + S_\alpha \tag{3.7}
\end{equation}

where $S_\alpha^{\beta\beta}$ is the total area of interface between $\beta$ particles per unit volume. The expected contiguity of the $\alpha$-phase in a random $\alpha$-$\beta$ mixture is equal to the surface fraction of that phase.

\begin{equation}
C_\alpha = \frac{S_\alpha}{S_\alpha + S_\beta} \tag{3.8}
\end{equation}

Since the volume to surface ratio of the $\alpha$-phase is related to the mean intercept length by Tomkeieff relation [72Und],
\begin{equation}
\frac{V_\alpha}{S_\alpha} = \frac{L_\alpha}{4} \tag{3.9}
\end{equation}

the following expression is obtained
\begin{equation}
C_\alpha = \frac{f_\alpha L_\beta}{f_\alpha L_\beta + f_\beta L_\alpha} \tag{3.10}
\end{equation}

If the grain size ($d_\alpha$ and $d_\beta$) is substituted for the mean intercept length ($L_\alpha = d_\alpha$, $L_\beta = d_\beta$), one obtains:
\begin{equation}
C_\alpha = \frac{f_\alpha d_\beta}{f_\alpha d_\beta + f_\beta d_\alpha} \tag{3.11}
\end{equation}

It is easy to see that if $d_\alpha = d_\beta$ in a random mixture of equiaxed $\alpha$ and $\beta$ particles.
\begin{equation}
C_\alpha = f_\alpha \tag{3.12}
\end{equation}
In addition, we can define the grain size ratio in a random mixture of equiaxed $\alpha$ and $\beta$ particles as $R$, i.e.,

$$R = \frac{d_\beta}{d_\alpha}$$  \hfill (3.13)

Then eqn (3.11) will become

$$C_\alpha = \frac{f_\alpha R}{f_\alpha R + f_\beta}$$  \hfill (3.14)

In the same way, the contiguity of the $\beta$-phase in an $\alpha$-$\beta$ mixture can be defined as the fraction of the total internal surface area of the $\beta$-phase shared with particles of the $\beta$-phase, denoted as $C_\beta$, which can be mathematically expressed by the following equations

$$C_\beta = \frac{2S_{V_\beta}}{2S_{V_\beta} + S_{V_\alpha}}$$  \hfill (3.15)

$$C_\beta = \frac{2N_{L_\beta}}{2N_{L_\beta} + N_{L_\alpha}}$$  \hfill (3.16)

Eqns (3.15) and (3.16) are valid for any combination of grain sizes and grain shapes. In a random mixture of equiaxed $\alpha$ and $\beta$ particles, the following equations can be obtained for the $\beta$-phase

$$C_\beta = \frac{S_{V_\beta}}{S_{V_\alpha} + S_{V_\beta}}$$  \hfill (3.17)

$$C_\beta = \frac{f_\beta d_\alpha}{f_\alpha d_\beta + f_\beta d_\alpha}$$  \hfill (3.18)

$$C_\beta = \frac{f_\beta}{f_\alpha R + f_\beta}$$  \hfill (3.19)

Under the assumption of a random mixture of equiaxed $\alpha$ and $\beta$ particles, eqn (3.14) and (3.19) can be used to calculate the contiguity of the $\alpha$- and $\beta$-phases, respectively, in terms of volume fraction and grain size of each constituent phase.

The contiguity in a microstructure with any volume fraction, grain size, grain shape and phase distribution can therefore be experimentally determined by using a standard metallographic method [72Und] according to eqns (3.3) and (3.16). This is schematically illustrated in Fig 3.2, where an array of circular test lines are employed to measure the intercepts of $N_{L_\alpha}$, $N_{L_\beta}$ and $N_{L_{\alpha\beta}}$. Experimentally determined contiguity data for each constituent phase in $\alpha$-$\beta$ brasses
after Werner and Stuwe [83Wer, 84Wer] and in ferrite-martensite dual-phase steels after Uggowitzer and Stuwe [80Ugg, 81Ugg, 82Ugg2, 82Ugg3] are shown in Figs 3.3 and 3.4, respectively.

**Fig 3.2.** Schematic illustration of metallographic measurement of $N_{L}^{\alpha\alpha}$, $N_{L}^{\beta\beta}$ and $N_{L}^{\alpha\beta}$ in random two-phase materials for determination of contiguity parameters using an array of circular test lines.

**Fig 3.3.** Experimentally determined contiguity data for the $\alpha$- and $\beta$-phases in $\alpha$-$\beta$ brasses after Werner and Stuwe [83Wer and 84Wer].
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1.0----------------------------------

0.8

Fe-C-Mn Dual-Phase Steels

[Graph showing contiguity data for different types of steels]

Fig 3.4. The experimentally determined contiguity data for ferrite-martensite dual-phase steels after Uggowitzer and Stuwe [80Ugg, 81Ugg, 82Ugg2, 82Ugg3]. Type A and Type B here refer to different heat treatments: Type A heat treatment results in a microstructure with isolated martensite embedded in a ferrite matrix; Type B heat treatment produces a microstructure with isolated ferrite embedded in a martensite network.

Fig 3.5. The calculated contiguity of the $\alpha$-phase in a random mixture of equiaxed $\alpha$ and $\beta$ grains as a function of volume fraction and grain size ratio, $R$, as indicated by the number attached to each line.

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The effects of volume fraction and grain size ratio $R$ on the contiguity of $\alpha$ and $\beta$ phases in a random mixture of equiaxed $\alpha$ and $\beta$ particles are illustrated in Fig 3.5 and Fig 3.6, respectively. Fig 3.5 shows that the contiguity of the $\alpha$-phase increases with increasing volume fraction of the $\alpha$-phase at a constant grain size ratio $R$, and that at constant volume fraction of the $\alpha$-phase the contiguity of the $\alpha$-phase increases with increasing grain size ratio $R$. Fig 3.6 reveals that the contiguity of the $\beta$-phase decreases with increasing volume fraction of the $\alpha$-phase at constant grain size ratio $R$, and that at a constant volume fraction of the $\alpha$-phase the contiguity of the $\beta$-phase decreases with increasing grain size ratio.

The conclusion from Figs 3.3–6 is that the contiguity of a phase in a two-phase microstructure can be controlled by a carefully designed heat treatment (Fig 3.4) or by proper choice of the grain size ratio $R$ (Figs 3.5–6).

3.3. The Separation of Phases in an $\alpha$-$\beta$ Phase Mixture

Analogous to the concept of contiguity described in the previous section, the separation of a phase in an $\alpha$-$\beta$ mixture is defined here as the fraction of the total surface areas of this phase shared with the particles of the other phase and designated as $S_\alpha$ and $S_\beta$ for the $\alpha$- and $\beta$-phases, respectively. $S_\alpha$ and $S_\beta$ can be expressed mathematically by the following equations.
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\[
S_{\alpha} = \frac{S_{\alpha}^{\alpha\beta}}{2S_{V}^{\alpha\alpha} + S_{V}^{\alpha\beta}} = \frac{N_{L}^{\alpha\beta}}{2N_{L}^{\alpha\alpha} + N_{L}^{\alpha\beta}} \tag{3.20}
\]

\[
S_{\beta} = \frac{S_{\beta}^{\alpha\beta}}{2S_{V}^{\beta\beta} + S_{V}^{\alpha\beta}} = \frac{N_{L}^{\alpha\beta}}{2N_{L}^{\beta\beta} + N_{L}^{\alpha\beta}} \tag{3.21}
\]

From the above analysis it is obvious that the sum of contiguity and separation of a phase is equal to unity, i.e.,
\[
C_{\alpha} + S_{\alpha} = 1 \tag{3.22}
\]
\[
C_{\beta} + S_{\beta} = 1 \tag{3.23}
\]

In a random mixture of the \(\alpha\)- and \(\beta\)-phases, the following equations can be obtained according to the previous analysis

\[
S_{\alpha} = \frac{S_{\beta}}{S_{\alpha} + S_{\beta}} \tag{3.24}
\]

\[
S_{\alpha} = \frac{f_{\beta}d_{\alpha}}{f_{\beta}d_{\alpha} + f_{\alpha}d_{\beta}} = \frac{f_{\beta}}{f_{\beta} + f_{\alpha}R} \tag{3.25}
\]

\[
S_{\beta} = \frac{S_{\alpha}}{S_{\alpha} + S_{\beta}} \tag{3.26}
\]

\[
S_{\beta} = \frac{f_{\alpha}d_{\beta}}{f_{\beta}d_{\alpha} + f_{\alpha}d_{\beta}} = \frac{f_{\alpha}R}{f_{\beta} + f_{\alpha}R} \tag{3.27}
\]

From the above equations, it is easy to obtain the following relationships between contiguity and separation of the \(\alpha\)- and \(\beta\)-phases in a random phase mixture with equiaxed grains,
\[
C_{\alpha} = S_{\beta} \tag{3.28}
\]
\[
C_{\beta} = S_{\alpha} \tag{3.29}
\]

The calculated separation of the \(\alpha\)- and \(\beta\)-phases in a random phase mixture with equiaxed grains are illustrated in Fig 3.7 and Fig 3.8, respectively, as functions of volume fraction and grain size ratio \(R\). At a constant grain size ratio, the separation of the \(\alpha\)-phase decreases with increasing volume fraction of the \(\alpha\)-phase, and the separation of the \(\alpha\)-phase decreases with increasing grain size ratio at constant volume fraction. Whereas the influence of volume fraction and grain size ratio on the separation of the \(\beta\)-phase is just opposite to that on the separation of the \(\alpha\)-phase.

3.4. Continuous Volume and Separated Volume

In 1978 Lee and Gurland [78Lee] proposed a new topological parameter, the continuous volume, in order to account for the extent of long range continuity of a phase in two-phase mixtures. This is defined as the volume fraction of interconnected particles of one phase, and denoted as \(f_{\alpha e}\) and \(f_{\beta e}\) for the \(\alpha\)- and \(\beta\)-phases, respectively. Their derivations for continuous volume can be summarised as follows:
Fig 3.7. The calculated separation of the $\alpha$-phase in a random mixture of equiaxed $\alpha$ and $\beta$ grains as a function of volume fraction and grain size ratio, $R$, as indicated by the number attached to each line.

Fig 3.8. The calculated separation of the $\beta$-phase in a random mixture of equiaxed $\alpha$ and $\beta$ grains as a function of volume fraction and grain size ratio, $R$, as indicated by the number attached to each line.
Fig 3.9. A representative two-phase microstructure used for the derivation of the continuous volume of the α-phase.

Fig 3.9 shows a representative microstructure for an α-β alloy. The continuous volume of the α-phase is calculated in terms of contiguity and volume fraction of the α-phase. Now, let us consider the volume element $i$ in Fig 3.9. The volume of this element $i$ is given by

$$v_i^\alpha = a_i^{\alpha \alpha} l_i \cos \phi_i,$$

where $a_i^{\alpha \alpha}$ is the interface area between two α grains, $l_i$ is the length of element $i$ and $\phi_i$ is the angle between the normal of area $a_i^{\alpha \alpha}$ and the axis of element $i$. The continuous volume is the sum of all elements

$$f_{\alpha c} = \frac{1}{V} \sum a_i^{\alpha \alpha} l_i \cos \phi_i = \frac{1}{2} \bar{l} \cos \phi \sum \frac{2a_i^{\alpha \alpha}}{V}.$$

(3.31)

where $V$ is the total volume, $\bar{l}$ are the average value of $l_i$ and $\bar{\phi}$ can be substituted by the linear mean intercept length, $\bar{\alpha}$, and $\cos \phi$ is the average value of $\cos \phi_i$. Evaluation of $\cos \phi$ can be visualized by means of a hemisphere [72Und], the result is

$$\cos \phi = \int_0^{\pi/2} P(\phi) \cos \phi d\phi = \int_0^{\pi/2} \sin \phi \cos \phi d\phi = \frac{1}{2}.$$

(3.32)

where $P(\phi)d\phi$ is the probability of the normal of an element lie between $\phi+d\phi$, and has a value of $\sin \phi d\phi$ [72Und].

By using the Tomkeieff equation (eqn (3.9)), one gets

$$f_{\alpha c} = \frac{1}{4} \bar{l} (2S_v^{\alpha \alpha}) = \frac{1}{4} \frac{4V_v^\alpha}{S_v^{\alpha \alpha}} (2S_v^{\alpha \alpha}) = \frac{2S_v^{\alpha \alpha}}{S_v^{\alpha}} = f_{\alpha} \frac{2S_v^{\alpha \alpha}}{S_v^{\alpha}} = f_{\alpha} C_{\alpha}.$$

Therefore,

$$f_{\alpha c} = f_{\alpha} C_{\alpha}.$$

(3.33)
In the same way, the mathematical expression for the continuous volume of the \( \beta \)-phase can be derived
\[
f_{\beta c} = f_\beta C_\beta
\] (3.34)

In a random phase mixture with equiaxed grains, it is easy to derive the following equations for the mathematical expressions of continuous volume of the \( \alpha \)- and \( \beta \)-phases
\[
f_{\alpha c} = \frac{f_\alpha d_\alpha}{f_\beta d_\alpha + f_\alpha d_\beta} = \frac{f_\alpha}{f_\beta + f_\alpha}
\] (3.35)
\[
f_{\beta c} = \frac{f_\beta d_\beta}{f_\beta d_\alpha + f_\alpha d_\beta} = \frac{f_\beta}{f_\beta + f_\alpha}
\] (3.36)

It is obvious that \( f_{\alpha c} + f_{\beta c} \neq 1 \).

The volume of a phase can be divided into two distinct parts, one is the continuous volume, which contains interconnected particles of this phase, as has been discussed previously, the other part is the separated volume which contains isolated particles of this phase. Therefore, the separated volume of a phase can be defined as the volume fraction of isolated particles of this phase and designated as \( f_{\alpha s} \) and \( f_{\beta s} \) for the \( \alpha \)- and \( \beta \)-phases, respectively. In a similar way for derivation of continuous volume, mathematical expressions for separated volume of the \( \alpha \)- and \( \beta \)-phases can be obtained as below
\[
f_{\alpha s} = f_\alpha S_\alpha
\] (3.37)
\[
f_{\beta s} = f_\beta S_\beta
\] (3.38)

In a random mixture of equiaxed \( \alpha \)- and \( \beta \)-grains, one has the following equations
\[
f_{\alpha s} = \frac{f_\alpha f_\beta d_\alpha}{f_\beta d_\alpha + f_\alpha d_\beta} = \frac{f_\alpha f_\beta}{f_\beta + f_\alpha}
\] (3.39)
\[
f_{\beta s} = \frac{f_\alpha f_\beta d_\beta}{f_\beta d_\alpha + f_\alpha d_\beta} = \frac{f_\alpha f_\beta}{f_\beta + f_\alpha}
\] (3.40)

It is obvious that the following relations between continuous volume and separated volume can be obtained according to the previous equations
\[
f_{\alpha s} + f_{\alpha c} = f_\alpha
\] (3.41)
\[
f_{\beta s} + f_{\beta c} = f_\beta
\] (3.42)
\[
f_{\alpha s} + f_{\alpha c} + f_{\beta s} + f_{\beta c} = f_\alpha + f_\beta = 1
\] (3.43)

The calculated continuous volume and separated volume of the \( \alpha \)-phase in a random \( \alpha \)-\( \beta \) phase mixture as functions of volume fraction and grain size ratio are presented in Figs 3.10 and 3.11, respectively. Fig 3.10 shows that \( f_{\alpha c} \) increases with increasing volume fraction of the \( \alpha \)-phase when the grain size ratio is kept constant and that \( f_{\alpha c} \) increases with increasing grain size ratio at a given volume fraction of the \( \alpha \)-phase. Fig 3.11 reveals that the influence of volume fraction and grain size ratio on the separated volume has an opposite trend to that on the continuous volume.
Fig 10. Calculated continuous volume of the $\alpha$-phase in a random mixture of equiaxed $\alpha$ and $\beta$ grains as a function of volume fraction and grain size ratio, $R$, as indicated by the number attached to each line.

Fig 11. Calculated separated volume of the $\alpha$-phase in a random mixture of equiaxed $\alpha$ and $\beta$ grains as a function of volume fraction and grain size ratio, $R$, as indicated by the number attached to each line.
3.5. Degree of Continuity and Degree of Separation

The degree of continuity of an \( \alpha-\beta \) phase mixture is defined here as the sum of the continuous volume of each constituent phase, and designated as \( F_c \), i.e.,
\[
F_c = f_\alpha C_\alpha + f_\beta C_\beta
\]  
(3.44)

The degree of separation of an \( \alpha-\beta \) phase mixture is defined here as the sum of the separated volume of each constituent phase, and designated as \( F_s \).
\[
F_s = f_\alpha S_\alpha + f_\beta S_\beta
\]  
(3.45)

According to eqns (3.44) and (3.45), the following expressions can be derived for random mixture of equiaxed \( \alpha \) - and \( \beta \)-grains,
\[
F_c = \frac{f_\alpha d_\beta + f_\beta d_\alpha}{f_\beta d_\alpha + f_\alpha d_\beta} = \frac{f_\alpha d_\beta + f_\beta}{f_\beta d_\alpha + f_\alpha d_\beta}
\]  
(3.46)

\[
F_s = \frac{f_\alpha \beta (d_\alpha + d_\beta)}{f_\beta d_\alpha + f_\alpha d_\beta} = \frac{f_\alpha \beta (1 + R)}{f_\beta + f_\alpha R}
\]  
(3.47)

It is obvious that the sum of \( F_c \) and \( F_s \) should be unity, i.e.,
\[
F_c + F_s = 1
\]  
(3.48)

The physical meaning of degree of separation is the probability of formation of long-range \( \alpha-\beta \) chains in an given \( \alpha-\beta \) phase mixture.
The calculated degree of continuity and degree of separation in a random mixture of equiaxed \( \alpha \)- and \( \beta \)-grains are presented in Figs 3.12 and 3.13, respectively, as functions of volume fraction and grain size ratio. The minimum and the maximum points are controlled by the values of grain size ratio, \( R \).

### 3.6. Directional Contiguity

Only the random two-phase microstructure has been described so far. However, oriented structures occur frequently in materials, such as lamellar eutectic alloys and aligned fibre reinforced metal matrix composites. This group of materials often exhibits anisotropy in mechanical properties, and therefore it is desirable to predict mechanical properties along a particular direction. In order to do so, a knowledge of microstructural parameters along particular directions is required. In this section, the concept of directional contiguity will be introduced.

For experimental determination of the topological parameters introduced in the foregoing sections, a random array of test lines is usually applied to a plane of polish (see Fig 3.2). The result of this kind of measurements offers only the average values of topological parameters along all possible directions, and no information can be obtained for a particular direction.
Therefore, the topological parameters measured by a random array of test lines on a plane of polish will be called the random contiguity, which is applicable for a random distribution of constituent phases. If an array of parallel test lines along a particular direction is applied on a plane of polish with an oriented structure, as schematically illustrated in Fig 3.14, the results of such measurements will give the topological properties along the direction parallel to the test lines. The results of this kind of measurements will be called the directional contiguity. The only difference between random contiguity and directional contiguity is therefore the arrangement of test lines for measurement. Thus, all the equations derived in the previous sections can be applied to both random contiguity and directional contiguity.

![Fig 3.14. Schematic illustration of the measurement of the directional contiguity of an oriented two-phase structure using an array of parallel test lines.](image)

### 3.7. Topological Transformation of Two-Phase Microstructure

A two-phase microstructure with any volume fraction, grain size, grain shape and phase distribution, as schematically illustrated in Fig 3.15(a) (denoted as Microstructure A), can be divided into three microstructural elements: element I (EI), the continuous α-phase, element II (EII), the continuous β-phase and element III (EIII), consisting of the separated α- and β-grains. Thus, EI has a volume fraction of \( f_{\alpha} \), a grain size of \( d_{\alpha} \), and contains only α-α grain boundaries; EII has a volume fraction of \( f_{\beta} \), a grain size of \( d_{\beta} \) and contains only β-β grain boundaries; the volume fraction of EIII is defined by the degree of separation, \( F_s \), and the grain size of EIII is defined by the volume-fraction-weighted-average grain size, \( \bar{d}_{\alpha\beta} \),

\[
\bar{d}_{\alpha\beta} = d_{\alpha} f_{\alpha\text{III}} + d_{\beta} f_{\beta\text{III}}
\]  

where \( f_{\alpha\text{III}} \) and \( f_{\beta\text{III}} \) are the volume fractions of the α- and β-phases in EIII. \( f_{\alpha\text{III}} \) and \( f_{\beta\text{III}} \) can be expressed mathematically as
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\[
f_{\alpha III} = \frac{f_{\alpha} - f_{\alpha c}}{F_s} = \frac{f_{\alpha s}}{F_s} \tag{3.50}
\]

\[
f_{\beta III} = \frac{f_{\beta} - f_{\beta c}}{F_s} = \frac{f_{\beta s}}{F_s} \tag{3.51}
\]

It is obvious that there is no contact between the grains of the same phase in EIII, and consequently there are no grain boundaries but only phase boundaries. These three microstructural elements with appropriate proportions can be now assembled to form a new microstructure, denoted as Microstructure B, which is schematically illustrated in Fig 3.15(b).

![Diagram of Microstructures A and B](image)

**Fig 3.15.** A schematic presentation of Microstructure A and B. It is important to note that this graph is just a schematic illustration of the topological transformation, it does not represent any quantitative information such as volume fraction, grain size and grain shape.

Topological properties of the microstructural features are those geometric properties which are invariant under continuous deformations or shape changes [72Ste]. If the concepts of random contiguity and directional contiguity are now applied to Microstructures A and B along the aligned direction, respectively, it will be found that both geometrical and topological parameters are identical in both microstructures. Therefore, the transformation from Microstructure A to Microstructure B can be regarded as a topological transformation, although it is difficult to do a rigorous analysis of this transformation based on the topological theory, since a complete topological classification for 3-dimensional Euler-Poincare equation is not available [72Ste]. However, this topological transformation does provide a quantitative description of the topological properties of two-phase microstructure and does offer an effective means for
correlation of mechanical properties to microstructural parameters, as will be shown in the latter chapters.

Perhaps the most interesting and most important aspect of this topological transformation is that Microstructures A and B are mechanically equivalent along the aligned direction. This means that the mechanical properties of Microstructure A (which is an isotropic medium) can be represented by those of Microstructure B (which is an anisotropic medium) in the aligned direction. This mechanical equivalence can be understood in the following ways. Firstly, they are mechanically equivalent along the aligned direction since the microstructural parameters along this direction are the same in both microstructures, and it is the microstructure which determines the mechanical properties. Secondly, this equivalence can be proved rigorously for certain limiting cases of microstructures. Thirdly, this equivalence can be tested in the more general cases of two-phase microstructures by comparing the mechanical properties of microstructure B along the aligned direction calculated by theory and those of Microstructure A obtained by experiments. Now let us consider the mechanical equivalence of microstructures A and B for some specific microstructural conditions:

(i) $f_\beta=0$. In this case, $f_{\beta c}=0$, $f_{\alpha c}=1$ and $F_S=0$, both microstructures A and B become single $\alpha$-phase alloys, and the mechanical equivalence of A and B is obvious;

(ii) $f_\beta=1$. In this case, $f_{\alpha c}=0$, $f_{\beta c}=1$ and $F_S=0$, both A and B are single $\beta$-phase alloys, and the mechanical equivalence of A and B is also obvious;

(iii) $f_\beta \neq 0$ and $f_\beta \neq 1$, but both $\alpha$- and $\beta$-phases are completely continuous. In this case, $f_{\alpha c}=f_{\alpha c}$, $f_{\beta c}=f_\beta$ and $F_S=0$, the 3-E body reduces to 2-E body, and the mechanical equivalence of A and B can be proved theoretically in Chapter 7;

(iv) $f_\beta \neq 0$ and $f_\beta \neq 1$, but both $\alpha$- and $\beta$-phases are completely separated. In this case, $f_{\alpha c}=0$, $f_{\beta c}=0$ and $F_S=1$, A and B are the same microstructure, of course they are mechanically equivalent;

(v) $f_\beta \neq 0$ and $f_\beta \neq 1$, but both $\alpha$- and $\beta$-phases are partially continuous, as schematically illustrated in Fig 15(a). In this case, $f_{\alpha c} \neq 0$, $f_{\beta c} \neq 0$ and $F_S \neq 0$. The mechanical equivalence of A and B in this case has to be a hypothesis. However, this hypothesis can be justified by comparing the theoretically calculated mechanical properties of microstructure B along the aligned direction with those of microstructure A obtained by experiment in Chapters 4, 6 and 7.

In addition, it is important to emphasise that the mechanical equivalence of microstructure A and B only refers to the aligned direction of B and that the topological transformation from A to B always refers to the direction in which the topological parameters of microstructure A are determined. If microstructure A is aligned and the mechanical property of A to be determined is
along the aligned direction, the measurement of topological parameters of A must be made along the aligned direction, and the topological transformation must also be made along the same direction.

Note that Fig 3.15 is just a schematic illustration of the topological transformation, and does not represent any quantitative information such as volume fraction, grain size and grain shape. In fact, the volume fraction of \( EII \) (i.e., the continuous volume of the \( \beta \)-phase \( f_{\beta_c} \)) in particulate \( \alpha \)-rich alloys is usually very small, for example, an \( \alpha-\beta \) brass with 50% \( \beta \)-phase has a continuous volume fraction of about \( f_{\beta_c} = 0.0625 \). It is also important to make a clear distinction between the different volume fractions, i.e., \( f_\alpha \), \( f_{\alpha \text{III}} \) and \( f_{\alpha \text{C}} \) are volume fractions of the \( \alpha \)-phase in the composite alloy, of the \( \alpha \)-phase in element III and of the continuous \( \alpha \)-phase in the 3-E body, respectively.

Such a topological transformation from Microstructure A to Microstructure B allows a complicated two-phase microstructure to be replaced by a much simpler but equivalent microstructure.

### 3.8. Concluding Remarks

1. Based on the concepts of contiguity and continuous volume, a series of topological parameters have been defined and the mathematical expressions have been derived for both experimental determinations and the theoretical calculations.

2. A combination of these topological parameters can offer a full description of the phase distribution in a two-phase structure with any volume fraction, grain size, grain shape and phase distribution.

3. All those topological parameters can be measured experimentally by using standard metallographic methods, and they can also be calculated theoretically from the grain size and volume fraction under the assumption of equiaxed grain and random phase distribution.

4. The concept of directional contiguity has been proposed for analysis of oriented microstructures along a particular direction.

5. A topological transformation of two-phase microstructure has been proposed, which allows a microstructure with any volume fraction, grain size, grain shape and phase distribution to be transformed into a three element body (3-E body) with three microstructural elements being aligned parallelly along a particular direction of interest.

6. The mechanical properties of the topologically transformed three-element body along the aligned direction can be proved to be equivalent to those of the original microstructure.
CHAPTER 4

Extension of the Hall-Petch Relation to Two-Ductile-Phase Alloys

4.1. Introduction

The objective of this chapter is to extend the Hall-Petch relation developed for single-phase alloys to two-ductile-phase alloys using the method developed in previous chapter for microstructural characterisation of two-ductile-phase alloys. It will be demonstrated that the extended Hall-Petch relation can separate the contributions to the overall efficiency of grain and phase boundaries as obstacles to dislocation motion from different kinds of boundaries. The extended Hall-Petch relation will be applied to α-β brasses, α-β Ti-Mn alloys and α-γ Fe-Cr-Ni stainless steels.

4.2. The Yield Strength of Two-Ductile-Phase Alloys

If a uniaxial tensile stress is applied along the aligned direction, the following equation can be obtained from the strain compatibility requirement [63For]

\[ \varepsilon_I = \varepsilon_T = \varepsilon_T = \varepsilon_T \]

(4.1)

where \( \varepsilon_I, \varepsilon_T \) and \( \varepsilon_T \) are the total strain in elements I, II, III and the whole composite (the 3-E body), respectively.

According to the virtual work principle, the following equation is obtained at the yielding point of the whole 3-E body

\[ \sigma^c = \sigma^c + \sigma^c + \sigma^c \]

(4.2)

where \( \sigma^c, \delta e^c_T \) and \( \delta e^c_T \) are the yielding stress and the total strain increment in the 3-E body at the yielding point of the 3-E body, \( \sigma^c, \sigma^c \) and \( \sigma^c \) are the in situ stresses at the yielding point of the 3-E body in EI, EIi and EIII, respectively. At this point, at least EI (the softest element) has yielded, and there will be a plastic strain incompatibility between EI and the other two elements. This plastic strain incompatibility can cause strong positive internal stresses in EIII and EI, which will aid the plastic deformation in them (see Chapter 6). In the case of the two-ductile-phase alloys concerned here, the yield strengths of the constituent phases are not very much different from each other, thus, the applied stress plus the internal stresses in EIII and EI are sufficient to introduce plastic deformation in EIII and EI [76Tom, also see Chapter 6]. Consequently, there is not much
difference between the in situ stress in one element and the yield stress of this element at the yielding point of the whole composite alloy. Therefore, to a good approximation, the in situ stresses $\sigma^\alpha$, $\sigma^\beta$ and $\sigma^{\alpha\beta}$ in Eqn (4.2) can be replaced by the corresponding yield stresses $\sigma_y^\alpha$, $\sigma_y^\beta$ and $\sigma_y^{\alpha\beta}$, and then we have the following equation

$$
\sigma_y F_s (4.3)
$$

Combining Eqns (4.1) and (4.3), one gets

$$
\sigma^c = \sigma^\alpha f_{ac} + \sigma^\beta f_{bc} + \sigma^{\alpha\beta} F_s (4.4)
$$

Eqn (4.4) is the general expression for the yield strength of two-ductile-phase alloys.

### 4.3. Extension of the Hall-Petch Relation to Two-Ductile-Phase Alloys

The Hall-Petch relation (eqn (4.5)) for single-phase alloys can now be applied to each microstructural element in turn, we get equations (4.6-8):

$$
\sigma_y = \sigma_y^\alpha + k_y d_{\alpha}^{-1/2} (4.5)
$$

$$
\sigma_y^\alpha = \sigma_y^\alpha + k_y d_{\alpha}^{-1/2} (4.6)
$$

$$
\sigma_y^\beta = \sigma_y^\beta + k_y d_{\beta}^{-1/2} (4.7)
$$

$$
\sigma_y^{\alpha\beta} = \sigma_y^{\alpha\beta} - k_y d_{\alpha\beta}^{-1/2} (4.8)
$$

where $\sigma_y^\alpha$, $\sigma_y^\beta$ and $\sigma_y^{\alpha\beta}$ are the friction stresses in EI, EII and EIII respectively, and $k_y^\alpha$, $k_y^\beta$ and $k_y^{\alpha\beta}$ are the Hall-Petch coefficients for EI, EII and EIII, respectively. Inserting eqns (4.6)-(4.8) into eqn (4.4), one gets

$$
\sigma^c = (\sigma_y^\alpha + k_y d_{\alpha}^{-1/2}) f_{ac} + (\sigma_y^\beta + k_y d_{\beta}^{-1/2}) f_{bc} + (\sigma_y^{\alpha\beta} + k_y d_{\alpha\beta}^{-1/2}) F_s (4.9)
$$

The yield strength of the 3-E body can also be expressed in terms of the volume-fraction-weighted-average grain size $\bar{d}$ ($\bar{d} = d_{\alpha} f_{\alpha} + d_{\beta} f_{\beta}$) as

$$
\sigma^c = \sigma_y^c + k_y \bar{d}^{-1/2} (4.10)
$$

Eqns (4.9) and (4.10) are the general equations which express the yield strength of an $\alpha$-$\beta$ phase mixture in terms of the yield strength and microstructural parameters of each microstructural element. All the required microstructural parameters can be experimentally determined by using standard metallographic methods. The parameters $\sigma_y^\alpha$, $k_y^\alpha$, $\sigma_y^\beta$, $k_y^\beta$, $\sigma_y^{\alpha\beta}$ and $k_y^{\alpha\beta}$ can be obtained from the single $\alpha$- and $\beta$-phase alloys and the $\alpha$-$\beta$ phase mixture, respectively. However, $\sigma_y^{\alpha\beta}$ and $k_y^{\alpha\beta}$ can not be obtained experimentally. In order to determine $\sigma_y^{\alpha\beta}$ and $k_y^{\alpha\beta}$, the following two cases are considered.

First, let $d_{\alpha} \rightarrow \infty$ and $d_{\beta} \rightarrow \infty$, and consequently $\bar{d}_{\alpha\beta} \rightarrow \infty$ and $\bar{d} \rightarrow \infty$. In this case, one obtains
Chapter 4: Extension of the Hall-Petch Relation to Two-Ductile-Phase Alloys

\[
\left[ \sigma_y \right]_{d \rightarrow \infty} = \left[ \sigma_y^c + k_y d^{-1/2} \right]_{d \rightarrow \infty} = \sigma_y^c
\]  (4.11)

and

\[
\left[ \left( \sigma_y^\alpha + k_y d^{-1/2} \right) f_{\alpha \alpha} + \left( \sigma_y^\beta + k_y d^{-1/2} \right) f_{\beta \beta} + \left( \sigma_y^{\alpha \beta} + k_y d^{-1/2} \right) F_s \right]_{d \rightarrow \infty} = \sigma_y^\alpha f_{\alpha \alpha} + \sigma_y^\beta f_{\beta \beta} + \sigma_y^{\alpha \beta} F_s
\]  (4.12)

Thus

\[
\sigma_y^c = \sigma_y^\alpha f_{\alpha \alpha} + \sigma_y^\beta f_{\beta \beta} + \sigma_y^{\alpha \beta} F_s
\]  (4.13)

or

\[
\sigma_y^{\alpha \beta} = (\sigma_y^c - \sigma_y^\alpha f_{\alpha \alpha} - \sigma_y^\beta f_{\beta \beta}) / F_s
\]  (4.14)

In eqn (4.14), all the parameters on the right-hand side can be experimentally determined, therefore, \( \sigma_y^{\alpha \beta} \) can be calculated.

Now, let \( d_{\alpha} = d_{\beta} = D \) (D is a constant grain diameter) and consequently \( d_{\alpha \beta} = D = D \). In this case, one has

\[
\sigma_y^c + k_y D^{-1/2} = \left( \sigma_y^\alpha + k_y D^{-1/2} \right) f_{\alpha \alpha} + \left( \sigma_y^\beta + k_y D^{-1/2} \right) f_{\beta \beta} + \left( \sigma_y^{\alpha \beta} + k_y D^{-1/2} \right) F_s
\]  (4.15)

Subtracting eqn (4.13) from eqn (4.15), one gets

\[
k_y^c = k_y^\alpha f_{\alpha \alpha} + k_y^\beta f_{\beta \beta} + k_y^{\alpha \beta} F_s
\]  (4.16)

or

\[
k_y^{\alpha \beta} = (k_y^c - k_y^\alpha f_{\alpha \alpha} - k_y^\beta f_{\beta \beta}) / F_s
\]  (4.17)

Eqns (16) and (17) relate the overall efficiency of a two-phase microstructure, \( k_y^c \), to the efficiencies of the different boundaries. Thus, \( k_y^{\alpha \beta} \) can be evaluated from the experimentally determined \( k_y^\alpha \), \( k_y^\beta \) and \( k_y^c \), along with the microstructural parameters.

4.4. Evaluation of \( \sigma_y^{\alpha \beta} \) and \( k_y^{\alpha \beta} \) in Two-Ductile-Phase Alloys

4.4.1. \( \alpha-\beta \) Brasses

Werner and Stuwe [84Wer] have studied a number of \( \alpha-\beta \) brasses with different grain sizes and phase volume fractions. The yield strength data from their work is plotted against the reciprocal square root of the volume-fraction-weighted-average grain diameter \( \bar{d}^{-1/2} \) for different volume fractions of the \( \beta \)-phase in Fig 4.1. This shows that the Hall-Petch relation is applicable to the two-ductile-phase alloys as long as the volume-fraction-weighted-average grain size is used. The \( k_y^c \) and \( \sigma_y^c \) values obtained for the \( \alpha-\beta \) brasses with different volume fractions of the \( \beta \)-phase are plotted in Figs 4.2 and 4.3, and are also summarized in Table 4.1.
Table 4.1. Summary of the obtained Hall-Petch constants and of the topological parameters averaged from the data of Werner and Stowe [84Wer] for α-β brasses. \( \sigma_y \) in MPa and \( k_y \) in MPa/\( \sqrt{\text{mm}} \).

<table>
<thead>
<tr>
<th>( f_\beta )</th>
<th>( f_{\alpha C} )</th>
<th>( f_{\beta C} )</th>
<th>( F_s )</th>
<th>( \sigma_y^c )</th>
<th>( k_y^c )</th>
<th>( \sigma_y^{\alpha \beta} )</th>
<th>( k_y^{\alpha \beta} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>1.00</td>
<td>0.00</td>
<td>0.00</td>
<td>35</td>
<td>11.9</td>
<td>35.0</td>
<td></td>
</tr>
<tr>
<td>0.18</td>
<td>0.56</td>
<td>0.00</td>
<td>0.44</td>
<td>38</td>
<td>13.0</td>
<td>41.8</td>
<td>14.2</td>
</tr>
<tr>
<td>0.25</td>
<td>0.45</td>
<td>0.00</td>
<td>0.55</td>
<td>40.5</td>
<td>13.4</td>
<td>45.1</td>
<td>14.7</td>
</tr>
<tr>
<td>0.40</td>
<td>0.19</td>
<td>0.03</td>
<td>0.78</td>
<td>48</td>
<td>13.8</td>
<td>50.2</td>
<td>14.5</td>
</tr>
<tr>
<td>0.58</td>
<td>0.05</td>
<td>0.12</td>
<td>0.83</td>
<td>58</td>
<td>14.5</td>
<td>57.1</td>
<td>15.8</td>
</tr>
<tr>
<td>0.90</td>
<td>0.00</td>
<td>0.67</td>
<td>0.33</td>
<td>80</td>
<td>12.0</td>
<td>90.2</td>
<td>13.3</td>
</tr>
<tr>
<td>1.00</td>
<td>0.00</td>
<td>1.00</td>
<td>0.00</td>
<td>75</td>
<td>11.4</td>
<td>75.0</td>
<td></td>
</tr>
</tbody>
</table>

Fig 4.1. The yield strength of α-β brasses plotted as a function of the reciprocal square root of the volume-fraction-weighted-average grain size. The experimental data is from Werner and Stuwe [84Wer].

From the obtained \( \sigma_y^c, k_y^c, \sigma_y^{\alpha}, k_y^{\alpha}, \sigma_y^{\beta} \), and \( k_y^{\beta} \) values together with the topological parameters averaged from the experimental results [84Wer], \( \sigma_y^{\alpha \beta} \) and \( k_y^{\alpha \beta} \) parameters can then be calculated from eqns (4.14) and (4.17), respectively. The \( \sigma_y^{\alpha \beta} \) values obtained in this way
are plotted against the volume fraction of the $\beta$-phase in Fig 4.4. The average $k_y^{\alpha \beta}$ value is 14.5 MPa/\%mm, which is slightly higher than the $k_y^\alpha = 11.9$ MPa/\%mm and $k_y^\beta = 11.4$ MPa/\%mm values.

Fig 4.2. The overall friction stresses of $\alpha$-\(\beta\) brasses as a function of volume fraction of the $\beta$-phase.

Fig 4.3. The overall Hall-Petch coefficients of $\alpha$-\(\beta\) brasses as a function of volume fraction of the $\beta$-phase.
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Fig 4.4. The calculated $\sigma^\alpha_{y1}$ for $\alpha$-$\beta$ brasses as a function of volume fraction of the $\beta$-phase.

Table 4.2. Summary of the obtained Hall-Petch constants and the topological parameters evaluated from the data of Uggowitzer et al [81Ugg, 82Ugg2] for $\alpha$-$\gamma$ Fe-Cr-Ni stainless steels. $\sigma_y$ in MPa and $k_y$ in MPa\(\sqrt{\text{mm}}\).

<table>
<thead>
<tr>
<th>$f_\alpha$</th>
<th>$f_{\gamma_c}$</th>
<th>$f_{\alpha_c}$</th>
<th>$F_\beta$</th>
<th>$\sigma^\alpha_{y1}$</th>
<th>$k_y$</th>
<th>$\sigma^\alpha_{y1}$</th>
<th>$k_y^{\alpha\gamma}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>1.00</td>
<td>0.00</td>
<td>0.00</td>
<td>48.4</td>
<td>14.5</td>
<td>48.8</td>
<td>11.4</td>
</tr>
<tr>
<td>0.32</td>
<td>0.33</td>
<td>0.00</td>
<td>0.67</td>
<td>96.8</td>
<td>12.4</td>
<td>120</td>
<td>11.3</td>
</tr>
<tr>
<td>0.51</td>
<td>0.14</td>
<td>0.05</td>
<td>0.81</td>
<td>258</td>
<td>11.6</td>
<td>283</td>
<td>11.3</td>
</tr>
<tr>
<td>0.67</td>
<td>0.05</td>
<td>0.19</td>
<td>0.80</td>
<td>339</td>
<td>10.5</td>
<td>313</td>
<td>10.1</td>
</tr>
<tr>
<td>1.00</td>
<td>0.00</td>
<td>1.00</td>
<td>0.00</td>
<td>444</td>
<td>8.9</td>
<td>444</td>
<td></td>
</tr>
</tbody>
</table>

4.4.2. $\alpha$-$\gamma$ Fe-Cr-Ni Stainless Steels

Tomota et al [76Tom] measured the yield strength of quenched $\alpha$-$\gamma$ Fe-Cr-Ni stainless steels with varying grain sizes and volume fractions. Their results are plotted against the reciprocal square root of the volume-fraction-weighted-average grain size in Fig 4.5. The overall friction stresses and overall Hall-Petch coefficients are shown in Figs 4.6 and 4.7, respectively, as functions of the volume fraction of the $\alpha$-phase. Because of the lack of contiguity data for $\alpha$-$\gamma$ Fe-Cr-Ni stainless steels, the contiguity data for ferrite-martensite dual-phase steels with Type A heat treatment reported in the literature [81Ugg, 82Ugg2] is adopted here (see Fig 3.2). The calculated $\sigma^\alpha_{y1}$ values are plotted against the volume fraction of the $\alpha$-phase in Fig 4.8. The average $k_y^{\alpha\gamma}$ value is 10.9 MPa$\sqrt{\text{mm}}$. All these parameters are also summarised in Table 4.2.
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Fig 4.5. The yield strength of $\alpha$-$\gamma$ Fe-Cr-Ni stainless steels plotted against $d^{-1/2}$ using the data of Tomota et al [76Tom]. The data in the legend box indicates the volume fraction of the $\alpha$-phase.

4.4.3. $\alpha$-$\beta$ Ti-Mn Alloys

Ankem and Margolin [86Ank] investigated the relationship between microstructure and mechanical properties of $\alpha$-$\beta$ Ti-Mn alloys. Their yield strength data together with the data of
Margolin and Vijayaraghavan [83Mar] are plotted against the reciprocal square root of the volume-fraction-weighted-average grain size in Fig 4.9, where the yield strength data has been corrected for the variation of volume fraction according to the linear law of mixture. The obtained $\sigma^c_y$ and $k_y^c$ values are shown in Figs 4.10 and 4.11 as functions of the volume fraction of the $\alpha$-phase, respectively, and are also tabulated in Table 4.3.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{fig4_6.png}
\caption{The overall friction stress of $\alpha-\gamma$ Fe-Cr-Ni stainless steel as a function of volume fraction of the $\alpha$-phase.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{fig4_7.png}
\caption{The overall Hall-Petch coefficients, $k_y^c$, of $\alpha-\gamma$ Fe-Cr-Ni stainless steels as a function of volume fraction of the $\alpha$-phase.}
\end{figure}
Chapter 4: Extension of the Hall-Petch Relation to Two-Ductile-Phase Alloys

Fig 4.8. The calculated $\sigma_{y}^{\alpha\gamma}$ of $\alpha-\gamma$ Fe-Cr-Ni stainless steels as a function of volume fraction of the $\alpha$-phase.

Fig 4.9. The yield strength of $\alpha-\beta$ Ti-Mn alloys as a function of the reciprocal square root of the volume-fraction-weighted-average grain size. The experimental data is from Ankem and Margolin [86Ank] and Margolin and Vijayaraghavan [83Mar]. The data in the legend box indicates the volume fraction of the $\alpha$-phase.
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![Graph](image)

**Fig 4.10.** The overall friction stresses, $\sigma_y^c$, of $\alpha$-$\beta$ Ti-Mn alloys as a function of volume fraction of the $\alpha$-phase.

![Graph](image)

**Fig 4.11.** The overall Hall-Petch coefficients, $k_y^c$, of $\alpha$-$\beta$ Ti-Mn alloys as a function of volume fraction of the $\alpha$-phase.

Friction stress and the Hall-Petch coefficients for single $\alpha$- and $\beta$-phase alloys are not available for the Ti-Mn alloy system. According to Ankem and Margolin [86 Ank], the grain size dependence of the yield strength of single $\alpha$-phase alloys is very small and the yield strength varies from 322 MPa to 344 MPa when the $\alpha$-grain size decreases from 280 $\mu$m to 11 $\mu$m. They
also reported that the grain size effect on the yield strength of single β-phase Ti-Mn alloys is negligibly small. This is in agreement with the experimental results of Bowen and Partridge [73Bow] in Ti-Al alloys. The data of An kem and Margolin [86Ank] gives a $k_\alpha$ value of 2.8 MPa$^{-\frac{1}{2}}$. The values of $\sigma_\alpha$ and $\sigma_\beta$ can be approximated to 300 MPa and 1000 MPa from the alloys with $f_\alpha=0.985$, and $f_\alpha=0.033$, respectively. The $k_\beta$ value is chosen as 2.0 MPa$^{-\frac{1}{2}}$, which is slightly less than $k_\alpha$.

Table 4.3. Summary of the obtained Hall-Petch constants and of the topological parameters for α-β Ti-Mn alloys. \(\sigma_y\) in MPa and \(k_y\) in MPa$^{-\frac{1}{2}}$.

<table>
<thead>
<tr>
<th>$f_\alpha$</th>
<th>$f_\alpha$</th>
<th>$f_\beta$</th>
<th>$F_s$</th>
<th>$\sigma_\alpha$</th>
<th>$k_\alpha$</th>
<th>$\sigma_\beta$</th>
<th>$k_\beta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>0.00</td>
<td>1.00</td>
<td>0.00</td>
<td>1000</td>
<td>2.0</td>
<td>1000</td>
<td>2.0</td>
</tr>
<tr>
<td>0.186</td>
<td>0.01</td>
<td>0.44</td>
<td>0.56</td>
<td>840</td>
<td>7.3</td>
<td>716</td>
<td>11.5</td>
</tr>
<tr>
<td>0.403</td>
<td>0.07</td>
<td>0.13</td>
<td>0.81</td>
<td>560</td>
<td>10.7</td>
<td>512</td>
<td>12.7</td>
</tr>
<tr>
<td>0.625</td>
<td>0.24</td>
<td>0.02</td>
<td>0.74</td>
<td>400</td>
<td>9.0</td>
<td>417</td>
<td>11.3</td>
</tr>
<tr>
<td>0.811</td>
<td>0.53</td>
<td>0.00</td>
<td>0.47</td>
<td>320</td>
<td>6.7</td>
<td>342</td>
<td>11.1</td>
</tr>
<tr>
<td>0.985</td>
<td>0.96</td>
<td>0.00</td>
<td>0.04</td>
<td>300</td>
<td>3.2</td>
<td>300</td>
<td>11.2</td>
</tr>
<tr>
<td>1.00</td>
<td>1.00</td>
<td>0.00</td>
<td>0.00</td>
<td>300</td>
<td>2.8</td>
<td>300</td>
<td>2.8</td>
</tr>
</tbody>
</table>

![Fig 4.12. The calculated $\sigma_\alpha \beta$ values of α-β Ti-Mn alloys as a function of volume fraction of the α-phase.](image)
Chapter 4: Extension of the Hall-Petch Relation to Two-Ductile-Phase Alloys

In order to evaluate $\sigma_{y}^{\alpha\beta}$ and $k_{y}^{\alpha\beta}$ we need the values of the topological parameters $f_{\alpha c}$, $f_{\beta c}$ and $F_{s}$ which have not been determined for the Ti-Mn alloy system. In principle, these topological parameters can be calculated from the experimentally measured grain size and volume fraction data if an equiaxed grain and random phase distribution are assumed. When this approach is used, the calculated $k_{y}^{\alpha\beta}$ values are not constant, but vary as a function of volume fraction. This is attributed to the unrealistic assumption that the microstructure of $\alpha-\beta$ Ti-Mn alloys is a random phase mixture of equiaxed grains.

An alternative approach is to follow Werner and Stuwe [84Wer] who suggested that, to a very good approximation, the contiguity parameters $C_{\alpha}$ and $C_{\beta}$ in $\alpha-\beta$ brasses can be presented by $f_{a}^{2}$ and $f_{b}^{3}$, respectively. This suggestion is therefore adopted here for the $\alpha-\beta$ Ti-Mn alloys. Thus,

$$f_{\alpha c} = C_{\alpha} f_{\alpha} = f_{\alpha}^{3}$$  \hspace{1cm} (4.18)

$$f_{\beta c} = C_{\beta} f_{\beta} = f_{\beta}^{4}$$  \hspace{1cm} (4.19)

$$F_{s} = 1 - f_{\alpha c} - f_{\beta c} = 1 - f_{\alpha}^{3} - f_{\beta}^{4}$$  \hspace{1cm} (4.20)

The calculated $\sigma_{y}^{\alpha\beta}$ values are plotted against the volume fraction of the $\alpha$-phase in Fig 4.12. The average value for $k_{y}^{\alpha\beta}$ is 11.6 MPa/\text{mm}. All these parameters are also listed in Table 4.3.

4.5. Discussion

4.5.1. The Strengthening Contributions from Different Boundaries

Eqn (4.16) indicates that the strengthening effects of all kinds of boundaries are linearly additive. As mentioned in Chapter 2, $k_{y}$ is a direct measure of the efficiency of grain (or phase) boundaries as obstacles to dislocation motion. Thus, $k_{y}^{c}$ is a direct measure of the overall efficiency of all kinds of boundaries to dislocation motion in an alloy with a given microstructure, while $k_{y}^{\alpha}$, $k_{y}^{\beta}$ and $k_{y}^{\alpha\beta}$ measure the efficiency of $\alpha-\alpha$ grain boundaries, $\beta-\beta$ grain boundaries and $\alpha-\beta$ phase boundaries, respectively. In a particular $\alpha-\beta$ alloy with a given microstructure, each kind of boundary makes independently its own contribution to the overall efficiency, and the separation of the individual contributions can be made according to eqn (4.16). The contributions from $\alpha-\alpha$ grain boundaries, $\beta-\beta$ grain boundaries and $\alpha-\beta$ phase boundaries are $k_{y}^{\alpha} f_{\alpha c}$, $k_{y}^{\beta} f_{\beta c}$ and $k_{y}^{\alpha\beta} F_{s}$, respectively. The calculated individual contributions as a function of volume fraction for $\alpha-\beta$ Ti-Mn, $\alpha-\beta$ brasses and $\alpha-\gamma$ Fe-Cr-Ni stainless steels are given in Fig 4.13, 4.14 and 4.15, respectively, and the individual $k_{y}$ values for these alloys are tabulated in Table 4.4 for comparison. It can be seen that these different two-ductile-phase alloys represent a variety of relative contributions from different kinds of boundaries.
Chapter 4: Extension of the Hall-Petch Relation to Two-Ductile-Phase Alloys

In α-β Ti-Mn alloys, the strengthening effects from α-grain boundaries and β-grain boundaries are very small. In other words, α-grain boundaries and β-grain boundaries are ineffective obstacles to the motion of dislocation during the plastic deformation. Therefore, the grain size strengthening effect in α-β Ti-Mn alloys is mainly from the phase boundaries over nearly the whole range of volume fractions. This explains why the phase boundaries provide the majority of sites for void nucleation during the ductile fracture of α-β Ti-alloys. This also
supports the ductile fracture mechanism for α-β Ti-alloys proposed by Fan and Miodownik [90Fan].

Table 4.4. Comparison of the $k_y$ values (MPa m$^{1/2}$) for three alloy systems.

<table>
<thead>
<tr>
<th>Alloy system</th>
<th>$\alpha$ $k_y$</th>
<th>$\beta$ $k_y$ (or $\gamma$ $k_y$)</th>
<th>$\alpha\beta$ $k_y$ (or $\alpha\gamma$ $k_y$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$-Ti-Mn</td>
<td>2.8</td>
<td>2.0</td>
<td>11.7</td>
</tr>
<tr>
<td>$\alpha$-Cu-Zn</td>
<td>11.9</td>
<td>11.4</td>
<td>14.5</td>
</tr>
<tr>
<td>$\alpha$-γ Fe-Cr-Ni</td>
<td>8.9</td>
<td>14.5</td>
<td>10.9</td>
</tr>
</tbody>
</table>

In $\alpha$-β brasses the strengthening effects from $\alpha$-grain boundaries, $\beta$-grain boundaries and $\alpha$-β phase boundaries are similar, although the phase boundaries have slightly higher strengthening ability than the grain boundaries. Thus a different deformation behaviour, in terms of dislocation motion, is expected for $\alpha$-β brasses. In the $\alpha$-rich alloys, the strengthening contribution is mainly from the $\alpha$-grain boundaries, in the $\beta$-rich alloys, the strengthening contribution is mainly from the $\beta$-grain boundaries, whereas if the volume fraction of the $\beta$-phase is close to 0.5, the strengthening contribution comes mainly from the $\alpha$-β phase boundaries. The indication here is that in the absence of further information the voids should have a similar chance to nucleate at any kind of boundary during the ductile fracture of $\alpha$-β brasses.

Fig 4.15. The calculated separate contributions to the overall efficiency of $\alpha$-γ Fe-Cr-Ni stainless steels from the different boundaries as obstacles to dislocation motion.

In $\alpha$-γ Fe-Cr-Ni stainless steels, the $\gamma$-grain boundaries are stronger obstacles to dislocation motion than either the $\alpha$-grain boundaries or $\alpha$-γ phase boundaries, as shown in Table 4.4. In
the literature, it is often assumed that the phase boundaries are stronger obstacles than grain boundaries to dislocation motion in two-ductile-phase alloys (e.g., [84Wer]). The results in Table 4.4 indicate that phase boundaries are not always the strongest obstacles to dislocation motion in two-ductile-phase alloys, as in the case of $\alpha$-$\gamma$ Fe-Cr-Ni stainless steels, where $k_\gamma > k_\alpha \gamma$.

![Cu-Zn](image1.png)

**Fig 4.16.** Comparison between the theoretical predictions and the experimental results of Werner and Stuwe [84Wer] for the $\alpha$-$\beta$ Cu-Zn system.

![Ti-Mn](image2.png)

**Fig 4.17.** Comparison between the theoretical predictions and the experimental results of Ankem and Margolin [86Ank] for the $\alpha$-$\beta$ Ti-Mn system.
Chapter 4: Extension of the Hall-Petch Relation to Two-Ductile-Phase Alloys

4.5.2. Prediction of Yield Strength of Two-Ductile-Phase Alloys

Once the evaluation of \(\sigma^c_y\) and \(k^c_y\) has been performed for a particular alloy system, eqn (4.9) can be used to predict the yield strength of the alloys from the properties of each constituent phase and the microstructural parameters. Predicted yield strengths of \(\alpha-\beta\) Cu-Zn alloys are compared with the experimental results of Werner and Stuwe [84Wer] in Fig 4.16. A similar comparison is also made in Fig 4.17 between the predictions and experimental data of Ankem and Margolin [86Ank] and Margolin and Vijayaraghavan [83Mar] for \(\alpha-\beta\) Ti-Mn alloys. Figs 4.16 and 4.17 show that the predictions are in good agreement with the experimental results. Since the experiments are carried out on microstructure A (Fig 3.15a) and the theoretical calculations are made on the corresponding microstructure B (Fig 3.15b), the above good agreement supports the mechanical equivalence between microstructures A and B proposed in Chapter 3.

4.6. Summary and Conclusions

(1) The Hall-Petch relation originally developed for single-phase alloys has been extended to two-ductile-phase alloys. The extended Hall-Petch relation expresses the yield strength of an \(\alpha-\beta\) dual-phase alloy in terms of the grain size and Hall-Petch parameters of each constituent phase by the following equation

\[
\sigma_y^c = \sigma_y^c + k_y^c \bar{d}^{-1/2} = (\sigma_y^\alpha + k_y^\alpha \bar{d}^{-1/2}) \sigma_{\alpha \gamma} + (\sigma_y^\beta + k_y^\beta \bar{d}^{-1/2}) \sigma_{\beta \gamma} + (\sigma_y^\alpha + k_y^\alpha \bar{d}^{-1/2}) \sigma_{\alpha \gamma} + (\sigma_y^\beta + k_y^\beta \bar{d}^{-1/2}) \sigma_{\beta \gamma} + \sigma_y^f \sigma_{\gamma \gamma} F_s
\]

(2) The overall friction stress \(\sigma_y^c\) and the overall Hall-Petch coefficient \(k_y^c\) of an \(\alpha-\beta\) dual-phase alloy can be expressed by the following eqns

\[
\sigma_y^c = \sigma_y^\alpha \sigma_{\alpha \gamma} + \sigma_y^\beta \sigma_{\beta \gamma} + \sigma_y^f \sigma_{\gamma \gamma} F_s
\]

\[
k_y^c = k_y^\alpha \sigma_{\alpha \gamma} + k_y^\beta \sigma_{\beta \gamma} + k_y^f \sigma_{\gamma \gamma} F_s
\]

(3) It is now possible to separate the individual contributions from grain boundaries and phase boundaries to the overall efficiency of all kinds of boundaries as obstacles to dislocation motion, and the extended Hall-Petch relation has been applied to \(\alpha-\beta\) Ti-Mn alloys, \(\alpha-\beta\) Cu-Zn alloys and \(\alpha-\gamma\) Fe-Cr-Ni stainless steels to evaluate the efficiency of phase boundaries as obstacles to dislocation motion.

(4) Phase boundaries are not always the strongest obstacles to dislocation motion in two-ductile-phase alloys.

(5) The extended Hall-Petch relation can be used to predict the yield strength of two-phase alloys once \(\sigma_y^c\) and \(k_y^c\) have been evaluated for the particular alloy system. It has been shown that the predicted yield strength by the extended Hall-Petch relation is in very good agreement with experimental data in \(\alpha-\beta\) Ti-Mn alloys and \(\alpha-\beta\) Cu-Zn alloys.
5.1. Introduction

A number of technologically important alloys consist of two ductile phases, where both phases have similar grain size. Examples of this type of alloy are α-β Ti-alloys, α-β Cu-alloys and dual-phase steels. As a group, these alloys offer a very good combination of strength, ductility and fracture toughness.

In order to understand and to predict the mechanical properties of the composite alloys in terms of the mechanical properties of constituent phases, the classical linear law of mixtures [1889Voi] is often used. However, the flow properties of most particulate two-phase alloys do not follow an empirical linear law of mixtures. A more general but still empirical law of mixtures (called the modified law of mixtures) relating to the average stresses and strains in each phase was suggested by Tamura et al [73Tam]. It has been found that the empirical use of the modified law of mixtures for stress and/or strain shows good agreement with experiment [77Fis, 87Fun, 87Su, 82Ank1], theoretical modelling [85Geo, 78Lee] and finite element analysis [82Ank1, 82Ank2, 75Kar]. However, this modified law of mixtures can only be justified at small strains [87Gur]. Ankem and Margolin [86Ank] proposed an alternative to the linear law of mixtures, which includes an explicit term describing the interaction between constituent phases. In a different way to find the answer to the question, Gurland [79Gur1, 79Gur2] tried to apply the shear-lag model [86Kel] to the two-phase composite with irregular hard particles, resulting in the mathematical expression for the flow stress of two-phase alloys.

The most promising approach to the deformation behaviour of two-phase composites is the continuum transformation theory first proposed by Eshelby [57Esh, 59Esh, 61Esh] and further developed later in a number of studies [70Tan, 71Bro, 78Ped, 81Tay1, 81Tay2, 83Ped, 89Wit]. The Eshelby's model has undergone the following four stages of development: (i) Eshelby's continuum transformation theory for an infinite composite with a single inclusion [57Esh, 59Esh, 61Esh]; (ii) independent inclusion approaches for homogeneous composites with low volume fraction of inclusion [70Tan, 71Bro]; (iii) mean field approaches for
Chapter 5: Deformation Behaviour of Two-Ductile-Phase Alloys. I. Deformation Theory

inhomogeneous composites [78Ped, 79Ped, 83Ped, 81Tayl, 81Tay2, 82Dai]; (iv) more recent development and applications of Eshelby's model [e.g., 88Wit, 89Wit, 90Ped].

A difficult problem, which is avoided by both the independent inclusion approaches and the mean field approaches, is the interaction between particles of the same phase. It is obvious that the number of contacts between inclusion particles will increase rapidly with increasing volume fraction of inclusion, and consequently, the interaction between inclusion particles cannot be neglected at high volume fractions. Therefore, it is important to find a solution to handle the interaction problem.

This chapter aims to develop a new approach to the deformation of two-ductile-phase alloys which can consider the effect of microstructural parameters on the deformation behaviour, based on the Eshelby's continuum transformation theory and the microstructural characterization in Chapter 3. This chapter also aims to take into account of the interaction between inclusion particles. Therefore, the mean internal stress-strain field and the various interactions in the two-phase alloys will be analysed first to find a way to the solutions of interaction problem, and then the true stress-true strain curves will be derived theoretically for the EIII body and the 3-E body.

5.2. The Mean Internal Stress-Strain Field and Interaction Problem

5.2.1. The Mean Internal Stresses inside and outside a Spherical Inclusion Embedded in an Infinite Matrix

Following Eshelby's [57Esh, 59Esh, 61Esh] continuum transformation theory, Brown and Stobbs [71Bro] derived the mathematical expressions (eqn (2.4.4) for the stresses inside and outside a spherical inclusion which is embedded in an infinite matrix and is subject to a uniform transformation shear strain, $-\varepsilon_p$. Their work revealed the following important concepts:

(i) the stress inside the inclusion is uniform [57Esh, 71Bro];

(ii) the stress outside the inclusion decreases dramatically through a power law with increasing distance from the inclusion, and will become zero when the distance is large enough. It has, therefore, been called the local fluctuating stress;

(iii) The mean value of each stress component outside the inclusion is zero [71Bro, 72Tan]. Therefore these local fluctuating stresses do not contribute to the strengthening effect of the inclusion.

From the above analysis it is obvious that the interaction between the stress-strain fields of two inclusion particles should be small enough to be neglected if the distance between these two inclusion particles is large enough.
Strategically, there are two ways to solve the problem of interaction between the inclusion particles:

(i) derive the exact stress-strain fields in terms of the individual particles, work out the disturbance to the stress-strain field of a given particle from those of the surrounding particles which have a degree of interaction with it. To do this, it is essential to have a knowledge of the exact particle distribution, which is not available at present stage. Consequently, this approach is not realistic;

(ii) derive an equivalent microstructure where the interaction among the particles of the same phase can be neglected to a reasonable degree of accuracy. This is the approach adopted for the following theoretical treatment.

5.2.2 The Image Stress and the Interaction between the Inclusions and the External Stress-Free Surfaces

Eshelby's [57Esh, 59Esh, 61Esh] continuum transformation theory is mathematically rigorous for an infinite composite with a single inclusion. But in order to apply it to a real composite, the effects caused by the finite nature of the composite must be considered, that is to satisfy the boundary conditions at the external surfaces of such finite composite. For this purpose, Eshelby [57Esh] introduced the concept of image stress. On finding that the fluctuating stress field calculated by the Eshelby's method about a spherical inclusion is averaged to zero throughout a spherical region of the matrix, Brown and Stobbs [71Bro] determined the image stress in terms of the inclusion stress through a stress balance. In fact, the image stress is the result of the interaction between the inclusion and the stress-free external surface. In other words, the finite homogeneous composite has to bear an extra stress field, the image stress field, because of the existence of the stress-free external surfaces.

5.3. The Mean Internal Stress-Strain Field

In Brown and Stobbs' [71Bro] independent inclusion approach, it is assumed that a volume fraction of ellipsoidal shaped hard particles (denoted as the β-phase), which are plastically non-deformable, are uniformly distributed in a plastically deformable soft matrix (denoted as the α-phase). It is also assumed that the elastic constants for both the α- and β-phases are isotropic and equal. When the isotropic matrix undergoes an essentially uniform axial plastic strain, $\varepsilon_{P33}^{\alpha}$, along the tensile direction, there are the following strain components along the other directions required by the incompressibility:

$$\varepsilon_{P33}^{\alpha} = \varepsilon_p$$
$$\varepsilon_{P11}^{\alpha} = \varepsilon_{P22}^{\alpha} = -\frac{1}{2} \varepsilon_p$$
$$\varepsilon_{P12}^{\alpha} = \varepsilon_{P23}^{\alpha} = \varepsilon_{P31}^{\alpha} = 0$$

where $\varepsilon_p$ is the tensile strain of the α-β phase mixture along the tensile direction.
Chapter 5: Deformation Behaviour of Two-Ductile-Phase Alloys. I. Deformation Theory

According to the Eshelby's [57Esh] continuum transformation theory, the plastic strain incompatibility between two phases can cause internal stresses within both phases. The elastic stress-strain fields within and around the hard particles can be computed by specifying Eshelby's "transformation strain" [57Esh]. The mean internal strain in the matrix of a finite homogeneous composite, \( \langle \varepsilon_{ij}^{I} \rangle_{M} \), is expressed by the following equation after Brown [73Bro]

\[
\langle \varepsilon_{ij}^{I} \rangle_{M} = f_{\beta} (\varepsilon_{ij}^{T} - S_{ijkl} \varepsilon_{kl}^{T})
\]

(5.2)

where \( f_{\beta} \) is the volume fraction of the \( \beta \)-phase, \( \varepsilon_{ij}^{T} \) and \( \varepsilon_{kl}^{T} \) are the Eshelby's transformation strains, and \( S_{ijkl} \) are the Eshelby's tensors which connect the constraint strains of inclusions with the transformation strains and are only functions of inclusion shape and Poisson's ratio, \( \nu \) (It is assumed here that \( \nu^{\alpha} = \nu^{\beta} = \nu^{c} \), where the superscripts \( \alpha \), \( \beta \), and \( c \) refer to the \( \alpha \)-phase, the \( \beta \)-phase and the \( \alpha-\beta \) composite).

For convenience, Brown and Clarke [75Bro] defined the two rank strain accommodation tensors and stress accommodation tensors in the deformation coordinate frame to connect the transformation strains with Eshelby's tensors. By this treatment, the mean strains and stresses in the matrix of a finite homogeneous composite, \( \langle \varepsilon_{ij}^{I} \rangle_{M} \) and \( \langle \sigma_{ij}^{I} \rangle_{M} \), can be expressed by the following equations

\[
\langle \varepsilon_{ij}^{I} \rangle_{M} = \gamma_{ij} f_{\beta} \varepsilon_{P}^{\alpha}
\]

(5.3a)

\[
\langle \sigma_{ij}^{I} \rangle_{M} = 2G \Sigma_{ij} f_{\beta} \varepsilon_{P}^{\alpha}
\]

(5.3b)

where \( G \) is the shear modulus of the composite, \( \gamma_{ij} \) and \( \Sigma_{ij} \) are the strain and stress accommodation tensors, respectively. The \( \gamma_{ij} \) and \( \Sigma_{ij} \) values for spheres and infinitely long fibres have been calculated by Brown and Clarke [75Bro]. \( \varepsilon_{P}^{\alpha} \) is the plastic incompatibility or the unrelaxed plastic strain between the inclusion and matrix, i.e.,

\[
\varepsilon_{P}^{\alpha} = \varepsilon_{P}^{\beta} - \varepsilon_{P}^{\alpha}
\]

(5.4)

Thus, for a randomly distributed \( \alpha-\beta \) phase mixture, the mean strains and stresses can be expressed by the following equations

\[
\langle \varepsilon_{ij}^{I} \rangle_{\alpha} = \gamma_{ij} f_{\beta} \varepsilon_{P}^{\alpha} = \gamma_{ij} f_{\beta} (\varepsilon_{P}^{\beta} - \varepsilon_{P}^{\alpha})
\]

(5.5a)

\[
\langle \varepsilon_{ij}^{I} \rangle_{\beta} = \gamma_{ij} f_{\alpha} \varepsilon_{P}^{\beta} = \gamma_{ij} f_{\alpha} (\varepsilon_{P}^{\alpha} - \varepsilon_{P}^{\beta})
\]

(5.5b)

\[
\langle \sigma_{ij}^{I} \rangle_{\alpha} = 2G \Sigma_{ij} f_{\beta} \varepsilon_{P}^{\alpha} = 2G \Sigma_{ij} f_{\beta} (\varepsilon_{P}^{\beta} - \varepsilon_{P}^{\alpha})
\]

(5.5c)

\[
\langle \sigma_{ij}^{I} \rangle_{\beta} = 2G \Sigma_{ij} f_{\alpha} \varepsilon_{P}^{\beta} = 2G \Sigma_{ij} f_{\alpha} (\varepsilon_{P}^{\alpha} - \varepsilon_{P}^{\beta})
\]

(5.5d)

where \( f_{\alpha} \) and \( f_{\beta} \) are volume fractions. These mean internal stresses and strains can be regarded as the results of the interaction between those two phases, which is quantitatively described by eqn (5.5).

To sum up, there are three kinds of interactions in an elastically homogeneous composite:

(i) interaction between constituent phases, the results of which are the mean internal stresses in each phase;
interaction between the inclusion and the stress-free external surfaces, which leads to the image stresses;
(ii) interaction between the particles of the same phase, which can be taken into account by an equivalent microstructural transformation, as will be shown in the following section.

5.4. The Mean Internal Stress-Strain Field in Each Microstructural Element of the 3-E Body

From Chapter 3, it is seen that a body with any grain size, grain shape and phase distribution can be topologically transformed into a 3-E body with three microstructural elements aligned parallelly along the tensile direction, and that the 3-E body is mechanically equivalent to the original body along the aligned direction. If the analysis is now confined to the three microstructural elements in the 3-E body, instead of the original α- and β-phases, the interaction between one element and the other two elements can be worked out in terms of the mean internal stresses and strains. This leads to the following equations:

\[
\langle \varepsilon_{11}^{(1)} \rangle_{E_{11}} = \langle \varepsilon_{22}^{(1)} \rangle_{E_{11}} = -\frac{1}{4(1-\nu)} \left\{ f_{\beta}\varepsilon_P^\beta + F_{SE}^\alpha (f_{\beta c} + F_S)\varepsilon_P^\alpha \right\} \\
\langle \varepsilon_{33}^{(1)} \rangle_{E_{11}} = f_{\beta}\varepsilon_P^\beta + F_{SE}^\alpha (f_{\beta c} + F_S)\varepsilon_P^\alpha \\
\langle \varepsilon_{11}^{(1)} \rangle_{E_{111}} = \langle \varepsilon_{22}^{(1)} \rangle_{E_{111}} = -\frac{1}{4(1-\nu)} \left\{ f_{\alpha}\varepsilon_P^\alpha + F_{SE}^\beta (f_{\beta c} + F_S)\varepsilon_P^\beta \right\} \\
\langle \varepsilon_{33}^{(1)} \rangle_{E_{111}} = f_{\alpha}\varepsilon_P^\alpha + F_{SE}^\beta (f_{\alpha c} + F_S)\varepsilon_P^\beta \\
\langle \varepsilon_{11}^{(1)} \rangle_{E_{1111}} = \langle \varepsilon_{22}^{(1)} \rangle_{E_{1111}} = -\frac{1}{4(1-\nu)} \left\{ f_{\alpha}\varepsilon_P^\alpha + f_{\beta}\varepsilon_P^\beta - (f_{\alpha c} + f_{\beta c})\varepsilon_P^\alpha \right\} \\
\langle \varepsilon_{33}^{(1)} \rangle_{E_{1111}} = f_{\alpha}\varepsilon_P^\alpha + f_{\beta}\varepsilon_P^\beta - (f_{\alpha c} + f_{\beta c})\varepsilon_P^\beta \\
\langle \sigma_{11}^{(1)} \rangle_{E_{11}} = \langle \sigma_{22}^{(1)} \rangle_{E_{11}} = \frac{(1-2\nu)E}{4(1-\nu)^2} \left\{ f_{\beta}\varepsilon_P^\beta + F_{SE}^\alpha (f_{\beta c} + F_S)\varepsilon_P^\alpha \right\} \\
\langle \sigma_{33}^{(1)} \rangle_{E_{11}} = \frac{(2-\nu)E}{2(1-\nu)^2} \left\{ f_{\beta}\varepsilon_P^\beta + F_{SE}^\alpha (f_{\beta c} + F_S)\varepsilon_P^\alpha \right\} \\
\langle \sigma_{11}^{(1)} \rangle_{E_{111}} = \langle \sigma_{22}^{(1)} \rangle_{E_{111}} = -\frac{(1-2\nu)E}{4(1-\nu)^2} \left\{ f_{\alpha}\varepsilon_P^\alpha + F_{SE}^\beta (f_{\alpha c} + F_S)\varepsilon_P^\beta \right\} \\
\langle \sigma_{33}^{(1)} \rangle_{E_{111}} = \frac{(2-\nu)E}{2(1-\nu)^2} \left\{ f_{\alpha}\varepsilon_P^\alpha + F_{SE}^\beta (f_{\alpha c} + F_S)\varepsilon_P^\beta \right\} \\
\langle \sigma_{11}^{(1)} \rangle_{E_{1111}} = \langle \sigma_{22}^{(1)} \rangle_{E_{1111}} = \frac{(1-2\nu)E}{4(1-\nu)^2} \left\{ f_{\alpha}\varepsilon_P^\alpha + f_{\beta}\varepsilon_P^\beta - (f_{\alpha c} + f_{\beta c})\varepsilon_P^\alpha \right\} \\
\langle \sigma_{33}^{(1)} \rangle_{E_{1111}} = \frac{(2-\nu)E}{2(1-\nu)^2} \left\{ f_{\alpha}\varepsilon_P^\alpha + f_{\beta}\varepsilon_P^\beta - (f_{\alpha c} + f_{\beta c})\varepsilon_P^\alpha \right\}
\]

\( \text{(5.6a)} \quad \text{(5.6b)} \quad \text{(5.6c)} \quad \text{(5.6d)} \quad \text{(5.6e)} \quad \text{(5.6f)} \quad \text{(5.6g)} \quad \text{(5.6h)} \quad \text{(5.6i)} \quad \text{(5.6j)} \quad \text{(5.6k)} \quad \text{(5.6l)} \)
where $\varepsilon_P^\alpha$, $\varepsilon_P^\beta$ and $\varepsilon_P^{\alpha\beta}$ are the plastic strain in elements EI, EII and EIII, respectively, $f_{\alpha\alpha}$, $f_{\beta\beta}$ and $F_s$ are the continuous volume of the $\alpha$-phase, the continuous volume of the $\beta$-phase and the degree of separation, or the volume fractions of EI, EII and EIII, respectively, and $E$ is Young’s modulus.

Now let us consider the internal stresses and strains in the $\alpha$- and $\beta$-phases in the EIII body when it alone is subject to a uniaxial applied stress. As discussed in the previous section, the interaction between the stress-strain fields of two particles of the same phase is small enough to be neglected if the distance between these two particles is large enough. While in the EIII body, all the $\alpha$-grains are surrounded by $\beta$-grains and all the $\beta$-grains are surrounded by $\alpha$-grains, and the average $\alpha$-grain spacing and average $\beta$-grain spacing are $d_\alpha$ and $d_\beta$, respectively. Thus, the distance between two grains of the same phase can be considered as sufficiently large that the interaction between the stress-strain fields of the grains of the same phase may be neglected. In such circumstances, the mean internal stresses and strains in each phase in the EIII body can be computed from the following equations when the EIII body alone is subject to uniaxial tension:

\[
\begin{align*}
\left\langle \varepsilon_{11} \right\rangle_{\alpha III} &= \left\langle \varepsilon_{22} \right\rangle_{\alpha III} = -\frac{7-5v}{30(1-v)} f_{\beta III} (\varepsilon_P^\beta - \varepsilon_P^\alpha) \\
\left\langle \varepsilon_{33} \right\rangle_{\alpha III} &= \frac{7-5v}{15(1-v)} f_{\beta III} (\varepsilon_P^\beta - \varepsilon_P^\alpha) \\
\left\langle \varepsilon_{11} \right\rangle_{\beta III} &= \left\langle \varepsilon_{22} \right\rangle_{\beta III} = -\frac{7-5v}{30(1-v)} f_{\alpha III} (\varepsilon_P^\alpha - \varepsilon_P^\beta) \\
\left\langle \varepsilon_{33} \right\rangle_{\beta III} &= \frac{7-5v}{15(1-v)} f_{\alpha III} (\varepsilon_P^\alpha - \varepsilon_P^\beta) \\
\left\langle \sigma_{11} \right\rangle_{\alpha III} &= \left\langle \sigma_{22} \right\rangle_{\alpha III} = -\frac{(7-5v)E}{30(1-v^2)} f_{\beta III} (\varepsilon_P^\beta - \varepsilon_P^\alpha) \\
\left\langle \sigma_{33} \right\rangle_{\alpha III} &= \frac{(7-5v)E}{15(1-v^2)} f_{\beta III} (\varepsilon_P^\beta - \varepsilon_P^\alpha) \\
\left\langle \sigma_{11} \right\rangle_{\beta III} &= \left\langle \sigma_{22} \right\rangle_{\beta III} = -\frac{(7-5v)E}{30(1-v^2)} f_{\alpha III} (\varepsilon_P^\alpha - \varepsilon_P^\beta) \\
\left\langle \sigma_{33} \right\rangle_{\beta III} &= \frac{(7-5v)E}{15(1-v^2)} f_{\alpha III} (\varepsilon_P^\alpha - \varepsilon_P^\beta)
\end{align*}
\]

where $f_{\alpha III}$ and $f_{\beta III}$ are the volume fractions of the $\alpha$- and $\beta$-phases in the EIII body. In contrast to eqns (5.2)-(5.5) which are only applicable to elastically homogeneous composites with low volume fraction, eqns (5.6) and (5.7) can be applied to an elastically homogeneous composite with any volume fraction, grain size, grain shape and phase distribution, since the interaction between the particles of the same phase has been taken into account as a consequence of the equivalent microstructural transformation.
5.5. Derivation of the Flow Curve of the 3-E Body

In this section, the flow curve of EIII will be derived first from the known flow curves of α- and β-phases, and then the flow curve for the whole 3-E body will be derived from the known flow curves of EI (α-phase), EII (β-phase) and the derived flow curve of EIII.

5.5.1 The flow curve of EIII

EIII is taken as a continuum body and subject to uniaxial tension. the whole deformation process of the EIII body can be divided into three stages delineated by two critical points, as summarized in Table 5.1, where \( Y(1) \) is the critical applied stress to start the plastic deformation in the α-phase in EIII body, \( Y(2) \) is the critical applied stress to start the plastic deformation in the β-phase in EIII body, ED and PD stand for elastic deformation and plastic deformation, respectively, and \( \sigma_F \) is the fracture stress of EIII body. α(III) and β(III) denote the α-phase and the β-phase in EIII body.

Table 5.1. Summary of the deformation stages for EIII body.

<table>
<thead>
<tr>
<th>Deformation Stage</th>
<th>The Applied Stress</th>
<th>Deformation Status</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stage 1</td>
<td>( 0 \leq \sigma_{33} \leq Y(1) )</td>
<td>( \alpha(III) ) ED</td>
</tr>
<tr>
<td>Onset of PD in α(III)</td>
<td>( \sigma_{33} \geq Y(1) )</td>
<td>PD</td>
</tr>
<tr>
<td>Stage 2</td>
<td>( Y(2) &lt; \sigma_{33} &lt; Y(2) )</td>
<td>PD</td>
</tr>
<tr>
<td>Onset of PD in β(III)</td>
<td>( \sigma_{33} \geq Y(2) )</td>
<td>PD</td>
</tr>
<tr>
<td>Stage 3</td>
<td>( Y(2) &lt; \sigma_{33} &lt; \sigma_F )</td>
<td>PD</td>
</tr>
</tbody>
</table>

(1) **Stage 1 Deformation and Onset of Plastic Deformation in the α-Phase**

With the assumption of equal elastic constants of both phases, Stage 1 deformation is homogeneous, and the critical stress at onset of plastic deformation of the α-phase is equal to the yield stress of the α-phase.

(2) **Stage 2 Deformation**

When the uniaxially applied stress reaches the yield stress of the softer phase (α-phase), it starts to deform plastically. As plastic deformation proceeds in the α-phase, there will be a plastic incompatibility between the two phases, which can cause the internal stresses in both phases, as described by eqn (5.7). When the plastic strain in the α-phase is increased by a small amount, \( \Delta \varepsilon_{ij} \) (Note that \( \Delta \varepsilon_{33} = -2\Delta \varepsilon_{11} = -2\Delta \varepsilon_{22} = \Delta \varepsilon_p \)) where \( \Delta \varepsilon_p \) is the plastic strain increment in the whole EIII body), the plastic work done on unit volume of the α-phase is
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\[
\sigma_{ij}^A + \sigma_{ij}^B \delta e_{ij} = \left\{ (\sigma_{33}^A + \langle \sigma_{33}^B \rangle_{\alpha} - \frac{1}{2} \langle \sigma_{11}^B \rangle_{\beta} + \langle \sigma_{13}^B \rangle_{\beta}) \delta e_{\alpha}^P \right\}
\]

This must be equal to the plastic work done per unit volume of the \( \alpha \)-phase in the bulk, \( \sigma_f^\alpha \delta e_{ij}^\alpha \), where \( \sigma_f^\alpha \) is the flow stress of the pure \( \alpha \)-phase when it is alone uniaxially strained to a plastic strain of \( \varepsilon_\alpha^P \). Thus, the flow stress of EIII during Stage 2 deformation is described by the following equation

\[
\sigma_{33}^A = \sigma_f^\alpha + A_f^\alpha \varepsilon_\alpha^P \tag{5.8}
\]

where \( A = \frac{E(7-5v)}{10(1-\nu^2)} \).

(3) Onset of Plastic Deformation in the \( \beta \)-Phase of EIII Body

The accumulation of the internal stress in the harder phase promotes plastic flow in it. At a critical applied stress, \( \varepsilon_\beta^P \), the internal stress together with the applied stress allow the harder phase to deform plastically. The condition for the onset of plastic flow in the hard phase is given by the virtual work principle as

\[
\sigma_{ij}^B \delta e_{ij}^B = \left\{ \frac{1}{2} \langle \sigma_{11}^B \rangle_{\beta} - \frac{1}{2} \langle \sigma_{13}^B \rangle_{\beta} \right\} \delta e_{\beta}^P \tag{5.9}
\]

where \( \delta e_{\beta}^P \) is the tensile plastic strain increment in the \( \beta \)-phase in the tensile direction. At the same time, eqn (5.8) must also be satisfied. This condition leads to the following simultaneous equations which define the onset of plastic deformation in the \( \beta \)-phase of EIII

\[
\begin{align*}
Y(2) &= \sigma_y^B - A_f^\alpha \varepsilon_\alpha^P(2) \\
Y(2) &= \sigma_f^\alpha \varepsilon_\alpha^P(2) + A_f^\alpha \varepsilon_\alpha^P(2) \tag{5.10}
\end{align*}
\]

where \( \varepsilon_\alpha^P(2) \) is the critical plastic strain in the \( \alpha \)-phase which is necessary to start the plastic flow in the \( \beta \)-phase.

(4) Stage 3 Deformation of EIII Body

When the applied stress, \( \sigma_{33}^A \), is beyond the critical point, \( Y(2)[\varepsilon_\alpha^P(2)] \), the internal stresses plus the applied stress must again satisfy the virtual plastic work principle. This condition leads to the following simultaneous equations

\[
\begin{align*}
\sigma_{33}^A &= \sigma_f^\alpha \varepsilon_\alpha^P - A_f^\alpha \varepsilon_\alpha^P(2) \\
\sigma_{33}^B &= \sigma_f^\beta \varepsilon_\beta^P + A_f^\beta \varepsilon_\beta^P(2) \tag{5.11}
\end{align*}
\]

Eqn (5.11) describes the flow curve of Stage 3 deformation of EIII body. The flow curve of Stage 3 deformation can be constructed by solving the above simultaneous equations for \( \sigma_{33}^A \) and \( \varepsilon_\beta^P \) at the prescribed value of \( \varepsilon_\alpha^P \).
(5) The Plastic Strain Distribution between Two Phases in EIII Body

Eqns (5.8), (5.10) and (5.11) describe the stresses and plastic strains in each phase, but leave the plastic strain in the EIII body still unknown. In Tomota's model [76Tom], it is assumed that the plastic strain of a phase mixture is distributed in its constituent phases according to the linear law of mixtures. In fact, the plastic strain distribution between two phases can be derived theoretically. Let us take Stage 3 deformation of the EIII body as an example. At certain point during Stage 3 deformation of EIII (the first point), the plastic strain in α- and β-phases are $\varepsilon_{p}^{\alpha}$ and $\varepsilon_{p}^{\beta}$. Assuming that there is a plastic strain increment in the EIII body by a small amount of $\Delta \varepsilon_{p}^{\alpha}$ (the second point), then, the plastic strains in the α- and β-phases would be $\varepsilon_{p}^{\alpha} + \Delta \varepsilon_{p}^{\alpha}$ and $\varepsilon_{p}^{\beta} + \Delta \varepsilon_{p}^{\beta}$. From the first point to the second point, the plastic work done by the whole EIII body should be equal to the sum of the plastic work done by each phase, as required by the virtual plastic work principle. This leads to the following equation

\[
\sigma_{33} \Delta \varepsilon_{p}^{\beta} = \left\{ (\sigma_{33}^{A} + A[\varepsilon_{p}^{\alpha} + \Delta \varepsilon_{p}^{\alpha} - (\varepsilon_{p}^{\alpha} + \Delta \varepsilon_{p}^{\alpha})]f_{\alpha III}^{\alpha} \right\} \Delta \varepsilon_{p}^{\alpha III} f_{\alpha III}^{\alpha III} \\
+ \left\{ (\sigma_{33}^{A} + A[\varepsilon_{p}^{\beta} + \Delta \varepsilon_{p}^{\beta} - (\varepsilon_{p}^{\beta} + \Delta \varepsilon_{p}^{\beta})]f_{\beta III}^{\beta} \right\} \Delta \varepsilon_{p}^{\beta III} f_{\beta III}^{\beta III}
\]  
(5.12)

Rearranging Eqn (5.12), leads to the following equation

\[
\Delta \varepsilon_{p}^{\alpha} = \Delta \varepsilon_{p}^{\alpha III} f_{\alpha III}^{\alpha} + \Delta \varepsilon_{p}^{\beta III} f_{\beta III}^{\beta} \\
+ \frac{A}{\sigma_{33}^{A}} \left\{ \left[ \varepsilon_{p}^{\beta III} + \Delta \varepsilon_{p}^{\beta III} - (\varepsilon_{p}^{\beta III} + \Delta \varepsilon_{p}^{\beta III}) \right] \Delta \varepsilon_{p}^{\alpha III} f_{\alpha III}^{\alpha} \right\} \\
+ \left\{ \left[ \varepsilon_{p}^{\alpha III} + \Delta \varepsilon_{p}^{\alpha III} - (\varepsilon_{p}^{\alpha III} + \Delta \varepsilon_{p}^{\alpha III}) \right] \Delta \varepsilon_{p}^{\beta III} f_{\beta III}^{\beta} \right\}
\]  
(5.13)

which describes the plastic strain distribution between the α- and β-phases in EIII.

(6) The Total Strain of EIII Body

Since the assumption has been made that the elastic constants of both phase are the same, the elastic strain is uniformly distributed between the α- and β-phases. Consequently, the total strains of each phase and the whole EIII body can be expressed by the following equations

\[
\varepsilon_{T}^{\alpha} = \varepsilon_{p}^{\alpha} + \frac{A}{\sigma_{33}^{A}} \varepsilon_{E}^{\alpha III} \\
\varepsilon_{T}^{\beta} = \varepsilon_{p}^{\beta} + \frac{A}{\sigma_{33}^{A}} \varepsilon_{E}^{\beta III} \\
\varepsilon_{T}^{\alpha \beta} = \varepsilon_{p}^{\alpha \beta} + \frac{A}{\sigma_{33}^{A}} \varepsilon_{E}^{\alpha \beta III}
\]

(5.14a)

where $\varepsilon_{T}^{\alpha}$, $\varepsilon_{T}^{\beta}$ and $\varepsilon_{T}^{\alpha \beta}$ are the total strains in the α-phase, the β-phase and the whole EIII body, respectively.
5.5.2. Derivation of the Flow Curve of the 3-E Body

In this section, the flow curve of the whole 3-E body will be constructed from the known flow curves of EI (α-phase) and EII (β-phase) and the derived flow curve of the EIII body in Section 5.4.1. The yield stresses of three microstructural elements are denoted as $\sigma_y^\alpha$, $\sigma_y^\beta$ and $\sigma_y^{\alpha\beta}$ for EI, EII and EIII, respectively, and $\sigma_y^{\alpha\beta}$ is assumed to follow the linear law of mixtures to a reasonable approximation in two-ductile-phase alloys, i.e.,

$$\sigma_y^{\alpha\beta} = \sigma_y^{\alpha}\gamma_{\alpha III} + \sigma_y^{\beta}\gamma_{\beta III}$$  (5.15)

In addition, $\sigma_y^{\alpha\beta}$ can be calculated more accurately from the extended Hall-Petch relation in Chapter 4.

The whole deformation process of the 3-E body will be divided into four stages which are delineated by three critical points, as summarized in Table 5.2, where $Y(i)$, $Y(ii)$ and $Y(iii)$ are the critical applied stresses to start the plastic deformation in EI, EIII and EII, respectively, and $\sigma_F$ is the fracture strength of the 3-E body.

In this section, the general equations will be derived first in Stage 4 deformation, and then the specific equations which describe the flow curve in other deformation stages and critical points will be deduced from the general equations.

Table 5.2. Summary of the deformation stages for 3-Element body

<table>
<thead>
<tr>
<th>Deformation Stage</th>
<th>The Applied Stress</th>
<th>Deformation Status</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$0.5\leq\sigma_{33}^A&lt;Y(i)$</td>
<td>EI: ED, EII: ED, EIII: ED</td>
</tr>
<tr>
<td>Onset of PD in EI</td>
<td>$\sigma_{33}^A=Y(i)$</td>
<td>PD: ED, EII: ED</td>
</tr>
<tr>
<td>Stage 2</td>
<td>$Y(i)&lt;\sigma_{33}^A&lt;Y(ii)$</td>
<td>PD: ED, EII: ED</td>
</tr>
<tr>
<td>Onset of PD in EIII</td>
<td>$\sigma_{33}^A=Y(ii)$</td>
<td>PD: ED, EII: PD</td>
</tr>
<tr>
<td>Stage 3</td>
<td>$Y(ii)&lt;\sigma_{33}^A&lt;Y(iii)$</td>
<td>PD: ED, EII: PD</td>
</tr>
<tr>
<td>Onset of PD in EII</td>
<td>$\sigma_{33}^A=Y(iii)$</td>
<td>PD: ED, EII: PD</td>
</tr>
<tr>
<td>Stage 4</td>
<td>$Y(iii)&lt;\sigma_{33}^A&lt;\sigma_F$</td>
<td>PD: ED, EII: PD</td>
</tr>
</tbody>
</table>

(1) Stage 4 Deformation

At a certain deformation point during the Stage 4 deformation, the *in situ* plastic strain in each microstructural element are $\varepsilon_P^\alpha$ in EI, $\varepsilon_P^\beta$ in EII and $\varepsilon_P^{\alpha\beta}$ in EIII; this point is denoted as point A. Assuming that the 3-E body undergoes a further plastic deformation, $\delta\varepsilon_P^c$, the plastic strain in each element at this new point will be $\varepsilon_P^\alpha + \delta\varepsilon_P^c$ in EI, $\varepsilon_P^\beta + \delta\varepsilon_P^c$ in EII and $\varepsilon_P^{\alpha\beta} + \delta\varepsilon_P^{\alpha\beta}$ in EIII, this point is denoted as point B. The internal stresses and strains in each element along each
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direction can be then calculated according to Eqn (5.6). Note that the strain components of each element must satisfy eqn (5.1). The internal stresses in each microstructural element plus the applied stress must satisfy the flow condition, which is expressed by the virtual work principle. This leads to the following simultaneous equations

\[
\sigma_f^\alpha (\varepsilon_p^\alpha + \varepsilon_p^\beta) \delta \varepsilon_p^\alpha = \left[ \left( \sigma_{33}^A + \left( \sigma_{ij}^A \right) \right) \right] \delta \varepsilon_p^\alpha
\]

\[
\sigma_f^\beta (\varepsilon_p^\beta + \varepsilon_p^\gamma) \delta \varepsilon_p^\beta = \left[ \left( \sigma_{33}^A + \left( \sigma_{ij}^A \right) \right) \right] \delta \varepsilon_p^\beta
\]

\[
\sigma_f^\alpha \varepsilon_p^\alpha + \sigma_f^\beta \varepsilon_p^\beta + \sigma_f^\gamma \varepsilon_p^\gamma = \left[ \left( \sigma_{33}^A + \left( \sigma_{ij}^A \right) \right) \right] \delta \varepsilon_p^\alpha \delta \varepsilon_p^\beta
\]

Where \( \sigma_f^\alpha \varepsilon_p^\alpha \), \( \sigma_f^\beta \varepsilon_p^\beta \) and \( \sigma_f^\gamma \varepsilon_p^\gamma \) are the bulk flow stresses of EI, EII and EIII at deformation point B when they alone deformed uniaxially to strains of \( \varepsilon_p^\alpha \), \( \varepsilon_p^\beta \) and \( \varepsilon_p^\gamma \), respectively. Inserting the internal stresses in eqn (5.16) from eqn (5.6) at point B, one obtains

\[
\sigma_f^\alpha (\varepsilon_p^\alpha + \varepsilon_p^\beta) = \sigma_{33}^A + B \left[ \left( f_{\beta C}(\varepsilon_p^\beta + \varepsilon_p^\gamma) + F_s(\varepsilon_p^\alpha + \varepsilon_p^\beta) - (f_{\beta C} + F_s)(\varepsilon_p^\alpha + \varepsilon_p^\beta) \right) \right]
\]

\[
\sigma_f^\beta (\varepsilon_p^\beta + \varepsilon_p^\gamma) = \sigma_{33}^A + B \left[ \left( f_{\alpha C}(\varepsilon_p^\alpha + \varepsilon_p^\gamma) + F_s(\varepsilon_p^\beta + \varepsilon_p^\gamma) - (f_{\alpha C} + F_s)(\varepsilon_p^\beta + \varepsilon_p^\gamma) \right) \right]
\]

\[
\sigma_f^\gamma (\varepsilon_p^\gamma) = \sigma_{33}^A + B \left[ \left( f_{\alpha C}(\varepsilon_p^\alpha) + f_{\beta C}(\varepsilon_p^\beta) + (f_{\alpha C} + f_{\beta C})(\varepsilon_p^\alpha + \varepsilon_p^\beta) \right) \right]
\]

Where \( B = \frac{(5-4\nu)E}{4(1-\nu^2)} \).

At the limit of \( \delta \varepsilon_p^\alpha \rightarrow 0 \), \( \delta \varepsilon_p^\beta \rightarrow 0 \) and \( \delta \varepsilon_p^\gamma \rightarrow 0 \), and rearranging eqn (5.17), giving the following simultaneous equations

\[
\sigma_{33}^A = \sigma_f^\alpha (\varepsilon_p^\alpha) - B \left( f_{\beta C} \varepsilon_p^\beta + F_s \varepsilon_p^\alpha - (f_{\beta C} + F_s) \varepsilon_p^\alpha \right)
\]

\[
\sigma_{33}^A = \sigma_f^\beta \varepsilon_p^\beta - B \left( f_{\alpha C} \varepsilon_p^\alpha + F_s \varepsilon_p^\beta - (f_{\alpha C} + F_s) \varepsilon_p^\beta \right)
\]

\[
\sigma_{33}^A = \sigma_f^\gamma (\varepsilon_p^\gamma) - B \left( f_{\alpha C} \varepsilon_p^\alpha + f_{\beta C} \varepsilon_p^\beta - (f_{\alpha C} + f_{\beta C}) \varepsilon_p^\gamma \right)
\]
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The above simultaneous equations describe the flow curve of the 3-E body in Stage 4 deformation. If the plastic strain in one element is prescribed, the applied stress and the plastic strain in the other two elements can be determined by solving the above simultaneous equations.

In addition, the plastic work done by the whole composite from point A to point B should be equal to the sum of the plastic work done by each element, i.e.,

\[
\sigma_{33} \delta \epsilon_p = \left[ A \left[ f_{\alpha c}(\epsilon_p + \delta \epsilon_p) + F_s(\epsilon_p + \delta \epsilon_p) - (f_{\beta c} + F_s)(\epsilon_p + \delta \epsilon_p) \right] \right] f_{\alpha c} \delta \epsilon_p
\]

\[
+ \left[ A \left[ f_{\alpha c}(\epsilon_p + \delta \epsilon_p) + F_s(\epsilon_p + \delta \epsilon_p) - (f_{\alpha c} + F_s)(\epsilon_p + \delta \epsilon_p) \right] \right] f_{\beta c} \delta \epsilon_p
\]

\[
+ \left[ A \left[ f_{\alpha c}(\epsilon_p + \delta \epsilon_p) + F_s(\epsilon_p + \delta \epsilon_p) - (f_{\alpha c} + F_s)(\epsilon_p + \delta \epsilon_p) \right] \right] F_s \delta \epsilon_p
\]

Rearranging eqn (5.19), one gets

\[
\delta \epsilon_p = f_{\alpha c} \delta \epsilon_p + f_{\beta c} \delta \epsilon_p + F_s \delta \epsilon_p
\]

Eqn (5.20) then describes the plastic strain distribution between the three microstructural elements of the whole 3-E body.

Next, the equations which describe the flow curve of other stages and critical points will be deduced. This will start from Stage 2 deformation as Stage 1 deformation is elastically homogeneous, which can be expressed by the Hooke's law, and consequently the onset of plastic deformation in EI is defined by \( Y(i) = \sigma_y \).

(2) Stage 2 Deformation of the 3-E Body

The condition for Stage 2 deformation of the 3-E body is defined in Table 5.2 as that EI deforms plastically while the deformation of EII and EIII remains elastic. Thus, \( \epsilon_p = \epsilon_{\beta} = 0 \).

It is obvious that eqns (5.18b) and (5.18c) are not relevant and from eqns (5.18a) and (5.20) one gets the following equations which describe the flow curve during Stage 2 deformation of the 3-E body

\[
\sigma_{33} = \sigma_f(\epsilon_p) + B(f_{\beta c} + F_s) \epsilon_p \tag{5.21}
\]

\[
\delta \epsilon_p = f_{\alpha c} \delta \epsilon_p - \frac{B}{A} \left[ (f_{\beta c} + F_s)(\epsilon_p + \delta \epsilon_p) \right] f_{\alpha c} \delta \epsilon_p \tag{5.22}
\]
(3) Onset of Plastic Deformation in EI and of the 3-E Body

When the applied stress, $\sigma_{33}^A$, reaches the critical point $Y(ii)$, the plastic deformation starts in EI and EI. The plastic strain in EI at this point is denoted as $\varepsilon_p^a(ii)$, and the following conditions are valid at this critical point:

$$\varepsilon_p^a = 0, \quad \sigma_{33}^A = Y(ii), \quad \sigma_f^a(\varepsilon_p^a) = \sigma_y^a = \sigma_y^f \alpha_{III} + \sigma_y^f \beta_{III}$$

Thus, the following two simultaneous equations can be derived from eqn (5.18) together with the above conditions

$$\begin{align*}
Y(ii) &= \sigma_f^a(\varepsilon_p^a(ii)) + B(f_{\beta c} + F_S \varepsilon_p^a)(ii) \\
Y(ii) &= \sigma_y^a - Bf_{\alpha c} \varepsilon_p^a(ii)
\end{align*}$$

(5.23)

The plastic strain increment in the 3-E body at this critical point can also be calculated by eqn (5.22).

(4) Stage 3 Deformation of the 3-E Body

During Stage 3 deformation of the 3-E body, EI and EI still remain elastic. Following a similar argument, one has the following simultaneous equations

$$\begin{align*}
\sigma_{33}^A &= \sigma_f^a(\varepsilon_p^a) - B\left\{F_S \varepsilon_p^a(\alpha_{III} - (f_{\beta c} + F_s) \varepsilon_p^a(\alpha_{III})\right\} \\
\sigma_{33}^A &= \sigma_f^a(\varepsilon_p^a) - B\left\{f_{\alpha c} \varepsilon_p^a - (f_{\alpha c} + f_{\beta c}) \varepsilon_p^a(\alpha_{III})\right\}
\end{align*}$$

(5.24)

The plastic strain increment in the 3-E body is determined by the following equation

$$\begin{align*}
\delta \varepsilon_p^c &= f_{\alpha c} \delta \varepsilon_p^a + F_S \delta \varepsilon_p^a \\
+ B \left\{ \left[f_S \varepsilon_p^a(\alpha_{III}) - (f_{\beta c} + F_s) \varepsilon_p^a(\alpha_{III})\right] f_{\alpha c} \delta \varepsilon_p^a \right\}
\end{align*}$$

(5.25)

Eqns (5.24) and (5.25) describes the flow curve of the 3-E body during Stage 3 deformation.

(5) Onset of Plastic Deformation in EI and of the 3-E Body

When the applied stress, $\sigma_{33}^A$, reaches a critical point $Y(iii)$, EI starts to deform plastically, and one has the following condition at this critical point:

$$\sigma_{33}^A = Y(iii), \quad \sigma_f^a(\varepsilon_p^a) = \sigma_y^a, \quad \varepsilon_p^a = \varepsilon_p^a(iii), \quad \varepsilon_p^a = \varepsilon_p^a(iii)$$

where $\varepsilon_p^a(iii)$ and $\varepsilon_p^a(iii)$ are the critical plastic strain in EI and EI, respectively. At this critical point, the following simultaneous equations can be deduced from eqn (18)

$$\begin{align*}
Y(iii) &= \sigma_f^a(\varepsilon_p^a(iii)) - B[F_S \varepsilon_p^a(\alpha_{III}) - (f_{\beta c} + F_s) \varepsilon_p^a(iii)] \\
Y(iii) &= \sigma_y^a - B[f_{\alpha c} \varepsilon_p^a(iii) + F_S \varepsilon_p^a(iii)] \\
Y(iii) &= \sigma_f^a(\varepsilon_p^a(iii)) - B[f_{\alpha c} \varepsilon_p^a(iii) - (f_{\alpha c} + f_{\beta c}) \varepsilon_p^a(iii)]
\end{align*}$$

(5.26)
The plastic strain increment in the whole 3-E body can be calculated from eqn (5.25).

(6) The Total Strain in the 3-E Body

Here again, only the plastic strains in each element and the 3-E body have been considered, since the total strains in EI, EII, EIII and the 3-E body can be conveniently expressed in terms of the sum of the plastic strain and elastic strain.

\[
\begin{align*}
\varepsilon^\alpha_T &= \varepsilon^\alpha_p + \frac{A_{33}}{E} \\
\varepsilon^\beta_T &= \varepsilon^\beta_p + \frac{A_{33}}{E} \\
\varepsilon^\alpha\beta_T &= \varepsilon^\alpha\beta_p + \frac{A_{33}}{E} \\
\varepsilon^c_T &= \varepsilon^c_p + \frac{A_{33}}{E}
\end{align*}
\]

for EI, EII, EIII and 3-E body, respectively.

where \(\varepsilon^\alpha_T, \varepsilon^\beta_T, \varepsilon^\alpha\beta_T\) and \(\varepsilon^c_T\) are the total strains in the EI, EII, EIII and 3-E body, respectively.

5.6. Summary

A new approach to the deformation behaviour of two-ductile-phase alloys has been developed based on the Eshelby's continuum transformation theory and the microstructural characterization in Chapter 3. In contrast to the existing theories of plastic deformation, the new theory can take into account the effects of microstructural factors, such as volume fraction, grain size, grain shape and phase distribution. The interactions in the two-phase materials have been analysed. There are three kinds of interactions in an elastically homogeneous composite: (i) the interaction between the elements (or phases), the results of which are the internal stresses and strains in each element (or phase); (ii) the interaction between the inclusion and the stress-free external surfaces, which leads to the image stresses; (iii) the interaction between the particles of the same phase, which has been taken into account by an equivalent microstructural transformation. The theoretical derivation has been made for the mean internal stresses and strains in each microstructural element and the true stress-true strain curve of the EIII body and 3-E body. In addition, the present approach has a provision for the calculation of the in situ stress and plastic strain distribution among three microstructural elements.
CHAPTER 6

Deformation Behaviour of Two-Ductile-Phase Alloys
—With Particular Reference to Microstructure
II: Applications of the Theory

6.1. Introduction

In this chapter, the stress-strain curves, the mean internal stress in each microstructural element and the in situ stress and plastic strain distribution among three microstructural elements will be calculated for various two-ductile-phase alloys by applying the theory developed in Chapter 5. The theoretical calculations will be compared with the experimental results drawn from the literature. The discussions will be made in terms of the effects of microstructural parameters and the difference between the mechanical properties of constituent phases on the flow curves, the mean internal stresses and the in situ stress and plastic strain partition among three microstructural elements.

6.2. Comparison between the Theoretically Calculated Flow Curves and the Experimental Results

In order to calculate the flow curves for the two-ductile-phase alloys, it is necessary to prescribe the flow curves of the individual phases. The flow curves of the \( \alpha \)- and \( \beta \)-phases are described here by the Ludwik equation [09Lud], i.e.,

\[
\sigma_f^\alpha = \sigma_y^\alpha + K_\alpha (\varepsilon_F^\alpha)^{n_\alpha}
\]

(6.1a)

\[
\sigma_f^\beta = \sigma_y^\beta + K_\beta (\varepsilon_F^\beta)^{n_\beta}
\]

(6.1b)

where \( \sigma_y^\alpha \) and \( \sigma_y^\beta \) are the yield stresses, \( K_\alpha \) and \( K_\beta \) are the work hardening coefficients and \( n_\alpha \) and \( n_\beta \) are the work-hardening exponents. The subscripts and superscripts \( \alpha \) and \( \beta \) here and hereafter refer to the \( \alpha \)- and \( \beta \)-phases.

The above parameters for the alloy systems in concern can be found or evaluated from the experimental results in the literature. The parameters for \( \alpha \)-\( \beta \) Ti-Mn alloys are evaluated from the experimental flow curves of near \( \alpha \)-phase and near \( \beta \)-phase alloys after Ankem and Margolin [86Ank], those for ferrite-martensite dual-phase steels are from Uggowitzer and Stuwe's [80Ugg, 81Ugg, 82Ugg2] experimental results, and those for \( \alpha \)-\( \gamma \) Fe-Cr-Ni stainless steels are evaluated from the experimental results for single phase steels after Tomota et al [76Tom]. The obtained parameters for these alloy systems are summarized in Table 6.1.
## Table 6.1. Summary of parameters used for the calculations.

<table>
<thead>
<tr>
<th>Alloy System</th>
<th>E(GPa)</th>
<th>ν</th>
<th>Kₐ(MPa)</th>
<th>K₈(MPa)</th>
<th>n₁</th>
<th>n₂</th>
<th>σᵧ(MPa)</th>
<th>σᵧ(MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ti-Mn</td>
<td>110</td>
<td>1/3</td>
<td>770</td>
<td>850</td>
<td>0.35</td>
<td>0.5</td>
<td>220</td>
<td>950</td>
</tr>
<tr>
<td>Fe-C-Mn</td>
<td>210</td>
<td>1/3</td>
<td>827</td>
<td>185</td>
<td>0.66</td>
<td>0.4</td>
<td>83</td>
<td>765</td>
</tr>
<tr>
<td>Fe-Cr-Ni (Q)</td>
<td>206</td>
<td>1/3</td>
<td>640</td>
<td>750</td>
<td>0.54</td>
<td>0.31</td>
<td>137</td>
<td>460</td>
</tr>
<tr>
<td>Fe-Cr-Ni (Q+A)</td>
<td>206</td>
<td>1/3</td>
<td>640</td>
<td>1460</td>
<td>0.54</td>
<td>0.31</td>
<td>137</td>
<td>820</td>
</tr>
</tbody>
</table>

Note: Q: Quench; Q+A: Quench + Age; α and β refer to the softer and the harder phases in the dual-phase mixture, respectively.

![Graphs](image)

**Fig 6.1.** Comparison between the theoretically predicted flow curves and the experimental results of four α-β Ti-Mn alloys with different volume fractions of the β-phase (as indicated in the brackets) in the lower strain region. Experimental data is from Ankm and Margolin [82Ank1, 82Ank2]. (a) fₜ = 0.836; (b) fₜ = 0.59; (c) fₜ = 0.366; (d) fₜ = 0.187.
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Fig 6.2. A comparison of the theoretically predicted flow curves of $\alpha$-$\gamma$ Fe-Cr-Ni stainless steels with different volume fraction of the $\alpha$-phase ($f_\alpha=0.51$ in (a) and $f_\alpha=0.67$ in (b)). The experimental values are taken from Tomota et al [76Tom]. Here Q stands for quenched, Q+A stands for Quenched and then Aged.

The topological parameters for Ti-Mn alloys were calculated from eqns (4.18-20) following the suggestion made by Werner and Stuwe [84Wer]. The topological parameters for ferrite-martensite dual-phase steels with two types of heat treatments are from Uggowitzer and Stuwe [80Ugg, 81Ugg, 82Ugg2] (see Fig 3.4). For $\alpha$-$\gamma$ Fe-Cr-Ni stainless steels, the topological
parameters are not available. In the absence of such information, the experimentally determined topological parameters for ferrite-martensite dual-phase steels with Type A heat treatment from Uggowitzer and Stuwe [80Ugg, 81Ugg, 82Ugg2] are adopted here.

A Fortran program (see Appendix) has been designed to perform all the calculations. During the calculations, the plastic strain in EI is always prescribed, and the plastic strain increment in EI is set at $10^{-5}$ for Stage 2, Y(ii), Stage 3 and Y(iii), and at $10^{-3}$ for Stage 4.

The calculated flow curves in the low strain region for four $\alpha$-\$\beta$ Ti-Mn alloys with different volume fraction of the $\beta$-phase are shown in Fig 6.1 in comparison with the experimentally measured flow curves by Ankem and Margolin [82Ank1, 82Ank2]. The comparisons in the low strain region are also made in Fig 6.2 for $\alpha$-$\gamma$ Fe-Cr-Ni stainless steels with different volume fractions of the $\alpha$-phase and different heat treatments. The experimental results used in Fig 6.2 are from Tomota and coworkers [76Tom]. Figs 6.1 and 6.2 indicate that the theoretical predictions are in good agreement with the experimental results in the lower strain region. The comparisons at strains up to fracture for $\alpha$-$\beta$ Ti-Mn alloys are shown in Fig 6.3, which also reveals a very good agreement. It is worth noting that the discrepancy between the theoretical calculations and the experimental results tends to be larger when the volume fraction of the $\beta$-phase changes towards 0.5. This may be a result of the increasing inaccuracy of the topological parameters used for the calculations.

![Fig 6.3. Comparison between the theoretical predictions and the experimental results of different $\alpha$-$\beta$ Ti-Mn alloys with different volume fraction of the $\alpha$-phase (as indicated in the legend box) in the high strain region. Experimental data was taken from Ankem and Margolin [86Ank].](image-url)
Uggowitzer and Stuwe [82Ugg2] investigated the effect of phase distribution on the mechanical properties of ferrite-martensite dual-phase steels and measured the flow curves of two ferrite-martensite dual-phase steels with same volume fraction of martensite phase, 50%, but with different topological parameters. The calculated flow curves for those two steels are compared with their experimental results in Fig 6.4. Although the experimental results show that there is a significant effect of the phase distribution on the flow curves of dual-phase steels, especially in the higher strain region, the calculated flow curves of these two steels show very little difference in Stage 4 deformation; a significant effect can only be found in stages 2 and 3 as indicated in Fig 6.5, in which the comparisons between the flow curves of Type A and Type B dual-phase steels are made in the low strain region for four Fe-C-Mn dual-phase steels with different volume fractions of martensite phase. The comparison in Fig 6.5 shows that although the topological parameters do not markedly affect the flow curves of two-ductile-phase alloys in Stage 4 deformation in the high strain region, they do have certain influence on the plastic deformation behaviour in the low strain region, especially on the Stage 2 deformation.

![Fe-C-Mn Dual-Phase Steels (0.5)](image-url)

**Fig 6.4.** The theoretically predicted flow curves for two ferrite-martensite dual-phase steels with the same volume fraction of martensite but different phase distributions in comparison with experimental results after Uggowitzer and Stuwe [82Ugg2].

It is quite possible that the difference in experimental flow curves [82Ugg2] for those two dual-phase steels in Fig 6.4 is caused by factors other than the difference in contiguity parameters, for example, by the difference in mechanical properties of each phase in those two steels caused by the difference in heat treatment.

Since the experiments are carried out on microstructure A (Fig 3.15a) and the theoretical calculations are made on the corresponding microstructure B (Fig 3.15b), the above good agreement offers a strong proof to the mechanical equivalence between microstructures A and B proposed in Chapter 3.
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6.3. Calculation of the Mean Internal Stresses in Three Microstructural Elements

It has long been recognised that the mean internal stresses play a very important role in the deformation process of two-phase materials. But the theoretical modeling is not well developed [88Wit]. However, the deformation theory developed in Chapter 5 has a provision for calculation of the mean internal stresses in three microstructural elements of two-ductile-phase alloys. The mean internal stresses in three microstructural elements can be calculated at each deformation point according to eqn (5.6) in Chapter 5. The calculated results of a ferrite-martensite dual-phase steel (Type A) with 50% of martensite phase are shown in Fig 6.6, which are representative of all the two-ductile-phase alloys.

![Fig 6.5](image_url)

**Fig 6.5.** The comparison between the calculated flow curves of Type A and Type B ferrite-martensite dual-phase steels with different volume fraction of martensite phase. (a) \( f_\beta = 0.3 \); (b) \( f_\beta = 0.4 \); (c) \( f_\beta = 0.5 \); (d) \( f_\beta = 0.75 \).
Fig 6.6. The theoretically calculated mean internal stresses in each microstructural element along the tensile direction and two transverse directions in a Fe-C-Mn dual-phase steel (Type A) with 50% of martensite phase.

Because \( \langle \sigma_{11} \rangle_i \) and \( \langle \sigma_{22} \rangle_i \) are related to \( \langle \sigma_{33} \rangle_i \) through eqn (5.6) in Chapter 5, hereafter, only the mean internal stress along the tensile direction, \( \langle \sigma_{33} \rangle_i \), will be calculated and discussed, where the subscript "i" stands for EI, EII and EIII. Fig 6.6 shows that the mean internal stress in the softest element (EI) along the tensile direction, \( \langle \sigma_{33} \rangle_{EI} \), is negative, that in the hardest element (EII), \( \langle \sigma_{33} \rangle_{EII} \), is positive, while that in EIII, \( \langle \sigma_{33} \rangle_{EIII} \), can either be negative or be positive depending on the volume fraction and contiguity of the constituent phases. Therefore, it is obvious that the mean internal stress in the softest element acts as a back stress, which impedes further plastic deformation in it, while the mean internal stress in hardest element acts as a load transfer stress, which aids further plastic deformation in the latter. To sum up, the role of the mean internal stresses is to make the plastic deformation of two-ductile-phase alloys tend to be more uniform throughout the 3-E body.

Furthermore, a close examination of the mean internal stress in EI in the low strain region (Fig 6.6) indicates that the magnitude of the mean internal stress in EI, \( \langle \sigma_{33} \rangle_{EI} \), increases rapidly with the increasing macroscopic plastic strain during Stage 2 deformation, and drop dramatically after the onset of plastic deformation in EIII, then increases again during Stage 3 and Stage 4 deformation, but with less increasing rate. The mean internal stress caused by the plastic strain incompatibility between EI and other two elements will be accumulated in EI in
the form of elastic energy during Stage 2 deformation. After the onset of plastic deformation in EIII, the plastic incompatibility between EI and EIII is suddenly reduced, so some of the stored elastic energy in EI has to be released in order to maintain the energy balance at a new level, which consequently causes a reduction of the mean internal stress in EI. This phenomenon is typical of all the two-ductile-phase alloys.

Fig 6.7. The theoretically calculated mean internal stresses in each microstructural element along the tensile direction in \( \alpha-\gamma \) Fe-Cr-Ni stainless steels with same volume fraction of the \( \alpha \)-phase (51\%) but with different heat treatments. (a) for Quenched steel in the low strain region; (b) for Quenched steel in the high strain region; (c) for Quenched and then Aged steel in the low strain region; (d) or Quenched and then Aged steel in the high strain region.
The calculated mean internal stresses in three microstructural elements in \( \alpha\)-\( \gamma \) Fe-Cr-Ni stainless steels which contain the same volume fraction of the \( \alpha \)-phase (51\%) but have been subjected to different heat treatments (leading to different mechanical properties of constituent phases, see Table 6.1) are shown in Fig 6.7. Fig 6.7 indicates that the difference between the mechanical properties of the constituent phases has a significant influence not only on the magnitude of the mean internal stress but also on the trend of variation with increasing plastic strain. The yield stress of the quenched \( \alpha \)-phase is about 3 times of that of the quenched \( \gamma \)-phase, while after aging, the yield stress of the \( \alpha \)-phase is about 6 times of that of the \( \gamma \)-phase. As a consequence of this difference, the magnitude of the mean internal stresses in EI and EII in the quenched and aged steel are much larger than those in the alloy which has only been quenched. In addition, for the quenched alloy, the magnitude of the mean internal stresses in EI and EII decrease with increasing macroscopic plastic strain during Stage 4 deformation, while the magnitude of the mean internal stresses in EI and EII in the quenched and aged steel tend to increase with the increasing macroscopic plastic strain, as shown in Fig 6.7.

![Graph showing the mean internal stresses in three microstructural elements for Type A and B ferrite-martensite dual-phase steels.](image)

**Fig 6.8.** The theoretically calculated mean internal stresses in three microstructural elements along the tensile direction for Type A and B ferrite-martensite dual-phase steels with 50\% of martensite phase.

Figs 6.8 and 6.9 show the comparison between the calculated mean internal stresses in two types of ferrite-martensite dual-phase steels with same volume fraction of martensite phase but having different phase distribution as described by the topological parameters presented in Fig 3.4. Figs 6.8 and 6.9 indicate that \( \left< \sigma_{33} \right>_{EI} \) and \( \left< \sigma_{33} \right>_{EII} \) in Stage 4 deformation are not
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affected markedly by the phase distribution, but the phase distribution does have a pronounced influence on $\sigma_{33}^{\text{III}}$. Furthermore, the phase distribution has a significant effect on the mean internal stresses in all three microstructural elements during Stages 2 and 3 deformation as shown in Figs 6.8 and 6.9.

It is necessary to point out that according to the present theory the mean internal stress in grains of the same phase can be very different, depending on the nature of their neighbours. For example, the mean internal stress in an $\alpha$-grain in EI will be quite different from that in an $\alpha$-grain in EIII. This makes it very difficult to determine experimentally the values of mean internal stresses in each constituent phase of two-ductile-phase alloys by X-ray techniques. Consequently, it is difficult to compare the theoretical calculations with the experimental results [70Nak, 80Tom].

![Image](image_url)

**Fig 6.9.** The theoretically calculated mean internal stresses in three microstructural elements along the tensile direction for Type A and B ferrite martensite dual-phase steels with 75% of martensite phase.

### 6.4. The Deformation Stages and the Macroscopic Yielding Start Point in Two-Ductile-Phase Alloys

The theoretically calculated flow curves in the very low strain region for $\alpha$-$\beta$ Ti-Mn alloys with different volume fractions of the $\beta$-phase are presented in Fig 6.10. Similar results for ferrite-martensite dual-phase steels (Type A and B) with different volume fraction of martensite are
Fig 6.10. The calculated true stress-strain curves of $\alpha-\beta$ Ti-Mn alloys with different volume fraction of the $\beta$-phase, also shown here are the theoretically calculated critical points for each alloy.

In contrast to the existing deformation theories, the present approach predicts that there are 4 deformation stages in the whole deformation process of two-ductile-phase alloys, of which Stage 1 is elastic, while Stages 2, 3 and 4 are plastic. This is in agreement with the experimental results in Fe-C-Mn-Si dual-phase steels obtained by Lawson and coworkers [81Law], who analysed the true stress-true plastic strain curves of several dual-phase steels with different volume fractions of martensite-phase by the Jaoul-Crussard method [57Jau]. Their results of analysis indicate that there are three plastic deformation stages during the plastic deformation process of dual-phase steels.

Broadly speaking, Stage 2 deformation has the largest work-hardening rate while Stage 4 has the smallest work-hardening rate, and the work-hardening rate in Stage 3 lies between those of Stage 2 and Stage 4. According to Chapter 5, the internal stress in one element caused by the presence of other elements is proportional to the plastic strain difference between those elements at a fixed volume fraction. During Stage 2 deformation, the plastic strain in EII and EIII is zero, so that a very large plastic strain incompatibility between EI and EII and EIII is expected, which can cause very large internal stresses in each element. The internal stress is
stored in each element in the form of elastic energy, which is responsible for the highest work hardening rate in Stage 2 deformation. When the plastic deformation reaches a critical point \( Y(ii) \), plastic deformation starts in EI\(\text{II} \), the plastic strain in EI\(\text{II} \) is not zero any more. As a consequence of the reduction of the plastic incompatibility, the work hardening rate will be reduced. The same argument explains the reduction of the work hardening rate in Stage 4 deformation.

In addition, since the flow stresses of the constituent phases in two-ductile-phase alloys are not very much different from each other, the extent of stage 2 and Stage 3 deformation is insignificant for large plastic deformation. Consequently, the critical points \( Y(ii) \) and \( Y(iii) \) are closely located in a small strain region, and the strains at which the critical point \( Y(i) \), \( Y(ii) \) and \( Y(iii) \) are reached are not very much different from each other, as indicated in Figs 6.10 and 6.11.

Another interesting point which emerges from Figs 6.10 and 6.11 is that there are always a drop in flow stress after the onset of plastic deformation in EI\(\text{II} \) and EI\(\text{I} \), especially after the onset of plastic deformation in EI\(\text{III} \). As discussed previously, there is a dramatic relief of the elastic energy stored in EI after the onset of the plastic deformation in EI\(\text{III} \), which is the cause of this flow stress drop. This argument also explains the flow stress drop after \( Y(iii) \), but because the reduction of the plastic strain incompatibility at \( Y(iii) \) is much less than that at \( Y(ii) \), the stress drop after \( Y(iii) \) will be much less than that after \( Y(ii) \). Since the onset of plastic deformation in EI marks the microscopic yielding point of 3-E body, the onset of plastic deformation in EI\(\text{III} \) can be regarded as the start of the macroscopic yielding, as the work hardening rate in Stage 2 deformation is so large that the flow curve in this stage hardly deviates from the linear section.

There is an experimental evidence which supports the above argument. Long and Yin [90Lon] recently investigated the acoustic emission (AE) during the plastic deformation of dual-phase steels. Their results show that the AE always reaches a peak at the point where the macroscopic yielding begins. This means that there is a dramatic elastic energy release at this point. If the second critical point, \( Y(ii) \), is taken as the macroscopic yielding start point, this AE peak is coincident with the flow stress drop after \( Y(ii) \). In their work, a second peak is also observed after the first peak in some dual-phase steels with high cooling rate. They believed that the second AE peak is caused by the cracking mainly at the ferrite-martensite interfaces, and secondly within the martensite phase. But it is quite possible that the second AE peak is caused by the sudden release of the elastic energy accumulated in EI and EI\(\text{III} \) during the stages 2 and 3 deformation after the onset of plastic deformation in EI\(\text{I} \), since void nucleation, growth and coalescence are all time-dependent processes, which are unlikely to happen just at one particular strain, and consequently cannot cause a single AE peak.
Fig 6.11. The calculated true stress-strain curves of Type A (a) and Type B (b) ferrite-martensite dual-phase steels with different volume fraction of martensite-phase in the low strain region, also shown here are the theoretically calculated critical points for each alloy.

6.5. The Plastic Inhomogeneity in Two-Ductile-Phase Alloys

The inhomogeneity of plastic deformation among three microstructural elements has been taken into account during the theoretical development in Chapter 5. In fact, the in situ stress-strain field in each microstructural element is the superposition of the applied stress-strain field and
the mean internal stress-strain field. Therefore, the in situ true stress and true plastic strain distribution in three microstructural elements can be calculated at each deformation point. The calculated in situ true stress and true plastic strain distribution in an α-β Ti-Mn alloy with 36.6% β-phase are presented in Fig 6.12 together with the calculated true stress-true plastic strain curve for the two-phase composite (denoted as C). Also shown in Fig 6.12 are the tie-lines which connect four simultaneous points (plastic strains and stresses in EI, EII, EIII and the whole composite). Fig 6.12 clearly show that the plastic deformation in the low strain region is inhomogeneous, the tie-lines in this region has a large negative slope, but with the increasing macroscopic plastic strain, the tie-lines move gradually towards the vertical position, this means that the plastic deformation becomes more and more uniform with increasing macroscopic plastic strain.

Fig 6.12. The theoretically calculated in situ stress and plastic strain distribution among three microstructural elements in an α-β Ti-Mn alloy with 36.6% of the β-phase. (a) in the low plastic strain region; (b) in the medium plastic strain region; (c) in the high plastic strain region.
Precisely, the plastic strain distribution is governed by eqn (5.20) in Chapter 5:

\[
\delta e_p = f_{ac} \delta e^a_p + f_{bc} \delta e^b_p + F_s \delta e^s_p
\]

\[
+ \left[ f_{bc}(\delta e^b_p + \delta e^a_p) + F_s(\delta e^s_p + \delta e^a_p) - (f_{ac} + F_s)(\delta e^s_p + \delta e^a_p) \right] f_{ac} \delta e^a_p
\]

\[
+ \left[ f_{ac}(\delta e^a_p + \delta e^a_p) + f_{bc}(\delta e^b_p + \delta e^a_p) - (f_{ac} + f_{bc})(\delta e^b_p + \delta e^a_p) \right] f_{bc} \delta e^b_p
\]

from (5.20)

It consists of two parts, the first part is the linear law of mixtures, while the second part is the terms in the curved brackets multiplied by \(B/c_{33}^A\). The contributions by these two parts were deliberately made separate during the calculation. It is interesting to find that the contribution from the second term is extremely small at a level of \(1/10^5\) of that from the first term at the whole plastic strain region for all the alloy systems concerned here. Therefore, to a very good approximation, the plastic strain distribution among the three microstructural elements can be described by the linear law of mixture for two-ductile-phase alloys, i.e.,

\[
\epsilon_p^c = \epsilon_p^a f_{ac} + \epsilon_p^b f_{bc} + \epsilon_p^s F_s
\]

(6.2)

The increasing homogeneity of plastic deformation in two-ductile-phase alloys can be explained by the role played by the internal stresses. During the plastic deformation, if there is a plastic strain difference between any two microstructural elements, this plastic strain incompatibility will cause a positive internal stress in the harder element and a negative internal stress in the softer element along the tensile direction, and thus these internal stresses will aid the plastic deformation in harder element and impede the further plastic deformation in the softer element. Therefore, it is the internal stresses which make the plastic deformation of two-ductile-phase alloys tend to be more and more homogeneous with the increasing macroscopic plastic strain.

To understand the effect of the difference between the mechanical properties of the constituent phases on the in situ stress and plastic strain distribution among three microstructural elements, the in situ stress and plastic strain distributions have been theoretically calculated for two \(\alpha\)\-\(\gamma\) Fe-Cr-Ni stainless steels with same volume fraction of the \(\alpha\)-phase (67%) but different heat treatments, which lead to the different flow properties of the \(\alpha\)-phase as shown in Table 6.1. The calculated results are shown in Figs 6.13 and 6.14.

A comparison between Figs 6.13 and 6.14 reveals that the difference between the mechanical properties of the constituent phases has a significant influence on the in situ stress and plastic strain distribution among three microstructural elements. If this difference is smaller, as is true for quenched \(\alpha\)-\(\gamma\) Fe-Cr-Ni stainless steel in Fig 6.13, the plastic strain tends to distribute more uniformly among three microstructural elements, as shown by the positions of the tie-lines in Fig 6.13, while if this difference is larger, like quenched and aged \(\alpha\)-\(\gamma\) Fe-Cr-Ni stainless steel
in Fig 6.14, the plastic strain tends to distribute more inhomogeneously, as shown by the large negative slope of the tie-lines in Fig 6.14.

Fig 6.13. The theoretically calculated in situ stress and in situ plastic strain distribution among three microstructural elements in a quenched $\alpha$-$\gamma$ Fe-Cr-Ni stainless steel with 67% of the $\alpha$-phase.

Fig 6.14. The theoretically calculated in situ stress and in situ plastic strain distribution among three microstructural elements in a quenched and aged $\alpha$-$\gamma$ Fe-Cr-Ni stainless steel with 67% of $\alpha$-phase.
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The comparison between Figs 6.13 and 6.14 also indicate that the difference between the mechanical properties of constituent phases also affects the stress partition among three microstructural elements. If the difference is small, the stress partition tends to become more uniform with increasing macroscopic plastic strain, if the difference is large, the stress partition tends to become less uniform.

---

**Fig 6.15.** The theoretically calculated in situ stress and in situ plastic strain distribution among three microstructural elements in a Type A ferrite-martensite dual-phase steel with 75% of martensite-phase.

---

**Fig 6.16.** The theoretically calculated in situ stress and in situ plastic strain distribution among three microstructural elements in a Type B ferrite-martensite dual-phase steel with 75% of the martensite-phase.
Figs 6.15 and 6.16 show the calculated in situ stress and strain distribution among three microstructural elements in Type A and B ferrite-martensite dual-phase steels with same volume fraction of martensite phase (75%). A comparison between Figs 6.15 and 6.16 shows that the phase distribution described by the topological parameters has a very strong influence on the in situ stress and plastic strain partition among three microstructural elements as indicated by the tie-lines. However, the true stress-true plastic strain curves for the whole α-β composite (denoted by C in Figs 6.15 and 6.16) do not depend very much on the phase distribution. Therefore, it can be concluded that the phase distribution described by the topological parameters has a very strong effect on the local deformation behaviour, such as the mean internal stresses and strains, the in situ stress and plastic strain distribution in three microstructural elements, but it does not affect very much the macroscopic deformation behaviour described by the true stress-true strain curves. This is in agreement with the results obtained by Le Hazif et al [83Haz]. These authors constructed a three dimensional model to calculate the flow properties of two-phase alloys by finite element analysis and their model can consider different parameters characteristic of the microstructure, such as volume fraction and continuity of constituent phases. Their results showed that the macroscopic behaviour described by the stress-strain curve does not depend very much on the continuity of each phase, but on the contrary, the local behaviour depends strongly on such parameters.

6.6. Concluding Remarks

(1) The true stress-true strain curves of various two-ductile-phase alloys have been theoretically calculated by applying the new approach developed in Chapter 5. The theoretical predictions are in good agreement with the experimental results draw from the literature not only in the low strain region but also in the high strain region.

(2) The mean internal stresses in three microstructural elements have been calculated for various two-ductile-phase alloys. The calculated results show that the mean internal stress in EI (the softest element) is negative, that in EII (the hardest element) is positive and that in EIII can be either negative or positive depending on the volume fraction and phase distribution. Therefore, it can be concluded that the role of the mean internal stresses is to imped the further plastic deformation in the softer element, to aid the further plastic deformation in the harder elements and consequently to make the plastic deformation of two-ductile-phase alloys tend to be more homogeneous throughout the whole microstructure.

(3) The present approach predicts that there are four deformation stages in the whole deformation process of two-ductile-phase alloys. This is different from the classical 3-stage deformation theory. The calculated stress-strain curves of various two-ductile-phase alloys show a flow stress drop after the onset of plastic deformation in EIII or EII, and
this stress drop has been explained in terms of the elastic energy release, which is supported by the experimental evidence in the literature. It is believed that the onset of the plastic deformation in EIII marks the starting point of macroscopic plastic deformation of the two-phase mixture.

(4) The in situ stress and the in situ plastic strain distributions among three microstructural elements in various two-ductile-phase alloys have been calculated. The results of the calculations reveal that the in situ plastic strain partition among three microstructural elements tends to be more and more homogeneous with the increasing macroscopic plastic strain, due to the role played by the internal stresses during the plastic deformation.

(5) The effect of the difference between the mechanical properties of the constituent phases on the stress-strain curves, the mean internal stresses and the in situ stress and plastic strain distribution in the three microstructural elements has been discussed. It is found that this difference has a significant influence on the mean internal stresses and the in situ stress and plastic strain distribution not only on the magnitude but also on the variation trend with increasing macroscopic plastic strain.

(6) The effect of phase distribution described by the topological parameters on the stress-strain curves, the mean internal stresses and the in situ stress and plastic strain distribution in the three microstructural elements has also been discussed. It is found that the phase distribution has a significant effect on the local deformation behaviour of two-ductile-phase alloys in terms of mean internal stresses and the in situ stress and plastic strain distribution in the three microstructural elements, although it does not affect markedly the macroscopic deformation behaviour described by the true stress-strain curves of two-phase composites during Stage 4 deformation.
7.1. Introduction

The search for light alloys of high stiffness has stimulated research in metal matrix composites (MMCs). Despite a drastic fall in ductility, large improvements in the stiffness on the unreinforced alloys have been achieved. Therefore, it would appear that the majority of engineering applications will exploit their excellent stiffness rather than their high ultimate tensile strength [88Wit]. Thus it is important that the elastic behaviour of composite materials is well understood, and that the upper limit of the attainable mechanical properties, especially the elastic moduli, can be predicted.

In this chapter, a new approach for predicting the Young's moduli of two-phase composites will be developed based on the mean field theory [83Ped] and the topological transformation in Chapter 3. The new approach can solve the interaction problem and can consider the effect of particle shape and distribution on the elastic moduli of the composite materials. The predictions from the present approach will be justified by comparison with the experimental results and other theoretical predictions for the Co/WCp, Al/SiCp and glass-filled epoxy composites. The effect of continuity of constituent phases in the particulate composites will also be discussed.

7.2. Development of the Approach

It is assumed that a uniaxial stress $\sigma^A (\sigma^A \leq \sigma_Y^c$, where $\sigma_Y^c$ is the yield stress of the composite) is applied along the aligned direction of the 3-E body in Fig 3.15b. From the strain compatibility requirement, we have

$$e^I = e^{II} = e^{III} = e^c$$  \hspace{1cm} (7.1)

where $e^I$, $e^{II}$, $e^{III}$ and $e^c$ are the in situ elastic strains in EI, EII, EIII and the whole 3-E body, respectively. For a subsequent elastic strain increment in the whole 3-E body, $\delta e^c$, we write the elastic strain increments in EI, EII and EIII as $\delta e^I$, $\delta e^{II}$ and $\delta e^{III}$, respectively. From eqn (7.1) it is obvious that

$$\delta e^I = \delta e^{II} = \delta e^{III} = \delta e^c$$  \hspace{1cm} (7.2)
Chapter 7: Prediction of the Young's Moduli of Particulate Two-Phase Composites

According to the virtual work principle

$$\sigma^c \delta e^c = \sigma^I \delta e^I f_{\alpha c} + \sigma^{II} \delta e^{II} f_{\beta c} + \sigma^{III} \delta e^{III} F_s$$  \hspace{1cm} (7.3)

where $\sigma^I$, $\sigma^{II}$, $\sigma^{III}$ and $\sigma^c$ are the in situ stresses in the three microstructural elements (I, II and III) and the 3-E body, respectively. Combining eqns (7.2) and (7.3) one gets

$$\sigma^c = \sigma^I f_{\alpha c} + \sigma^{II} f_{\beta c} + \sigma^{III} F_s$$  \hspace{1cm} (7.4)

Dividing both sides of eqn (7.4) by the in situ elastic strain in the whole 3-E body, $e^c$, and noting that $e^I = e^{II} = e^{III} = e^c$, gives

$$\sigma^c / e^c = (\sigma^I / e^c) f_{\alpha c} + (\sigma^{II} / e^c) f_{\beta c} + (\sigma^{III} / e^c) F_s$$  \hspace{1cm} (7.5)

Then, applying Hooke's law, eqn (7.5) is written as

$$E = E^I f_{\alpha c} + E^{II} f_{\beta c} + E^{III} F_s$$  \hspace{1cm} (7.6)

where $E^I$, $E^{II}$, and $E^{III}$ are the Young's moduli of elements I, II, III and the whole 3-E body, respectively. It is obvious that $E^I = E^\alpha$ and $E^{II} = E^\beta$, where $E^\alpha$ and $E^\beta$ are the Young's moduli of single $\alpha$-phase and $\beta$-phase alloys and can be determined from tests of single phase specimens, while $E^{III}$ can be calculated theoretically. Eqn (7.6) relates the Young's modulus of the $\alpha$-$\beta$ composite to the Young's moduli of three microstructural elements.

Since $E^{III}$ contains only phase boundaries, there is no direct contact between the particles of the same phase, and consequently, interactions between particles of the same phase in $E^{III}$ can be neglected (see Chapter 5). Thus the mean field approach [78Ped, 83Ped] can be applied to $E^{III}$ body in a more rigorous manner. By applying the mean field theory, the following equation can be derived

$$G^{III} = \frac{G^\alpha}{1 + f_{\beta III} D / (1 - f_{\beta III} \gamma D)}$$  \hspace{1cm} (7.7)

where

$$D = \frac{G^\alpha - G^\beta}{(G^\beta - G^\alpha)(1 - \gamma) + G^\alpha}$$  \hspace{1cm} (7.8)

$$\gamma = \frac{7 - 5 \nu^\alpha}{15 (1 - \nu^\alpha)}$$  \hspace{1cm} (7.9)

where $G^\alpha$ and $G^\beta$ are the shear moduli of the $\alpha$- and $\beta$-phases, $G^{III}$ is the shear modulus of the $E^{III}$ body, $\gamma$ is the strain accommodation tensor for a sphere, $\nu^\alpha$ is the Poisson's ratio of the matrix.

Once $G^{III}$ is obtained from eqn (7.7), $E^{III}$ can be calculated from the following equation

$$E^{III} = 2G^{III} (1 + \nu^{III})$$  \hspace{1cm} (7.10)

where $\nu^{III}$ is the Poisson's ratio of the $E^{III}$ body and it is assumed that $\nu^{III} = \nu_{\alpha III} + \nu_{\beta III}$. From the above equations the Young's modulus of the two-phase composite can be calculated.
7.3. Discussion

7.3.1 Comparison Between the Theoretical Calculations and Experimental Results

In the following part of this chapter, the effectiveness of the new approach developed in the previous sections will be explored by comparing predictions various theories and experimental results from Co/WCp and Al/SiCp composites as well as glass-filled epoxy composites. The mechanical properties of the constituent phases in the three composite systems used for all the theoretical calculations are summarised in Table 7.1. Also listed in Table 7.1 are the stiffness ratio of the constituent phases in each composite system.

Table 7.1. The mechanical properties of constituent phases used for all the theoretical calculations.

<table>
<thead>
<tr>
<th>System</th>
<th>Phase</th>
<th>G (GPa)</th>
<th>K (GPa)</th>
<th>E (GPa)</th>
<th>ν</th>
<th>E/G/EEp</th>
</tr>
</thead>
<tbody>
<tr>
<td>Co/WCp [60Pau, 70Doi]</td>
<td>Co</td>
<td>80.2</td>
<td>184.2</td>
<td>210.0</td>
<td>0.31</td>
<td>3.3</td>
</tr>
<tr>
<td></td>
<td>WC</td>
<td>293.1</td>
<td>381.3</td>
<td>700.0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Al/SiCp [88Wit, 91Dav]</td>
<td>Al</td>
<td>26.1</td>
<td>75</td>
<td>70.0</td>
<td>0.34</td>
<td>6.0</td>
</tr>
<tr>
<td></td>
<td>SiC</td>
<td>189.0</td>
<td>181.1</td>
<td>420.7</td>
<td>0.19</td>
<td></td>
</tr>
<tr>
<td>Glass-filled Epoxy [90Ahm]</td>
<td>Glass beads</td>
<td>28.7</td>
<td>41.59</td>
<td>70.0</td>
<td>0.22</td>
<td>18.9</td>
</tr>
<tr>
<td></td>
<td>Epoxy</td>
<td>1.33</td>
<td>5.66</td>
<td>3.7</td>
<td>0.39</td>
<td></td>
</tr>
</tbody>
</table>

In many investigations of the mechanical properties of two-phase materials the Co/WCp system has been used as a model system in which numerous experimentally measured E values are available in the literature. A further reason for the choice of this system in the present investigation is that the topological parameters required for the calculation can be found in the experimental work of Fischmeister and Exner [66Fis] and Lee and Gurland [78Lee]. Their contiguity data for the WC phase (denoted as the β-phase) in Co-WC composites can be expressed as

\[ C_\beta = f_\beta^3 \]  

(7.11)

Here it is assumed that the contiguity of Co (denoted as the α-phase) follows the same trend of variation with the volume fraction of Co-phase, i.e.,

\[ C_\alpha = f_\alpha^3 \]  

(7.12)

The other required parameters can be calculated from eqns (7.11) and (7.12) according to Chapter 3. In general, the continuous volumes of α- and β-phases are assumed to follow the following equations

\[ f_{\alpha c} = f_{\alpha}^m \]  

(7.13)

\[ f_{\beta c} = f_{\beta}^n \]  

(7.14)

The elastic constants and Poisson’s ratios of Co and WC are taken from the experimental results of Paul [60Pau] and Doi et al [70Doi], as listed in Table 7.1.
Fig 7.1. A comparison of the theoretically predicted Young's moduli of the Co-WC system by the present approach with the theoretical predictions by HS upper and lower bounds [63Has], the FEM calculations [72Jae] and the experimental results [63Has, 70Doi, 60Nis, 60Vor, 53Kei]. (a) the whole volume fraction range; (b) the high volume fraction range.

The calculated Young's moduli of the Co-WC system by the present approach are shown in Fig 7.1 as a function of the volume fraction of the WC phase. Also shown in Fig 7.1 are the experimental data from a number of investigators [63Has, 70Doi, 60Nis, 60Vor, 53Kei], the results from finite element method (FEM) calculations [72Jae] and the theoretical predictions by
HS lower and upper bounds [63Has]. It is necessary to be reminded that the HS bounds for the effective composite modulus in Fig 7.1 also represent the bounds predicted by the mean field theory [88Wit, 83Ped] as mentioned in Section 7.1. Fig 7.1(b) shows the high volume fraction region in Fig 7.1(a). In Fig 7.1 the theoretical predictions of the present approach are well within the HS lower and upper bounds. Furthermore, there is a better agreement between the present theoretical predictions and the experimental results [63Has, 70Doi, 60Nis, 60Vor, 53Kei] especially at high volume fractions of the WC phase (Fig 7.1(b)) than the theoretical predictions of both the HS bounds and the mean field theory.

In recent years there has been a great deal of interest in the Al/SiCp composites and experimental data on the Young's modulus is available. Therefore it is interesting to justify the present approach in light of the experimental data for the Al/SiCp composites. Because the topological parameters required by the present approach are not available for Al/SiCp system, it will be assumed that, just as in the Co-WC system, the continuous volumes of the α-phase and β-phase are represented by $f_\alpha^4$ and $f_\beta^4$, respectively, i.e., $m=n=4$ in eqns (7.13) and (7.14). The other required microstructural parameters can then be calculated according to Chapter 3.

A comparison of the Young's moduli of Al/SiCp composites is made in Fig 7.2(a) among the theoretical predictions by the present approach, the HS bounds (which also represent the theoretical prediction by the mean field theory) and the recently reported experimental results [86Mcd, 87Mah, 87DWA, 87Tru, 91Har, 91Gei]. The small variations in the Young's moduli of the different unreinforced aluminium alloys have not been considered in this comparison. Fig 7.2(b) shows the predicted Young's moduli of Al/SiCp composites by the present approach and by the HS bounds over the whole volume fraction range. Fig 7.2 indicates that the theoretical predictions by the present approach are in a better agreement with the experimental results than those by the other two theories.

Another group of composites of commercial interests are the polymer matrix composites, where the stiffness ratio of the two constituent phases is normally large. In a glass-filied epoxy composite $E_p/E_\alpha$ can be as large as 20. Therefore, from both theoretical and practical view points, it is interesting to apply the present approach to this system. The required topological parameters are calculated by assuming $m=n=4$.

The predicted Young's moduli for graded glass beads filled epoxy composites as a function of volume fraction of glass beads are shown in Fig 7.3(a) and in comparison with the experimental data from [84Spa] and [90Ahm2], as well as with the theoretical predictions by the HS bounds. The HS lower bound in Fig 7.3(a) is closer to the experimental data than the predictions by the present approach under the above microstructural condition. However, in graded glass bead filled epoxy system the filler normally tends to be much more separated than the matrix. If the continuous volume of the glass beads is assumed to be 0 ($f_{pc}=0$) and that of
the \( \alpha \)-phase to be \( f_{\alpha c} = f_{\alpha}^3 \) (m=3), the calculated Young’s moduli of this system are much closer to the experimental data. A comparison between the predictions of the present approach and of the HS bounds over the whole volume fraction range for this system is shown in Fig 7.3(b).

Fig 7.2. (a) A comparison among the theoretically predicted Young’s moduli of Al/SiCp composites by the present approach, the theoretical predictions by HS upper and lower bounds [63Has] and experimental results [86Mcd, 87Mah, 87DWA, 87Tru, 91Har, 91Gei], (a) at low volume fraction of SiC particles; (b) at the whole volume fraction range.
Fig 7.3. A comparison the theoretically predicted Young's moduli of glass-filled epoxy composites by the present approach, the theoretical predictions by HS upper and lower bounds [63Has] and experimental results [90Ahm, 84Spa]. (a) in low volume fraction of glass beads among; (b) in the whole volume fraction range.

Furthermore, as shown in Figs 7.1(a), 7.2(b) and 7.3(b), the Young’s moduli predicted by the present approach are close to the HS lower bound at low volume fractions and approach the HS upper bound at high volume fraction. This is an interesting result, because one would
Chapter 7: Prediction of the Young's Moduli of Particulate Two-Phase Composites

intuitively expect the lower bound to be more closely followed at low volume fractions, whereas the stiffness could approach the upper bound at high volume fractions, as pointed out by Withers [88Wit].

Since the experiments were carried out on microstructure A (Fig 3.15a) and the theoretical calculations were made on the corresponding microstructure B (Fig 3.15b), the above results offers further support to the mechanical equivalence between microstructures A and B proposed in Chapter 3.

7.3.2. The Effect of Phase Continuity on the Young's Moduli of the Composites

An advantage of the present approach over the HS bounds and the mean field theory is that it can take into account the microstructural effects on the elastic constants of two phase composites. Unfortunately, no experimentally determined contiguity data has been found in the literature for the constituent phases of composite materials, thus, it is impossible to compare quantitatively the theoretical predictions with experimental results. However, the predicted effects of phase continuity on the Young's moduli of composite materials do offer some useful information for understanding the elastic behaviour of composites. The effect of phase continuity on the composites' stiffness will be discussed in some detail.

If the continuous volume of the matrix is fixed at a given volume fraction (i.e., making the assumption that $f_{ae} = f_{a}^{d}$ and $m=4$), the effect of the continuity of the reinforcing phase on the Young's modulus can be calculated by varying the value of $n$ in $f_{be} = f_{b}^{n}$ (the smaller $n$ is the more continuous the reinforcing phase). Fig 7.4(a) is a plot of the calculated Young's moduli by the present approach for different $n$ values for the Co-WCp system. Similar results for Al/SiCp and glass-filled epoxy composites are presented in Figs 7.4(b) and 7.4(c), respectively. As shown in Fig 7.4, at a fixed volume fraction of the reinforcing phase, the predicted Young's moduli increase with increasing continuity of the reinforcing phase when the continuity of the matrix phase is kept constant. The predicted Young's moduli reaches the Paul's upper bound (eqn (2.38)) when the reinforcing phase becomes completely continuous and perfectly aligned ($n=1$). In fact, increasing the continuity of reinforcing phase in particulate composites is equivalent to increasing the aspect ratio in aligned short fibre composites, in which the effect of the fibre aspect ratio on the stiffness of the composites has been investigated intensively. It is well known that the stiffness of the aligned short fibre composites increases with increasing fibre aspect ratio [89Wit]. Furthermore, Fig 7.4 show that at a fixed volume fraction, the increase in composites' stiffness with increasing continuity of the reinforcing phase is dependent on the stiffness ratio of the constituent phases. The larger the stiffness ratio, the larger the increment in stiffness with increasing continuity of the reinforcing phase at a fixed volume fraction.
Chapter 7: Prediction of the Young's Moduli of Particulate Two-Phase Composites

Fig 7.4. A plot of the theoretically predicted Young's moduli of Co/WCp (a), Al/SiCp (b) and glass-filled epoxy composites (c) by the present approach against volume fraction of the reinforcing phase showing the effect of varying continuity of the reinforcing phase (by changing the $n$ value in eqn (7.14), as indicated in the legend box) on the composite's stiffness when the continuity of the matrix is fixed at a given volume fraction ($m=4$ in eqn (7.13)).

Fig 7.5. A plot of the theoretically predicted Young's moduli of Co/WCp (a), Al/SiCp (b) and glass-filled epoxy composites (c) by the present approach against volume fraction of the reinforcing phase showing the effect of varying contiguity of matrix (by changing the $m$ value in eqn (7.13) as indicated in the legend box) on the composites' stiffness when the reinforcing phase is completely separated ($f_{bc}=0$).
Now, the present approach will be applied to predict the composites' Young's moduli in the two extreme cases of microstructures. In the first case, both the matrix and the included phase are assumed to be continuous, which is equivalent to a perfectly aligned continuous fibre composite. Then \( f_{ac} = f_a, f_{bc} = f_b \) and \( F_s = 0 \), i.e., \( m=n=1 \) in eqns (7.13) and (7.14). Eqn (7.6) is now reduced to the classical linear law of mixtures, i.e.,

\[
E^c = E^\alpha f_a + E^\beta f_b
\]  

(7.15)

Therefore, the classical linear law of mixtures is just a specific case of eqn (7.6). However, in contrast to the classical linear law of mixtures, eqn (7.6) can be derived theoretically from continuum mechanics and can be applied to a composite with any volume fraction, grain shape and phase distribution. In the second case of extreme microstructure, the reinforcing phase is assumed to be completely separated, i.e., \( C_p = 0 \), and the continuous volume of matrix can be changed by varying \( m \) in \( f_{ac} = f_m^a \). Thus, the smaller the \( m \) value is, the more continuous the matrix is. The predicted \( E \) values by the present approach as a function of volume fraction of reinforcing phase and the \( m \) values are shown in Figs 7.5(a), 7.5(b) and 7.5(c) for the Co-WC\(_p\), Al/SiC\(_p\) and glass-filled epoxy composites, respectively. Fig 7.5 shows that at fixed volume fraction of the reinforcing phase and \( f_p = 0 \), the predicted \( E \) values of the composite materials increase with the decreasing \( m \) values. If \( m=1 \), the material becomes a continuously reinforced composite which has been discussed previously.

7.4. Concluding Remarks

(1) A new approach for predicting the Young's modulus of two-phase composites has been developed and has been applied to the Co-WC\(_p\), Al/SiC\(_p\) and glass filled epoxy composites;

(2) It has been shown that the theoretical predictions by the present approach are well within Hashin and Shtrikmans' lower and upper bounds and in better agreement with the experimental results in the corresponding composite systems than both the HS bounds and the predictions of the mean field theory;

(3) An advantage of the present approach over other continuum approaches is that it can predict not only the effect of volume fraction of the reinforcing phase but also the effects of microstructural parameters such as grain shape and phase distribution on the composites' Young's moduli;

(4) It is shown that the classical linear law of mixtures is a specific case of the present approach, where the reinforcement is perfectly aligned continuous fibre. However, in contrast to the classical linear law of mixtures, the present approach is also applicable to a two-phase composite with any volume fraction, grain shape and phase distribution;

(5) In particulate composite systems, the present approach predicted that the Young's modulus increases with the increasing continuity of the constituent phases. This increase in \( E \) values is dependent on the stiffness ratio of the reinforcement to matrix. The larger the stiffness ratio, the larger the increment in stiffness with increasing continuity of the reinforcing phase at fixed volume fraction;

(6) The present approach can also provide a simple but effective solution to the problem of interaction between the particles of the same phase.
CHAPTER 8

Experimental Investigation of Microstructure and Mechanical Properties of In Situ Ti/TiB Metal Matrix Composites Obtained by Rapid Solidification Process

Part I. Experiments and Results

8.1. Introduction

Metal matrix composites (MMCs) have been investigated since the early 1960s with the impetus at that time being the high potential structural properties that would be achieved with materials engineered to specific applications [65Kel, 65Cra]. Initial MMC work was stimulated by the high performance needs of the aerospace industry which places performance ahead of the price, at least in developmental programs. However, after 20 years of development, it has become obvious that an MMC must also be cost-competitive with other materials. Thus emphasis of research in MMC is now toward novel processing techniques, cheaper reinforcement and near net shape production techniques [91Mor, 88Mor].

Until now, the available techniques for MMC fabrication include: (i) diffusion bonding with thin sheet interleaved with ceramic fibres [84Smi, 85Rho1, 85Rho2]; (ii) mechanical processing of ceramic suspensions in molten metal ("compocasting") [78Lev, 82Qui]; (iii) premixing and consolidation of metallic powders and ceramic whiskers, fibres and particles [81Div, 84Fro]; (iv) melt infiltrating into a preform consisting of an assembly of ceramic fibres [85Cly, 85Cor, 87Cly]; (v) spray codeposition of metal droplets and ceramic particles [84Nam, 87Hug, 87Tsu, 87Jac]; (vi) in situ growth MMCs from molten alloys [89Koc]. The main concerns in the first five processing techniques are the following two recurrent problems: nearly all the commercially significant reinforcements are poorly wetted by molten Al, Ti and their alloys, and several significant reinforcements react severely with molten Al and Ti. The art and science of fabrication of MMCs mainly consist of overcoming this two problems.

From a processing view point, the advantages of in situ processing compared with the conventional powder metallurgy or casting particulate approaches include: (i) cost-effective; (ii) homogeneous distribution of reinforcement; (iii) finer particles; (iv) improved wetting between the particulate and matrix; (v) elimination of the deleterious interface reaction.

Rapid solidification (RS) processing can provide a novel route for fabrication of MMCs, especially for the systems in which the reinforcement react strongly with the matrix during their
fabrication process, for instance, SiC or boride reinforced Ti-alloy matrix. As discussed in Chapter 2, rapid solidification of liquid alloys at cooling rate greater than $10^4$Ks$^{-1}$ results in significant undercooling of the melt and leads to several metastable effects, which has been conveniently divided into constitutional and microstructural effects [91Sur]. RS allows large departures from equilibrium constitution and results in large extension of solid solubility, as shown in Table 2.3. The equilibrium solid solubility of B in both $\alpha$ and $\beta$ Ti is less than 1 at.-%. By applying RS, this solubility can be extended beyond 10 at.-% [86Wha1]. This advantage of RS processing over conventional ingot metallurgy can offer us an effective way of producing Ti solid solution with supersaturated B in it. Upon subsequent aging of the RS product, a ceramic phase (in this case TiB) will nucleate within the titanium matrix to form a fine and uniform distribution of ceramic reinforcement [86Sas].

In this chapter, the results from an experimental investigation of microstructure and mechanical properties of in situ Ti/TiB composites produced by rapid solidification processing will be presented. The experimental procedure will be briefly introduced in section 2. The experimental results on microstructures and mechanical properties of materials obtained under different thermomechanical conditions will be presented in section 3.

8.2 Experimental Procedure

8.2.1. Alloy Chemistry

Pure titanium provides greater than 1 at.-% maximum equilibrium solid solubility for about 45 elements [91Sur]. In addition, since pure titanium exhibits two allotropic forms (cph $\alpha$ and bcc $\beta$) to the melting point, an additional degree of freedom is available for developing microstructures and hence the mechanical properties. The alloying elements for Ti can be broadly divided into two groups [65Mol], one is the $\alpha$-stabilizers, e.g., Al, the other is the $\beta$-stabilizer, e.g., V, depending on whether the alloying element enhances the stability of the $\alpha$-phase or that of the $\beta$-phase. In an $\alpha$-$\beta$ alloy, such as Ti-6Al-4V, both $\alpha$- and $\beta$-stabilizers are present. The final microstructure will depend upon the relative efficiency and the amount of addition of each stabilizer, and the thermomechanical history [82Bro]. Ti-6Al-4V is a widely used commercial alloy, and has been extensively studied in the past [82Bro]. Thus there is a better understanding of microstructural evolution and the response of mechanical properties to the microstructural changes in this alloy. Therefore, Ti-6Al-4V has been chosen here as the matrix alloy for fabrication of in situ Ti/TiB MMCs. The calculated Ti-Al-V ternary phase diagrams (isothermal sections) [91Cha] are presented in Fig 8.1, and the calculated vertical section at constant Al (6 wt.-%) and V (4 wt.-%) contents for Ti-Al-V-B system [91Cha] in Fig 8.2. The predicted $\alpha/\beta$ transus is 935°C for pure Ti-6Al-4V alloy without B addition [91Cha], which is in good agreement with the experimental result (937°C) of Kahveci and Welsch [86Kah]. The predicted eutectic reaction temperature (L→$\beta$(Ti) + TiB) for Ti-6Al-4V-XB alloys is 1597°C, and the eutectic composition for the liquid phase is 1.58 wt.-% B. Fig 8.3 shows the calculated Ti-B binary phase diagram [91Cha], in which the
Chapter 8: Experiments on In Situ Ti/TiB MMCs. Part I. Experiments and Results

eutectic reaction takes place at 1.6 wt.-% B, 1540°C. A comparison between Fig 8.2 and Fig 8.3 indicates that the addition of α- and β-stabilizers does not significantly change the stability of TiB phase, due to the low solubility of B in both α- and β-Ti. The B contents chosen for the composite production are 0.1, 0.2, 0.5, 0.8, 1.5 and 2.0 wt.-% B. The alloys with less than 1.5 wt.-% are hyper-eutectic alloys, Ti-6Al-4V-1.5B is a near eutectic alloy and Ti-6Al-4V-2B is a hypo-eutectic alloy.

![Diagram](image1)

**Fig 8.1.** Calculated isothermal sections of Ti-Al-V ternary phase diagram [91Cha].
(a) 800°C; (b) 900°C; (c) 1000°C; (d) 1100°C.

Notes: 1: α; 2: α+β; 3: α+α2; 4: α+β+α2; 5: α2+γ; 6: α2+β+γ.

Two Ti-Mn alloys with the same amount of B (0.5wt.-%) were also made to produce different volume fractions of β-phase in the matrix. These binary Ti-Mn alloys contained 3.0 and 7.5 wt.-% Mn, and were designed to have 25% and 75% β-phase (at 700°C), respectively. Although the Ti-Mn binary phase diagram in Fig 8.4 shows that Mn is a eutectic former, Mn is normally behaved like a β-isomorphous element, such as V and Mo, because of the very sluggish eutectic reaction [91Sur].
Chapter 8: Experiments on In Situ Ti/TiB MMCs. Part I. Experiments and Results

**Fig 8.2.** Calculated vertical section of Ti-Al-V-B quaternary phase diagram at constant Al (6 wt.-%) and V (4 wt.-%) [91Cha].

**Fig 8.3.** Calculated Ti-B binary phase diagram [91Cha].
Chapter 8: Experiments on In Situ Ti/TiB MMCs. Part I. Experiments and Results

8.2.2. Melting and Melt-Spinning

Melting and melt-spinning of all the alloys designed for this project were performed in a MARKO's advanced melt spinner (Model 5T), which has been designed with a capability to melt spin metals, alloys and compounds with melting temperature up to 3000°C. Melting and chill casting (melt-spinning) of alloys was carried out in a stainless steel chamber with a high vacuum/inert gas atmosphere. The alloy was melted in a water cooled copper hearth using a non-consumable tungsten electrode under a high purity argon atmosphere. The melt was then delivered at a controlled rate to contact the circumferential surface of a rotating Mo wheel whereby the melt is rapidly solidified as long fibres with a crescent-shaped cross-section, typically 100-300 μm in width and 40-100 μm in thickness. A schematic illustration of the Marko's Model 5T melt spinner is shown in Fig 8.5. The operating parameters used in this work were as follows: the wheel speed was 2500 rpm, the gap between the V-notch on the copper hearth and the tip of the Mo wheel was about 1.5 mm. The total time for melting and melt-spinning was less than 6 min.

The alloys containing boron were produced by melting together Ti-6Al-4V ingots and TiB₂ powder (99.5% purity supplied by Johnson Matthey Ltd.) wrapped in 0.025 mm thick 99.6% pure titanium foil. To ensure the chemical homogeneity of the melted alloy, the ingots were melted at least three times before final melt spinning. Each melting and melt-spinning operation

Fig 8.4. Ti-Mn binary phase diagram [89Mur].
Chapter 8: Experiments on In Situ Ti/TiB MMCs. Part I. Experiments and Results

was conducted after evacuation to at least $10^{-5}$ tor followed by two flushes of the chamber with argon of 99.998% purity. Chemical analysis of the melt spun products showed that no severe B loss and no pickup of interstitials during the melting and melt-spinning processes (see Tables 8.1 and 8.3).

The Ti-Mn-B alloys were produced by melting together commercially pure (CP) titanium ingot, CP Mn tablets and TiB$_2$ powders (99.5% purity). All other procedures were the same as for the production of Ti-6Al-4V-XB alloys.

![Key to the diagram:
(1) Stainless steel vacuum chamber  
(2) Rotating casting wheel  
(3) Water-cooled copper hearth  
(4) Copper electrode holder  
(5) Tungsten electrode  
(6) DC power generator  
(7) Ionization gauge control  
(8) Heavy duty steel stand  
(9) Variable speed drive  
(10) Air inlet valve  
(11) Sub-atmosphere pressure gauge  
(12) Argon inlet valve  
(13) Quartz view point  
(14) Nylon handle  
(15) Power stat for DC power generator  
(16) Switch for tilting motor  
(17) Tilting motor  
(18) Tow-stage gas regulator  
(19) Argon tank  
(20) Mechanical vacuum valve  
(21) Filter  
(22) Mechanical pump  
(23) Gate valve  
(24) Foreline Valve  
(25) Diffusion pump  
(26) Thermocouple gauge

Fig 8.5. Schematic diagram of the MARKO 5T melt-spinner. The arrows in this diagram indicate the water in and out directions.
8.2.3. Comminution of Melt-Spun Fibres

To enable further consolidation of the melt-spun fibres and produce suitable specimens for microstructure examination and mechanical property determination, it is necessary to comminute first the melt spun fibres into finer particles, which also ensure the chemical and microstructural homogeneity of the consolidated products. Fig 8.6 shows the diagram of the comminuting apparatus used for this purpose. It consists of a set of tungsten carbide hammers mounted on the end of a rotary shaft and free to move inside the annular space provided by the casing. When the first chopped melt-spun fibres (<30 mm in length) are fed into the mill through the hopper, comminution is achieved by repeating the crushing of the melt spun fibres against the casing wall by hammers rotating at high speed. The resulting comminuted product was sieved to obtain the -200μm fraction. In most of the cases it was possible to comminute all the melt spun fibres to this size fraction. The whole set-up is enclosed in a glove-box which allows the milling operation to be conducted in an argon atmosphere, thereby avoiding the possibility of spontaneous combustion and any further oxygen pick-up by the alloys. The chemical analysis of the comminuted powders in a later section showed that there is a slight increase of oxygen during the comminution process in most of the cases.

Fig 8.6. Schematic diagram of the comminuting apparatus.
8.2.4. Pre-Hipping Heat Treatment of Comminuted Powders

For pre-hipping heat treatment of the comminuted powders, approximately 5g alloy was wrapped by pure titanium foil and put into a quartz tube with about 30g sponge titanium for gettering residual oxygen within the quartz tube. The tube was then evacuated to about $10^{-3}$ tor and sealed. The samples were heat treated in a pre-heated furnace at designed temperature and time, and then quenched in water or air.

8.2.5. Consolidation

The procedure adopted for consolidation was as follows. The comminuted alloy powders were filled into cylindrical Ti-alloy cans which have a hole drilled into their lids. The lids were first electron-welded leaving the hole open. These cans were then baked in the temperature range of 250-300°C in vacuum ($10^{-5}$ tor) before the holes were sealed by electron beam welding. This procedure was adopted to remove occluded gases in the powder compacts. They were then loaded into a High Temperature Isostatic Presser (hipper). The consolidation of different rapidly solidified Ti-alloys were carried out at 900-920°C, 150-300 MPa for two hours. Some of the consolidated alloys were then subjected to a further 40% reduction in cross section by hot pressing (forging) the can at 900°C. Flat test specimens were made from these flat pieces after removing the can material by milling.

8.2.6. Microstructural Examination

1) Optical and Scanning Electron Microscopy

Samples for optical microscopy (OM) and scanning electron microscopy (SEM) were taken from the products at different stages of processing. These samples were then cut into suitable shape and mounted in conductive bakelite. Mounting conditions to minimise degradation were approximately 150°C for 5 minutes. The mounted specimens were prepared using a Struers automatic polishing machine, PLANOPOL-2 with PEDEMAX head. The samples were polished by grinding on grinding papers down to 4000 grit followed by polishing on 3μm and 1μm diamond wheels. The polished specimens were chemically etched by using Kroll’s reagent (2ml HF (40 vol.-%) and 3 ml HNO₃ in 95 ml distilled water). A Zeiss optical microscope and a Cambridge 250 SEM were used to examine the microstructure of these samples.

(2) Transmission Electron Microscopy

Specimens for transmission electron microscopy (TEM) can be prepared by both ion beam milling and electrolytical polishing. To produce electropolished specimens, a thin section (about 200 μm in thickness) was sliced from the bulk material by an ACUTOM machine and then punched into 3mm diameter discs. These discs were then ground down to a thickness of about 100 μm on 4000 grit paper to finish. To prevent their loss they were held in a special brass holder during this process. Finally, the discs were electrolytically polished to
performance in a "TENUPOL" unit using an electrolyte of 5 vol.-% perchloric acid in methanol. The polishing temperature was -40°C, the voltage 30V, the current about 30 mA and the flow rate was set at 0.5. To prepare thin foils by ion beam milling, the discs before milling were undergone the same route as mentioned above for electropolished specimens. The discs were then ion beam milled in a Cambridge GATON dual ion beam miller at room temperature. The Voltage was set at 6 kV and the specimen current was about 1 A. The thinning angle was 12°.

Thin foils for TEM examination were also prepared from melt spun fibres. The copper-plating technique developed by Adkins et al [91Adk] for preparation of TEM thin foils from gas atomised powders was adapted here. 20-30 melt spun fibres were cut into 30 mm in length and bundled together at two ends by copper wire. The fibre bundle was plated using a current density of 200 mA/cm² at room temperature for 20 hours. The plating bath contained 100 g copper sulphate (BDH analar), 32 ml H₂SO₄ (BDH analar), 0.25 g naphthalene tri-sulphonic acid (BDH analar) and 0.0025 g thiourea (BDH analar) plus water to 500 ml. After plating the fibres were firmly bound together by the plated copper between them. The final product was a round bar similar to a copper matrix composite reinforced by the continuous titanium fibres. This composite bar was sliced by an ACUTOM machine into discs of 200 μm in thickness. These discs were then punched into 3 mm in diameter and ground down to a thickness of 100 μm on 4000 grit paper to finish. Finally, the same procedure described above for ion beam thinning was used to mill the discs to performance. The thin foils prepared by this method contained the cross-sections of melt spun fibres.

(3) X-Ray Diffraction

X-ray diffraction (XRD) was used to identify the phases present in different samples and to monitor changes during the different stages of composite fabrication. For this purpose, a PHILIPS diffractometer was used and operated under a voltage of 46 kV and a current of 35 mA. The diffraction angle (2θ) was scanned from 15 to 130° for every sample at 2°/min. scan rate. For more accurate determination of peak position the lower scan rates were also used (1°/min., 1/2°/min. and 1/4°/min.). The phases were characterised using JCPDS files [87JCPDS].

Samples were also analysed at DRA by X-ray diffraction using Cu Kα radiation at 40 kV and 40 mA and employing a PSA detector set at a resolution of 0.01 degree 2θ. Scans were made over the range of 20-70 degree 2θ. The raw data was smoothed before a peak search was carried out.

8.2.7. Tensile Testing

Tensile test pieces were milled from the consolidated and forged material. The final test piece was a thin sheet with a thickness of approximately 1.5-2.5 mm and a gauge length of 14-16 mm. The other dimensions are illustrated in Fig 8.7. All the tensile tests were performed at DRA at room temperature on a Mayes 100 kN Servo-Hydraulic Uniaxial Tensile/Fatigue Testing Machine fitted with hydraulic grips. The strain rate used was 10⁻³/sec.. Load-
elongation curves were obtained on an X-Y plotting table which was linked to the load cell of the machine and to an extensometer clamped to the test piece.

\[\text{Fig 8.7. Schematic drawing of the thin specimen used for tensile testing. All the dimensions are in mm.}\]

8.2.8. Microhardness Measurement

Specimens prepared for optical microscopy were also used for microhardness tests. After optical microscopy examination, the etched specimens were polished on the 1µm diamond wheel to ensure the accuracy of the hardness measurement. The hardness test was carried out on a AKASHI microhardness tester using a 500g load. All quoted results represent the average of 5~10 measurements.

8.3. Experimental Results

8.3.1. Microstructure of as Received Ti-6Al-4V Ingot

Ti-6Al-4V is a widely used commercial Ti-base alloy. Its microstructure can be tailored by annealing at different temperature above or below the β-transus, 935°C for this alloy [91Cha]. The final volume fractions of the α- and β-phases depend on both the annealing temperature and the cooling rate. The microstructure of as received Ti-6Al-4V ingot from DRA is shown in Fig 8.8(a). The microstructure of the same alloy after being annealed at 900°C for 1 hour and cooled in the furnace is presented in Fig 8.8(b). The volume fraction of the β-phase in as received state (Fig 8.8(a)) and the annealed state (Fig 8.8(b)) are 0.09 and 0.08, respectively. Results of the chemical analysis of the as received Ti-6Al-4V alloy ingot are listed in Table 8.1. The interstitial oxygen was 2050 ppm in this master alloy.

A sample was cut from as received Ti-6Al-4V ingot and annealed at 800°C for 2 hours. The cooling from 800°C was programmed to produce a cooling rate of 38°C/hr. The slowly cooled sample was X-rayed and the results are presented in Fig 8.9. Note that the position of the (110)β peak is between (0002)α and (01T1)α. The peak position is a function of the lattice parameter which in turn is a function of phase composition. This diffractograph was used as a relative standard for later work.
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Fig 8.8. Microstructure of Ti-6Al-4V alloy. (a) as received from DRA; (b) annealed at 900°C for 1 hr and furnace-cooled.

Fig 8.9. X-Ray diffractograph of Ti-6Al-4V alloy annealed at 800°C for 2 hrs and then cooled in furnace with a cooling rate of 38°C/hr.

Table 8.1. Results of chemical analysis of as received Ti-6Al-4V alloy ingot.

<table>
<thead>
<tr>
<th>Element</th>
<th>Ti</th>
<th>Al</th>
<th>V</th>
<th>Fe</th>
<th>C</th>
<th>N₂</th>
<th>O₂</th>
<th>H₂</th>
</tr>
</thead>
<tbody>
<tr>
<td>wt.-%</td>
<td>Balance</td>
<td>6.43</td>
<td>4.02</td>
<td>0.19</td>
<td>0.01</td>
<td>0.0075</td>
<td>0.205</td>
<td>0.002</td>
</tr>
</tbody>
</table>
8.3.2. Microstructure of Alloy Ingot Solidified in the Water-Cooled Copper Hearth

Samples of different alloys were taken from ingots solidified in the water-cooled copper hearth. The solidification condition here is different from that of conventional ingot metallurgy. The former has a higher cooling rate than the latter. X-ray analysis was used to identify phases present in alloy ingots. To enhance the reflections from the boride phase at low volume fractions, the samples for X-ray analysis were deeply etched using the Kroll's reagent which attacks preferentially the matrix. In this way, the volume fraction of borides sampled by the X-ray can be increased. Typical results are shown in Fig 8.10 for the Ti-6Al-4V-2B ingot. The X-ray analysis of the alloy ingots shows that the boride phase is TiB which has an orthorombic structure (B27) with a=6.12Å, b=3.06Å and c=4.56Å. This result was also confirmed by electron diffraction work on the transmission electron microscope (TEM), as shown by the [011], [112], [111] and [101] zone axis patterns (ZAPs) in Fig 8.11. There is no evidence for the presence of Ti₃B₄ and TiB₂ phases. This result is in agreement with a recent work on Ti-9.7 at.-% B and Ti-1.3 at.-% B alloys by Hyman et al [89Hym], where the boride phase was also identified as TiB.

Fig 8.10. X-Ray diffractograph of Ti-6Al-4V-2B ingot solidified in the water-cooled copper hearth.

Fig 8.11 is an optical micrograph showing the microstructure of Ti-6Al-4V-0.1 B ingot solidified in the water-cooled copper hearth. The prior β-grains have been transformed into a fully martensitic structure. Nearly all the boron atoms added to the alloy have been rejected into the grain boundary regions where they exist as TiB. The detailed morphology of boride phase formed at grain boundaries is shown in Fig 8.13 for Ti-7.5Mn-0.5B alloy ingot. The needle-shaped TiB particles are clustered at the prior β-grain boundaries.
Both Ti-6Al-4V-0.1 B and Ti-7.5Mn-0.5B are hypereutectic alloys. The solidification route can be seen clearly from the binary Ti-B phase diagram (Fig 8.3). During the cooling process, the $\beta$-grains will first nucleate and grow from the liquid alloy, which leads to the further enrichment of B in the remaining liquid alloy. With further cooling, TiB particles will nucleate and grow from the liquid alloy by coupled growth with $\beta$ when the composition of the liquid phase reaches the eutectic point (approximately 1.58 wt-% B at 1597°C [91Cha]). This explains why TiB particles are always located in the prior $\beta$-grain boundaries in the slowly solidified hypoeutectic alloy ingots.

The microstructure of Ti-6Al-4V-2B ingot solidified in the water-cooled copper-hearth is shown in Fig 8.14. The TiB phase in this alloy has two distinguished morphologies: one is large TiB needles with low aspect ratio which are believed to form first from the liquid alloy and called primary TiB, the other is in the form of finer needle-shaped TiB particles with high aspect ratio formed from the coupled growth with $\beta$. Because Ti-6Al-4V-2B is a hyper-eutectic
alloy (see Fig 8.3), the nucleation and growth of primary TiB from the liquid alloy make the composition of the liquid alloy change towards the eutectic composition and eventually lead to the coupled growth of needle-shaped TiB and β-grains to form an eutectic structure.

Fig 8.12. Optical micrograph showing the microstructure of Ti-6Al-4V-0.1B ingot solidified in the water-cooled copper hearth.

Fig 8.13. SEM micrograph of Ti-7.5Mn-0.5B ingot solidified in the water-cooled copper hearth showing the needle-shaped TiB particles clustered at prior grain boundaries.

Depending on the cooling rate, the cross-section of the alloy ingot solidified in the water-cooled copper hearth can be roughly divided into three regions: the region near the water-cooled copper hearth which has the highest cooling rate; the region near the free side has a medium cooling rate; the region in the middle part of the ingot has the lowest cooling rate. This difference of the cooling rate in different parts of the ingot offers us an easy way to investigate the effect of cooling rate on the morphology of the TiB-phase. The microstructures of these three parts are presented in Fig 8.14(a), 14(b) and 14(c), respectively. If the cooling rate is high, a uniform distribution of finer TiB phase is obtained (Fig 8.14(a)); if the cooling rate is reduced (corresponding to the medium cooling rate), large blocky primary TiB and fine eutectic
TiB particles are formed, as shown in Fig 8.14(b); if the cooling rate is low, both the primary and the eutectic TiB particles are very coarse (Fig 8.14(c)). This effect of cooling rate on the TiB morphology is important to finding an effective way for controlling the TiB morphology to ensure the desired mechanical properties.

Fig 8.14. Optical micrographs showing the microstructure of Ti-6Al-4V-2B ingot solidified in the water-cooled copper hearth. (a) near the bottom of the water-cooled copper hearth with the highest cooling rate; (b) near the top of the ingot with an intermediate cooling rate; (c) at the centre of the ingot with the slowest cooling rate.
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Fig 8.15. (a) TEM image of a cross-section of the needle-shaped TiB particle showing that the needle is along the \([010]_{\text{TiB}}\) direction and is bounded by \((100), (101)\) and \((10\overline{1})\) planes; (b) \([010]_{\text{TiB}}\) ZAP taken from the TiB phase in (a).

The needle-shaped TiB particles observed in these alloys are single crystals with occasional stacking faults. Detailed TEM work revealed the following characteristics of these needles: (i) the needles are single crystals with their axis parallel to the \([010]\) direction of the B27 unit cell. In tilting to this zone axis all the fringing effects at the particle/matrix interface disappear, giving rise to sharply defined boundaries which indicating that the planes enclosing this crystal lie perpendicular to the plane of image; (ii) the cross-sections of these needles are bounded either by \((100), \{101\}\) type planes or by \((100), \{102\}\) type planes. A typical example is given in Fig 8.15, which shows the BF TEM image of the cross-section of a TiB needle with the \([010]\) ZAP shown in Fig 8.15(b). Analysis of this ZAP indicated that the crystallographic facets in this needle are \((100), (101)\) and \((10\overline{1})\) planes as indicated in Fig 8.15(a). In fact, it is easy to identify the planes which enclosing the cross-sections of TiB needles even with the SEM images, due to the characteristic angles between relevant planes, as listed in Table 8.2.
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Two examples are presented in Fig 8.16. Figs 8.16(a) and 16(c) are the SEM images of the TiB particles, while Figs 8.16(b) and 16(d) are schematic illustrations of the cross-sections and bounding faces of the corresponding TiB particles in Figs 8.16 (a) and 16(c), respectively. The TiB particle in Fig 8.16(a) is enclosed by (100) and {101} types of planes, while that in Fig 8.16(c) is bounded by (100) and {102} types of planes. This result is different from the recent work of Hyman et al [89Hym] who concluded that the cross-section of the needle-shaped TiB is always enclosed by (100) and (10$\bar{1}$) types of planes.

Table 8.2. The characteristic angles between the relevant planes of TiB phase.

<table>
<thead>
<tr>
<th>Planes</th>
<th>(101)-(10$\bar{1}$)</th>
<th>(100)-{101}</th>
<th>(102)-(102)</th>
<th>(100)-{102}</th>
</tr>
</thead>
<tbody>
<tr>
<td>Angle (deg.)</td>
<td>106.4</td>
<td>126.4 or 53.6</td>
<td>41</td>
<td>110.5 or 69.5</td>
</tr>
</tbody>
</table>

Fig 8.16. (a) and (c) are SEM micrographs showing morphology of the TiB phase in Ti-6Al-4V-2B ingot solidified in the water-cooled copper hearth; (b) and (d) are schematic illustrations of the cross-sections of the boride in (a) and (c), respectively. The boride in (a) is bounded (100), (101) and (10$\bar{1}$) planes, while that in (c) by (100), (102) and (102) planes.
Another interesting phenomenon observed in this study is that some of the primary TiB particles are hollow (Fig 8.16). The reason for this phenomenon is not known [89Hym]. Further work is required to understand this interesting phenomenon.

Fig 8.17. X-ray diffractographs of as melt-spun Ti-6Al-4V-0.2B (A) and Ti-6Al-4V-2B (B) fibres.

8.3.3. Microstructure of melt spun Fibres

The chemical analysis of the impurities present in the alloys after melt-spinning are listed in Table 8.3. A comparison of the impurity levels between the master Ti-6Al-4V alloy (Table 8.1) and the melt spun fibres (Table 8.3) indicates that there is no severe pickup of the impurities during the melt spinning process. The reductions of the oxygen level in some of the melt spun fibres may result in the intrinsic error of analysis. The results of X-Ray diffractions show that only the hcp α-phase can be detected in melt spun fibres of Ti-6Al-4V alloys with B additions up to 1.5 wt.-%. The absence of TiB and β reflections is due to their low volume fractions in melt spun state. However, for melt spun Ti-6Al-4V-2B fibres, TiB phase can be detected by X-ray, due to the increased volume fraction of the TiB phase formed during the RS process. Typical X-ray results for melt spun fibres are presented in Fig 8.17 for Ti-6Al-4V-0.2B and Ti-6Al-4V-2B alloys. A comparison between the X-ray diffractographs for these two alloys also reveals the significant effect of boron addition on the refinement of the matrix structure as indicated by the peak broadening effects in the X-ray diffractograph of Ti-6Al-4V-2B alloy (curve B in Fig 8.17).

The microstructure of the cross-section of melt spun Ti-7.5Mn-0.5B alloy is shown in Fig 8.18 by two optical micrographs. Similar results for as melt-spun Ti-6Al-4V-0.5B alloy are presented in Fig 8.19 and for Ti-6Al-4V-2B alloy in Fig 8.20. Figs 8.18-19 show that the
cross-section of melt spun fibres all have a crescent shape with the concave face at wheel side and the convex face at the free side. Figs 8.18-19 also show that the cross-section of melt spun fibres shows two zones: a columnar grain zone at the wheel side and an equiaxed dendritic zone at the free side. This two-zone structure of the cross-section of melt spun fibres is typical of all the boron containing Ti-base alloys. The only difference is the relative thickness of each zone. Qualitatively speaking, the thickness of the equiaxed dendrite zone increases with increasing B content in the bulk alloy provided the cooling rate is kept constant.

Fig 8.18. Optical micrographs showing the microstructure of the cross-section of melt spun Ti-7.5Mn-0.5B fibres.

Table 8.3. The results of the chemical analysis of melt spun fibres.

<table>
<thead>
<tr>
<th>Alloy</th>
<th>Chemical analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>C</td>
</tr>
<tr>
<td>Ti-6Al-4V</td>
<td>0.015</td>
</tr>
<tr>
<td>Ti-6Al-4V-0B</td>
<td>NA</td>
</tr>
<tr>
<td>Ti-6Al-4V-0.2B</td>
<td>0.027</td>
</tr>
<tr>
<td></td>
<td>0.032</td>
</tr>
<tr>
<td>Ti-6Al-4V-0.8B</td>
<td>NA</td>
</tr>
<tr>
<td>Ti-6Al-4V-2.0B</td>
<td>0.059</td>
</tr>
</tbody>
</table>

Note: NA-not analysed
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Fig 8.19. Backscattered SEM image of the cross-sections of a melt spun Ti-6Al-4V-0.5B fibre. "w" indicates the wheel side and "f" indicates free side.

Fig 8.20. Secondary SEM image of the cross-sections of a melt spun Ti-6Al-4V-2B fibre (near the free side).

One possible cause for the two-zone microstructure is the departure of the fibre from the spinning wheel while it is still hot [82Jon]. However, the multilayer melt spun fibres in which good thermal contact is continuously maintained with the Mo wheel also have a two-zone microstructure in each layer, as shown in Fig 8.21. It therefore appears that the two-zone microstructure of the cross-section of melt spun fibre is a characteristic feature of the solidification process of B-containing Ti-alloys [86Row], rather than loss of the thermal contact.

The backscattered SEM images of the different regions in the cross-section of melt spun Ti-6Al-4V-0.5B fibre are presented in Fig 8.22. Fig 8.22 indicates that there is a B concentrated
region between the equiaxed dendrites and between the columnar grains. A detailed TEM examination of this region in the Ti-6Al-4V-0.8B alloy (Fig 8.23 (a)) indicates that this region has an eutectic structure (TiB+β) where needle-shaped TiB particles are interleaved by the β-phase. This is supported by the SAD pattern in Fig 8.23(b). This eutectic structure can also be seen from the SEM images of the cross-section of melt spun Ti-6Al-4V-2B alloy fibre in Fig 8.24. Therefore, it can be concluded that the cooling rate achieved during the melt spinning process in the present investigation is not enough to keep all the boron atoms in solid solution. Furthermore, there is an indication that the maintained β-phase has been decomposed partially into α-precipitates which do not obey the well-known Burger’s orientation relation and have been called type 2 α phase [75Rhol], symbolised by arched reflections in the SAD pattern as shown in Fig 8.23(b).

Another systematic change in the microstructure of melt-spun fibres with increasing B concentration is that the size of the equiaxed dendrite decreases with increasing B concentration in the bulk alloys. This can be seen by comparing the microstructure in Figs 8.19 and 8.20. This grain refining effect of B is a result of the segregation of TiB at grain boundaries.

In addition, the microstructural examination showed that there is no martensite in fine melt spun fibres (see Figs 8.19-20). This means that the martensitic transformation start temperature (Ms) has been depressed below the room temperature due to the high cooling rate of the melt spinning process. However, the martensitic transformation does occur in thicker melt spun fibres (thickness>200μm, with a lower cooling rate), as shown by the optical and TEM micrographs in Fig 8.25. These results indicate that cooling rate has a strong effect on Ms. If the cooling rate is beyond a critical point, the martensitic transformation will be completely suppressed (Ms is below room temperature).
Fig 8.22. Backscattered SEM images of the cross-section of a melt spun Ti-6Al-4V-0.5B fibre. (a) near the free side; (b) in the middle; (c) near the wheel side.
8.3.4. Microstructure of Alloy Powders after Pre-Hipping Heat Treatment

In order to understand the formation of TiB from B supersaturated α- and β-titanium solid solutions and to control the morphology of this phase, pre-hipping heat treatments were carried out on both the melt spun fibres and the comminuted powders. Two alloy compositions, namely, Ti-6Al-4V-0.5B and Ti-6Al-4V-2B, were chosen for this purpose. The annealing temperature and time are listed in Table 8.4. In this section, the results from the pre-hipping heat treatment of the comminuted powders are presented in detail, only a simple comparison will be made between the microstructure resulted from annealing of melt spun fibres and that of the comminuted powders.

Fig 8.23. (a) TEM image of the equiaxed dendrite boundary; (b) electron diffraction pattern taken approximately from (a).
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The comminuted alloy powders were chemically analysed and the results are listed in Table 8.5. This shows that after comminution the impurity level has been increased. For example, the oxygen concentration of Ti-6Al-4V alloy has been increased from 2300 ppm in melt spun fibres to 3100 ppm in the comminuted state. This is probably caused by the higher oxygen partial pressure in the pulverising chamber and the increased temperature due to friction. This problem remains to be solved.

The microstructure of the comminuted Ti-6Al-4V-0.5B powders is shown in Fig 8.26. Near the free side, the microstructure of the comminuted powders is very similar to that of melt spun fibres, although a few fine equiaxed TiB particles are present within the α-matrix (8.26(a)). However, the volume fraction of the fine equiaxed TiB particles at the wheel side appears larger in the comminuted state (Fig 8.26(b)) than that in melt spun state (Fig 8.22(c)). A similar result is also observed in the comminuted Ti-6Al-4V-2B powder. Therefore, the comminution process appears to have certain beneficial effect on the precipitation of TiB phase from the B supersaturated Ti solid solution. Possible reasons are as follows: (i) the stress-strain field within the samples accumulated during the comminution process can enhance B diffusion in the B supersaturated solid solution; (ii) the increased temperature caused by the friction between the hammers and the samples can also enhance the kinetics of TiB precipitation.

Fig 8.24. Secondary SEM image of the cross-section of a melt spun Ti-6Al-4V-2B fibre showing the eutectic structure at equiaxed dendrite boundaries. The thickness of this fibre is more than 100 μm.

The microstructure of comminuted Ti-6Al-4V-0.5B powders after pre-hipping heat treatment at different temperature for 4 hours are presented in Figs 8.27. A uniform distribution of fine equiaxed TiB particles is observed after pre-hipping heat treatment at temperatures between 300-700°C (see Figs 8.27(a-e)). However, a small population of the needle-shaped TiB particles are also present in the samples heat treated at a temperature above 800°C (Figs 8.27(f-)
probably due to the enhanced boron diffusivity in the B supersaturated Ti solid solution at higher temperature.

To check the stability of the fine equiaxed TiB particles formed during the low temperature annealing, a step annealing experiment was carried out. The comminuted Ti-6Al-4V-0.5B powders were first annealed at 600°C for 4 hrs to produce the fine equiaxed TiB precipitates, and then quickly heated up to a temperature of 800°C and kept for 24 hrs and finally they were furnace cooled. The final microstructure resulted from this step annealing experiment is presented in Fig 8.28, which indicates that the size of the majority of fine TiB particles formed at 600°C remains more or less unchanged, although some needle-shaped TiB particles are also present within the sample.

![Fig 8.25. Optical micrograph of Ti-6Al-4V-0.5B powder produced during the melt spinning process (a) and BF TEM image of melt spun Ti-6Al-4V-0.8B fibre (thickness > 200 μm) (b) showing the martensitic structure.](image)
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Typical microstructure of melt spun Ti-6Al-4V-0.5B fibre after the pre-hipping heat treatment is presented in Fig 8.29. A comparison between the microstructures of melt spun fibre (Fig 8.29) and that of the comminuted powder (Fig 8.27(d) after a similar pre-hipping heat treatment indicates that the TiB particle size in the heat treated fibre (Fig 8.29) is substantially bigger than that in heat treated powder (Fig 8.27(d)). This difference in TiB size is probably caused by the enhanced diffusion kinetics for TiB nucleation in the comminuted state due to the high dislocation population.

The microstructures of comminuted Ti-6Al-4V-2B powders after pre-hipping heat treatment at different temperatures for 4 hours are presented in Fig 8.30. Again, a uniform distribution of fine TiB particles is observed after pre-hipping heat treatment at temperatures between 300-700°C (see Figs 8.30(a-e)), and a small population of needle-shaped TiB particles is also present in the samples heat treated at temperatures above 800°C (Figs 8.30(f-g)). The same step annealing experiment was also carried out on the comminuted Ti-6Al-4V-2B powders. The resulted microstructure is presented in Fig 8.31, which indicates that the size of the
majority of the TiB particle formed at 600°C remains more or less unchanged, although some needle-shaped TiB particles are also present within the sample.

The X-ray diffraction results for the comminuted Ti-6Al-4V-2B powders after different pre-hipping heat treatments are given in Fig 8.32. A comparison among the X-ray diffractographs of the comminuted Ti-6Al-4V-2B powders in the comminuted state (Curve A), after annealing at 600°C/4hrs (Curve B) and annealing at 600°C/4hrs plus 800°C/24hrs (Curve C) reveals the following facts: (i) after annealing the matrix grains have not grown appreciably due to the locking effect of TiB particles to the grain boundary migration, as indicated by the intensity of the α-phase reflections; (ii) the sharpness of (210)_{TiB} and (102)_{TiB} peaks is increased after annealing because of the increased volume fraction and particle size of TiB phase.

Table 8.4. Summary of pre-hipping heat treatment conditions for melt spun fibres and comminuted alloy powders.

<table>
<thead>
<tr>
<th>Heat Treatment</th>
<th>Ti-6Al-4V-0.5B as melt-spun</th>
<th>Ti-6Al-4V-0.5B comminuted</th>
<th>Ti-6Al-4V-2B comminuted</th>
</tr>
</thead>
<tbody>
<tr>
<td>300°C, 4 hrs</td>
<td>√</td>
<td>√</td>
<td>√</td>
</tr>
<tr>
<td>400°C, 4 hrs</td>
<td>√</td>
<td>√</td>
<td>√</td>
</tr>
<tr>
<td>450°C, 4 hrs</td>
<td>√</td>
<td>√</td>
<td>√</td>
</tr>
<tr>
<td>500°C, 4 hrs</td>
<td>√</td>
<td>√</td>
<td>√</td>
</tr>
<tr>
<td>550°C, 4 hrs</td>
<td>√</td>
<td>√</td>
<td>√</td>
</tr>
<tr>
<td>600°C, 4 hrs</td>
<td>√</td>
<td>√</td>
<td>√</td>
</tr>
<tr>
<td>650°C, 4 hrs</td>
<td>√</td>
<td>√</td>
<td>√</td>
</tr>
<tr>
<td>700°C, 4 hrs</td>
<td>√</td>
<td>√</td>
<td>√</td>
</tr>
<tr>
<td>800°C, 4 hrs</td>
<td>√</td>
<td>√</td>
<td>√</td>
</tr>
<tr>
<td>900°C, 4 hrs</td>
<td>√</td>
<td>√</td>
<td>√</td>
</tr>
<tr>
<td>600°C, 4 hrs and then 800°C, 4 hrs</td>
<td>√</td>
<td>√</td>
<td>√</td>
</tr>
</tbody>
</table>

Table 8.5. The results of chemical analysis for comminuted powders.

<table>
<thead>
<tr>
<th>Alloy</th>
<th>Chemical analysis (wt.-%)</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>C</td>
<td>N₂</td>
</tr>
<tr>
<td>Ti-6Al-4V-0B</td>
<td>0.02</td>
<td>0.014</td>
</tr>
<tr>
<td>Ti-6Al-4V-0.8B</td>
<td>0.04</td>
<td>0.019</td>
</tr>
<tr>
<td>Ti-6Al-4V-1.5B</td>
<td>0.04</td>
<td>0.018</td>
</tr>
<tr>
<td>Ti-6Al-4V-2.0B</td>
<td>0.059</td>
<td>0.02</td>
</tr>
<tr>
<td>Ti-6Al-4V-2.0B</td>
<td>0.11</td>
<td>0.022</td>
</tr>
<tr>
<td>Ti-6Al-4V-0.5B</td>
<td>0.03</td>
<td>0.026</td>
</tr>
<tr>
<td>Ti-6Al-4V-0.5B</td>
<td>0.02</td>
<td>0.042</td>
</tr>
</tbody>
</table>

Notes: ?: not analysed;
IMI Anal: IMI analysis;
Fig 8.27. Optical micrographs showing the microstructures of the comminuted Ti-6Al-4V-0.5B powders after annealing at different temperatures for 4 hrs.

(a) 300°C;
(b) 400°C;
(c) 500°C;
(d) 600°C;
(e) 700°C;
(f) 800°C;
(g) 900°C.
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Fig 8.28. SEM micrograph showing the microstructure of a comminuted Ti-6Al-4V-0.5B powder after annealing at 600°C/4hrs and then at 800°C/24hrs.

Fig 8.29. Two optical micrographs showing the typical microstructure of melt spun Ti-6Al-4V-0.5B fibres after annealing at 550°C/4hrs.
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Fig 8.30. Optical micrographs showing the microstructures of the comminuted Ti-6Al-4V-2B powders after annealing at different temperatures for 4 hrs.

(a) 300°C;
(b) 400°C;
(c) 500°C;
(d) 600°C;
(e) 700°C;
(f) 800°C;
(g) 900°C.
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Fig 8.31. SEM micrograph showing the microstructure of a comminuted Ti-6Al-4V-2B powder after annealing at 600°C/4hrs and then at 800°C/24hrs.

Fig 8.32. X-ray diffractographs of the comminuted Ti-6Al-4V-2B powders after different heat treatments. (A) comminuted powders; (B) annealed at 600°C/4hrs; (C) annealed at 600°C/4hrs and then at 800°C/24hrs.
8.3.5. Microstructure of Consolidated Alloys

The microstructures of consolidated alloys were examined by X-Ray diffraction, OM, SEM and TEM. Because of the large number of alloys studied, only some of the typical results will be presented here.

Fig 8.33. X-ray diffractographs of B-containing alloys consolidated at a temperature of 900°C and a pressure of 250MPa for 2 hrs and then forged at 900°C to 40% reduction in cross-section. (1) Ti-6Al-4V-2B; (2) Ti-6Al-4V-1.5B; (3) Ti-6Al-4V-0.8B; (4) Ti-6Al-4V-0.5B; (5) Ti-6Al-4V-0.2B; (6) Ti-6Al-4V-0.1B.

(1) X-Ray Analysis

The results of X-ray analysis of consolidated Ti-6Al-4V-XB alloys are presented in Fig 8.33, which revealed the following facts: (i) the boride phase in consolidated alloys is identified as TiB, which is also confirmed by the electron diffraction (Fig 8.52); (ii) the intensity of the TiB
phase reflections increases with increasing B concentration, due to the increased volume fraction of the TiB phase; (iii) the intensity of the β-phase reflections decreases with increasing B concentration, as shown by the intensity of (110)β peak in Fig 8.33; (iv) α- and β-grain sizes decrease with increasing B addition, as indicated by the peak width of the α- and β-phase reflections due to inhibition of grain boundary migration by TiB.

The results of X-ray analysis for consolidated Ti-3.0Mn-0.5B and Ti-7.5Mn-0.5B alloys are presented in Fig 8.34, where the boride phase is also identified as TiB phase (Fig 8.34).

![Fig 8.34. X-ray diffractographs of Ti-3Mn-0.5B (A) and Ti-7.5Mn-0.5B (B) alloys consolidated at a temperature of 900°C and a pressure of 250MPa for 2 hrs and then forged at 900°C to 40% reduction in cross-section.](image)

![Fig 8.35. Secondary SEM image of Ti-6Al-4V alloy consolidated at a temperature of 925°C and a pressure of 300MPa for 2 hrs.](image)
The identification of the boride phase in RS Ti-base alloys containing B has not been very precise in the prior literature. For example, TiB has been reported in Ti-6Al-3Sn-3Zr-0.25B [86Row] and in Ti-10Zr-6B (in at.-%) alloys [88Lu] without giving any detailed evidence; TiB₂ was also reported in Ti-15V-3Cr-3Sn-3Al-0.1-0.2B alloys [88Kai] and in Ti-25V-2Er-0.02B alloy [88Lee1]. One possible reason for this inconclusive identification is that other alloying elements may have a strong influence on the structure of boride. For instance, Hyman et al [89Hym] have suggested that increasing Al content may be favourable to TiB₂ formation.

(2) Microstructure of Consolidated Ti-6Al-4V alloys without Boron

A Ti-6Al-4V alloy was hipped at a temperature of 925°C, a pressure of 300 MPa for 2 hrs. The resulted microstructure is shown in Fig 8.35. The α-matrix has an equiaxed grain shape with an approximate grain diameter of about 6 μm, while the β-grains have an irregular shape and are located at the α-grain boundaries. The fine white bits within the α-grains are an artefact due
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to the etching effect. This has been confirmed by the TEM examination (Fig. 8.36(a)). Fig 8.36(b) shows the detailed microstructure of the β-phase in the as consolidated Ti-6Al-4V alloy.

A series zone axis patterns (ZAPs) taken from the β-grain with the beam directions within the bcc unit stereographic triangle are presented in Fig 8.37. The ZAPs presented in Fig 8.37 indicates that the ω-phase, type 1 α and type 2 α precipitates coexist within the β-matrix. Further evidence for such coexistence is presented in Figs 8.38 and 8.39.

Fig 8.37. A series SADPs with different beam directions within the bcc unit stereographic triangle taken from the β-grain in Ti-6Al-4V alloy consolidated at a temperature of 925°C and a pressure of 300MPa for 2 hours, showing the coexistence of ω-phase and α precipitates within the β matrix.
Fig. 8.38. [012]_B ZAP taken from the β-grain in Ti-6Al-4V alloy consolidated at a temperature of 925°C and a pressure of 300MPa for 2 hrs showing the coexistence of the ω-phase and the α precipitates within the β matrix.

Fig. 8.39. The [111]_B ZAPs taken from the β-grain in Ti-6Al-4V alloy consolidated at a temperature of 925°C and a pressure of 300MPa for 2 hrs showing the presence of Type 1 α (a) and Type 2 α (b) precipitates within the β matrix.

Apart from the ω-phase which will be discussed in Section 9.5, there are two more products of metastable β decomposition: Type 1 α and Type 2 α phases [75Rho1]. Both types of α-phases have been shown to have a hexagonal structure with the same lattice parameters, but differ in
orientation relation to β-phase [75Rho1]. Of these, it appears that the type 1 α-phase, which obeys the well-known Burger's orientation relation \( ((0001)_\alpha // (110)_\beta; <11\overline{2}0>\alpha // <1\overline{1}\overline{1}>\beta) \), is metastable with respect to the type 2 α-phase, which has a twin related orientation relation to the type 1 α-phase and, therefore, has a complex orientation relation to the β-phase [78Wil]. The relative stability of the two types of α-phase is reported to be affected by the alloy composition, especially by the Al content [75Rho1, 75Rho2]. The two types of α-phase are readily distinguished because they have distinct diffraction effects associated with them [75Rho1, 75Rho2]. The discrete extra spots (see Fig 8.39(a)) are indication of the presence of the type 1 α-phase, while the arced reflections are due to the presence of type 2 α-phase (see Fig 8.39(b)).

(3) Microstructure of Consolidated Ti-6Al-4V-XB Alloys

The microstructures of consolidated Ti-6Al-4V-XB alloys are shown in Figs 8.40-42. Full densification has been achieved for all Ti-6Al-4V-XB alloys, and porosity in the consolidated alloys was rarely observed. All B-containing alloys show a uniform distribution of fine TiB particles with a nearly equiaxed morphology. Some needle-shaped TiB particles are also observed in the alloys containing higher B concentration (see Figs 8.41 and 8.42). This is also evidenced by detailed TEM observations. A typical example is given in Figs 8.43 and 8.44, which are BF TEM images of the consolidated Ti-6Al-4V-0.8B and Ti-6Al-4V-2B alloys, respectively. Both the fine equiaxed and the needle-shaped TiB particles can be clearly seen in Figs 8.43 and 8.44. It is believed that the needle-shaped TiB particles are mainly formed during the solidification processing, while the equiaxed TiB particles are formed during the pre-hipping heat treatment.

The effect of the pre-hipping heat treatment of the comminuted powders on the morphology of TiB phase after consolidation is shown in Fig 8.45 by two SEM micrographs. The TiB particles in consolidated Ti-6Al-4V-0.8B alloy with pre-hipping heat treatment (Fig 8.45(a)) mainly have an equiaxed morphology with the occasional presence of the needle-shaped TiB particles, whereas the TiB particles in consolidated Ti-6Al-4V-0.8B alloy without the pre-hipping heat treatment (Fig 8.45(b)) mainly have a needle-shaped morphology.

BF and (110)\(_\beta\) DF TEM images of the β-grain in consolidated Ti-6Al-4V-0.8B alloy are shown in Fig 8.46. The SAD patterns taken from the β-grains are presented in Fig 8.47. Besides the strong bcc reflections, these SAD patterns show the diffused streaking which is characteristic of the presence of the ω-phase [73Wil]. There has been no prior report of the ω-phase transformation in rapidly solidified Ti-6Al-4V-XB alloys in the literature. In addition, the BF image of the β-phase in Fig 8.48(a) and the [001]\(_\beta\) SAD pattern (Fig 8.48(b)) show that Type 1 α precipitates are also present in the β-matrix.
Fig 8.40. Optical micrographs showing the microstructure of the B-containing titanium alloys after consolidation at a temperature of 900°C and a pressure of 250MPa for 2 hrs and then forged at 900°C to 40% reduction in cross-section. (a) Ti-6Al-4V-0.1B; (b) Ti-6Al-4V-0.2B; (c) Ti-6Al-4V-0.5B; (d) Ti-6Al-4V-0.8B.
Fig 8.41. Two SEM micrographs with different magnifications showing the microstructure of Ti-6Al-4V-1.5B alloy after consolidation at a temperature of 900°C and a pressure of 250MPa for 2 hrs and then forged at 900°C to 40% reduction in cross-section.

Fig 8.42. Two SEM micrographs with different magnifications showing the microstructure of Ti-6Al-4V-2B alloy after consolidation at a temperature of 900°C and a pressure of 250MPa for 2 hrs and then forged at 900°C to 40% reduction in cross-section.
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Fig 8.43. BF TEM images of Ti-6Al-4V-0.8B alloy after consolidation at a temperature of 900°C and a pressure of 250MPa for 2 hrs showing the uniform distribution of nearly equiaxed and needle-shaped TiB particles in the α (-β) matrix.

Fig 8.44. Bright field TEM images of Ti-6Al-4V-2B alloy after consolidation at a temperature of 900°C and a pressure of 250MPa for 2 hrs showing (a) the uniform distribution of nearly equiaxed and needle-shaped TiB particles in the α (-β) matrix and (b) a cluster of TiB particles at α-grain boundaries.
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Fig 8.45. A comparison between the microstructures of Ti-6Al-4V-0.8B alloy after consolidation at a temperature of 900°C and a pressure of 250MPa for 2 hrs (a) with and (b) without a pre-hipping heat treatment (annealing at 600°C/4hrs).

Fig 8.46. Bright field (a) and (110)\(\beta\) dark field (b) TEM images showing the detailed microstructure of \(\beta\)-phase in Ti-6Al-4V-0.8B alloy after consolidation at a temperature of 900°C and a pressure of 250MPa for 2 hrs.
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Fig 8.47. SADPs taken from the β-grains in Ti-6Al-4V-0.8B alloy after consolidation at a temperature of 900°C and a pressure of 250MPa for 2 hrs with electron beam (a) close to [110]_β and (b) close to [135]_β; showing diffused streakings which are characteristic of the presence of the ω-phase within the β-matrix.

TEM examinations also show that the consolidated alloys are heavily dislocated; some typical examples of the dislocation structures are presented in Fig 8.49. Fig 8.50 shows that there is a strong interaction between the TiB particles and moving dislocations during the consolidation process.

Fig 8.51 is a BF TEM image of a grain boundary region in consolidated Ti-6Al-4V-2B alloy. The TiB phase in Fig 8.51 has two distinct morphologies: one is the needle-shaped TiB, the other is the equiaxed. The needle-shaped TiB particles lie parallel to each other and are only occasionally faulted or twinned. On other hand, the equiaxed ones are heavily faulted and/or twinned. It is believed that needle-shaped TiB particles are formed during the solidification process at the region between the equiaxed dendrites (see Fig 8.24), whereas the equiaxed TiB particles are formed by solid state reaction during the pre-hipping heat treatment or the consolidation process.

The SAD patterns from nearly equiaxed TiB particles with the electron beam close to [010] show a strong streaking effect along the <102> directions, as shown in Fig 8.52(a) (a key is given in Fig 8.52(b). Closer examination of <102> streaks in Fig 8.52(a) reveals that they consist of a number of closely spaced sports, which indicates that these streaks are caused by the presence of both {102} stacking faults and {102} twins in the TiB particles. This argument
is further demonstrated in Fig 8.53 by the BF image (Fig 8.53(a)) and (102)\textsubscript{TiB} dark field image (Fig 8.53(b)) of an equiaxed TiB particle. \{102\} stacking faults and \{102\} twins can be seen clearly in Fig 8.54. \{100\} stacking faults and/or \{100\} twins have also been observed in the fine equiaxed TiB particles, as shown in Fig 8.55 by the SAD pattern taken from the \(\beta\)-phase containing fine TiB particles. Furthermore, the SAD work indicates that the cross-sections of those nearly equiaxed TiB particles are enclosed either by \{100\} and \{102\} types of planes (Figs 8.53 and 54) or by \{100\} and \{101\} types of planes (Fig 8.56). This is in agreement with the results obtained from the ingot samples (Figs 8.15-16).

The TiB/matrix interfaces are sharp with no visible evidence of the presence of any chemical reaction zone, as indicated by the BF and DF TEM images presented in Figs 8.53–54 and 56.

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Fig 8.48. (a) BF TEM image of the \(\beta\)-phase in Ti-6Al-4V-0.8B alloy after consolidation at a temperature of 900\(^\circ\)C and a pressure of 250MPa for 2 hrs; (b) \([001]_\beta\) ZAP indicating the presence of the Type 1 \(\alpha\) precipitates in the \(\beta\)-matrix.
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Fig 8.49. Two BF TEM images showing the dislocation structures in Ti-6Al-4V-0.8B alloy consolidated at 900°C, 250MPa for 2 hrs.

Fig 8.50. BF TEM image showing the interaction between the TiB particles and the moving dislocations during the consolidation of Ti-6Al-4V-0.8B alloy.
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Fig 8.51. BF TEM image of an α-grain boundary region in Ti-6Al-4V-2B alloy consolidated at 900°C, 250MPa for 2 hrs showing two distinct morphologies of TiB phase: one is the needle-shaped TiB which is rarely faulted or twinned, as indicated in the micrograph by “n”; the other is the equiaxed TiB which has the faulted or twinned substructure, as indicated by “e”.

(4) Microstructure of Consolidated Ti-Mn-B alloys

Comminuted Ti-3.0Mn-0.5B and Ti-7.5Mn-0.5B powders were annealed at 600°C for 2 hours and then hipped at a temperature of 900°C and a pressure of 250 MPa for 2 hours, and then forged at 900°C to a further 40% reduction in cross-section. The microstructures of these two alloys after consolidation are shown in Figs 8.57(a) and 57(b), respectively. A uniform distribution of fine equiaxed TiB particles in the α+β matrix was obtained in both alloys. Fig 8.58 shows the BF TEM image of the consolidated Ti-7.5Mn-0.5B alloy. Needle-shaped TiB particles are also observed as in the B-containing Ti-6Al-4V alloys.

Selected area diffractions were conducted on the β-grains in the consolidated Ti-7.5Mn-0.5B alloy. A series of ZAPs with B within the bcc unit stereographic triangle are presented in Fig 8.59. Fig 8.59 shows clearly that besides the strong bcc β reflection, the diffused streaking are present in all the ZAPs which are characteristic of the ω-phase reflection [73Wil].
8.3.6. Mechanical Properties of Consolidated Alloys

The specimens for tensile testing were prepared according to the procedure described in the section 8.2.7. Unfortunately, nearly all the samples failed in the elastic range of the stress-strain curve showing no ductility. Therefore, the only information which could be obtained from these tensile tests are Young's moduli, which are summarised in Table 8.6.

Microhardness tests were also performed on the consolidated alloys. The Vicker's hardness obtained from samples of different thermomechanical histories is listed in Table 8.7. Each data point is the average of 5-10 measurements.

Fig 8.52. (a) SAD pattern with electron beam close to [010]_TIB showing the streaking and sport splitting along the [102]_TIB and [021]_TIB directions; (b) key to (a).
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Fig 8.53. BF (a) and (102)_{TiB} CDF (b) images of a nearly equiaxed TiB particle in Ti-6Al-4V-2B alloy consolidated at 900°C, 250MPa for 2 hrs showing the (102) and (102) twin structure, presumably the (102) and (102) stacking faults are also present.

Fig 8.54. BF images of the nearly equiaxed TiB particles in consolidated Ti-6Al-4V-0.8B alloy (a) and Ti-6Al-4V-2B alloy (b) indicating that these two TiB particles are bounded by (100), (102) and (102) planes, as shown in each micrograph.
Fig 8.55. SAD pattern taken from the β-grain containing fine TiB particles in consolidated Ti-6Al-4V-0.8B alloy showing the streaking along the $[100]_{\text{TiB}}$ direction.

Fig 8.56. BF images of the nearly equiaxed TiB particles in consolidated Ti-6Al-4V-0.8B alloy indicating that these two TiB particles are bounded by (100), (101) and (10$ar{1}$) planes, as shown in each micrograph.
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Fig 8.57. Optical micrographs showing the microstructure of (a) Ti-3Mn-0.5B alloy and (b) Ti-7.5Mn-0.5B alloy consolidated at a temperature of 900°C and a pressure of 250MPa for 2 hrs and then forged at 900°C.

Fig 8.58. BF image of Ti-7.5Mn-0.5B alloy consolidated at a temperature of 900°C and a pressure of 250MPa for 2 hrs and then forged at 900°C showing the uniform distribution of the fin TiB particles in the α-β matrix.
Fig 8.59. A series SADPs with different beam directions within the bcc unit stereographic triangle taken from the β-grain in Ti-7.5Mn-0.5B alloy consolidated at a temperature of 900°C and a pressure of 250MPa for 2 hrs indicating the presence of the ω-phase.
### Table 8.6. Summary of the Young's moduli of consolidated alloys

<table>
<thead>
<tr>
<th>Alloy</th>
<th>Expected TiB Vol.-%</th>
<th>Processing Condition</th>
<th>E (GPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ti-6Al-4V</td>
<td>0.0</td>
<td>Hipped at 900°C, 300 MPa, 2 hrs.</td>
<td>116.6</td>
</tr>
<tr>
<td>Ti-6Al-4V-0.1B</td>
<td>0.5</td>
<td>Pre-heat treated at 600°C 2 hrs, hipped at 900°C, 250 MPa, 2 hrs and forged at 900°C</td>
<td>116.2</td>
</tr>
<tr>
<td>Ti-6Al-4V-0.2B</td>
<td>1.0</td>
<td>Pre-heat treated at 600°C 2 hrs, hipped at 900°C, 250 MPa, 2 hrs and forged at 900°C</td>
<td>121.0</td>
</tr>
<tr>
<td>Ti-6Al-4V-0.5B</td>
<td>2.4</td>
<td>Pre-heat treated at 600°C 2 hrs, hipped at 900°C, 250 MPa, 2 hrs and forged at 900°C</td>
<td>129.7</td>
</tr>
<tr>
<td>Ti-6Al-4V-0.8B</td>
<td>3.9</td>
<td>Hipped at 925°C, 150 MPa, 2 hrs.</td>
<td>123.5</td>
</tr>
<tr>
<td>Ti-6Al-4V-0.8B</td>
<td>3.9</td>
<td>Pre-heat treated at 600°C 2 hrs, hipped at 900°C, 250 MPa, 2 hrs and forged at 900°C</td>
<td>128.2</td>
</tr>
<tr>
<td>Ti-6Al-4V-0.8B</td>
<td>3.9</td>
<td>Pre-heat treated at 600°C 2 hrs, hipped at 900°C, 250 MPa, 2 hrs and forged at 900°C</td>
<td>131.8</td>
</tr>
<tr>
<td>Ti-6Al-4V-1.5B</td>
<td>7.1</td>
<td>Hipped at 900°C, 300 MPa, 2 hrs.</td>
<td>128.6</td>
</tr>
<tr>
<td>Ti-6Al-4V-1.5B</td>
<td>7.1</td>
<td>Pre-heat treated at 600°C 2 hrs, hipped at 900°C, 250 MPa, 2 hrs and forged at 900°C</td>
<td>132.90</td>
</tr>
<tr>
<td>Ti-6Al-4V-2.0B</td>
<td>9.9</td>
<td>Pre-heat treated at 600°C 2 hrs, hipped at 900°C, 250 MPa, 2 hrs and forged at 900°C</td>
<td>139.4</td>
</tr>
<tr>
<td>Ti-3.0Mn-0.5B</td>
<td>2.4</td>
<td>Pre-heat treated at 600°C 2 hrs, hipped at 900°C, 250 MPa, 2 hrs and forged at 900°C</td>
<td>136.6</td>
</tr>
<tr>
<td>Ti-7.5Mn-0.5B</td>
<td>2.4</td>
<td>Pre-heat treated at 600°C 2 hrs, hipped at 900°C, 250 MPa, 2 hrs and forged at 900°C</td>
<td>125.0</td>
</tr>
</tbody>
</table>
Table 8.7. The results of the microhardness tests of consolidated alloys

<table>
<thead>
<tr>
<th>Alloy</th>
<th>Expected TiB Vol.-%</th>
<th>Processing Condition</th>
<th>Vicker's Hardness</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ti-6Al-4V</td>
<td>0.0</td>
<td>As received ingot from RAE</td>
<td>297</td>
</tr>
<tr>
<td>Ti-6Al-4V</td>
<td>0.0</td>
<td>Hipped at 900°C, 300 MPa, 2 hrs.</td>
<td>464</td>
</tr>
<tr>
<td>Ti-6Al-4V</td>
<td>0.0</td>
<td>Hipped at 900°C, 300 MPa, 2 hrs, heat treated at 900°C WQ</td>
<td>393</td>
</tr>
<tr>
<td>Ti-6Al-4V-0.1B</td>
<td>0.5</td>
<td>Pre-heat treated at 600°C 2 hrs, hipped at 900°C, 250 MPa, 2 hrs and forged at 900°C</td>
<td>470.0</td>
</tr>
<tr>
<td>Ti-6Al-4V-0.2B</td>
<td>1.0</td>
<td>Pre-heat treated at 600°C 2 hrs, hipped at 900°C, 250 MPa, 2 hrs and forged at 900°C</td>
<td>476.0</td>
</tr>
<tr>
<td>Ti-6Al-4V-0.3B</td>
<td>2.4</td>
<td>Pre-heat treated at 600°C 2 hrs, hipped at 900°C, 250 MPa, 2 hrs and forged at 900°C</td>
<td>488.4</td>
</tr>
<tr>
<td>Ti-6Al-4V-0.8B</td>
<td>3.9</td>
<td>Hipped at 925°C, 150 MPa, 2 hrs.</td>
<td>453.0</td>
</tr>
<tr>
<td>Ti-6Al-4V-0.8B</td>
<td>3.9</td>
<td>Hipped at 925°C, 150 MPa, 2 hrs, annealed at 700°C 4 hrs</td>
<td>446.0</td>
</tr>
<tr>
<td>Ti-6Al-4V-0.8B</td>
<td>3.9</td>
<td>Hipped at 925°C, 150 MPa, 2 hrs, annealed at 600°C 15hrs</td>
<td>455.0</td>
</tr>
<tr>
<td>Ti-6Al-4V-0.8B</td>
<td>3.9</td>
<td>Pre-heat treated at 600°C 2 hrs, hipped at 900°C, 250 MPa, 2 hrs and forged at 900°C</td>
<td>525.5</td>
</tr>
<tr>
<td>Ti-6Al-4V-1.5B</td>
<td>7.1</td>
<td>Hipped at 900°C, 300 MPa, 2 hrs.</td>
<td>548.0</td>
</tr>
<tr>
<td>Ti-6Al-4V-1.5B</td>
<td>7.1</td>
<td>Hipped at 900°C, 300 MPa, 2 hrs, annealed at 700°C 4 hrs, WQ.</td>
<td>523.0</td>
</tr>
<tr>
<td>Ti-6Al-4V-1.5B</td>
<td>7.1</td>
<td>Pre-heat treated at 600°C 2 hrs, hipped at 900°C, 250 MPa, 2 hrs and forged at 900°C</td>
<td>550.0</td>
</tr>
<tr>
<td>Ti-6Al-4V-2.0B</td>
<td>9.9</td>
<td>Ingot solidified in the water cooled copper hearth with a higher cooling rate</td>
<td>449.0</td>
</tr>
<tr>
<td>Ti-6Al-4V-2.0B</td>
<td>9.9</td>
<td>Ingot solidified in the water cooled copper hearth with a lower cooling rate</td>
<td>432.0</td>
</tr>
<tr>
<td>Ti-6Al-4V-2.0B</td>
<td>9.9</td>
<td>Ingot solidified in the water cooled copper hearth with an average cooling rate</td>
<td>440.5</td>
</tr>
<tr>
<td>Ti-6Al-4V-2.0B</td>
<td>9.9</td>
<td>Pre-heat treated at 600°C 2 hrs, hipped at 900°C, 250 MPa, 2 hrs and forged at 900°C</td>
<td>580.0</td>
</tr>
<tr>
<td>Ti-3.0Mn-0.5B</td>
<td>2.4</td>
<td>Pre-heat treated at 600°C 2 hrs, hipped at 900°C, 250 MPa, 2 hrs and forged at 900°C</td>
<td>410.0</td>
</tr>
<tr>
<td>Ti-7.5Mn-0.5B</td>
<td>2.4</td>
<td>Pre-heat treated at 600°C 2 hrs, hipped at 900°C, 250 MPa, 2 hrs and forged at 900°C</td>
<td>458.0</td>
</tr>
</tbody>
</table>
CHAPTER 9

Experimental Investigation of Microstructure and Mechanical Properties of In Situ Ti/TiB Metal Matrix Composites Obtained by Rapid Solidification Process

Part II. Discussion

9.1. The Feasibility of Fabricating of In Situ Ti/TiB MMCs via RS

The microstructural examination of in situ Ti/TiB composites produced through RS route in Chapter 8 allows the following conclusions to be drawn: (i) there is a very good bonding between the TiB phase and Ti-alloy matrix; (ii) there is no chemical reaction at the TiB/matrix interface; (iii) a fine TiB phase is uniformly distributed throughout the Ti-alloy matrix. These advantages are inherent of production of Ti/TiB composite through RS route. As the reinforcement (TiB) is formed either from liquid alloy during RS process or from B super-saturated solid solution, it is expected to have a good bonding between the TiB-phase and the matrix. The TiB-phase formed in such way is also thermodynamically stable. After the pre-hipping heat treatment and consolidation process, all the phases formed during these thermomechanical processes are more or less in equilibrium, and there is no driving force for any other chemical reaction to occur in this system. In addition, due to the extremely low equilibrium solid solubility of B in both α- and β-Ti, there are hardly any extra B atoms left to support any chemical reaction at the TiB/matrix interface. Therefore, the formation of other borides (such as Ti₃B₄ or TiB₂) at the TiB/matrix interface is completely excluded. Pre-hipping heat treatment can produce fine TiB particles which are uniformly distributed throughout the entire matrix. Thus after hipping the uniform distribution of the reinforcing phase in the matrix should be expected. From these results, it is obvious that the production of Ti/TiB composite through RS route can avoid some of the problems encountered by other production routes.

Of course, much more work is needed to improve the fabrication process and the mechanical properties of in situ Ti/TiB composite produced by this route, especially in relation to the low ductility that has been found sofar. Until now, the maximum volume fraction of TiB-phase produced by this production route is 0.1. It is desirable to increase this volume fraction to 0.2–0.3. Further increase in TiB volume fraction is controlled by the capacity of the
equipment, for example the increased power supply to fulfil the requirement of melting Ti-alloys with higher B content, rather than by any theoretical problem.

9.2. The Effect of B Addition on the $\alpha/\beta$ Transus

Although B has been classified as an $\alpha$-stabiliser [65Mol], in practice, it has a negligible effect on $\alpha/\beta$ transition temperature in conventional ingot metallurgy due to its low solid solubility in both $\alpha$- and $\beta$-phases. On the other hand, solubility extensions as high as 10 at.-% B in $\alpha$-Ti have been reported by rapid solidification and ascribed to increased interstitial solubility [86Wha1]. Thus, it would be expected that with such an extended solubility of B in Ti, B could have a stronger effect on $\alpha/\beta$ transus, i.e., B could significantly stabilize the $\alpha$-phase. Because the variation of volume fraction of the $\beta$-phase can be used as an indication of the change in $\alpha/\beta$ transus at a given alloy composition, this is supported by the experimental observation that the volume fraction of the $\beta$-phase in as melt spun Ti-6Al-4V-XB fibres is so low that it could not be detected by X-ray diffraction (see Fig 8.17). This effect can even be detected in the consolidated Ti-6Al-4V-XB alloys, in which the volume fractions of the $\beta$-phase (approximately 0.03–0.05) are substantially less than it would be expected (~0.1 [66Cas]) for Ti-6Al-4V alloys. Further evidence of the effect of B addition on the $\alpha/\beta$ transition temperature is given by the experimental result that the volume fraction of the $\beta$-phase decreases with increasing B content as indicated by the results of X-ray diffraction presented in Fig 8.33.

9.3. The Influence of Cooling Rate on $M_S$

The microstructural examination in Section 8.3.3 indicates that fine melt spun fibres of thickness <$100 \mu$m for all alloys investigated here have an $\alpha$ (+$\beta$) matrix, and no martensite is present (Figs 8.18-24). On the other hand, it was found that melt spun fibres of thickness $>$ 200 $\mu$m have a fully martensitic structure (Fig 8.25). After consolidation the $\alpha$-phase has an equiaxed morphology (see Fig 8.43). These experimental observations are in good agreement with previous experimental results in literature [87Wha, 86Wha3, 85Bro] and support the suggestion made by Whang [87Wha, 86Wha3] that very high cooling rate can suppress the martensitic transformation.

Earlier studies indicated that in general martensitic transformation is independent of cooling rate provided that cooling rate is beyond a critical point [70Jep]. However, Whang [87Wha, 86Wha3] has shown that martensitic transformation start temperature $M_S$ is lowered as cooling rate increase on rapid quenching from liquid state. Consequently, for a given alloy composition no martensitic structure was observed at a cooling rate considerably above the critical cooling rate, while at lower cooling rate (a result of the increased section thickness), a clear martensitic structure was observed. This effect of cooling rate on martensitic transformation can also affect the morphology of the $\alpha$-phase after consolidation at the $\alpha + \beta$ phase region, as supported by the experimental results of Broderick et al [85Bro]. They showed that in alloys cooled at
≤5×10^3°C/sec martensite transformed to lenticular \( \alpha \) after consolidation, while alloys cooled at ≥5×10^5°C/sec developed an equiaxed \( \alpha \) morphology. This change in morphology was explained in terms of the high dislocation density and grain size refinement, both of which resulted from high cooling rate, because martensitic transformation is known to be depressed by ultrafine grain size [91Sur].

From the above discussion, it can be concluded that increasing cooling rate decreases \( \text{Ms} \). For a given alloy composition, martensitic transformation can be completely suppressed if cooling rate is beyond a critical point.

Fig 9.1. Schematic illustration of the arrangement of Ti and B atoms in TiB.

9.4. The Growth Habit of the TiB Phase under Different Thermal Conditions

9.4.1. Crystallographic Considerations

TiB has a B27 structure (FeB type), characterised by zig-zag chains of boron atoms parallel to the b direction, with each B atom lying at the centre of a trigonal prism of six titanium atoms [54Dec]. The arrangement of B and Ti atoms in B27 structure is schematically illustrated in Fig 9.1. In fact, crystal structures of many refractory borides are all based on the same building block: the trigonal prismatic array of six Ti atoms with a B atom at the centre [77Lun] (see Fig 9.2(a)). Different boride structures can then be built by arranging these trigonal prisms in various patterns [77Lun]. B27 structure consists of trigonal prisms stacked in columnar arrays sharing only two of their three rectangular faces with neighbouring prisms, as depicted in Fig 9.2(b). Boron atoms in this case form a zig-zag chain along the [010] direction. To account for the stoichiometry, the columns of prisms are connected only at their edges forming boron-free
Chapter 9: Experiments on In Situ Ti/TiB MMCs. Part II. Discussion

"pipes" of metal atoms with a trapezoidal cross section, as shown by the shaded area in Fig 9.2(b) and the (010) projection of the atomic arrangement of B27 structure in Fig 9.2(c).

Fig 9.2. Schematic illustration of the atomic packing in TiB. (a) the basic trigonal prismatic arrangement of Ti atoms around each B atom; (b) the arrangement of the basic trigonal prisms to form the TiB structure; (c) the projection of the TiB structure on the (010) plane. The shaded area in (b) and (c) represents the boron-free "pipe".

Since the crystals growing from melt are usually bound by the slowest-growing facets, one would anticipate that the stacking pattern shown in Fig 9.2(b) should lead to a well defined TiB crystal morphology. Typically, growth normal to planes containing both metal and B atoms in the same stoichiometry as the crystal should be faster than growth along directions involving alternating planes of metal and B atoms. Moreover, planes with a higher density of strong bonds (B-B>B-M>M-M) or "periodic Bond Chains" [73Har] also tend to grow at a faster rate. Based on the above argument, TiB should exhibit much faster growth along [010] direction.
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than normal to (100), (101), (102) and (001) planes, therefore developing a needle-shaped morphology.

Fig 9.3. The relationship between the atomic arrangement in TiB (the projection on (010) plane) and the microscopic facets ((100), (101) and (101̅) planes) shown by TiB particles.

9.4.2. Microstructural Evidence

The TiB morphology can be rationalised, to some extent, based on its crystal structure and the thermal conditions in which they evolve. The preferred growth direction in TiB is always [010], which is the axis of the boron chains formed by stacking the trigonal prisms (Fig 9.2(b)). The (010) planes perpendicular to this direction also have a 1:1 stoichiometry, providing sites for attachment of both Ti and B atoms on the same plane. Now let us consider the facets which enclose the cross section of the needle crystal. If the bond strength in TiB is B-B>Ti-B>Ti-Ti, as estimated from interatomic distances [54Dec], one would expect Ti planes bounding the boron chains in the crystal to have lower interfacial energy and, thus, be exposed
to the liquid. From the crystal structure of TiB phase, it is also clear that the packing density of the Ti atoms decreases in the following order: (100)>(101), (10\(\bar{1}\))>(102), (\(\bar{1}02\))>(001), and that the growth along [100], <101> and <102> directions involves alternating Ti and B planes, whereas the growth along [001] direction involves planes of equiatomic stoichiometry. One can thus conclude that (001) planes are likely to exhibit a faster growth rate than any other planes in this group, leaving the crystal bounded by (100), (101), (10\(\bar{1}\)), (102) and (\(\bar{1}02\)) type faces. Arrangement of B and Ti atoms in (010) projections of TiB enclosed both by (100) and (10\(\bar{1}\)) and by (100) and (102) type of planes are schematically illustrated in Figs 9.3 and 9.4. The experimental results presented in Section 8.3.2 are in very good agreement with the predictions by the crystal growth theory.

9.4.3. The Effect of Thermal Condition on the Morphology of TiB

It is clear from the above discussion that TiB phase will have a needle-shaped morphology if it grows from liquid alloy. However, the morphology of the TiB phase formed from the B supersaturated titanium solid solution can be either needle-shaped or nearly equiaxed depending upon the thermal condition in which the TiB phase nucleates and grows. The results of pre-
hipping heat treatments in Section 8.3.4 showed that TiB phase has a nearly equiaxed morphology if annealing temperature is below 800°C and that a small population of the TiB particles can also have a needle-shaped morphology if the annealing temperature is above this temperature. This dependence of TiB morphology on the thermal condition can be explained by the role of B diffusion during TiB nucleation and growth process. If TiB phase is to be formed from liquid alloy, because of the high diffusion rate of B in liquid alloys, the difference in growth rates among different metallographic directions is controlled by the condition for atomic attachment, such as stoichiometry of constituent atoms, interplanar spacing and periodic bonding chain (PBC) [73Har]. In this case, the morphology of TiB phase is most likely to be large needles.

However, if the TiB phase is to be formed from the B super-saturated titanium solid solution at a lower temperature (such as 500°C), the diffusion of B in solid solution will become the controlling factor. Difference between the growth rates along different metallographic directions of TiB structure will largely depend on the supply of B atoms from the solid solution. Furthermore, the nucleation of TiB phase is highly favourable because of the high instability of B super-saturated titanium solid solution at this temperature. The consumption of B atoms from the solid solution by formation of a large number of TiB nuclei makes the growth of TiB even more difficult, and eventually impossible when B concentration in solid solution reaches equilibrium. Thus, the difference between the growth rates along different metallographic directions will be substantially reduced, and the TiB phase will eventually develop a nearly equiaxed morphology with the longer axis being parallel to the [010] direction. Annealing the B super-saturated solid solution at a higher temperature (such as 900°C), because of the improved condition for B diffusion, will produce TiB precipitates with relatively higher aspect ratio and a small population of fine TiB needles by coalescence of equiaxed TiB particles which are touching. The Ostwald ripening mechanism is not expected to play an important role during the growth of TiB particles. For the growth of a bigger particle by consumption of smaller ones which are not in touch with the bigger particle, the smaller ones have to be dissolved into the solid solution, while the amount of energy required for this dissolving process is possibly much larger than the surface energy difference between the smaller and bigger particles which is the driving force for Ostwald ripening. However, some of the TiB particles may grow if they form a cluster and are in touch with each other. In this case, a needle-shaped TiB may form from this kind of cluster of equiaxed TiB particles. This explains why after annealing at higher temperature the majority of the TiB particles remain fine and have a nearly equiaxed morphology and only a small population of the fine TiB particles will grow into needles. This also explains the difference in TiB morphology in the consolidated Ti-6Al-4V-XB alloys with and without pre-hipping heat treatment.

### 9.5. The ω-Phase Transformation

The ω-phase is one of the metastable β-phase decomposition products in bcc (β-phase) alloys based on Ti, Zr and Hf [73Wil]. Occurrence of ω-phase was first encountered by Frost et al [54Fro] in aged Ti-Cr binary alloys. Since then, the ω-phase has received extensive study, initially because of its deleterious effects on mechanical and physical properties and later
because the $\beta$ to $\omega$ transformation exemplifies an interesting class of phase transformation: displacive transformation. In this part of the discussion, we shall focus on the mechanism of $\omega$-phase transformation in order to explain the observed electron diffraction patterns.

It is well known that the $\omega$-phase can form by either quenching from high temperature (athermal $\omega$-phase) or by ageing at low temperature (isothermal $\omega$-phase) [73Wil]. Both the athermal and the isothermal $\omega$-phase have a hexagonal structure, belonging to space group $D_{6h}^1$ (P6/mmm), and with a well defined orientation relationship with the parent bcc phase, as first reported by Silcock [55Sil, 58Sil]: $(111)_\beta$ // $(0001)_\omega$, $(1\overline{1}0)_\beta$ // $(11\overline{2}0)_\omega$. It can be seen from this orientation relationship that there are four $\omega$ variants depending on which $(111)_\beta$ is parallel to the $(0001)_\omega$ plane. Similar to the martensitic transformation, athermal $\omega$-phase transformation starts from a well defined temperature ($\omega_s$) [73Wil], and it has been demonstrated that the athermal $\omega$ to $\beta$ transformation is totally reversible [71Fon]. The kinetics of isothermal $\omega$-phase formation is extremely rapid, even in solute-rich alloys which do not contain pre-existing athermal $\omega$-phase [68Hic].

![Schematic representation of the mechanism of the $\beta$ to $\omega$ transformation showing the (111) planes on edge](image)

Fig 9.5. Schematic representation of the mechanism of the $\beta$ to $\omega$ transformation showing the (111) planes on edge after [73Wil].

As first suggested by Hatt and Roberts [60Hat] and later by de Fontaine [70Fon], the athermal $\beta$ to $\omega$ transformation can be accomplished formally by collapsing a pair of neighbouring (111) planes to an intermediate position, leaving the next (111) plane unaltered, collapsing the next pair and so on. This operation produces a structure of hexagonal symmetry which, in the limit of complete double plane collapse, can be called the "ideal $\omega$ structure" as reported by Silcock [55Sil]. A schematic illustration of the formal mechanism of the $\beta$ to $\omega$ transformation is shown in Fig. 9.5 where the (111) planes are shown edge on. Note that submitting the bcc lattice to a 2/3 (111) longitudinal displacement wave of proper amplitude and phase produces the required transformation [73Wil]. An atomic description of the $\beta$ to $\omega$ transformation has
been suggested by Williams et al [73Wil], as schematically illustrated in Fig 9.6. In Fig 9.6(b) the (111) bcc planes are shown edge on, and the bcc stacking sequence is ABCABC…. The projected bcc structure is shown in Fig 9.6(a), where symbols (0) represent atoms in the plane of the figure, (+) above and (-) below the plane. Upon double plane collapse, ω structure is formed and the resulting structure, projected on (111) planes, is shown in Fig 9.6(c). The stacking sequence is now AB'AB'…, with filled circles representing the atoms in the collapsed B' plane. Note the change of three fold symmetry around [111] direction in the bcc structure (Fig 9.6(a)) to a six fold symmetry in ideal ω structure (Fig 9.6(c)).

Fig 9.6. Atomistic representation of the mechanism of the β to ω transformation [73Wil]. (a) bcc structure projected on (111) plane, (0) atoms in the plane of the figure, (+) above and (-) below the plane; (b) (111) planes on edge showing the bcc stacking sequence ABCABC; (c) (111) plane projection after double plane collapse to give ω structure with stacking sequence now AB'AB', filled circles representing the B' plane.

It is reasonable to assume [73Wil] that displacement of one atom along a nearest neighbour direction will introduce like displacement of its neighbours along the atomic row, creating a displaced row segment or linear defect, a schematic representation of which is given in Fig 9.7. Dots represent atomic positions, the vertical lines denoting the equilibrium β position of the bcc lattice. The atomic positions at +1/3 and -1/3 of the nearest neighbour distance are denoted as ω+ and ω-, respectively. Row segments in ω positions are separated from row segments in β positions by transition regions of compression and dilatation, as indicated.

Based on this mechanism, Williams et al [73Wil] tried to predict qualitatively the diffraction patterns expected under various conditions. Above the ω transition temperature, the linear defects belonging to a given <111> variant are uncorrelated and each displaced row thus diffracts almost independently of its neighbours. The diffused intensity should be therefore concentrated in {111} reciprocal planes, perpendicular to the variant in question. As two dimensional correlations make their appearance in the vicinity of the ω transition temperature, the diffused intensity must become modulated within the {111} reciprocal planes and must
eventually peak at the $2/3<111>$ positions and other crystallographically equivalent points in reciprocal lattice. Thus far only a single $<111>$ (or $\omega$) variant has been considered, but interaction between variants must also be taken into account. For instance, a $[11\overline{1}]$ displaced row segment will tend to produce a longitudinal displacement along the intersecting $[111]$ row, which will induce further displacements along $[11\overline{1}]$, and so on. An example of the resulting zig-zag defect is illustrated in Fig 9.8. Thus, when the intervariant interaction is significant, diffused intensity will tend to depart from the octahedron formed by $[111]$ reciprocal planes and lie, instead, in spheres inscribed in the octahedron. One such "sphere of intensity" is shown in Fig 9.9, which is a unit cell of the reciprocal lattice (fcc) of the real bcc lattice.

Fig 9.7. Schematic representation of the linear defect showing displaced row segments [73Wil].

Fig 9.8. Zig-zag defect resulting from interaction of linear displacement defects along intersecting rows [73Wil].

From the above constructed reciprocal lattice of real bcc $\beta$ containing hexagonal $\omega$-phase (Fig 9.9) deduced from the proposed transformation mechanism of $\omega$-phase by Williams et al [73Wil], the different ZAPs can be obtained. Two examples of such ZAPs are presented in Figs 9.10 and 11 for $[001]_\text{ZN}$ and $[110]_\text{ZN}$, respectively. Comparisons between the predicted ZAPs in Figs 9.10-11 and the corresponding experimental ZAPs in Fig 8.59 show that the predicted ZAPs are identical to the corresponding ZAPs obtained by electron diffraction. This result offers solid experimental evidence for the proposed $\beta$ to $\omega$ transformation mechanism by Williams et al [73Wil].
This transformation has rarely been observed in Ti-6Al-4V alloys, and there is no prior report of its formation in RS Ti-6Al-4V-XB alloys. Only one report has been found in the literature that the $\beta$ to $\omega$ transformation can occur in a Ti-6Al-4V alloy quenched from 800°C and aged at 360°C for 30–60 minutes [79Las].

![Reciprocal space of bcc real lattice showing one example of the spheres of diffused intensity inscribed in the octahedron formed by {111} type of reciprocal planes. All the octahedron sites (centres of the inscribed spheres) are indicated by the open circles.](image)

9.6. The Effect of Microstructure on Strength

9.6.1. The Strength of the Matrix Alloy

One of the advantages of rapid solidification processing is the great grain refinement which can be achieved over the conventional ingot metallurgy [91Sur]. The strengthening effect of RS is demonstrated in Fig 9.12 by the increase in Vicker's hardness of the consolidated Ti-6Al-4V alloy over the conventional ingot metallurgy product of the same alloy. Fig 9.12 shows that the consolidated RS alloy has an increment of 167 in Vicker's hardness. Also shown in Fig 9.12 is the Vicker's hardness of the consolidated alloy after annealing at 700°C for 4 hrs. The hardness reduction after the annealing is 71 in Vicker's hardness. This variation in Vicker's hardness can be explained by the well-known Hall-Petch relation between the strength and the grain size (eqn (4.33)). The high hardness of the as consolidated alloy is caused by the fine grain size obtained through the rapid solidification processing, and the hardness reduction after heat treatment of the as consolidated alloy is due to grain growth during the heat treatment.
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Fig 9.10. The reciprocal lattice section, [001]_β zone normal, showing the ω reflections (filled ellipses) moved toward the octahedral sites (open circles at 100 type points) and away from the rectilinear streaks (dotted lines).

Fig 9.11. The reciprocal lattice section, [110]_β zone normal, showing the ω reflections (filled ellipses) moved toward the octahedral sites (open circles at 100 and 111 type point) and away from the rectilinear streaks (dotted lines).
However, the matrix grain growth in alloys with B addition is also substantially reduced. The retardation of the matrix grain growth by B addition is demonstrated in Fig 9.13 which is a comparison of the Vicker's hardness between the consolidated alloy and the consolidated alloy after annealing at 700°C/4hrs for different composite alloys. Fig 9.13 shows that the reduction in Vicker's hardness is much smaller for alloys with B addition than the plain Ti-6Al-4V alloy. As discussed before, the volume fraction and the particle size of the TiB-phase is more or less invariant during the post-hipping heat treatment. Therefore, the hardness reduction of the B containing alloys is largely controlled by the matrix grain size. Little reduction in Vicker's hardness means little matrix grain growth. Thus the presence of uniformly distributed fine TiB particles can prevent matrix grains from growing quickly. This grain refining effect of TiB phase can be explained in terms of pinning of grain boundaries by TiB particles during high temperature exposure.

9.6.2. The Strengthening Effect of TiB-phase

The presence of the TiB particles can substantially strengthen the Ti-6Al-4V matrix. This effect is demonstrated in Figs 9.14 and 9.15, where the Vicker's hardness data of the as consolidated alloys is plotted against the B content (in weight percent) and the volume fraction of TiB-phase, respectively. The Vicker's hardness (VH) of the consolidated alloys increases almost linearly with the B concentration or the volume fraction of TiB-phase within the B content studied here (≤2wt.%). These relationships can be expressed by the following two equations:

\[ VH = 465.2 + 69.0 \text{ (B wt.%) } \]  \hspace{1cm} (9.1)

\[ VH = 465.2 + 11.9 \text{ (TiB vol.-%) } \]  \hspace{1cm} (9.2)

![Graph showing Vicker's hardness of Ti-6Al-4V alloy after different thermomechanical processes](image)

**Fig 9.12.** Vicker's hardness of Ti-6Al-4V alloy after different thermomechanical processes showing the hardness gain of the rapid solidification over the conventional ingot metallurgy and the reduction of hardness after annealing.
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Fig 9.13. Vicker's hardness of rapidly solidified Ti-6Al-4V-XB alloys showing the hardness reduction after annealing at 700°C/4hrs.

Fig 9.14. Vicker's hardness of rapidly solidified Ti-6Al-4V-XB alloys as a function of B content.

Pre-hipping heat treatment of the comminuted alloy powders can produce an extra strengthening effect of TiB phase. The measured Vicker's hardness of the consolidated Ti-6Al-4V-0.8B and Ti-6Al-4V-1.5B alloys with and without pre-hipping heat treatment are compared in Fig 9.16, which indicates that the Vicker's hardness of the alloys with pre-hipping heat treatment is higher than that without the pre-hipping heat treatment. As we have discussed in Section 8.3.4, the pre-hipping heat treatment can produce a finer and more uniform distribution
of TiB phase than the alloys without pre-hipping heat treatment, although the volume fractions of the TiB phase should be the same in these two alloys. Therefore, it can be concluded that the extra strengthening effect of pre-hipping heat treatment is due to the finer particle size of TiB phase.

![Ti-6Al-4V-XB Alloys](image)

**Fig 9.15.** Vicker's hardness of rapidly solidified Ti-6Al-4V-XB alloys as a function of volume percentage of TiB phase.

The strengthening effect of TiB phase and the extra strengthening effect of pre-hipping heat treatment can be explained by the Orowan theory of dispersion strengthening [48Oro]. The yield stress has been calculated by Orowan [48Oro] using the model shown in Fig 9.17, when the particles are by-passed but leave residual dislocation loops around each particle. A simplified form of the relationship for the initial flow stress of this model is

\[
\tau_0 = \tau_s + \frac{T}{b\Lambda/2}
\]  

\(9.3\)

where \(\tau_0\) is the yield stress of the composite alloy, \(\tau_s\) is the critical resolved shear stress of the matrix, \(T\) is the line tension of the dislocation, \(b\) is the Burger's vector of the dislocation, \(\Lambda\) is the average particle spacing. For a random distribution of the second-phase, the average particle spacing \(\Lambda\) can be related to the volume fraction and the average particle radius, \(r\) by the following equation [64Ash]

\[
f = \frac{r^2}{(\Lambda/2)^2}
\]  

\(9.4\)

Then

\[
\tau_0 = \tau_s + \frac{T\sqrt{f}}{br}
\]  

\(9.5\)

Eqn 9.5 reveals the following facts: (a) the yield stress of the composite alloy increases with increasing volume fraction of the second-phase at a constant particle size; (b) the yield stress of the composite alloy increases with decreasing particle size at a constant volume fraction of the
second phase. Therefore, the alloys with pre-hipping heat treatment should exhibit a higher hardness (or strength) due to their decreased TiB particle size.

![Graph showing Vicker's hardness of T-6Al-4V-XB alloys with and without pre-hipping heat treatment](image)

**Fig 9.16.** Vicker's hardness of rapidly solidified Ti-6Al-4V-XB alloys showing the effect of pre-hipping heat treatment.

**Fig 9.17.** Schematic illustration of the interaction of dislocations with second phase particles after Orowan [48Oro].

Another interesting phenomenon indicated by Fig 9.16 is that the strengthening effect due to pre-hipping heat treatment is much lower in alloys with higher boron content than those with lower boron content. This phenomenon can be explained by the relative amount of borides formed from the liquid alloys. The higher the boron content in the bulk alloy, the higher the volume fraction of the borides formed from the liquid alloy provided that the cooling rates are the same. The pre-hipping heat treatment can hardly affect the borides formed from the liquid alloy but only borides precipitated from the supersaturated solid solution. This phenomenon is further complicated by the effect of B content in the bulk alloy on the extent of under-cooling which can be achieved during the RS process. The undercooling will be substantially reduced by the increasing B content in the bulk alloy because primary TiB particles can act as heterogeneous nucleation sites for the prior β-grains. This in turn reduces the extended...
solubility of B in the Ti solid solution. Therefore, the effectiveness of pre-hipping heat treatment will be further reduced by the lowered undercooling in the alloys with high B content. It is expected that the pre-hipping heat treatment will produces no strengthening effect when a critical value of B content is exceeded.

9.7. An Iteration Approach to Young's Modulus of Multi-phase Composites

In Chapter 7, an approach has been developed to predict the Young's modulus of two-phase composites. To a reasonable approximation, this approach can be extended to predict Young's modulus of a multi-phase composite by an iteration method.

Generally speaking, there are two classes of multi-phase composites: (i) a single phase matrix with several reinforcing phases; (ii) both the matrix and the reinforcement are multi-phase systems. Let us consider first the Young's modulus \(E^C\) of the first class of composites (C), which has a single matrix (denoted as M) with 2 reinforcing phases (denoted as R1 and R2). The volume fractions of M, R1 and R2 phases are \(f^M, f^{R1}\) and \(f^{R2}\), respectively. If the R2 phase is hypothetically excluded from the composite system, the remaining composite (denoted as C1) only consists of M (denoted as M1) and R1. The volume fractions of M1 and R1 in composite C1 \(f^{M1}\) and \(f^{R1}\) can be calculated by the following equations:

\[
\frac{f^{R1}}{f^{C1}} = \frac{f^{R1}}{f^M + f^{R1}} \quad (9.6)
\]

\[
f^{M1} = 1 - f^{R1} \quad (9.7)
\]
where $f$ is volume fraction, superscripts denote phases and subscripts denote the composites. The approach described in Chapter 7 can be used to calculate the Young’s modulus of composite $C_1$, $E^{C_1}$ (see Fig 9.18). The composite $C_1$ is then treated as a uniform matrix (denoted as $M_2$) with a $E$ value of $E^{C_1}$. $M_2$ and $R_2$ form a new composite $C_2$. The volume fractions of $M_2$ and $R_2$ in $C_2$ can be obtained by the following equations:

\[
f_{C_2} = f_{C_2}^{R_2}
\]

\[
f_{C_2} = f_{C_2}^{M_2} + f_{C_2}^{R_1}
\]

The Young’s modulus of $C_2$ ($E^{C_2}$) can be calculated by applying the same standard procedure described in Chapter 7. $E^{C_2}$ is the expected Young’s modulus for the entire composite ($C$). This iterative approach for determination of the Young’s modulus of a composite with one matrix and two reinforcing phases is schematically illustrated in Fig 9.18. If there are more than two reinforcing phases, more iterative steps are needed, but the principle remains the same.

For the determination of the Young’s modulus for the second class of composites the approach described above needs to be first applied to both matrix and reinforcement phases to calculate the effective Young’s moduli of matrix ($E^M$) and reinforcement ($E^R$). Finally, the approach for two-phase composite is used to determine the Young’s modulus of the whole composite ($E^C$).

It should be pointed out that porosity in a composite can also be treated as a phase with zero Young’s modulus. Thus, this iterative approach can be also used to evaluate the effect of porosity on the Young’s modulus of the composite.

9.8. Prediction of Young’s Modulus of In situ Ti/TiB Composites

9.8.1. The Young’s Modulus of the Matrix

Elastic constants are generally insensitive to heat treatment, deformation and microstructure. However, the Ti-6Al-4V alloy behaves differently in this respect, because the volume fractions of different phases can be altered by heat treatment [91Lee]. Modulus variation is also caused by texture [74Lar], oxygen concentration [85Lee] and precipitates produced by certain heat treatment [90Lee]. For the purpose of the present work only the effect of phase transformations will be discussed here.

Because the chemical composition of the $\alpha$-phase varies little with heat treatment temperature [88Lee2, 66Cas], its mechanical properties are hardly affected by heat treatment [88Liu]. However the $\beta$-phase experiences large compositional variations [88Lee2] which is reflected in significant mechanical property changes. Vanadium enrichment in the $\beta$-phase occurs in
proportion to the reduction of the volume fraction of the β-phase [90Lee, 66Cas]. At vanadium content of 15 wt-% or more, the bcc β-phase is stabilised on quenching. Vanadium-lean β-phases will transform into α" and α' martensites. When a β-phase with 10 wt.-% vanadium is quenched, it partly retains the bcc structure, and partly transforms into orthorhombic α" martensite [85Lee, 90Lee]. The higher the solution-treatment temperature, the smaller the V enrichment in β-phase; it transforms into α' martensite upon quenching.

Recently, Lee et al [91Lee] studied the effect of heat treatment on the Young's modulus of Ti-6Al-4V alloy. Their results are presented in Fig 9.19 in which the Young's moduli are plotted as a function of the quenching temperature. The Young's modulus of Ti-6Al-4V alloy exhibits a minimum at about 800°C, which correspond to the temperature of transition from retained β to martensite. From these results, we can obtain the Young's modulus as a function of volume fraction of the prior β-phase using the correlation between the volume fraction of the prior β-phase and the quenching temperature obtained by Castro and Sepaphin [66Cas]. The obtained results are shown in Fig 9.20. It is interesting to note that the Young's modulus of Ti-6Al-4V alloys is a complex function of the volume fraction of the prior β-phase, which differs from other composite systems. This abnormal behaviour of Ti-6Al-4V alloy is caused by the phase transformation in the prior β-phase as discussed previously. In the following part of this section, the iterative approach developed in Section 9.7 together with the knowledge of phase transformation in this alloy will be employed to predict the Young's modulus of Ti-6Al-4V alloy.

![Fig 9.19. Young's modulus of Ti-6Al-4V alloy as a function of quenching temperature. The experimental data is from Lee et al [91Lee].](image-url)
Fig 9.20. Young's modulus of Ti-6Al-4V alloys as a function of volume fraction of β-phase. The experimental data (filled circles) are from Lee et al [91Lee], and the volume fraction of β-phase is converted from the quenching temperature using the experimental results of Castro and Sepaphin [66Cas]. The solid line represents the prediction by the iterative approach.

According to their experimental work on β-titanium alloys, Elfer and Copley [85Elf] found that the dependence of modulus on composition can be described by the following equation:

$$E^\beta = 73 + 0.48V\% + 1.0Al\% + 1.5Cr\% + 2.0Mo\% \text{ (GPa)}$$ (9.10)

where the amount of alloying elements is in weight per cent. It is assumed that the β-phase will be retained after quenching from 800°C and the β composition is Ti-4Al-10V following [91Lee]. The Young's modulus of β-phase can be calculated from eqn (9.10), i.e., $$E^\beta = 82 \text{ GPa}$$. From this value, the Young's modulus of α-phase can then be extrapolated from the E value of the α+β composite at room temperature in Fig 9.19 by using the law of mixtures. The extrapolated result is $$E^\alpha = 117 \text{ GPa}$$. The young's modulus of the martensite phase (assuming that $$E^\alpha = E^\alpha$$) can also be obtained from Fig 9.19, because the quenched microstructure will be fully martensitic if the heat treatment temperature is above the α/β transus (935°C) [91Lee]. Thus $$E^\omega = E^\alpha = 113 \text{ GPa}$$. The young's modulus of ω-phase ($$E^\omega$$) can be approximated from $$E^\beta$$ following the suggestion of $$E^\omega = 2E^\beta$$ made by Bowen [71Bow, 80Bow]. The various Young's moduli and Poisson's ratios obtained for different phases in Ti-6Al-4V alloy are summarised in Table 9.1.

The E and v values listed in Table 9.1 can now be used to calculate the Young's modulus of the Ti-6Al-4V "composite" by applying the iterative approach described previously. This alloy can be considered as consisting of an α-matrix reinforced by the β-phase which itself is a
composite system (β-matrix reinforced by α:, α' and ω-phases). As discussed in Chapter 7, the Young's moduli predicted by the approach described in Chapter 7 and the law of mixtures will be very close provided that the Young's moduli of constituent phases only differ by a factor less than 2. This is the case of Ti-6Al-4V. For simplicity, the law of mixtures will therefore be applied here. It is also assumed that quenching from a temperature above 800°C will always produce retained β-phase with a volume fraction of 1/4 \( f_\beta \), which is the proportion of \( \alpha \) to \( \beta \) phase at 800°C, i.e., \( f_\beta = 0.25 f_\alpha \). The predicted results are presented in Fig 9.20, which indicates that the theoretical predictions are in fairly good agreement with the experimental results Lee et al [91Lee].

**Table 9.1. Summary of the Young's moduli and Poisson's ratios obtained for different phases in Ti-6Al-4V.**

<table>
<thead>
<tr>
<th>Phase</th>
<th>E (GPa)</th>
<th>ν</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>β-phase</td>
<td>82 [85Elf]</td>
<td>0.27</td>
<td>Assumed ( \nu_\beta = \nu_\alpha )</td>
</tr>
<tr>
<td>α-phase</td>
<td>117 [91Lee]</td>
<td>0.27</td>
<td>Extrapolated</td>
</tr>
<tr>
<td>ω-phase</td>
<td>165 [80Bow]</td>
<td>0.333</td>
<td>( \nu ) is assumed to be 1/3</td>
</tr>
<tr>
<td>martensite (α'' and α')</td>
<td>113 [91Lee]</td>
<td>0.355</td>
<td>Experimental results</td>
</tr>
</tbody>
</table>

In the case of the consolidated Ti-6Al-4V alloy, the volume fraction of prior β-phase is approximately 0.05 and the experimentally determined E value is 116.6 GPa. This result can be confirmed by theoretical calculation by assuming that 50% of the prior β-phase will be retained, 25% transformed to \( \alpha \) precipitates and the rest transformed into ω-phase.

**9.8.2. Prediction of the Young's Modulus of the In Situ Ti/TiB Composites**

The Young's modulus of TiB is not available in the literature. However, the Young's modulus of TiB\(_2\) has been theoretically determined (550GP) by applying a thermodynamic approach developed by Miodownik [92Mio]. For the present calculation, it is assumed that \( E_{TiB} = E_{TiB2} \). The Poisson's ratio of TiB-phase (0.14) was adopted from a value quoted for ZrB\(_2\) [60Lan], but the calculations are not sensitive to this value. For the matrix alloy, the Young's moduli have been calculated in the previous section, and the Poisson's ratio is chosen as 0.27, which is the Poisson's ration of α-phase [91Lee]. The reason for choosing this value is the high volume of α-phase in the matrix alloy. These parameters are listed in Table 9.2.

**Table 9.2. List of the parameters used for prediction of Young's modulus of in situ Ti/TiB composites.**

<table>
<thead>
<tr>
<th>Phase</th>
<th>E (GPa)</th>
<th>ν</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Matrix (Ti-6Al-4V)</td>
<td>116.7</td>
<td>0.27</td>
<td>Calc. from exp. data of [91Lee]</td>
</tr>
<tr>
<td>TiB</td>
<td>550 [92Mio]</td>
<td>0.14 [60Lan]</td>
<td>( \nu ) is Adopted from ZrB(_2) phase</td>
</tr>
</tbody>
</table>
Because of the extremely fine size of TiB phase, it is difficult to determine experimentally the topological parameters required for the calculation. However, we can follow the assumptions made in Chapter 7 for other composite systems, i.e.,

\[
\begin{align*}
\text{f}_{\text{TiBc}} &= \text{f}^2_{\text{TiB}} \\ 
\text{f}_{\text{Mc}} &= \text{f}^3_{\text{M}}
\end{align*}
\]  

(9.11) (9.12)

where \( \text{f}_{\text{TiBc}} \) and \( \text{f}_{\text{Mc}} \) are the continuous volume of TiB and the matrix phases, respectively. The calculated Young's moduli of the in situ Ti/TiB composites are compared in Fig 9.21 with the experimental results from this investigation (open circles) and those obtained by Saito and Furuta [92Sai] from reaction sintering (open squares). The predictions using the law of mixtures are also presented in this figure for comparison. Fig 9.21 shows that the theoretical predictions by the present approach are in fairly good agreement with the experimental results and better than the predictions by the law of mixtures.

![Fig 9.21. Comparison of the theoretical predictions of Young's modulus of Ti-6Al-4V-XB composites by the present approach (the solid line) and by the law of mixtures (the dashed line) with the experimental results from this investigation (the open circles) and from Saito and Furuta [92Sai] by reaction sintering (the open squares).](image)

**9.9. The Ductility of in situ Ti/TiB Composites**

As has been mentioned in Section 8.3.6, nearly all the samples for tensile test failed within the elastic range of the flow curve, even for the consolidated Ti-6Al-4V powders without any B addition. It was expected that these samples should have appreciable ductility according to the microstructures observed in Section 8.3.5. Furthermore, the Ti-6Al-4V alloy produced by conventional ingot metallurgy has an elongation of 11% [86Ray1], and elongations of 16–17%
have been obtained from the gas atomized and hipped Ti-6Al-4V alloy after annealing at 720°C [86Mol]. A 2.5 % elongation has also been quoted for the melt spun and extruded Ti-6Al-4V-TiB alloy [86Rayl]. Therefore, it is clear that the zero elongation is not an inherent characteristics of the Ti-6Al-4V-XB composites, there must have been an inappropriate step in the thermomechanical history.

Eylon and Froes [86Ey1] investigated the feasibility of hipping RS Ti-6Al-4V powders at high pressure and low temperature. In their experimental work, two sets of hipping conditions were chosen for the consolidation. One sample was hipped at 650°C/300MPa/24hrs, the other at 595°C/300MPa/24hrs. The tensile test results showed that the sample hipped at 650°C had an elongation of 8%, while the sample hipped at 595°C had an elongation of only 0.2%. They also reported that the post-hipping annealing can substantially improve the ductility of these samples. An elongation as high as 22% was obtained for the sample hipped at 650°C and 8% for the other sample. They considered that sample hipped at 595°C/300MPa/24hrs produced poor bonding at the powder interface, while the sample hipped at 650°C/300MPa/24hrs led to improved bondings between the alloy powders, although in both cases a densified material had been achieved. The post-hipping heat treatment can enhance the condition for interdiffusion between alloy powders, and consequently resulted in even better bonding at the alloy powder interface. This argument was confirmed by the fractographic examinations of samples undergone different thermomechanical processings [86Ey1].

From the above experimental results of Eylon and Froes [86Ey1], it can be concluded that the ductility of the hipped RS alloys is very sensitive to the bonding between alloy powders, while the bonding in turn is controlled by the adequateness of the diffusion between the alloy powders which is a function of the product of temperature and time. It is very likely that the zero elongation of the present samples is caused by poor bonding between the alloy powders resulting from inadequate time and temperature chosen for the hipping process.

9.10. Summary

(1) A series of Ti-6Al-4V and Ti-Mn alloys with different levels of B addition have been melt-spun and consolidated. These materials have been characterised by microstructural examination and mechanical testing to investigate the feasibility of production of Ti-alloy based in situ TiB composites through the rapid solidification route. Production of in situ Ti/TiB metal matrix composites through the rapid solidification has been found to be a feasible method, which can exclude the wetting and chemical reaction problems encountered by other production route, such as diffusion bonding and squeeze casting. However, further work is needed to increase the possible volume fraction of TiB, to improve the ductility and to lower the production costs before the process can be optimised and scaled up.

(2) All the borides present in the compositions under investigation have been identified as TiB with a B27 (Pnma) structure by both electron and X-ray diffractions. There is no evidence for the presence of Ti3B4 and TiB2.
Chapter 9: Experiments on In Situ Ti/TiB MMCs. Part II. Discussion

(3) Pre-hipping heat treatment on the RS product at a temperature below 800°C can lead to the precipitation of fine equiaxed TiB particles from the B super-saturated Ti solid solution. The TiB precipitates are uniformly distributed throughout the α+β matrix. Microhardness test indicates that a pre-hipping heat treatment can also lead to an extra strength due to a refinement of the TiB particle size, but this extra strengthening effect is less significant for alloys with higher B content.

(4) TiB phase in the consolidated composites exhibits both needle-shaped and near equiaxed morphologies. It is believed that the needle-shaped TiB is formed from the liquid alloy during the RS process, while the near equiaxed TiB is precipitated from the B super-saturated Ti solid solution during the pre-hipping heat treatment performed on the RS product. The needle-shaped TiB phase always grows along the [010] direction of the B27 unit cell, leaving the cross-section of the needles consistently enclosed either by (100) and (101) type planes or by (100) and (102) type planes. The cross-section of the nearly equiaxed TiB particle is also bounded by the same planes. These interesting phenomena can be explained by a combination of the crystallographical characteristics of TiB phase and crystal growth theory.

(5) The β to ω transformation has been observed in all the composite alloys after consolidation. Further work is needed to account for the origin of β to ω transformation in the Ti-6Al-4V base composite. Nevertheless, systematic electron diffraction work on the β-phase has provided strong experimental evidence for the β to ω transformation mechanism proposed by Williams et al [73Wil]. In addition to the ω-phase, type 1 and type 2 α phases have also been observed within the β-phase in the Ti-6Al-4V-XB composites.

(6) An iterative approach has been proposed for prediction of Young's modulus of the multi-phase composite and has been applied to the Ti-6Al-4V matrix alloy and the Ti-6Al-4V-XB in situ composites. It is found that the theoretical predictions by the iterative approach are in fairly good agreement with the experimental results.
CHAPTER 10

General Conclusions

(1) Based on the concepts of contiguity and continuous volume proposed by Gurland et al [58Gur, 78Lee], a series of topological parameters have been defined. All those parameters can be either measured experimentally by using a standard metallographic method or calculated theoretically from the grain size and volume fraction (under the assumption of equiaxed grain and random phase distribution). A combination of these topological parameters with well defined geometrical quantification methods can offer a full quantitative characterisation of two-phase microstructures with any volume fraction, grain size, grain shape and phase distribution. The concept of directional contiguity has also been defined for the analysis of oriented microstructures along a particular direction.

(2) A topological transformation of two-phase microstructure has been proposed, which allows a microstructure with any volume fraction, grain size, grain shape and phase distribution to be transformed into a three-element body with three microstructural elements being aligned in parallel along a particular direction of interests. It has been shown that the topologically transformed 3-element body along the aligned direction is mechanically equivalent to the original microstructure before the transformation.

(3) The Hall-Petch relation originally developed for single-phase alloys has been extended to two-ductile-phase alloys. The extended Hall-Petch relation expresses the yield strength of an $\alpha-\beta$ dual-phase alloy in terms of the grain size and Hall-Petch parameters of each microstructural element. It is now possible to separate the individual contributions from grain and phase boundaries to the overall efficiency of all kinds of boundaries as obstacles to dislocation motion. The extended Hall-Petch relation has been applied to $\alpha-\beta$ Ti-Mn alloys, $\alpha-\beta$ Cu-Zn alloys and $\alpha-\gamma$ Fe-Cr-Ni stainless steels to evaluate the efficiency of phase boundaries as obstacles to dislocation motion. It is found that phase boundary is not always the strongest obstacles to dislocation motion in two-ductile-phase alloys.

(4) The extended Hall-Petch relation can also be used to predict the yield strength of two-phase alloys once $\sigma_{\gamma}^{\alpha\beta}$ and $k_{\gamma}^{\alpha\beta}$ have been evaluated for a particular alloy system. It has been shown that the predictions of the yield strength by the extended Hall-Petch relation are in very good agreement with experimental data in $\alpha-\beta$ Ti-Mn and $\alpha-\beta$ Cu-Zn alloys.
5. A new approach to the deformation behaviour of two-ductile-phase alloys has been developed based on the Eshelby's continuum transformation theory. In contrast to the existing theories of plastic deformation, the new theory can consider the effect of microstructural factors, such as volume fraction, grain shape and phase distribution. The interactions between particles of the same phase has been taken into account by an equivalent microstructural transformation. In addition, the present approach has a provision for the calculation of the in situ stress and plastic strain distribution among three microstructural elements.

6. The true stress-true strain curves of various two-ductile-phase alloys have been calculated and theoretical predictions are in good agreement with the experimental results draw from the literature, not only in the low strain region but also in the high strain region. The mean internal stresses in three microstructural elements have also been calculated for various two-ductile-phase alloys.

7. It has been shown that the role played by the mean internal stresses during the deformation process is to imped the further plastic deformation in the softer element, to aid the further plastic deformation in the harder element and consequently to make the plastic deformation of two-ductile-phase alloys tend to be more homogeneous throughout the microstructure with increasing plastic strain.

8. The theory predicts that there are four deformation stages in the overall deformation process of two-ductile-phase alloys, which differs from the classical 3-stage deformation theory. The calculated stress-strain curves of various two-ductile-phase alloys show a flow stress drop after the onset of plastic deformation in EIII or EI, and this stress drop has been explained in terms of the elastic energy release, which is supported by the experimental evidence in the literature. It is believed that the onset of the plastic deformation in EIII marks the starting point of macroscopic plastic deformation of the two-phase mixture.

9. It has been found that the difference between the mechanical properties of the constituent phases has a significant influence on the mean internal stresses and the in situ stress and plastic strain distribution not only on the magnitude but also on the variation trend with increasing macroscopic plastic strain.

10. The phase distribution has a significant effect on the local deformation behaviour of two-ductile-phase alloys in terms of mean internal stresses and the in situ stress and plastic strain distribution in the three microstructural elements, although it does not affect markedly the macroscopic deformation behaviour described by the true stress-strain curves of two-phase composites during Stage 4 deformation.

11. A new approach for predicting the Young's moduli of two-phase composites has been developed based on mean field theory and microstructural characterisation in Chapter 3.
and has been applied to the Co-WC$_p$, Al/SiC$_p$ and glass filled epoxy composites. It has been shown that the theoretical predictions by the present approach are well within Hashin and Shtrikmans' lower and upper bounds and in better agreement with the experimental

(12). The classical linear law of mixtures appears as a specific case of the present approach, where the reinforcement is perfectly aligned continuous fibres. However, in contrast to the classical linear law of mixtures, the present approach can be applied to a two-phase composite with any volume fraction, grain shape and phase distribution.

(13). The Young’s modulus of particulate composite increases with the increasing contiguity of the constituent phases. This increase in E values is dependent on the stiffness ratio of reinforcement to matrix. The larger the stiffness ratio, the larger the increment in stiffness with increasing continuity of the reinforcing phase at fixed volume fraction.

(14). An iterative approach has been proposed for determination of Young’s modulus of the multi-phase composite and has been applied to the Ti-6Al-4V matrix alloy and Ti-6Al-4V-XB in situ composites. It is found that the theoretical predictions are in good agreement with the experimental results.

(15). A series of Ti-6Al-4V and Ti-Mn alloys with different levels of B addition have been melt-spun and consolidated, and characterised by microstructural examinations and mechanical. It is found that production of in situ Ti/TiB metal matrix composites through the rapid solidification is a very promising method, which can exclude the wetting and chemical reaction problems encountered by other production routes, such as diffusion bonding and squeeze casting.

(16). All the borides in the alloy composition range under present investigation have been identified as TiB with a B27 (Pnma) structure by both electron and X-ray diffractions. There is no evidence for the presence of Ti$_3$B$_4$ and TiB$_2$ in these composite alloys. Microstructural examination indicates that TiB phase in the consolidated composites exhibits two distinguished morphologies: needle-shaped TiB and near equiaxed TiB. It is believed that the needle-shaped TiB is formed mainly from the liquid alloy during the RS process, while the nearly equiaxed TiB is precipitated from the B super-saturated Ti solid solution during the pre-hipping heat treatment on the RS products.

(17). Needle-shaped TiB always grows along the [010] direction of the B27 unit cell, leaving the cross-sections of the needles consistently enclosed either by (100) and (101) type planes or by (100) and (102) type planes. It is also found that the cross-sections of near equiaxed TiB particles are also bounded by the same planes although the growth rate along the [010] direction has been considerably reduced. This has been explained by the combination of the crystallographical characteristics of TiB phase and the crystal growth theory.
(18). Pre-hipping heat treatment on the RS product at a temperature below 800°C can lead to the precipitation of fine equiaxed TiB particles from the B super-saturated Ti solid solution, which are uniformly distributed throughout the \( \alpha + \beta \) matrix. This heat treatment also results in extra strength due to the finer TiB particles. Ostwald ripening of TiB is inhibited even after long time exposure at higher temperature (e.g., 800°C) due to the extremely low solid solubility of B in both \( \alpha \) and \( \beta \) titanium solid solutions.

(19). Hardness changes due to boron additions have been satisfactorily explained in terms of the intrinsic properties of TiB and the effect of this phase on the grain size of the matrix.

(20). The \( \omega \)-phase has been observed in RS Ti-6Al-4V alloys with and without B addition after consolidation. This transformation has also been observed in RS Ti-Mn-B alloys after consolidation. Systematic electron diffraction work on the \( \beta \)-phase offers a strong experimental evidence for the transformation mechanism proposed by Williams et al.
Suggestions for Further Work

In trying to understand correlations between microstructure and mechanical properties of multiphase materials, the work described in this thesis has opened several avenues for further study, and has generated more questions than it has solved. The followings are considered to be the most important ones:

**Further Development of the Theories**

1. Although the topological parameters defined in Chapter 3 can be calculated from the known grain size and volume fraction of each constituent phase under the assumption of random distribution of equiaxed grains, it is desirable to establish a systematic method for the calculation of these topological parameters for random or aligned structures with non-equiaxed grains. This would be beneficial to the extension of the deformation theory described in Chapter 5 and 6 to account for the deformation behaviour of short fibre MMCs.

2. The mechanical equivalence between the topologically transformed 3-E body and the original microstructure needs further justifications in terms of both mathematical and experimental supports.

3. Further work is needed to develop the deformation theory described in Chapter 5 and 6 to predict the deformation behaviour of short fibre MMCs by considering the relaxation process occurred at fibre/matrix interfaces during the plastic deformation stage.

**Experiments on In Situ Ti/TiB MMCs Obtained through RS Route**

1. So far, the volume fraction of TiB phase in the in situ Ti/TiB MMCs produced by RS route is less than 0.1. It is desirable to increase the TiB volume fraction to 0.2-0.3. As discussed in Chapter 8, this increase in TiB volume fraction is limited by the capacity of the present equipment rather than any theoretical problem.

2. As discussed in Chapter 9, the zero ductility is not an inherent characteristics of the in situ Ti/TiB MMCs produced through the RS route. Further work is required to improve the ductility of these materials by allowing adequate interdiffusion between the comminuted powders to assure the good bonding between them.
Appendix: Fortran Program for Calculation of the Flow Curves for Two-Ductile-Phase Alloys

Appendix

Fortran Program for Calculation of the Flow Curves of Two-Ductile-Phase Alloys

This Fortran program is designed to calculate the true stress-true strain curves of two-ductile-phase alloys according to the approach developed in Chapter 5.

This is the main program which reads the necessary data from a data file, calls the relevant subroutines to execute the calculations and transfer the calculated results into output data files.

```
COMMON /VOL/FA,FB,FA3,FB3,FAC,FBC,FS
COMMON /SIGY/SIGYA,SIGYB,SIGY3,E
COMMON /ABC/AB(4),B(4),C(4)
COMMON /AB/AA,BB
COMMON /SIGF/SIGF3(10000,2),SIGF(10000,2)
COMMON /ABN/NA,NB
COMMON /ABK/KA,KB
COMMON /MNK/M,N,KS
COMMON /EPSX/EPS1,EPS2,EPU
COMMON /TOP
REAL NA,NB,KA,KB
OPEN(1,FILE='INPUT',STATUS='OLD')
OPEN(2,FILE='OUTE3',STATUS='NEW')
OPEN(3,FILE='OUTAB',STATUS='NEW')
OPEN(4,FILE='OUT123',STATUS='NEW')
OPEN(5,FILE='OUTINT',STATUS='NEW')
OPEN(6,FILE='OUTEPC',STATUS='NEW')
READ(1,*END=50)FA,FB,FAC,FBC,SIGYA,SIGYB,
+ SIGY3,E,P,NA,NB,KA,KB,EPS1,EPS2,TOP,EPU
FS=1.-FAC-FBC
FA3=(FA-FAC)/FS
FB3=(FB-FBC)/FS
M=0
N=0
KS=1
AA=(E*(7.-5.*P)/(10.*(1.-P**2.)))
```
Appendix: Fortran Program for Calculation of the Flow Curves for Two-Ductile-Phase Alloys

BB=(E*(19.-14.*P)/(16.*(I.-P**2.))
CALL FCEIII
CALL FCAB
DO 80 I=1,N
80 WRITE(2,*)SIGF3(I,1),SIGF3(I,2)
DO 100 J=1,M
ET=SIGF(J,1)+SIGF(J,2)/E
JJ=J+1
WHR=(SIGF(JJ,2)-SIGF(J,2))/(SIGF(JJ,1)-SIGF(J,1))
100 WRITE(3,*)J,ET,SIGF(J,1),SIGF(J,2),WHR
CLOSE(1,STATUS='KEEP')
CLOSE(2,STATUS='KEEP')
CLOSE(3,STATUS='KEEP')
CLOSE(4,STATUS='KEEP')
CLOSE(5,STATUS='KEEP')
CLOSE(6,STATUS='KEEP')
STOP
END

C THIS SUBROUTINE IS DESIGNED TO EXECUTE THE CALCULATION OF
C FLOW CURVES OF THE EIII BODY
SUBROUTINE FCEIII
COMMON /VOL/FA,FB,FA3,FB3,FAC,FBC,FS
COMMON /SIGY/SIGYA,SIGYB,SIGY3,E
COMMON /ABC/A(4),B(4),C(4)
COMMON /AB/AA,BB
COMMON /SIG/SIGF3(10000,2),SIGF(10000,2)
COMMON /EP33/EPA3(2),EPB3(2)
COMMON /ABN/NA,NB
COMMON /ABK/KA,KB
COMMON /MNK/M,N,KS
COMMON /EPSX/EP81,EP82,EP8
COMMON /T/TOP
REAL NA,NB,KA,KB
A(1)=SIGYA
A(2)=FB3*AA
A(3)=KA
B(1)=SIGYB
B(2)=FA3*AA
B(3)=0.
CALL BIE2(Y23,EPA23,0.,NA,0.,1)
N=1
EPA3(1)=0.
EPB3(1)=0.
SIGF3(1,1)=0.
Appendix: Fortran Program for Calculation of the Flow Curves for Two-Ductile-Phase Alloys

```
SIGF3(1,2) = SIGYA
10  N = N + 1
    NN = N - 1
    EPA3(2) = EPA3(1) + EPS1/3.
    EPB3(2) = 0.
    IF(EPA3(2),GE.EPA23) GOTO 20
SIG33A = SIGYA + KA*(EPA3(2))**NA + FB3*AA*EPA3(2)
CALL INCRE3(SIG33A, DEP3)
SIGF3(N,1) = SIGF3(NN,1) + DEP3
SIGF3(N,2) = SIG33A
WRITE(*,1) N, SIGF3(N,1), SIGF3(N,2)
1  FORMAT(1X,5H*****,I5,2F14.7)
EPA3(1) = EPA3(2)
EPB3(1) = EPB3(2)
GOTO 10
20  EPA3(2) = EPA23
    EPB3(2) = 0.
    CALL INCRE3(Y23, DEP3)
SIGF3(N,2) = Y23
SIGF3(N,1) = SIGF3(NN,1) + DEP3
WRITE(2,*), SIGF3(N,1), SIGF3(N,2)
EPA3(1) = EPA3(2)
EPB3(1) = EPB3(2)
30  N = N + 1
    NN = N - 1
    EPA3(2) = EPA3(1) + EPS2/6.
    IF(SIGF3(NN,1),GE.TOP+.01) GOTO 40
A(1) = SIGYA + KA*(EPA3(2))**NA + FB3*AA*EPA3(2)
A(2) = -FB3*AA
A(3) = 0.
B(1) = SIGYB - FA3*AA*EPA3(2)
B(2) = FA3*AA
B(3) = KB
CALL BIE2(SIG33A, EPB3(2), EPB3(2), NB, 0, 1)
CALL INCRE3(SIG33A, DEP3)
SIGF3(N,1) = SIGF3(NN,1) + DEP3
SIGF3(N,2) = SIG33A
WRITE(*,2) N, SIGF3(N,1), SIGF3(N,2)
2  FORMAT(1X,15,12H$$$$$$$$$$$,2F14.7)
EPA3(1) = EPA3(2)
EPB3(1) = EPB3(2)
GOTO 30
40  N = NN
RETURN
END
```
THIS SUBROUTINE IS DESIGNED TO EXECUTE THE CALCULATION OF THE PLASTIC STRAIN INCREMENT IN THE EIII BODY

SUBROUTINE INCRE3(SIG33A,DEP3)
COMMON /VOLF/FA,FB,FA3,FB3,FAC,FBC,FS
COMMON /AB/AA,BB
COMMON /SIGF/SIGF3(10000,2),SIGF(10000,2)
COMMON /EP33/EPA3(2),EPB3(2)
COMMON /MNK/M,N,KS
DEPA3=EPA3(2)-EPA3(1)
DEPB3=EPB3(2)-EPB3(1)
DEP31=DEPA3*FA3+DEPB3*FB3
DEP32=(AA/SIG33A)*(EPB3(2)-EPA3(2)-EPA3(2)*DEPA3+(EPA3(2)-EPB3(2))*DEPB3)*FA3*FB3
DEP3=DEP31+DEP32
RETURN
END

THIS SUBROUTINE IS DESIGNED TO EXECUTE THE CALCULATION OF FLOW CURVES OF THE 3-E BODY

SUBROUTINE FCAB
COMMON /VOLF/FA,FB,FA3,FB3,FAC,FBC,FS
COMMON /SIGY/SIGYA,SIGYB,SIGY3,E
COMMON /ABC/A(4),B(4),C(4)
COMMON /AB/AA,BB
COMMON /SIGF/SIGF3(10000,2),SIGF(10000,2)
COMMON /EPAB3/EPA(2),EPB(2),EP3(2)
COMMON /SIN/SIGINA,SIGINB,SIGIN3
COMMON /ABN/NA,NB
COMMON /ABK/KA,KB
COMMON /MNK/M,N,KS
COMMON /EPSX/EPS1,EPS2,EPU
COMMON /TOP
REAL KA,KB,NA,NB
B(1)=SIGY3
B(2)=BB*FAC
B(3)=0.
A(1)=SIGYA
A(2)=BB*(FBC+FS)
A(3)=KA
D=0.
CALL BIE2(YII,EPAII,0.,NA,D,1)
M=1
EPA(1)=0.
EP3(1)=0.
Appendix: Fortran Program for Calculation of the Flow Curves for Two-Ductile-Phase Alloys

EPB(1)=0.
SIGF(M,1)=0.
SIGF(M,2)=SIGYA
CALL INT(EPA(1),EPB(1),EP3(1))
WRITE(4,7)

7 FORMAT(1X,2X,3HNO.,4X,2HEC,6X,2HSC,6X,2HEA,
+6X,2HSA,6X,2HEB,6X,2HSB,6X,2HE3,6X,2HS3)

10 M=M+1
MM=M-1
EPA(2)=EPA(1)+EPS
EPB(2)=0.
EP3(2)=0.
IF(EPA(2).GE.EPAl)GOTO 20
IF(SIGF(MM,1).GT.TOP)GOTO 70
SIG33A=SIGYA+KA*(EPA(2)*)**NA+BB*(FBC+FS)*EPA(2)
CALL INCREASE(SIG33A,DEP)
SIGF(M,1)=SIGF(MM,1)+DEP
SIGF(M,2)=SIG33A
CALL INT(EPA(2),EPB(2),EP3(2))
Y1=SIG33A+SIGINA
Y2=SIG33A+SIGIN3
Y3=SIG33A+SIGINB
WRITE(4,9)M,SIGF(M,1),SIGF(M,2),EPA(2),Y1,EPB(2),Y3,EP3(2),Y2

9 FORMAT(1X,I5,4(2X,F6.1))
WRITE(* ,5)M,SIGF(M,1),SIGF(M,2)

5 FORMAT(1X,5H===,I5,5H=====,2F14.7)
EPA(1)=EPA(2)
EPB(1)=EPB(2)
EP3(1)=EP3(2)
GOTO 10

20 EPA(2)=EPAII
EPB(2)=0.
EP3(2)=0.
CALL INCREASE(YII,DEP)
SIGF(M,1)=SIGF(MM,1)+DEP
SIGF(M,2)=YII
CALL INT(EPA(2),EPB(2),EP3(2))
WRITE(3,* )M,SIGF(M,1),SIGF(M,2)
EPA(1)=EPA(2)
EPB(1)=EPB(2)
EP3(1)=EP3(2)
A(1)=SIGYA
A(2)=BB*(FBC+FS)
A(3)=KA
A(4)=BB*FS
Appendix: Fortran Program for Calculation of the Flow Curves for Two-Ductile-Phase Alloys

\begin{verbatim}
B(1)=SIGYB
B(2)=-BB*FAC
B(3)=0.
B(4)=-BB*FS
C(1)=0.
C(2)=-BB*FAC
C(3)=0.
C(4)=BB*(FAC+FBC)
D=0.
CALL BIE3(YIII,EPAIII,EP3III,EPAII,0.0,NA,D)
WRITE(*,*) YIII,EPAIII,EP3III
EPA(1)=EPAII
EP3(1)=0.
M=M+1
MM=M-1
EPA(2)=EPA(1)+EPS1
EPB(2)=0.
IF(EPA(2).GT.EPAIII)GOTO 50
IF(SIGF(MM,1).GT.TOP)GOTO 70
A(1)=SIGYA+KA*(EPA(2)**NA+BB*(FBC+FS)*EPA(2)
A(2)=-BB*FS
A(3)=0.
B(1)=0.
D=-BB*FAC*EPA(2)
B(3)=0.
B(2)=BB*(FAC+FBC)
CALL BIE2(SIG33A,EP3(2),EP3(1),0.,D,2)
CALL INCREA(SIG33A,DEP)
SIGF(M,1)=SIGF(MM,1)+DEP
SIGF(M,2)=SIG33A
CALL INT(EPA(2),EPB(2),EP3(2))
Y1=SIG33A+SIGINA
Y2=SIG33A+SIGIN3
Y3=SIG33A+SIGINB
WRITE(4.9)M,SIGF(M,1),SIGF(M,2),EPA(2),Y1,EPB(2),
+ Y3,EP3(2),Y2
WRITE(*,45)M,SIGF(M,1),SIGF(M,2)
45 FORMAT(1X,10H..........,I5,2F14.7)
EPA(1)=EPA(2)
EPB(1)=EPB(2)
EP3(1)=EP3(2)
GOTO 40
50 EPA(2)=EPAIII
EP3(2)=EP3III
EPB(2)=0.
\end{verbatim}
Appendix: Fortran Program for Calculation of the Flow Curves for Two-Ductile-Phase Alloys

```
WRITE(*,*)EPA(2),EP3(2),EPB(2)
CALL INCREA(YIII,DEP)
SIGF(M,1)=SIGF(MM,1)+DEP
SIGF(M,2)=YIII
WRITE(3,*)M,SIGF(M,1),SIGF(M,2)
CALL INT(EPA(2),EPB(2),EP3(2))
EPA(1)=EPA(2)
EPB(1)=EPB(2)
EP3(1)=EP3(2)
M=M+1
MM=M-1
EPA(2)=EPA(1)+EPS2
IF(SIGF(MM,1).GE.TOP)GOTO 70
A(1)=SIGYA+KA*(EPA(2)**NA+BB*(FBC+FS)*EPA(2)
A(2)=-BB*FBC
A(3)=0.
A(4)=-BB*FS
B(1)=SIGYB-BB*FAC*EPA(2)
B(2)=BB*(FAC+FS)
B(3)=KB
B(4)=-BB*FS
C(1)=0.
D=-BB*FAC*EPA(2)
C(2)=-BB*FBC
C(3)=0.
C(4)=BB*(FAC+FBC)
CALL BIE3(SIG33A,EPB(2),EP3(2),EPB(1),EP3(1),NB,D)
WRITE(*,*)SIG33A,EPA(2),EPB(2),EP3(2)
CALL INCREA(SIG33A,DEP)
SIGF(M,1)=SIGF(MM,1)+DEP
SIGF(M,2)=SIG33A
CALL INT(EPA(2),EPB(2),EP3(2))
Y1=SIG33A+SIGINA
Y2=SIG33A+SIGIN3
Y3=SIG33A+SIGINB
WRITE(4,9)M,SIGF(M,1),SIGF(M,2),EPA(2),Y1,EPB(2),
+ EP3(2),Y2
WRITE(*,65)M,SIGF(M,1),SIGF(M,2)
```

65 FORMAT(1X,15H,I5,2F14.7)
EPA(1)=EPA(2)
EPB(1)=EPB(2)
EP3(1)=EP3(2)
GOTO 60
Appendix: Fortran Program for Calculation of the Flow Curves for Two-Ductile-Phase Alloys

70 M=M
RETURN
END

C
THIS SUBROUTINE IS DESIGNED TO EXECUTE THE CALCULATION OF
C
THE PLASTIC STRAIN INCREMENT IN THE WHOLE 3-E BODY.
SUBROUTINE INCREA(SIG33A,DEP)
COMMON /VOL/FA,FB,FA3,FB3,FAC,FBC,FS
COMMON /AB/AA,BB
COMMON /SIGF/SIGF3(10000,2),SIGF(10000,2)
COMMON /EPAB3/EPA(2),EPB(2),EP3(2)
COMMON /MNK/M,N,KS
DEPA=EPA(2)-EPA(1)
DEPB=EPB(2)-EPB(1)
DEP3=EP3(2)-EP3(1)
DEP1=DEPA*FAC+DEPB*FBC+DEP3*FS
DEP2=(BB/SIG33A)*(
  + (FBC*EPB(2)+FS*EP3(2)-(FBC+FS)*EPA(2))*FAC*DEPA
  + +(FAC*EPA(2)+FS*EP3(2)-(FAC+FS)*EPB(2))*FBC*DEPB
  + +(FAC*EPA(2)+FBC*EPB(2)-(FAC+FBC)*EP3(2))*FS*DEP3)
DEP=DEP1+DEP2
WRITE(6,*)M,DEP,DEP1,DEP2
RETURN
END

C
THIS SUBROUTINE IS DESIGNED TO EXECUTE THE CALCULATION OF
C
INTERNAL STRESS IN EACH MICROSTRUCTURAL ELEMENT.
SUBROUTINE INT(EPA,EPB,EP3)
COMMON /VOL/FA,FB,FA3,FB3,FAC,FBC,FS
COMMON /SIGY/SIGYA,SIGYB,SIGY3,E
COMMON /SIN/SIGINA,SIGINB,SIGIN3
COMMON /AB/AA,BB
COMMON /SIGF/SIGF3(10000,2),SIGF(10000,2)
COMMON /MNK/M,N,KS
SIGINA=(BB/1.5)*(FBC*EPB+FS*EP3-(FBC+FS)*EPA)
SIGINB=(BB/1.5)*(FAC*EPA+FS*EP3-(FAC+FS)*EPB)
SIGIN3=(BB/1.5)*(FAC*EPA+FBC*EPB-(FAC+FBC)*EP3)
WRITE(5,*)M,SIGF(M,1),SIGINA,SIGINB,SIGIN3
RETURN
END

C
THIS SUBROUTINE IS DESIGNED TO SEARCH THE FLOW STRESS OF THE
C
EIII BODY AT GIVEN PLASTIC STRAIN FROM A DATA FILE.
SUBROUTINE SEARCH(SIG3K,EP3K)
COMMON /SIGF/SIGF3(10000,2),SIGF(10000,2)
Appendix: Fortran Program for Calculation of the Flow Curves for Two-Ductile-Phase Alloys

COMMON /MNK/M,N,KS
DO 10 I=1,N

40 FORMAT(1X,I5)
IF((SIGF3(I,1)-EP3K).GE.0.)GOTO 20
10 CONTINUE
20 SIG3K=SIGF3(I,2)
RETURN
END

C THIS SUBROUTINE IS DESIGNED TO SOLVE THE TWO SIMULTANEOUS
C EQUATIONS

SUBROUTINE BIE2(YR,XR,XS,XN,D,KK)
COMMON /SIGF/SIGF3(10000,2),SIGF(10000,2)
COMMON /ABC/A(4),B(4),C(4)
COMMON /MNK/M,N,KS
Y1(X)=A(1)+A(2)*X+A(3)*X**XN
Y2(X)=B(1)+B(2)*X+B(3)*X**XN
POLY(X)=Y1(X)-Y2(X)
DX=1.E-5
X1=XS
IF(KK.EQ.1)GOTO 20
CALL SEARCH(SIG3K,X1)
B(1)=D+SIG3K

20 F1=POLY(X1)
IF(ABS(F1).LT.1.)GOTO 2
1 X2=X1+DX
IF(KK.EQ.1)GOTO 30
CALL SEARCH(SIG3K,X2)
B(1)=D+SIG3K

30 F2=POLY(X2)
IF(ABS(F2).LT.1.)GOTO 4
IF(F1*F2).LT.1.)GOTO 4
IF(F1*F2).LT.1.)GOTO 4
3 IF(F1*F2).LT.1.)GOTO 4
IF(F1*F2).LT.1.)GOTO 4
6 X1=X2
F1=F2
GOTO 1

5 CONTINUE
8 XM=(X1+X2)/2.
IF(KK.EQ.1)GOTO 40
CALL SEARCH(SIG3K,XM)
B(1)=D+SIG3K

40 FM=POLY(XM)
IF(ABS(FM).LT.1.)GOTO 11
9 IF(F1*FM).LT.1.)GOTO 11
12 X2=XM
F2=FM

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Appendix: Fortran Program for Calculation of the Flow Curves for Two-Ductile-Phase Alloys

```
GOTO 5
13 X1=XM
F1=FM
GOTO 5
2 XR=X1
YR=Y1(X1)
GOTO 7
4 XR=X2
YR=Y1(X2)
GOTO 7
11 XR=XM
IF(KK.EQ.1)GOTO 7
YR=Y1(XM)
YA=Y1(XR)
YB=Y2(XR)
7 RETURN
END

C THIS SUBROUTINE IS DESIGNED TO SOLVE THE THREE SIMULTANEOUS EQUATIONS

SUBROUTINE BIE3(YR,X1R,X2R,X1S,X2S,XN,D)
COMMON /SIGF/SIGF3(10000,2),SIGF(10000,2)
COMMON /ABC/A(4),B(4),C(4)
COMMON /MNK/M,N,KS
Y1(X1,X2)=A(1)+A(2)*X1+A(3)*X1**XN+A(4)*X2
Y2(X1,X2)=B(1)+B(2)*X1+B(3)*X1**XN+B(4)*X2
Y3(X1,X2)=C(1)+C(2)*X1+C(3)*X1**XN+C(4)*X2
X21(X1)=(A(1)-C(1)+(A(2)-C(2))*X1+(A(3)
+ -C(3))*X1**XN)/(C(4)-A(4))
X22(X1)=(C(1)-B(1)+(C(2)-B(2))*X1+(C(3)
+ -B(3))*X1**XN)/(B(4)-C(4))
X2A(X1)=(X21(X1)+X22(X1))/2.
POLY(X1)=X21(X1)-X22(X1)
DX1=1.E-2
X11=X1S
X2K=X2S
CALL SEARCH(SIG3K,X2K)
C(1)=D+SIG3K
20 F1=POLY(X11)
IF(ABS(F1).LT.1.E-5)GOTO 2
1 X12=X11+DX1
X2K=X2A(X12)
CALL SEARCH(SIG3K,X2K)
C(1)=D+SIG3K
30 F2=POLY(X12)
```
Appendix: Fortran Program for Calculation of the Flow Curves for Two-Ductile-Phase Alloys

```
IF(ABS(F2).LT.1.E-5)GOTO 4
3   IF(F1*F2).LT.5.6 GOTO 5,5,6
6   X11=X12
    F1=F2
    GOTO 1
5   CONTINUE
8   X1M=(X11+X12)/2.
    X2K=X2A(X1M)
    CALL SEARCH(SIG3K,X2K)
    C(1)=D+SIG3K
40  FM=POLY(X1M)
    IF(ABS(FM).LT.1.E-5)GOTO 11
9   IF(F1*FM).LT.12,12,13
12  X12=X1M
    F2=FM
    GOTO 5
13  X11=X1M
    F1=FM
    GOTO 5
2   X1R=X11
    X2R=X2A(X11)
    GOTO 7
4   X1R=X12
    X2R=X2A(X12)
    GOTO 7
11  X1R=X1M
    X2R=X2A(X1M)
7   YR=(Y1(X1R,X2R)+Y2(X1R,X2R)+Y3(X1R,X2R))/3.
    RETURN
END
```
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