Investigation into Factors Affecting the Timing and Capacity of Offshore Oil Production Projects

A Thesis Submitted to the University of Surrey for the degree of Doctor of Philosophy by Richard J Barnes under the supervision of Professor Antonis Kokossis

November 2008

The Centre for Process and Information Systems Engineering School of Engineering, University of Surrey Guildford, GU2 7XH, United Kingdom
Abstract

This thesis investigates the effect of different design capacities on the economic return of a number of hypothetical offshore oilfields. In particular, it investigates the optimum design capacity of both a main field and a satellite field being produced over the main field platform. The optimum timing of first oil from the satellite field is also investigated.

During the course of the work, the sensitivity of the model simulation to a number of other parameters was investigated to determine the stability of the solution and its independence from these parameters. Parameters such as oil price, increased well cost and uncertainty in the cost estimates and the reserves estimates were investigated.

The problem had the potential to develop into a very complex non-linear problem that would be difficult to solve. One of the objectives throughout the course of the work was to maintain a linear model and to reduce complexity to widen the potential applicability of the method.

The problem was solved by the development of a field model that stepped through the years of production in sequence until all the reserves had been produced. The model then determined economic return of the particular case. A wide range of cases were investigated to improve the understanding of the response of the model to changes in parameters.

A commercial algebraic solver was used to solve the large number of equations that the model generated.
Acknowledgements

I would like to acknowledge the help, advice and encouragement provided by Professor Antonis Kokossis throughout the project. His guidance has resulted in what I trust is a useful contribution to this field of knowledge.

Also I am extremely grateful to my long-suffering wife, Jennie, who has encouraged me throughout the years of work in preparing this work. She has been very tolerant and understanding. Without her, the project may possibly have never started - it certainly would never have been finished.

My thanks to David Blenkinsop who suggested the original topic of research.

Finally, I thank the Halliburton Corporation for sponsoring me in the first years of this work.
# Table of Contents

Section 1  Introduction  
Section 2  Optimum Drilling Centre  
Section 3  Single Field Model  
Section 4  Model Based Sensitivity Analysis  
Section 5  Two Field Model  
Section 6  Results and Conclusions  
Appendix 1  Description of Hardware and Software  
Appendix 2  Drilling Geometry  
Appendix 3  Cost Algorithms  
Appendix 4  Triangular Distribution  
Appendix 5  Description of Two Field Model  
Appendix 6  Shell Sorting Method  
Appendix 7  References
Abbreviations

ADCF    Annual Discounted Cash Flow
bbl     Barrel
BP      British Petroleum
BPD     Barrels per day
CAPE    Computer Aided Process Engineering
CAPEX   Capital Expenditure
CDF     Cumulative Distribution Function
DCF     Discounted Cash Flow
DRILLEX Drilling Expenditure
GAMS    General Algebraic Modelling System
GB      Gigabyte
GHz     Gigahertz
IRR     Internal rate of return
MB      Megabyte
MILP    Mixed Integer Linear Program
MM      Million
NACF    Net Annual Cash Flow
NPV     Net Present Value
OPEX    Operating Expenditure
P_{10}  Event with a 10% probability of occurring
P_{50}  Event with a 50% probability of occurring
P_{90}  Event with a 90% probability of occurring
RAM     Random Access Memory
TSP     Total Specific Production
UR      Unrisked
US      United States
Major Cost Centres

In the oil and gas industry some of the major budgetary cost centres are known by abbreviations that may not be as well known outside the industry. For the sake of clarity these are defined below.

CAPEX

CAPEX is an abbreviation for Capital Expenditure. Capital Expenditure is the cost of designing, building and installing the major cost elements of the offshore development such as the jacket, topsides and pipelines.

DRILLEX

DRILLEX is an abbreviation for Drilling Capital Expenditure. Drilling Capital Expenditure contains all the cost of drilling a well, including: design, rig lease, drill crew costs, casing and tubing, cement, and logging.

OPEX

OPEX is an abbreviation for Operating Cost. Operating Cost contains all the costs, normally quoted on an annual basis, incurred during the operation phase of field life, including: operator’s salaries, production chemicals, maintenance, spares, helicopter flights and accommodation.
1.0 Introduction

Offshore oil and gas fields are commonly developed from a platform. The platform supports all the process, utility and accommodation modules. The wells are also drilled either from the main platform, from a wellhead platform alongside the main platform, or from a subsea template. Larger platforms also have their own drilling rig permanently located on the platform.

The investment in an offshore platform, even if the platform is small and is located in benign waters, represents a significant expenditure and risk. Investment in large platforms such as in the Brent and Forties fields represents major investments even for the “super majors”.

By definition, investment in any oilfield development involves risk. The biggest single risk is the reservoir itself. Despite extensive seismic survey, appraisal drilling and sophisticated modelling, it is only when the field is finally abandoned that the recoverable reserves will be known accurately. Yet it is the reserves that are the fundamental input to a field development plan.

This project investigates the next stage of the field development plan. Based on recoverable reserves, what capacity should the surface facilities be designed for? Should the field be produced quickly with large capacity facilities, or slowly with smaller facilities? How should any adjacent field be tied in to the main facility? This study attempts to answer these questions.

Satellite fields are frequently developed over an existing, operational field, particularly in the North Sea, where it is believed that all the larger fields have been discovered and only the smaller fields are left to be developed that could not economically support a large and expensive production and export infrastructure.
1.0 Introduction

Scheduling an offshore field development and determining an optimum drilling programme is a complex and imprecise task. Only limited information is available about the reservoir performance parameters, so that the objective is to use the best information available, and to use a thorough and consistent method of determining the best option based on the information available at the time.

Whilst this area of research has not been extensively investigated, a number of previous and current workers have built a basis that has guided this current work.

Bohannon (1970) proposed a mixed-integer linear programming (MILP) models for the oil field design and its production planning.

Frair and Devine (1973) proposed an optimisation model that simultaneously addressed the location-allocation of wells, the scheduling of the facility operations and the production rates for different time periods.

Williams (1986) describes the stages that a project will pass through with particular emphasis of the feasibility stage. The primary tasks at this stage of development are described and the method of approach to the studies is discussed.

Iyer et al (1998) presented a multi-period MILP model for the planning and scheduling of investment and operation of offshore facilities. They assume that the locations of platforms were given. The model considers different reservoirs and different fields as separate entities. Both the reservoir pressure and the gas oil ratio are represented by piecewise linear interpolation rather than a continuous function. The piecewise representation is reported to add to computational time. The problem is decomposed by aggregation of wells and time periods to obtain an initial, approximate solution. This approximate solution is then used to target a more precise solution.
Van den Heever and Grossmann (2000, 2001) have proposed a mixed-integer non-linear programming (MINLP) with a simultaneous approach for the oil field planning which directly deals with non-linearities. The authors state that the problem can be solved in a reasonable computer time. The model assumes that the operating conditions are constant across the planning horizon so that the productivity index can be assumed constant for a given period of time. This work is an extension of previous work by lyer et al (1998).


Zollotukhin and Gudmestad (2001) describe the uncertainties that are inherent in the development of a hydrocarbon resource. The effect of uncertainty in the reservoir performance and reserves; in the number and location of wells; in the development, such as process equipment size and cost; and in project design and fabrication are considered. They then discuss the application of these uncertainties to the Statfjord field in Norway. Different production profiles are generated for the $P_{10}$, $P_{50}$ and $P_{90}$ reserves estimates (that is reserves that have a 10%, 50% or 90% probability of occurring). In each case a 10% depletion rate is set at peak or plateau production.

The authors then use this data to identify an uncertainty span for the concept and also determine the effect of an uncertainty level of 10% in overall cost, in all variables and with a two-year slippage. From this information, project cash flows can be calculated. The authors state that their analysis is far from complete; however, their analysis supports the conclusions reached in this work on uncertainty. Their analysis does not investigate the selection and location of wells within the field, and only considers a single field.
1.0 Introduction

Lin and Floudas (2003) developed an MINLP model using continuous-time modelling to optimise the development of a number of gas fields.

Carvalho and Pinto (2006a) have developed an MINLP model representing the installation of platforms and drilling wells in discrete periods to maximise NPV. The resulting MINLP is complex and requires additional assumptions to be made in order to obtain a solution.

Bieker et al (2006) provide a technology survey related to methods of well allocation, gas lift and gas/water injection optimisation, and updating of the models. The elements described include data acquisition, data storage, processing facility model updating, well model updating, reservoir model updating, production planning, reservoir planning and strategic planning.

Mockridge and Akhatar (1985) discuss the logical approach to determine the optimum location of topside facilities, particularly with two or more locations.

Comparatively little work has concentrated on the optimisation of well and drilling centre location. The purpose of the current work has been to investigate this area and provide a functional model that is flexible in field layout and well location characteristics, but at the same time is not computationally intense. Barnes et al (2002) have reported the initial findings of a single field optimisation model. Barnes and Kokossis (2004) have reported further investigations into two field production systems with a main and satellite platform.

The author is of the opinion that it is less common to allow the reservoir pressure to dramatically decline, Iyer et al (1998), than to use water injection to maintain reservoir pressure at or close to the original pressure. The assumption of constant productivity index over field life is more justified at constant reservoir pressure.
1.0 Introduction

It is also considered more practical and common to drill at least a number of wells from a wellhead platform, rather than only using subsea tiebacks. If only subsea tiebacks are used, it would more logical to tieback to the production platform, perhaps with some form of subsea manifolding to reduce the number of flowlines.

1.1 Sub-surface Description

The platform is located over the reservoir. Wells are drilled from the platform into the more productive reservoir areas, as determined by the Reservoir Engineer.

An understanding of the reservoir characteristics is built up from seismic survey, and exploratory and appraisal wells. This information allows the Reservoir Engineer to develop a three dimensional model of the reservoir. An estimate of the oil originally in-place is made from the oil-bearing rock volume and porosity. These parameters vary over the reservoir and hence a knowledge of the properties is required over as wide an extent as possible. The reservoir model is continuously updated as new wells are drilled enabling more accurate estimates of the reserves to be made over the full extent of the reservoir.

From the type of rock, porosity, and experience, an estimate is made of the recovery factor for the reservoir. The recovery factor is simply the fraction of the oil initially in-place that is estimated can actually be produced to the surface. Recoverable reserve is the total volume of oil that it is estimated can be produced.

All deviated wells were assumed to be drilled as “build and hold” trajectory. Figure 1.1 shows a number of the common type of well configurations, Mian (1992).

Well costs were expressed as a function of the distance along the well bore. A description of the method used to calculate this distance for a specific well is described in Appendix 2.0 Drilling Geometry. A vertical well is a special case of a
1.0 Introduction

build and hold well with zero build angle. Build, hold and drop, and horizontal wells are an extension of the build and hold configuration; these wells were not considered in the current work, and were therefore not investigated further.

The models could be extended to include these other types of wells

Different well designs are used to reach different locations and to achieve particular well characteristics. Vertical wells are the cheapest and quickest to drill. Build and hold wells are the easiest way to reach a well target that is not vertically under the drilling location. Build, hold and drop reaches a particular well target avoiding the area near the reservoir below the drilling centre. Horizontal wells provide the largest well bore area within the oil zone. They are useful for thin reservoirs or for producing viscous crudes.

Deviated wells are more expensive to drill because they are longer and take more drilling time. The longer along-hole length requires more casing and hence results in greater material costs. Also the well takes longer to drill due to its greater length. Deviating and tracking a well also takes additional drilling time. There is an optimum location for a platform over an oil field that will minimise the cost of drilling wells to the reservoir.
1.2 Surface Description

The platform consists of the jacket and topsides. The jacket is the tubular structure piled to the seabed and supporting the topsides above the sea. The topsides consist of the well area, production facilities, export pumps, power generation, utilities and accommodation. All these facilities are required to be able to operate the production facilities in a safe manner and efficiently export the oil and gas to their destination.

A simplified diagram of an offshore platform is shown in Figure 1.2. The reservoir is shown with both a gas cap and a lower water zone. The oil lies between the two interfaces. A vertical and two deviated wells penetrate the reservoir zone. All the wells have been drilled from the platform which has an integral drilling rig.

The presence of the rig allows workover of wells as well as the drilling of new wells to be performed without the requirement to contract a jack-up or other mobile rig from a drilling contractor.

Dependent on the size of the field, up to about 40 wells may be drilled from a single platform. Most of these wells will be deviated to target particular locations in the reservoir. There can only be one truly vertical well. However, dependent on the deviation required and the reservoir depth, several wells will be able to be drilled with a small deviation. Figure 1.3 shows a plan view of a typical field.

The rate at which the field is produced is a function of the size of the platform and the number of wells drilled; that is, the design capacity of the platform. A schematic of a typical three stage separation process train on a platform is shown in Figure 1.4, Arnold and Stewart (1998).
1.0 Introduction

Figure 1.2 Simplified View of a Production Platform over a Reservoir.

The figure shows a cross-section of a typical offshore platform and oil reservoir. The platform is located over the reservoir at a location that gives advantageous access to the reservoir underneath. Wells are then drilled vertically and deviated into the oil-bearing structure. Oil reservoirs frequently have a gas cap above them of associated gas and a water level below them. Often the water zone pressurises the oil zone and assists in its production. The location of the platform in relation to the reservoir is investigated as part of this work.
1.0 Introduction

Figure 1.3 Plan view of a Typical Field

The plan of a typical oilfield shows how, dependent on distance, a larger area of the reservoir can be reached by drilling deviated wells. The distribution of well targets is very much dependent on the reservoir characteristics. Not all wells will be oil producing wells. Some may be water or gas injection.

The design capacity of a platform or process facility is the design rate that is used in the engineering design to size each piece of equipment and pipe. An operator will normally specify the capacity that he requires from a facility; this is sometimes known as the "nameplate" capacity. This capacity will be the maximum throughput that the operator requires the facility to safely handle and achieve the product specifications. It is also the minimum throughput that the engineering contractor will be required to guarantee that the facility will process. The engineering contractor will add a design margin to be sure that his facilities will be able to meet the nameplate capacity. Typically an engineering contractor will add a 10% margin; this throughput is known as the design capacity. In this work, for the sake of simplicity and to avoid confusion, it is assumed that the nameplate and design capacities are equal.

The wellhead fluids that are produced to the surface may be at high pressure and frequently contain produced water and dissolved gas. The water and gas are
removed in a series of production separators that operate at progressively lower pressures. Operating conditions of the separators are maintained such that the resulting oil meets the pipeline or tanker specification for the platform.

At each separation stage, the gaseous phase is removed leaving the oil less volatile and at a lower pressure. The first and third stages are three phase separators that also remove produced water from the oil. A desalting/dehydrator vessel may follow the third stage separator to remove any remaining water and to reduce the crude salt content. A heat exchanger heats the feed to the third stage separator to ensure the export crude meets the pipeline or tanker specification. The crude leaving the desalter is cooled before entering the crude export system.

Gas from each of the stages of separation is compressed up to export pipeline pressure. The export pressure is typically about 1200 psig (83 barg) to remain within the 600 lb flange rating on the pipeline system. The gas is dehydrated within the compression train to avoid hydrate formation in the pipeline. Some gas streams may also require CO₂ and H₂S to be removed before entering the pipeline system.

Dependent on pipeline specification the gas may also be treated to reduce the hydrocarbon dew point to avoid any liquid drop out in the pipeline system.

Typically, the gas is chilled by either expansion across a Joule-Thompson valve or through a turbo-expander. The heavier hydrocarbon fraction is removed in a liquid separator and after heat exchange with the gas feed to the expansion process, the lean gas is recompressed up to pipeline export pressure.

Where facilities do not exist for gas export, or where the gas volumes are relatively small, the gas may be reinjected into the reservoir for disposal.
Figure 1.4 Platform Process Schematic
1.0 Introduction

It is common for treated water to be reinjected into the reservoir to replace the volume of crude produced and to maintain reservoir pressure. A water volume equivalent to about 120% of the produced oil volume must be reinjected to compensate for the dissolved gas.

One parameter that must be decided early in the design is the design capacity of the platform. This decision is driven by a number of variables, including:

- Recoverable reserves;
- Reservoir areal extent;
- Well productivity;
- Anticipated field life;
- Water depth;
- Cost of the platform and the topsides;
- Capacity constraints within the export system.

If the facilities are built smaller than the optimum size, the revenue from the field will be lower and the cost of the installation will not be recovered as quickly as from optimally sized facilities.

If the facilities are built too big, the capital cost of the installation will be higher than it need be and, despite the increased revenue, cost recovery will be slower than the optimum due to the higher initial capital expenditure.

The optimum design capacity will be judged by determining the Net Present Value, NPV, of each case and seeking the maximum NPV.
1.0 Introduction

During the course of this work, reference is made to the "design capacity" of the platform. During the detailed design stage a required capacity will be specified by the field operator. The design contractor will then add a design margin, typically 5 to 10%, to give a design capacity that will be used to size equipment. Due to additional design factors and conservative techniques, the actual capacity of a particular plant can frequently exceed its design capacity. For the purposes of this project it has been assumed, for sake of simplicity and clarity, that the design capacity and actual capacity of the platform are identical.

1.3 Economic Analysis Method

The economic value of investing in a particular field is far more complex than selecting the most economic wells to be drilled. It requires an analysis of the field's performance over its entire life. As such, the true value of the field can only be determined after the field is abandoned at the end of its productive and economic life.

Revenue is obtained by selling the produced oil. In some fields additional revenue can be generated by the sale of gas. In other fields the gas cannot be sold and is either flared or reinjected. For oil fields remote from infrastructure, the disposal of gas can be a major economic and environmental problem.

Expenditure on the field breaks down into two broad categories: capital expenditure, CAPEX; and operating expenditure, OPEX.

CAPEX includes all the costs of the facilities: platform, topsides, pipelines, design engineering, project management and construction costs. The drilling cost of new wells is also normally included in CAPEX.
1.0 Introduction

OPEX includes all the running costs of the field: operating and maintenance staff, treatment chemicals, spare parts, helicopter flights and support vessels. Workover costs of existing wells are included as a maintenance item within OPEX.

The NPV calculation takes the time value of money into account when determining the profitability of a project. Money spent early in the project has greater cost to the project than money spent later in the project. Similarly, revenue earned early in the project is credited higher than revenue earned later in the project.

For each year, expenditure and revenue is multiplied by a discount factor which is equal to $1/e^{it}$. Where $i$ is the assumed discount rate, and $t$ is the year number.

For the purposes of this study, a 10% discount rate was taken. This is the typical number used by oil companies to assess different projects. The discount rate in the first year is 1.0 and decreases exponentially in subsequent years. The discount rates for the first 5 years are shown in Table 1.1 and for the first 20 years in Figure 1.5.

<table>
<thead>
<tr>
<th>Year</th>
<th>Discount Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.000</td>
</tr>
<tr>
<td>2</td>
<td>0.905</td>
</tr>
<tr>
<td>3</td>
<td>0.819</td>
</tr>
<tr>
<td>4</td>
<td>0.741</td>
</tr>
<tr>
<td>5</td>
<td>0.670</td>
</tr>
</tbody>
</table>

From Table 1.1 and Figure 1.5, it can be seen that capital or revenue incurred in Year 5 is only 2/3 (0.670) of that incurred in Year 1. Similarly, in Year 17 the discount factor is only 0.2. Thus revenue or capital expenditure later in field life has less effect on the profitability of the project than revenue or capital expenditure early in field life.
The discount factor falls rapidly initially, and then more slowly. Project economics can be improved by delaying capital expenditure and accelerating revenue.

The project NPV is equal to the sum of the discounted revenues minus the sum of the discounted costs.

$$NPV = \sum_{i=1}^{n} R_i \cdot \delta_i - \sum_{i=1}^{n} C_i \cdot \delta_i - \sum_{i=1}^{n} D_i \cdot \delta_i - \sum_{i=1}^{n} O_i \cdot \delta_i$$

Where:

- $C_i$ = Capital cost of platform, topsides and pipelines in Year $i$;
- $D_i$ = Drilling expenditure in Year $i$;
- $O_i$ = Operating and maintenance expenditure in Year $i$;
- $R_i$ = Revenue (from oil sales) in Year $i$;
- $\delta_i$ = Discount factor for Year $i = 1/e^{\delta(1)}$.

The discount factor is calculated as $1/e^{\delta(1)}$ rather than $1/e^{\delta}$ to give a discount factor of 1.0 in Year 1.
1.0 Introduction

1.4 The Problem

The problem investigated in this work was to find and develop a method of estimating the optimum design capacity of an offshore development. To enable the method to be used readily, the method should be simple to use and to calculate. If possible the method should be explicit. The method should be sufficiently flexible to permit its use on any offshore field.

The development considered for this work was a main field with a satellite field connected to it. Both the main and satellite field produce oil and gas. A platform is located at each field. The platform on the main field has full processing facilities to make stable crude and pipeline specification gas. Two export lines from the platform transport the crude and gas to the export trunk system. Figure 1.6 shows the outline scheme of the fields.

The satellite platform has a single stage of separation so that oil and gas can be piped to the main platform in separate lines to avoid two phase flow problems.

All wells are assumed to be drilled from the two platforms.

1.5 Field Development Uncertainty

Throughout this work, it must be remembered that an inexact problem is being investigated. There is no “right” answer. A total understanding of the field performance can only be approached at the end of field life when a true value of the recoverable reserves becomes available and progressive testing and analysis has better defined the extent and characteristics of each section of the reservoir.

Development of any oil reservoir is an inexact science at best, and frequently benefits from experience and intuition.
1.0 Introduction

The diagram shows the layout of a typical oil field with a satellite field located alongside. Production from both fields is processed over the main platform and the pipeline quality oil and gas is exported from the area through trunk lines. Both fields are drilled from their respective platforms located over each field. The satellite field fluids are separated into gas and liquid phases and sent through separate pipelines to the main platform for processing.

The models described in this work were developed to enable a better understanding of a field to be obtained and to assist in early investigation of different development plans. In the early, conceptual, stage of the development of a field, the biggest single unknown is the size of the reservoir itself and other
uncertainties are less significant or are assumed to be rolled up into the reservoir uncertainty.

Most oil exploration companies will determine three reserves values for a given field. These are the $P_{90}$, $P_{50}$, and $P_{10}$ reserves. Respectively these are the minimum reserves that are estimated to exist in a reservoir with 90%, 50% and 10% probability. The reserves values are obtained by Monte Carlo simulation using a range of expected values of several reservoir parameters, including:

- Reservoir extent;
- Pay zone thickness;
- Net/gross rock volumes;
- Reservoir porosity;
- Oil viscosity;
- Hydrocarbon saturation;
- Reservoir pressure.

The ratio of the $P_{90}$ to the $P_{10}$ reserves is an indication of the uncertainty, and therefore risk, of developing the particular reservoir. Typically, the value of the $P_{90}/P_{10}$ ratio is interpreted as:

- 1 to 10 Appraisal Stage;
- 10 to 70 Exploration Stage;
- 50 to 120 Wildcat Drilling (pre-Exploration Stage);
- 100 to 200 Frontier Drilling (high risk well in undeveloped area).
1.0 Introduction

The development of any oil reservoir goes through a number of stages:

In the first stage very little will be known about the reservoir. Some seismic may have been shot over the area, but this may be of limited extent. At this stage the oil company is assessing whether and how much to bid for the licence to develop the block containing the reservoir. It is probable that at this stage, no well has been drilled into the reservoir, so there is no certainty that oil even exists. Wells drilled in this stage are regarded as Frontier or Wildcat Wells.

In the Exploratory Stage, one or more exploration wells are drilled at locations that are deemed to be the most likely to contain oil. When oil (or gas) is found, the well becomes a discovery well and the field is designated as discovered. However, there is still only limited knowledge of the reservoir, and the field cannot be declared "commercial".

The field now enters the third or Appraisal Stage in which an attempt is made to delineate the field and to improve the estimate of the reserves by reducing the ratio between the $P_{90}$ and $P_{10}$ reserves estimates. At the appraisal stage, wells are drilled to define the lateral extent and thickness of the reservoir. Cores of the reservoir formation are recovered to determine the physical properties of the rock to reduce the range of uncertainty of individual parameters. Collectively, this reduction in uncertainty of individual parameters reduces the overall uncertainty as measured by the $P_{90}/P_{10}$ ratio.

If the uncertainty is considered to be too great, additional appraisal wells may be drilled to reduce specific areas of uncertainty, such as investigating the closure or sealing of the reservoir at a particular location or determining the oil-water contact at a particular location. This work continues until sufficient understanding
of the reservoir has been obtained to justify the investment required to develop the field.

No further well data is now obtained until development wells are drilled as the field passes into the Development Stage. This is the stage in which a development plan and concept is put into practice, the platform is built and the wells drilled from there. During the Development Stage performance of the reservoir is continuously monitored to follow the response of the reservoir to production. In this way the reservoir engineers knowledge of the field is continuously improved so that the field may be developed in the most efficient and economic manner.

1.6 Model Applicability

The model developed in this work is designed to enable a development engineer to better understand the response of an oilfield to changes in drilling activity and the development of adjacent fields. It also provides a method of investigating the effect of alternative development options.

For the development of a particular oilfield to be considered a success, it must be economically successful and make a good return on the capital invested. Therefore the optimisation of the economic performance of the field has been a significant factor in this work.

The objective of the study has been to develop a comparatively simple model that a development engineer can use without long and involved training, to allow reliable and consistent development decisions to be taken, and also to enable the development engineer to better understand the response of the field economics to changes from the optimum. Such changes may be required due to unsuitable conditions on the sea bed, location of a shipping lane, limited capacity in third party infrastructure or other factors outside of the engineer's control.
1.0 Introduction

Work by others has linked the production field to the shipment of oil and refining through to finished products, Neiro et al (2004). It is considered that this type of modelling crosses a natural boundary. Crude oil is sold on the open market and is traded internationally. It is a commodity in which the refiner is not directly interested in its origin, and, to some extent, does not care what quality it is. His main interest is whether he can refine the product slate that he requires at an acceptable profit margin.

There is a preference for low sulphur crude oil, but refineries that can process high sulphur crude oil can buy this quality crude at a discount. This discount, in part reflects the extra cost of processing. Similarly, a refinery may want to blend its feedstocks so that these can be refined into products in the required proportions. These factors add significantly extra complication to the problem and are not directly linked to factors that affect production of oil from the reservoir.

Therefore it was decided to limit this work to considering the production of oil from the reservoir, through the well, over the platform and through the production facilities and into the export pipeline. Shipment by tanker and refining of the crude oil was excluded from this study.

1.7 Thesis Structure

This section, Section 1, provides a general description of the problem being investigated, provides a brief description of how the oil industry decides whether to develop a particular offshore oil prospect, and provides background into the assessment of a prospect.

Section 2 describes three methods that were developed to determine the optimum location of the platform and drilling centre within the field. The optimum drilling centre needs only to be determined once for each field, but its location is required
1.0 Introduction

to provide a base for the main thrust of this study which is described in subsequent sections.

Section 3 describes models develop to determine the optimum drilling sequence in a field to achieve the specified production profile over field life at the lowest cost. The single field studies provide the basis of understanding of the development models and provide the basis for the sensitivity analysis.

Section 4 investigates the sensitivity of the project to changes in a number of variables. The sensitivity analysis was undertaken to determine if the conclusions reached for a particular, realistic set of parameters were valid over a wider range to cover likely values that would be experience in practical applications. The analysis also provides a method of determining the sensitivity of the NPV to variations in the main parameters.

Section 5 expands the original problem to incorporate a second, satellite, field and investigates the interaction of the two fields to determine the optimum configuration to achieve the specified production profile. The timing of first oil from the satellite field and the production rates of each field will have a significant effect on the profitability of the combined development.

Section 6 provides a summary of the conclusions of the work and describes its potential practical application in how an oil exploration company might investigate individual prospects or groups of prospects to maximise the economic return on a project.
2.0 Optimum Drilling Centre

An important decision in developing an offshore field is the selection of the drilling centre from where the majority of the wells can be drilled. The principal selection parameter is the cost of drilling all the wells from the location. The lowest cost location is preferred. Other factors that may affect the location are: condition of the seabed, distance from shore or other key infrastructure, and depth of water.

At the present time in the oil industry, the method of locating the drilling centre is relatively subjective with a location being selected that is deemed to be in the centre of the most prolific part of the field. This is probably adequate for simple, uniform fields. However, where a reservoir is badly faulted and is divided into several areas of differing productivity, the location of the drilling centre should be determined in a more structured manner.

Dogru (1987) describes a nonlinear mixed-integer model that minimises the total drilling cost whilst maximising the productive potential of a field.

Watson et al (1989) developed a model for determining the optimum platform location based on the cost of drilling and the cost of the platform. The cost of drilling was a function of the total well trajectory. The cost of the platform was a function of the water depth at the location. The paper gives a general description of the model, but does not provide the model structure.

Hansen et al (1994) describe a model that simultaneously determines the location, size and well selection for a platform. The model must be repeatedly applied to optimise the location by minimising the cost per barrel of oil produced.

Scheinder et al (2002), describes a model applied to a gas field in Sumatra. The model integrates reservoir data, material balance, well performance, surface
2.0 Optimum Drilling Centre

pipeline configuration and facilities in a single model. The model optimises the system to meet contractual gas sales targets. The optimisation criterion in this model is achieving production targets rather than optimising project financial return.

2.1 Introduction

An offshore oilfield is developed by drilling a number of wells into the reservoir to drain the oil from the reservoir to achieve the maximum recovery of the reserves. For all but the small fields, the wells will be drilled from a platform or central location in the field. The wells will deviate from the vertical in order to reach the downhole targets specified by the reservoir engineer.

In this present work only the drilling cost has been considered since the other factors that may affect the platform location are specific to each case and could be overriding in location considerations. The surface location that gives the lowest cost of drilling the required number of wells is described as the Optimum Drilling Centre, ODC.

The cost of drilling a well is a function of several different parameters, but one of the most important is the length of the well. Longer wells take a longer time to drill and require more casing and tubing to complete the well. Other factors such as the hardness of the rock and any drilling difficulties such as stuck tubing also affect well cost. However, these other parameters have not been considered in this section since they are again specific to the location or are a random occurrence. Random variations on drilling cost are investigated in Section 4.5.

For a fixed number of wells at specified reservoir locations, the location of the platform can be varied within the field and still achieve the downhole targets. However, the well bore lengths will vary with location, and hence the cost of
drilling the required wells will vary with the location of the drilling centre. The productivity of individual wells also varies over the field and this can affect the number of wells required to meet a particular production target. The objective of the first part of this work is to determine the optimum location of the drilling platform to drill the required number of wells to achieve a specified initial production rate.

A production target is a production rate that the facilities are to be designed to be capable of achieving. For the purposes of determining the optimum drilling centre, this is a fixed value. In the later sections of this study it refers to a target production rate that is determined for each year of field life to achieve a specified production profile.

Dependent on water and reservoir depth, field location and method of drilling (semi-submersible, jack-up or platform rig) a single well can cost between about $10 and 30 million. For a field that may be developed with 40 wells, if a saving of 10% on drilling each well could be made, the overall saving in development of the field could be in the order of $40 million to $120 million.

Figure 2.1 shows the relationship between the surface grid on which the platform drilling centre is located and the subsurface grid containing the downhole targets in the reservoir. For simplicity, these two grids directly overlay each other.

The problem is to determine the drilling centre location from which sufficient wells can be drilled to achieve the specified production target at minimum cost.

This can be expressed more formally as:

**Objective function:** Minimise cost of drilled wells

**Subject to:** Achieving specified production rate.
2.0 Optimum Drilling Centre

The challenge was to develop a computational method that would determine the ODC quickly, accurately and efficiently in terms of the number of iterations and computational time, when it was not known how many wells would actually be required for a specified development scenario.

Three different methods of estimating the optimum drilling centre location were developed. These were:

- The Sequential Approach;
- The Total Specific Production Approach; and,
- The Simultaneous Approach.

These three approaches are explained in the following Sections.
2.0 Optimum Drilling Centre

2.1.1 Common Assumptions

Certain assumptions were made that were common to all three approaches. It is assumed that each potential drilling centre is viable and that the cost of drilling a similar well in any location is the same. In particular, this means that there are no gullies, or other sudden changes in seabed profile or conditions. Additionally, the overburden to be drilled through should be similar in all locations and not have local salt domes or pockets of shallow gas that may delay drilling.

The effect of drilling delays due to stuck pipe, lost circulation or mechanical failure is random and therefore not addressed in this analysis.

It is also assumed that as the top of reservoir rises, the pay zone thickens and thus accounts for the increase in well productivity.

For the Sequential and Simultaneous models to be described shortly, a production target was set for each field. These targets were the same for each model, but varied with the field size. This production targets were set to a value representing between two and five times the plateau production or design capacity of the field. This multiple was selected to ensure that more wells than were required to meet the first plateau production rate were required.

This multiple ensured that the drilling centre location considered the wells that were required in the first few years of production in order to determine the optimum location. In practice, the production target would be set at the plateau production rate or design capacity of the facilities and the optimum drilling centre determined. The location of the optimum drilling centre would then be determined by increasing the production target to investigate the effect on the drilling centre location. This sensitivity analysis would enable a robust location to be determined.
2.0 Optimum Drilling Centre

2.1.2 Example Fields

A number of hypothetical fields that are typical of real life field examples were used to test and evaluate the three models.

2.1.2.1 Example Field 1

For initial work to develop the models and to provide a relatively simple test field, a small field was used. This is shown diagrammatically in Figure 2.2. This field only has a total of 29 potential well site locations in a 1,000 m by 1,000 m grid.

The figure shows the layout of a small, hypothetical offshore oil field. The reservoir slopes upwards from location P12 to P18. The most productive well is P18.

The most productive well location is P18 whilst the least productive is P12. The top of reservoir slopes upwards from west to east with the most productive portion of the reservoir being to the east and presumed thicker pay zone.

A production target of 60,000 BPD is assumed for this field since it would require about 10 wells or about 35% of all the potential well locations productivity.
2.0 Optimum Drilling Centre

One possible location for the drilling centre would be P15 which is in the centre of the field. Using P15 as the drilling centre, it would be natural to select P15 and the 8 surrounding wells to give the lowest drilling cost for that drilling centre.

Selection of these nine wells gives a total production of 54,000 BPD. A tenth well is required to meet the production target of 60,000 BPD. The natural well to select is P17 with a productivity of 7,000 BPD, since this is the nearest high production well to the P15 drilling centre. Therefore the total potential production from these wells is 61,000 BPD, 1,000 BPD over target. The total cost of drilling these wells is $76.503 million. A summary of the calculations is given in Table 2.1.

Table 2.1 Well Cost Summary, Drilling Centre P15

<table>
<thead>
<tr>
<th>Well No.</th>
<th>Horizontal Deviation, m</th>
<th>Total Length, m</th>
<th>Well Cost, $MM</th>
<th>Well Productivity, BPD</th>
</tr>
</thead>
<tbody>
<tr>
<td>P8</td>
<td>1,414</td>
<td>3,411</td>
<td>7.945</td>
<td>5,500</td>
</tr>
<tr>
<td>P9</td>
<td>1,000</td>
<td>3,189</td>
<td>7.426</td>
<td>6,000</td>
</tr>
<tr>
<td>P10</td>
<td>1,414</td>
<td>3,324</td>
<td>7.742</td>
<td>6,500</td>
</tr>
<tr>
<td>P14</td>
<td>1,000</td>
<td>3,235</td>
<td>7.535</td>
<td>5,500</td>
</tr>
<tr>
<td>P15</td>
<td>0</td>
<td>3,000</td>
<td>6.987</td>
<td>6,000</td>
</tr>
<tr>
<td>P16</td>
<td>1,000</td>
<td>3,142</td>
<td>7.318</td>
<td>6,500</td>
</tr>
<tr>
<td>P17</td>
<td>2,000</td>
<td>3,622</td>
<td>8.436</td>
<td>7,000</td>
</tr>
<tr>
<td>P20</td>
<td>1,414</td>
<td>3,411</td>
<td>7.945</td>
<td>5,500</td>
</tr>
<tr>
<td>P21</td>
<td>1,000</td>
<td>3,189</td>
<td>7.426</td>
<td>6,000</td>
</tr>
<tr>
<td>P22</td>
<td>1,414</td>
<td>3,324</td>
<td>7.742</td>
<td>6,500</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
<td>76.503</td>
<td>61,000</td>
</tr>
</tbody>
</table>

The wells selected are shown in heavy outline in Figure 2.3. The well selection was based on a visual inspection of the field layout and a certain amount of logical thinking. It is possible that a lower cost solution exists that still meets the minimum production criteria.
2.0 Optimum Drilling Centre

Alternative drilling centres must be similarly modelled to be able to identify the lowest drilling cost location within the field. The lowest cost location is the ODC.

Figure 2.3 Manual Estimate of Optimum Drilling Centre

<table>
<thead>
<tr>
<th>Depth.m</th>
<th>Production, BPD</th>
</tr>
</thead>
<tbody>
<tr>
<td>2,850</td>
<td>7,500</td>
</tr>
<tr>
<td>2,900</td>
<td>7,000</td>
</tr>
<tr>
<td>2,950</td>
<td>6,500</td>
</tr>
<tr>
<td>3,000</td>
<td>6,000</td>
</tr>
<tr>
<td>3,050</td>
<td>5,500</td>
</tr>
<tr>
<td>3,100</td>
<td>5,000</td>
</tr>
<tr>
<td>3,150</td>
<td>4,500</td>
</tr>
</tbody>
</table>

The wells selected within the heavy outline meet the production target of 60,000 BPD. Well selection was manually by inspection. The drilling centre was located at P15, centrally within the field.

2.1.2.2 Example Field 2

Once a model had been developed, it could be tested against a more complex field with several hundred potential drilling centre locations. Such a field is shown in Figure 2.4. The West reservoir has a total of 224 potential well locations. The reservoir shown is typical of an anticline were a dome rises to the centre of the reservoir. Oil, being lighter than water rises and accumulates in the porous rock or sands at the crest. Spreading out from the high point of the anticline, the top of the reservoir dips and reduces the pay zone or oil thickness, and therefore the productivity of a well drilled at that location. The outer limits of the reservoir are either the oil water contact or the location of sealing faults. To the north of the anticline, a small fault block provides a number of isolated high productivity wells. The field is divided into a grid of 250 m by 250 m cells.
2.0 Optimum Drilling Centre

Figure 2.4 West Field Layout

The West field was used as a medium sized field. The shallowest part of the reservoir, and the highest productivity wells, are located in the centre of the field and in a small accumulation off the north end of the main field.

2.1.2.3 Example Field 3

To provide a more stringent test of the different techniques to determine the ODC, a large field with 580 potential well locations was also defined. The layout of the field is shown in Figure 2.5. This field is again based on an anticline structure, but is not as uniform. There are three separate peaks in the centre of the field yielding three areas of high productivity. A fault lies between the eastern and western peak, accounting for the low productivity region in between these two peaks. An area of secondary peak productivity lies to the west of the main peaks, but this area is low productivity compared with the main peaks in the centre of the field. The field is divided into a grid of 250 m by 250 m cells.
2.0 Optimum Drilling Centre

Figure 2.5 Central Field Layout

The figure shows the layout and well productivities of a large, complex field. The field contains a total of 580 potential well locations. The field contains three areas of high production towards the centre, and thins out to a low production area to the west.

2.2 The Sequential Model

This approach was fairly simple since it calculated the optimum well selection for a single, fixed, specified drilling centre. The model did not optimise the drilling centre location, it only optimised the well selection for the specified drilling centre location. The model was built to prove the basic modelling method and to provide a starting point for the more complex modelling.

The method is an exhaustive enumeration approach inasmuch as the calculations must be repeated with different drilling centre locations until the optimum location has been determined. Not every location need be modelled as several locations may be eliminated by either inspection because they lay in the less...
productive region of the field or by analysis of the trend of drilling cost which can be used to guide the selection towards the optimum location.

Use of an exhaustive enumeration method can result in an excessively large number of calculations for fields with a large number of potential locations.

2.2.1 The Sequential Approach

In the sequential approach, a drilling location and a production target are specified. Using this data, the model determines the minimum cost of drilling sufficient wells from the location to meet the production target. Other drilling locations are then specified and the model rerun. New locations are selected until the user is confident that the optimum has been found. In a small field it may be practical to calculate the well selections for all drilling locations. Such exhaustive enumeration ensures that all possible locations are considered.

2.2.1.1 Problem Data

To determine the location of the ODC for an individual field, sufficient parametric data must be provided to describe the reservoir and its anticipated performance. This data consists of the X, Y and Z coordinates of each downhole well target and the anticipated productivity from the well. An equation to determine the cost of drilling each well is also provided.

The target production rate is also be specified. This value will determine the number of wells required based on the productivity of individual wells selected.

Finally, the location of the drilling centre must be specified for each run.
2.2.1.2 Decision Variables

Using the problem data, sufficient wells can be selected to achieve the specified production target. The model must then iterate through the well selection to minimise the cost of drilling the wells whilst ensuring the production target is achieved.

The output from the model is the minimum cost of drilling sufficient wells to meet the specified production from the specified location.

The approach adopted was to determine the minimum drilling cost from the specified drilling location, and then to select other drilling centres and determine the minimum drilling cost from these centres. The centres would be varied until sufficient had been considered that the optimum location had been identified.

For small fields, with only a relatively small number of drilling centre locations it is possible to model all the locations sequentially and select the lowest drilling cost location as the ODC. For larger fields, likely locations should be modelled until it is reasonably certain that the optimum has been located.

As stated earlier, this model was deliberately simple to prove that the well cost calculation and the optimisation technique worked correctly. The later methods were developed to solve the problem of drilling centre location in a single stage.

The model selects wells from the specified drilling centre to meet the production target and then optimises this selection to minimise the cost of drilling the wells.

2.2.2 Local Search Model

The field to be investigated is divided into a regular spaced grid for convenience. Potential drilling centres are located at the seabed in these grid locations. Potential well downhole targets are located vertically below the grid locations at
2.0 Optimum Drilling Centre

the top of reservoir. Figure 2.1 shows the layout of a generic field. The surface and subsurface grids overlay each other, so that a location on the surface grid lies vertically over the corresponding location on the subsurface grid. Both locations can therefore be referred to using the same reference, a well number.

From any drilling location, one vertical well can be drilled and the remaining wells must be deviated. The more deviated wells are more expensive to drill because they are longer and the sharper kick-off angle takes longer to generate. Since only some wells are required out of the total possible well locations, there is an ODC location from which all the required wells can be drilled at minimum cost.

The model consists of the set:

\[ i \] the set of wells.

Scalar input parameters are:

\[ T \] the target field production;
\[ p_i \] the initial production from well \( i \).

An additional set of well parameters is calculated at run time:

\[ c_i \] the cost of drilling well \( i \).

A binary variable is used during the calculation:

\[ z_i \] selects or deselects each well (\( z_i = 1 \) selects the well).

The only continuous variable is:

\[ \text{Cost} \] the cost of drilling sufficient wells to meet or exceed the target production, 
\[ T. \]
2.2.3 Mathematical Model

The problem can be expressed in two parts. These functions can be expressed as:

Minimise:

\[
Cost = \sum_i (z_i c_i) \tag{2.1}
\]

The objective function is to minimise the cost of drilling sufficient wells to meet the production target. Equation (2.1) calculates the cost of meeting the production target using the wells selected through the binary variable, \( z_i \).

Subject to:

\[
\sum_i (z_i p_i) \geq T \tag{2.2}
\]

Equation (2.2) constrains the total production from the selected wells to equal or exceed the specified production target.

The optimum well selection for each drilling centre location can then be determined by a dedicated run. The model output is the minimum cost of drilling sufficient wells from that location to meet the specified initial production rate, and the wells required.

Equations (2.1) and (2.2) constitute an MILP problem which requires the selection of a number of wells sufficient to produce the production target at minimum cost.

For simple problems with only a few wells, say maximum of 20 or 30, in the field it would be feasible to do this by a trial and error process. However, for fields containing several hundred potential well locations this would be impractical.
2.0 Optimum Drilling Centre

The MILP problem can be solved using a commercial solver package. Use of such a solution technique ensured that all possible well combinations are investigated to ensure the well selection is the optimum for the specified drilling centre location.

2.2.4 Implementation

The MILP problem was set up in GAMS with an objective function to minimise the total drilling cost from a single drilling centre, and the single constraint of meeting or exceeding the specified production rate.

The problem was solved using the CPLEX solver in the GAMS package. A data file containing the downhole well location coordinates and assigned well productivity for each location in the field was read as input data. Additionally, the drilling centre location and the target production were specified.

The solution output was a list of the selected wells, the cost of drilling the wells and the total production from the field. Since no optimisation of the drilling centre location was performed, it was necessary to repeat the calculations on sufficient locations to ensure that all likely locations had been investigated and that the optimum drilling centre had in fact been located.

2.2.5 Results

The Sequential model was tested with the three fields: Simple, West and Central.

2.2.5.1 Example 1 Results

The Sequential model was run first with the Simple field, as this field was used to develop and test the more sophisticated models. The Sequential model was run for each well location in the Small field to determine the lowest cost drilling centre. The results are shown in Table 2.2 and Figure 2.6.
### Table 2.2 Total Drilling Cost for Different Drilling Centres

<table>
<thead>
<tr>
<th>Drilling Centre Location</th>
<th>Total Drilling Cost, $MM</th>
<th>Drilling Centre Location</th>
<th>Total Drilling Cost, $MM</th>
<th>Drilling Centre Location</th>
<th>Total Drilling Cost, $MM</th>
</tr>
</thead>
<tbody>
<tr>
<td>P01</td>
<td>83.988</td>
<td>P11</td>
<td>68.571</td>
<td>P21</td>
<td>73.630</td>
</tr>
<tr>
<td>P02</td>
<td>92.023</td>
<td>P12</td>
<td>100.746</td>
<td>P22</td>
<td>67.991</td>
</tr>
<tr>
<td>P03</td>
<td>81.917</td>
<td>P13</td>
<td>90.217</td>
<td>P23</td>
<td>68.571</td>
</tr>
<tr>
<td>P04</td>
<td>76.653</td>
<td>P14</td>
<td>80.564</td>
<td>P24</td>
<td>92.023</td>
</tr>
<tr>
<td>P05</td>
<td>71.387</td>
<td>P15</td>
<td>73.324</td>
<td>P25</td>
<td>81.917</td>
</tr>
<tr>
<td>P06</td>
<td>72.661</td>
<td>P16</td>
<td>67.618</td>
<td>P26</td>
<td>76.653</td>
</tr>
<tr>
<td>P07</td>
<td>90.354</td>
<td>P17</td>
<td>67.953</td>
<td>P27</td>
<td>71.387</td>
</tr>
<tr>
<td>P08</td>
<td>80.564</td>
<td>P18</td>
<td>73.044</td>
<td>P28</td>
<td>72.661</td>
</tr>
<tr>
<td>P09</td>
<td>73.630</td>
<td>P19</td>
<td>90.354</td>
<td>P29</td>
<td>83.988</td>
</tr>
<tr>
<td>P10</td>
<td>67.991</td>
<td>P20</td>
<td>80.564</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

This figure shows the distribution of drilling costs for different drilling centre locations. The lowest cost location is P16, details of which are provided in the text.

The results in Table 2.2 show that the location P16 gave the lowest drilling cost to achieve the target production of 60,000 BPD. The minimum drilling cost is $67.618 MM. The wells selected for the P16 drilling centre are shown in Table 2.3 and are
2.0 Optimum Drilling Centre

shown diagrammatically in Figure 2.7. P21 could have been selected instead of P09 since the two wells are symmetrical about P16 and have the same parameters.

Table 2.3 Optimised Well Cost Summary, Drilling Centre P16

<table>
<thead>
<tr>
<th>Well No.</th>
<th>Horizontal Deviation, m</th>
<th>Total Length, m</th>
<th>Well Cost, $MM</th>
<th>Well Productivity, BPD</th>
</tr>
</thead>
<tbody>
<tr>
<td>P10</td>
<td>1,000</td>
<td>3,142</td>
<td>7.318</td>
<td>6,500</td>
</tr>
<tr>
<td>P11</td>
<td>1,414</td>
<td>3,281</td>
<td>7.642</td>
<td>7,000</td>
</tr>
<tr>
<td>P15</td>
<td>1,000</td>
<td>3,189</td>
<td>7.426</td>
<td>6,000</td>
</tr>
<tr>
<td>P16</td>
<td>0</td>
<td>2,950</td>
<td>6.871</td>
<td>6,500</td>
</tr>
<tr>
<td>P17</td>
<td>1,000</td>
<td>3,096</td>
<td>7.210</td>
<td>7,000</td>
</tr>
<tr>
<td>P18</td>
<td>2,000</td>
<td>3.584</td>
<td>8.348</td>
<td>7,500</td>
</tr>
<tr>
<td>P21</td>
<td>1,414</td>
<td>3.368</td>
<td>7.843</td>
<td>6,000</td>
</tr>
<tr>
<td>P22</td>
<td>1,000</td>
<td>3,142</td>
<td>7.318</td>
<td>6,500</td>
</tr>
<tr>
<td>P23</td>
<td>1,414</td>
<td>3,281</td>
<td>7.642</td>
<td>7,000</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
<td>67.618</td>
<td>60,000</td>
</tr>
</tbody>
</table>

Figure 2.7 Small Field Optimised Drilling Centre

This figure shows the final, optimised drilling centre at P16. By moving the drilling centre from P15 to P16, the cost of drilling P10, P11, P16, P17, P18, P22 and P23 has been reduced at the expense of increased cost for P15 and P21.
2.0 Optimum Drilling Centre

The most expensive drilling centre location is P12 at a cost of $100.746. This location is in the lowest productivity area and is on the periphery of the field so that long, and therefore more expensive wells, must be drilled to achieve the specified production. Since the wells in this location have a low productivity, more wells must be drilled to meet the target, again increasing the cost of the drilling centre.

P16 is the lowest CAPEX location at a cost of $67.618 million, 30% cheaper than the most expensive location. P16 is able to take advantage of the high productivity wells surrounding the location; therefore the well count is lower.

2.2.5.2 Example 2 Results

Each of the drilling centre locations was selected in turn so that all 224 possible locations were investigated. In practise, it would not be necessary to investigate all the potential locations for the drilling centre, since the low productivity areas could be shown to be significantly sub-optimal.

Each location was selected in turn as the drilling centre and Sequential model was used to solve the resulting MILP problem. For each location, the minimum cost of achieving an initial production of 60,000 BPD was determined. From these results, the minimum cost location could be determined. This location is the Optimum Drilling Centre, ODC. The results of these calculations are shown in Figure 2.8.

The red coloured locations show, that in this particular case, there is a large central area where the drilling costs differ little between locations. The lowest cost location and hence the ODC is located at W079 where the total cost of drilling sufficient wells to meet the production target was $65.697 MM.
2.0 Optimum Drilling Centre

Figure 2.8 West Field Cost of Drilling using the Sequential Model

The Sequential model was used to determine the distribution of cost of drilling sufficient wells at each location to meet the 60,000 BPD production target. The central red area shows that in this particular case a relatively large area of minimum cost exists. The lowest cost location is W079 at a cost of $65.697 million.

2.2.5.3 Example 3 Results

The model for the Central field was modified slightly from the other two fields due to its size. Referring to Figure 2.5, it can be seen that there are two areas that have 9 wells each that can produce 10,000 BPD from each well. These areas are shown in red, representing the individual well production. Thus for target productions equal or less than 90,000 BPD, these wells would be selected, and the ODC would be in the centre of the cluster. For this reason, and to use a more realistic production from a large field, a target of 120,000 bpd was selected.

The model determined the ODC to be at C218, and required 12 wells to produce 120,000 BPD. The well selection is shown in Figure 2.9. The ODC is not over any of
2.0 Optimum Drilling Centre

the selected wells. Besides the 9 high productivity wells forming a group to the east of the field, a further 3 high productivity wells to the north were selected. This is an interesting selection, as wells further from the drilling centre were selected in preference to nearer but lower productivity wells. The increased productivity from the more remote wells outweighs the extra drilling cost.

The Optimum Drilling Centre located at C218. 12 wells are required to meet the 120,000 BPD target. All selected wells are high productivity, 10,000 BPD, wells. No well is required at C218 itself.

The cost of drilling from each location in the Central field is shown in Figure 2.10. It is clear that there is a large area where there is only a small difference in cost to achieve the specified initial production rate. This lower sensitivity to cost is caused by the three areas of high well productivity and that the target production cannot be achieved by only drilling in one area of high productivity wells.

Figure 2.9 Central Field ODC and Selected Wells Using the Sequential Model
The area of lowest drilling cost to achieve the target production of 120,000 BPD is shown in red. As a result of the layout of the particular field, there is a large area where the drilling cost does not vary greatly and could be potential locations for the drilling centre when external factors are considered.

### 2.3 Total Specific Production Model

The Total Specific Production model was developed in an attempt to devise a method of determining the ODC without recourse to the solution of an MILP problem.

The Total Specific Production, TSP, method determines the cost of each well that can be drilled from a particular drilling centre to the downhole well targets. The Specific Well Production, defined as the initial production rate divided by the well cost, is calculated for each well target.

\[
\text{Specific Well Production} = \frac{\text{Production from well}}{\text{Cost of drilling well}}
\]  

(2.3)

The units of Specific Well Production are barrels per day of production per $ million of drilling cost. The Specific Well Production parameter is a simple method
of taking account of the productive "efficiency" of a well since it gives a value of the production rate achieved from each million dollars invested in a well. The higher the value of the parameter, the more oil that can be produced for each million dollars invested in that well.

Thus an expensive well, perhaps with a long horizontal section, becomes attractive compared with a cheap vertical well with a low production, provided that the extra cost of drilling the horizontal well produces a proportionate increase in production. Therefore, from any drilling location, wells with the highest Specific Production are the preferred selection.

2.3.1 The Problem

In Section 2.2.1, the lowest cost location for drilling sufficient wells to meet the production target was determined using an MILP model. Although this method gives acceptable results, it does not take into account wells that are required later in field life to maintain production rates. The method could also result in a complex model if large fields with several hundred well locations were investigated. Additionally, the model did not determine the optimum location in a single simulation.

2.3.2 Outline Approach

The required input data is the same as for the Sequential model described in Section 2.2.

The objective is to determine the optimum drilling location using the Total Specific Production criteria.

No further assumptions were made from those described in Section 2.1.2.
2.0 Optimum Drilling Centre

The model uses exhaustive enumeration to determine the TSP parameter for each drilling location. The location with the highest TSP is taken as the optimum drilling centre since this location represents the most productive location based on production per unit cost of drilling.

The challenge with this model was to develop a method of determining the location of the ODC without requiring the solution of an MILP problem. Such a method would not then require an algebraic solver to determine the solution.

The Total Specific Production is calculated for each drilling centre, and the centre with the highest Total Specific Production is selected as the Optimum Drilling Centre. The method is not computationally efficient, inasmuch as the method requires exhaustive enumeration of all possible locations. However, the method has the potential of being much quicker than the solution of an MILP problem, particularly for large fields, since the solution is explicit and not iterative.

The Total Specific Production method involves a solution without recourse to an MILP. For each drilling location and specified maximum stepout, a certain number of wells can be drilled at a cost that can be calculated for each well. The productivity for each well is specified as part of the input data.

The TSP model uses a different criterion from the Sequential model which only optimised the well selection to achieve a specified production. The TSP model identifies all feasible wells that can be drilled from each location and therefore also considers production later in field life when in-fill wells are required to maintain production.

The model uses an input data file of well parameters that is provided in GAMS include file format so that the data can be used in subsequent models. The model was written in C++. 
2.0 Optimum Drilling Centre

The model output is the drilling centre location which has the highest Total Specific Production. For each location, the TSP is reported so that a distribution diagram can be plotted.

2.3.3 Design Approach

The sum of the Specific Productions for each well location for each potential drilling centre location, the Total Specific Production, TSP, is calculated. The optimum drilling centre can then be selected as the location that has the largest TSP, since this location can reach the highest productive value of wells.

The drilling centre will be located over one of the downhole targets, thus ensuring one well will be vertical. A maximum well step out is specified to avoid including excessively long wells, and to limit the area of reservoir that a drilling centre can drain. Such limits apply in the oil industry.

By only selecting the well targets that can be drilled from a specific location, the number of well locations that are investigated for each potential drilling centre is restricted to only feasible targets. Infeasible targets are those which exceed the specified step-out. High step-out or long reach wells are more expensive to drill and are greater risk of delays and extra expenditure. The maximum step-out is a function of the size of the field and the cost of facilities. In shallow, benign waters several cheap platforms can be built and a short step-out used. In deeper water it will be uneconomic to build several platforms, and it will be cheaper to drill longer reach wells. A common step-out limit that is used in this work is 4,000 m.

2.3.4 Problem Nomenclature

The model consists of the set:
2.0 Optimum Drilling Centre

Set of wells.

Scalar input parameters are:

\( H_{\text{MAX}} \) Maximum well stepout from drilling centre;

\( XC \) X coordinate of the drilling centre location;

\( YC \) Y coordinate of the drilling centre location.

The well parameters are:

\( x_i, y_i \) the X and Y coordinates of well \( i \);

\( p_i \) the initial well production from well \( i \).

A run time set of variables is calculated for the current drilling centre:

\( c_i \) Cost of drilling well \( i \) from the current drilling centre.

2.3.5 Modelling Constraints

For each drilling location, the Specific Production, \( p_i / c_i \), is calculated for each well. The Total Specific Production is the sum of all the Specific Productions of all feasible wells drilled from the drilling centre.

\[
\text{Total Specific Production} = \sum_i \left( \frac{p_i}{c_i} \right)
\]  

Equation (2.4) sums the “well efficiencies” of all wells that can be drilled from a location. It therefore takes into account the productivity of the wells, the cost of drilling the wells, the reservoir extent and the number of wells that can be drilled from the location.
2.0 Optimum Drilling Centre

The selection of each well is subject to the constraint that the maximum stepout is not exceeded:

\[ H_{\text{MAX}} \geq \sqrt{(x_i - XC)^2 + (y_i - YC)^2} \]  

(2.5)

Equation (2.5) limits the horizontal displacement of the well target from the drilling centre not to exceed the maximum step out. If the horizontal displacement exceeds the maximum, the Specific Production is set to zero.

2.3.6 Solution Approach

The model loops through all the drilling centre locations. For each potential drilling centre, the model loops through all the downhole targets, calculating the horizontal step-out. If this step-out is within the maximum specified step-out, the well cost and Specific Production are calculated. If the step-out exceeds the specified maximum step-out, the Specific Production is set to zero since the well cannot be drilled. A running total is maintained of the TSP for each drilling centre.

A flowchart of the model is shown in Figure 2.11.
2.0 Optimum Drilling Centre

Figure 2.11 Specific Production Model Flowchart

START

1. Number of wells

j. Number of wells

N. Location within Step-out limit?

Y. Calculate Well Cost

Calculate Specific Production Accumulate Total

Find Maximum Total Specific Production

END
2.0 Optimum Drilling Centre

2.3.7 Total Specific Production Model Results

The same fields that were used to test the Sequential model were also used to test the TSP model.

2.3.7.1 Example 1 Results

The Small field was initially used to test the TSP model. Using the same data that had been used for the Sequential model, the TSP model determined that the ODC to be P16. The Total Specific Production for this location was 3,083 BPD/$ million. This is the same location which was determined using the Sequential model.

2.3.7.2 Example 2 Results

The TSP method was used to calculate the optimum drilling centre. It identified W095 as the optimum location. This well is close to W079. The cumulative Specific Productions for locations W079 and W095 are 261,856 and 264,590 BPD/$MM. These Total Specific Productions are within 1% of each other. It is not surprising that the two different methods of determining the optimum give different results. However, they both identify drilling centres that are very close to each other.

Figure 2.12 shows a plot of the Total Specific Production for each of the drilling centre locations in the field. Well W095 is in the centre of the most prolific wells in the field.

2.3.7.3 Example 3 Results

The TSP model was then used to determine the location of the ODC for the large, Central field. The model determined the optimum drilling centre location for this
field was C274. Figure 2.13 shows a plot of the Total Specific Production for each of the locations in the Central field.

The figure shows the distribution of Total Specific Production over the field. The highest values are, as expected, around the high productivity wells. The optimum drilling centre calculated by this method, W095 is very close to the optimum calculated using the Sequential model, W079.

The optimum drilling centre is located between the three areas of high productivity, shown in red in Figure 2.5. The distribution of TSP values over the field is shown in Figure 2.13. The red area in the figure shows that there is an area of high TSP in the field which contains the optimum drilling centre location, C274.

The maximum stepout specified in the TSP model was 4,000 m. Therefore almost all well locations can be reached from C274; the only ones that could not be were the low production wells to the west of the field. In this example, there is a wide scope for the selection of the drilling centre location and still to remain in an area of high TSP. The large area of high TSP is caused by the reservoir characteristics,
in particular the three areas of high productivity wells. This flexibility of drilling centre location would allow other factors, such as water depth and sea bed quality to have a greater influence on the final selection of the platform location.

The figure shows the distribution of TSP across the Central field. The area of highest TSP is concentrated around the high productivity wells (See Figure 2.5). The area of highest TSP (coloured red) is relatively small when compared to the area generated by the MILP method (Figure 2.10). The location of highest TSP is C274.

2.4 Simultaneous Model

A third method was developed, which was based on the Sequential model described in Section 2.2, but which automatically optimised the drilling centre location selection in a single determination.

2.4.1 The Problem

This model was developed to ensure a comprehensive coverage of all potential locations and avoided repeated specification of potential drilling centre locations as is required with the Sequential model.
2.4.1.1 Input Data

The required input data is the same as that for the Sequential model described in Section 2.2.

2.4.1.2 Optimisation Objective

The objective of this model was to determine the optimum drilling location in a single computational run.

2.4.2 Challenges

The principal challenge in this model was to avoid a complex solution that was difficult to solve in a field with a large number of well locations.

2.4.3 Assumptions

No further assumptions were made from those described in Section 2.1.2.

2.4.4 Approach

The approach was based on the same method as employed in the Sequential model. That is determining the drilling centre location that results in the lowest drilling cost to meet a specified target production. However, the model was structured so that not only the well selection, but also the drilling centre location was optimised simultaneously.

2.4.5 Problem Nomenclature

The problem was set up in a similar manner to the Sequential model.

The model consists of two sets:

\[ i \quad \text{the set of wells}; \]
2.0 Optimum Drilling Centre

\( j \) the set of drilling centres.

Scalar input parameters are:

\( T \) the target field production.

The set, \( i \), of well parameters is:

\( p_i \) the initial well production.

An additional well parameter that is calculated at run time is:

\( w_{i,j} \) the cost of drilling well \( i \) from location \( j \).

The binary variables are:

\( z_i \) selects or deselects each well \( (z_i=1 \) selects the well\);

\( y_j \) selects or deselects each drilling centre location \( (y_j=1 \) selects the location\).

The variables are:

\( Cost \) the total cost of drilling sufficient wells to meet or exceed the specific target production, \( T \);

\( c_{i,j} \) the cost of drilling a selected well \( i \) from location \( j \). The value is set to zero if the well has not yet been drilled. \( c_{i,j} \) is defined as a positive variable.

2.4.6 Problem Formulation

The formulation of the objective function is to minimise:

\[
Cost = \sum_i \sum_j c_{i,j} \tag{2.6}
\]
The objective function is to minimise the cost of drilling sufficient wells from location \( j \) to meet the production target. Equation (2.6) calculates the cost of meeting the production target from each drilling location to enable the lowest cost location to be determined.

The solution is subject to a number of constraints:

\[
c_{i,j} \geq (z_i + y_j - 1)w_{i,j} \tag{2.7}
\]

Equation (2.7) sets the cost of drilling Well \( i \) from location \( j \) to zero, unless both \( z_i \) and \( y_j \) are equal to 1, that is both the Well \( i \) and the drilling location \( j \) have been selected. If either \( z_i \) or \( y_j \) are zero, \( (z_i + y_j - 1) \) is zero (but not both), then \( c_{i,j} \) is zero. If both \( z_i \) and \( y_j \) are zero then \( (z_i + y_j - 1) \) is -1, but since \( c_{i,j} \) is defined as a positive variable, \( c_{i,j} \) is set to zero. Therefore, only the cost of the wells that are actually drilled from each location are totalled in Equation (2.6).

\[
\sum_i (z_i, p_i) \geq T \tag{2.8}
\]

Equation (2.8) ensures the production target is met or exceeded for each drilling location.

\[
\sum_j y_j = 1 \tag{2.9}
\]

Equation (2.9) ensures that there is only one drilling centre. This could be changed in the future to investigate the effect of multiple drilling centres.

The formulation determines the ODC directly in a single simulation run.
2.4.7 Solution Approach

The Simultaneous MILP model is based on the Sequential model, but in a single run optimises the drilling centre location. In this manner, all drilling locations are considered, and the lowest drilling cost location is found directly.

Unlike the Sequential model described in Section 2.2, which only considers the set \( i \) of different well locations this model simultaneously considers the set \( i \) of well locations and the set \( j \) of drilling centres. This results in a more difficult problem to solve, since for each drilling centre location there is an optimisation to determine the well selection. There is then a second optimisation to select the drilling location that has the lowest drilling cost to meet the specified production target.

2.4.8 Implementation

The multi-variant MILP problem was solved using the GAMS solver, CPLEX.

The input file was the same as used by the other models providing data on all the wells. The GAMS model contained the target production parameter as well as the structure of the model.

The output file identified the Optimum Drilling Centre location in a single run and also reported the well selection for that location.

2.4.9 Simultaneous Model Results

The Simultaneous model was tested using the three fields that were used to test the previous two models: Small, West and Central fields.
2.0 Optimum Drilling Centre

2.4.9.1 Example 1 Results

The Simultaneous MILP model should give results that are the same as those obtained by repeated use of the Sequential model since the Simultaneous MILP model is essentially an automated version of the Sequential model.

The results for the Small field were that P16 was the optimum location for the drilling centre. The only difference compared with the Sequential model was that well P09 was selected instead of P21. From Figure 2.2, it can be seen that these two wells have the same parameters and are symmetrical about P16. Therefore the selection of either well is arbitrary.

2.4.9.2 Example 2 Results

The West field results using the Simultaneous model were identical to those obtained using the Sequential model. The ODC was W079, and the selected wells were exactly the same. The advantage of the Simultaneous model is that only one run is required, whereas with the Sequential model, a total of 224 runs were required to fully investigate the potential drilling centre locations.

2.4.9.3 Example 3 Results

The Central field results obtained using the Simultaneous model were identical to those obtained using the Sequential model. The ODC was C218, and the selected wells were exactly the same.

2.5 Model Performance

All three models determined the Small field ODC to be P16.

The Sequential and Simultaneous models determine the West field ODC to be the same location, W079. The TSP model determined the ODC location to be W095.
The two locations are very similar.

The location of C274 determined by the TSP model compared with C218 determined by the Sequential and Simultaneous models initially appears to be quite different. However closer examination of the field layout, as shown in Figure 2.14, shows that the two selections are logical considering the different approaches used in their determination.

![Figure 2.14 Central Field ODC Locations for Different Models](image)

The figure shows the ODCs determined by the two calculation methods. Not unsurprisingly, the two centres are at different locations. The MILP method only considers wells to meet the specified initial production. The Total Specific Production considers all feasible wells that can be drilled from the location, and therefore considers much higher productions.

Both locations are on the edge of similar high productivity blocks and biased towards the same northern high productivity block. The results are comparable. Figures 2.10 and 2.13 show that relatively large areas which contain locations that are close to the optimum location. The relatively large area containing near
2.0 Optimum Drilling Centre

Optimum solutions is a characteristic of the reservoir, and are generated, in particular, from the three separate areas of high productivity.

A summary of the Simultaneous MILP model performance for the three fields is given in Table 2.4.

Figure 2.15 shows a logarithmic plot of the time taken to solve the MILP problem against the number of potential locations for the ODC. From this graph, it can be seen that at the number of potential locations increases, the computation time required increases, approximately by the power of 2.5.

Table 2.4 Simultaneous MILP Model Performance

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Small Field</th>
<th>West Field</th>
<th>Central Field</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total wells</td>
<td>29</td>
<td>224</td>
<td>580</td>
</tr>
<tr>
<td>Matrix size</td>
<td>843 x 899</td>
<td>50,178 x 50,624</td>
<td>336,402 x 337,560</td>
</tr>
<tr>
<td>Non-zero elements</td>
<td></td>
<td>150,976</td>
<td>1,010,360</td>
</tr>
<tr>
<td>Iterations</td>
<td>5,329</td>
<td>18,420</td>
<td>61,049</td>
</tr>
<tr>
<td>Computer time</td>
<td>0 h 0 m 3 s</td>
<td>0 h 3 m 33 s</td>
<td>1 h 56 m 46 s</td>
</tr>
</tbody>
</table>

2.6 Conclusions

Two different methods of estimating the location of the Optimum Drilling Centre have been described and investigated. The Sequential and Simultaneous MILP models estimate the location by determining the optimum, lowest drilling cost, location to meet a specified production rate.

The Total Specific Production model determines the location that has the highest Total Specific Production. The Total Specific Production is the sum of the production from individual wells divided by the cost of drilling all the feasible wells from that location. Therefore the TSP model is biased more to the total number of wells that can be drilled from the location, whereas the MILP model is biased
towards a specific production.

Figure 2.15 Computation Time for Different Size Fields

The graph shows the computational time against field size as the green line. Computational time is a logarithmic scale. The trend line plotted in black shows that computational time increases by approximately the power of 2.5 with increasing field size. Thus large fields require 1 or 2 hours, dependent on the number of well locations.

It should be borne in mind that there is no “correct” solution to this problem. The drilling centre must be determined early in field life when there is relatively little reliable well information. Whilst data for each well location can be specified, the range of uncertainty will be high, declining slowly over the field life.

The ODC locations determined by each method are summarised in Table 2.5. The costs for drilling sufficient wells to meet the production target in the Sequential and Simultaneous models are identical, since they both select the same drilling centre.

The TSP model does not select wells to achieve a specified production so that results cannot be directly compared. However, the selected location can be
2.0 Optimum Drilling Centre

compared on a comparable basis with the other two models. This comparison is summarised in Table 2.6.

Table 2.5 ODC Locations

<table>
<thead>
<tr>
<th>Method</th>
<th>Sequential</th>
<th>Total Specific Production</th>
<th>Simultaneous MILP</th>
</tr>
</thead>
<tbody>
<tr>
<td>Small Field</td>
<td>P16</td>
<td>P16</td>
<td>P16</td>
</tr>
<tr>
<td>West Field</td>
<td>W079</td>
<td>W095</td>
<td>W079</td>
</tr>
<tr>
<td>Central Field</td>
<td>C218</td>
<td>C274</td>
<td>C218</td>
</tr>
</tbody>
</table>

Table 2.6 Comparison of Drilling Cost to Achieve Production Target

<table>
<thead>
<tr>
<th>Method</th>
<th>Production Target, BPD</th>
<th>Sequential and Simultaneous</th>
<th>Total Specific Production</th>
</tr>
</thead>
<tbody>
<tr>
<td>Small Field</td>
<td>60,000</td>
<td>$67.618 million</td>
<td>$67.618 million</td>
</tr>
<tr>
<td>West Field</td>
<td>60,000</td>
<td>$65.697 million</td>
<td>$70.136 million</td>
</tr>
<tr>
<td>Central Field</td>
<td>120,000</td>
<td>$107.380 million</td>
<td>$107.316 million</td>
</tr>
</tbody>
</table>

The models are capable of investigating different cases to better understand the potential performance of the field. The TSP model may be modified by increasing or decreasing the permitted step out distance of the well. The Sequential and Simultaneous models can be modified by increasing or decreasing the specified minimum production.

The models provide a systematic and consistent method of analysing a field's performance parameters to identify potential locations for the platform.

It should also be remembered that the location determined by these methods may be impractical due to a number of different external reasons, such as:

- Water depth;
- Seabed quality;
2.0 Optimum Drilling Centre

- Presence of existing pipelines;
- Location of shipping lanes; or,
- Presence of a salt dome, for example.
3.0 Single Field Model

The optimisation of single platform offshore fields has been studied by a number of workers. Jardine (1985) gives an overview of simulation techniques that can be used to investigate this problem.

Morel (1989) describes a method of optimising the number of wells in an offshore gas field to meet production targets and rates. The application is demonstrated for a Central North Sea gas field. The method does not consider the cost of the platform and pipelines, nor does it investigate different production rates.

Garcia-Diaz et al (1996) describe a solution method based on an implicit enumeration method of optimising the location of as number of offshore facilities and allocating wells to them. The method was tested on typical field development cases.

Nesvold et al (1996) describes two different techniques of coupling the reservoir simulation with linear programming to optimise the selection of wells. The method was used for a new wellhead platform designed for an eventual maximum of 50 wells to be deployed on the Ekofisk field.

3.1 Introduction

In order to investigate the technical and economic behaviour of an offshore field, it was necessary to build a model that would determine the production levels, facilities size, capital and operating costs, and the revenue of the field.

Once a drilling centre has been selected using the method described in Section 2, the next parameter that must be decided is the design capacity of the production facilities: the larger the design capacity, the greater the potential revenue earning
3.0 Single Field Model

capability of the development. However, larger production facilities involve larger separators, larger diameter piping, heavier equipment and therefore a heavier and stronger jacket. Consequently, the cost of the facility is greater.

There is a trade-off between capital expenditure early in the field life and revenue during the field’s productive life. It is this balance that is investigated for a single field in this section of the work. Higher CAPEX can be compensated, in discounted cash flow terms, by accelerated revenue from higher production rates.

Two models were built to determine the economic value of a particular sized development and to enable the effect of different parameters on the economic value to be investigated. The two models took slightly different approaches.

Both models were built to provide a large degree of flexibility, so that although the examples were based on hypothetical fields in the North Sea, the models could equally well be used in other offshore locations throughout the world. The first model worked in a stepwise manner, sequentially modelling and optimising the drilling programme for each year before passing on to the next year. This is referred to as the Sequential Single Field Model.

The second model was built to optimise the entire drilling programme through the life of the field in a single stage so that the drilling programme was the most economic over field life. This is referred to as the Simultaneous Single Field Model.

3.2 Sequential Single Field Model

The initial objective of this study was to consider the development of a hypothetical oilfield in the North Sea and determine the optimum size of the surface facilities. Once this optimum size had been identified, an attempt would
be made to identify rules which could be used on fields in other locations and of
different characteristics to identify their optimum size.

The plan to investigate the optimum design capacity for a particular field was:

2. Develop cost equations for drilling the wells, constructing and installing the
   platform and topsides, and building the pipelines. These would be used to
   build the project CAPEX estimates for different design capacities.

3. Develop a field grid to provide a realistic representation of a reservoir.

4. Select a drilling centre using the method described in Section 2.

5. Develop a target production profile based on the specified production rate
   and the reserves.

6. Select sufficient wells to most economically produce the target production
   for the current year.

7. Repeat Step 6 for each year, incrementing the year count, until the
   reserves have been produced.

8. Perform an economic analysis on the results to determine the project NPV.

9. Repeat the process for different design production rates to investigate a
   series of case studies.

The results would then be analysed to identify the most economic production rate
and to suggest rules that would enable this rate to be identified explicitly.

3.2.1 The Problem

The problem solved by the Sequential Single Field Model is to optimise the drilling
schedule of the wells in such a manner that the cost of drilling is minimised for
3.0 Single Field Model

each individual year in sequence, and that the specified production profile is achieved for each year.

Net Present Value, NPV, was selected as a measure of economic efficiency of the different production configurations to be studied. Even if the absolute value of NPV is not accurate, different development options will be analysed in the same manner and therefore the comparative conclusions will yield a reliable indication of the most economic option.

Expenditure and revenue was discounted at a rate of 10% based on the year that the cash movement occurs. In this way the time value of money is accounted for, and that expenditure, particularly for drilling, is only incurred at the most economic time. Revenue is generated in proportion to the production profile using a fixed oil price.

3.2.1.1 Input Data

The input parameters define the problem to be solved and are specified as part of the problem input data. These parameters are:

- The reserves of the field;
- The design capacity of the facilities;
- Location and productivity of all well locations;
- Location of selected drilling centre;
- Production target for each year of operation;
- Cost equations for wells, platform and pipelines.
3.0 Single Field Model

3.2.1.2 Optimisation Objective

The problem objective is to minimise the cost of drilling sufficient wells each year to achieve the specified production target for that year.

3.2.2 Challenges

The problem of optimising an oil field development could easily become very complex taking into account reservoir performance, vertical multi-phase flow and surface process simulation.

The objective of this study has been to develop a model to enable comparative economic assessments to be performed quickly. Previous work by others have built complex non-linear models that have then had to be decomposed into a series of problems that can be solved by making a large number of simplifying assumptions.

3.2.3 Assumptions

The recoverable reserves are assumed to remain constant for each individual field. The reserves define the size of the field and the limit to the design capacity to maintain at least one year at the plateau production rate. It must be realised that the reserves will be an estimate until the field has been completely produced and finally abandoned. Only then will the reserves be accurately known.

The oil price of $20/bbl, discount rate of 10%, and the operating cost as 4% of installed CAPEX, remained unchanged for all cases run. However, the model has the flexibility to change these values, if required.

Drilling costs are incurred in the year before the well is required in production. Jacket and topsides costs are distributed in the profile: 15% in Year 1, 25% in Year 2, and 30% in Years 3 and 4. Pipeline costs are distributed in the profile: 50% in Year 3 and 50% in Year 4. First oil production always commences in Year 5.
3.0 Single Field Model

The design capacity of the facility is taken as the peak production rate. The target production is set at 50% of the peak rate in Year 5 (the first year of production) and 100% in Year 6. Production is then set at the peak rate for subsequent years until 85% of the reserves have been produced. The year is rounded down so that the production to the end of plateau does not exceed 85% of the reserves. The remaining reserves after the last peak rate year are then distributed over 7 years of decline in the ratio of \((8 - N)/28\), where \(N\) is the decline year, \(1 \leq N \leq 8\). This distribution ensures that the target production exactly equals the reserves.

Production commences in Year 5, at or above the target rate. Sufficient wells are drilled to at least meet the target production profile. This may result in a small excess production as the actual production from an integral number of wells may exceed the target. Production continues each year until all the reserves have been produced. Total production is limited to exactly the recoverable reserves, so that the undiscounted revenue generated by each different case is equal.

3.2.4 Sequential Single Field Model Outline Approach

The approach to the solution to the single field problem was in two stages. In the first stage a sequence of production target constraints is solved by optimising the drilling sequence for each year in succession by selecting the lowest cost drilling schedule that will achieve the specified production profile. Wells that have already been drilled are preferentially selected and then any additional wells that are required are selected in the optimisation. The actual production and well selections are stored for retrieval in the second part of the solution. Subsequent years are then modelled until all the reserves have been produced.

In the second part of the solution, the CAPEX, drilling costs, OPEX and revenue are determined for each year from commencement of engineering in Year 1, to first
3.0 Single Field Model

production in Year 5 and through to production of all the reserves in the final year of production. These costs and revenue are then discounted back to Year 1 to calculate the NPV of the project under the parameters of the specified scenario. The results are then written to a detail report for subsequent analysis and review.

3.2.5 Problem Nomenclature

The Sequential model contains two sets:

\[ i \] the set of wells;

\[ t \] the set of years within the field development.

The scalar input parameters are:

\[ F^O \] Discount factor (taken as 0.1 for this work);

\[ F^{OPEX} \] OPEX as a percentage of CAPEX;

\[ F^\text{PIPE}_t \] Fraction of gas and oil pipelines costs incurred in Year \( t \). \( t \leq N-1 \);

\[ F^\text{PLAT}_t \] Fraction of jacket and topsides costs incurred in Year \( t \). \( t \leq N-1 \);

\[ N \] Number of years to initial production (taken as \( N = 5 \));

\[ P^O \] Oil price (taken as \$20/bbl for economic analysis);
3.0 Single Field Model

\( Q \) Recoverable reserves in field;

\( T_t \) Target production for Year \( t \);

\( w_i \) Productivity of Well \( i \).

The cost of the facilities was built up from cost equations as a function of the

design capacity specified as an input variable. The costs are:

\( c^{GPL} \) Gas export pipeline cost;

\( c^{JACK} \) Jacket cost;

\( c^{GPL} \) Oil export pipeline cost;

\( c^{TOP} \) Topsides cost.

Binary variables used within the program are:

\( y_i \) Indicates whether Well \( i \) is drilled in the current year;

\( z_i \) Indicates whether Well \( i \) has been drilled in previous years.

Variables calculated within the program are:

\( Cost \) The total cost of drilling wells in the current year;
3.0 Single Field Model

$c_t^{CAPEX}$ Total CAPEX expenditure in Year $t$;

$c_t^{CUML}$ Cumulative CAPEX up to and including Year $t$;

$c_t^{DRILL}$ Cost of drilling wells in Year $t$;

$c_t^{OPEX}$ Operating cost in Year $t$;

$c_i^{WELL}$ Cost of drilling well $i$;

$F_t^{PD}$ Production decline factor in Year $t$;

$m_t^{ANN}$ Annual cash flow in Year $t$;

$NPV$ Project Net Present Value;

$p_t$ Production in Year $t$;

$p_t^{CUML}$ Cumulative oil production up to but excluding Year $t$;

$r_t^{GROSS}$ Gross revenue in Year $t$;

$r_t^{NET}$ Net revenue in Year $t$. 
3.0 Single Field Model

3.2.6 Problem Formulation

The problem was built up from a series of equations that determined the variables to formulate the optimisation equations for each year in a sequential manner.

3.2.6.1 CAPEX Profile

The CAPEX amounts for the first three years are:

\[ c_{t}^{CAPEX} = F_{t}^{PLAT}(c^{JACK} + c^{TOP}) + F_{t}^{PIPE}(c^{OPL} + c^{GPL}) \]  

(3.1)

Values of \( F_{t}^{PLAT} \) and \( F_{t}^{PIPE} \) are summarised in Table 3.1, based on the assumptions described in Section 3.2.1.2.

The CAPEX for Year \( N-1 \) is similar to Equation (3.1), except it includes the drilling cost of the wells required to be in production to meet the Year \( N \) production target:

\[ c_{t}^{CAPEX} = F_{t}^{PLAT}(c^{JACK} + c^{TOP}) + F_{t}^{PIPE}(c^{OPL} + c^{GPL}) + c_{t}^{DRILL} \]  

(3.2)

The CAPEX for the remaining years is only the DRILLEX for the subsequent year, since all other CAPEX has been expended.

\[ c_{t}^{CAPEX} = c_{t}^{DRILL} \]  

(3.3)

\( N \) has been taken equal to 5 in all cases in this current work. However, the number of years to first oil can be changed, if required.
3.0 Single Field Model

Table 3.1 Fraction of CAPEX Costs During Construction Period

<table>
<thead>
<tr>
<th>Year</th>
<th>Platform CAPEX Fraction, ( F_{t}^{\text{PLAT}} )</th>
<th>Pipeline CAPEX Fraction, ( F_{t}^{\text{PIPE}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.15</td>
<td>0.00</td>
</tr>
<tr>
<td>2</td>
<td>0.25</td>
<td>0.00</td>
</tr>
<tr>
<td>3</td>
<td>0.30</td>
<td>0.50</td>
</tr>
<tr>
<td>4</td>
<td>0.30</td>
<td>0.50</td>
</tr>
</tbody>
</table>

3.2.6.2 Production Period

In order to model the decline in production from individual wells, a Production Decline Factor was introduced. The Production Decline Factor, \( F^{PD} \) is defined as:

\[
F^{PD} = \frac{\text{Reserves} - 0.5 \times \text{Cumulative Production}}{\text{Reserves}}
\]  

(3.4)

It is assumed that the facilities are in production for 350 days each year and that the remaining days are accounted for by scheduled and unscheduled shutdowns.

The Production Decline Factor in Year \( t \) is based on the cumulative production up to and including \( t-1 \):

\[
F^{PD}_t = \frac{Q - 0.5 \sum_{i} P_{i}^{\text{CUML}}}{Q}
\]  

(3.5)

The purpose of the factor is to reduce the productivity of all the wells each year to reflect changes in reservoir performance as a result of reservoir pressure decline and the decline in production of wells with age. \( F^{PD} \) is set at 1.0 for the first year of production, and is calculated at the end of each year to provide a new value for the subsequent year. The factor decreases to 0.5 when all the reserves have been produced.

At the beginning the modelling of each year, an input file will be written containing data from the previous year and the current production target to enable
3.0 Single Field Model

the model to determine which wells, if any, should be brought into production in the year.

The cost of drilling new wells in Year $t$ is included in the CAPEX of Year $t-1$:

For $t = N-1$, equation (3.2) applies.

For $t \geq N$, equation (3.3) applies.

The Cumulative CAPEX up to Year $t$ is given by:

$$c_t^{\text{CUML}} = \sum_{i} c_i^{\text{CAPEX}}$$  \hspace{1cm} (3.6)

3.2.6.3 Annual Gross Revenue

The annual gross revenue generated is the product of the oil production in the current year and the oil price that is provide as an input parameter. For this part of the study, a fixed oil price was used. Section 4 reports the investigation into the sensitivity to variations in input parameters, including oil price.

The production in Year $t$ is:

$$p_t = \sum_i (F_i^{PD}.(w_i.x_i + w_i.y_i))$$  \hspace{1cm} (3.7)

Therefore the Gross Revenue in Year $t$ is:

$$r_t^{\text{GROSS}} = p_t.P^O$$  \hspace{1cm} (3.8)

3.2.6.4 Annual Operating Cost

The annual operating cost, OPEX, is taken as a fixed percentage of the cumulative capital expenditure to date. For this study, it was taken that the operating cost
3.0 Single Field Model

was 4% of total CAPEX up to and including the current year each year. Therefore
the Annual Operating Cost is given by:

$$C_{t}^{\text{OPEX}} = F_{t}^{\text{OPEX}} \cdot C_{t}^{\text{CUML}}$$  \hspace{1cm} (3.9)

3.2.6.5 Annual Net Revenue

Net revenue is defined as the Gross Revenue less the Annual Operating Cost:

$$r_{t}^{\text{NET}} = r_{t}^{\text{GROSS}} - C_{t}^{\text{OPEX}}$$  \hspace{1cm} (3.10)

3.2.6.6 Cash Flow and NPV

The Annual Cash Flow is the Net Revenue less the Total CAPEX expenditure in Year
\( t \). In early years, when revenue is zero or small, the annual cash flow is negative.
In later years when the CAPEX expenditure in the year is only for drilling, the cash
flow is strongly positive. The Annual Cash Flow in Year \( t \), \( m_{t}^{\text{ANN}} \), is:

$$m_{t}^{\text{ANN}} = r_{t}^{\text{NET}} - C_{t}^{\text{CAPEX}}$$  \hspace{1cm} (3.11)

The Net Present Value, NPV, of the project is the sum of the discounted Annual
Cash Flows over the life of the project:

$$NPV = \sum_{t} \frac{m_{t}^{\text{ANN}}}{e^{\delta_{t}(t-1)}}$$  \hspace{1cm} (3.12)

NPV is the measure used in this work to measure the profitability of the project.

3.2.7 Design Approach

The modelling approach was to step through each year of the project, totalling
capital expenditure, CAPEX; drilling expenditure, DRILLEX; operating cost, OPEX;
and revenue for each year in succession.
CAPEX was determined based on the CAPEX estimates for each component and the fraction chargeable in the year based on the distribution shown in Table 3.1. DRILLEX was determined by optimisation of well selection to meet the specified target production for the current year. OPEX was calculated as a fixed percentage of total cumulative CAPEX plus DRILLEX for the current year. Revenue was calculated from the annual production multiplied by the specified oil price.

When the reserves had been depleted, the economic value of the particular case was calculated by determining the NPV of the case.

It was decided to use a time increment of one year. This value was selected to give a reasonable planning interval and to coincide with the annual increment used for financial calculations. It would be possible to extend the model to use a period of months or even weeks, but it would lead to fractional well drilling, which would make economic analysis more difficult.

The model allows different peak production rates to be investigated. It was built to be able to model different field layouts and for different recoverable reserves to be specified.

Neither the capital cost of the facilities nor oil price were escalated. Capital cost escalation could be incorporated into the original CAPEX estimate and the distribution over the construction period. The effect of changes in oil price is investigated in Section 4.

First Oil is fixed to be in Year 5 to allow four years before this date for design and construction of the platform and pipelines.
3.0 Single Field Model

For each year of operation, an objective function and constraint was developed based on the case conditions and input parameters. The objective function is to minimise the cost of meeting the production target in the current year:

$$\text{Cost} = \sum_i (c_i^{\text{WELL}} \cdot y_i) \quad (3.13)$$

Subject to:

$$T_i \leq \sum_j (F_i^{PD} \cdot (p_i \cdot z_i + p_i \cdot y_i)) \quad (3.14)$$

The binary variable $z_i$ selects the production from all wells drilled previously; the binary variable $y_i$ selects the production from all new wells that are required to meet the current year's production target.

The model structure is shown diagrammatically in Figure 3.1. The order of execution of the model is as follows:

1. Using the input data of reserves, peak production, capital cost equation coefficients and well data, the values of capital cost and the production profile for the specified peak production rate and reserves are calculated.

2. Commencing in Year 1, the first year of the project, the CAPEX for each year of construction is calculated.

3. An input file for the current year is written using data from the initial conditions or previous year and the production target for the current year. The input file contains data on the target production rate, wells already drilled and cumulative production.

4. The model determines if any new wells are required to be drilled to meet the current year's target. If new wells are required, it determines which
combination of new wells is the lowest cost to meet the production target. The model stores this information to an intermediate output file.

5. The model reads the output file to obtain the well data. The remaining reserves are calculated by deducting the current year's production. If the remaining reserves are positive, the procedure is repeated from step 3 by incrementing the year count. If the reserves are negative, the production in the current year is reduced to give zero reserves at the end of the year.

6. The model then calculates the cashflow for each year. The discounted cash flow is then calculated at the specified discount rate. The sum of the discounted cashflows for each year is the NPV.

7. The production and financial data is written to an output file and the calculation stops.

8. The financial results can then be compared with other cases and the case with the highest NPV can be identified.

The model was developed in a modular form and in stages in order that each module could be easily checked. The well selection optimisation module was based on the model that had been developed for the platform location model described in Section 2. The correct operation of the model was checked by building a spreadsheet model. Comparison of the two models showed that the full model was performing correctly.
3.0 Single Field Model

Figure 3.1 Model Flowchart

Start

Read Simulation Run Data

Calculate Annual Production Targets

Set Year = 5 (First year of production)

Write Input File for Current Year

Calculate new wells

Read Output File

Reserves Produced?

Increment Year

N

Y

Calculate NPV for simulation case

Write Results File

End
3.2.8 Remarks and Implementation

The model was based on the model used to optimise the drilling centre location, but was extended in several ways to meet the requirement of model production over the whole of the field life.

3.2.8.1 Field Layout

A number of hypothetical offshore oil reservoirs were selected for the investigation. Each reservoir was laid out in a rectangular grid with each square representing a potential downhole target and also representing a surface location, vertically above the downhole location, for the platform location. Such a layout gave, in some cases, several hundred possible locations for the platform and a similar number of potential downhole drilling targets.

3.2.8.2 Capital Cost

The cost of individual wells was calculated from a cost equation as a function of the along hole length from the drilling centre to the downhole well target. The method of calculating the well length is described in Appendix 2.

The jacket, topsides, and export pipeline costs are all flowrate dependent. Cost estimates were made at different flowrates using the cost estimating tool, QUESTOR. This package is described in Appendix A1.2.3. The costs were then curve fitted to linear equations to enable the cost of these facilities to be determined at any flowrate. The equations are described in Appendix 3.

3.2.8.3 Model Structure

Equations (3.14) and (3.15) are an MILP problem which can be solved using a commercial MILP solver such as that provided by the GAMS platform. The solver is
3.0 Single Field Model

used to determine the number and location of wells to be drilled each year. It was found that the CPLEX solver was the most efficient at solving these particular problems. An optimisation must be made for each year of field life.

The model steps through each year of field life in a sequential manner starting from Year 1. In the first four years the facilities are designed and built. Therefore there is no oil production, only capital expenditure. In Year 4 the first production wells must be drilled to enable the Year 5 production target to be met.

Year 5 is the first year in which an MILP problem is formulated. The target production is determined from the specified production profile. The Production Decline Factor, $F_5^{PD}$, is set to 1.0 to reflect the first year of production when the cumulative production is zero. Equations (3.14) and (3.15) define the MILP problem that is then solved using the solver provided in the GAMS platform. The cost of drilling the required wells is then added to the Year 4 CAPEX, reflecting that the wells have to be operational for Year 5.

The cumulative production, Production Decline Factor and target production are then calculated for Year 6. A new MILP problem is then formulated with the Year 6 data and a list of wells that are already drilled is passed to the solver to determine any additional wells that are required to meet the Year 6 production target. The cost of these new wells is then added to the CAPEX in Year 5.

The cumulative production is compared with the reserves and if it is less than the reserves, the calculations are repeated for the next year. If the cumulative production exceeds the reserves, the production in the current year is reduced so that the cumulative production equals the reserves and the production sequence is determined. Calculation proceeds to the economic analysis.
3.0 Single Field Model

Using the data that has been calculated during the sequential production simulation, the economic parameters are determined using Equations (3.2) to (3.13). These calculations are made sequentially from Year 1 to the end of the final year of production. The result is the NPV for the project.

In order to determine the life of field costs, it was necessary to perform these optimisation calculations over an average of about 20 years of field life. This amount of data had the possibility of creating an unwieldy data set.

Costs were discounted dependent on the year in which the cost is incurred in order to take account of the time-value of money.

3.2.8.4 Model Implementation

It was concluded that the best model structure would be to write an “executive” program in C++ that would write the GAMS input file for each year based on a specific case description. This hierarchical approach proved to give a good distribution of computational tasks between the two parts of the model. In particular, it provided a logical interface for checking and debugging of the model.

The method enabled the MILP problem for each year to be solved consecutively and each year’s results to be saved. At the end of the simulation of field life, the data could be retrieved to determine the overall economic value of the project.

3.2.8.5 Solution Output

The output is a discounted cash flow forecast based on a specified peak production rate. The NPV and IRR of the project are reported for several different oil prices. In addition, the drilling schedule is reported for individual wells. The schedule provides an indication of the maximum number of rigs required in any year and the
practicality of achieving the schedule. The schedule also permits a comparison between the two different models that have been developed as part of this work.

3.2.8.6 Model Verification

During the course of development, optimisations were run and cross-checked against spreadsheet calculations. The process was then automated and then again checked against spreadsheet calculations. After the elimination of bugs, the two solutions were identical, indicating that the model was functioning correctly.

3.2.9 Results

To demonstrate the capability of the approach, typical results for the West field at a peak production rate of 100,000 BPD are described. Based on reserves of 800 MM bbl and the peak production rate, the target production rate is shown in Figure 3.2. In this particular model, the build up of the production to the peak value was over four years and not two years as described in Section 3.2.1.2. The model was constructed so that it provided flexibility to investigate a number of different scenarios, including different production profiles.

Actual production in each year is that achieved from all operational wells, and can exceed the target production. Production is not constrained to the target, but is set equal or greater than the target. All selected wells produce to their maximum. Therefore production will normally be a little higher than the target each year.

During the decline period it is possible that no new wells are required to be drilled since production exceeds the target in each year. As a result, the total reserves can be produced earlier than specified by the seven year target decline profile.

Figure 3.2 also shows the actual production profile from the 100,000 BPD simulation. Actual production exceeds the target production each year by an
average of almost 5,000 BPD. From Year 21, no more new wells were required to be drilled. Total reserves were produced by Year 24 instead of the target Year 28.

Figure 3.2 Target Production Profile

The actual field production meets or exceeds the target production in all years until the field comes off production in Year 22. Due to the small excess production over target in earlier years and the higher production capability of existing wells in the decline period, the total reserves are actually produced earlier than the target profile.

Figure 3.3 shows the wells that were selected to meet the production target during field life. The optimum drilling centre had previously been determined as W093, using the method described in Section 2.0. This location was selected as a vertical high production well could be drilled from there. The eight wells surrounding the drilling centre were also selected as they were nearly all high production wells (except W106). The remaining wells were selected as a trade off between longer and therefore higher cost wells, but with a higher productivity; and shorter and therefore lower cost wells, but with slightly lower productivity. The model selection was biased towards the shorter wells.
3.0 Single Field Model

3.2.9.1 Development Optimisation

A series of optimisation runs was made at peak rates of between 50,000 BPD and 450,000 BPD for constant recoverable reserves of 500 MM bbl. 450,000 BPD was the highest rate that the defined profile could be achieved with the specified reserves. These runs optimised the field development to achieve the target production for each year of field life. The optimisation results included actual annual production and the new wells that were required to generate the production.

![Figure 3.3 Well Selection for 100,000 BPD Production](image)

The drilling centre is located at W093. To meet the 100,000 BPD production target, the vertical well at the drilling centre is selected together with the 8 surrounding wells. The balance is made of a mixture of 6,000 and 5,000 BPD wells with an emphasis on short distance from the drilling centre.

3.2.9.2 Economic Analysis

The NPV of the project at a fixed discount rate of 10% and the internal rate of return were calculated for a range of crude oil prices between $5 and $30 / barrel. These results are shown in Figure 3.4.
The results of the economic analysis were unexpected. For all oil prices except $5/bbl, the NPV and IRR increased for increased production, indicating that the largest facilities that could be built were the most economic. At an oil price of only $5/bbl, the revenue was more comparable with the cost of the facilities and a maximum NPV occurred at about 250,000 BPD capacity.

It had been expected that a maxima would occur at all oil prices. However, what was happening was that, with higher oil prices, the project generates so much revenue that it could profitably finance the size of facilities. The field model is loosely based on the Shell Fulmar field in the Central North Sea. The field was developed during the period when the oil price was about $15/bbl. At higher prices, the development would become increasingly more profitable.

These results show the NPV increasing with design capacity at all oil prices, except the very low $5/bbl. At all other oil prices, the NPV continuously increases with design capacity.

With current oil prices reaching over $80/bbl, this conclusion is even more valid, although costs have also risen significantly. The model has limits to maximum
capacity: at least one year at peak production is required, and only 85% of the reserves can be produced by the end of the first year of peak production.

3.3 Life Time Drilling Optimisation

The second model tackled the problem from a different direction from the Sequential model just described. Where the Sequential model stepped through each year in turn, optimising production for the current year before moving on to the next year, the second model optimised the drilling programme over the life of the field in a single run. The second model was referred to as the Simultaneous Model.

The model was built to only optimise the drilling programme and did not determine the economic performance of the scenario. This decision was taken to avoid building a model that had extraneous calculations in it that could be easily performed subsequently in a simple spreadsheet.

3.3.1 The Problem

The problem tackled by the Simultaneous Model is to determine the drilling programme to meet the specified target production over the life of the field, and to minimise the total discounted drilling cost over the field life. The Sequential model only minimised the drilling cost for the current year.

A time increment of one year was used.

3.3.1.1 Input Data

The input parameters define the problem to be solved and are specified as part of the problem input data. These parameters are:

- The reserves of the field;
3.0 Single Field Model

- The design capacity of the facilities;
- Location and productivity of all well locations;
- Location of selected drilling centre;
- Production target for each year of operation;
- Cost equations for wells.

This was the same input data as the Sequential model, excluding the facilities cost data.

3.3.1.2 Outputs

The output is a drilling schedule reporting the wells to be drilled in each year to meet the production profile and to minimise the total discounted drilling cost. The model did not calculate the project NPV. Either the model could be extended to do this, or, more practically, a separate short program could analyse the results from the Simultaneous model and determine the project NPV.

3.3.1.3 Optimisation Objective

The problem objective was to minimise the total cost over the life of the field of drilling sufficient wells to achieve the specified production target in each year.

3.3.2 Challenges

The main challenge in developing the model was to formulate the problem without non-linear terms that would impose difficulties in combination with the large number of binary variables that are required by the model. For example, some of the fields being investigated had several hundred potential well locations, and production extended over more than a 20 year period. Since the Simultaneous
model optimised all the years at the same time, the problem was, on average, 20 times larger than the Sequential model problem.

### 3.3.3 Assumptions

The same assumptions were made as were made for the Sequential model and are described in Section 3.2.4, except for two drilling assumptions:

1. The first year of production was Year 1, since the model did not consider the design and construction phase.

2. A well is available for production in the year that it is drilled, and that it delivers a full year’s production. This assumption was made to avoid a zero index for drilling cost, since production started in Year 1.

### 3.3.4 Simultaneous Single Field Model Outline Approach

The model was developed to determine the optimum drilling schedule over field life. It did not determine the CAPEX profile for the jacket, topsides and pipelines; nor did it determine the revenue and operating costs. Therefore the model did not determine the NPV of the project. If required, NPV could be calculated separately in a simple spreadsheet.

The single field model was built as part of the development of a simultaneous two field model that is described in Section 5. Equations were built up to express the drilling requirement over field life to be solved simultaneously.

### 3.3.5 Problem Nomenclature

The Simultaneous Single Field model nomenclature is the same as described in Section 3.2.4 for the Sequential Single Field model, except as described below. The differences are primarily in the index range of parameters and variables.
3.0 Single Field Model

The model consists of two sets:

\[ i \] The set of wells;

\[ t \] The set of years of production.

The following parameters define the input data of the model:

\[ D \] Design capacity of facilities;

\[ DCF \] Discount factor for NPV calculation. Taken as 10%;

\[ F_t^D \] Financial discount factor for Year \( t \). \( F_t^D = 1/(1+DCF)^t \). This factor is calculated for each year and provided as input data;

\[ F_t^{PD} \] Production decline factor for Year \( t \). Defined in Equation 3.5;

\( G_t \) Constrained production from field in Year \( t \);

\( T_t \) Production target for Year \( t \). The production target is set for each year of production;

The binary variables that have not been defined previously or are redefined are:

\( x_{it} \) Indicates Well \( i \) has been drilled and is available for production in Year \( t \);

\( z_{it} \) Indicates Well \( i \) commences first production in Year \( t \).

The variables are:

\( c_i \) Cost of drilling Well \( i \);

\( Cost \) the total cost of drilling sufficient wells to meet or exceed the specific target production, \( T_t \);

\( p_{it} \) Production from Well \( i \) in Year \( t \) (defined as a positive variable);

\( w_i \) Maximum production from Well \( i \).
3.0 Single Field Model

3.3.6 Model Formulation

Minimise:

\[ \text{Cost} = \sum_{i} \sum_{t} z_{i,t} c_{i} F_{t}^{D} \]  

(3.15)

Equation 3.15 is the objective function to minimise the discounted cost of drilling all wells required to meet the production forecast throughout field life.

The \( F_{t}^{D} \) term discounts the cost of each well dependent on when the well is drilled. In this way, wells that are drilled later in field life cost less in discounted terms than wells drilled early in field life. There is, therefore, an incentive to delay drilling as much as possible. The discount term is calculated from Equation 3.16.

\[ F_{t}^{D} = \frac{I}{e^{DCF(t-1)}} \]  

(3.16)

Subject to:

\[ p_{i,t} \leq x_{i,t} w_{i} F_{i}^{PD} \]  

(3.17)

Equation 3.17 determines the production from Well \( i \) in Year \( t \).

\[ T_{t} = \sum_{i} p_{i,t} \]  

(3.18)

Equation 3.18 sets the production in Year \( t \) to the Production target in that Year.

\[ x_{i,t} \geq x_{i,t-1} \]  

(3.19)

Equation 3.19 sets \( x_{i,t} \) to be greater than or equal to \( x_{i,t-1} \). Hence if \( x_{i,t-1} = 1 \), then \( x_{i,t} = 1 \). This ensures that if a well is in production in Year \( t-1 \), it is also available for production in Year \( t \).

\[ z_{i,t} = x_{i,t} \quad [t=1] \]  

(3.20)

Equation 3.20 sets \( z_{i,t} \) to the value of \( x_{i,t} \) for Year \( 1 \). This sets \( z_{i,t} \) to 1 if Well \( i \) is drilled in Year \( 1 \), else it sets it to zero if the well has not been drilled.

\[ z_{i,t} = x_{i,t} - x_{i,t-1} \quad [t>1] \]  

(3.21)
3.0 Single Field Model

Equation (3.21) determines if a well is drilled in Year $t, t > 1$. There are three possible cases:

- The well has not yet been drilled in Year $t$. In which case, both $x_{i,t}$ and $x_{i,t-1}$ are zero, therefore $z_{i,t}$ will be zero.

- The well is drilled in Year $t$. In which case, $x_{i,t} = 1$, but $x_{i,t-1} = 0$, and therefore $z_{i,t} = 1$.

- Finally if the well was drilled before Year $t$, Equation 3.19 ensures that $x_{i,t} = x_{i,t-1} = 1$, and therefore $z_{i,t} = 0$.

Hence, $z_{i,t} = 1$ only in the first year that the well goes into production, as is required.

$$G_i \leq \sum_i P_{i,t}$$ (3.22)

Equation (3.22) recalculates the total production for Year $i$.

$$D \geq G_i$$ (3.23)

Equation (3.23) then limits the production to no greater than the specified peak production or design capacity. Variable $G_i$ is generated to enable the constrained value to be reported.

3.3.7 Design Approach

The optimum solution was the one that met the production profile at a minimum total drilling cost over the life of the field. The model plan that was adopted was:

1. Specify the drilling centre by using one of the optimisation models described in Section 2.

2. Calculate the cost of drilling each possible well.
3.0 Single Field Model

3. Develop the target production profile based on the specified production rate and the reserves.

4. Formulate a set of equations to describe the drilling requirements for each year, based on Equations 3.15, 3.17 and 3.18.

5. Solve the optimisation problem with the objective of minimising the total discounted well cost over field life.

3.3.8 Implementation

The objective function was to minimise the total cost of drilling wells over the field life, subject to meeting the specified annual production for each year of production.

A set of equations were formulated using Equations 3.17 and 3.18 for each year of production.

The problem can be structured and solved as an MILP problem, and can be solved using one of the MILP solvers provide in the GAMS platform.

The model was built to use the same input well data as the Sequential model. The model runs exclusively in the GAMS environment and does not make use of an executive program in the manner of the Sequential model.

3.3.9 Results

The Simultaneous model was first tested on the Small field with 29 wells. It was then used on the West field which had 223 wells. The second field results were compared in detail with those obtained from the Sequential model.
3.0 Single Field Model

3.3.10.1 Field 1 Results

In order to test the model during development, a small field with a total of 29 wells was used. The specified production profile produced all the reserves over a 20 year period. The profile built to the peak production rate in 4 years and the held the plateau rate for a total of 10 years and then declines for a period of 7 years.

The resultant production profile is shown in Figure 3.5. The drilling schedule is shown in Figure 3.6.

Figure 3.5 Simultaneous Model Production Profile

The production profile has been forced to exactly match the target production. Total production therefore matches the recoverable reserves since the target production is calculated to produce the reserves over field life.

Production from the field exactly matches the target production because, indirectly, the production is set to the target. However, the model first determines the unconstrained production to equal or exceed the target and satisfies the objective function by selecting the lowest CAPEX option to achieve the target production over field life.
Drilling is spread over the first 12 years of field life, ensuring wells are only drilled when required. No drilling is required once the profile goes into decline.

### 3.3.10.2 Field 2 Results

The Sequential model used a stepwise approach to optimise the well selection for each year before moving on to the optimisation of the next year. Optimisation was therefore performed repeatedly, in a sequential manner.

The Simultaneous model optimised the well selection for the field life in one single optimisation. The well selection is the optimum for the field over the whole of field life. Therefore there was the potential for different solutions to the problem.

The West field was used to obtain a comparison. This field contains 223 wells. The reserves were set at 500 million bbls and a peak production rate of 150,000 BPD was selected for both models. The same drilling centre, W095, was used for each model. The well selections are shown in Figure 3.7.
The well selections by the two models are near identical. The Sequential model selected W045, whilst the Simultaneous model selected W084, which is a lower cost well. The only other difference was the selection of W054 instead of W056. These two wells are similar and cost the same to drill, so the selection is arbitrary.

The well selections were nearly identical. Each model required a total of 41 wells. Of these all except two wells were selected by both models.

The Sequential model selected well W045, whilst the Simultaneous model selected well W084. W045 is a 6,000 BPD well which is a long distance from the drilling centre. W084 is a 5,000 BPD well that is much closer to the drilling centre. The new model was able to make a slight economic improvement in the selection since one higher capacity well was not required in its configuration.

The other well difference was between W054 and W056. These two wells have the same productivity and are located symmetrically about the drilling centre. They therefore have the same drilling cost and are equivalent.

Therefore, it can be concluded that there is very little difference between the two calculation methods.

The timing of drilling the wells in the two models is shown in Table 3.2. No new wells were required after Year 9.
3.0 Single Field Model

Table 3.2. Comparison of Drilling Sequences

<table>
<thead>
<tr>
<th>Year</th>
<th>Sequential</th>
<th>Simultaneous</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>W065, W066, W095, W107, W122, W124</td>
<td>W065, W066, W079, W095, W107, W122</td>
</tr>
<tr>
<td>5</td>
<td>W069, W125</td>
<td>W063, W121</td>
</tr>
<tr>
<td>6</td>
<td>W059, W121</td>
<td>W074, W098</td>
</tr>
<tr>
<td>7</td>
<td>W047, W074</td>
<td>W047, W059</td>
</tr>
<tr>
<td>8</td>
<td>W054, W055, W098</td>
<td>W055, W056, W084</td>
</tr>
<tr>
<td>9</td>
<td>W048, W060, W112</td>
<td>W060, W069, W134</td>
</tr>
</tbody>
</table>

Year 1

The well selections for the two different methods are shown in Figure 3.8. The only difference between the two selections is that W079 is selected in the Sequential model, whilst W124 is selected in the Simultaneous model. These two wells have the same drilling cost and productivity as they are the same distance from the drilling centre and have the same productivity, 6,000 BPD.

Figure 3.8 Year 1 Well Selections
3.0 Single Field Model

Year 2
The well selections for the two models are the same except that W124 is selected in the Sequential model, whilst W079 is selected in the Simultaneous model. This results in identical well selections for each model at the end of this year. The well selections are shown in Figure 3.9.

Figure 3.9 Year 2 Well Selections

Year 3
The Year 3 well selections are identical for each model. The selections are shown in Figure 3.10.

Figure 3.10 Year 3 Well Selections
3.0 Single Field Model

Year 4

The well selections for the two different methods are shown in Figure 3.11. The only difference between the two selections is that W063 is selected in the Sequential model, whilst W125 is selected in the Simultaneous model. Well W063 is further from the drilling centre and more expensive to drill than well W125, but has a higher productivity at 6,000 BPD compared with 5,000 BPD productivity from well W125.

Year 5

In Year 5, the Sequential model selects wells W069 and W125, whilst the Simultaneous model selects wells W063 and W121. Thus both models have now selected wells W063 and W125, albeit in different years. Wells W069 and W121 are equidistant either side of the drilling centre and both have the same well productivity and are therefore equivalent. The selection is shown in Figure 3.12.
The Sequential model selects wells W059 and W121 to meet the production target, whilst the Simultaneous model selects wells W074 and W098. Well W121 has now been selected by both models. The differences between the two models are the selection of W059 instead of W074 and W069 instead of W098. There is little difference in cost between these wells and no difference in productivity. The Year 6 selection is shown in Figure 3.13.
3.0 Single Field Model

**Year 7**

In Year 7, both models select well W047. The Sequential model also selects well W074 which was selected by the Simultaneous model in Year 6. The Simultaneous model selects well W059 which was selected by the Sequential model in Year 6. The selections for the two models are shown in Figure 3.14, are the same in this year except for wells W069 and W098.

**Figure 3.14 Year 7 Well Selections**

- **Year 8**

In this year, the Sequential model selects wells W054, W055 and W098. The Simultaneous model selects wells W055, W056 and W084. Well W055 is selected by both models. Wells W054 and W056 are symmetrical about the drilling centre and are therefore equivalent. Wells W084 and W098 are adjacent and are similar in cost. The well selection is shown in Figure 3.15.
Year 9

In Year 9, the final year of drilling, the Sequential model selects wells W048, W060 and W112; the simultaneous model selects W060, W069 and W134. Well W060 is common to both models and W069 was selected in Year 5 by the Sequential model. The well selections are shown in Figure 3.16. Thus finally, the wells that are not common to both models are wells W048, W054 and W112 selected by the Sequential model; and wells W056, W084 and W134 selected by the simultaneous model.
3.0 Single Field Model

The well pairs W054 and W056, and W112 and W084 are symmetrical pairs about the drilling centre. Therefore the only difference in the final selection is between well W047 selected by the Sequential model and W134 selected by the Simultaneous model. The well selections for the two models are, therefore near identical.

3.4 Conclusions

The development of the model demonstrates that a practical and realistic model can be built to determine the optimal life of field drilling schedule in a single simulation run.

The model was developed and proven with a field containing 29 possible well locations, but is capable of modelling a field of unlimited locations.

The modelling method requires the production profile and production decline factor to be defined as input data to avoid the problem becoming non-linear.

The Sequential model perhaps more closely follows actual oilfield operations since it is based only on historical and current data. However the Simultaneous model determines the maximum optimisation that can be achieved. Comparison between the two methods gives an indication of the benefit of considering the whole of field life in determining the optimal drilling schedule.

The Sequential model was not developed to determine the full economic return of the project by determining project NPV. However, this could easily be added if it were decided that it would be beneficial.
4.0 **Model Based Sensitivity Analysis**

A number of investigations were made to determine the effect of changes in the main variables on the economic performance of the development schemes. Solis et al (2004) investigated the effect of reservoir uncertainty, schedule and capital expenditure risk on an offshore gas field development in Mexico.

The risk associated with optimization of the overall economic return of a project was investigated by Lane et al (1994) for both mature fields and new projects.

The different investigations are described in the following section. In several cases the Single Field Model that was built to complete the work described in Section 3 was used with appropriate modifications.

Two types of analysis were performed. Initially, a simple parametric analysis changed the value of certain parameters. Subsequently a Monte Carlo simulation was performed to determine the effect of uncertainty in the input parameters.

The parametric analysis investigated the effect of changes in the following parameters:

- Oil price (Section 4.1);
- Well cost (Section 4.2);
- Discount rate for NPV calculations (Section 4.3);
- Individual well decline factors (Section 4.4).

Two Monte Carlo simulations were performed to investigate the effect of random changes in some input parameters. The parameters investigated were:

- Cost of the facilities (Section 4.5);
4.0 Investigation of Model Sensitivity

- Size of the reserves (Section 4.6).

Finally, the alternative methods of presenting the effect of uncertainty in parameters are described in Section 4.7.

4.1 Effect of Fluctuating Oil Price

All previous work had been based on using a constant oil price for the duration of the field life. This obviously does not reflect real life where the oil price can gyrate over a wide range in a seemingly random manner.

Jornsten (1992) discusses the need for a Decision Support System as an analysis tool for offshore oil and gas fields with field lives of 20 to 30 years. The uncertainty of future oil and gas prices results in high uncertainty levels. The levels of uncertainty can have a large effect on the substantial investment made in the field’s development.

4.1.1 The Problem of Fluctuating Oil Price

Figure 4.1 shows the variation of oil price over the last 29 years, BP (2005). The effect of the price profile on the economics of a field development could be very significant with such wide variation in the value of the product.

During the course of this work, the oil price has risen steadily until at the completion of the work, the oil price has remained over $90/bbl for several months, BP (2007) and has been approaching $100/bbl, Oil & Gas Journal (2008).

The increase in the price of oil has been driven by several factors that are totally out of the control of the oil industry. These factors include political uncertainty, particularly over events in Iraq; greatly increased demand, in particular from developing countries such as China and India; and to some extent by speculation within the market.
4.0 Investigation of Model Sensitivity

4.1.2 The Fluctuating Oil Price Model

The model was based on the Sequential Single Field model described in Section 3, except that it was extended to allow a variable oil price profile.

The oil price profile was provided as an input file that was read as part of the run parametric input data. For each year of production, the appropriate oil price was read from the file and this value was used to determine the revenue credited to the project for that year.

Figure 4.1 Brent Crude Oil Price

The annual average oil price has varied considerably over the last 29 years. The data was extracted from the BP Statistical Review. Actual swings during each year have been even greater, and the current oil price has been over $70 for several months.

4.1.3 Model Assumptions

The effect of changing oil price was investigated using a series of oil price profiles. A set of oil price profiles was generated using the oil price between 1981 and 2000.
4.0 Investigation of Model Sensitivity

This profile was repeated a number of times to give sufficient data to cover all the years of simulation. Three sets of profiles were developed based on the maximum, minimum and average oil prices over the years selected. The profiles were then built to start with the maximum, minimum or average oil price and then follow the sequence of price changes. This is shown in Figure 4.2.

Simulations using the three different oil price profiles were run from 50,000 to 450,000 BPD production capacity in increments of 50,000 BPD. The resulting IRRs for these three cases are shown in Figure 4.3.

![Figure 4.2 Oil Price Profiles](image)

The figure shows the oil price profiles that were used to study the sensitivity of project NPV to changes in oil price. The fluctuations are based on actual values over the last 20 years. The pattern repeats after this period.

4.1.4 Model Results

The IRR for the three different oil price scenarios is shown in Figure 4.3 at different design capacities. The general shape of the curves is similar to that
found for the single field models in Section 3: the slope of the curve decreases as the design capacity increases, but the IRR never reaches a maximum.

The values for a constant oil price of $20/bbl are also shown in Figure 4.3. The IRR closely follows the minimum oil price curve. By examination of Figure 4.2, it is clear that in the early years, before the discount factor takes a significant effect, the minimum oil price is approximately $20/bbl. The discount factor minimises the effect of changes that occur more than 5 or so years in the future.

In Figure 4.3, the IRR curve for the average price is close to that of both the minimum price and the fixed price of $20/bbl. The reason for this is the effect of discounting revenue in later years. In each case, the revenue in early years has much more effect on the IRR and NPV of the project.

Figure 4.3 Effect of Oil Price on IRR

Whilst IRR changes with different oil price scenarios, the shape of the curve remains the same and the economic return continuously increases with increasing Design Capacity. Therefore the results are not changed by a fluctuating oil price as the relative effect on economic return remains the same.
4.0 Investigation of Model Sensitivity

In the minimum, average and fixed oil price cases the high oil price is deferred for several years, see Figure 4.2. Therefore these all have similar IRRs. The maximum case has the high oil price in the initial years, and therefore the discounted revenue is significantly higher for the same production volumes.

4.2 Effect of Increased Well Cost

As a proportion of the total investment in the field, the drilling cost is approximately 25% of the overall cost of the projects used in this study. Table 4.1 provides an example of the relative costs of the major components in a typical development model.

The effect of increased well costs on the overall project economics was investigated in this section of the study.

Table 4.1 Major Development Costs

<table>
<thead>
<tr>
<th>Cost Centre</th>
<th>Cost, $MM</th>
<th>Cost, %</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jacket</td>
<td>78.076</td>
<td>13.2</td>
</tr>
<tr>
<td>Topsides</td>
<td>310.486</td>
<td>52.6</td>
</tr>
<tr>
<td>Oil pipeline</td>
<td>41.698</td>
<td>7.1</td>
</tr>
<tr>
<td>Gas pipeline</td>
<td>26.012</td>
<td>4.4</td>
</tr>
<tr>
<td>Drilling</td>
<td>133.721</td>
<td>22.7</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>589.993</strong></td>
<td><strong>100.0</strong></td>
</tr>
</tbody>
</table>

4.2.1 Method

The well costs were increased by multiplying the well cost calculated within GAMS by a factor of 3. Such a large increase in well costs could arise from higher than anticipated offshore rig charter rates due to sudden increase in demand, and also to more difficult drilling conditions. The increase resulted in the well cost increasing from approximately 22% to 47%.
4.0 Investigation of Model Sensitivity

The higher well costs were included in the well selection optimisation. The Single Field Model was modified to use this higher cost. As usual, the recoverable reserves were set at 500 MM bbl, and the production rate was varied between 50,000 and 450,000 BPD in increments of 50,000 BPD.

4.2.2 Results

The NPVs of each design capacity case for both the original well costs and the trebled costs are summarised in Table 4.2. Well cost are in millions of dollars.

<table>
<thead>
<tr>
<th>Design Capacity, BPD</th>
<th>Well Cost x 1</th>
<th>Well Cost x 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>450,000</td>
<td>4,799</td>
<td>4,292</td>
</tr>
<tr>
<td>400,000</td>
<td>4,730</td>
<td>4,203</td>
</tr>
<tr>
<td>350,000</td>
<td>4,683</td>
<td>4,253</td>
</tr>
<tr>
<td>300,000</td>
<td>4,573</td>
<td>4,184</td>
</tr>
<tr>
<td>250,000</td>
<td>4,431</td>
<td>4,132</td>
</tr>
<tr>
<td>200,000</td>
<td>4,193</td>
<td>3,941</td>
</tr>
<tr>
<td>150,000</td>
<td>3,818</td>
<td>3,626</td>
</tr>
<tr>
<td>100,000</td>
<td>3,226</td>
<td>3,098</td>
</tr>
<tr>
<td>50,000</td>
<td>1,993</td>
<td>1,931</td>
</tr>
</tbody>
</table>

The original results were compared with those with the higher well costs. In each case, the well selections were the same. The NPVs of the two sets of results are shown in Figure 4.4.

4.2.3 Analysis

The NPV has been reduced by the increased well costs, as would be expected. The drilling schedule remains the same in the two different cases as the production target and well productivities remain the same. Since the relative well cost also
remains the same, the well selection remains unchanged in both the original and the high well cost cases. The lowest costs wells remain the lowest cost, albeit that they are higher CAPEX. Only the DRILLEX increases, resulting in a reduction in the project NPV.

Figure 4.4 NPV at Different Well Costs

The graph shows the effect on NPV of trebling the well cost. At low design capacity, the effect is small as the well count is small. At higher design capacities, the effect is greater, resulting in decreased NPV as costs increase.

This simple adjustment to the well cost still does not limit the production of oil from an individual well. The effect of the increased well cost is more marked at the higher rates, but the effect of the discounting on costs and revenues in later years reduces or dampens a large proportion of the effect.

Figure 4.5 shows the exponential decline of the discount factor with project life at a discount rate of 10%. At this rate, after 8 years of project life, the discount factor is 50% of original and is only 25% of original after 15 years. Thus the effect of costs and revenues in later years is significantly reduced by the discount factor.
4.3 Effect of Different Discount Rates

There is a possibility that the discount rate may affect the selection of wells, particularly during later field life when the discount factor has more effect. In order to determine this effect one case was run at four different discount rates to determine the well selections in each case.

![Figure 4.5 Discount Factor Decline at 10%](image)

The discount factor reduces the effect on NPV of future costs and revenues. Therefore to improve NPV, expenditure needs to be deferred and revenue accelerated.

The discount rates selected were: 8%, 10%, 12% and 15%. The case selected was the East field with reserves of 350 million barrels and a peak production rate of 100,000 BPD. The drilling centre was located at E069. For all four discount rates, the well selection remained the same throughout field life. The wells selected are shown in Figure 4.6. The discount rate does not affect the well selection, since the discount rate only effects the economic analysis performed after the field development optimisation. The discount rate reflects different rates of return on the capital employed.
4.0 Investigation of Model Sensitivity

Figure 4.6 Well Selection

For the four different discount rates, the well selection was exactly the same both in well selection and timing of specific wells.

The well timing also remains the same since the change in the discount rate does not affect the drilling schedule. The only change was the obvious one of decreased NPV as the discount rate increased. The changes in the NPV are summarised in Table 4.3. It can therefore be concluded that the discount rate does not affect the well selection. This is because the relative cost of the wells remains the same for each year, irrespective of the value of the discount rate.

Table 4.3 Change in NPV with Discount Rate

<table>
<thead>
<tr>
<th>Discount Rate</th>
<th>NPV, $MM</th>
</tr>
</thead>
<tbody>
<tr>
<td>8%</td>
<td>2,249</td>
</tr>
<tr>
<td>10%</td>
<td>1,778</td>
</tr>
<tr>
<td>12%</td>
<td>1,397</td>
</tr>
<tr>
<td>15%</td>
<td>956</td>
</tr>
</tbody>
</table>
4.4 Effect of Individual Well Decline Factors

The objective of this model was to implement and investigate the effect of a more precise well productivity decline model. The model is based on the Single Field Model in order to investigate the effect of a single change to the original model and to eliminate the additional effect of the previous parametric investigations.

The objective of this extension to the investigation was to determine the effect of limiting the production and productivity from individual wells as a function of the cumulative production in previous years.

4.4.1 Individual Production Reduction Factors

In the Single Field Model the Production Decline Factor was defined as:

\[ F_{PD} = \frac{Reserves - 0.5 \times Cumulative Production}{Reserves} \]  \hspace{1cm} (4.1)

This factor was applied to all wells in the field irrespective of the quantity of oil produced from any individual well.

An individual well can realistically only produce the oil that is trapped in the surrounding formation. Therefore, to all practical purposes, there is a limit to the total volume of oil that an individual well can produce. This volume is determined by porosity, viscosity, reservoir pressure, reservoir thickness and well geometry. The volume is referred to as the total well production.

Well productivity is the rate that a well can produce at. Well productivity declines as a function of time. As more oil is produced from the well, the reservoir pressure tends to decline, the near well pores may become blocked and the well may start
4.0 Investigation of Model Sensitivity

to produce water. All these factors reduce the rate at which a well can produce oil.

Reducing the well productivity on a global basis could be an unrealistic, although expedient method of implementing a well decline mechanism. By restricting production from individual wells, the new model would be driven to drilling more wells and extending the area of the reservoir from which the oil is produced, as wells drilled early in field life declined in production. This would probably make the model more realistic.

4.4.2 Implementation

A new model was built based on the model described in Section 3, but modified to implement this enhanced functionality of assigning individual Production Decline Factors, $F_{PD}$, to each well. The production was then reduced by a constant factor after each year of production to give an exponential decay to production. The constant factor was set to 0.85, although this is a variable which is user selectable.

An alternative method of implementing the decline, which was not used, would be to specify a maximum production from each well and to reduce the productivity as a function of the volume of oil produced from the individual well.

A matrix for all the wells in the field contains the individual well $F_{PD}$. These are all set to 1.0 initially. After each year that a well has been in production, the $F_{PD}$ is multiplied by the constant decay factor. Thus wells that first come into production in the same year have the same $F_{PD}$ value throughout the remainder of field life and wells that have not been put in production yet continue to have values of 1.0.
4.0 Investigation of Model Sensitivity

The matrix of $F_{PD}$ values is stored in a GAMS include file which is read directly by GAMS. The executive program writes the include file initially and updates the file after each GAMS run. The GAMS platform is used to optimise the well selection for each year in a sequential manner.

4.4.3 Results

The effect of the revised algorithm in the new model is shown in Figure 4.7. This figure shows the yearly Production Decline Factors with 500 MM bbl reserves and 100,000 BPD design capacity. Individual well productivity decays more rapidly with the new model and is in an exponential form. Such a decay is closer to that which would be expected from a typical field.

A series of runs were then made at design capacities of between 50,000 and 450,000 BPD to determine the effect over the full range of design rates.

![Figure 4.7 Production Decline Factor, $F_{PD}$, Decay at 100,000 BPD Design Capacity](image)

In the Single Field Model, productivity decline is approximately linear. In the revised model not only is it exponential, but, by the choice of factor, is significantly greater. However, well selection was not affected.
Figure 4.8 shows the number of wells required to meet each production capacity and
4.0 Investigation of Model Sensitivity

Figure 4.9 shows the NPV at an oil price of $20/bbl. In each case earlier results from Single Field Model are included for comparison.

Well selections were similar, with the model requiring a larger number of wells for a particular flowrate selecting all those selected by the other model at that rate before selecting other wells to make up the balance.

The effect of the two modelling techniques is only significant at design capacities below about 150,000 BPD for the model investigated.

The original Single Field Model had a linear decline mechanism. The revised model had an exponential decline. The model with the exponential decline requires more wells to be drilled at the lower design capacities. However, at higher design rates, the universal decline factor reduces the productivity even from new wells and therefore more wells are required for the linear decline model.
4.0 Investigation of Model Sensitivity

The NPV of the Exponential decline model is greater than the Single Field Model because, although more wells are drilled, the production from the wells early in their live is higher since the universal decline factor is not applied to individual wells.

4.4.4 Interpretation of Results

The Exponential Decline Model has produced a more realistic decay pattern for each well as indicated in the comparison shown in Figure 4.7.

The reason why the total well numbers cross in Figure 4.14 is that in the new algorithm a new well produces at its maximum, normally 15,000 BPD in the first year, whereas in the old model, using a common decline factor, even production from new wells was reduced by the $F_{PD}$ for that year. This means that in Exponential Decline Model, new wells drilled late in field life produce significantly more oil when first started up.

At low production rates, more wells are required in the Exponential Decline model because with the long field life the new decline mechanism has more effect. At higher production rates, the NPVs become similar as field life is shortened.
4.0 Investigation of Model Sensitivity

4.5 Effect of Uncertainty in Costs

The effect of reservoir parameters, capital and operating costs, and oil price has been investigated by Motta et al (2000). Project risk in the development of mature North Sea hydrocarbon reservoirs and the effect on NPV is discussed by Pedersen et al (2006).

Previous work on this project has been based on fixed capital costs for each of the three major cost centres of platform, pipelines and drilling, and has searched for an optimum design capacity for these facilities. This previous work has indicated that the investment return, measured by both NPV and IRR, is relatively insensitive to design capacity above a minimum capacity, and that a clear optimal peak in the design capacity financial return relationship does not exist with the parameters being currently modelled.

The next stage of the current study has been to make the economic modelling more realistic by incorporating capital cost uncertainty into the economic models. The unrisked costs from the economic model are summarised in Table 4.4.

Table 4.4 Unrisked Cost of Facilities and Drilling, $MM

<table>
<thead>
<tr>
<th>Capacity, BPD</th>
<th>Platform</th>
<th>Pipeline</th>
<th>Drilling</th>
</tr>
</thead>
<tbody>
<tr>
<td>50,000</td>
<td>315.453</td>
<td>54.383</td>
<td>48.655</td>
</tr>
<tr>
<td>100,000</td>
<td>351.270</td>
<td>61.004</td>
<td>90.832</td>
</tr>
<tr>
<td>150,000</td>
<td>388.563</td>
<td>67.710</td>
<td>133.721</td>
</tr>
<tr>
<td>200,000</td>
<td>427.330</td>
<td>74.502</td>
<td>171.898</td>
</tr>
<tr>
<td>250,000</td>
<td>467.573</td>
<td>81.380</td>
<td>193.034</td>
</tr>
<tr>
<td>300,000</td>
<td>509.290</td>
<td>88.342</td>
<td>261.185</td>
</tr>
<tr>
<td>350,000</td>
<td>552.483</td>
<td>95.391</td>
<td>292.697</td>
</tr>
<tr>
<td>400,000</td>
<td>597.150</td>
<td>102.524</td>
<td>349.164</td>
</tr>
<tr>
<td>450,000</td>
<td>643.293</td>
<td>109.744</td>
<td>329.841</td>
</tr>
</tbody>
</table>
4.0 Investigation of Model Sensitivity

4.5.1 The Problem

The costs used by an oil company at the project sanction stage are obviously estimates of the final cost. The estimates are developed using in-house estimating procedures, experience and cost data. However, actual final costs will vary from these estimates in a random manner dependent on various internal and external factors.

All these random factors combine to affect the overall project cost. This stage of the study investigated the effect of random variations in cost of the facilities and drilling on the overall economics of the development.

4.5.1.1 Input Data

The model used the output data from the Single Field Sequential model as its base input data. The results from the Single Field Sequential model are the unrisked NPVs for different design capacities. The output data was read from the economic summary file. Since the study was investigating the effect of CAPEX changes to project NPV, the drilling schedule and well selection remained constant.

4.5.1.2 Model Objective

The objective of the model was to provide an indication of the sensitivity of the NPV of the projects at different design capacities to realistic changes in the main input variables. The results provide an indication of the risk of the project becoming sub-economic or conversely, having an economic return that is higher than expected.

The model output is the risked NPVs, sorted into equal ranges to give the specified number of divisions to enable the results to be plotted on an Excel histogram for further analysis.
4.0 Investigation of Model Sensitivity

4.5.2 Challenges

The challenge in this part of the study was to develop a model that would enable the effect of changes in key input parameters on project economics to be determined.

4.5.3 Assumptions

It is assumed that the actual costs of the three cost centres vary randomly and independently of each other. For each cost centre, minimum and maximum values can be estimated based on experience and an understanding of market conditions. The distribution of costs can therefore be better represented by a triangular rather than a normal distribution. A triangular distribution allows upper and lower limits to be set such that all costs are distributed within these two limits. Normal distribution could result in unrealistically high or low values.

4.5.3.1 Platform Costs

Platform costs can vary as a result of changes in the price of steel and equipment. It is also not unusual for the construction manhours, and hence cost, to be underestimated due to change orders and late modifications. It is possible that some of these costs could be less than estimated. For example, there could be a fall in the world steel price as demand decreased.

For these reasons, the variation in actual cost is assumed to be between 90% and 130% of the unrisked cost.

4.5.3.2 Pipeline Costs

Pipeline cost can be affected by weather conditions being better or worse than assumed, resulting in lower or higher pipeline installation times, and hence, costs than originally estimated. Another major factor can be the day-rate negotiated for
4.0 Investigation of Model Sensitivity

the lay barge. The rate finally negotiated for the lay barge will be very much dependent on world-wide demand at the time of the installation of the pipeline. Similarly the cost of the pipe material will be heavily dependent on demand at the pipe mills and the pipe size and wall thickness.

The variation in actual cost is taken to be between 80% and 120% of the base cost.

4.5.3.3 Drilling Costs

Drilling costs are potentially the most variable cost of the entire project. Less wear on the drill bit can result in the drill string having to be pulled less frequently. On the other hand a stuck or broken drill string can result in significant extra drilling time, including the need to sidetrack the well. Such effects can be independent of the experience on previous wells.

The variation in actual drilling cost is assumed to be between 75% and 200% of the base cost and to be applied independently to each year of drilling.

4.5.4 Monte Carlo Simulation Outline Approach

The Monte Carlo simulation technique is frequently used to model the effect of random changes in parameters, and this method was selected for this part of the study.

The NPV of a project is the sum of the discounted cash flows for each year of project life. In turn, the cash flow in each year is the revenue from the oil production less the capital expenditure in the year, and operating expenses.

The cash flow in each year is given by:

\[
\text{Annual Cash Flow} = \text{Oil Revenue} - (\text{Platform CAPEX} + \text{Pipeline CAPEX} + \text{Drilling Expenditure}) - \text{OPEX}
\]  

\[ (4.2) \]
4.0 Investigation of Model Sensitivity

Hence, the NPV is defined as:

\[ NPV = \sum_{N} \left( \text{Annual Cash Flow} \right) \cdot \left( \text{Discount Factor} \right) \]  

Where \( N \) = the number of years of field life.

The parameters selected to investigate the effect of variation on the project NPV were:

- Platform CAPEX;
- Pipeline CAPEX; and,
- Drilling Expenditure.

OPEX is also a dependent variable because it is a fixed percentage of the cumulative project CAPEX.

4.5.5 Problem Nomenclature

The model inputs are:

- \( i \) The number of Monte Carlo simulations;
- \( t \) The set of years of development and operation of the field.

The scalar input parameters read from the Single Field model are:

- \( c_{\text{PLAT}} \) Total platform and topsides cost;
- \( c_{\text{PIPE}} \) Total oil and gas export pipeline cost.

The annual parameters are:

- \( c_{t}^{\text{DRILL}} \) DRILLEX in Year \( t \);
4.0 Investigation of Model Sensitivity

$p_i$ Production in Year $t$.

Scalar parameters set within the program are:

$P_o$ Oil price (taken as $20/bbl);

$F_{\text{OPEX}}$ OPEX as a percentage of CAPEX (taken as 4%);

$F^d$ Discount factor (taken as 0.1 for this work)

$c_{L,\text{PLAT}}$, $c_{M,\text{PLAT}}$, $c_{H,\text{PLAT}}$ The low, medium and high platform costs in the triangular distribution;

$c_{L,\text{PIPE}}$, $c_{M,\text{PIPE}}$, $c_{H,\text{PIPE}}$ The low, medium and high pipeline costs in the triangular distribution;

$c_{L,\text{DRILL}}$, $c_{M,\text{DRILL}}$, $c_{H,\text{DRILL}}$ The low, medium and high drilling cost in the triangular distribution;

$F_{t,\text{PLAT}}$ The fraction of the Platform CAPEX occurring in Year $t$. $t \leq 4$;

$F_{t,\text{PIPE}}$ The fraction of the Pipeline CAPEX occurring in Year $t$. $t \leq 4$.

The variables calculated within the program are:

$c_{i,t,\text{DRILL}}$ The risked drilling cost in Year $t$ for Monte Carlo simulation $i$;

$c_{i,\text{PLAT}}$ The risked platform cost for Monte Carlo simulation $i$;

$c_{i,\text{PIPE}}$ The risked pipeline cost for Monte Carlo simulation $i$;

$r_{i,\text{GROSS}}$ The Annual Gross Revenue for Year $t$;

$c_{i,t,\text{RCUML}}$ The Risked Cumulative CAPEX Year $t$;

$c_{i,t,\text{RCAPEx}}$ The risked total CAPEX for Monte Carlo simulation $i$ for Year $t$;
4.0 Investigation of Model Sensitivity

- $c_{i,t}^{\text{ROPEX}}$ The Annual Risked OPEX in Year $t$;
- $m_{i,t}^{\text{ANN}}$ Annual Risked cashflow for Monte Carlo simulation $i$ for Year $t$;
- $r_{i,t}^{\text{DRILL}}$ The random number for the drilling cost for simulation $i$, for Year $t$;
- $r_{i,t}^{\text{PIPE}}$ The random number for the pipeline cost for Monte Carlo simulation $i$;
- $r_{i,t}^{\text{PLAT}}$ The random number for the platform cost for Monte Carlo simulation $i$;
- $r_{i,t}^{\text{RNET}}$ The net risked revenue in Year $t$;
- $\text{RNPV}_i$ Risked project NPV for simulation $i$.

4.5.6 Problem Formulation

The method of calculating the risked cost from the unrisked cost and upper and lower cost limits for a triangular distribution is described in Appendix 4.

The risked costs are a function of the high and low limits of the cost in the triangular distribution, the unrisked cost and the random number generated to determine the probability of the particular cost. The risked platform cost is:

$$c_{i,t}^{\text{PLAT}} = f(c_{L}^{\text{PLAT}}, c_{M}^{\text{PLAT}}, c_{H}^{\text{PLAT}}, r_{i,t}^{\text{PLAT}})$$  \hspace{1cm} (4.4)

The risked pipeline cost is:

$$c_{i,t}^{\text{PIPE}} = f(c_{L}^{\text{PIPE}}, c_{M}^{\text{PIPE}}, c_{H}^{\text{PIPE}}, r_{i,t}^{\text{PIPE}})$$  \hspace{1cm} (4.5)

The risked annual drilling cost in Year $t$ is:

$$c_{i,t}^{\text{DRILL}} = f(c_{L}^{\text{DRILL}}, c_{i,t}^{\text{DRILL}}, c_{M}^{\text{DRILL}}, c_{H}^{\text{DRILL}}, r_{i,t}^{\text{DRILL}})$$  \hspace{1cm} (4.6)
4.0 Investigation of Model Sensitivity

The CAPEX profiles for the first four years are built up in an identical manner to that described in Section 3 for the Sequential Single Field model, except that the risked costs are used instead of the unrisked. The cost fractions of CAPEX expended in each year are shown in Table 4.5.

<table>
<thead>
<tr>
<th>Year</th>
<th>Platform CAPEX Fraction, $F_{i}^{PLAT}$</th>
<th>Pipeline CAPEX Fraction, $F_{i}^{PIPE}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.15</td>
<td>0.00</td>
</tr>
<tr>
<td>2</td>
<td>0.25</td>
<td>0.00</td>
</tr>
<tr>
<td>3</td>
<td>0.30</td>
<td>0.50</td>
</tr>
<tr>
<td>4</td>
<td>0.30</td>
<td>0.50</td>
</tr>
</tbody>
</table>

The risked costs are used to build the annual risked CAPEX values for the first three years of project life:

$$c_{i,t}^{RCAPEX} = F_{i}^{PLAT} \cdot c_{i}^{PLAT} + F_{i}^{PIPE} \cdot c_{i}^{PIPE} \quad [1 \leq t \leq 3]$$  \hspace{1cm} (4.7)

Equation (4.7) is used for Years 1, 2, and 3 when the only expenditure is on the platform, consisting of the jacket and topsides, and the pipeline.

The risked CAPEX for Year 4, Equation (4.8), is similar except it includes the drilling cost of wells required to be in production to meet the Year 5 production target:

$$c_{i,4}^{RCAPEX} = F_{i}^{PLAT} \cdot c_{i}^{PLAT} + F_{i}^{PIPE} \cdot c_{i}^{PIPE} + c_{i,4}^{DRILL} \quad [t = 4]$$  \hspace{1cm} (4.8)

The risked CAPEX for subsequent years is only the annual DRILLEX, since all the other CAPEX has been expended constructing and installing the facilities:

$$c_{i,t}^{RCAPEX} = c_{i,t}^{DRILL} \quad [t > 4]$$  \hspace{1cm} (4.9)

The Risked Cumulative CAPEX for Year $t$ is:
4.0 Investigation of Model Sensitivity

\[
c_{i,t}^{\text{RCUML}} = \sum c_{i,t}^{\text{RCAPEX}}
\]  

(4.10)

The Annual Gross Revenue for each simulation is the same, since neither the annual oil production nor the oil price is risked:

\[
r_{i,t}^{\text{GROSS}} = p_t \cdot p^O
\]  

(4.11)

The Risked Annual OPEX in Year \( t \) is calculated from the Risked Cumulative CAPEX and the OPEX fraction:

\[
c_{i,t}^{\text{ROPEX}} = f^{\text{OPEX}} \cdot c_{i,t}^{\text{RCAPEX}}
\]  

(4.12)

The Net Risked Revenue in Year \( t \) is the Gross Revenue less the Risked Annual Operating Cost:

\[
r_{i,t}^{\text{RNET}} = r_{i,t}^{\text{GROSS}} - c_{i,t}^{\text{ROPEX}}
\]  

(4.13)

The annual risked cashflow is the Net Risked Revenue in Year \( t \) less the Risked Total CAPEX for simulation \( i \) for Year \( t \):

\[
m_{i,t}^{\text{ANN}} = r_{i,t}^{\text{RNET}} - c_{i,t}^{\text{RCAPEX}}
\]  

(4.14)

The Risked Net Present Value, NPV, of the project is the sum of the discounted Risked Annual Cash Flows over the life of the project:

\[
\text{RNPV}_i = \sum \frac{m_{i,t}^{\text{ANN}}}{e^{\rho (t-t_i)}}
\]  

(4.15)

4.5.7 Outline Approach

In order to investigate the effect of financial risk on the actual cost of key cost centres that make up the total cost of an offshore development, a model was built to determine the risked NPV of a project.
4.0 Investigation of Model Sensitivity

The model was built to use the output from the unrisked model described in Section 3.2 as input data. This data consisted of the CAPEX, DRILLEX and revenue of each optimised case. A total of 9 design flows rates between 50,000 BPD and 450,000 BPD were used in the simulations.

The model uses random numbers to generate risked costs within the triangular distributions described in Section 4.5.3 to vary the value of parameters to be risked between maximum and minimum values to simulate random changes in their costs.

The parameters that were selected to be varied were:

- Platform cost (jacket and topsides);
- Pipeline cost (oil and gas);
- Drilling cost.

These costs represent the main CAPEX inputs and are the parameters that are most likely to vary in practice.

The program logic can be followed by referring to the flowchart shown in Figure 4.10. The model was written in C++. The program was initialised by setting the limits for the triangular distribution. Variable data and the total number of simulations were then entered to initiate the simulations.

The appropriate data file was read, dependant on the design capacity being simulated. This data included the mean or $P_{50}$ CAPEX values. The high and low values (maximum and minimum) were then calculated from the fixed percentage ranges described in Section 4.5.3. These values correspond to the $P_{10}$ and $P_{90}$ values. Collectively they define the triangular distribution for each cost centre.
4.0 Investigation of Model Sensitivity

Figure 4.10 Monte Carlo Simulation Program Flowchart

START

Open Model 11 Output file and read run results

Calculate CAPEX high and low values

Use RNG to calculate each CAPEX element value

Calculate NPV of project with risked CAPEX values

Count number of simulations

Sort simulation NPV values in ascending order

Sort NPVs into intervals and count occurrences

END
4.0 Investigation of Model Sensitivity

The risked costs were calculated using random numbers to determine the risked costs within the triangular distribution for each cost centre. In the cases of the platform and pipeline, a single risked total cost was calculated for each of these centres. This cost was then distributed over the same number of years and in the same proportions as used in previous models.

A different risked drilling cost was calculated for each year based on individual random numbers. This method was selected for the drilling costs since some of the events described in Section 4.5.3.3 could occur independently from year to year.

Using these risked costs, the NPV of the project was then determined as previously described at a fixed oil price of $20/bbl. A discount rate of 10% was been used throughout.

The pipeline, platform and drilling costs were all varied independently in each simulation and these risked costs were used to calculate the NPV for each simulation.

The resultant NPVs were stored at the end of each simulation. The simulation was then repeated with a new set of risked costs calculated from new random numbers.

Once the required number of Monte Carlo simulations had been completed and the resultant NPVs determined, the NPVs were sorted in ascending order using the Shell sorting method described in Appendix 6. The results were then allocated to a specified number of intervals so that histograms of the number of occurrences of the NPV value ranges could be plotted.

The Project Risked NPV was calculated using a set of random numbers to generate triangularly distributed costs for each simulation. By recalculating the NPV from costs generated from different sets of costs within the triangular distributions, a
Monte Carlo simulation can be made of the effect of the risked costs on the NPV of the project. Initially, a total of 1,000 simulations were made for each design capacity. However, subsequently the number of simulations for each design capacity was increased to 10,000 to provide a smoother distribution histogram. By calculating the risked NPV with each set of risked costs, it is possible to assess the effect of realistic changes in the input parameters on the resultant NPV.

4.5.8 Results

The risked NPVs for each design capacity between 50,000 and 450,000 BPD were then divided into steps of $5 million and were plotted as a series of histograms shown in Figure 4.11. These histograms show the distribution of NPV, the location of the unrisked NPV (UR) which was calculated using the model described in Section 3, and the $P_{10}$, $P_{50}$ and $P_{90}$ NPVs.

The $P_{90}$ NPV is the NPV that has a 90% probability of occurring. That is, there is a 90% certainty that the NPV will equal or exceed this value. The $P_{50}$ NPV is the NPV that has a 50% probability of the project achieving or exceeding. The $P_{10}$ NPV is the NPV that has only a 10% probability of the project achieving or exceeding.

The histograms show that the Unrisked NPV occurs close to the $P_{10}$ value. This means that there is only approximately a 10% possibility of the unrisked NPV being equal to the risked NPV. It is far more probably that the Risked NPV will be lower than the Unrisked NPV.

In all cases, the $P_{50}$ NPV occurs at a significantly lower NPV than the $P_{10}$. Consequently it can be concluded, in the cases studied, that the NPV outturn of the project will be significantly less than the unrisked NPV. That is that the project will be less profitable than original, unrisked estimate.
Figure 4.11 NPV Histograms
The shape of the histograms is similar, although the range of NPV increases as the design capacity increases. This can be concluded by comparing the skew and kurtosis of each histogram (Martinez, 2002).

A distribution has a positive skewness if the mass of the distribution is concentrated on the left of the distribution peak, or that there is more data in the right tail of the distribution than on the left. A distribution has a negative skewness if the mass of the distribution is concentrated on the right of the distribution peak, or that there is more data in the left tail of the distribution than on the right. The coefficient of skewness is zero for symmetric distributions. A normal distribution has a coefficient of skewness of zero.

Kurtosis measures the extent of the peak or the degree of flatness near the distribution's centre. A positive ratio indicates more values in the region of the mean, that is, more peaked than the normal distribution. A negative ratio indicates that the distribution is flatter than the normal distribution. A normal distribution has a kurtosis equal to zero.

Both the coefficient of skewness and the kurtosis were calculated using the Excel add-in statistical functions.

\[ \text{Skewness} = \frac{n}{(n-1)(n-2)} \sum \left( \frac{x_i - \bar{x}}{s} \right)^3 \quad (4.16) \]

\[ \text{Kurtosis} = \frac{n(n+1)}{(n-1)(n-2)} \sum \left( \frac{x_i - \bar{x}}{s} \right)^4 - \frac{3(n-1)^2}{(n-2)(n-3)} \quad (4.17) \]

Where:

- \( x_i \) = univariate data;
- \( \bar{x} \) = mean value of \( x \);
4.0 Investigation of Model Sensitivity

\[ s = \text{the standard deviation}; \]
\[ n = \text{number of data points}. \]

Values of both the coefficient of skewness and the Kurtosis for each histogram are shown in Figure 4.12. From these plots it can be seen from these two parameters, the shape of each histogram is almost identical. The similarity between the different cases is a result of using the same triangular distributions for each of the cost centres. Although the values of the cost centres change for each design rate case, the shape of the distribution is identical.

**Figure 4.12 Histogram Characteristics**

The graph shows that there is little variation in either the skew or the kurtosis over the range of design flowrates investigated. This indicates that the shape of the distributions are very similar.

Figure 4.13 shows the distribution of cost variation of the three input parameters. All variations were defined as being triangular. The pipeline cost varies from 80 to
120%, and therefore the distribution will be evenly distributed around the 100% value used to calculate the Unrisked NPV. Both the platform and drilling costs are, however, biased to higher costs. The platform cost varies from 90 to 130%, whilst the drilling cost varies from 75 to 200%.

![Figure 4.13 Cost Triangular Distribution](image)

The pipeline range of actual costs are equally distributed around the estimated cost. However, the costs for both the platform and drilling are biased to overrun. Consequently the risked costs exceed the unrisked costs and the $P_{50}$ NPV is a lot lower than the unrisked NPV.

The range of risk selected for each cost centre shows that it is anticipated that the cost outturns are more likely to be higher than estimated, rather than lower. Therefore the project NPV is expected to be less than predicted using the unrisked costs.

In order to better match the $P_{50}$ and unrisked NPVs, oil companies add a contingency to the unrisked cost to account for unknown but anticipated cost.
4.0 Investigation of Model Sensitivity

overruns. The contingency is a percentage which decreases as a project progresses and the project definition improves. At the conceptual stage it could be 50% whilst at the construction phase it will have decreased to about 10%. Different companies and different projects have different contingency factors based on experience and the project complexity and location.

The contingency level can be set to bring the value of the Unrisked NPV to be equal to the $P_{30}$ NPV.

The final project costs are more likely to exceed the values used to calculate the Unrisked NPV than to be less than the unrisked costs. As a consequence, the NPV in the risked cases will be less than in the unrisked. This has been shown to be correct.

The Monte Carlo simulation for this project has shown the unrisked NPV is a good indication of the $P_{10}$ NPV value. This clearly is in line with expectation that the higher costs generated by the risked simulation result in a lower NPV.

The technique indicates the importance of considering the range of anticipated costs from a probabilistic analysis rather than assuming a single outcome from a deterministic approach. The analysis does show that the two analytical methods are consistent and compatible.

4.6 Effect of Uncertainty in Reserves

The actual volume of reserves in a particular field is probably the largest parameter with the largest range of uncertainty. The volume of stock tank oil initially in place in a field, STOIIIP, can only be estimated from the reservoir physical dimensions and properties. The actual recoverable reserves are dependent on the STOIIIP and on the means of recovery that will determine the
4.0 Investigation of Model Sensitivity

recovery factor. The value of the recoverable reserves will only be known at the time of abandonment of the field.

Throughout this work, only recoverable reserves have been considered. This eliminates one variable from the study. Recovery factors are primarily affected by reservoir engineering considerations and as such lie out of the scope of this study.

Goel and Grossmann (2004), and Goel et al (2006) developed a branch and bound algorithm for optimising the development of gas fields assuming uncertainties in size and initial deliverabilities of gas fields.

Litvak et al (2005) developed a model that history matched field production data of a Gulf of Mexico oil field. This allowed the production of uncertainty forecasts for various production scenarios. Litvak et al (2007) developed a complex reservoir and surface modelling method using public and proprietary techniques. The model is claimed to lead to significant improvements in oil recovery and sweep efficiency. The main emphasis of the model is the optimisation of reservoir performance under uncertainty.

The effects of uncertainty in reservoir properties which lead to uncertainties in recoverable reserves were investigated by Hayashi et al (2007).

The objective of this part of the study was to investigate the effect of uncertainty in the reserves estimate, and not to investigate the cause of the uncertainty, which is a reservoir engineering problem.

The Single Field Model used fixed CAPEX values and was based on fixed reserves. To investigate the effect of reservoir uncertainty, the Single Field Model was modified in a similar way to that described in Section 4.5 to take account of this uncertainty by performing a Monte Carlo simulation on the recoverable reserves.
4.0 Investigation of Model Sensitivity

4.6.1 The Problem

It is normal for Reservoir Engineers to quote three reserves figures. These are the $P_{10}$, $P_{50}$ and $P_{90}$ reserves corresponding respectively to the 10, 50 and 90% probability values.

The project economics must therefore be assessed over a range of probable reserves.

4.6.2 The Challenges

The challenge in this section of the study was to define a continuous function to represent the probability of a particular reserves value occurring and retaining the original $P_{10}$, $P_{50}$ and $P_{90}$ reserves values within the distribution.

The $P_{10}$ and $P_{90}$ reserve figures need not be, and frequently are not, symmetrical about the $P_{50}$ value. The distribution of reserves cannot therefore normally be represented by a symmetrical distribution function such as the Gaussian (normal) distribution. The Gaussian distribution is symmetrical about the mean value.

A triangular distribution can be used to represent the asymmetrical nature of the distribution, but are not easily be set to represent the $P_{10}$ and $P_{90}$ values and retain the $P_{50}$. The triangular distribution will also result in definite maximum and minimum values just either side of the $P_{10}$ and $P_{90}$ values.

4.6.2.1 Input Data

The model will use the output data from the Single Field Sequential model as its base input data. The results from the Single Field Sequential model are the unrisked NPVs for different design capacities.
4.0 Investigation of Model Sensitivity

4.6.2.2 Model Objective
The objective of building this model was to investigate the sensitivity of the project NPV to variations in the actual volume of recoverable reserves in the field. Since there is always an uncertainty in the actual volume of oil that can be produced from a field, there is a risk to the revenue and hence NPV of the project. This model provided a method of investigating the effect of different reserves based on their probability distribution.

4.6.3 Assumptions
In order to produce a simple function to represent the risked reserve values, two quadratic polynomials were used. Using this method, it is logical to directly represent the inverse cumulative distribution function so that risked reserves can be calculated directly corresponding to the probability from the output of the random number generator.

In order to represent this solution, the two end points are also specified to define the shape of the curve at the end points. On the reasonable assumption that a field will pass through the exploration and appraisal drilling phases, it will have been proven that a minimum volume of oil is present. For the purpose of this model, it is assumed that a minimum of 40% of the $P_{90}$ reserves is present. This corresponds to a $P_{100}$ value.

In a similar manner, it is assumed unlikely that the reserves of a field will be more than 1.25 times the $P_{10}$ reserves. This corresponds to a $P_0$ value. These multipliers can be changed at any time in the model since actual $P_0$ and $P_{100}$ values are input. These two additional reserve figures give a total of 5 points over the inverse cumulative distribution curve. By breaking the curve at the $P_{50}$ point, two quadratic equations can be defined to exactly pass through the three values in
4.0 Investigation of Model Sensitivity

each section of the curve. Specifying the $P_0$ and $P_{100}$ values ensures that errors are not introduced by extrapolation beyond the $P_{10}$ and $P_{90}$ values. Specifying 5 points over the full length of the curve ensures that the general shape of the curve is controlled.

As an example, assume the data shown in Table 4.6 for a particular field.

Table 4.6 - Example Reserve Cumulative Probabilities

<table>
<thead>
<tr>
<th>Estimate</th>
<th>Cumulative Probability</th>
<th>Reserves, MM bbl</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_0$</td>
<td>0.0</td>
<td>1,062.5</td>
</tr>
<tr>
<td>$P_{10}$</td>
<td>0.1</td>
<td>850</td>
</tr>
<tr>
<td>$P_{50}$</td>
<td>0.5</td>
<td>500</td>
</tr>
<tr>
<td>$P_{90}$</td>
<td>0.9</td>
<td>250</td>
</tr>
<tr>
<td>$P_{100}$</td>
<td>1.0</td>
<td>100</td>
</tr>
</tbody>
</table>

The data in Table 4.6 is plotted in Figure 4.14. The first section of the curve is the section containing the $P_{50}$, $P_{10}$ and $P_0$ values. Three simultaneous equations can be written down to represent the quadratic equation of the cumulative probability going through the three values.

These equations are of the form:

$$ R_i = a_0 i + a_1 i^2 + a_2 $$ \hspace{1cm} (4.18)

Specifically, for the $P_0$, $P_{10}$ and $P_{50}$ reserves estimates:

$$ R_{50} = a_0 + 0.5 \cdot a_1 + 0.25 \cdot a_2 $$ \hspace{1cm} (4.19)

$$ R_{10} = a_0 + 0.1 \cdot a_1 + 0.01 \cdot a_2 $$ \hspace{1cm} (4.20)

$$ R_0 = a_0 $$ \hspace{1cm} (4.21)
Where:

- $P_i$: $i^{th}$ probability as a percentage;
- $R_i$: The recoverable reserves corresponding to the $i^{th}$ probability;
- $i$: The probability value, $0 \leq i \leq 1$;

![Figure 4.14 Probability Distribution](image)

The graph shows the probability of the reserves being at least a certain value. There is a probability of 1 that the reserves will be greater than 100, 0.5 that the reserves will be greater than 500, and 0.0 that the reserves will be greater than 1,062.5.

$a_n$: Coefficients in the cubic correlation equation, $n = 0, 1, 2$.

A second set of simultaneous equations can be developed in a similar manner for the $P_{50}$, $P_{90}$ and $P_{100}$ values. Solving these two sets of equations yields the equations of the two parts of the cumulative distribution function.

The solution of Equations (4.19), (4.20) and (4.21) is:
4.0 Investigation of Model Sensitivity

\[ R_i = 1062.5 - 2375 \cdot r_i^{RES} + 2500 \cdot r_i^{RES}^2 \quad [r_i^{RES} \leq 0.5] \quad (4.22) \]

The solution to the second set of simultaneous equations is:

\[ R_i = 25 + 1825 \cdot r_i^{RES} - 1750 \cdot r_i^{RES}^2 \quad [r_i^{RES} > 0.5] \quad (4.23) \]

These two equations can be used to generate risked values of the recoverable reserves from a set of random numbers in the defined distribution from random input numbers representing probability values.

4.6.4 Solution Approach

The approach to the solution of this model was to use the Single Field Sequential model described in Section 3.2 with variable reserve values. The design production rate and production profile over field life would remain constant for the different reserve values. The program would then limit the cumulative production to be equal to the reserves. In this manner the production in each case would be the same, irrespective of the value of the reserves, until the specified reserves had been depleted.

This structure varied only one parameter, the reserves, and therefore enables direct comparison of each simulation. The calculation needs to be performed many times to generate sufficient data so that the calculated NPVs can be divided into a number of equal ranges and the distribution of the values determined.

4.6.5 Problem Nomenclature

This problem uses the same nomenclature as described in Section 4.5.5, with the following exceptions:

\[ R_i \quad \text{Risked Reserves of Monte Carlo simulation } i \; ; \]
4.0 Investigation of Model Sensitivity

The random number for the reserves for Monte Carlo simulation \( i \);

4.6.6 Problem Formulation

The problem is formulated in a similar manner to the method used to investigate variation in cost parameters described in Section 4.5.6, except that the actual reserves value is varied randomly and the costs are held constant at the unrisked values.

For the example shown in Table 4.6, the two portions of the inverse CDF represented by Equations (4.22) and (4.23) can be used to generate reserves values, distributed in accordance with the distribution represented by the inverse functions from randomly distributed input probability values.

These calculated values of the reserves are then used to determine the NPV of the run in exactly the same manner as described in Section 3.2.

4.6.7 Solution Approach

The method of determining the risked reserves is:

1. Generate a random number such that \( 0.0 < r_i^{RES} < 1.0 \).

2. Dependent on the value of \( r \), use either Equations 4.22 or 4.23 to calculate \( R_i \).

3. \( R_i \) represents the value of the reserves corresponding to the value \( r_i^{RES} \) and is distributed in accordance with the CDF expressed by the two quadratic equations.

4. This value of reserves is then used to determine the NPV of the project producing the specified volume of reserves. The value of the reserves and the NPV are stored.
4.0 Investigation of Model Sensitivity

5. The calculations are repeated for the specified number of simulations.

6. When the specified number of simulation runs has been completed, the values of NPV are sorted in ascending order using the Shell sorting technique described in Appendix 6.

7. The NPV values are then sorted into 40 equal intervals between the minimum and maximum values. The number of instances in each interval is determined and a histogram is plotted to show the distribution of values.

4.6.8 Implementation

As previously stated, this model is based on the Single Field Model, described in Section 3.2. An outer loop to control the number of Monte Carlo simulations was added to the model. For each simulation, a random number was generated that in turn was used to generate the risked reserves value for that simulation. The wells required to produce the specified production rate were then determined in the usual manner using the original code instructions.

At the end of each simulation, the NPV for the case was calculated and stored in a matrix. The original random number and the risked reserves were also stored for subsequent analysis.

Each of the simulations was performed at a fixed design capacity which was specified, together with the number of simulations, at the beginning of the simulation run.

At the completion of the simulation run, the matrices of random numbers, reserves and NPVs were sorted into ascending order of reserves, divided into equal data ranges for the histograms and printed to a comma delimited file. The actual histograms are drawn using Excel plotting routines.
4.0 Investigation of Model Sensitivity

4.6.9 Results

The shape of the reserves distribution curve with a near horizontal section between about 0.4 and 0.6 and comparatively steep ends either side, see Figure 4.14, results in a distribution that has an extremely pronounced peak. A typical reserves distribution is shown in Figure 4.15. This peak is a function of the reserves values used to generate the curve and the actual shape of the curve. A small change in the $P_0$ or $P_{100}$ values can make a large change to the horizontal portion of the curve in the $P_{50}$ region, thus making a significant change the reserves distribution.

![Figure 4.15 Reserves Distribution](image)

The histogram shows the reserves distribution generated from the cumulative distribution curve shown in Figure 4.13. The very sharp peak is generated by the flat portion of the cumulative distribution curve around the $P_{50}$ value.

The distribution of the NPVs using these reserves is shown in Figure 4.16.

Figure 4.16 shows a very similar distribution pattern for the NPV as Figure 4.15 does for the reserves. The NPV at a fixed reserve value of 500 MM bbl was $3,818
4.0 Investigation of Model Sensitivity

million. This value is in the same increment as the peak NPV. The results at other
design rates were similar. This indicates that while the NPV is obviously affected
by the reserves value, the distributed reserves and distributed NPVs are consistent
and that for the purposes of further work, a fixed reserves value can be used.

The distribution of reserves shows a rapid reduction in occurrences lower or higher
than the peak. The lower reserve values directly affect the volume of oil that can
be produced from the reservoir and therefore the revenue that can be earned.
The loss of revenue directly reduces the NPV of the case.

The higher reserve values do not generate a correspondingly higher NPV since the
extra production only occurs later in field life, when their effect on NPV is reduced
by the discount factor.

Figure 4.16 NPV Distribution

![NPV Distribution 150,000 BPD Design Capacity](image)

The sharp peak in the NPV distribution is again caused by the peak in the reserves. Each value of
reserves will generate a specific NPV, so that the peak reserves frequency corresponds to the peak
NPV frequency.
4.0 Investigation of Model Sensitivity

4.7 Parameter Sensitivity

An alternative method of representing and presenting the sensitivity of different parameters is to determine the effect of equal percentage changes in the parameters. This form of analysis is frequently used in the economic analysis of projects to determine their sensitivity to changes in different parameters.

The effects of ±10% and ±20% changes on the NPV of the project were determined. Table 4.7 shows the values of the parameters used in this analysis. The model was based on a field containing 500 MM bbl reserves and produced at a peak rate of 150,000 BPD. Only one parameter was changed at a time.

The results shown in Table 4.8 can be plotted in a number of ways. Different oil companies use different formats to present economic analysis. In this particular example, with the parameters selected, the conclusions are fairly obvious. However, in more complex problems, where, for example tariffs and taxation have to be considered, the effect of changes in key parameters are more difficult to determine and need not be linear.

The resulting changes in NPV are shown Table 4.8.

Table 4.7 Parameter Changes

<table>
<thead>
<tr>
<th>Parameter</th>
<th>-20%</th>
<th>-10%</th>
<th>0%</th>
<th>+10%</th>
<th>+20%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Oil Price, $/bbl</td>
<td>16</td>
<td>18</td>
<td>20</td>
<td>22</td>
<td>24</td>
</tr>
<tr>
<td>Platform Cost, $MM</td>
<td>310.850</td>
<td>349.706</td>
<td>388.562</td>
<td>427.418</td>
<td>466.274</td>
</tr>
<tr>
<td>Pipeline Cost, $MM</td>
<td>54.168</td>
<td>60.939</td>
<td>67.710</td>
<td>74.481</td>
<td>81.252</td>
</tr>
<tr>
<td>Drilling Cost</td>
<td>-20%</td>
<td>-10%</td>
<td>0%</td>
<td>+10%</td>
<td>+20%</td>
</tr>
<tr>
<td>Operating Cost</td>
<td>3.2%</td>
<td>3.6%</td>
<td>4.0%</td>
<td>4.4%</td>
<td>4.8%</td>
</tr>
<tr>
<td>Discount Rate, %</td>
<td>8</td>
<td>9</td>
<td>10</td>
<td>11</td>
<td>12</td>
</tr>
</tbody>
</table>
4.0 Investigation of Model Sensitivity

Table 4.8 Effect of Parameter Changes on NPV, $ million

<table>
<thead>
<tr>
<th>Parameter</th>
<th>-20%</th>
<th>-10%</th>
<th>0%</th>
<th>+10%</th>
<th>+20%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Oil Price</td>
<td>2,942</td>
<td>3,380</td>
<td>3,818</td>
<td>4,255</td>
<td>4,693</td>
</tr>
<tr>
<td>Platform Cost</td>
<td>3,897</td>
<td>3,857</td>
<td>3,818</td>
<td>3,778</td>
<td>3,738</td>
</tr>
<tr>
<td>Pipeline Cost</td>
<td>3,831</td>
<td>3,824</td>
<td>3,818</td>
<td>3,811</td>
<td>3,805</td>
</tr>
<tr>
<td>Drilling Cost</td>
<td>3,837</td>
<td>3,827</td>
<td>3,818</td>
<td>3,808</td>
<td>3,798</td>
</tr>
<tr>
<td>Operating Cost</td>
<td>3,837</td>
<td>3,827</td>
<td>3,818</td>
<td>3,808</td>
<td>3,798</td>
</tr>
<tr>
<td>Discount Rate</td>
<td>4,535</td>
<td>4,160</td>
<td>3,818</td>
<td>3,504</td>
<td>3,217</td>
</tr>
</tbody>
</table>

A simple X-Y plot is shown in Figure 4.17. The percentage change in each parameter is plotted against the resultant NPV of the project.

Figure 4.17 X-Y Plot of effect of Changes in Parameters

The x-y plot shows the effect changes in different parameters has only the overall economic performance of a project. The plot helps to identify key parameters in a particular project.

The X-Y plot shows that the only parameter which has a positive effect on NPV, causing an increase in NPV as the value of the parameter increases, is the oil price. Increases in discount rate have a large, negative effect on NPV. The cost of the
facilities and wells has a relatively small impact on NPV. This helps to explain why the increase in design capacity continues to increase the NPV of the project.

An alternative method of presenting the information is the use of a “Spider Diagram”. This method is shown in Figure 4.18. This method of presentation is preferred by at least one multi-national oil company.

**Figure 4.18 Spider Diagram of the Effect of Parameter Changes**

The “Spider Diagram” shows where the effect of changes in the value of parameters. A large separation forming the rings of a web indicates a large effect, whereas a small change is indicated by the data points coinciding.

Values of the NPV are shown on radials for each parameter. The wider the separation of the data points on each radial arm, the larger the variation in NPV caused by that parameter. The greater the separation of the +20% and -20% points on the radial, the greater is the effect of the parameter value. The diagram clearly shows that changes in the oil price and discount rate have a large effect on the NPV, but that changes in the drilling cost and OPEX only have a small effect as
4.0 Investigation of Model Sensitivity

is shown by the near coincidence of the 5 points on these axes. The platform cost has a small effect shown by the slight separation of the points on this axis.

Finally, the data can be presented in the form of a "Tornado Diagram", shown in Figure 4.19. This form of presentation probably most clearly shows the parameter that causes the largest changes. In the case of this diagram, the changes must be sorted into ascending order to be plotted in the tornado format.

Figure 4.19 Tornado Diagram of the Effect of Parameter Changes

The "Tornado Diagram" emphasises the parameters that have the biggest effect on the NPV of the project. Data must be sorted in ascending order.

The data is sorted with the parameter having the lowest effect on the bottom, to the one having the greatest effect on the top. Reductions in NPV are shown on the left of the vertical axis and increases are shown on the right of the vertical axis.

In the tornado diagram, only the largest and smallest changes are shown. In this example, the +20% and -20% values are used to build the diagram.
4.0 Investigation of Model Sensitivity

It can be clearly seen that Pipeline Cost has the least effect on NPV and Oil price has the greatest effect. The name "Tornado Diagram" is derived from the conical shape of a tornado vortex.

4.8 Conclusions

The investigations into the sensitivity of the project to changes in project parameters have shown that the NPV of the project is not very sensitive to project capital cost. The biggest effects on NPV are the oil price and project discount rate. At the conceptual stage of a project, both these parameters are set by company management based on corporate policy.

For this project with the selected parameters, it has been shown that there is no optimum design size. The facilities should be built as large as possible to provide one year's production at the peak rate. This is due to the comparative insensitivity of the project to capital cost. The effect of increased revenue outweighs the effect of increased capital expenditure resulting from larger facilities.

The Single Field Model has worked successfully and has produced results that although not entirely anticipated can be logically explained.

The model has permitted investigation of sensitivities in a number of different areas. However, it has been shown that although these sensitivities obviously affect the economic viability of the project, they do not affect the comparative ranking of the different cases of design capacity.

4.8.1 Options for Extension of the Technique

Exactly the same technique can be used for gas fields as was used for oil fields. The program would, however, need some small adjustments to be able to handle gas volumes instead of oil volumes.
4.0 Investigation of Model Sensitivity

During the course of the present work, a uniform, rectangular grid distribution of well targets has been used. This type of distribution was selected as the most simple to define and to demonstrate the results. However, since the model works with well coordinates, a random or non-uniform distribution could be used provided it reasonably models the reservoir.

The model was developed for use with offshore fields, but, with certain limitations, it could be used for onshore fields. Where access to the entire area of an onshore field is possible, the model may have relatively limited application. This is because, under these circumstances, well targets will probably be reached by drilling vertical wells and then running surface flowlines between the well and the gathering facilities. This configuration would be selected since it is normally more economic to run surface flowlines than to drill highly deviated wells.

However, where factors restrict the number of wellsites, wells are then drilled from a common well pad. In this case, the model can be used for selecting the location of the well pad.

Well costs were assumed to vary linearly with length of the well bore. This is a simplification in as much as more highly deviated wells require more sophisticated drilling equipment and techniques, and are susceptible to greater risk of failure or protracted drilling times. The well cost can be better represented by a quadratic or cubic equation. Provision for describing the drilling costs by a second or third order polynomial has been included in the model.

The model uses the length of well bore to describe the well rather than the horizontal displacement of the well target from the drilling centre. The well bore length calculation is more complex. This method was selected so that comparison between wells with different True Vertical Depths could be made. If well cost data is available
for the required reservoir depth, the calculations can be simplified by considering horizontal displacement.
5.0 Two Field Model

It is common for a new offshore development to commence operations with only one producing field and one platform. This field will normally be able to supply sufficient oil to meet the platform's design capacity for several years. However, production will eventually start to decline as well productivity decreases as a result of increasing water cut and a decline in reservoir pressure.

When production starts to decline, additional production can be obtained from a satellite field and in this manner the production plateau through the main field facilities can be maintained longer. Consequently, the facilities are used to their design capacity for longer and the overall project rate of return is improved.

In the models described in this section, production from the satellite field is processed through the main platform facilities. Therefore production from the satellite field competes with that from the main field. Additionally, the satellite field is developed to fill ullage in the main field process facilities and thereby extend the overall field life. The three questions the work attempts to answer are:

- What is the optimum capacity of the main field?
- What is the optimum capacity of the satellite field?
- What is the optimum time to start production from the satellite field?

Several workers have studied the optimisation of the location of multiple platforms in multiple fields. Friar and Devine (1975) developed a model to investigate the timing and location of platforms in an offshore development. Dogru et al (1977) built a model that also investigated the optimisation of the interconnecting pipeline network.
Grimmett and Startzman (1988) describe a model and computational technique to minimise investment in an offshore development.

Hansen et al (1992) describe a method of determining the capacity and location of standard size platforms in a field and the assignment of wells to these platforms.

Barker et al (1994) describe a method of estimating the optimum order, timing installed capacity and offtake profile for a number of developments.

Bittencourt and Horne (1997) developed an algorithm integrating economic analysis, simulation and process design for reservoir development optimisation. The algorithm was used to optimise the relocation of 33 wells by optimising the cash flow.

Nygreen et al (1998) describe the model used by the Norwegian Petroleum Directorate to study the long-term planning of Norwegian petroleum production and transport.

Carvalho and Pinto (2006a) and (2006b) developed a model using bilevel decomposition techniques applied to an MILP problem. The master level assigned platforms to wells and a planning sub-problem calculated timing for fixed assignments.

5.1 Introduction

The Two Field Models have individual drilling centres located in each field to allow the distance between the fields to exceed the maximum drilling stepout distance.

The models are based on the concept of a main platform with full production and export facilities and a satellite platform with single stage separation facilities feeding liquid and gas streams to the main platform for processing. Total production is limited to the capacity of the main facilities.
5.0 Two Field Model

As in the case of the Single Field model, two models were built: the Sequential Two Field Model and the Simultaneous Two Field Model.

The Sequential Two Field Model steps through each year in sequence, optimising the well selection for the current year, before moving on to the next year. This model is described in Section 5.2.

The Simultaneous Two Field Model minimises the lifetime drilling cost by considering the discounted cost of the wells required to meet the lifetime production profile. This model is described in Section 5.3.

Each individual run is based on the satellite being available for production from the specified year. The model is used to determine the lowest cost production wells, irrespective of their location. It is therefore possible for a satellite facility to be installed in a particular year, but no wells to be drilled until subsequent years. Such a schedule would be penalised by the pre-investment and would be less economic than a schedule that has the satellite developed when it is required to meet the drilling schedule.

Production from the satellite field is limited to the maximum design capacity of the satellite.

Three different cases can exist in modelling the main and satellite fields:

- Only the main field in production before the satellite field has been commissioned;
- Both the main and satellite fields in production;
- Only the satellite field in production because recoverable reserves in the main field already produced.
5.0 Two Field Model

The option of the satellite field being in production before the main field is not possible since the processing facilities would not be available, and therefore is not viable.

The model was built to change case during the life of the facilities, reflecting the particular operating mode at the time.

5.1.1 Common Assumptions

The Recoverable Reserves normally remain constant for each individual field. The reserves define the size of the field and the limit to the Design Capacity to maintain at least one year at the plateau production rate. Recoverable Reserves is the total volume of oil produced from the field over the full field life.

The oil price of $20/bbl, discount rate of 10%, and the operating cost as 4% of installed CAPEX, remained unchanged for all cases run. However, the program has the flexibility to change these values if required.

It is assumed that all processing is on the main field. There are no facilities on the satellite platform, except a single stage separator to separate the well fluids into gas and liquid streams to avoid two phase flow problems.

It is assumed that the facilities are in production for 350 days each year and that the remaining days are accounted for by scheduled and unscheduled shutdowns.

5.2 Sequential Two Field Model

The Two Field model operates in a similar manner to the Sequential Single Field model with the added feature of including details of the satellite field and the year of first production from the satellite. A model was built which selects the combination of the cheapest wells from either the main field or the satellite field to meet the specified production. The satellite field is only included if the year is
5.0 Two Field Model

greater than or equal to the year of first production from the satellite. This
feature is set by the Two Field model including or excluding the satellite field as
appropriate.

The main platform is always commissioned in Year 5 as in previous models. The
satellite platform is commissioned at a specified time from Year 5 onwards.

5.2.1 The Problem

The problem solved by the Sequential Two Field model is to schedule the drilling of
wells in such a manner that the cost of drilling is minimised each year, and that
the specified annual production profile is achieved for each year. Once the
satellite field has been commissioned, wells may be selected from either the main
or satellite field to achieve the lowest annual drilling cost, subject to meeting the
constraints.

Expenditure and revenue is discounted based on the year that the cash movement
occurs. In this way the time value of money is accounted for, and that
expenditure, particularly for drilling, is only incurred at the most economic time.
Revenue is generated in proportion to the production profile using a fixed oil price.

5.2.1.1 Problem Description

In order to fully describe the case being modelled, several parameters are
required. The recoverable reserves in each field are required to define the total
production from the fields. The design capacity of the main field production
facilities defines the peak production capacity from the fields and facilities. The
design capacity of the satellite field limits the production rate from this field. The
earliest year that the satellite field can be put into production is also specified.

The cost of drilling each well remains fixed for the specified drilling centre. The
productivity of each well is specified in the input data.
5.0 Two Field Model

The CAPEX of the main and satellite jacket and topsides; the oil and gas export pipelines; and the intrafield flowlines between the satellite and production fields are all capacity dependent and are calculated from cost equations. These equations are described in Appendix 3.

5.2.1.2 Outputs

The output is a discounted cash flow forecast based on the specified peak production rates and satellite first production date. The NPV of the project is reported for the specified oil price. The results are reported separately for both the main and satellite fields, and then a consolidated report for the combined fields is provided.

5.2.2 Challenges

The problem of modelling two fields with a variable start date for one of the fields significantly increased the challenge of keeping the model relatively simple and linear. A secondary challenge was to keep the model as flexible as possible.

5.2.3 Assumptions

Drilling costs are assumed to be incurred the year before the well is required in production.

The satellite production can start in any year from Year 5. The latest year that the satellite can commence production is the final year of the main field production, since there is no logic in shutting the main field facilities down and then restarting them later to process the satellite production.

The same method of estimating the costs of the jacket, topsides, and oil and gas pipelines was used for both the main and satellite fields, as had been used for the single field models.
5.0 Two Field Model

5.2.4 Approach

Net Present Value, NPV, was again selected as a measure of economic efficiency of the different production configurations to be studied.

5.2.5 Problem Implementation

The original format of the model was revised during this work to reduce computational effort and to structure input better. The original drilling centre optimisation model took the GAMS include file and wrote a modified include file for the Two Field model. The new drilling centre model now produces a new format include file containing well name, well productivity and well cost for the ODC. This structure avoids repetitive well geometry and well cost calculations being performed in the Two Field model and significantly speeds computation.

5.2.6 Problem Nomenclature

The model uses the following sets:

- \( i \) The set of wells in the main field;
- \( j \) The set of wells in the satellite field;
- \( t \) The set of years of the overall project.

Scalar input parameters are:

**Main Field**

- \( F_{i}^{MPD} \) Production decline factor for Year \( t \);
- \( p_{i}^{M} \) Initial production from Well \( i \);
- \( w_{i}^{M} \) The cost of drilling Well \( i \).

**Satellite Field**
5.0 Two Field Model

\( F^{SPD}_t \)  Production decline factor for Year \( t \);

\( p^S_j \)  Initial production from Well \( j \);

\( P^S \)  Satellite design capacity;

\( w^S_j \)  The cost of drilling Well \( j \).

**Common**

\( F^O \)  Discount rate, taken as 10% per year;

\( F^{OPEX} \)  OPEX as a fraction of CAPEX;

\( N \)  Year of first production. In this study \( N = 5 \);

\( P^O \)  Oil price (taken as $20/bbl for economic analysis);

\( T_t \)  Annual production target for Year \( t \).

Binary variables calculated by the model during simulation are:

**Main Field**

\( y^M_i \)  Binary to indicate that Well \( i \) is selected for production in the current year and must be drilled in the previous year;

\( z^M_i \)  Binary to indicate that Well \( i \) was in production in the previous year.

**Satellite Field**

\( y^S_j \)  Binary to indicate that Well \( j \) is selected for production in the current year and must be drilled in the previous year;

\( z^S_j \)  Binary to indicate that Well \( j \) was in production in the previous year.

Runtime variables are:

**Main Field**

\( c^{MD}_t \)  Main field DRILLEX in Year \( t \);
5.0 Two Field Model

\( c_t^{MP} \)  CAPEX of main platform in Year \( t \).

**Satellite Field**

\( c_t^{SD} \)  Satellite field DRILLEX in Year \( t \);

\( c_t^{SP} \)  CAPEX of satellite platform in Year \( t \).

**Common**

*Cost*  The cost of drilling both the main and satellite wells in the current year;

\( c_t^{CAPEX} \)  Total CAPEX of both fields in Year \( t \);

\( c_t^{CUML} \)  Cumulative CAPEX up to and including Year \( t \) for both fields;

\( c_t^{OPEX} \)  Operating cost for both fields in Year \( t \);

\( c_t^{P} \)  CAPEX of pipelines in Year \( t \);

\( m_t^{ANN} \)  Annual cash flow from both fields in Year \( t \);

*NPV*  The Net Present Value of the project;

\( p_t \)  Total production from both fields in Year \( t \);

\( r_t^{GROSS} \)  Gross revenue from production from both fields;

\( r_t^{NET} \)  Net cash flow from both fields in Year \( t \).

5.2.7  Problem Formulation

Different objective functions and limits are required for the single and multiple field cases.

5.2.6.1  Main Field Only Case

For the main field only case, the equations are the same as have been used previously:
5.0 Two Field Model

Minimise:

\[ Cost = \sum_i \left( y_i^M \cdot w_i^M \right) \]  \hfill (5.1)

Subject to:

\[ \sum_i \left( \left( y_i^M \cdot p_i^M + z_i^M \cdot p_i^M \right) \cdot F_i^{MPD} \right) = T_i \]  \hfill (5.2)

Equation (5.1) calculates the drilling cost in Year \( t \) for the main field only.

Constraint (5.2) is required to ensure production meets the target for Year \( t \). A small change from the Single Field model was that the Two Field model production was not permitted to exceed the Production Target.

5.2.6.2 Main and Satellite Fields Case

For two fields, these equations are extended to:

Minimise:

\[ Cost = \sum_i \left( y_i^M \cdot w_i^M \right) + \sum_j \left( y_j^S \cdot w_j^S \right) \]  \hfill (5.3)

Subject to:

\[ \sum_i \left( \left( y_i^M \cdot p_i^M + z_i^M \cdot p_i^M \right) \cdot F_i^{MPD} \right) + \sum_j \left( \left( y_j^S \cdot p_j^S + z_j^S \cdot p_j^S \right) \cdot F_j^{SPD} \right) = T_i \]  \hfill (5.4)

and:

\[ P^S \geq \sum_j \left( \left( y_j^S \cdot p_j^S + z_j^S \cdot p_j^S \right) \cdot F_j^{SPD} \right) \]  \hfill (5.5)

Equation (5.3) calculates the drilling cost in Year \( t \) for both the main and satellite fields.
5.0 Two Field Model

Constraint (5.4) ensures that the combined production from both fields equals the production target for Year $i$.

Constraint (5.5) is required to limit production from the satellite field to within the design capacity of the satellite facilities.

5.2.6.3 Satellite Field Only Case

For the case where all the main field reserves have been produced and only the satellite field is in production, these equations are modified to:

Minimise:

$$\text{Cost} = \sum_j \left( y^S_j \cdot w^S_j \right)$$

(5.6)

Subject to:

$$\sum_j \left( (y^S_j \cdot p^S_j + z^S_j \cdot p^S_j \cdot F_i^{\text{SPD}}) \right) = T_i$$

(5.7)

and:

$$P^S \geq \sum_j \left( (y^S_j \cdot p^S_j + z^S_j \cdot p^S_j \cdot F_i^{\text{SPD}}) \right)$$

(5.8)

Equation (5.6) calculates the drilling cost in Year $i$ for the satellite field only.

Constraint (5.7) ensures that the satellite production meets the production target for Year $i$.

Constraint (5.8) is required to limit production from the satellite field to within the design capacity of the satellite facilities and is the same constraint as (5.5).
5.0 Two Field Model

5.2.6.4 CAPEX Profile

The CAPEX phasing is exactly the same as described for the Single Field Model, except that the timing of the Satellite CAPEX is variable, dependent on when the satellite field commences production. For both fields the phasing is shown in Table 5.1 relative to the date of first production for that field.

<table>
<thead>
<tr>
<th>Year</th>
<th>Jacket and Topsides</th>
<th>Pipelines</th>
</tr>
</thead>
<tbody>
<tr>
<td>First Production - 4</td>
<td>15%</td>
<td>0%</td>
</tr>
<tr>
<td>First Production - 3</td>
<td>25%</td>
<td>0%</td>
</tr>
<tr>
<td>First Production - 2</td>
<td>30%</td>
<td>50%</td>
</tr>
<tr>
<td>First Production - 1</td>
<td>30%</td>
<td>50%</td>
</tr>
<tr>
<td>First Production</td>
<td>0%</td>
<td>0%</td>
</tr>
</tbody>
</table>

In the case of the main field, first production commences in Year $N$. In this work, $N$ has been set at 5, however, this can be varied, if required.

DRILLEX is incurred in the year before a well is required to be in production. DRILLEX for the main field commences in Year $N-1$. DRILLEX for the satellite field commences in the year before first production from the satellite field.

The total CAPEX for both fields in Year $t$ is:

$$C_t^{CAPEX} = C_t^{MP} + C_t^{SP} + C_t^{P} + C_t^{MD} + C_t^{SD}$$  \hfill (5.9)

Equation (5.9) includes the cost of drilling wells in both the main and satellite field.

The Cumulative CAPEX up to Year $t$ is given by:

$$C_t^{CUM} = \sum_t C_t^{CAPEX}$$  \hfill (5.10)
5.0 Two Field Model

5.2.6.5 Production Profile

The production profile in the Two Field model was modified so that the production built up to the peak production over a period of 4 years. The production target during the build up phase is shown in Table 5.2. This profile is independent of the first year of production from the satellite field.

<table>
<thead>
<tr>
<th>Year</th>
<th>Percentage of Design Capacity</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>25%</td>
</tr>
<tr>
<td>6</td>
<td>50%</td>
</tr>
<tr>
<td>7</td>
<td>75%</td>
</tr>
<tr>
<td>8</td>
<td>100%</td>
</tr>
</tbody>
</table>

Production is then set at the peak rate for subsequent years until 85% of the reserves have been produced. The year is rounded down so that the production to the end of plateau does not exceed 85%. The remaining reserves after the last peak rate year are then distributed over 7 years of decline in the ratio of \((8 - D)/28\), where \(D\) is the decline year, \(1 \leq D \leq 8\). This distribution ensures that the target production exactly equals the reserves.

5.2.6.6 Production Period

Production commences in Year 5 from the main field. Production commences from the satellite field in the year specified in the input data. Production is from whichever fields are available. Sufficient wells are drilled to meet the target production profile in all fields that are available for production. Production continues each year until all the reserves of the field have been produced. Total production is limited to exactly the recoverable reserves from each field.
5.0 Two Field Model

The Production Decline factor in Year \( t \) is based on the cumulative production from each field up to and including Year \( t-1 \). Separate decline factors are used for each field calculated from the reserves and cumulative production from that field.

The Production Decline factor is described in Section 3.2.3.

5.2.6.7 Annual Gross Revenue

The annual gross revenue generated is the product of the oil production in the current year from all operational fields, and the oil price that is provided as an input parameter.

The production in Year \( t \) is:

\[
p_t = \sum_i \left( Y_i^M \cdot P_i^M + Z_i^M \cdot P_i^M \cdot F_M^{MPD} \right) + \sum_j \left( Y_j^S \cdot P_j^S + Z_j^S \cdot P_j^S \cdot F_j^{SPD} \right)
\]

(5.11)

If one of the fields has ceased production, the value of the Production Decline factor, \( F_i^{MPD} \) or \( F_i^{SPD} \), is set to zero to exclude any further contribution to the total production.

Therefore the Gross Revenue in Year \( t \) is:

\[
r_t^{\text{GROSS}} = p_t \cdot P^O
\]

(5.12)

5.2.6.8 Annual Operating Cost

The annual operating cost, OPEX, is taken as a fixed percentage of the cumulative capital expenditure to date. For this study, it was taken that the operating cost was 4% of total CAPEX each year. Therefore the Annual Operating Cost is given by:

\[
c_t^{\text{OPEX}} = F^{\text{OPEX}} \cdot C_t^{\text{CUMUL}}
\]

(5.13)

Since the cumulative CAPEX value includes the facilities and drilling CAPEX for both fields, the operating cost is for both fields.
5.0 Two Field Model

5.2.6.9 Annual Net Revenue

Net revenue is defined as the Gross Revenue less the Annual Operating Cost:

\[ r_t^{\text{NET}} = r_t^{\text{GROSS}} - c_t^{\text{OPEX}} \]  

(5.14)

5.2.6.10 Cash Flow and NPV

The Annual Cash Flow is the Net Revenue less the Total CAPEX expenditure in Year \( t \). The Annual Cash Flow in Year \( t \), \( m_t^{\text{ANN}} \), is:

\[ m_t^{\text{ANN}} = r_t^{\text{NET}} - c_t^{\text{CAPEX}} \]  

(5.15)

The Net Present Value, NPV, of the project is the sum of the discounted Annual Cash Flows over the life of the project:

\[ NPV = \sum_t \left( m_t^{\text{ANN}} \cdot \frac{m_t^{\text{ANN}}}{e^{P(t-1)}} \right) \]  

(5.16)

NPV is the measure used in this work to measure the profitability of the project.

5.2.8 Design Approach

The model was developed using two hypothetical fields in the North Sea, however, the model can be applied to any offshore location. The objective of the study was to determine the optimum timing for commissioning production from satellite field in relation to the commissioning of the main field. Optimum timing was defined as that which gives the highest combined NPV for the development.

The investigation of the optimum design capacities and satellite field first production for a particular pair of fields followed the plan:

1. Cost equations were used to determine the costs of drilling the wells, constructing the platform and topsides, and installing the pipelines. These allow the project CAPEX estimates to be built for different design capacities. The
5.0 Two Field Model

equations are a function of design capacity and apply to both the main and satellite fields. These equations are described in Appendix 3.

2. The field grid was developed to provide a realistic, but hypothetical representation of a reservoir.

3. A drilling centre was selected using the method described in Section 2 for both the main and satellite fields.

4. A target production profile was developed based on the specified production rate and the reserves in both fields.

5. Sufficient wells were selected to most economically produce the target production for the current year. This is an MILP problem that will be solved using the CPLEX solver in the GAMS platform. Production from the satellite field is possible once the year of first production for the satellite has been reached.

6. Step 5 is repeated for each year until all the reserves from both fields have been produced.

7. An economic analysis is then be performed on the results to determine the project NPV.

8. The entire process can then be repeated for different design production rates for each field and different years when satellite production commences.

The results obtained can then be analysed to identify the most economic production rates and satellite timing, and to suggest rules that would enable this rate to be identified explicitly.

5.2.9 Remarks and Implementation

The method of implementation of the model was similar to that used to develop the Single Field Sequential model. A time increment of one year was selected.
5.0 Two Field Model

The total reserves of the two fields must be produced before the production facility at the main field is abandoned. At the same time, the reserves of an individual field cannot be exceeded. Thus if the main field is depleted first, production from this field will cease while the remaining reserves from the satellite field continue to be produced.

A similar hierarchical structure of a C++ executive building input files containing the MILP and using the GAMS solver to find the solution to the problem was used.

In the year that a field reaches its total reserves, the production for the year is set to the difference between the cumulative production up to the end of the previous year and the reserves. This ensures that all the reserves are produced and that total production does not exceed the reserves. For subsequent years, the value of the Production Decline Factor, $F_{t, MPD}$ or $F_{t, MPD}$ is set to zero to ensure no further production from that field. Production continues until the reserves from both fields have been produced.

At high production rates from the satellite field, particular when the satellite production does not occur until the main field is approaching decline, the satellite production requirement can be relatively very high to meet the production target. This high production requirement can produce an impractical demand for new satellite wells. In one case, 127 wells were required to meet a small production volume in the penultimate year before the field was depleted. Obviously, such a drilling programme is impractical and grossly uneconomic. For that reason, an additional feature was included in the program input file to restrict the number of satellite wells that can be drilled each year. The number is set as input data and can be varied. A maximum of 8 wells drilled in one year was specified during this work. If a rig takes 1 ½ months to drill and complete each well, then the maximum number of wells it can drill in a year is eight.
5.0 Two Field Model

First Oil from the main field is fixed to be in Year 5 to allow four years before this date for design and construction of the platform and pipelines. The order of execution of the model is as follows:

1. The model reads the input data of reserves, peak production, capital cost algorithms and well locations for both fields.

2. Using this data, the runtime parameters are calculated, including capital cost and production profiles.

3. The input file for the next year is written using data from the previous year as appropriate and the production target for the current year. The model includes the data for the satellite if the satellite field is available for production.

4. The solver determines if any new wells are required to be drilled to meet the current year's target. If new wells are required, it determines which combination of new wells is the lowest cost to meet the required production. GAMS then writes this information to the output file.

5. The executive program reads the output file to obtain the well data, which it stores for future calculation.

6. The remaining reserves are calculated by deducting the current year's production. If the remaining reserves are positive, the procedure is repeated from step 3 by incrementing the year count. If the reserves are negative, the production in the year is reduced to give zero reserves at the end of the year. Steps 3, 4 and 5 are repeated until the reserves from both fields have been produced.

7. The program then calculates the cashflow for each year for the main field and satellite field separately, and then as combined fields. The discounted cash
flow is then calculated at the specified discount rate. The sum of the discounted cashflows for each year is the NPV.

8. The production and financial data is written to an output file and the calculation stops.

9. The financial results can then be compared with other cases and the case with the highest NPV can be identified.

5.2.10 Results

The study of multiple drilling centres was based on the two hypothetical oilfields: West and East that have been used in previous optimisation studies.

Simulations were made over a wide range of both main and satellite field design capacities with different start dates for the satellite production.

Main field design capacity ranged from 100,000 BPD to 350,000 BPD. The maximum design limit of 350,000 BPD was selected on the basis that the combined reserves would be produced within 6.5 years at an annual depletion rate of 15%. This was considered to above the maximum rate that a country’s ministry of energy would normally permit a field of this size to be developed. It is also a rate at which significant reservoir damage could occur that would lead to a lower recovery that expected. Finally, approximately 30 wells need to be drilled before the first year of production and 40 before the second year of production. These numbers of wells significantly exceed the number that could practically be drilled in an offshore field within these time constraints.

Consequently, these limits cover the full range of practical design capacity for this particular development.
Satellite field capacity also ranged from 100,000 to 350,000 BPD. For each design capacity for the main and satellite fields, the satellite field became operational in a range of years between Years 5 and 20. A limit of 50 years was imposed on the overall project.

For main field production at higher rates, all the reserves can be produced before the satellite field is operational. The facility could remain idle for a year or more before satellite production commences. This is obviously impractical and uneconomic. In these cases, late start up of the satellite has not been considered. Table 5.3 shows the year in which all the main field reserves would have been produced with no satellite production. These dates are the limits for satellite year start up.

Table 5.3 Latest Year for Satellite Start Up

<table>
<thead>
<tr>
<th>Main Field Design Capacity, BPD</th>
<th>Year of Latest Satellite Start Up</th>
</tr>
</thead>
<tbody>
<tr>
<td>100,000</td>
<td>20</td>
</tr>
<tr>
<td>150,000</td>
<td>15</td>
</tr>
<tr>
<td>200,000</td>
<td>13</td>
</tr>
<tr>
<td>250,000</td>
<td>12</td>
</tr>
<tr>
<td>300,000</td>
<td>11</td>
</tr>
</tbody>
</table>

Simulations were performed at each of the main field design capacities with the satellite field starting up in each of the possible years. For each simulation the NPV was calculated. This enabled the optimum year for the satellite field to come into production to be determined. The results of these simulations are shown for each main field design capacity in the following sections. Each design capacity is shown as the different cases show a number of interesting characteristics which enable a better understanding of the dynamics of the cases.
5.0 Two Field Model

5.2.10.1 Optimum Drilling Centres

The first stage in investigating the optimisation of a field development consisting of multiple drilling centres was to identify the optimum drilling centre in each individual field. This was achieved using the drilling centre optimisation method described in Section 2. The locations for the two fields are shown in Table 5.4.

Table 5.4 Optimum Drilling Centre Locations

<table>
<thead>
<tr>
<th>Reservoir</th>
<th>Total Well Locations</th>
<th>Optimum Drilling Centre</th>
<th>Recoverable Reserves, MM bbl</th>
</tr>
</thead>
<tbody>
<tr>
<td>West</td>
<td>224</td>
<td>W095</td>
<td>500</td>
</tr>
<tr>
<td>East</td>
<td>184</td>
<td>E069</td>
<td>300</td>
</tr>
</tbody>
</table>

5.2.10.2 Independent Production from Each Field

The economic performance of each independent field was investigated. For each field, the NPV at a constant oil price of $20/bbl was investigated over a range of design capacities. The NPVs are summarised in Tables 5.5 and 5.6.

Table 5.5 West Field NPV at Different Design Capacities, $MM

<table>
<thead>
<tr>
<th>Design Capacity, M BPD</th>
<th>West Field</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>1,624</td>
</tr>
<tr>
<td>100</td>
<td>2,852</td>
</tr>
<tr>
<td>150</td>
<td>3,435</td>
</tr>
<tr>
<td>200</td>
<td>3,837</td>
</tr>
<tr>
<td>250</td>
<td>4,087</td>
</tr>
<tr>
<td>300</td>
<td>4,236</td>
</tr>
<tr>
<td>350</td>
<td>4,354</td>
</tr>
<tr>
<td>400</td>
<td>4,403</td>
</tr>
<tr>
<td>450</td>
<td>4,485</td>
</tr>
</tbody>
</table>

Table 5.6 East Field NPV at Different Design Capacities, $MM

<table>
<thead>
<tr>
<th>Design Capacity, M BPD</th>
<th>East Field</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>-2</td>
</tr>
<tr>
<td>40</td>
<td>208</td>
</tr>
<tr>
<td>60</td>
<td>280</td>
</tr>
<tr>
<td>80</td>
<td>292</td>
</tr>
<tr>
<td>100</td>
<td>300</td>
</tr>
<tr>
<td>120</td>
<td>247</td>
</tr>
<tr>
<td>140</td>
<td>274</td>
</tr>
<tr>
<td>160</td>
<td>195</td>
</tr>
</tbody>
</table>

5.2.10.3 Main Field Capacity 100,000 BPD

The variation in NPV at 100,000 BPD main field capacity is shown in Figure 5.1. At 100,000 BPD design capacity for the main facilities, the maximum NPV is achieved in Year 15 at 100,000 BPD satellite capacity. The value is $2,736 million.
At 100,000 BPD design capacity for the main facilities, the largest NPV is achieved with a satellite field also designed for 100,000 BPD. The satellite field should not be commissioned until Year 15.

If the satellite field is commissioned before Year 15, production from the satellite is delayed since production from the main field is cheaper. This is shown in Figure 5.2 for Year 10 with a satellite design capacity of 100,000 BPD. Despite the satellite being available in Year 10, there is only limited production from Year 14.

It should also be noted that when the main field comes off plateau production, the limit of 8 new satellite wells each year restricts the build up of satellite production and results in three years of production below the plateau in Years 21, 22 and 23.

Figure 5.3 shows the production profile in for a satellite production of 100,000 BPD commencing in Year 15, the maximum NPV case. In this case, the satellite field facilities are used in the same year that they are commissioned. This leads to a far more efficient use of investment, and hence higher NPV.
5.0 Two Field Model

Figure 5.2 Year 10 Production Profile, 100,000 Main Capacity, 100,000 BPD Satellite Capacity

Building the satellite field early, in Year 10, results in the facilities being unused until Year 14. This pre-investment is sub-optimal.

Figure 5.3 Year 15 Production Profile, 100,000 Main Capacity, 100,000 BPD Satellite Capacity

Commissioning the satellite facilities in Year 15 results in the satellite facilities being used immediately. Note that when the production from the main field ceases after Year 21, the restriction on the number of wells drilled in a year limits production in Year 22.
5.0 Two Field Model

Figure 5.4 shows the effect of delaying the commissioning of the satellite facilities. In this figure, satellite production is delayed until Year 18. There is insufficient time for the satellite to build production up to the plateau level. Therefore there is a significant drop in production and revenue is deferred.

By delaying the commissioning of the satellite field until Year 18, insufficient satellite wells can be built after the main field ceases production. There is a marked reduction in production in Years 21 and 22.

Figure 5.5 shows the effect of decreasing the satellite design capacity to 50,000 BPD whilst the main field facility remains 100,000 BPD. After the main field ceases production, the plateau drops to that of the satellite design value. Therefore early revenue is lost and the resultant NPV is lower than when the satellite design capacity was the same as the main field's value.

The satellite plateau rate varies around the specified maximum, because the satellite is permitted to produce at the level above the design capacity.
corresponding to the full production achieved by the number of wells required to meet the design capacity. This is similar to actual operation, where full production is taken from all wells that are operational, but new wells are not drilled until required.

Installing satellite facilities with a design capacity greater than the main field does not generate any benefit. The satellite field can never operate at its design capacity because the main field will restrict it.

Figure 5.5 Year 16 Production Profile, 100,000 Main Capacity, 50,000 BPD Satellite Capacity

If the satellite design capacity is only 50,000 BPD, when the main field ceases production, the satellite design capacity restricts production to only 50,000 BPD and revenue is deferred with a resulting decrease in NPV.

5.2.10.4 Main Field Capacity 150,000 BPD

The variation in NPV at 150,000 BPD main field capacity is shown in Figure 5.6.

At 150,000 BPD design capacity for the main facilities, the maximum NPV is achieved in Year 8 at 100,000 BPD satellite capacity. The value is $3,686 million.
5.0 Two Field Model

Figure 5.6 Main Capacity 150,000 BPD

Delaying the commissioning of the satellite field until Year 9 results in a large loss of revenue as production is restricted by the satellite field capacity.

Figure 5.7 shows the production profile for the case when the satellite facilities are available for operation in Year 6. They are not required for one year, therefore the NPV is lower than the maximum value.

Figure 5.8 shows the production profile for the maximum NPV case, when the satellite is commissioned in Year 8 with a design capacity of 100,000 BPD. Figure 5.9 shows the effect of slipping the satellite start up by one year to Year 9. The Year 9 start up requires the main field to meet the first year plateau rate, and therefore between years 9 and 16, the main field provides a far greater proportion of the production. Therefore the plateau is two years shorter than in the Year 8 case.

Year 8 shares the production more evenly and therefore results in a longer plateau and a larger discounted revenue.
Commissioning the satellite facilities two years early, in Year 6, results in them being unused for a year. This pre-investment reduces the NPV of the case.

The cases with 50,000 BPD satellite design capacity result in a longer field life due to the lower plateau rate after the main field has ceases production. In the case when the satellite starts production in Year 8, the 50,000 BPD case has a field life of 26 years, compared with only 23 years for the 100,000 BPD case. These lower rates result in lower discounted revenue. Hence the 100,000 BPD case has the greater NPV.

Figure 5.10 shows the effect of increasing the satellite design capacity to 150,000 BPD. Compared with the maximum for this set of cases: 100,000 BPD capacity from Year 8, Figure 5.8, there are only small differences in the production profiles. The field lives are the same at 23 years. The plateaus are both from Year 8 to Year 18. This indicates that more capacity is being installed with the 150,000 BPD design than can be economically used.
5.0 Two Field Model

**Figure 5.8 Year 8 Satellite Production, 150,000 Main Capacity, 100,000 BPD Satellite Capacity**

Commissioning the facilities in Year 8 results in the satellite facilities being used immediately and therefore improving the economic return of the case.

**Figure 5.9 Year 9 Satellite Production, 150,000 Main Capacity, 100,000 BPD Satellite Capacity**

Delaying commissioning the satellite facilities until Year 9 results in a significant deferral of production when the main field ceases production. This deferral significantly reduces the NPV of the case.
5.0 Two Field Model

Figure 5.10 Year 8 Production Profile, 150,000 Main Capacity, 150,000 BPD Satellite Capacity

The satellite design capacity of 150,000 BPD is never achieved. Therefore excess capacity has been built and the case is sub-optimal.

5.2.10.5 Main Field Capacity 200,000 BPD

The variation in NPV at 200,000 BPD main field capacity is shown in Figure 5.11. The maximum NPV is achieved in Year 7 at 100,000 BPD satellite capacity. The value is $4,381 million. The value at the same satellite design capacity and first satellite production in Year 8 is almost identical at $4,380 million. The two production profiles are shown in Figures 5.12 and 5.13.

The Year 7 profile achieves an extra year of plateau production at the expense of accelerating the satellite construction costs by one year. These two effects cancel each other out. Consequently there is little to choose between these two cases.

If the satellite production is delayed by one year, to Year 9, effectively one further year at plateau production is lost. This effect explains the sudden drop in NPV in Year 9 shown in Figure 5.11. The production profile for Year 9 is shown in Figure 5.14 for comparison.
The highest NPV is achieved commissioning the satellite facilities in Year 7 or 8. The two NPVs are almost identical.

Figure 5.12 Year 7 Satellite Production, 200,000 Main Capacity, 100,000 BPD Satellite Capacity

Commissioning the satellite field in Year 7 increases the discounted CAPEX but also accelerates the revenue by producing the total reserves in 18 years compared with the 19 years when the satellite is commissioned in Year 8.
5.0 Two Field Model

Figure 5.13 Year 8 Satellite Production, 200,000 Main Capacity, 100,000 BPD Satellite Capacity

Commissioning the satellite facilities in Year 8 instead of year 7 results in a deferral of expenditure by a year, but also extends the production by one year. The two effects essentially cancel each other out.

Figure 5.14 Year 9 Satellite Production, 200,000 Main Capacity, 100,000 BPD Satellite Capacity

By deferring commissioning the satellite facilities until Year 9, the profile is extended by one more year, production is limited to the satellite design capacity one year earlier.
The variation in NPV at 250,000 BPD main field capacity is shown in Figure 5.15.

At 250,000 BPD design capacity, the highest NPV occurs with a satellite commissioned in Year 7 with a capacity of 100,000 BPD.

At 250,000 BPD design capacity, the maximum NPV is achieved in Year 7 at 100,000 BPD design capacity. The NPV at these conditions is $4,800 million. The NPV in Year 6 is $4,781 million and in Year 9 is $4,763 million. The production profile for Year 8 is shown in Figure 5.16.

This profile is particularly interesting in that in the first three years of production, the main field capacity ramps up by 25% each year, but that in Year 8 when the satellite becomes operational, only 8 satellite wells are drilled. The GAMS program is calling for more satellite wells rather than main field wells, but the executive program is limiting the maximum number of satellite wells. As a result, the combined production does not reach the plateau rate of 250,000 BPD until Year 10,
two years later than planned. This appears to be the effect of the very fast build up in production to reach the high plateau rates.

5.2.10.7 Main Field Capacity 300,000 BPD

The variation in NPV at 300,000 BPD main field capacity is shown in Figure 5.17. The largest NPV for this design capacity is $5,128 million which occurs at 150,000 BPD satellite design capacity with the satellite facilities commissioned in Year 6.

The production profile for this case is shown in Figure 5.18. It should be noted that although the case specified a satellite design flowrate of 150,000 BPD, the limit on the number of wells that can be drilled in any one year limited the actual maximum flowrate through the satellite facilities to 126.463 BPD. Thus the 100,000 BPD and 150,000 BPD cases are both similar. The NPV of the 100,000 BPD satellite design capacity and year of commissioning is Year 6 is $5,108 million.
5.0 Two Field Model

Figure 5.17 Main Capacity 300,000 BPD

The highest NPV occurs in Year 6 at 150,000 BPD. The NPV rapidly declines if the satellite commissioning is delayed beyond this date.

Figure 5.18 Year 6 Satellite Production, 300,000 BPD Main Capacity, 150,000 BPD Satellite Capacity

Production from the satellite field is required early in field life to meet the rapid ramp up in production. The maximum production from the satellite field does not use all the available design capacity.
5.0 Two Field Model

5.2.11 Analysis of Simulations

The results of the simulations are summarised in Table 5.7 where, for each main field design capacity, the maximum NPV, and the corresponding first year of satellite production and satellite production rate.

Table 5.7 Simulation Optimum Configurations

<table>
<thead>
<tr>
<th>Main Field Design Capacity, BPD</th>
<th>Satellite Field Design Capacity, BPD</th>
<th>Year Satellite Commissioned</th>
<th>Project NPV, $MM</th>
</tr>
</thead>
<tbody>
<tr>
<td>100,000</td>
<td>100,000</td>
<td>15</td>
<td>2,736</td>
</tr>
<tr>
<td>150,000</td>
<td>100,000</td>
<td>8</td>
<td>3,686</td>
</tr>
<tr>
<td>200,000</td>
<td>100,000</td>
<td>7</td>
<td>4,381</td>
</tr>
<tr>
<td>250,000</td>
<td>100,000</td>
<td>7</td>
<td>4,800</td>
</tr>
<tr>
<td>300,000</td>
<td>150,000</td>
<td>6</td>
<td>5,128</td>
</tr>
</tbody>
</table>

5.2.10.1 Main Field Design Capacity

Figure 5.19 shows the result of plotting the highest NPV for each main field design capacity and indicates that, for this field simulation, a design capacity of 300,000 BPD yields the largest NPV. A design capacity of 300,000 BPD corresponds to a depletion rate of 21.0% from the main field. A 10% depletion rate is frequently considered by reservoir engineers and ministries of energy as being the maximum that can be achieved without causing reservoir damage or loss of recoverable reserves. 10% depletion corresponds to a design capacity of 143,000 BPD.

5.2.10.2 Optimum Year for Satellite Commissioning

The optimum year for the satellite field to be commissioned, corresponding to the maximum NPV for each design capacity is shown in Figure 5.20. This indicates that the optimum is a function of the design capacity with the value declining as the design capacity increases.
For development of the main field alone, the maximum NPV is achieved by building facilities for the maximum capacity.

Except for the lowest main field design capacity, the satellite facilities are required early in main field life, decreasing from Year 8 to Year 6 at the highest capacity.
This conclusion is intuitive. At low design rates, production from the satellite is not required and production can be concentrated on the more productive wells in the main field. As design capacity increases, more wells are required and it becomes economic to develop the satellite field earlier.

Additionally, since production builds up to design capacity over a period of four years, from Year 5, it indicates that as design capacity increases, there is benefit in accelerating the satellite commissioning at higher production rates to be able to commission high production satellite wells early.

5.2.10.3 Optimum Satellite Design Capacity

The optimum design capacity of the satellite field is shown in Figure 5.21. This graph indicates that the design capacity of the satellite facility should be of the order of 100,000 to 150,000 BPD.

Only at the maximum main field design capacity is the optimum satellite design capacity higher than 100,000 BPD.

5.2.12 Conclusions

The conclusions drawn from this analysis are only valid for this particular case and configuration since the field reserves, the productivity of wells, and the cost correlations will all affect the inter-relationship of the different parameters.

Within the constraint of not exceeding the limit of producing more than 10% of a field's reserves in any one year, the maximum production from the main field is limited to 143,000 BPD and the satellite field to 86,000 BPD. Rounding these two values to the nearest 50,000 BPD to match the steps used in the simulations: the main field production should not exceed 150,000 BPD and the satellite field production should not exceed 100,000 BPD.
The optimum size for the satellite design capacity is 100,000 BPD, except when the main facilities have a design capacity of 300,000 BPD when it is 150,000 BPD.

The value of 100,000 BPD for the optimum satellite field production equals the optimum found by analysis. The value of 150,000 BPD for the main field design capacity gives an optimum NPV of 72% of that theoretically achieved by producing at the maximum rate of 300,000 BPD. However, the lower rate reduces the risk of reservoir damage and therefore is the preferred option.

The satellite timing indicates that ideally the satellite field should be commissioned as soon as the main field reaches plateau. Such timing will provide the operator with maximum flexibility of selecting to produce from the main or satellite fields, and also commissions two drilling centres to allow simultaneous operation of two rigs.
5.3 Simultaneous Two Field Model

The objective of the Simultaneous Two Field model is to optimise the drilling of wells in two fields over the life of the field. The Sequential Two Field Model only optimised the drilling for the current year, and sequential stepped through the life of the field. As in the case of the Sequential Two Field model, the Simultaneous model optimises the development of a main field with a satellite field that will be produced over the main field platform using the main field production facilities.

5.3.1 The Problem

The problem needed to be structured so that a series of equations were built up describing the production target and drilling programme for each year, discounting drilling cost to enable the whole of field life to be optimised in a single simulation. This also required logic to be built in to ensure that production rates and field reserves were not exceeded.

Well data was provided for all potential well locations and the location of the ODC, as in the Sequential Two Field model. Additionally, field reserves, the maximum production from the satellite field, production profile, Production Decline factor and the years that the satellite field was in production were specified.

The output consisted of an optimised, minimum cost drilling schedule for the life of both fields and the cost of drilling the individual wells.

The main challenge in building the logic was to ensure that once a well was put into production, it was used in subsequent years in preference to drilling a new well, and that the year the well was drilled could be identified.

It is assumed that the production profile will be similar to the previous profiles, building up to peak production in four years. Dependent on when production starts
from the satellite field, the production can be from either the main or satellite field provided the production profile is met.

In the Simultaneous Two Field model, production is assumed to commence from the Main field in Year 1.

The Production Decline factor had to be specified in the input data and was common for both fields, being calculated on total production. This was necessary because otherwise the factor would have been an exogenous variable, and the problem could not be solved using the MILP solver.

In order to simplify the modelling, and to avoid additional calculation outside the optimisation of the drilling schedule, it was decided to exclude calculation of the facilities CAPEX profiles, oil revenue and OPEX. The model does not, therefore, calculate the NPV of the different projects. However, it is a simple task to determine this from the input data and the optimised drilling schedule.

5.3.2 Problem Nomenclature

The model was developed as an MILP. The model consisted of the following sets:

\[ i \ldots \] the set of wells in Field P;

\[ j \ldots \] the set of wells in Field Q;

\[ t \ldots \] The set of years.

Scalar input parameters are:

Main Field

\[ c_i^M \] Cost of drilling Well \( i \);
5.0 Two Field Model

$R^M$ Recoverable reserves;

$w_i^M$ Initial production from Well $i$.

**Satellite Field**

$c_j^S$ Cost of drilling Well $j$;

$P^S$ Satellite design capacity;

$R^S$ Recoverable reserves;

$w_j^S$ Initial production from Well $j$;

**Common**

$F_i^{D}$ Discount factor for Year $t$, based on 10% discount rate;

$F_i^{PD}$ Production Decline factor in Year $t$;

$T_i$ Target production rate for Year $t$;

A binary parameter shows that the satellite field was available for production:

$S_i$ Indicate the satellite field is available for production in Year $t$.

Binary variables calculated by the model during simulation are:

**Main Field**

$y_{i,t}^M$ Binary to indicate that Well $i$ has been drilled;

$z_{i,t}^M$ Binary to indicate in which year Well $i$ was drilled;

**Satellite Field**

$Y_{j,t}^S$ Binary to indicate that Well $j$ has been drilled;
5.0 Two Field Model

$z_{ji}^S$  Binary to indicate in which year Well $j$ was drilled.

Runtime variables are:

**Main Field**

$p_{ji}^M$  Production from Well $i$ in Year $t$;

$Q_{it}^M$  Cumulative production up to and including Year $t$;

**Satellite Field**

$p_{j}^S$  Production from Well $j$ in Year $t$;

$Q_{it}^S$  Cumulative production up to and including Year $t$.

**Common**

Cost  The discounted cost of drilling all the wells in both fields;

5.3.3 Problem Formulation

The objective function is to minimise:

$$Cost = \sum_i \sum_t z_{i,t}^M \cdot c_i^M \cdot F_t^D + \sum_j \sum_t z_{j,t}^S \cdot c_j^S \cdot F_t^D$$

Equation (5.17) equates the cost of drilling all the wells required in both fields to the product of the cost of drilling the well, the binary indicating which year the well was drilled and the discount factor. $z_{i,t}^M$ and $z_{j,t}^S$ are only equal to 1 in the year that the well is drilled, all other values of the binaries are zero.

Subject to the constraints described below.

$$p_{ji}^M \leq y_{ji}^M \cdot w_i^M \cdot F_t^{PD}$$

(5.18)
5.0 Two Field Model

\[ p_{j,t}^S \leq y_{j,t}^S . w_j^S . F_t^{PD} . S_t \]  
(5.19)

Constraints (5.18) and (5.19) determine the production from each field, each year and take into account the Production Decline factor. \( y_{j,t}^M \) and \( y_{j,t}^S \) binaries are set to equal 1 when a well is drilled and remain set to 1 throughout field life to indicate that the well is in production.

\[ T_t = \sum_i p_{j,t}^M + \sum_j p_{j,t}^S \]  
(5.20)

Constraint (5.20) sets the total production rate from the two fields to equal the target production rate for each year of production.

For \( \forall t \):

\[ p^S \geq \sum_j p_{j,t}^S \]  
(5.21)

Constraint (5.21) limits the production from the satellite field to not exceed the rate specified in the input parameter.

For \( t > t' \):

\[ y_{j,t}^M \geq y_{j,t'-1}^M \]  
(5.22)

\[ y_{j,t}^S \geq y_{j,t'-1}^S \]  
(5.23)

Constraints (5.22) and (5.23) ensure that if a well has been available for production in the previous year, it is also available in the current year.
5.0 Two Field Model

The next four equations are used to identify which year a well was drilled in. The year is assumed to be the same year as it enters production. It is necessary to determine the year to apply the correct discount factor.

For $t = 1$:

$$z_{i,t}^M = y_{i,t}^M$$

(5.24)

$$z_{j,t}^S = y_{j,t}^S$$

(5.25)

Equations (5.24) and (5.25) identify if a well is drilled in Year 1 and set $Z_{i,t}^M$ or $Z_{j,t}^S$ to 1 to indicate that the well was drilled in Year 1, if $Y_{i,t}^M$ or $Y_{j,t}^S$ equals 1.

For $\forall t > 1$:

$$z_{i,t}^M = y_{i,t}^M - y_{i,t-1}^M$$

(5.26)

$$z_{j,t}^S = y_{j,t}^S - y_{j,t-1}^S$$

(5.27)

Equations (5.26) and (5.27) detect if a well has been drilled in the current year, $t > 1$. There are three cases, which are described by referencing the main field, but equally apply to the satellite field with suitable changes in sub and superscripts.

(a) Well has not yet been drilled:

$$y_{i,t}^M = 0 \text{ and } y_{i,t-1}^M = 0$$

Therefore, $z_{i,t}^M = 0 - 0 = 0$

(b) Well has already been drilled:

$$y_{i,t}^M = 1 \text{ and } y_{i,t-1}^M = 1$$

Therefore, $z_{i,t}^M = 1 - 1 = 0$
5.0 Two Field Model

(c) Well was drilled in the current year:

\[ y_{i,t}^M = 1 \text{ and } y_{i,t-1}^M = 0 \]

Therefore, \( z_{i,t}^M = 1 - 0 = 1 \)

Hence the logic correctly identifies that a well was drilled in the current year.

For \( t = 1 \):

\[ Q_t^M = \sum_i \left( \frac{P_{i,t}^M \times 350}{1000} \right) \]

(5.28)

\[ Q_t^S = S_t^* \sum_j \left( \frac{P_{j,t}^S \times 350}{1000} \right) \]

(5.29)

Equations (5.28) and (5.29) set the cumulative production from the two fields for the first year of production when there is no total to carry forward. Satellite production is only counted if the satellite field has been commissioned as indicated by the value of \( S_t^* \).

\( \forall t > 1 \):

\[ Q_t^M = Q_{t-1}^M + \sum_i \left( \frac{P_{i,t}^M \times 350}{1000} \right) \]

(5.30)

\[ Q_t^S = Q_{t-1}^S + S_t^* \sum_j \left( \frac{P_{j,t}^S \times 350}{1000} \right) \]

(5.31)

Equations (5.30) and (5.31) calculate the cumulative production from each field when \( t > 1 \), and there is a total to carry forward.

\[ R^M \geq Q_t^M \]

(5.32)
Equations (5.32) and (5.33) limit the cumulative production to each field to the reserves for that field.

5.3.4 Solution Approach

The formulation constitutes a MILP problem. For each year, the selection of wells in the main field and satellite field (if $S_i = 1$) is chosen so that the discounted cost of drilling totalled over field life is a minimum.

Additionally, the Annual production targets must be met without exceeding the satellite field capacity or the reserves of either field. Note that if the production target and the satellite field design capacity are met, but not exceeded, then the main field design capacity cannot be exceeded.

5.3.5 Model Implementation

The model is an MILP problem and was developed using the GAMS development environment. Two fields: P and Q, were used to develop the model. The model was developed with the field data in data input files so that any two fields could be used to give the model flexibility.

The model minimises the discounted cost of drilling sufficient wells to meet the specified production profile in each year of field life. Costs were discounted to avoid pre-investment and to ensure that wells were only drilled when required.

5.3.6 Two Field Simultaneous Model Results

Satellite timings from Year 1 to Year 4 were modelled. There was no feasible solution for Years 5 and higher since the satellite field was unable to meet the combined production profile when the main field was in its final years of decline.
The combined production profiles for the four years that were modelled are shown in Figures 5.22, 5.23, 5.24 and 5.25.

Figure 5.22 shows production from the satellite field commencing in Year 1 at the same time as production from the main field.

Satellite production commences in Year 1 and continues in all years except Year 13, where production is met completely from the main field. Production from the satellite is not required in Year 13.

Figure 5.23 shows production from the satellite field commencing in Year 2, one year after production from the main field commences.
5.0 Two Field Model

Figure 5.23 Distribution of Production between Fields. Satellite Start in Year 2

- Satellite production commences in Year 2 and continues in all years except Year 11, where production is met completely from the main field. Production from the satellite is not required in Year 11.

- Figure 5.24 shows production from the satellite field commencing in Year 3, two years after production from the main field commences.

- Note that in all cases, production in Years 17 to 20 is exclusively from the satellite field, since the reserves of the main field have already been produced.

- Figure 5.25 shows production from the satellite field commencing in Year 3, three years after production from the main field commences. Again all production in Years 17 to 20 is from the satellite field. In Year 16, 6,505 BPD are produced from the main fields and 49,999 BPD is produced from the satellite field. The production from the satellite field is limited to 50,000 BPD.
5.0 Two Field Model

Figure 5.24 Distribution of Production between Fields. Satellite Start in Year 3

Satellite production commences in Year 3 and continues in all years. Production is shared between the fields from Year 3 to Year 17 when the main field reserves have been exhausted.

If the satellite field is delayed by a further year, an additional 50,000 BPD is required from the main field in Year 4. This is taken from the production in Year 16 and the problem becomes insoluble since the production in Year 16 of 56,505 BPD can only come from the satellite field which is restricted to a maximum of 50,000 BPD. The problem is therefore insoluble.

In this particular problem, the discounted drilling costs for all three cases were very similar. For the case where the satellite was available for production in Year 1, the cost was $341 million. In the remaining three cases, the cost was $347 million.
Satellite production commences in Year 4 and continues in all years. Production is share between the fields from Year 4 to Year 17 when the main field reserves have been exhausted.

5.3.7 Limitations of Simultaneous Model

The main limitation to the Simultaneous Two Field model is that the Production Decline factor must be applied to the total production from both fields and not to the production from individual fields since if it were applied the problem would become non-linear since the factor would be dependent on production from the individual field. In the model, this is overcome by calculating the Production Decline factor based on the productions specified in the target production from the combined fields.

Additionally, whilst the simultaneous model optimises the well selection for an individual case in a similar manner to the original model, the overall economics of the project must be considered in discounted cash terms to determine the most
5.0 Two Field Model

economic size of development of both fields and the timing of the satellite field production.

This particular model had 99 potential well locations in the main field and 29 locations in the satellite field. The field life extended over a total of 20 years production. The computational time for the solution was 16 minutes 45 seconds. Currently, no attempt has been made to accelerate conversion.
6.0 Results and Conclusions

The results and conclusions described in this section only apply directly to the fields that have been modelled and based on the performance and cost models that have been described. However, it is possible to draw more general conclusions, but caution must be applied in extending the range of applicability.

6.1 Results

This work has shown that a mathematical model can be developed to describe the performance of a hypothetical offshore oilfield over a wide range of operating conditions. The model has permitted the economic performance of the field to be assessed under these different operating conditions.

The model enables a production build up, plateau and decline to be specified. These are used to specify target production rates for each year of production.

The model selects the optimum (lowest CAPEX) wells required to be able to meet the specified production rates for each year. Later versions of the model were designed to restrict production to exactly that of the design capacity of the facilities. Any extra production available from wells was deferred to subsequent years.

A Production Decline Factor, \( P^{PD} \), was defined which was used to model the decline in production from a well during its productive life. As an individual well is produced, the production from the well, in general, declines. This reduction in production is caused by a number of factors, including: drainage of oil from the area surrounding the well, restriction of the flow path to the wellbore by sand or other debris (either natural or from drilling), and the increase in water production.

Factors that have been investigated include:
6.0 Results and Conclusions

- The main field design capacity;
- The satellite field design capacity;
- The year the satellite facility is commissioned in relation to the main facility;
- The effect of different oil prices;
- The effect of more expensive wells;
- The effect of random variations in platform, pipeline and drilling costs.

A method has been developed that enables an optimum drilling location to be calculated quickly and efficiently. The method does not require the use of a sophisticated linear programming package. The method uses a parameter referred to as the Specific Production to determine the most cost-effective location for the drilling centre based on drilling cost from each possible location. Drilling costs were based on a function of along-hole distance.

The costs of platforms, topsides, pipelines and drilling were determined using a commercial package, QUESTOR, produced by IHS.

6.2 Conclusions

A linear program can be developed to estimate the optimum size and timing for a two field offshore development. It was concluded that the optimum production rate should be selected as producing 10% of the recoverable reserves each year. This proportion corresponds to a limit frequently applied by national hydrocarbon resource agencies.

This conclusion was reached based on studying the economic return over a range of oil prices from $5/bbl to $30/bbl. Recent excursions of the oil price to a peak
$65/bbl only support the conclusion of producing the reserves as quickly as possible.

The satellite field production rate should be equal or slightly less than the capacity of the main field. There is, obviously, no benefit in designing the satellite facilities to a higher capacity than the main field facilities, since the extra capacity will not be able to be used.

The satellite field facilities should be commissioned shortly after the main field reaches plateau. This will then allow the operator to select the most cost effective well, either in the main field, or in the satellite field to maintain plateau production.

These conclusions can only be applied to the particular fields that have been modelled. Different cost curves and well performances may well result in different conclusions.

However, this work has demonstrated a method of modelling and investigating offshore two reservoir oil fields. By using field specific reservoir and cost data, the model can be used to assess the economic performance of other offshore fields.

The model can also be adapted to model onshore fields where multiple wells are drilled from a common well pad.
Appendix 1.0 Description of Hardware and Software

A1.1 Hardware

Initial work on this project used a Pentium laptop running Windows 95. The machine had 32 MB of RAM and a 40 GB hard disc. This proved to be too slow for the larger models and a faster machine had to be used.

The new machine was a 2.4 GHz Pentium laptop running Windows 2000. The computer had 512 MB RAM and a 40 GB hard drive.

A1.2 Software

The study made use of several software packages to enable the large number of calculations to be completed most efficiently. The three primary packages used were:

- Borland C++ Builder;
- GAMS;
- IHS Energy QUESTOR Offshore.

The use of these packages is described in the following sections.

A1.2.1 Borland C++ Builder

The main programs were written as a console application using Borland's C++ Builder 6.0, Swart et al (2003). C++ was selected because of familiarity with the language and package availability. The program was not written as a Windows application because of the overhead it writing this type of application. If the work
Appendix 1.0 Description of Hardware and Software

were to be commercialised, it would be a relatively simple task for a programmer to write the Windows front end.

The programs were written in a structured manner with the executive program being short and primarily calling subroutines to perform specific, clearly defined tasks. In this manner, as the programs were developed, large amounts of earlier work could be reused with little or no modification.

The modular construction also facilitated program development.

A1.2.2 GAMS

General Algebraic Modelling System, GAMS, is an industry standard linear and non-linear solver, Brook et al (1998). The package was used to solve the well selection optimisation.

The original version of GAMS used was version 19.6. This was upgraded during the study to version 20.7.

Originally, the standard solver was used, but as the models became larger, it was necessary to use the CPLEX solver within GAMS.

A1.2.3 IHS QUE$TOR Offshore

QUESTOR Offshore 7.9b is a standard Oil Industry cost estimating package marketed by IHS Energy, Que$tor User Manual, (2003). The package enable cost estimates of offshore installations to be quickly and accurately prepared for all locations in the world.

The package was used to prepare a series of cost estimates for different size facilities. This data was then curve fitted and incorporated in the main programs
Appendix 1.0 Description of Hardware and Software

to enable cost estimates for drilling and facilities to be rapidly calculated during the optimisation runs.

During the course of this work, QUESTOR has been updated several times. The current issue is version 9.5. The updating includes revision of the cost database. In order not to have to regularly change the cost equations, the work is based inclusively on version 7.9b. This is consistent with the lower oil price used in the work compared with the price prevailing at the present.
Appendix 2.0 Drilling Geometry

Several different well geometries can be used to drill a reservoir. The more common configurations are shown in Section 1, Figure 1.1. The work in this study has been based on the vertical, and “build and hold” configurations. The solution presented here was developed from first principles, but matches that described in the literature, Mian (1992), Devereux (1998).

A vertical well is a special case of a build and hold well, where the build angle is zero. The model can be easily extended to cater for other geometry wells. This section derives the formula for the total length of a build and hold well. The along hole distance is used in the cost equation to estimate the cost of drilling the well.

Figure A2.1 shows the configuration of a well drilled on the build and hold principle. The well is drilled vertically to a specified depth before reaching the kick-off point. At this point, the well starts to deviate from the vertical at a specified rate of deviation angle build-up, expressed as degrees per fixed distance, for example, 4° / 100 ft. Once the required angle has been established, drilling is continued in a straight trajectory towards the downhole target.

Let

\[ D = \text{Total horizontal displacement} \]

\[ R = \text{Radius of build section} \]

\[ V = \text{True vertical depth of well} \]

\[ K = \text{Depth of Kick-off point} \]

\[ L = \text{Length of build section} \]

\[ \theta = \text{Deviation angle} \]
The diagram shows a typical build and hold well profile. The well is drilled vertically until the kick-off point, K, where sufficient angle is built so that from the point B, a straight line extension reaches the target downhole location, T.
Appendix 2.0 Drilling Geometry

Figure A2.2 Detail of Build Profile

The diagram shows the detail in the region of the build portion of the well trajectory.

A2.1 Deviation Angle

The geometry around the build up section is shown enlarged in Figure A2.2. The radius of the build section is calculated from the deviation angle:

\[ R = \frac{L}{\theta} \]

The build horizontal displacement, \( BD = KC - EC \)

\[ = R - EC \]

\[ = R \cdot (1 - \cos \theta) \]

The build vertical displacement, \( KB = ED \)

\[ = R \cdot \sin \theta \]

\[ \therefore \] the total horizontal deviation is given by:

\[ D = \text{build horizontal displacement} + \text{hold horizontal displacement} \]
Appendix 2.0 Drilling Geometry

\[ D = R \ast (1 - \cos \theta) + (OV - OK - R \ast \sin \theta) \ast \tan \theta \]

\[ D - R = - R \ast \cos \theta + (OV - OK - R \ast \sin \theta) \ast \tan \theta \]

\[ (D - R) \ast \cos \theta = -R \ast \cos^2 \theta + (OV - OK) \ast \sin \theta - R \ast \sin^2 \theta \]

\[ (D - R) \ast \cos \theta = -R \ast \cos^2 \theta + (OV - OK) \ast \sin \theta - R + R \ast \cos^2 \theta \]

\[ (D - R) \ast \cos \theta = (OV - OK) \ast \sin \theta - R \]

\[ (D - R) \ast \sqrt{1 - \sin^2 \theta} = (OV - OK) \ast \sin \theta - R \]

\[ 1 - \sin^2 \theta = \frac{(OV - OK) \ast \sin \theta - R)^2}{(D - R)^2} \]

\[ = \frac{(OV - OK)^2 \ast \sin^2 \theta - 2 \ast R \ast (OV - OK) \ast \sin \theta + R^2}{(D - R)^2} \]

\[ (D - R)^2 - (D - R)^2 \ast \sin^2 \theta = (OV - OK)^2 \ast \sin^2 \theta - 2 \ast R \ast (OV - OK) \ast \sin \theta + R^2 \]

\[ \{(OV - OK)^2 + (D - R)^2\} \ast \sin^2 \theta(D - R)^2 - 2 \ast R \ast (OV - OK) \ast \sin \theta + R^2 \ast (D - R)^2 = 0 \]

\[ \therefore \sin \theta = \frac{2 \ast R \ast (OV - OK) \pm \sqrt{4 \ast R^2 \ast (OV - OK)^2 - 4 \ast ((OV - OK)^2 + (D - R)^2) \ast (2 \ast D \ast R - D^2)}}{2 \ast ((OV - OK)^2 + (D - R)^2)} \]

\[ \therefore \sin \theta = \frac{R \ast (OV - OK) \pm \sqrt{R^2 \ast (OV - OK)^2 - ((OV - OK)^2 + (D - R)^2) \ast (2 \ast D \ast R - D^2)}}{(OV - OK)^2 + (D - R)^2} \]

This expression gives the value of the angle of inclination of the hold section of the well. The positive root is taken if \( D > R \), and the negative root is taken if \( D < R \).
A2.2 Total Measured Length

The total drilled length of the well is made up of three sections:

Length = Vertical section + Curved build section + straight inclined lower section

The length of the vertical section is simply the depth of the kick off point, OK.

The length of the curved build section is the length of the arc of the curve, which is given by $R \theta$, where $\theta$ is in radians.

The vertical height of the inclined lower section is:

$$BV = OV - OK - KB$$

$$= OV - OK - R \sin \theta$$

Therefore the length of the inclined section is:

$$ST = (OV - OK - R \sin \theta)/\cos \theta$$

Therefore, the total measured or drilled length of the well is:

$$Length = KO + R \theta + (OV - OK - R \sin \theta)/\cos \theta$$

This length is then used in the cost equations described in Appendix 3.
Appendix 3.0 Cost Algorithms

In order to be able to calculate the CAPEX value of the components of the field development, cost algorithms were determined for different well configurations and different facility design capacities.

A3.1 Well Cost

Drilling cost for wells with different deviations were estimated using the cost estimating package QUESTOR. This package is described in Appendix 1. Well costs were determined for a number of well lengths using QUESTOR. Initially, the well cost was taken as $2,329/m of length. This algorithm was used for the Small field case. For the larger fields a correlation using a fixed cost and a variable cost element dependent on well total length was used:

\[ \text{Cost} = 1.611 + 0.001897 \times \text{Well Length} \]

Where:

- **Well length** is in metres.
- **Cost** is in $ millions.

The well costs are shown in Table A3.1 and plot of the data with the regression line and equation are shown in Figure A3.1.

Table A3.1 Well Cost Data

<table>
<thead>
<tr>
<th>Well Length, m</th>
<th>Cost, $MM</th>
</tr>
</thead>
<tbody>
<tr>
<td>2,500</td>
<td>6.472</td>
</tr>
<tr>
<td>3,000</td>
<td>7.163</td>
</tr>
<tr>
<td>3,500</td>
<td>8.106</td>
</tr>
<tr>
<td>4,000</td>
<td>9.145</td>
</tr>
<tr>
<td>4,500</td>
<td>10.334</td>
</tr>
</tbody>
</table>
Appendix 3.0 Cost Algorithms

Figure A3.1 Well Cost Regression

Curve fit of well drilling cost for wells with a total length of between 2,500 and 4,500 m.

A3.2 Jacket Cost

The jacket cost for design capacities of between 50,000 BPD and 400,000 BPD were determined using QUE$TOR. The costs include the cost of the jacket, fabrication, tow-out, installation and piling. The costs are shown in Table A3.2.

<table>
<thead>
<tr>
<th>Design Capacity, BPD</th>
<th>Cost, $MM</th>
</tr>
</thead>
<tbody>
<tr>
<td>50,000</td>
<td>163.449</td>
</tr>
<tr>
<td>100,000</td>
<td>163.428</td>
</tr>
<tr>
<td>200,000</td>
<td>170.361</td>
</tr>
<tr>
<td>300,000</td>
<td>175.481</td>
</tr>
<tr>
<td>400,000</td>
<td>183.344</td>
</tr>
</tbody>
</table>

These values were curve fitted to a quadratic equation using the Microsoft Excel regression feature. The resulting plot is shown in Figure A3.2.
Appendix 3.0 Cost Algorithms

Figure A3.2 Jacket Cost

Curve fit of Jacket cost for facilities design capacity of between 50,000 and 400,000 BPD. Values are in $ thousands. The cost of the jacket for 50,000 and 100,000 BPD capacity are almost the same.

The jacket cost can be represented by:

\[
Jacket \ Cost = 162,453 + 15.228 \cdot Capacity + 0.0940 \cdot Capacity^2
\]

Where:

Capacity is in thousand BPD design capacity.

Jacket Cost is in $ thousands.

A3.3 Topsides Cost

The topsides cost for design capacities of between 50,000 BPD and 400,000 BPD were determined using QUESTOR. The costs include the process facilities, utilities accommodation and helipad. Fabrication, sail out and installation on to the jacket are included. The costs are shown in Table A3.3.
Appendix 3.0 Cost Algorithms

Table A3.3 Topsides Cost Data

<table>
<thead>
<tr>
<th>Design Capacity, BPD</th>
<th>Cost, $MM</th>
</tr>
</thead>
<tbody>
<tr>
<td>50,000</td>
<td>369.263</td>
</tr>
<tr>
<td>100,000</td>
<td>408.198</td>
</tr>
<tr>
<td>200,000</td>
<td>441.784</td>
</tr>
<tr>
<td>300,000</td>
<td>524.120</td>
</tr>
<tr>
<td>400,000</td>
<td>659.755</td>
</tr>
</tbody>
</table>

These values were curve fitted to a quadratic equation using the Microsoft Excel regression feature. The resulting plot is shown in Figure A3.3.

The topsides cost can be represented by:

\[
\text{Topsides Cost} = 369,263 + 786.014 \cdot \text{Capacity} - 0.1462 \cdot \text{Capacity}^2
\]
Appendix 3.0 Cost Algorithms

Where:

Capacity is in thousand BPD design capacity.

Topsides Cost is in $ thousands.

A3.4 Oil Pipeline Cost

The oil pipeline cost for design capacities of between 50,000 BPD and 400,000 BPD were determined using QUESTOR. The costs include pipe material, coating material and installation. The costs are shown in Table A3.4.

Table A3.4 Oil Pipeline Cost Data

<table>
<thead>
<tr>
<th>Design Capacity, BPD</th>
<th>Cost, $MM</th>
</tr>
</thead>
<tbody>
<tr>
<td>50,000</td>
<td>66.151</td>
</tr>
<tr>
<td>100,000</td>
<td>71.250</td>
</tr>
<tr>
<td>200,000</td>
<td>95.147</td>
</tr>
<tr>
<td>300,000</td>
<td>109.468</td>
</tr>
<tr>
<td>400,000</td>
<td>125.820</td>
</tr>
</tbody>
</table>

These values were curve fitted to a quadratic equation using the Microsoft Excel regression feature. The resulting plot is shown in Figure A3.4.

The oil pipeline cost can be represented by:

\[
\text{Oil Pipeline Cost} = 57.267 + 178.414 \cdot \text{Capacity} - 0.0147 \cdot \text{Capacity}^2
\]

Where:

Capacity is in thousand BPD design capacity.

Oil Pipeline Cost is in $ thousands.
Appendix 3.0 Cost Algorithms

Figure A3.4 Oil Pipeline Cost

Curve fit of oil pipeline cost for facilities design capacity of between 50,000 and 400,000 BPD. Values are in $ thousands.

A3.5  Gas Pipeline Cost

The gas pipeline cost for design capacities of between 50,000 BPD and 400,000 BPD were determined using QUE$TOR. The costs include pipe material, coating material and installation. The costs are shown in Table A3.5.

Table A3.5 Gas Pipeline Cost Data

<table>
<thead>
<tr>
<th>Design Capacity, BPD</th>
<th>Cost, $MM</th>
</tr>
</thead>
<tbody>
<tr>
<td>50,000</td>
<td>45.120</td>
</tr>
<tr>
<td>100,000</td>
<td>55.809</td>
</tr>
<tr>
<td>200,000</td>
<td>66.086</td>
</tr>
<tr>
<td>300,000</td>
<td>69.815</td>
</tr>
<tr>
<td>400,000</td>
<td>75.369</td>
</tr>
</tbody>
</table>
These values were curve fitted to a quadratic equation using the Microsoft Excel regression feature. The resulting plot is shown in Figure A3.5.

The gas pipeline cost can be represented by:

\[ \text{Gas Pipeline Cost} = 47,984 + 80.991 \cdot \text{Capacity} - 0.0274 \cdot \text{Capacity}^2 \]

Where:

- \( \text{Capacity} \) is in thousand BPD design capacity.
- \( \text{Gas Pipeline Cost} \) is in $ thousands.

![Figure A3. Gas Pipeline Cost](image)

Curve fit of gas pipeline cost for facilities design capacity of between 50,000 and 400,000 BPD. Values are in $ thousands

### A3.6 Satellite Field Cost

The satellite development also requires cost estimates for satellite wellhead platforms to feed the main platform. The wellhead platforms were specified with...
Appendix 3.0 Cost Algorithms

production and test separator facilities to enable the fluids to be transported to the main platform by two single phase pipelines. The platforms also had water injection facilities located on them. The platforms were specified to be unmanned.

Cost estimates were prepared for satellite facilities with capacities between 50,000 and 500,000 BPD. This range required between one and three platforms to maintain the well count on individual platforms to 40 or less. The quadratic cost equations, in thousands of dollars are summarised in Table A3.6. It was assumed that 10 km pipelines were required to connect each satellite to the main platform.

The method of determining the regressional equations was exactly the same as that used for the main platform.

Table A3.6 Satellite Platform Cost Estimate Equations

<table>
<thead>
<tr>
<th></th>
<th>$a_2$ Coefficient</th>
<th>$a_1$ Coefficient</th>
<th>$a_0$ Coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jacket</td>
<td>0.3827</td>
<td>-9.7591</td>
<td>89,563</td>
</tr>
<tr>
<td>Topsides</td>
<td>0.3122</td>
<td>289.09</td>
<td>64,007</td>
</tr>
<tr>
<td>Oil Export Line</td>
<td>0.0053</td>
<td>63.256</td>
<td>18,168</td>
</tr>
<tr>
<td>Gas Export Line</td>
<td>0.0601</td>
<td>14.363</td>
<td>22,321</td>
</tr>
</tbody>
</table>
Appendix 4.0 Triangular Distribution

The triangular distribution of probability is frequently used in engineering analysis where data is sparse, and minimum, maximum and most probable values can be estimated, Allen (2006), Kotz et al (2004). It also has the advantage that the minimum and maximum values cannot be exceeded. Therefore, very high or low values cannot occur as can be the case with the normal distribution which can generate very high or low values, albeit with very low probability.

Consider a triangular distribution where \( a \) is the lower limit value, \( b \) is the most probable value and \( c \) is the upper limit value. Then the probability distribution function is as shown in Figure A4.1.

![Figure A4.1 Triangular Probability Distribution Function.](image)

The area under the curve for a probability density function is equal to 1. This area is equal to the area of the triangle \( ach \).

\[
\text{Area} = \frac{1}{2} \cdot \text{base} \cdot \text{perpendicular height}
\]
\[
= \frac{1}{2} \cdot (c - a) \cdot h
\]

\[225\]
Appendix 4.0 Triangular Distribution

\[ \therefore \frac{1}{2} \cdot (c - a) \cdot h = 1 \]

or \[ h = \frac{2}{(c - a)} \]

To determine the probability of a particular value occurring within a triangular distribution, the probability function, \( f(x) \), must be determined for the range of outcome values.

For \( \forall x < a \), \( f(x) = 0 \).

For \( a \leq x \leq b \):

The general equation of the line passing through the points \( (x_1, y_1) \) and \( (x_2, y_2) \) is:

\[ y - y_1 = \frac{(y_2 - y_1)}{(x_2 - x_1)} \cdot (x - x_1) \]

The equation of the line passing through \( (x_1, y_1) = (a, 0) \) and \( (x_2, y_2) = (b, h) \) is:

\[ f(x) = \frac{h}{(b - a)} \cdot (x - a) \]

Substituting for \( h \):

\[ f(x) = \frac{2(x - a)}{(b - a)(c - a)} \]

For \( b \leq x \leq c \):

The equation of the line passing through \( (x_1, y_1) = (b, h) \) and \( (x_2, y_2) = (c, 0) \) is:

\[ f(x) - h = \frac{-h}{(c - b)} \cdot (x - b) \]

\[ \therefore f(x) = h - \frac{h}{(c - b)} \cdot (x - b) \]
Substituting for \( h \):

\[
f(x) = \frac{2}{c-a} - \frac{2(x-b)}{(c-a)(c-b)}
\]

\[
f(x) = \frac{2(c-x)}{(c-a)(c-b)}
\]

For \( \forall x > c \), \( f(x) = 0 \).

In order to evaluate the value of \( x \) corresponding to a probability represented by a random number between 0 and 1, the cumulative distribution function is required.

For \( a \leq x \leq b \):

\[
f(x) = \frac{2(x-a)}{(b-a)(c-a)}
\]

Therefore the cumulative distribution function is given by:

\[
F(x) = \int_{a}^{x} \frac{2(x-a)}{(b-a)(c-a)} \, dx
\]

Where \( u \) is the random number representing a value for which the probability of the value occurring is required to be calculated, such that \( a \leq u \leq b \).

\[
\therefore F(x) = \frac{1}{(b-a)(c-a)} \int_{a}^{u} 2(x-a) \, dx
\]

\[
= \frac{1}{(b-a)(c-a)} \left[ x^2 - 2ax \right]_{a}^{u}
\]

\[
= \frac{1}{(b-a)(c-a)} \left( u^2 - 2au - a^2 + 2a^2 \right)
\]
Appendix 4.0 Triangular Distribution

\[
\frac{(u-a)^2}{(b-a)(c-a)}
\]

For \(b \leq x \leq c\):

\[
f(x) = \frac{2(c-x)}{(c-a)(c-b)}
\]

Therefore the cumulative distribution function is given by:

\[
F(x) = \int_a^x \frac{2(c-x)}{(b-a)(c-a)} \, dx + \int_x^c \frac{2(c-x)}{(c-a)(c-b)} \, dx
\]

Where \(b \leq u \leq c\):

\[
= \frac{(b-a)^2}{(b-a)(c-a)} + \frac{1}{(c-a)(c-b)} \left[ 2(c-x) \right]_b^u
\]

\[
= \frac{(b-a)}{(c-a)} + \frac{1}{(c-a)(c-b)} \left[ 2cx - x^2 \right]_b^u
\]

\[
= \frac{(b-a)}{(c-a)} + \frac{2cu - u^2 - 2bc + b^2}{(c-a)(c-b)}
\]

\[
= \frac{(b-a)(c-b) + 2cu - u^2 - 2bc + b^2}{(c-a)(c-b)}
\]

\[
= \frac{ab + 2cu - ac - u^2 - bc}{(c-a)(c-b)}
\]

\[
= \frac{c^2 - bc - ac + ab - c^2 + 2cu - u^2}{(c-a)(c-b)}
\]

\[
= \frac{(c-a)(c-b)-(c-u)^2}{(c-a)(c-b)}
\]
In the Monte Carlo simulation, the random number from the random number generator is \( r = F(u) \) and we require to know the variable \( u \) to obtain values within the triangular distribution. The inverse of the cumulative distribution function is required.

For \( 0 \leq r \leq (b-a)/(c-a) \):

\[
r = \frac{(u-a)^2}{(b-a)(c-a)}
\]

\[
(u-a)^2 = r \cdot (b-a) \cdot (c-a)
\]

\[
\therefore u = a + \sqrt{r \cdot (b-a) \cdot (c-a)}
\]

For \( r > (b-a)/(c-a) \):

\[
r = 1 - \frac{(c-u)^2}{(c-a)(c-b)}
\]

\[
1-r = \frac{(c-u)^2}{(c-a)(c-b)}
\]

\[
(c-u) = \sqrt{(1-r) \cdot (c-a)(c-b)}
\]

\[
\therefore u = c - \sqrt{(1-r) \cdot (c-a)(c-b)}
\]

These two equations for \( u \) in terms of \( r \) are used with the low, mid and high values, \( a, b \) and \( c \), to calculate the risked value of the variable from the random numbers.
Appendix 5.0 Description of Two Field Model

A detailed description of the final two field model is provided in this appendix to indicate the method of modelling. The operation and results obtained from Two Field model have already been described in Sections 5 and 6. This appendix describes the logic and structure of the program.

A5.1 Description of Modules

The Two Field model has been developed in a structured manner for ease of programming and development. The executive procedure, main(), controls all the sub-procedures.

A5.1.1 Procedure main()

The executive program does minimal calculation. A flowchart of the procedure’s structure is shown in Figure 1. The programming sequence is described below.

1. The program initialises the combined field producible reserves (TotalReserves) to zero.

2. Procedure getdata() is called to read the input data file.

3. Procedure target() is called to calculate the annual target productions for each year.

4. Procedure initcost() is called to calculate the CAPEX costs for both the Main and Satellite Field costs and to calculate the annual CAPEX profiles.

5. The program then loops through the next 6 steps to run GAMS for each year of production in sequence.

6. Procedure gamsif() is called to prepare and assemble the main GAMS input file for each individual year.
7. GAMS is called and the GAMS program determines the minimum cost of additional wells to meet the specified production targets.

8. The program then opens a new output file using the current model year in the name to store year-specific data.

9. Procedure `newwells()` is called to search the GAMS output file for any new wells that have been commissioned to meet the current year production.

10. The program then calls procedure `production()` to calculate the total cumulative production from each field.

11. Using the Total Production from each field, the Field Reduction Factors, FPR for each field are calculated for the next year:

\[
FPR = \frac{\text{Field Reserves} - 0.5 \times \text{Field Total Production}}{\text{Field Reserves}}
\]

If FPR < 0.5, FPR is set to zero as the Field Reserves for the field have been produced.

12. Steps 5 to 11 are repeated for each until the reserves of both fields have been produced.

13. The simulations results are then assembled into the output file in procedure `results()`.

14. The main program terminates.

**A5.1.2 Procedure `getdata()`**

The procedure is used to read the input file data. The input file describes the case to be studied, specifying the run time parameters. A flowchart of the procedure's structure is shown in Figure 2. An example of an input file is shown in Section A4.4.
The procedure checks that the correct keyword is read before the data item is read. If the keyword is not correct, a warning message is issued and the program halts.

The procedure sequence is:

1. Prompt for case name. The procedure then builds the full file name and attempts to open the appropriate input file. Failure to open the file results in a fatal error.

2. Reads and stores case title.

3. Reads and stores oil price.

4. Reads and stores discount rate for economic analysis.

5. Reads and stores the OPEX rate as a percentage of CAPEX.

6. Reads the case configuration. This is either "STANDALONE" or "MULTIPLE". STANDALONE refers to a case where data is only provided for the main platform. MULTIPLE refers to a case consisting of the main platform plus one satellite field.

7. If the case is MULTIPLE, the first year that the satellite is operational is read.

8. The program then loops through a set of read instructions for the main and satellite fields as appropriate.

9. The first data read is the name of the GAMS include file that contains the field data.

10. Reads the drilling centre location and well coordinates.

11. Reads the recoverable reserves for the field.

12. Reads the peak (maximum) production for the case.

13. For the satellite field, reads the maximum number of wells that can be drilled in one year.
14. Reads a block of CAPEX coefficients for the Jacket, topsides, oil pipeline and gas pipeline. The coefficients are the $a_0$ and $a_1$ terms for the linear cost equations.

15. Steps 8 to 14 are then repeated for the satellite field.

16. The procedure terminates and control returns to the executive program.

All the variables are stored as global variables to enable simple transfer between the different procedures.

A5.1.3 Procedure target()

Procedure target() is used to set the daily target production for each year for the main field. A flowchart of the procedure’s structure is shown in Figure 3. The sequence of instructions is described below:

1. The arrays Target[l], Production[l] and AnnualProduct[l] are all set to zero.

2. Target rates are then set for the first four years of production starting with Year 5. These targets are all based on the Peak production rate for the main field.

   Target[5] = 0.25 * PeakRate[0]

   Target[6] = 0.5 * PeakRate[0]

   Target[7] = 0.75 * PeakRate[0]

   Target[8] = PeakRate[0]

3. The remaining years are all set to the peak rate. Earlier, single field, models had a decline curve for the tail end years. However, for the two field model this was not necessary as the satellite field makes up the decline in main field plateau rate.

4. The procedure then terminates and control returns to the executive program.
Appendix 5.0 Description of Two Field Model

A5.1.4 Procedure initcost()

Procedure initcost() is used to calculate the CAPEX of the platforms and pipelines, and then to set the CAPEX profiles for the appropriate years. A flowsheet describing the procedure is provided in Figure 4. The sequence of instructions for the procedure is:

1. For each of the fields, calculate the Jacket, Topsides, Oil Pipeline and Gas Pipeline CAPEXs using the quadratic cost coefficients from the input file.

2. Calculate Platform CAPEX as the sum of the Jacket and Topsides CAPEXs, and the Pipeline CAPEX as the sum of the Oil and Gas Pipeline CAPEXs.

3. Set the annual CAPEXs for Platform, Pipeline and Drilling to zero for all years.

4. Generate the CAPEX profile for the main field as shown in Table A5.1.

<table>
<thead>
<tr>
<th>Year</th>
<th>Platform, %</th>
<th>Pipelines, %</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>15</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>25</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>30</td>
<td>50</td>
</tr>
<tr>
<td>4</td>
<td>30</td>
<td>50</td>
</tr>
</tbody>
</table>

5. Generate the CAPEX profile for the Satellite Field. The profile is similar to that shown in Table 1, except that Year 1 is the first year of satellite production.

6. The procedure then terminates and control returns to the executive program.

A5.1.5 Procedure gamsif()

This procedure writes the GAMS input file from standard sections and run-specific data. The procedure also has to determine how many of the fields are in production for the specific year. A flowchart of the procedure's structure is shown in Figure 5. The sequence of instructions for the procedure is:
1. Build the input file name and open the file to receive the data.

2. Write the title and general model description common to all files.

3. Determine which case applies to the current year:
   - Main field only;
   - Main and Satellite fields;
   - Satellite field only.

4. If only the Main field is in production:
   - Write the Scalar variables:
     
     FPR1, the Main Field Reduction Factor;
     Minimum production required from the production facilities.
   - Write the Main Field only GAMS model.
   - Write the name of the Main Field GAMS include file to be used.
   - Write the binary variables. These are the variables indicating which well is already in operation. Inclusion of this data ensures that all producing wells are in production before any new wells are selected.

5. If both the Main and Satellite fields are in production:
   - Write the Scalar variables:
     
     FPR1, the Main Field Reduction Factor;
     FPR2, the Satellite Field Reduction Factor;
Appendix 5.0 Description of Two Field Model

Minimum production required from the production facilities;

Design capacity of the Satellite Field.

- Write the name of the Main Field GAMS include file to be used.

- Write the name of the Satellite Field GAMS include file to be used.

- Write the Main plus Satellite Field GAMS model.

- Write the Main Field binary variables.

- Write the Satellite Field binary variables.

6. If only the Satellite field is in production:

- Write the Scalar variables:

  FPR2, the Satellite Field Reduction Factor;

  Minimum production required from the production facilities;

  Design capacity of the Satellite Field.

- Write the name of the Satellite Field GAMS include file to be used.

- Write the Satellite Field only GAMS model.

- Write the Satellite Field binary variables.

7. Write the third, common section of the GAMS file.

8. The procedure then terminates and control returns to the executive program.
A5.1.6 Procedure newwells()

The `newwells()` procedure is used to count the number of wells drilled in the current year, and to determine the cumulative cost of drilling these wells. The cumulative cost is set to the `DRILLEX` of the previous year, since all wells are deemed to be drilled in the preceding year. A flowchart of the procedure's structure is shown in Figure 6. The sequence of instructions for the procedure is:

1. Search the GAMS output file *.LST for the beginning of the binary variable data.
2. Step through the Main field binary data searching for active wells.
3. Increment active well count for each active well found.
4. Check if the active well is also a new well. If it is, increment new well count and accumulate drilling cost.
5. When Main Field wells have all been checked, repeat steps 2, 3 and 4 for the Satellite Field.
6. The procedure then terminates and control returns to the executive program.

A5.1.7 Procedure production()

This procedure calculates the total cumulative production for each field at the end of the current year and checks that the production limits have not been exceeded. A flowchart of the procedure's structure is shown in Figure 7. The sequence of instructions for the procedure is:

1. The procedure loops through the two fields starting with the Main Field and then following with the Satellite Field.
2. The appropriate `include` file is opened to retrieve the production data.
3. Go to the start of the well listing.

4. Search the *include* file for the first active well.

5. Read the well productivity.

6. Accumulate the product of well productivity and Reduction Factor.

7. If Field is Main Field:

   Check if annual production exceeds Target, if so, set annual production equal to target.

8. If Field is Satellite Field:

   Check if annual production exceeds Target, if so, set annual production to spare capacity in Main facilities.

9. Accumulate total field production.

10. Repeat steps 2 to 9 for the Satellite Field.

11. The procedure then terminates and control returns to the executive program.

**A5.1.8 Procedure results()**

Procedure `results()` prepares and writes the run summary file. An example of a results file is shown in Section A4.9. The file summarises results for the Main Field, Satellite Field and the combined development.

A flowchart of the procedure's structure is shown in Figure 8. The sequence of steps in the procedure is:

1. Calculate the combined CAPEXs for the jacket, topsides, oil pipeline and gas pipeline.
2. Loop through all years and calculate the combined platform, pipeline and drilling costs for each year of field life.

3. Open a new file for the results output.

4. Loop through Main Field, Satellite Field and combined fields and prepare output.

5. Write header lines containing date, time, field, platform and pipeline costs, economic parameters and run data summary.

6. Write table headers for main annual output.

7. Write the annual CAPEXs for Years 1 to 4 when there is no production.

8. Loop through from Year 5 to end of field life writing each year's results in turn and accumulating CAPEX, DRILLEX, production and revenue.

9. Repeat steps 5, 6, 7 and 8 for the Satellite Field and the combined fields.

10. Calculate the NPV at the specified discount rate.

11. Calculate the IRR for the combined field.

12. The procedure then terminates and control returns to the executive program.
A5.2 Example Input File

<table>
<thead>
<tr>
<th>Section</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>TITLE</td>
<td>West Main Reservoir, East Satellite, input file CB08</td>
</tr>
<tr>
<td>OIL PRICE</td>
<td>20</td>
</tr>
<tr>
<td>DISCOUNT RATE</td>
<td>10</td>
</tr>
<tr>
<td>OPEX RATE</td>
<td>4</td>
</tr>
<tr>
<td>CONFIGURATION</td>
<td>MULTIPLE</td>
</tr>
<tr>
<td>FIRST YEAR</td>
<td>8</td>
</tr>
<tr>
<td>MAIN</td>
<td>INCLUDE West.inc</td>
</tr>
<tr>
<td>DRILLING CENTRE</td>
<td>SPECIFY W095 -2500 -1000</td>
</tr>
<tr>
<td>RECOVERABLE RESERVES</td>
<td>500</td>
</tr>
<tr>
<td>PEAK PRODUCTION</td>
<td>150</td>
</tr>
<tr>
<td>CAPEX COEFFICIENTS</td>
<td>JACKET</td>
</tr>
<tr>
<td></td>
<td>0.0940   15.228   162453</td>
</tr>
<tr>
<td></td>
<td>TOPSIDES</td>
</tr>
<tr>
<td></td>
<td>-0.1462  786.014   369263</td>
</tr>
<tr>
<td></td>
<td>OILPIPE</td>
</tr>
<tr>
<td></td>
<td>-0.0147  178.414   57267</td>
</tr>
<tr>
<td></td>
<td>GASPIPE</td>
</tr>
<tr>
<td></td>
<td>-0.0274  80.991   47984</td>
</tr>
<tr>
<td>SATELLITE</td>
<td>INCLUDE East.inc</td>
</tr>
<tr>
<td>DRILLING CENTRE</td>
<td>SPECIFY E069 1500 -250</td>
</tr>
<tr>
<td>RECOVERABLE RESERVES</td>
<td>300</td>
</tr>
<tr>
<td>PEAK PRODUCTION</td>
<td>100</td>
</tr>
<tr>
<td>MAXIMUM DRILLING</td>
<td>8</td>
</tr>
<tr>
<td>CAPEX COEFFICIENTS</td>
<td>JACKET</td>
</tr>
<tr>
<td></td>
<td>0.3827   -9.7591   89563</td>
</tr>
<tr>
<td></td>
<td>TOPSIDES</td>
</tr>
<tr>
<td></td>
<td>0.3122   289.09   64007</td>
</tr>
<tr>
<td></td>
<td>OILPIPE</td>
</tr>
<tr>
<td></td>
<td>0.0053   63.256   18168</td>
</tr>
<tr>
<td></td>
<td>GASPIPE</td>
</tr>
<tr>
<td></td>
<td>0.0601   14.363   22321</td>
</tr>
</tbody>
</table>
A5.3 Example Include File

* Well site data for the East Reservoir

Sets
  k Well locations / E001*E184 /
  l Well coordinates / Productl, WellCostl /

Table
  Satl(k,l) Coordinates of Production and Cost

<table>
<thead>
<tr>
<th></th>
<th>Productl</th>
<th>WellCostl</th>
</tr>
</thead>
<tbody>
<tr>
<td>E001</td>
<td>5000</td>
<td>8.424</td>
</tr>
<tr>
<td>E002</td>
<td>5000</td>
<td>8.208</td>
</tr>
<tr>
<td>E003</td>
<td>6000</td>
<td>7.868</td>
</tr>
<tr>
<td>E004</td>
<td>6000</td>
<td>7.717</td>
</tr>
<tr>
<td>E005</td>
<td>5000</td>
<td>8.248</td>
</tr>
<tr>
<td>E006</td>
<td>5000</td>
<td>8.024</td>
</tr>
<tr>
<td>E007</td>
<td>5000</td>
<td>7.835</td>
</tr>
<tr>
<td>E008</td>
<td>6000</td>
<td>7.517</td>
</tr>
<tr>
<td>E009</td>
<td>5000</td>
<td>8.106</td>
</tr>
<tr>
<td>E010</td>
<td>5000</td>
<td>7.878</td>
</tr>
<tr>
<td>E011</td>
<td>5000</td>
<td>7.684</td>
</tr>
<tr>
<td>E012</td>
<td>5000</td>
<td>7.529</td>
</tr>
<tr>
<td>E013</td>
<td>5000</td>
<td>7.416</td>
</tr>
<tr>
<td>E014</td>
<td>5000</td>
<td>7.347</td>
</tr>
<tr>
<td>E015</td>
<td>6000</td>
<td>7.143</td>
</tr>
<tr>
<td>E016</td>
<td>6000</td>
<td>7.167</td>
</tr>
<tr>
<td>E017</td>
<td>4000</td>
<td>8.163</td>
</tr>
<tr>
<td>E171</td>
<td>1500</td>
<td>10.409</td>
</tr>
<tr>
<td>E172</td>
<td>1500</td>
<td>10.366</td>
</tr>
<tr>
<td>E173</td>
<td>1500</td>
<td>10.351</td>
</tr>
<tr>
<td>E174</td>
<td>1500</td>
<td>10.366</td>
</tr>
<tr>
<td>E175</td>
<td>500</td>
<td>11.050</td>
</tr>
<tr>
<td>E176</td>
<td>500</td>
<td>10.955</td>
</tr>
<tr>
<td>E177</td>
<td>500</td>
<td>10.887</td>
</tr>
<tr>
<td>E178</td>
<td>1500</td>
<td>10.723</td>
</tr>
<tr>
<td>E179</td>
<td>1500</td>
<td>10.709</td>
</tr>
<tr>
<td>E180</td>
<td>1500</td>
<td>10.723</td>
</tr>
<tr>
<td>E181</td>
<td>500</td>
<td>11.249</td>
</tr>
<tr>
<td>E182</td>
<td>500</td>
<td>11.210</td>
</tr>
<tr>
<td>E183</td>
<td>500</td>
<td>11.622</td>
</tr>
<tr>
<td>E184</td>
<td>500</td>
<td>11.584</td>
</tr>
</tbody>
</table>
A5.4 GAMS Model Sections of Input File

A5.4.1 Main Field Only

Variables
  z Total drilling cost
  TotProd "Total production from field, BPD";

Equations
  cost Total drilling cost
  field Meet minimum production criteria;

  cost..  z =e= sum(i, Main(i, 'WellCost') * s(i));
  field.. sum(i, Main(i,'Product')*s(i)*FPR1) =g= MinProd;

Option
  optcr = 0.0
  mip = cplex;

Model
  Reservoir /all/;

Solve
  Reservoir using mip minimising z;

Option
  decimals=3;

A5.4.2 Main and Satellite Fields

Variables
  z Total drilling cost
  TotProd "Total production from field, BPD";

Equations
  cost Total drilling cost
  field Meet minimum production criteria
  satellite Design capacity of satellite field;

  cost..  z =e= sum(i, Main(i, 'WellCost') * s(i)) + sum(k, Satl(k, 'WellCost1') * t(k));
  field.. sum(i, Main(i,'Product') * s(i) * FPR1) +sum(k, Satl(k, 'Product1') * t(k) * FPR2) =g= MinProd;
  satellite.. sum(k, Satl(k, 'Product1') * t(k) * FPR2) =1= SatProd;

Option
  optcr = 0.0
  mip = cplex;

Model
  Reservoir /all/;

Solve
Appendix 5.0 Description of Two Field Model

Reservoir using mip minimising z;

Option
  decimals=3;

A5.4.3 Satellite Field Only

Variables
  \( z \) Total drilling cost
  \( \text{TotProd} \) "Total production from field, BPD";

Equations
  cost Total drilling cost
  field Meet minimum production criteria;

  cost.. \( z =e= \sum(k, \text{Sat}(k, 'WellCost1') * t(k)) \);
  field.. \( \sum(k, \text{Sat}(k, 'Product1') * t(k) * FPR2) =g= \text{MinProd} \);

Option
  optcr = 0.0
  mip = cplex;

Model
  Reservoir /all/;

Solve
  Reservoir using mip minimising z;

Option
  decimals=3;
A5.5 Example of GAMS Input File

$title Field Production Model - Model 19Y09.
$ontext
West Main Reservoir, East Satellite, input file BB06
Year 9. Production target 100000 BPD.
The model calculates the NPV for a main and satellite combination of fields.
Drilling costs are a function of fixed and variable costs.
There is no restriction on well reach.
$offtext

* Selection parameter: 2
Scalar
    FPR1 Productivity Reduction Factor / 0.9196 /
    FPR2 Productivity Reduction Factor / 0.9881 /
    MinProd Minimum production required from field / 100000 /
    SatProd Design capacity of satellite facility / 100000 /
$include West.inc
$include East.inc

Binary variable
    s(i) Selects or deselects individual main wells
    t(k) Selects or deselects individual satellite wells;

s. fx('W065') = 1;
s. fx('W066') = 1;
s. fx('W079') = 1;
s. fx('W080') = 1;
s. fx('W092') = 1;
s. fx('W093') = 1;
s. fx('W094') = 1;
s. fx('W095') = 1;
s. fx('W107') = 1;
s. fx('W108') = 1;
s. fx('W109') = 1;
s. fx('W122') = 1;
s. fx('W123') = 1;
s. fx('W124') = 1;
s. fx('W135') = 1;
t. fx('E015') = 1;
t. fx('E031') = 1;
t. fx('E043') = 1;

Variables
    z Total drilling cost
    TotProd "Total production from field, BPD"

Equations
    cost Total drilling cost
    field Meet minimum production criteria
    satellite Design capacity of satellite field;

    cost..  z =e= sum(i, Main(i, 'WellCost') * s(i)) + sum(k, Satl(k, 'WellCostl') * t(k));
    field..  sum(i, Main(i, 'Product') * s(i) * FPR1) + sum(k, Satl(k, 'Productl') * t(k) * FPR2) =g= MinProd;
    satellite.. sum(k, Satl(k, 'Productl') * t(k) * FPR2) =l= SatProd;

Option
    optcr = 0.0
Appendix 5.0 Description of Two Field Model

mip = cplex;

Model
   Reservoir /all/;

Solve
   Reservoir using mip minimising z;

Option
   decimals=3;
Appendix 5.0 Description of Two Field Model

A5.6 Example of GAMS Output File (Part listing)

GAMS Rev 133 Windows NT/95/98

Page 1

Field Production Model - Model 19Y09.

10 * Selection parameter: 2
12 Scalar
13 FPR1 Productivity Reduction Factor / 0.9196 /
14 FPR2 Productivity Reduction Factor / 0.9881 /
15 MinProd Minimum production required from field / 100000 /
16 SatProd Design capacity of satellite facility / 100000 /

INCLUDE C:\DOCUMENTS AND SETTINGS\RICHARD J BARNES\MY DOCUMENTS\PGS\MODELS\MODEL 19 TWO FIELD SINGLE PRODUCTION\WEST.INC

18 * Well site data for the West Reservoir
19
20 Sets
21 i Well locations
22 / W001*W224 /
23
24 j Well coordinates / Product, WellCost /
25
26 Table
27 Main(i,j) Coordinates of Production and Cost
28 Product WellCost
29 W001 3000 9.470
30 W002 3000 9.420
31 W003 2000 9.540

. .

246 W219 3000 10.296
247 W220 3000 10.486
248 W221 2000 10.744
249 W222 3000 10.812
250 W223 2000 11.086
251 W224 2000 11.259

INCLUDE C:\DOCUMENTS AND SETTINGS\RICHARD J BARNES\MY DOCUMENTS\PGS\MODELS\MODEL 19 TWO FIELD SINGLE PRODUCTION\EAST.INC

253 * Well site data for the East Reservoir
254
255 Sets
256 k Well locations
257 / E001*E184 /
258
259 l Well coordinates / Product1, WellCost1 /
260
261 Table
262 Sat1(k,l) Coordinates of Production and Cost
263 Product1 WellCost1
264 E001 5000 8.424
265 E002 5000 8.208
266 E003 6000 7.868

. .

445 E182 500 11.210
446 E183 500 11.622
447 E184 500 11.584

246
Appendix 5.0 Description of Two Field Model

448
449 Binary variable
450 s(i) Selects or deselects individual main wells
451 t(k) Selects or deselects individual satellite wells;
452
453 s.fx('W065') = 1;
454 s.fx('W066') = 1;
455 s.fx('W079') = 1;
456 s.fx('W080') = 1;
457 s.fx('W092') = 1;
458 s.fx('W093') = 1;
459 s.fx('W094') = 1;
460 s.fx('W095') = 1;
461 s.fx('W107') = 1;
462 s.fx('W108') = 1;
463 s.fx('W109') = 1;
464 s.fx('W122') = 1;
465 s.fx('W123') = 1;
466 s.fx('W124') = 1;
467 s.fx('W135') = 1;
468 t.fx('E015') = 1;
469 t.fx('E031') = 1;
470 t.fx('E043') = 1;

471 Variables
472 z Total drilling cost
473 TotProd "Total production from field, BPD";
474
475 Equations
476 cost Total drilling cost
477 field Meet minimum production criteria
478 satellite Design capacity of satellite field;
479
480 cost.. z =e= sum(i, Main(i, 'WellCost') * s(i)) + sum(k, Satl(k, 'WellCost1') * t(k));
481 field.. sum(i, Main(i, 'Product') * s(i) * FPR1) + sum(k, Satl(k, 'Product1') * t(k) * FPR2) =g= MinProd;
482 satellite.. sum(k, Satl(k, 'Product1') * t(k) * FPR2) =l= SatProd;
483
484 Option
485 optcr = 0.0
486 mip = cplex;
487
488 Model
489 Reservoir /all/;
490
491 Solve
492 Reservoir using mip minimising z;
493
494 Option
495 decimals=3;
496
497
498 ---- VAR s Selects or deselects individual main wells

LOWER LEVEL UPPER MARGINAL
### Appendix 5.0 Description of Two Field Model

#### W001 - W080

<table>
<thead>
<tr>
<th>Well</th>
<th>Lower</th>
<th>Level</th>
<th>Upper</th>
<th>Marginal</th>
</tr>
</thead>
<tbody>
<tr>
<td>W001</td>
<td>1.000</td>
<td>9.470</td>
<td></td>
<td></td>
</tr>
<tr>
<td>W002</td>
<td>1.000</td>
<td>9.420</td>
<td></td>
<td></td>
</tr>
<tr>
<td>W003</td>
<td>1.000</td>
<td>9.540</td>
<td></td>
<td></td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>W065</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>7.119</td>
</tr>
<tr>
<td>W066</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>7.046</td>
</tr>
<tr>
<td>W067</td>
<td>...</td>
<td>1.000</td>
<td>7.207</td>
<td></td>
</tr>
<tr>
<td>W068</td>
<td>...</td>
<td>1.000</td>
<td>7.230</td>
<td></td>
</tr>
<tr>
<td>W069</td>
<td>...</td>
<td>1.000</td>
<td>7.301</td>
<td></td>
</tr>
</tbody>
</table>
| ...   | ...   | ...   | ...   | ...

#### W070 - W222

<table>
<thead>
<tr>
<th>Well</th>
<th>Lower</th>
<th>Level</th>
<th>Upper</th>
<th>Marginal</th>
</tr>
</thead>
<tbody>
<tr>
<td>W070</td>
<td>1.000</td>
<td>7.594</td>
<td></td>
<td></td>
</tr>
<tr>
<td>W071</td>
<td>...</td>
<td>1.000</td>
<td>8.004</td>
<td></td>
</tr>
<tr>
<td>W072</td>
<td>...</td>
<td>1.000</td>
<td>8.267</td>
<td></td>
</tr>
<tr>
<td>W073</td>
<td>...</td>
<td>1.000</td>
<td>8.559</td>
<td></td>
</tr>
<tr>
<td>W074</td>
<td>...</td>
<td>1.000</td>
<td>8.740</td>
<td></td>
</tr>
<tr>
<td>W075</td>
<td>...</td>
<td>1.000</td>
<td>7.582</td>
<td></td>
</tr>
<tr>
<td>W076</td>
<td>...</td>
<td>1.000</td>
<td>7.706</td>
<td></td>
</tr>
<tr>
<td>W077</td>
<td>...</td>
<td>1.000</td>
<td>7.333</td>
<td></td>
</tr>
<tr>
<td>W078</td>
<td>...</td>
<td>1.000</td>
<td>7.167</td>
<td></td>
</tr>
<tr>
<td>W079</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>7.046</td>
</tr>
<tr>
<td>W080</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>6.972</td>
</tr>
<tr>
<td>W081</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>7.136</td>
</tr>
<tr>
<td>W222</td>
<td>...</td>
<td>1.000</td>
<td>10.812</td>
<td></td>
</tr>
<tr>
<td>W223</td>
<td>...</td>
<td>1.000</td>
<td>11.086</td>
<td></td>
</tr>
<tr>
<td>W224</td>
<td>...</td>
<td>1.000</td>
<td>11.259</td>
<td></td>
</tr>
</tbody>
</table>

#### E001 - E224

<table>
<thead>
<tr>
<th>Well</th>
<th>Lower</th>
<th>Level</th>
<th>Upper</th>
<th>Marginal</th>
</tr>
</thead>
<tbody>
<tr>
<td>E001</td>
<td>...</td>
<td>...</td>
<td>1.000</td>
<td>8.424</td>
</tr>
<tr>
<td>E002</td>
<td>...</td>
<td>...</td>
<td>1.000</td>
<td>8.208</td>
</tr>
<tr>
<td>E003</td>
<td>...</td>
<td>...</td>
<td>1.000</td>
<td>7.868</td>
</tr>
<tr>
<td>E015</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>7.143</td>
</tr>
</tbody>
</table>
| ...   | ...   | ...   | ...   | ...
| E182  | ...   | 1.000 | 11.210 |
| E183  | ...   | 1.000 | 11.622 |
| E184  | ...   | 1.000 | 11.584 |

<table>
<thead>
<tr>
<th>Well</th>
<th>Lower</th>
<th>Level</th>
<th>Upper</th>
<th>Marginal</th>
</tr>
</thead>
<tbody>
<tr>
<td>E015</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>7.143</td>
</tr>
</tbody>
</table>
| ...   | ...   | ...   | ...   | ...
| E182  | ...   | 1.000 | 11.210 |
| E183  | ...   | 1.000 | 11.622 |
| E184  | ...   | 1.000 | 11.584 |

#### VAR t

Selects or deselects individual satellite wells

<table>
<thead>
<tr>
<th>Well</th>
<th>Lower</th>
<th>Level</th>
<th>Upper</th>
<th>Marginal</th>
</tr>
</thead>
<tbody>
<tr>
<td>E001</td>
<td>...</td>
<td>...</td>
<td>1.000</td>
<td>8.424</td>
</tr>
<tr>
<td>E002</td>
<td>...</td>
<td>...</td>
<td>1.000</td>
<td>8.208</td>
</tr>
<tr>
<td>E003</td>
<td>...</td>
<td>...</td>
<td>1.000</td>
<td>7.868</td>
</tr>
<tr>
<td>E015</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>7.143</td>
</tr>
<tr>
<td>E182</td>
<td>...</td>
<td>1.000</td>
<td>11.210</td>
<td></td>
</tr>
<tr>
<td>E183</td>
<td>...</td>
<td>1.000</td>
<td>11.622</td>
<td></td>
</tr>
<tr>
<td>E184</td>
<td>...</td>
<td>1.000</td>
<td>11.584</td>
<td></td>
</tr>
</tbody>
</table>

#### VAR z

Total drilling cost

**z** Total drilling cost
A5.7  Example of Two Field Model Output File

NPV Analysis
Date: 16/11/2004
Time: 13:46:19
Page: 1 of 3

West Main Reservoir, East Satellite, input file CD07
Main Field Results

<table>
<thead>
<tr>
<th>Recoverable Reserves, MM bbl: 500.0</th>
<th>Platform Cost, $MM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Design Capacity, M BPD: 150.0</td>
<td>Pipeline Costs, $MM</td>
</tr>
<tr>
<td>Satellite production First Year: 7</td>
<td>Economic Parameters</td>
</tr>
<tr>
<td></td>
<td>Oil: 83.698</td>
</tr>
<tr>
<td></td>
<td>OPEX, % of CAPEX: 4.0</td>
</tr>
<tr>
<td></td>
<td>Discount Rate: % 10.0</td>
</tr>
<tr>
<td>Total: 650.728</td>
<td></td>
</tr>
<tr>
<td>Total: 143.215</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Topside</td>
<td>Pipeline</td>
<td>Drillex</td>
<td>Capital</td>
<td>Capital</td>
<td>Product</td>
</tr>
<tr>
<td>Yr</td>
<td>$MM</td>
<td>$MM</td>
<td>$MM</td>
<td>$MM</td>
<td>$MM</td>
</tr>
<tr>
<td>1</td>
<td>97.609</td>
<td>97.609</td>
<td>97.609</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>162.682</td>
<td>162.682</td>
<td>260.291</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>195.218</td>
<td>71.607</td>
<td>266.826</td>
<td>527.117</td>
<td>13.125</td>
</tr>
<tr>
<td>5</td>
<td>0.000</td>
<td>0.000</td>
<td>42.415</td>
<td>42.415</td>
<td>885.283</td>
</tr>
<tr>
<td>6</td>
<td>0.000</td>
<td>0.000</td>
<td>42.994</td>
<td>42.994</td>
<td>928.277</td>
</tr>
<tr>
<td>7</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>928.277</td>
</tr>
<tr>
<td>8</td>
<td>0.000</td>
<td>0.000</td>
<td>7.333</td>
<td>7.333</td>
<td>935.610</td>
</tr>
<tr>
<td>9</td>
<td>0.000</td>
<td>0.000</td>
<td>14.296</td>
<td>14.296</td>
<td>949.906</td>
</tr>
<tr>
<td>10</td>
<td>0.000</td>
<td>0.000</td>
<td>7.160</td>
<td>7.160</td>
<td>957.066</td>
</tr>
<tr>
<td>11</td>
<td>0.000</td>
<td>0.000</td>
<td>14.414</td>
<td>14.414</td>
<td>971.480</td>
</tr>
<tr>
<td>12</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>971.480</td>
</tr>
<tr>
<td>13</td>
<td>0.000</td>
<td>0.000</td>
<td>14.460</td>
<td>14.460</td>
<td>985.940</td>
</tr>
<tr>
<td>14</td>
<td>0.000</td>
<td>0.000</td>
<td>7.230</td>
<td>7.230</td>
<td>993.170</td>
</tr>
<tr>
<td>15</td>
<td>0.000</td>
<td>0.000</td>
<td>14.602</td>
<td>14.602</td>
<td>1007.772</td>
</tr>
<tr>
<td>16</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>1007.772</td>
</tr>
<tr>
<td>17</td>
<td>0.000</td>
<td>0.000</td>
<td>14.625</td>
<td>14.625</td>
<td>1022.397</td>
</tr>
<tr>
<td>18</td>
<td>0.000</td>
<td>0.000</td>
<td>7.324</td>
<td>7.324</td>
<td>1029.721</td>
</tr>
<tr>
<td>19</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>1029.721</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Oil Price</th>
<th>$5</th>
<th>$10</th>
<th>$15</th>
<th>$20</th>
<th>$25</th>
<th>$30</th>
</tr>
</thead>
<tbody>
<tr>
<td>NPV $MM</td>
<td>-108.463</td>
<td>776.580</td>
<td>1661.623</td>
<td>2546.666</td>
<td>3431.709</td>
<td>4316.751</td>
</tr>
<tr>
<td>IRR, %</td>
<td>7.99</td>
<td>20.09</td>
<td>27.54</td>
<td>33.10</td>
<td>37.58</td>
<td>41.34</td>
</tr>
<tr>
<td>------</td>
<td>------------------------</td>
<td>-----------------------</td>
<td>-------------------------</td>
<td>------------------------</td>
<td>---------------</td>
<td>----------</td>
</tr>
<tr>
<td>1</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>2</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>3</td>
<td>35.585</td>
<td>0.000</td>
<td>35.585</td>
<td>35.585</td>
<td>-35.585</td>
<td>-35.585</td>
</tr>
<tr>
<td>4</td>
<td>59.308</td>
<td>0.000</td>
<td>59.308</td>
<td>94.893</td>
<td>-59.308</td>
<td>-59.308</td>
</tr>
<tr>
<td>5</td>
<td>71.170</td>
<td>0.000</td>
<td>100.484</td>
<td>195.377</td>
<td>-71.170</td>
<td>-71.170</td>
</tr>
<tr>
<td>7</td>
<td>0.000</td>
<td>54.822</td>
<td>54.822</td>
<td>357.890</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>8</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>9</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>10</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>11</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>12</td>
<td>0.000</td>
<td>7.230</td>
<td>7.230</td>
<td>365.120</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>13</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>14</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>15</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>16</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>17</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>18</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>19</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>20</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>21</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>22</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>23</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
</tbody>
</table>
### NPV Analysis
West Main Reservoir, East Satellite, input file CD07
Combined Field Results

<table>
<thead>
<tr>
<th>Jckt + Topside Pipeline</th>
<th>Total</th>
<th>Cumul.</th>
<th>Cumul.</th>
<th>Net Cash Flow</th>
<th>Cash Inflow</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yr</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Annual</td>
<td>Cumul.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$MM</td>
<td>$MM</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$MM</td>
<td>$MM</td>
</tr>
<tr>
<td>1</td>
<td>97.609</td>
<td>97.609</td>
<td>97.609</td>
<td>-97.609</td>
<td>-97.609</td>
</tr>
<tr>
<td>2</td>
<td>162.682</td>
<td>162.682</td>
<td>260.291</td>
<td>-162.682</td>
<td>-260.291</td>
</tr>
<tr>
<td>3</td>
<td>230.803</td>
<td>71.607</td>
<td>302.410</td>
<td>-302.410</td>
<td>-562.702</td>
</tr>
<tr>
<td>4</td>
<td>254.526</td>
<td>71.607</td>
<td>48.926</td>
<td>-375.060</td>
<td>-937.761</td>
</tr>
<tr>
<td>5</td>
<td>71.170</td>
<td>29.314</td>
<td>42.415</td>
<td>108.060</td>
<td>180.474</td>
</tr>
<tr>
<td>6</td>
<td>71.170</td>
<td>29.314</td>
<td>50.201</td>
<td>150.685</td>
<td>250.996</td>
</tr>
<tr>
<td>7</td>
<td>0.000</td>
<td>0.000</td>
<td>54.822</td>
<td>126.617</td>
<td>221.439</td>
</tr>
<tr>
<td>8</td>
<td>0.000</td>
<td>0.000</td>
<td>7.333</td>
<td>9.375</td>
<td>10.747</td>
</tr>
<tr>
<td>9</td>
<td>0.000</td>
<td>0.000</td>
<td>14.296</td>
<td>19.778</td>
<td>34.075</td>
</tr>
<tr>
<td>10</td>
<td>0.000</td>
<td>0.000</td>
<td>7.160</td>
<td>11.314</td>
<td>18.477</td>
</tr>
<tr>
<td>11</td>
<td>0.000</td>
<td>0.000</td>
<td>14.414</td>
<td>129.141</td>
<td>248.585</td>
</tr>
<tr>
<td>12</td>
<td>0.000</td>
<td>0.000</td>
<td>7.230</td>
<td>9.460</td>
<td>16.730</td>
</tr>
<tr>
<td>13</td>
<td>0.000</td>
<td>0.000</td>
<td>14.460</td>
<td>14.260</td>
<td>28.720</td>
</tr>
<tr>
<td>14</td>
<td>0.000</td>
<td>0.000</td>
<td>14.531</td>
<td>14.631</td>
<td>30.162</td>
</tr>
<tr>
<td>15</td>
<td>0.000</td>
<td>0.000</td>
<td>14.602</td>
<td>14.893</td>
<td>31.595</td>
</tr>
<tr>
<td>16</td>
<td>0.000</td>
<td>0.000</td>
<td>14.671</td>
<td>14.962</td>
<td>32.633</td>
</tr>
<tr>
<td>17</td>
<td>0.000</td>
<td>0.000</td>
<td>21.927</td>
<td>22.974</td>
<td>44.901</td>
</tr>
<tr>
<td>18</td>
<td>0.000</td>
<td>0.000</td>
<td>21.974</td>
<td>23.971</td>
<td>45.945</td>
</tr>
<tr>
<td>19</td>
<td>0.000</td>
<td>0.000</td>
<td>58.633</td>
<td>58.633</td>
<td>117.266</td>
</tr>
<tr>
<td>20</td>
<td>0.000</td>
<td>0.000</td>
<td>55.771</td>
<td>55.771</td>
<td>111.542</td>
</tr>
<tr>
<td>21</td>
<td>0.000</td>
<td>0.000</td>
<td>53.044</td>
<td>53.044</td>
<td>106.087</td>
</tr>
<tr>
<td>22</td>
<td>0.000</td>
<td>0.000</td>
<td>54.695</td>
<td>54.695</td>
<td>110.390</td>
</tr>
<tr>
<td>23</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
</tbody>
</table>

**Oil Price** $5 $10 $15 $20 $25 $30

**NPV $MM** $134.417 1112.511 2359.438 3606.365 4853.292 6100.220

**IRR, %** 8.13 20.39 27.78 33.29 37.75 41.50

251
Appendix 6.0 Shell Sorting Method

The Shell sort method is named after Donald Shell, and is considerably faster than the bubble sort, particularly for long lists of data, Kassab (1984).

In a bubble sort, items are only compared with adjacent items. Therefore, if an item is a long way from its sorted location, it will require many moves to reach its correct location.

Consider the unsorted data:

5 7 3 8 1 2

In the first pass, the first element is compared with the second element and if larger is swapped, the second element is compared with the third and so on until the end of the list is reached:

Before: 5 7 3 8 1 2  After: 5 7 3 8 1 2  No swap necessary
Before: 5 7 3 8 1 2  After: 5 3 7 8 1 2
Before: 5 3 7 8 1 2  After: 5 3 7 8 1 2  No swap necessary
Before: 5 3 7 8 1 2  After: 5 3 7 1 8 2
Before: 5 3 7 1 8 2  After: 5 3 7 1 2 8

In the second pass:

Before: 5 3 7 1 2 8  After: 3 5 7 1 2 8
Before: 3 5 7 1 2 8  After: 3 5 7 1 2 8  No swap necessary
Before: 3 5 7 1 2 8  After: 3 5 1 7 2 8
Before: 3 5 1 7 2 8  After: 3 5 1 2 7 8
Appendix 6.0 Shell Sort Method

Before: 351278 After: 351278 No swap necessary

In the third pass:

Before: 351278 After: 351278 No swap necessary
Before: 351278 After: 315278
Before: 315278 After: 312578
Before: 312578 After: 312578 No swap necessary
Before: 312578 After: 312578 No swap necessary

In the fourth pass:

Before: 312578 After: 132578
Before: 132578 After: 123578
Before: 123578 After: 123578 No swap necessary
Before: 123578 After: 123578 No swap necessary
Before: 123578 After: 123578 No swap necessary

The bubble sort algorithm continues until there are no swaps in the final pass. Therefore a fifth pass is necessary to prove the list has been sorted.

In this example, the bubble sort required 5 passes and a total of 10 swaps.

The sorting of data that must be moved to the right of the list is fairly quick, but data that must travel right to left moves more slowly.
Appendix 6.0 Shell Sort Method

The Shell sort consists of a series of bubble sorts, initial over a wide gap and then progressively smaller gaps until adjacent elements are compared. Using the same example, the initial gap for a six element list is \( \text{int}(6/2) = 3 \).

5 7 3 8 1 2

In the first pass:

Before: 5 7 3 8 1 2  After: 5 7 3 8 1 2  No swap necessary

Before: 5 7 3 8 1 2  After: 5 1 3 8 7 2

Before: 5 1 3 8 7 2  After: 5 1 2 8 7 3

In the second pass is \( \text{int}(3/2) = 1 \):

Before: 5 1 2 8 7 3  After: 1 5 2 8 7 3

Before: 1 5 2 8 7 3  After: 1 2 5 8 7 3

Before: 1 2 5 8 7 3  After: 1 2 5 8 7 3  No swap necessary

Before: 1 2 5 8 7 3  After: 1 2 5 7 8 3

Before: 1 2 5 7 8 3  After: 1 2 5 7 3 8

In the third pass, with a gap of 1 again:

Before: 1 2 5 7 3 8  After: 1 2 5 7 3 8  No swap necessary

Before: 1 2 5 7 3 8  After: 1 2 5 7 3 8  No swap necessary

Before: 1 2 5 7 3 8  After: 1 2 5 7 3 8  No swap necessary

Before: 1 2 5 7 3 8  After: 1 2 5 3 7 8
Appendix 6.0 Shell Sort Method

Before: 1 2 5 3 7 8  After: 1 2 5 3 7 8 No swap necessary

In the fourth pass:

Before: 1 2 5 3 7 8  After: 1 2 5 3 7 8 No swap necessary
Before: 1 2 5 3 7 8  After: 1 2 5 3 7 8 No swap necessary
Before: 1 2 5 3 7 8  After: 1 2 3 5 7 8
Before: 1 2 3 5 7 8  After: 1 2 3 5 7 8 No swap necessary
Before: 1 2 3 5 7 8  After: 1 2 3 5 7 8 No swap necessary

A fifth pass is required to prove that the list is sorted.

The Shell sort required 5 passes and 8 swaps, so that, even on a short list, the Shell method is faster than the bubble sort.
### Appendix 7.0 References

<table>
<thead>
<tr>
<th>Reference</th>
<th>Title</th>
<th>Publisher/Location</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reference</td>
<td>Description</td>
<td></td>
</tr>
<tr>
<td>-----------</td>
<td>-------------</td>
<td></td>
</tr>
<tr>
<td>Reference</td>
<td>Details</td>
<td></td>
</tr>
<tr>
<td>-----------</td>
<td>---------</td>
<td></td>
</tr>
<tr>
<td>Reference</td>
<td>Description</td>
<td></td>
</tr>
<tr>
<td>-----------------------------------</td>
<td>-----------------------------------------------------------------------------</td>
<td></td>
</tr>
<tr>
<td>Reference</td>
<td>Title</td>
<td></td>
</tr>
<tr>
<td>-----------</td>
<td>-------</td>
<td></td>
</tr>
</tbody>
</table>