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2\(^2\)H(\(d, \gamma\))\(^4\)He reaction and the \(^4\)He D state

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The \(2\)(\(d, \gamma\))\(^4\)He reaction observables resulting from \(M1, E1, M2,\) and \(E2\) transitions are studied quantitatively. The calculations are in agreement with recent data for the reaction vector \((A_x)\) and tensor \((A_y)\) analyzing powers at 5 MeV center of mass energy and also with the best available theoretical predictions for the \(^4\)He D state wave function.

Since the recent first measurement of \(T_{20}\) for \(2\)(\(d, \gamma\))\(^4\)He by Weller et al.,\(^1\) it has been clear that the \(^4\)He D state plays a significant role in the capture reaction. Subsequent to these data and associated theoretical work\(^2\) there has been intense experimental interest in this reaction for incident deuteron energies from 50 keV to 100 MeV.\(^4\)\(^6\) A common aim of the experiments is to understand quantitatively these D-state contributions to the reaction and thus to extract an empirical measure of the magnitude of the D state from the data. Analyses to date\(^7\)\(^8\) assume the reaction is pure \(E2\) in nature.

In this Rapid Communication we confine discussion to the very recent data of Mellema et al.\(^9\) for the reaction vector \((A_x)\) and tensor \((A_y)\) analyzing powers at \(E_{\text{c.m.}} = 5\) MeV. These data are significant for two reasons. First, they unambiguously, through a large measured \(A_y\), the presence of multipole transitions (e.g., \(E1\) and \(M2\)) other than simply \(E2\). Second, \(A_y\) is the analyzing power least sensitive to ambiguities in the present theoretical treatment of the initial state interaction\(^2\) and thus a good observable to study \(^4\)He D-state effects. This Rapid Communication reports calculations which address these two points and which incorporate the most reliable available theoretical estimates of the \(^4\)He D-state amplitude.\(^8\)\(^9\)

The probability amplitude for transition from a continuum two-deuteron initial state \(|d^2\rangle|d^2\rangle_k\) to the \(J^\pi = 0^+\) \(^4\)He ground state \(|a^0\rangle\) with the emission of a photon of circular polarization \(e_q\) \((q = \pm 1)\) relative to the photon momentum \(k\), is

\[
T(\sigma_1, \sigma_2; k \rightarrow a k, \sigma_q) = \langle a^0| H_e(k, e_q)|d^2\rangle_k (d^2\rangle_k). \tag{1}
\]

Here \(\sigma_1\) and \(\sigma_2\) are the projections of the intrinsic spins \((S_1 = S_2 = 1)\) of the incident \((2)\) and target \((1)\) deuterons and \(k\) their asymptotic wave number in the c.m. frame. The interaction Hamiltonian for emission is

\[
H_e(k, e_q) = - \sum_{LMs} \sum_{I} q^2 T_{LM}^I(\pi) \mathcal{D}_{Iq}(R)^* \tag{2}
\]

where the \(T_{LM}^I(\pi)\) are multipole operators for electric \((e, \pi = 0)\) and magnetic \((m, \pi = 1)\) transitions.\(^10\)\(^11\) The rotation \(R\) takes the fixed coordinate system \(z\) axis into \(k\), and, in the Madison system \((z\) axis along \(k\), \(y\) axis along \(k \times k_y\), \(R = (0, \theta, 0)\) where \(\theta = \cos^{-1}(k \cdot k_y)\). Viewed as a one-step process, the capture amplitude can be expressed, using the Wigner-Eckart theorem,\(^11\) as a sum of terms involving matrix elements of the form

\[
\langle a^0| T_{LM}^I(\pi)|2s^+1I^J; JM'\rangle = -(2L + 1)^{-1/2} \delta_{IJ} \delta_{MM'} \langle a^0| T_L^I(\pi)|2s^+1I\rangle, \tag{3}
\]

where \(|2s^+1I^J; JM'\rangle\) is a two-deuteron initial state with channel spin \(s\), orbital angular momentum \(l\), and total angular momentum \(J\). Symmetry of the \(d + d\)-wave function also requires that \(+\) + even. Thus, for \(L \leq 2\) the reaction can proceed only through the following transitions \(|a^1| E1\rangle, \langle a^1| M1\rangle \left| D_1\rangle, \langle a^1| E2\rangle \left| S_2\rangle, \langle a^1| E2\rangle \left| D_2\rangle, \langle a^2| E2\rangle \left| G_2\rangle, \langle a^2| M2\rangle \left| P_2\rangle, \right. \text{ and} \right| a^2| M2\rangle \left| F_2\rangle. \right. \text{ These are listed Table I.}\)

We need to consider the explicit forms of the \(T_{LM}\). These are\(^11\)

\[
T_{LM}(\pi) = a_{I} \sum_i \left[ Q_{LM}(r_i) + Q_{LM}(r_i) \right], \tag{4}
\]

\[
T_{LM}(m) = a_{I}' \sum_i \left[ M_{LM}(r_i) + M_{LM}(r_i) \right], \tag{5}
\]

for electric and magnetic transitions, respectively. They are sums over all nucleons of one-body operators, functions of the position \(r_i\) of nucleon \(i\) relative to the c.m. of the sys-
TABLE I. Tabulation, by increasing multipole order, of the coefficients \( C_l^i(l_s; l's') \) in the radial overlaps \( \Delta^i_l^0(l_s; l's') \) of Eq. (12).

| Multipole | \( L \times n \) | \( a_l^i \) | Transition \( (2^+1_l \rightarrow 2^+1'_0) \) | Coefficient \( C_l^i(l_s; l's') \)
<table>
<thead>
<tr>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td>( E1 )</td>
<td>1 0</td>
<td>( ik \gamma )</td>
<td>( 3P_1 \rightarrow 1S_0 )</td>
<td>( k_{1/2}(2S+4\pi ) )</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>( 3P_1 \rightarrow 3D_0 )</td>
<td>( -k_{1/2}(4S+4\pi ) )</td>
</tr>
<tr>
<td>( M1 )</td>
<td>1 1</td>
<td>( k \gamma )</td>
<td>( 5D_1 \rightarrow 5D_0 )</td>
<td>( \sqrt{15}(\mu - \beta/2)/\sqrt{8\pi} )</td>
</tr>
<tr>
<td></td>
<td>2 0</td>
<td>( -k_1/(2\sqrt{3}) )</td>
<td>( 1D_2 \rightarrow 1S_0 )</td>
<td>( -\sqrt{3\epsilon/(4\sqrt{6})} )</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>( 3S_2 \rightarrow 3D_0 )</td>
<td>( -\sqrt{3\epsilon/(2\sqrt{6})} )</td>
</tr>
<tr>
<td></td>
<td>2 0</td>
<td>( -k_1/(2\sqrt{3}) )</td>
<td>( 3D_2 \rightarrow 1S_0 )</td>
<td>( +\sqrt{3\epsilon/(4\sqrt{6})} )</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>( 3G_2 \rightarrow 3D_0 )</td>
<td>( -9\sqrt{3}/(2\sqrt{6}) )</td>
</tr>
<tr>
<td>( M2 )</td>
<td>2 1</td>
<td>( ik_1/(2\sqrt{3}) )</td>
<td>( 3P_2 \rightarrow 1S_0 )</td>
<td>( -\sqrt{3\mu/(\sqrt{2\pi})} )</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>( 3P_2 \rightarrow 3D_0 )</td>
<td>( -\sqrt{3\mu/(10\sqrt{3\pi})} )</td>
</tr>
<tr>
<td></td>
<td>2 1</td>
<td>( 3F_2 \rightarrow 3D_0 )</td>
<td>( 3F_2 \rightarrow 3D_0 )</td>
<td>( 3\sqrt{21\mu/(10\sqrt{3\pi})} )</td>
</tr>
</tbody>
</table>

The constants \( a_l^i \) are collected in Table I for the \( EL \) and \( ML \) transitions of interest here. Under the assumption that the nucleon-nucleon interaction is charge independent, and hence, that isospin is a good quantum number, both the initial and final states have total isospin \( T=0 \). The capture therefore proceeds only through the isoscalar components of these operators. In the long wavelength approximation, these are

\[
Q_{LM}(\Delta T = 0) = (e/2) r_{c}^{i} C_{LM}(\tilde{r}_{i}) ,
\]

(6)

\[
Q_{LM}(\Delta T = 0) = -k_{p}(L+1)_{i} \left[ i r_{c}^{i} C_{LM}(\tilde{r}_{i}) \right] s_{i} ,
\]

(7)

\[
M_{LM}(\Delta T = 0) = \beta(L + 1) \nabla_{i} \left[ i r_{c}^{i} C_{LM}(\tilde{r}_{i}) \right] s_{i} ,
\]

(8)

\[
M'_{LM}(\Delta T = 0) = \mu \nabla_{i} \left[ i r_{c}^{i} C_{LM}(\tilde{r}_{i}) \right] s_{i} ,
\]

(9)

where \( \nabla_{i} \), \( I_{i} \), and \( s_{i} \) are gradient, orbital, and spin angular momentum operators for nucleon \( i \), \( \beta \) in the nuclear magneton, and \( \mu = (\mu_{p} + \mu_{p}) \beta, \mu_{p}, \mu_{p} \) in nuclear magnetons, the sum of the neutron and proton magnetic moments. The \( C_{LM} \) are the normalized spherical harmonics.\(^{11}\) In making the long wavelength approximation we assume \( (k_{p}r_{c})^{2} \) is small compared with unity. This approximation should be reasonable in the present application, where the \( r_{i} \) are restricted by the finite extent of the \( ^{4}\text{He} \) wave function, and at the energy of interest \( k_{p} \approx 0.15 \text{ fm}^{-1} \). Referring to the allowed one-step transitions, it is clear that in this long wavelength, \( \Delta T = 0 \) limit, \( \sum_{l} Q_{LM} = 0 \), and hence, that the \( \langle a | E1 | P_{i} \rangle \) transition must proceed via the spin-term \( Q \) of \( E1 \). No other terms vanish due to isospin considerations.

When calculating the \( EL, ML \) matrix elements we assume, as in previous work,\(^{1,2} \) that the nucleon coordinates \( r_{i} \) in the \( T_{LM} \) are proportional to the vector \( \rho = [r_{1} + r_{2}] - (r_{1} + r_{2})/2 \) joining the center of mass of deuteron 1 to deuteron 2. That is, the deuterons are “pointlike” for the purposes of estimating the transition operators. So, subsequent to operating with \( I_{i} \) in Eq. (7) and \( \nabla_{i} \) in Eqs. (8) and (9) we set \( r_{1} = r_{2} = -\rho/2 \). We also must set \( I_{i} = \lambda/4 \) in Eq. (8), where \( \lambda \) is the spin-orbit relative orbital angular momentum operator. With this “point-deuteron” approximation, and assuming that the internal wave functions of the deuterons are their dominant \( ^{3}S_{1} \) configurations, the structure of \( ^{4}\text{He} \) appears only through the two-deuteron-\( ^{4}\text{He} \) overlap

\[
\langle \rho; d_{l}^{0} d_{2}^{0} | a; 0^{+} \rangle = \frac{1}{2} \sum_{l'' = 0,2} (-1)^{l''} ((l'M'S_{2}S_{2}) | S_{1} - \sigma_{1} ) u_{1}(\rho) Y_{l''}^{i} d_{l''}^{0} ,
\]

(10)

which is an admixture of \( 1_{S_{0}} \) and \( 5_{D_{0}} \) two-deuteron configurations. The \( M2 \) transitions, Table I, thus require a deuteron spin-flip (change in channel spin) and therefore only the \( M' \) term of \( T_{LM}(m) \) can contribute.

To describe the initial state distortions we utilize the results of the one-channel resonating group model (RGM) calculations of Chwieroth, Tang, and Thompson,\(^{12,13} \) which give a good account of low energy \( d + d \) phenomena.\(^{14,15} \) The channel wave functions of that analysis, which conserves both \( l \) and channel spin \( s \), are not however explicitly \( J \) dependent and are thus denoted \( \chi_{ls} \) with phase shifts \( \sigma_{ls} \).

In this model (and the Madison coordinate system) all \( EL, ML \) matrix elements reduce to the following form:

\[
\langle a; 0^{+} | T_{LM}^{i}(\pi) | d_{l}^{0} d_{2}^{0} \kappa: k \rangle = \sum_{l's'} \alpha_{l's'}((S_{1} \sigma_{1} S_{2} \sigma_{2}) s M) \langle 10 s M | LM \rangle \Delta^i_{l}^{0}(l_{s}; l's') ,
\]

(11)

where the radial overlaps \( \Delta^i_{l}^{0}(l_{s}; l's') \) for transition from the \( 2^{+}1_{L} \) component of the initial state to the \( 2^{+}1_{0} \) component of \( ^{4}\text{He} \) are

\[
\Delta^i_{l}^{0}(l_{s}; l's') = C^i_{l}(l_{s}; l's') \int d\rho \rho^{2 + L - \epsilon_{s}} \chi_{l}(\rho) u_{l}(\rho) .
\]

(12)
The coefficients \( C_l \) are given in Table I. For reference, the last column of the table contains each amplitude a single letter identifier (\( A-H \)). Amplitudes \( A-D \), for the \( E2 \) transition, are precisely those of Ref. 2 but for an \( i^j \) phase factor, Eq. (11). As regards the description of the initial state, our treatment of \( A-D \) follows exactly the technique of Ref. 2. The required \( \chi_{\text{ini}} \) (\( l_s = 02,20,22,42 \)) are calculated from separable potentials fitted to the RGM phase shifts of Chwieroth et al.\(^ {12} \) Only two additional \( 2^+_1 \) channels are introduced by the \( E1 \), \( M1 \), and \( M2 \) multipoles, \( \chi_{\text{31}} \) in amplitudes \( E (E1) \) and \( F (M2) \), and \( \chi_{\text{131}} \) in \( G (M2) \). Amplitude \( H (M1) \) uses \( \chi_{\text{232}} \) which enters \( C (E2) \). The \( ^3F \) channel is very weakly distorted, with \( \delta_{\text{111}} \approx 1^\circ \); we thus set \( \chi_{\text{111}} = 4\pi j_3(k, \rho) \). The \( ^3P \) channel, however, is strongly distorted with \( \delta_{\text{111}} \approx 110^\circ \). In this channel \( \chi_{\text{111}} \) is calculated in an attractive spherical square well chosen to reproduce \( \delta_{\text{111}} \). To simulate the interaction of two extended deuterons \( (\rho^2)^{1/2} \approx 2 \) fm) we take a well of radius 4 fm, although calculations show little sensitivity to this choice.

The two-deuteron\(^ {4} \)He overlap functions \( u' \), Eq. (10), have recently been the subject of two theoretical studies.\(^ {8,9} \) We will utilize the analysis of Schiavilla, Pandharipande, and Wiringa\(^ {9} \)'s \( ^{4} \)He wave function, which that used in Ref. 6, includes the effects of realistic three-nucleon forces and is in better agreement with the experimental \( ^{4} \)He g.s. energy. They tabulate the \( u' \), in momentum space, for two realistic two-body interaction models, the Argonne and Illinois interactions, which yield \( ^{4} \)He D-state parameters \( D_z(d,a) \) (Ref. 14) of \(-0.16 \) fm\(^2 \) and \(-0.24 \) fm\(^2 \), respectively. Here the \( u' \) are calculated in Woods-Saxon wells,

\[ V_{\text{dd}}(\rho) = V_0 (1 + \exp[(\rho - \rho_0) / a_0]), \]

with geometries \( (\rho_0 \text{ fm, } a_0 \text{ fm}) \) chosen to model, as closely as possible, the momentum space forms of Ref. 9 and depths adjusted to reproduce the \( d-d \) separation energy.

For the Argonne interaction the geometries are \( (2.11, 0.75) \) for \( l_0 = 0 \) and \( (2.65, 0.9) \) for \( l_0 = 2 \). These normalized \( u' \) are now scaled to the tabulated values of Ref. 9 at low momenta.

We now consider the reaction observables. In the case of \( A_y \), the largest contributions will arise due to \( E2/E1 \) \[ \text{Im}(E/A) \] and \( E2/M2 \) \[ \text{Im}(F/A) \] interference terms, in particular the indicated cross terms of \( E \) and \( F \) with the dominant \( E2 \) amplitude \( A \). Both the \( E1 (E) \) and \( M2 (F) \) amplitudes are in fact dominated by the transition to the \( ^1S_0 \) state of \( ^{4} \)He. In the present model, which contains no \( ^3P_1 \) phase shift splitting, these large \( S \)-state contributions to the interference terms are in fact equal and have a \( -\cos \theta / \sin(2\theta) \) angular distribution.

For \( A_y \), \( E2/E2 \) interference terms dominate, and explicitly

\[ A_y \equiv 4/\sqrt{7} \text{Re}[C/A - 5D/(12A)] . \quad (13) \]

The important point is that, whereas all \( T_{24} \) contain amplitude \( B \), to first order \( A_y = -(T_{20} + \sqrt{6} T_{22}) / \sqrt{2} \) is independent of \( B \). Amplitudes \( A, C, \) and \( D \) are only weakly distorted, show very little sensitivity to the detailed short range behavior of the \( \chi_{\text{ini}} \) (provided they are regular at the origin and have the same phase shift), and are thus expected to be well described by the model used. Amplitude \( B \), on the other hand, is strongly distorted and should include (potentially large) contributions from the deuteron \( D \) state to the \( \langle a^0(S_0) | E2 | ^3S_2 \rangle \) transition, leading to the dominant \( S \)-state of \( ^{4} \)He. This contribution vanished upon making the “point deuteron” approximation for \( E2 \). These uncertainties in amplitude \( B \) are not present in the observable \( A_{yy} \).

In Fig. 1 the solid curve shows the calculated \( A_y \) which result from \( E1 + M1 + E2 + M2 \) transitions using the model detailed above. The dashed and dot-dashed curves show the results of the \( E2+E1 \) and \( E2+M2 \) calculations only. As stated above, these are essentially equal in the present model, except near \( 90^\circ \) where other small interference terms contribute. In the presence of \( E2 \) alone the calculated \( A_y \) has modulus \( \leq 0.01 \) and is asymmetric about \( 90^\circ \). The \( M1 \) contributions are very small. The slight asymmetry seen in the angular distribution is the result of small cross terms of \( E \) and \( F \) with \( B \) and \( C \). The agreement with the data is quite satisfactory, though there appears to be a small overestimation (25-30%) in the \( F \) and/or \( E \) amplitudes of the present calculation.

Figure 2 shows the calculated \( A_{yy} \). The solid curve corresponds to the same \( E1 + M1 + E2 + M2 \) calculation as shown for \( A_y \), for the \( ^{4} \)He wave function of the Argonne interaction \( D_z(d,a) = -0.16 \) fm\(^2 \). The dashed curve shows just the \( E2 \) contribution in this calculation. Clearly, and as expected, there being no \( \Re(E/A) \) and \( \Re(F/A) \) type cross terms in the tensor observables, the \( E1 \) and \( M2 \) contributions to \( A_{yy} \) are small. They are, however, responsible for the small asymmetry about \( 90^\circ \) observed in the calculation and evident in the data. The dot-dashed curve is the full \( E1 + M1 + E2 + M2 \) calculation with \( D_z(d,a) = -0.24 \) fm\(^2 \) of the Illinois interaction. It appears that with only moderately improved \( A_{yy} \) data very useful limits could be placed upon \( D_z(d,a) \), which

![FIG. 1. Calculated vector analyzing power \( A_y \) for the \( ^3H(d,\gamma)^4He \) reaction at \( E_{cm} = 5 \) MeV obtained when including \( E1+M1+E2+M2 \) transitions (solid curve), \( E1+E2 \) transitions (dashed curve), and \( M2+E2 \) transitions (dot-dashed curve). The \( ^4He \) wave function is that of the Argonne interaction (Ref. 9). The data are from Ref. 7.](image-url)
could, in turn, allow one to reject certain otherwise realistic two-body nucleon-nucleon interaction models.

We remark in concluding that the present calculations, which reproduce $A_y$ and $A_{yy}$, seriously underpredict the $T_{20}$ data of Weller et al., as shown in Fig. 3. This indicates that the $E2$ amplitude $B$, which plays an important role in $T_{20}$ (Ref. 2) but which is absent (in first order) from $A_{yy}$, is very poorly described by the present model, as was discussed earlier. Clearly a microscopic calculation of the $S_2 = d-d$ channel would clarify this point. Alternatively, given that the $E2$ amplitudes $A$, $C$, and $D$ are essentially model-independent with regard to the initial state distortions; together, the $A_y$ and $T_{20}$ data are sufficient to determine $B$ empirically [actually Re($B/A$)]. The $B$ so determined not only reproduces $T_{20}$ but produces a positive going peak in $A_y$ near 90° as required by the data (Fig. 2) but absent from the present calculation. We will report fully on the results of this investigation in a subsequent article.

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