(d,α) reaction and the α particle D state

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(Received 12 May 1983)

The recent suggestion that the effects of a D-state component of the α particle are being seen in (d,α) reaction tensor analyzing power data is reexamined using exact finite range distorted-wave Born approximation calculations. Results for 32S(d,α)30P(g.s.), at E = 16 MeV, are at considerable variance with two-nucleon-core relative motion amplitudes derived from shell model calculations.

\[ \langle \vec{R} \mid \phi_\alpha \mid \phi_\alpha \rangle \right| V(\text{dd}) \right| \phi_\alpha \right] \]

\[ = \frac{1}{\hbar} \sum \left( -1 \right)^{L}(1\lambda_3\sigma_1|s_2-\sigma_2)v(R)Y_\lambda(R), \] (1)

where \( V(\text{dd}) \) is the interaction between the transferred and spectator deuterons. The earlier investigation of Santos et al.1 which was based upon a plane wave peripheral model for the reaction amplitude, indicated that calculated Cartesian tensor analyzing powers, \( A_\alpha \) and \( A_\gamma \), in particular, are expected to show very marked changes as a result of the presence of the α particle D state. These calculations are, however, lacking in three respects. Firstly, they use a simple model for the already approximate one-step amplitude. Secondly, they do not include spin-orbit distortion in the deuteron channel. This will lead to nonzero tensor analyzing powers even in the absence of an α particle D-state component. The magnitude of this effect clearly needs to be investigated. Finally, as the transferred spin in a (d,α) reaction is one, \( J^P = (\text{odd})^+ \) transitions on a 0+ target have the property that two orbital angular momentum configurations \( L = J \pm 1 \), with amplitudes \( F(J,L) \), are allowed for the deuteron in the target nucleus. These two \( L \) contributions add coherently to the full transfer amplitude, as do the α particle S- and D-state amplitudes. The calculations of Santos et al.7 take only the dominant L component, as determined by the DWBA analysis of the corresponding vector analyzing power, \( IT_\alpha \), data.8 The \( IT_\alpha \) data are, in general, well reproduced in the one-step transfer model.

The direct observation of the D-state component for the α particle is more complicated than that of the deuteron, triton, and 3He in the (d,α) reaction, as the spin transfer is one, the simplest DWBA calculation, which neglects spin-dependent channel distortions, will generate nonzero tensor analyzing powers even in the absence of the D state. The one case in which this is not so is when the deuteron is initially bound in the target with orbital angular momentum \( L = 0 \), and thus \( J = 1 \). Analysis of the \( IT_\alpha \) data indicates that the dominant configuration of a deuteron cluster, in the ground state of 32S, is of this form. We therefore pay particular attention to the reaction 32S(d,α)30P(g.s.), for which data exist. In the absence of \( L = 2 \) admixtures in 32S, and deuteron spin-orbit distortion, the calculated tensor analyzing powers for this reaction should be zero in the absence of a d-d relative D state.

In this Rapid Communication we present exact finite range calculations for the data previously analyzed by Santos et al.7 The accuracy of the local energy approximation (LEA) prescription for the calculation of finite range effects8 is also investigated. We consider, in some detail for 32S, the effects which arise due to the admixture of different \( L \) components in the target system. In the calculations presented, the α optical potentials were taken from the original data analysis of Ludwig et al.8 The deuteron potentials are taken from the global analysis of Daenick, Childs, and Vrecl.10 Calculations were performed using a modified version of the computer program TDFN.11

In Figs. 1 to 3, the dashed and solid curves show the results of exact finite range calculations, for

\[ ^{32}\text{S}(d,\alpha)^{30}\text{P}(g.s.;1^+,L=0), \]

\[ ^{38}\text{Ar}(d,\alpha)^{36}\text{Cl}(1.60 \text{ MeV;1}^+,L=2), \] and

\[ ^{36}\text{Ar}(d,\alpha)^{34}\text{Cl}(2.18 \text{ MeV;3}^+,L=2) \]

at 16 MeV, which include the \( S \) and \( S + D \) states, respectively, of the α particle. Calculations correspond to a \( D_\alpha(d,\alpha) \) value of \(-0.31 \text{ fm}^2\), which gives a reasonable description of the data. The \( L \) values indicated are the
dominant components, as deduced from the $iT_{11}$ angular distributions, and those used in these calculations.

In the absence of a microscopic form for the overlap, Eq. (1), the calculations use a simple model form factor constructed as follows. The dominant $1^3D_0$ state of the $\alpha$ particle is expected to be that reached from the ground state by the single action of the nucleon-nucleon tensor force, thus, the $S$- and $D$-state components of the d-d relative motion are assumed to be $0S$ and $0D$ states. These $S$- and $D$-state radial functions are obtained as eigenfunctions of a simple potential well, the depth of which was adjusted to reproduce the d-d separation energy (23.85 MeV). In the case of a Wood-Saxon well, for a fixed diffuseness parameter ($a = 0.5$ fm), the radius parameter was adjusted until $(r_0 = 1.5$ fm) the generated $S$-state radial form factor had a range parameter, $\beta = 1.46$ fm$^{-1}$, consistent with that used by Santos et al. The amount of $D$-state admixture could then be adjusted so that the total wave function corresponded to any given $D_j(d, \alpha)$ value. Generating the wave function in the same way, but using a Gaussian interaction $V(dd) = V_0 \exp[-(r/\gamma)^2]$, $\gamma = 2.0$ fm, which corresponds to $\beta = 1.46$ fm$^{-1}$, produced the same calculated observables, for the same $D_j(d, \alpha)$.

To further check the insensitivity of the calculations to the precise, and presently unknown, short-range behavior of the overlap, a LEA calculation was performed. Here, the configuration space form factor, Eq. (1), was constructed as

**FIG. 1.** Calculated Cartesian tensor analyzing powers, $A_{xx}$ and $A_{yy}$, for the reaction $^{30}Si(d, \alpha)^{30}P(E_x=0.00$ MeV, $J^\pi=1^+$) at 16 MeV. The curves are finite range DWBA calculations which include (i) the $\alpha$ particle $S$ state only (dashed curves) and (ii) the $\alpha$ particle $S$ and $D$ states (solid curves). In cases (i) and (ii) the $\alpha$ particle wave function, generated in a Wood-Saxon well, corresponds to $\beta = 1.46$ fm$^{-1}$ and $D_2(d, \alpha) = -0.31$ fm$^2$. In curves (iii) the $S$ and $D$ states were treated in the LEA with the same $D_2(d, \alpha)$ and $\beta$ values (dot-dashed curves). In all cases a pure $L=0$ configuration [$F(1,0) = 1, F(1,2) = 0$] for the deuteron in $^{38}Ar$ is assumed.

**FIG. 2.** Calculated $A_{xx}$ and $A_{yy}$ for the reaction $^{38}Ar(d, \alpha)^{36}Cl(E_x=1.60$ MeV, $J^\pi=1^+$) at 16 MeV. The curves are as in Fig. 1 and a pure $L=2$ configuration [$F(1,2) = 1, F(1,0) = 0$] for the deuteron in $^{38}Ar$ is assumed.

**FIG. 3.** Calculated $A_{xx}$ and $A_{yy}$ for the reaction $^{38}Ar(d, \alpha)^{34}Cl(E_x=2.18$ MeV, $J^\pi=3^+$) at 16 MeV. The curves are as in Fig. 1 and a pure $L=2$ configuration [$F(3,2) = 1, F(3,4) = 0$] for the deuteron in $^{38}Ar$ is assumed.
A sum of Gaussians,

\[ v_i(R) = \sum B_i R^\gamma \exp \left\{ - (R/\gamma)^2 \right\}, \]

and the \( B_i, \gamma_i \) adjusted such that, in \( K \) space, for small \( K_i \),

\[ v_0(K) = D_0 \left[ 1 - K^6/\beta^6 + O(K^6) \right] + \cdots, \]

\[ v_2(K) = D_2K^2 \left[ 1 + O(K^4) \right] + \cdots, \]

where

\[ v_i(K) = \sqrt{4\pi} \int dR R^{i-2} |\langle KR \rangle| v_i(R) . \]

The solid and dot-dashed curves in Fig. 1 show the calculations performed using the Wood-Saxon and LEA form factors, respectively, to be in good agreement. Clearly the calculated tensor analyzing powers in low energy \((d,\alpha)\) reactions depend, to good approximation, upon the \( S \) and \( D \) states of the \( \alpha \) particle only through \( D_2(d,\alpha) \) and \( \beta \). Uncertainties in the precise microscopic form of the overlap are therefore of no concern for the analysis of these data.

It is clear from Fig. 1 that the effect of deuteron spin-orbit distortion cannot, in the absence of the \( \alpha \) particle \( D \) state, explain the experimental analyzing power data. Spin-orbit distortion alone produces small and oscillatory analyzing powers (dashed curves), whereas the data are large and primarily of one sign. The gross features of the data are, however, reproduced when the \( \alpha \) particle \( D \)-state component is included with \( D_2(d,\alpha) = -0.31 \, \text{fm}^{-2} \). Comparison of these calculations with those of Santos et al.7 shows the model calculations employed there to be surprisingly reliable. The present calculations differ only in there being oscillatory structure built upon the smooth angular dependence obtained by Santos et al.

The present calculation for \( ^{32}\text{S} \) looks the most encouraging and, under the assumption that the deuteron was initially bound in \( ^{32}\text{S} \) in a relative \( L = 0 \) state \( \{F(1,0) = 1, F(1,2) = 0\} \), would appear to provide clear evidence of the observation of the \( \alpha \) particle \( D \) state. However, in Fig. 4 we present calculations in which we relax the \( L = 0 \), or \( F(1,2) = 0 \), condition. We find that results very similar to the \( F(1,2) = 0 \) situation can be obtained by introducing an amplitude \( F(1,2) = -0.34 \) for finding the deuteron in a \( L = 2 \) state in \( ^{32}\text{S} \) and no \( D \) state component in the \( \alpha \) particle, \( D_2(d,\alpha) = 0 \, \text{fm}^{-2} \). [With the adopted phase convention a negative \( F(1,2)/F(1,0) \) corresponds to \( L = 0 \) and \( L = 2 \) radial wave functions in \( ^{32}\text{S} \) with the same phase asymptotically.] Also, the dot-dashed curve shows that intermediate situations, with the \( D \)-state strength shared between \( ^{32}\text{S} \) and the \( \alpha \) particle, e.g., \( F(1,2) = -0.2 \), \( D_2(d,\alpha) = -0.2 \, \text{fm}^{-2} \), produce very similar results. In all cases, \( L = 2 \) admixtures of these magnitudes produced only small changes in the calculated cross section and vector analyzing power angular distributions. There exists, therefore, because of the symmetry in spin structure in this reaction, \( 1^+ + 0^- \rightarrow 0^+ + 1^- \), symmetry in the calculation to whether the required \( D \) state is included in the target or projectile bound states. No significant improvement in agreement with the \( A_\alpha \) and \( A_\phi \) data was obtained when an \( L = 0 \) (\( L = 4 \)) admixture was introduced in the \( ^{36}\text{Ar}(^{20}\text{Ar}) \) target system.

Faced with this situation we appeal to nuclear structure calculations to help clarify the dominant components of the transferred nucleons. These configurations have recently been studied, for the \( ^{28}\text{S}(d,\alpha)^{30}\text{P}(g.s.) \) reaction, by de Meijer et al.13 With use of the shell model wave functions for \( ^{20}\text{P} \) and \( ^{32}\text{S} \), generated using \( i \) the free-parameter surface \( \delta \) interaction (FPSDI) of Wildenthal et al.14 and \( ii \) the effective interaction of Chung and Wildenthal, the single particle occupancies of the transferred nucleons about \( ^{20}\text{P} \) were evaluated. The Bayman-Kallio projection method15 was then used to calculate the two-nucleon-core relative motion amplitudes under the assumption that, in the two-nucleon cluster, the neutron and proton move in a relative triplet \( S \) state. These cluster amplitudes, \( G(JL) \), can be compared to the amplitudes \( F(JL) \), introduced above, which represent the probability amplitude for finding a deuteron in a normalized state \( (JL) \) about the core.

The shell model calculations give, in the \( ^{28}\text{S}(g.s.) \) case, \( G(1,2)/G(1,0) = 0.57 \) and \( G(1,2)/G(1,0) = 1.43 \) for the FPSDI and Chung-Wildenthal interactions, respectively. These values are considerably larger, and of opposite sign, to the value \( F(1,2)/F(1,0) = -0.34 \) which describes the \( (d,\alpha) \) tensor analyzing power data in the absence of the \( \alpha \) particle \( D \) state. The effect of changing the sign of \( F(1,2)/F(1,0) \), in the reaction calculations, is essentially to reverse the sign of the calculated tensor analyzing powers. The experimental data cannot, therefore, be reproduced by calculations in which the \( L = 2 \) cluster-core component is as predicted by the shell model.

There is little clear evidence, from the reaction data presently analyzed, for the direct observation of the \( \alpha \) particle \( D \) state. For the \( ^{20}\text{Ar}(d,\alpha) \) reaction the deuteron-target
core relative motion is dominantly $L=2$, thus, the $\alpha$ particle $D$ state introduces only a small additional effect upon the calculated tensor analyzing powers. In the case of the $^{36}Ar(d,\alpha)$ reaction the agreement with data is not sufficiently good to draw conclusions. The $^{32}S(d,\alpha)^{30}P(g.s.)$ reaction warrants further study. The available data at 16 MeV do not discriminate between the observed $D$-state effects arising from the projectile or the target bound state systems. In either instance, the data cannot be reconciled with the predictions of the Wildenthal et al. and Chung-Wildenthal shell model states for $^{32}S$ and $^{30}P$. Additional precision data for the $^{32}S(d,\alpha)^{30}P$ reaction, at a different low bombarding energy, might help clarify this situation. Further data, in an energy region where the LEA remains valid, would mean that the $(d,d|\alpha)$ form factor enters the calculation in the same way as at 16 MeV. The $L=0$ and $L=2$ components of the $(^{30}P,d|^{32}S)$ form factor, on the other hand, will be populated differently as a function of the incident deuteron energy. Two such data sets would provide a consistency check for the derived $\alpha$ particle and $^{32}S$ $D$-state strengths. It is clear that experimental and theoretically calculated tensor analyzing powers, for low energy $(d,\alpha)$ reactions, provide a very sensitive test for two-nucleon cluster-core amplitudes deduced from nuclear structure studies. It is not clear, however, that useful results can be obtained using available single particle shell model configurations as input to cluster transfer reaction calculations.

I would like to thank Dr. N. Kishida for supplying a copy of the program TWOPNR. The financial support of the Science and Engineering Research Council is gratefully acknowledged.