Tensor interaction effects in the $^4\text{He}(^2\text{H},\gamma)^6\text{Li}$ capture reaction

R. Crespo, (1), A. M. Eiró, (2) and J. A. Tostevin (1)

(1) Department of Physics, University of Surrey, Guildford, Surrey, GU2 5XH, United Kingdom;
(2) Centro de Física Nuclear, Universidade de Lisboa, 1699 Lisboa Codex, Portugal

(Received 26 March 1990)

The spin dependence of the $^4\text{He}(^2\text{H},\gamma)^6\text{Li}$ capture reaction is studied in a deuteron-alpha-particle direct capture model. Nucleon-nucleon tensor interaction effects, manifest through the deuteron and $^6\text{Li}$ internal $D$ states and through the deuteron-alpha-particle tensor interaction, are included within the $d+\alpha$ cluster model. The entrance channel tensor force, calculated from the folding model, is shown to produce relatively minor effects on the calculated reaction tensor analyzing powers compared to those of the $^6\text{Li} D$ state for center-of-mass energies of proposed experiments ($\approx 10$ MeV).

I. INTRODUCTION

The total reaction cross section of the $^4\text{He}(^2\text{H},\gamma)^6\text{Li}$ reaction at low (astrophysical) incident deuteron energies has recently been the subject of comprehensive theoretical calculations. $^1$–$^3$ Experimental total cross section data and differential cross section angular distributions have also been available for some time $^1$ for this system. The fact that measurements of the capture reaction tensor analyzing powers are being planned $^4$ through the use of a tensor polarized deuteron beam, is very exciting. It is widely thought that such data, sensitive in first order to tensor amplitudes in the process, should help to clarify the role of the nucleon-nucleon tensor interaction in the $^6\text{Li}$ nucleus, as would be revealed through a small $D$-state amplitude in the nuclear wave function. This has certainly proved to be the case in the analysis of the now extensive experimental data for the analogous $^2\text{H}(^2\text{H},\gamma)^4\text{He}$ reaction (e.g., Ref. 5). Presently, there are contradictory theoretical estimations as to even the sign of the $D$-state amplitude in the case of the $^6\text{Li}$ nucleus. $^6$, $^7$

In a recent short communication, Crespo $et$ $al.$ $^3$ investigated the role of the $^6\text{Li} D$ state in the $^4\text{He}(^2\text{H},\gamma)^6\text{Li}$ reaction within a simplified deuteron-alpha-particle direct capture model. In particular, the resonant nature of the entrance channel and the strong state dependence of the entrance channel interactions revealed by the experimental $d+\alpha$ phase shifts, $^9$ was neglected. In the absence of this spin-dependent entrance channel distortion, the calculated tensor analyzing powers vanish identically in the absence of the $^6\text{Li} D$-state amplitude. As a result, the $^6\text{Li} D$-state effects appear very clearly delineated, suggesting that the reaction may be used to obtain unambiguous information about the deformation of the Lithium nucleus. This $D$-state signal might not be expected to appear so clearly in the presence of a realistic entrance channel description.

In this paper we present calculations of the effects of the small $^6\text{Li} D$-state components, predicted by theory, on the reaction tensor analyzing powers in the presence of realistic $d+\alpha$ distortions. Additional tensor effects, originating in the nucleon-nucleon tensor interaction, namely the deuteron $D$-state component and the resulting $d+\alpha$ tensor interaction, which are likely to affect strongly the calculated tensor analyzing powers, are treated consistently through the use of a realistic deuteron wave function together with the folding model for the $d+\alpha$ interaction. The folding model has been used successfully in the past $^1$ to model accurately the low energy properties of the $d+\alpha$ system.

II. THE TRANSITION AMPLITUDE

The transition amplitude for the capture of an incident deuteron (with center-of-mass momentum $k$ and spin projection $\sigma_d$) and the $\alpha$ particle to produce the $^6\text{Li}$ ground state in spin projection $\Sigma$ with the emission of a photon of polarization $\epsilon_q$ ($q = \pm 1$) and wave number $k_e$ is written $^{11}$

$$T(\alpha \sigma_d, k \rightarrow ^6\text{Li} \Sigma, k_e, \epsilon_q)$$

$$= \langle ^6\text{Li} \Sigma | H_e(k_\Sigma, \epsilon_q) | \alpha \sigma_d k \rangle .$$  \hspace{1cm} (1)

The interaction Hamiltonian for photon emission, $H_e$, is written, in first order perturbation theory,

$$H_e(k_\Sigma, \epsilon_q) = - \sum_{L, \nu} q T_{L, \nu}^{+} \langle \nu \nu | D''_{\nu \nu} (\hat{R})^* \rangle,$$  \hspace{1cm} (2)

where the sum extends over electric ($e=0$) and magnetic ($m, \nu = 1$) transitions of all multipole orders $L$. The $T_{L, \nu} (\pi)$ are the appropriate multipole transition operators$^{11}$ and $D$ is the rotation matrix. In this work we adopt analyzing powers $T_{2y}$ referred to the Madison coordinate system$^{12}$ and thus the rotation $\hat{R}$ entering Eq. (2), which takes the reaction $z$ axis into the outgoing photon direction $k_\Sigma$, is simply $\hat{R} = (0, \theta, 0)$ where $\theta = \cos^{-1}(\langle \hat{k} \cdot \hat{k}_\Sigma \rangle)$.

In the $d+\alpha$ cluster model, allowing for a tensor interaction between the deuteron and $\alpha$, with separation $p = [1/2 (r_\alpha + r_\Sigma) - r_\alpha]$, the entrance channel wave function is written...
where we have defined

\[ \chi_{LL'}^{\kappa}(k, p) = 4\pi i e^{i\sigma_L} \{ F_L(k, p) \delta_{LL'} + T_{LL'}^{\sigma_L} [G_L(k, p) + iF_L(k, p)] \} / k \rho. \]  

Here \( F_L \) and \( G_L \) are the regular and irregular Coulomb functions, \( T_{LL'}^{\sigma_L} \) are the partial wave transition amplitudes, \( \sigma_L \) is the Coulomb phase, and the \( \alpha \) and deuteron ground state wave functions have been denoted \( \phi_\alpha \) and \( \phi_d^\rho \), respectively.

It follows that

\[ \langle ^6\text{Li}; 1 | H_\alpha(k, \sigma_d) | \alpha, d \sigma_d; k \rangle = \sum_{J'M} g_{LL'}(k, \sigma_d) \langle ^6\text{Li}; 1 | T_{J'M}(\pi) | \alpha, d \sigma_d; k \rangle, \]

where

\[ \langle ^6\text{Li}; 1 | T_{J'M}(\pi) | \alpha, d \sigma_d; k \rangle = \sum_{J'M} (L \alpha 1 \sigma_d | JM) Y_{L\alpha}(\hat{k}) \langle ^6\text{Li}; 1 | T_{J'M}(\pi) | kL; J(L'1)M \rangle. \]

It also proves convenient to define a reduced matrix element, by the relation

\[ \langle ^6\text{Li}; 1 | T_2^+(\pi) | kL; J(L'1)M \rangle = (L, M) \langle ^6\text{Li}; 1 | T_2^+(\pi) | kL; J(L'1) \rangle. \]

The \( ^6\text{Li} \) ground state wave function is similarly written, in the cluster model, as

\[ \langle \rho | \alpha, d \sigma_d; k \rangle = \sum_{JMLAL'} (L \alpha 1 \sigma_d | JM) Y_{L\alpha}(\hat{k}) \times \langle \rho | kL; J(L'1)M \rangle, \]

where

\[ \chi_{LL'}^{\kappa}(k, p) = 4\pi i e^{i\sigma_L} \{ F_L(k, p) \delta_{LL'} + T_{LL'}^{\sigma_L} [G_L(k, p) + iF_L(k, p)] \} / k \rho. \]  

and where asymptotically, the radial functions \( \chi_{LL'}^{\kappa} \) are normalized to

\[ Y_{L\alpha}(\hat{\rho}) \phi_\alpha \phi_d^\rho. \]

Within the model presented, the reduced matrix elements for the two components of the \( E2 \) transition operator reduce to the forms

\[ \langle ^6\text{Li}; 1 \| T_2^+(\rho) \| kL; J(L'1) \rangle = \alpha_2^E \sum_{l=0, 2} \frac{2}{\sqrt{3}} k \rho \sum_{L020} (L020|10) W(2IJ1; L'1)J(4, JLL'), \]

and

\[ \langle ^6\text{Li}; 1 \| T_2^+(\rho) \| kL; J(L'1) \rangle = \alpha_2^E \frac{3\sqrt{10}}{2} eQ_d W(21L'; 1J)J(2, L'JLL'), \]
where $Q_d$ is the deuteron ground state quadrupole moment and $J$ represents the radial overlap

$$I(n,JLLL') = \int dp p^n u_l(r) \chi_L^J(k,r)$$

(16)

describing the dynamics of the process. We note that, whereas the $d-\alpha$ relative motion amplitude, Eq. (14), introduces the $n=4$ radial moment in the relevant overlap integral, the $n-p$ relative motion term, arising from the deuteron D state, Eq. (15), produces only an $n=2$ radial weighting. Given therefore the weak binding (1.473 MeV) of the $d+\alpha$ system and the resulting long range tail of the $d-\alpha$ relative motion wave functions, $u_l(r)$, one expects that the first of these amplitudes will completely dominate the capture mechanism, and will probe the asymptotic region of the $u_l(r)$. Indeed we find that the contribution to the reaction observables arising from the deuteron D-state term, Eq. (15), is completely negligible when compared with other uncertainties in the model. Similar results were obtained by Langanke et al.²

Also evident from Eq. (14) is that the presence of a tensor interaction in the entrance channel distortion opens a route whereby the incident $^3S_1$ wave, through its tensor coupling to the $^3D_1$ state, will couple by the $E2$ multipole to the $S$-state component of the $^6Li$ ground state. In the absence of a tensor interaction of course $L'=L$ and the incident $^3S_1$ state can couple only with the ground state $D$ wave component. We obtain a quantitative estimate of the importance of this additional amplitude in the following by taking the folding model prediction as a reasonable theoretical estimate of the entrance channel tensor interaction.

III. THE $d-\alpha$ RELATIVE MOTION WAVE FUNCTIONS

In treating the entrance channel, the radial functions $\chi_{LL'}$, have been calculated using two distinct local potential descriptions, one with and one without a tensor interaction. In the case where we neglect the tensor interaction the $d-\alpha$ wave functions will be calculated from the phenomenological potential model of McIntyre and Haebelir,¹³ which contains only central and spin-orbit components, but which provides an accurate reproduction of the phase shift data.

In the case in which we include the tensor interaction, the deuteron-alpha interaction is calculated numerically according to the folding model. Now the deuteron-alpha interaction takes the form

$$U_{d\alpha}(\rho) = \langle \phi_d | V_n(|\rho - r/2|) + V_p(|\rho + r/2|) | \phi_d \rangle$$

$$= V_C(\rho) + V_S(\rho)\mathbf{L} \cdot \mathbf{S} + V_T(\rho) T_R$$

(17)

where $V_n$ and $V_p$ are the underlying neutron- and proton-alpha-particle interactions, $T_R = (\mathbf{S} \cdot \hat{\mathbf{p}})^2 - \frac{1}{2}$ characterizes the tensor interaction arising from the deuteron D state and $S$ the deuteron spin operator.

This folded potential was constructed, in configuration space, using the methods of Keaton et al.¹¹ The nucleon-alpha potentials were taken from the analysis of Batty et al.¹² and the deuteron wave function used was that of the Reid soft core¹⁶ interaction. In order to make a meaningful comparison between calculations of reaction observables made with these two interactions, and in particular their energy dependence, it is vital that both interactions should reproduce the positions of the $^3D_3$, $^3D_2$, and $^3D_1$ resonances in the entrance channel. In order to achieve this, in the case of the parameter free folded interactions, we scaled the strength of the central part of the calculated interaction by a factor $C_{LL}$ so as to best reproduce the $d-\alpha$ scattering phase shifts. Only small adjustments from unity were required, the fitted coefficients being $C_{13} = 1.006$, $C_{22} = 0.98$, and $C_{21} = C_{01} = 1.08$. The final results for the phase shifts obtained from the folding model and the McIntyre and Haebelir potentials are shown by the solid and dashed curves, respectively, in Fig. 1.

Following Nishioka et al.⁶ the $^6Li$ bound state radial wave functions $u_l(\rho)$, treated as the relative motion of a free alpha particle and deuteron, can be calculated as the $2S$ and $1D$ states in a central Woods-Saxon potential well of radius $R = 1.9$ fm and diffuseness $a = 0.65$ fm. These parameters reproduce the $^6Li$ root mean square radius. The central potential well depths are adjusted to reproduce the $d-\alpha$ separation energy in the $l=0$ and $l=2$ states individually. Within this model, in which the deuteron has its free quadrupole moment $Q_d$, the amplitude of $d-\alpha$ relative D-wave motion, $A_2$, required to reproduce the $^6Li$ quadrupole moment is $A_2 = -0.08$.⁶ Three-body calculations based on separable descriptions of the nucleon-alpha interaction, on the other hand, produce amplitudes $A_2$ of roughly equal magnitude but of opposite sign.⁷ In the following the reaction observables are calculated taking these two extreme values of $A_2$ as an indication of our present theoretical understanding of the $^6Li$ D state.

IV. THE REACTION OBSERVABLES

Because of these theoretical uncertainties in the $^6Li$ wave function, the possibility of an experimental determination of the D-state component is of great interest. However, because the amplitude of the D state concerned is so small, it is important to analyze realistically the sensitivity of the observables to other tensor and spin-dependent effects in the system.

The capture reaction cross section and tensor analyzing powers were calculated using the scattering wave functions derived from the folding model and McIntyre and Haebelir potentials and with the $^6Li$ D-state amplitudes described above. Figure 2 shows the angular distributions for the tensor analyzing powers $T_{20}$ and $T_{21}$ at the deuteron center-of-mass energy $E_{c.m.} = 4$ MeV. These calculations were obtained using the folding model potential and included in the $d-\alpha$ $T_R$ tensor interaction. The two curves correspond to the choices $A_2 = -0.08$ (solid curves) and $A_2 = +0.08$ (dashed curves). We choose this particular deuteron energy to allow comparison with the corresponding calculations presented in Ref. 8. The state dependence of the entrance channel interaction, included here, has had a major effect on the calculations. The previous analyzing power calculations were essentially isotropic in angle³ and had a magnitude...
proportional to the $D$-state amplitude in the $^6\text{Li}$ nucleus. We see here that in the presence of realistic spin-dependent entrance channel distortion, the calculated tensor analyzing powers are large, even in the absence of a $^6\text{Li} D$-state component making the $D$-state signature much less evident. Similar results were found when using the McIntyre-Haeberli phenomenological potential with no tensor forces in the entrance channel and so this is not a specifically tensor interaction effect.

To clarify the role of the entrance channel tensor interaction, in Fig. 3 we show the contributions of the various contributing entrance channels to the total capture reaction cross section for an $E2$ transition. Also shown are the experimental reaction cross section data. The theoretical curves do not involve any renormalization in the reaction calculations. The entrance channel contributions are of course incoherent. In the upper part of the figure the potential used was the folding model potential, including the tensor force, and in the lower part the McIntyre-Haeberli potential with no tensor forces present. As was discussed earlier, the primary effect of the tensor force is to introduce an amplitude for capture to the $^6\text{Li} S$-state wave function from the $S$, entrance channel configuration, raising the contribution to the cross section from the $S$ channel by about 2 orders of magnitude. Nevertheless, for a tensor interaction of real-
istic strength, as deduced from the folding model, the contribution is too small to have any significant effect on the total reaction cross section calculations. The long standing disagreement of such direct capture calculations with the experimental total cross section data, for deuteron energies above 3 MeV in the center of mass (see Ref. 3), persists in our calculations also.

What then is the possibility of observing the $^6$Li $D$-state component empirically? To attempt to shed light on this question, in Fig. 4 we present the calculated energy dependence of the tensor analyzing powers $A_{yy}$ (upper part) and $T_{20}$ (lower part) at $\theta_{c.m.}=45^\circ$, at which angle the analyzing powers tend to be maximal. Here the solid and dashed curves show the results of calculations, including the $d-\alpha$ tensor interaction, with the $^6$Li $D$-state amplitudes $A_2=-0.08$ and $A_2=+0.08$, respectively. The dotted curves show the calculated energy dependences corresponding to the value $A_2=-0.08$, but using the McIntyre-Haeberli potential with no tensor force. Once again these calculations can be compared with the corresponding figures in Ref. 8. The entrance channel distortion has generated rapid energy dependence in the tensor analyzing powers through the energy region of the $d-\alpha$ resonance states. Also, the importance of the $d-\alpha$ tensor force in comparison with the $^6$Li $D$-state effects changes significantly as a function of incident deuteron energy. In general, however, for deuteron center-of-mass

![Graph](image-url)

**FIG. 3.** Contributions to the capture reaction total cross section from each entrance channel partial wave, as a function of deuteron center-of-mass energy $E_{c.m.}$. Parts (a) and (b) of the figure show the results obtained with the folding model potential, including the tensor interaction, and the McIntyre and Haeberli potential (Ref. 13), respectively. The experimental data are from Ref. 3.

![Graph](image-url)

**FIG. 4.** Calculated energy dependence of the tensor analyzing powers $A_{yy}$ (a) and $T_{20}$ (b) at $\theta=45^\circ$. The solid and dashed curves, corresponding to $^6$Li $D$-state amplitudes $A_2=-0.08$ and $A_2=+0.08$, respectively, were obtained using the folding model $d-\alpha$ potential, including the tensor interaction. The dotted curves, corresponding to a $^6$Li $D$-state amplitude $A_2=-0.08$, were obtained using the McIntyre and Haeberli potential (Ref. 13).
energies $E_{\text{c.m.}} \leq 6$ MeV, the analyzing powers are calculated to be large and to show no strong sensitivity to either the $^6\text{Li} D$ state or the $d - \alpha$ tensor force, as would provide a good experimental signature. To the extent that the direct capture model at the higher deuteron energies is not missing any significant spin dependence, it appears, however, that, for deuteron energies $E_{\text{c.m.}} \geq 6$ MeV, a certain simplicity is restored. The effects of the tensor interaction become small and by comparison the $^6\text{Li} D$-state effects, indicated by the differences between calculations using the amplitudes $A_2 = -0.08$ and $A_2 = +0.08$, remain significant.

V. SUMMARY AND CONCLUSIONS

The small theoretically predicted, but ill determined, $D$-state component in the $^6\text{Li}$ wave function is presently the subject of experimental investigation. In this paper we study the $^6\text{He}(^2\text{H}, \gamma)^6\text{Li}$ reaction and in particular its spin dependence within the deuteron + alpha particle direct capture model, with a view to understanding the role of the $^6\text{Li} D$ state in this system. The $d - \alpha$ entrance channel distortions are treated realistically and the role of the $d - \alpha T_R$ tensor interaction, originating from the deuteron $D$-state component, is estimated consistently within the deuteron + alpha model, by use of the folding model.

We find no clear $^6\text{Li} D$-state signature in the calculated tensor analyzing powers at low incident deuteron energies, $E_{\text{c.m.}} \leq 6$ MeV. In this energy region $^6\text{Li} D$-state and $T_R$ interaction effects are of comparable magnitude but present a small perturbation to the large analyzing powers generated by the entrance channel spin dependence. At such energies there appears little chance of direct experimental observation. At higher energies, above the $^3D_0, ^3D_2$, and $^3D_1 d - \alpha$ resonances in the entrance channel, our calculations show that the $T_R$ tensor interaction effects become insignificant and that $^6\text{Li} D$-state effects are once again significant. Given, however, the lack of detailed agreement of direct capture model calculations with the measured total reaction cross section at these higher energies, more detailed calculations of the role of the $0^+ (T = 1)$ state\textsuperscript{1} and the $n-p$ breakup continuum in the reaction need to be carried out at these center-of-mass energies.

ACKNOWLEDGMENTS

The financial support of the Instituto Nacional de Investigação Científica and the British Council (for R.C.) and the Science and Engineering Research Council (U.K.), through research Grant Nos. GR/F/4105.1 and GR/F/1086.6 (for J.A.T.) is gratefully acknowledged.

---

\*On leave from Physics Department, Instituto Superior Técnico, Lisbon, Portugal.


