High-$K$ isomers as probes of octupole collectivity in heavy nuclei

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Abstract

The influence of the octupole deformation on the structure of high-$K$ isomeric states in the region of heavy even-even actinide nuclei is studied through a reflection asymmetric deformed shell model (DSM). Two-quasiparticle states with high-$K$ values are constructed by taking into account the pairing effect through a DSM+BCS procedure with constant pairing interaction. The behaviour of two-quasiparticle energies and magnetic dipole moments of $K^\pi = 6^+, 6^-$ and $8^-$ configurations, applicable to mass numbers in the range $A = 234 - 252$, was examined over a wide range of quadrupole and octupole deformations. A pronounced sensitivity of the magnetic moments to the octupole deformation is found. The result suggests a possibly important role for high-$K$ isomers in determining the degree of octupole deformation in heavy actinide nuclei.

keywords: heavy nucleus, octupole shape, dipole moment

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The octupole shape degree of freedom in atomic nuclei gives rise to a number of observables, such as parity doublets and enhanced electric-dipole transition rates [1]. Most studies have focussed on ground-state and rotational-vibrational properties. Indeed, rotational motion often appears to enhance the octupole features [2].

The present work addresses the complementary issue of broken-pair, two-quasiparticle excitations. In axially symmetric, quadrupole-deformed nuclei, two-quasiparticle states may be isomeric [3] due to the approximate conservation of the $K$ quantum number (i.e. the angular momentum projection on the symmetry axis) and such isomers can also be expected in quadrupole-octupole deformed nuclei. The presence of isomers opens a variety of experimental opportunities, including the measurement of electromagnetic moments, for example by laser hyperfine spectroscopy [4].

There are now a good number of isomers known in heavy even-even actinide nuclei [5, 6, 7] where octupole-deformed shapes may be encountered. The present work examines the theoretical sensitivity of isomer excitation energies and magnetic dipole moments to non-zero octupole deformations, in the expectation that future experiments will be able to make appropriate measurements.

A single-particle deformed shell model, allowing reflection asymmetry [8, 9, 10], is applied to calculating the nuclear shell structure, in order to examine the possibility for the coupling of high-$K$ orbitals near the Fermi level. The particular realization of the model includes a Woods-Saxon potential with axial quadrupole and octupole deformations for which a numerical code is available [11]. The Hamiltonian of the model is

$$H_{sp} = T + V_{ws} + V_{s.o.} + V_c,$$

where $V_{ws}(r, \hat{\beta}) = V_0 \left[ 1 + \exp \left( \frac{\text{dist}_2(r, \hat{\beta})}{a} \right) \right]^{-1}$ is the Woods-Saxon potential with $\hat{\beta} \equiv (\beta_2, \beta_3, \beta_4, \beta_5, \beta_6)$ and $\text{dist}_2(r, \hat{\beta})$ being the distance between the point $r$ and the nuclear surface represented by $R(\theta, \hat{\beta}) = c(\hat{\beta}) R_0 \left[ 1 + \sum_{\lambda=2,3,\ldots} \beta_\lambda Y_\lambda^0(\cos \theta) \right]$ ($c(\hat{\beta})$ is a scaling factor to keep the volume fixed). $V_{s.o.}$ and $V_c$ are the spin-orbit and Coulomb terms whose analytic form is given in [11]. The Hamiltonian (1) is diagonalized in the axially symmetric, deformed harmonic oscillator (ADHO) basis $|N n z \Lambda \Omega\rangle$, with the single-particle wave function being obtained as the expansion

$$\mathcal{F}_\Omega = \sum_{N n z \Lambda} C_{N n z \Lambda}^\Omega |N n z \Lambda \Omega\rangle.$$

In the case of non-zero octupole deformation the wave function (2) appears with mixed parity. Then the parity of a given single-particle orbital is characterized by the expectation (average) value of the parity operator

$$\langle \hat{\pi}_{sp} \rangle = \langle \mathcal{F}_\Omega | \hat{\pi}_{sp} | \mathcal{F}_\Omega \rangle = \sum_{N n z \Lambda} (-1)^N |C_{N n z \Lambda}^\Omega|^2.$$

In the present work we are restricted to the situation where the average parity remains close to the good values of $+1$ or $-1$, i.e. where the parity is still an asymptotically good
quantum number. More extended discussion of the parity mixing and the construction of a total collective+intrinsic state with good parity is given in Ref. [12].

The pairing effect is taken into account through a BCS procedure with constant pairing interaction applied to the DSM single-particle levels. The pairing constants $G_{n/p}$ for neutrons(n)/protons(p) are taken as [13] (see page 311):

$$G_{n/p} = \left( g_0 \mp g_1 \frac{N - Z}{A} \right) / A.$$  \hspace{1cm} (4)

In [13] it is suggested that $g_0 = 19.2 \text{ MeV}$ and $g_1 = 7.4 \text{ MeV}$. Here a slightly refined value for the first parameter $g_0 = 17.8 \text{ MeV}$ is used, providing a better overall energy scale to examine the $K^\pi = 8^-$ isomeric state of $^{244}\text{Pu}$ and its dependence on quadrupole and octupole deformations (see below). The BCS procedure is applied, as suggested in [13], within energy windows including $(15N)^{1/2}$ orbitals for neutrons and $(15Z)^{1/2}$ orbitals for protons below and above the Fermi surface. It should be noted that in the deformation regions under consideration, the values obtained for the neutron pairing gap $\Delta_n$ in $^{244}\text{Pu}$ vary over a narrow range between 0.6 and 0.7 MeV.

The energy of a two-quasiparticle configuration for a broken pair is taken as $E_{2qp}^{K\pi} = E_{1qp}^{\Omega_1\pi_1} + E_{1qp}^{\Omega_2\pi_2}$, with $E_{1qp}^{\Omega_\pi} = \sqrt{(E_{sp}^{\Omega_\pi} - \lambda)^2 + \Delta^2}$ being the one-quasiparticle energy. The $K$-value is determined as $K = \Omega_1 + \Omega_2$, while the parity of the configuration is $\pi = \pi_1 \cdot \pi_2$ (or $\pi = \text{sign}(\pi_1) \cdot \text{sign}(\pi_2)$, in the case of non-zero octupole deformation).

The magnetic moment in the two-quasiparticle configuration is determined as [14]

$$\mu = \mu_N \left[ g_R \frac{I(I + 1) - K^2}{I + 1} + g_K \frac{K^2}{I + 1} \right],$$  \hspace{1cm} (5)

with $\mu_N = e\hbar/(2mc)$, $g_R = Z/A$ and

$$g_K = \frac{1}{K} \sum_{n=1,2} \langle F_{\Omega_n} | g_\Sigma \cdot \Sigma + g_l \cdot \Lambda | F_{\Omega_n} \rangle,$$  \hspace{1cm} (6)

where $\Sigma = \Omega \mp \Lambda$ is the intrinsic spin projection, and $g_l$ and $g_\Sigma$ are the standard gyromagnetic ratios. The proton and neutron $g_\Sigma$ values are attenuated by a commonly used factor of 0.6 compared to the free values. As mentioned above, the present consideration is restricted to the situations with only small parity mixing in the single-particle states. Typically the parity admixtures in the wave function do not exceed 10-15\%, which means that the contribution of the mixed components to the gyromagnetic ratio $g_K$, Eq. (6), is of second order and can be neglected at the current stage of the study. In this sense, the present situation corresponds to an approximate parity projection. In the general case of larger parity mixing, projection techniques can be applied as considered in [1] and [12].

The above model considerations have been applied to high-$K$ excited states in several heavy even-even actinide nuclei. Specifically, calculations have been performed for two-quasi-neutron configurations in $^{234}\text{U}$ with $K^\pi = 6^-$, in $^{244}\text{Pu}$ with $K^\pi = 8^-$, and in $^{244}\text{Cm}$ with $K^\pi = 6^+$. These are considered to be representative examples, with quasiparticle configurations that may occur also in other nuclides in the same mass region [6, 7, 15].
In $^{244}\text{Pu}$, analysis of the single-particle states near the neutron Fermi level suggests a $K^\pi = 8^-$ coupling of the orbitals $7/2[624]$ and $9/2[734]$. Moreover it is found that with increasing octupole deformation, $\beta_3$, these two orbitals remain close to the Fermi level and even cross each other. This is illustrated in Fig. 1 for the neutron levels of $^{244}\text{Pu}$ obtained for fixed $\beta_2 = 0.293$ [16]. It is seen that the two orbitals cross near $\beta_3 = 0.13$. The analysis of the two-quasiparticle energy for this configuration around $\beta_2 = 0.29$ indicates the presence of a minimum in the same octupole deformation region, $\beta_3 \sim 0.1$, with a depth between 50 and 100 keV. To obtain a global view of the quadrupole-octupole dependence of the $\beta$ energy, an extended calculation over a fine net in the $(\beta_2, \beta_3)$-plane was performed. The result is shown in Fig. 2 where the presence of a two-dimensional energy minimum in the region around $\beta_2 = 0.26$ and $\beta_3 = 0.08$ is clearly seen.

Also, the magnetic dipole moment (5) of the same $^{244}\text{Pu}$ configuration was examined as a function of $\beta_2$ and $\beta_3$. The result is presented in Fig. 3 as a two-dimensional plot in the $(\beta_2, \beta_3)$-plane. This plot indicates a pronounced sensitivity of the magnetic moment to the octupole deformation. This is especially well seen for quadrupole deformations up to about $\beta_2 = 0.26$. For the higher $\beta_2$-values the behaviour of the magnetic moment is flatter in the $(\beta_2, \beta_3)$-plane, but still with considerably different values depending on the octupole deformation. This result suggests consideration of different $\beta_2$ regions. One is the already considered region (as in Fig. 1) close to the experimentally established value $\beta_2 = 0.293$ [16]. Another is the one near $\beta_2 = 0.22$ which is suggested by the global macroscopic-microscopic calculation of Möller et al. [17]. Also, an interesting region seems to be the one near $\beta_2 = 0.26$, where the two-dimensional minimum in the two-quasiparticle energy is observed as shown in Fig. 2. The particular behaviour of the magnetic moment as a function of the octupole deformation for the different $\beta_2$ values is shown in Fig. 4a. Indeed, it is seen that the lowest value $\beta_2 = 0.22$ provides much stronger variation of the magnetic moment as a function of $\beta_3$ compared to the case of $\beta_2 = 0.293$, while for $\beta_2 = 0.26$ the variation of the magnetic moment is intermediate. In Fig. 4b it is illustrated that the magnetic moment has only a weak dependence on the quadrupole deformation when the octupole deformation is zero.

A similar situation applies to the other $N = 150$, $K^\pi = 8^-$ states with the same two-quasi-neutron configuration, extending to $^{252}\text{No}$ [7]. Calculations have also been performed for the $K^\pi = 6^-$, $\{\nu 5/2[622] \otimes \nu 7/2[624]\}$ configuration in $^{244}\text{Cm}$. The $\beta_2$ dependence of the magnetic moment is extremely weak (at least at zero $\beta_3$) whereas the $\beta_3$ dependence is strong, as illustrated in Fig. 5a.

Finally, calculations for $^{234}\text{U}$ show that the formation of the $K^\pi = 6^-$, $\{\nu 5/2[633] \otimes \nu 7/2[743]\}$ state is favoured (i.e. obtained at lower energy) near the “macroscopic-microscopic” value of $\beta_2 = 0.215$ [17], where both orbitals $5/2[633]$ and $7/2[743]$ cross each other at $\beta_3 = 0.1$, rather than at the experimentally suggested value of $\beta_2 = 0.272$ [16] for which the two orbitals are far from each other and furthermore diverge with $\beta_3$. Here the hexadecapole deformation is also found to be important, and we use the Möller et al. [17] value of $\beta_3 = 0.110$. This result suggests that the present calculations are compatible with the previous study [18] in the region of the lower quadrupole deformation $\beta_2 = 0.215$. The $\beta_3$-dependence of the two-quasiparticle magnetic moment is illustrated in Fig. 5b. Again,
considerable $\beta_3$ sensitivity is evident, although there is a turning point at $\beta_3 \approx 0.03$.

With all three of the considered high-$K$, two-quasi-neutron configurations, the sensitivity of the magnetic dipole moments to $\beta_3$ suggests that measurements of the dipole moments would provide useful constraints on the degree of octupole deformation. Since these high-$K$ states are known to be isomeric, with half-lives in the range of microseconds to seconds, a variety of experimental techniques would appear to be open to exploitation. It is notable that Greenlees et al. [19] have measured the $^{250}\text{Fm}$, $K^\pi = 8^-$ rotational-band $g$-factors. While these establish two-quasi-neutron structure, they are not yet of the accuracy needed to probe or usefully constrain the octupole degree of freedom.

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References

Figure 1: Neutron single-particle levels near the Fermi level $E_f$ of $^{244}$Pu as a functions of the octupole deformation $\beta_3$, with $\beta_2 = 0.293$ taken from experiment [16]. Note that the parity label only strictly applies at $\beta_3 = 0$. 
Figure 2: Calculated two-quasiparticle energy of the $K^\pi = 8^-$, $\{\nu 7/2[624] \otimes \nu 9/2[734]\}$ configuration in $^{244}$Pu as a function of $\beta_2$ and $\beta_3$. 
Figure 3: Calculated magnetic moment, in units of $\mu_N$, of the $K^\pi = 8^-$, $\{\nu 7/2[624] \otimes \nu 9/2[734]\}$ configuration in $^{244}$Pu as a function of $\beta_2$ and $\beta_3$. 
Figure 4: Calculated magnetic moment of the $K^\pi = 8^−, \{\nu 7/2[624] \otimes \nu 9/2[734]\}$ configuration in $^{244}$Pu as a function of (a) $\beta_3$ for three different values of $\beta_2$; and (b) $\beta_2$ for $\beta_3 = 0$.

Figure 5: Calculated magnetic moments of (a) the $K^\pi = 6^+, \{\nu 5/2[622] \otimes \nu 7/2[624]\}$ configuration in $^{244}$Cm as a function of $\beta_3$ for two different values of $\beta_2$; and (b) the $K^\pi = 6^−, \{\nu 5/2[633] \otimes \nu 7/2[743]\}$ configuration in $^{234}$U as a function of $\beta_3$ for fixed values of $\beta_2$ and $\beta_4$. 