Elastic and quasielastic scattering of $^8\text{He}$ from $^{12}\text{C}$

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We present new calculations and experimental measurements of the quasielastic cross section angular distribution for $^8\text{He}$ scattering from $^{12}\text{C}$ at 60 MeV/nucleon. $^8\text{He}$ is treated as a five-body $\alpha+4n$ system and the six-body $^8\text{He}+$target scattering calculations make use of the eikonal few-body method and the cluster orbital shell model approximation for the $^8\text{He}$ wave function. The qualitative features of the new data are successfully described without parameter variation. The sensitivity of the calculations to correlations in the $^8\text{He}$ wave function is assessed.

We may consider $^8\text{He}$ as a prototype for reaction studies of heavier neutron dripline systems with a many-neutron skin.

In this Rapid Communication we present new experimental and theoretical studies of exotic light nuclei with a normal, localized nuclear core and a dilute few-neutron halo or skin are now well advanced and are becoming increasingly sophisticated. Earlier inclusive and total cross section analyses at very high energies [1] are giving way to exclusive and differential cross section measurements, many at energies of between 30 and 100 MeV/nucleon. In this energy regime eikonal methods have been successfully applied quite extensively for the calculation of scattering angular distributions and of breakup momentum distributions of projectiles with a predominantly binary or three-body structure, such as $^{11}\text{Li}$ [3–5], $^8\text{B}$ [6], $^{11}\text{Be}$ [7] and $^{14}\text{Be}$ [8]. A class of non-eikonal corrections to the lowest order theory have also been investigated with very promising results [7] for extending their range of applicability.

In this Rapid Communication we present new experimental and theoretical results for the quasielastic scattering of $^8\text{He}$ from $^{12}\text{C}$ at an energy of 60 MeV/nucleon. The $^8\text{He}$ nucleus is of intrinsic interest. It is thought to be the lightest nuclear system to display a neutron skin in which four valence neutrons move about a localized $\alpha$ particle core, as distinct from a one- or two-neutron-halo nucleus. A simple theoretical model [9] yields a root mean squared (rms) separation of the centers of mass of each neutron-$\alpha$ pair of 3.47 fm in $^8\text{He}$, as compared with the $\alpha$ core rms matter radius of 1.45 fm. The model thus generates a two component ground state density, with $T=0$ core and $T=2$ neutron skin contributions, and is consistent with proton+$^8\text{He}$ scattering [10] and with the measured momentum distribution of $^8\text{He}$ following the dissocation of $^9\text{He}$ [9]. We can consider $^9\text{He}$ as a prototype for reaction studies of heavier neutron dripline systems with a many-neutron skin.

Here we extend the application of the few-body eikonal model [3,4] to $^8\text{He}+^{12}\text{C}$ scattering, treated as an $\alpha+4n+$target six-body system. We also report and compare our calculations with new measurements of the quasielastic cross section angular distribution for this system. The role of correlations in the composite projectile is also assessed.

Full details of the experimental setup are given in Refs. [11] and [12]. The measured $^8\text{He}+^{12}\text{C}$ cross section angular distribution is quasielastic, as the experimental energy resolution of 7.5 MeV full width at half maximum (FWHM) did not permit the low lying ($2^+$ and $3^+$) states of the $^{12}\text{C}$ target to be resolved from the elastic channel. The data are therefore an incoherent sum of elastic and inelastic cross section contributions. The measured angular distribution (ratio to Rutherford) is shown in Fig. 1 by the full circle symbols. The vertical error bars include both the statistical errors and an estimate of the systematic uncertainties due to the angular resolution of the detector, indicated by the horizontal error bars in this figure. The absolute normalization of the experimental data has a systematic uncertainty of 15%.

We note that the $^8\text{He}$ cross section data are significantly larger in ratio to the Rutherford cross section than the recently reported $^9\text{Li}$ measurements [12] made at the same incident energy per nucleon. These are shown, for comparison, by the open circle symbols. They reveal a significant
change in the measured scattering due to the addition of a single proton and suggest quite different projectile-target \( S \)-matrix calculations. The open points show the measured \( ^{6}\text{Li}+^{12}\text{C} \) quasi-elastic cross section angular distribution (ratio to Rutherford) measured at 540 MeV.

Using the eikonal model, simplifications to the quantum few-body problem stem from two sources. The first is the adiabatic treatment of the internal degrees of freedom of the composite projectile. The second is the approximation that the incident particles follow straight line paths through the interaction field of the target. Within this model the amplitude for the elastic scattering of the \( ^{8}\text{He} \) nucleus, through angle \( \theta \), is 

\[
f_{\text{el}}(\theta) = -iK\int_{0}^{\infty} db db J_{0}(gb)[S_{8}(b)-1], \tag{1}
\]

an integral over all impact parameters \( b \) of the projectile’s center of mass (c.m.). Here \( g = 2K \sin(\theta/2) \) is the momentum transfer and \( K \) is the projectile’s incident wave number in the c.m. frame. The treatment of the projectile’s Coulomb interaction within the eikonal model, and the resulting modifications made to Eq. (1) for computational efficiency, are discussed fully elsewhere \([4,13]\). In the present work we assume the Coulomb interaction acts on the c.m. of the projectile and thus we neglect possible Coulomb breakup contributions.

In Eq. (1) the composite nature of the projectile appears through \( S_{8}(b) \), the eikonal approximation to the elastic \( S \)-matrix for the \( ^{8}\text{He}+\text{target} \) system, expressed as a function of impact parameter. This is

\[
S_{8}(b) = \langle \Phi_{8} | S_{\alpha}(b_{\alpha}) \prod_{i=1}^{4} S_{i}(b_{i}) | \Phi_{8} \rangle, \tag{2}
\]

where \( \Phi_{8} \) is the ground state wave function of the projectile. The bra-ket notation here implies integration over all space and spin coordinates internal to the projectile. The interactions of the constituent \( \alpha \) and four neutrons (\( i = 1,...,4 \)) with the target enter through the eikonal \( S \)-matrix for that constituent. Given their interactions \( V_{ij} \) (\( j = \alpha,1,...,4 \)) with the target these are computed, at each impact parameter \( b \), (see Fig. 2) according to

\[
S_{j}(b_{j}) = \exp\left[-\frac{i}{\hbar v} \int_{-\infty}^{\infty} V_{jT}(\sqrt{b_{j}^{2}+z^{2}})dz \right], \tag{3}
\]

where \( v \) is the incident projectile (and constituent) velocity in the c.m. frame. Equation (2) expresses transparently the underlying adiabatic assumption, that the constituent particle coordinates within the projectile are assumed fixed for the duration of the scattering event. The composite projectile elastic \( S \)-matrix is seen to be the appropriate (ground state) weighted average of these position dependent constituent amplitudes.

We compute the twelve dimensional spatial integral involved in the calculation of the \( ^{8}\text{He} \) \( S \)-matrix of Eq. (2) by use of random sampling (Monte Carlo) integration. We also make use of the harmonic oscillator-based cluster orbital shell model approximation (COSMA) wave function for \( ^{8}\text{He} \) \([9]\). While not an essential ingredient, this wave function does provide an analytic expression for the spin integrated four-neutron correlation function entering Eq. (2). It includes correlations associated with the antisymmetrization of the four valence neutrons amongst themselves, each in an assumed \( p_{3/2} \) oscillator orbital with respect to the \( \alpha \) core. Explicitly

\[
\langle \Phi_{8} | \Phi_{8} \rangle_{\text{spin}} = f_{\text{corr}}(r_{1}, r_{2}, r_{3}, r_{4}), \tag{4}
\]

where \( f_{\text{corr}} \), given by Eq. (6) of Ref. [9], is

\[
f_{\text{corr}}(r_{1}, r_{2}, r_{3}, r_{4}) = \left( \prod_{i=1}^{4} \frac{\phi(r_{i})^{2}}{4\pi} \right) A(1,2,3,4), \tag{5}
\]
with

$$A(1,2,3,4) = \frac{3}{4}[S_{12}^2S_{13}^2 + S_{12}^2S_{14}^2 + S_{13}^2S_{14}^2].$$

and where $\phi$ is the nodeless $p$-wave oscillator wavefunction. The $r_i$ are the position vectors of the neutrons relative to the $\alpha$ particle core, see Fig. 2, and $S_{ij}^2 = 1 - (r_i \cdot r_j)^2$ is the square of the sine of the angle between vectors $r_i$ and $r_j$.

The calculations sample at random the four neutron position vectors $r_i$ at each $^8$He c.m. impact parameter $b$ and $f_{\text{corr}}(r_1, r_2, r_3, r_4)$ is calculated. The positions $x_{a,b} = \frac{-\sum_{i=1}^{4} m_i r_i}{(4m_n + m_a)$ and $x_{a,b} = r_i + r_a$ of the core and neutrons relative to the projectile c.m. can then be computed and therefore the impact parameter of each constituent, $b_j$. In each such configuration the constituent $S$-matrices $S_j(b)$ are interpolated from a precalculated lookup table.

The simple form of the COSMA wave function and the procedure detailed above makes clear that the present calculations include two sources of correlations associated with the valence neutrons. These are (i) the angular and antisymmetrization correlations, contained within the factor $A$ in Eq. (5), and (ii) the c.m. correlations, with the finite mass of the $\alpha$ core, and expressed by the vector relationships imposed between the $x_i$ and $x_a$. Also clear is that these effects may be removed, progressively, by (I) replacing the factor $A$ by unity; this yields a modified correlation function $f_{\text{corr}}$ with associated $S$-matrix $S_{\text{corr}}^{ii}(b)$, which retains the c.m. correlations only, and (II) fixing the $\alpha$ core at the $^8$He c.m. by setting $x_a = 0$, which leads to an uncorrelated four-neutron skin, $f_{\text{corr}}$, and a resulting $^8$He $S$-matrix

$$S_{\text{corr}}^{ii}(b) = S_b(b)|\phi(S_a(b_a))|\phi|^4.$$  

The subscript $n$ now refers to any neutron coordinate. We investigate the relative importance of these two effects in the following.

We apply the formalism developed above to the elastic scattering of $^8$He from $^{12}$C at 60 MeV/nucleon. The required inputs to the theoretical description of elastic scattering, in addition to the chosen model for the $^8$He ground state wave function, are the projectile constituent-target interactions; that is an $\alpha + ^{12}$C and $n + ^{12}$C optical interaction at 60 MeV/nucleon. For consistency with earlier work the $n + ^{12}$C optical potential used was that tabulated in Ref. [4] and used previously for $^{11}$Li [4] and $^{13}$Be [7] systems at similar energies. For the $\alpha + ^{12}$C system there are no available data at 240 MeV incident energy. To avoid the dangers associated with extrapolations of phenomenological optical potential parameters, from data below 172.5 MeV, we make use of theoretically motivated density dependent double folding model calculations of the $\alpha$ optical potential due to Khoa et al. [14]. This approach has been highly effective in reproducing $\alpha$ particle elastic scattering observables at similar energies per nucleon with a largely energy-independent parameterization. The real part of the interaction was calculated using the BDM3Y1-Paris effective interaction [15]. This is obtained by introducing an appropriate density dependence, with parameters adjusted to the binding energy of nuclear matter, into the M3Y-Paris $G$-matrix effective interaction for finite nuclei derived from the Paris free nucleon-nucleon interaction. This real part was renormalized by a factor $N_R = 1.25$, as done in earlier analyses. The imaginary part of the interaction was of volume Woods-Saxon form with strength 22 MeV, radius parameter 0.96 fm, and diffuseness 0.7 fm [16]. There are no available data to guide possible potential parameter variations for the $\alpha$ fragment from these values.

The moduli of the input derived eikonal $S$-matrices are shown in Fig. 3. The figure shows the calculated $S$-matrices for the neutron $|S_n|$, (dot-dashed curve), alpha $|S_\alpha|$ (long dashed curve) and $^8$He $|S_\alpha|$ (solid curve), each as a function of its own impact parameter. The effects of averaging the constituent amplitudes $S_n$ and $S_\alpha$ over the extended ground state probability density are apparent, as is the highly absorptive nature of the $^8$He+$^{12}$C effective interaction which would generate this $|S_\alpha|$. This local interaction is calculated numerically from the eikonal phase shift function $\chi(b) = -i \ln S_b(b)$ using the expression given in Eq. (7) of Ref. [17]. Its real and imaginary form factors are shown by the solid curves in Fig. 4. The absorptive potential is seen to be of order 60 MeV deep, to be compared with the input $\alpha$ potential absorptive strength of 22 MeV. The theoretical elastic scattering cross section angular distribution (ratio to Rutherford) calculated using this few-body $S$-matrix is shown by the long dashed curve in Fig. 1.

The presented $^8$He+$^{12}$C experimental angular distribution includes contributions due to the inelastic excitation of the $^{12}$C target. As done in [4] for $^{11}$Li scattering, we estimate explicitly, in distorted wave Born approximation (DWBA), these inelastic contributions and add them to the calculated elastic cross section for comparison with the data. We calculate the DWBA cross sections to the $^2+$ and $^3-$ states of $^{12}$C only. The first-excited $^0+$ state also lies within the experimental energy resolution. However, as a monopole excitation, it is not expected to be strongly populated in an inelastic scattering process and is therefore ignored, as in Refs. [4]...
and [12]. We assume rotational model couplings of derivative form [4], and calculate the $2^+$ and $3^-\text{ inelastic transitions of } ^{12}\text{C}$ by deforming the local potential of Fig. 4. Following [18], we use deformation lengths $\delta_S = 1.648 \text{ fm}$ and $\delta_A = 1.00 \text{ fm}$ for the $2^+$ and $3^-\text{ transitions, respectively.}$ The DWBA calculations are performed using the computer code FRESCO [19].

The calculated $2^+$ and $3^-\text{ inelastic cross sections are shown by the dot-dashed and short dashed curves, respectively, in Fig. 1.}$ The sum of the elastic and inelastic cross sections is shown by the solid curve which we now compare with the experimental data. We observe that the magnitude and forward angle oscillations in the data are reasonably reproduced and that the inelastic channel contributions are important for generating a cross section of the required magnitude at the larger angles. Given the uncertainties in the present data, relating to the strengths with which the states of $^{12}\text{C}$ are actually excited and the accuracy of the use of DWBA, no attempt was made to improve the description of the data by variation of the neutron and/or $\alpha + ^{12}\text{C}$ interactions. Elastic scattering data for the $\alpha + ^{12}\text{C}$ system at the same energy per nucleon would surely clarify, empirically, the quality of the currently theoretical potential input in this subsystem.

It is of interest to assess the sensitivity of our results to the angular and c.m. correlations present within the few-body description. We find that the effects of the angular correlations, resulting from the antisymmetrized four neutron COSMA state, are in fact rather small. Using $f^{(II)}_{\text{core}}$, which is essentially indistinguishable from the solid curve in Fig. 3, and is not shown. It produces a modified $^8\text{He}$ potential given by the dashed curves in Fig. 4 with small changes from the full calculations (solid curves) only at the lowest radii. The radii of the c.m. correlations on the other hand, which are included carefully in the present approach, are large. Using the uncorrelated function $f^{(III)}_{\text{core}}$, Eq. (7) yields the $S^{(III)}_8$ shown in modulus by the short-dashed curve in Fig. 3 with changes at all impact parameters. The local equivalent $^8\text{He}$ potential is shown by the dot-dashed curves in Fig. 4 with large changes in the radial formfactors. The predicted elastic scattering angular distributions in these three cases are shown by the solid (full COSMA), dashed ($f^{(II)}_{\text{core}}$) and dot-dashed ($f^{(III)}_{\text{core}}$) curves in Fig. 5.

The calculations, and therefore the expected elastic scattering angular distribution, are clearly sensitive to these few-body correlations.

In summary, the $^8\text{He} + ^{12}\text{C}$ quasielastic scattering angular distribution has been measured and calculated at an incident energy of 60 MeV per nucleon. The measured ratio of the differential cross section angular distribution to the Rutherford cross section is found to be consistently larger than that for $^9\text{Li} + ^{12}\text{C}$ scattering at the same incident energy per nucleon, suggesting a quite different effective interaction in the case of the neutron skin nucleus $^8\text{He}$. Theoretical calculations are presented which include, for the first time, the six-body, $\alpha + 4n + \text{target}$, nature of the reacting system.

The approach presented makes such calculations practical by exploiting the simplifications brought about by the eikonal reaction model and its underlying adiabatic treatment of the motions of the projectile constituents. In the present work we also make use of the simplifications brought about by the use of the (analytic) COSMA wave function for the $^8\text{He}$ ground state; however, this is only a convenience. The $\alpha + ^{12}\text{C}$ interaction was taken from a careful double folding model theoretical analysis. There were therefore no free or adjusted parameters in the calculation; however, experimental elastic scattering data for the $\alpha$ core fragment, at the same incident energy per nucleon, would be invaluable in assessing this particular input. The magnitude and angular distribution of the measured $^8\text{He}$ quasielastic cross section are well explained by the presented few-body model of the process.
The calculations show considerable sensitivity to a correct treatment of the c.m. correlations in the composite projectile but rather weak sensitivity to the angular correlations present within the COSMA model. It would be very interesting to investigate further this sensitivity to details of the neutron skin structure by incorporating more sophisticated microscopic descriptions for the $^8$He ground state. Accurate elastic, rather than quasielastic, scattering data at similar energies would be invaluable in assessing these quantitative theoretical questions further.

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