Sizes of the He isotopes deduced from proton elastic scattering measurements

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Abstract. Glauber theory provides a microscopic formulation of reactions of composite nuclei at high energies. Two approaches, recently used for the treatment of the proton–He systems, are discussed and contrasted. The observed sensitivity of few-body calculations to the nuclear size and structure inputs used is discussed.

It has been demonstrated that reaction calculations which include an explicit treatment of the few-body nature of halo nuclei result in an increased transparency in their high energy collisions with *massive* targets [1]. The resulting reductions in the calculated cross sections then suggest that larger halo extensions are required to reproduce the already enhanced cross section data. Such an analysis for ^{6}He+^{12}C is consistent with a ^{6}He rms matter radius of order 2.5 fm [2]. This is encouraging since three-body models, with physically sensible inputs in each two-body channel, can produce ^{6}He nuclei which differ appreciably from this size only by over- or under-binding the two halo neutrons.

Stimulated by recent data on elastic ^{6}He and ^{8}He scattering from protons at 700 MeV/nucleon [3] we consider such few-body calculations of observables in the case of a *nucleon* target. An understanding of the sensitivity of elastic scattering and reaction cross sections to the assumed projectile structure for a nucleon target is of interest in assessing the spectroscopic value of such data.

In Glauber's multiple scattering theory the proton+A elastic amplitude, for incident proton wave number k and momentum transfer $\boldsymbol{q}$, is the integral over proton–target center of mass (c.m.) impact parameters

$$f(\boldsymbol{q}) = \frac{ik}{2\pi}\int d^2\boldsymbol{b}\, e^{i\boldsymbol{q}\cdot\boldsymbol{b}}\, [1-\mathcal{S}_A(b)]\ , \qquad \mathcal{S}_A(b) = \langle \Phi_A | \prod_{j=1}^{A} S_j(b_j) | \Phi_A \rangle\ , \tag{1}$$

where b_j is the incident proton impact parameter on target nucleon j. The profile function $\mathcal{S}_A$ (the eikonal elastic S-matrix) is thus an A-body matrix element

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of the (translationally invariant) target ground state wave function Φ_A. $\mathcal{S}_A$ also determines the reaction cross section observable. The $S_j(b_j) = 1 - \Gamma_{pj}(b_j)$ specify completely the nucleon-nucleon scattering/dynamics input, have been determined [4,5] from fits to small angle pp and pn scattering data, and are assumed here to be spin-independent.

The required nuclear structure input is the target *many-body* density integrated over spin coordinates $\rho_A(\boldsymbol{x}_1, \ldots, \boldsymbol{x}_A) \equiv \langle \Phi_A | \Phi_A \rangle_{\rm spins}$, a function of $A-1$ target nucleon position coordinates $\boldsymbol{x}_j$ referred to the target center of mass. Dependent on the structure model used, ρ_A will contain cluster, dynamical, antisymmetrisation, and/or short range correlations in addition to the (already assumed) c.m. correlations.

I APPROXIMATION SCHEMES

The p+^{6,8}He scattering data of [3] have been analysed using a minimally (c.m.) correlated density description [3] and a few-body description [6] of the nuclear structure. To be definite we write equations in the case of ^{6}He, a Borromean two-neutron halo nucleus with a well developed α+n+n three-body structure. We think it useful to present clearly the approximations used in the two cases.

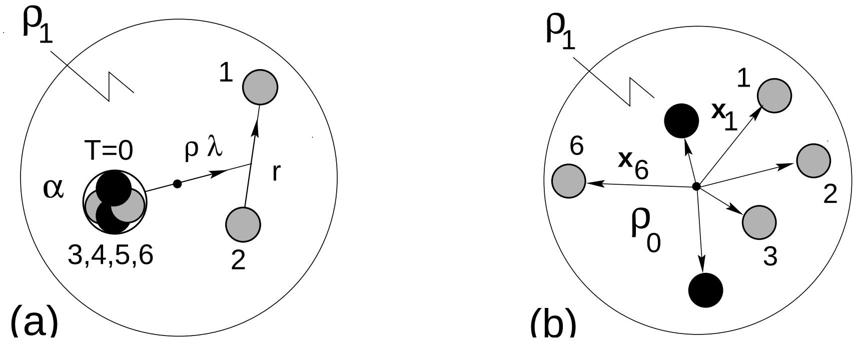

FIGURE 1. Schematic representation of the treatments of ^{6}He (a) in the few-body model, and (b) when including only c.m. correlations (see text). It is understood that the overall ^{6}He one-body density $\rho_1(x)$ is the same in the two cases and that $\sum_i \boldsymbol{x}_i = 0$.

In the few-body model, ^{6}He is treated as shown schematically in Fig. 1(a). The α+n+n relative motion wave function $\psi^{(3)}_{\rm rel}$ is obtained by solution of a three-body Schrödinger equation. Denoting the orbital angular momenta in coordinates $\boldsymbol{r}$ ($\boldsymbol{\rho}$) by l (λ), with total L, the dominant components ϕ_L in the ^{6}He ground state have $L(=\lambda=l)=0$ and 1. The ^{6}He many-body density is then [7]

$$\rho_6(\boldsymbol{x}_1, \ldots, \boldsymbol{x}_6) = \langle |\psi^{(3)}_{\rm rel}(\boldsymbol{\rho}, \boldsymbol{r})|^2 \rangle_{\rm spins}\; |\Phi_4|^2$$

$$\langle |\psi^{(3)}_{\rm rel}(\boldsymbol{\rho}, \boldsymbol{r})|^2 \rangle_{\rm spins} = \frac{1}{(4\pi)^2}\left[\phi_0^2(\rho, r) + \phi_1^2(\rho, r) - \phi_1^2(\rho, r) P_2(\hat{\boldsymbol{\rho}} \cdot \hat{\boldsymbol{r}})\right] \quad (2)$$

where Φ_4 is a (translationally invariant) α particle wave function and the assumed clustering in the projectile is clear. The corresponding one-body density $\rho_1(x)$ (normalised to unity) and hence the rms radius of ^{6}He can be computed as detailed in [1]. The profile function is

$$\mathcal{S}_6(b) = \int d\boldsymbol{\rho} \int d\boldsymbol{r} \ \langle|\psi^{(3)}_{\rm rel}(\boldsymbol{\rho},\boldsymbol{r})|^2\rangle_{\rm spins}\ \mathcal{S}_4(b_\alpha) S_n(b_1) S_n(b_2)\ , \tag{3}$$

where $\mathcal{S}_4(b_\alpha)$, given by Eq. (1), is the free p+α elastic S-matrix at the same incident energy per nucleon and should be consistent with such data [6]. Alpha particle core polarisation effects are assumed to be negligible [8]. These formulae summarise the physical basis of the presented few-body calculations for ^{6}He. For ^{8}He, Eq. (2) is revised to use instead a $\langle|\psi^{(5)}_{\rm rel}|^2\rangle_{\rm spins}$ in $\rho_8(\boldsymbol{x}_1,\ldots,\boldsymbol{x}_8)$ [6,9].

At the other extreme, if one neglects *all* correlations in the ^{6}He ground state, the uncorrelated 6-body density consistent with a given one-body density $\rho_1(x)$ is $\tilde{\rho}_6(\boldsymbol{r}_1,\ldots,\boldsymbol{r}_6) = \prod_{j=1}^{6}\rho_1(r_j)$, where all $\boldsymbol{r}_j$ are independent and refer to a fixed center. This is certainly inadequate for light systems, e.g. [5]. Including c.m. correlations (only) requires the use of the minimally correlated many-body density

$$\rho_6(\boldsymbol{x}_1,\ldots,\boldsymbol{x}_6) = \mathcal{N} \prod_{j=1}^{6} \rho_0(x_j)\, \delta(\sum_{i=1}^{6} \boldsymbol{x}_i) \tag{4}$$

where the imposed c.m. constraint is explicit and where $\mathcal{N}$ is a normalisation. Clearly $\rho_0(x)$, see Fig. 1(b), is the assumed position probability density of each nucleon about the ^{6}He c.m. such that the resulting ρ_6 derives a given one-body density $\rho_1(x)$. For a proper assessment of the importance of the cluster and other correlations explicit in Eq. (2) it would be helpful to compare calculations of different observables when using Eqs. (2) and (4). The relationship of ρ_0 to ρ_1 however is non-trivial and calculations using Eq. (4) have not been performed. Only when ρ_1 has a Gaussian form are the c.m. effects easily treated [5].

In the analysis presented in [3] the above mentioned c.m. correlations are included, but only approximately. In that analysis a number of model one-body densities ρ_1 are assumed for the ^{6}He and ^{8}He systems. In all cases the c.m. correlations are nevertheless treated as if ρ_1 *is* a Gaussian distribution with the rms matter radius of ρ_1. The accuracy of this procedure is untested, particularly for halo nuclei like ^{6}He, where an essential feature of realistic wave functions with Borromean three-body asymptotics will be an extended component in the density.

II CALCULATIONS OF OBSERVABLES

In [3], on the of basis fits to the experimental data obtained using the approximate (minimally correlated) theoretical model above, it is concluded that the p+^{6,8}He elastic scattering data determine "essentially model independent" values for the rms radii for the He isotopes. Values are quoted with small errors. We show that this is manifestly not the case and that reaction calculations are highly sensitive to details of the structure (wave function) inputs beyond simply their rms radii.

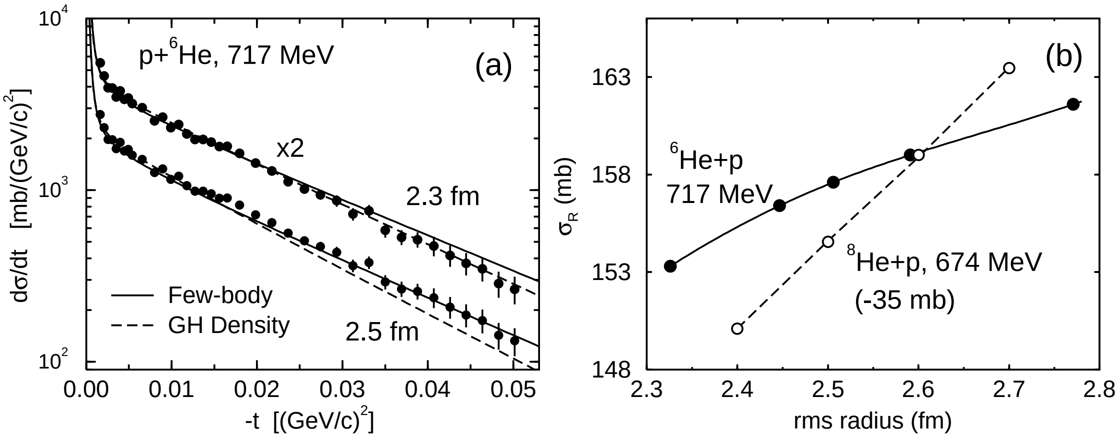

FIGURE 2. (a) Calculated and experimental elastic differential cross sections versus q^2 $(=-t)$ for p+^{6}He at 717 MeV. (b) Calculated reaction cross sections for p+^{6}He, ^{8}He as a function of the nuclear rms radius.

Fig. 2(a) contrasts the results of the few-body approach and those of [3] for p+^{6}He. The solid curves are the few-body results for ^{6}He structures with rms radii of 2.33 fm (upper) and 2.5 fm (lower). These include cluster correlations and realistic 2n-halo asymptotics. The dashed curves show the results of the approach of [3] using model (GH) ^{6}He densities with radii of 2.3 fm (upper) and 2.5 fm (lower). The results from the two models are quite different. The GH density-based calculations suggest the radius of 2.5 fm is too large (manifest as too steep a cross section with q^2). On the other hand the ^{6}He wave function with this rms radius reproduces the measured ^{6}He+^{12}C reaction cross section [2] and is consistent with the elastic scattering data within the few-body analysis.

The calculated reaction cross sections from the few-body model are shown in Fig. 2(b) for both ^{6}He and ^{8}He systems. Calculations of this observable when using the approximate treatment of c.m. correlations [3] have not been presented. Based on results for ^{12}C targets [1,2], it is expected that this observable will show considerable sensitivity to the cluster correlations included in the few-body approach. A comparison of such calculations would therefore be interesting. The sensitivity of the reaction cross section to rms radius is significant and a measurement of this observable could be very valuable.

The differences noted in Fig. 2(a) may arise from many sources since, within the few-body model, c.m. correlations are treated exactly, we include the granular nature of the nucleus and so use wave functions with realistic asymptotics. However, since these wave functions are exact solutions of three-body calculations, specific features of the wave function are not easily controlled to assess different sensitivities.

For instance, the wave function with rms radius 2.33 fm used in Fig. 2(a) has 2n-separation energy ≈1.2 MeV. While it has realistic three-body asymptotics the spatial fall off will be incorrect in detail.

For ^{8}He we use the COSMA wave function for $\psi^{(5)}_{\rm rel}$ which gives a simple expression for $\rho_8(\boldsymbol{x}_1,\ldots,\boldsymbol{x}_8)$ [9] while including exactly c.m., cluster, and those correlations associated with the antisymmetrisation of the four valence neutrons, amongst themselves. Each is assumed in a $p_{3/2}$ orbital with respect to the alpha core. In the original COSMA model these have oscillator radial wave functions. Here we also match these functions appropriately to (p-wave) Hankel function tails for an assumed single particle separation energy. This simple wave function is now flexible enough to allow construction of families of ^{8}He wave functions with the same asymptotic forms and different rms radii, or with the same rms radius and different asymptotic forms. The results of calculations using such wave functions are

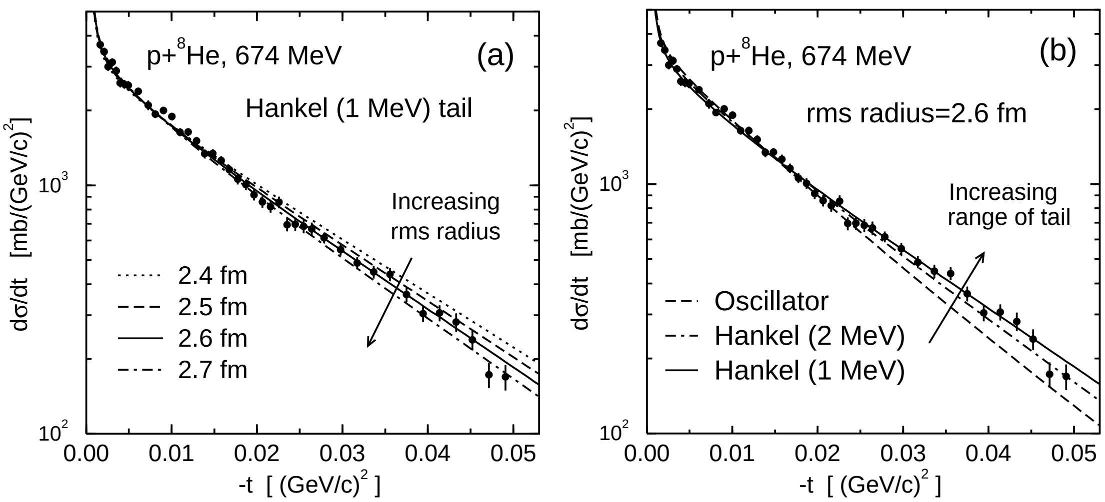

FIGURE 3. Calculated and experimental elastic differential cross sections versus q^2 ($=-t$) for p+^{8}He at 674 MeV for wave functions with (a) different rms radii but a fixed asymptotic form, and (b) a fixed rms radius but different asymptotic forms, for the p-wave valence neutrons.

shown in Fig. 3. Fig. 3(a) shows the calculated differential cross sections for fixed valence nucleon asymptotics, a Hankel function of 1 MeV separation energy, and the rms radii indicated. An increase in the slope of the cross section with rms radius is obtained, suggesting that high quality data might accurately determine an rms size. Evident from Fig. 3(b) however is that calculations for wave functions with this same rms radius, but different functional asymptotic forms for the valence n wave functions, show greater variation. Moreover, extending the range of the asymptotics, from Gaussian toward less weakly bound Hankel forms, the calculated slopes of the differential cross section move in the opposite direction to those in Fig. 3(a). It follows that a suitably chosen wave function with a small rms radius and Gaussian asymptotics can produce a similar result to a wave function with larger rms radius and Hankel function asymptotics.

This clarifies a very basic model dependence in the differential cross section calculations. While the experimental data can be used to assess the consistency of the data and a given structure model, one requires a confidence in the nuclear structure model used, in its asymptotics, and in the full and accurate treatment of this structure in the reaction calculation, to go further. We believe our few-body model treatment of both the structure and the scattering of ^{6}He is the most accurate attempt yet to do so. It would be interesting to engineer three-body wave functions for ^{6}He with the same (physical) 2n-separation energy but different rms sizes to delineate more carefully the sensitivity to the rms size in this case. The practicality of performing 6-body ^{6}He calculations using Eq. (4), to clarify the role of correlations beyond the trivial c.m. effects should also be considered.

III SUMMARY

We have discussed the 'few-body' and 'minimal correlations' models as applied in analyses of high energy proton elastic scattering from the helium isotopes. We show that the few-body calculations reveal very significant dependence on the structure model assumed and that the available elastic scattering data do not, by themselves, determine the rms radii of these isotopes in any model independent sense. Few-body calculations show clear sensitivity to the rms radius but also to the wave function asymptotics assumed in calculating the target many-body density. The use of simple model descriptions of the structures will thus lead to significant ambiguities in extracted spectroscopic information. Within the few-body model description for both the structure and the reaction, the elastic scattering data are consistent with a ^{6}He rms radius of 2.5 fm, and so with few-body calculations of the ^{6}He+^{12}C reaction cross section. To assess the specific role played by the clustering in the system it will be necessary to formulate differential and reaction cross section calculations which are able to treat exactly the c.m. correlations only. The observed sensitivity to the asymptotic behaviour of the wave functions suggests a more sophisticated wave function is needed in the case of the ^{8}He system.

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