Noneikonal calculations for few-body projectiles

J. M. Brooke, J. S. Al-Khalili, and J. A. Tostevin
Department of Physics, School of Physical Sciences, University of Surrey, Guildford, Surrey, GU2 5XH, United Kingdom
(Received 15 September 1998; revised manuscript received 18 November 1998)

Calculations which improve upon the eikonal model description of the scattering of loosely bound n-cluster composite nuclei at low and medium energies are studied. Each cluster-target eikonal phase shift is replaced by the continuation of the corresponding exact partial wave phase shift to noninteger angular momenta. Comparisons with fully quantum mechanical calculations for two-body projectiles show that this yields an accurate practical alternative to few-body adiabatic model calculations. Calculations are shown to be accurate for projectile energies as low as 10 MeV/nucleon at which the eikonal approximation is no longer reliable. [S0556-2813(99)04703-2]

PACS number(s): 21.45.+v, 24.50.+g, 25.60.−t, 25.70.Bc

I. INTRODUCTION

Semiclassical approximations have been used extensively in nuclear physics for approximate solutions of the small wavelength scattering problem. Theoretical formulations involve the phase shifts introduced by the projectile's interaction with a target, expressed as a function of the projectile's impact parameter b. Such models were developed extensively by Glauber and co-workers, e.g., Refs. [1,2], for the scattering of both elementary and composite systems. In the Glauber diffraction theory, the interaction of an incident nucleon with a composite nucleus is described by a multiple scattering series in which the incident nucleon scatters from an ensemble of fixed target nucleons. In the absence of three-body forces, the total projectile-target phase shift is also the sum of the phase shifts due to each target nucleon. The individual nucleon-nucleon (NN) scatterings are described by free NN scattering phase shifts. The use of phase shifts is, however, usually not discussed since, when applied at several hundred MeV, this explicit reference to phase shifts is recast in favor of the NN scattering amplitude. This is then parametrized directly from extensive small angle NN scattering and, through the optical theorem, total cross section data. The nucleon-nucleus scattering amplitude is obtained as the average of these elementary, impact parameter-dependent amplitudes, over the target ground state many-body density. Implicit is that the incident energy is sufficiently high that the target nucleons can be considered fixed during the passage of the projectile, the sudden or adiabatic approximation.

In this paper our interest is the scattering of very loosely bound composite projectiles from a stable target nucleus at energies of less than 100 MeV/nucleon. The composite projectile is assumed to be composed of n clusters (where n is less than the number of projectile nucleons). For halo nuclei these clusters are the core and the valence particles. Here it is the study of the cluster relative motion degrees of freedom in the projectile, and hence excitation and breakup effects, which are of interest. The projectile-target scattering is now described as a $(n+1)$-body problem [3,4], the projectile's n-body ground state density must be averaged over once the cluster-target phase shifts have been evaluated, and the adiabatic approximation is made at the level of the $(n+1)$-body Schrödinger equation [4]. In this lower-energy regime the scattering is not highly forward angle focused. The natural expression for the required scattering amplitudes is therefore in terms of the cluster-target (impact parameter dependent) phase shifts, and the accuracy with which this representation reproduces exact cluster-target scattering amplitudes, and so can be directly connected with experimental observables, needs to be reexamined. We consider this quantitative question in Sec. II. Generally speaking, however, experimental data are insufficient to allow an unambiguous determination of the cluster-target scattering amplitude or phase shifts.

Most recently, semiclassical few-body calculations of scattering and reactions in this lower-energy regime have made extensive use of the eikonal approximation, e.g., Refs. [3–5]. The assumption is that, for the purpose of calculating each cluster-target phase shift, the cluster's trajectory can be approximated by a straight line path through an assumed interaction potential with the target. This use of a potential description is extremely useful for making theoretical predictions for exotic and halo systems. Then global optical potential parametrizations, incorporating data systematics, or tested theoretical potential models, can be used for individual cluster-target systems when data are very limited or unavailable.

These approximate calculations, including those for ^{8}He scattering [6], treated as a six-body problem, show that the eikonal model provides an efficient basis for reaction calculations of few- and many-body projectiles. This efficiency arises from the additivity of phases property of the eikonal theory and means that attempts to extend its accuracy are of interest. In a recent Rapid Communication [7] noneikonal modifications to the phase shift of each cluster were introduced, but to third order in $\epsilon\,(\propto k^{-2})$, where k is the cluster-target center of mass wave number. In applications to ^{11}Be + ^{12}C scattering above 25 MeV/nucleon, these changes improved the accuracy of the calculations to lower energies and larger scattering angles.

Here we assess a simpler procedure. Rather than develop and sum the expansion for the phase shift in powers of ϵ we solve directly the radial Schrödinger equation for each cluster-target two-body system at the required impact parameters or noninteger orbital angular momenta λ. We therefore no longer make the eikonal approximation, but retain the adiabatic and additivity of phases approximations. Corrections to the latter, manifest as phase shift contributions due to simultaneous cluster-target potential overlaps, have been discussed by Feshbach [8]. We have studied such overlap terms quantitatively for halo nuclei using simple potential parametrizations. The results will be presented elsewhere. These estimates of overlap contributions are very small for spatially extended systems and the results presented here are consistent with these findings.

This paper deals only with corrections to the eikonal approximation. The adiabatic approximation is expected to be reasonable when the relevant excitation energies of the projectile are small compared with its incident energy. As in Ref. [7] we will use full three-body quantum-mechanical calculations, which make the adiabatic approximation but not the eikonal or additivity of phases assumptions, to assess the importance and accuracy of the noneikonal modifications. We present calculations of applications to two-body projectile scattering, namely, the deuteron and ^{11}Be (^{10}Be $+$neutron).

II. STRUCTURELESS PROJECTILE SCATTERING

A. Neutral point particle scattering

Glauber, Franco, and Wallace [1,2,9] have discussed in detail the mathematical and physical relationship of the discrete exact (partial wave sum) representation

$$F(\theta)=\frac{1}{2ik}\sum_{l=0}^{\infty}(2l+1)P_l(\cos\theta)[S_l-1], \tag{1}$$

and the Fourier-Bessel (impact parameter) integral representation of the scattering amplitude for a point particle. Here k is the projectile incident wave number in the center of mass frame. The exact partial wave S matrix $S_l=\exp(2i\delta_l)$ is obtained by solution of the radial Schrödinger equation for a given orbital angular momentum l in the presence of the assumed projectile-target interaction $V(r)$. Upon continuing these discrete l values to continuous angular momenta λ, and associating the physical angular momenta l with impact parameters b according to $bk=l+1/2$, one can write [2,9]

$$f(\theta)=-ik\int_0^{\infty}b\,db\;J_0(qb)[S(b)-1], \tag{2}$$

where $q=2k\sin(\theta/2)$ is the momentum transfer. $S(b)$ in Eq. (2) is the continuation of S_l for real noninteger angular momenta, and can be obtained by solution of the radial Schrödinger equation for angular momentum $\lambda=bk-1/2$. Explicitly, asymptotically in r

$$\psi_\lambda(r)\rightarrow\frac{i}{2}[H_\lambda^{(-)}(kr)-S_\lambda H_\lambda^{(+)}(kr)], \tag{3}$$

where the $H^{(\pm)}$ are the usual in- and out-going waves radial asymptotic solutions, but for noninteger λ. Thus $S(b)=S_\lambda=\exp[i\mathcal{X}(b)]$ coincides with the exact S_l for all integer λ, with $\mathcal{X}(b)=2\delta_\lambda$. We refer to $S(b)$ as the exact continued (EC) S matrix. It is important that Eq. (2) has not made the eikonal approximation to the scattering phase shift.

The amplitudes f and F are not formally equal. In writing Eq. (2), in addition to the discrete to continuous variable transformation, only the leading term in the small forward angle expansion of the Legendre function P_λ, has been retained (e.g., Appendix A of Ref. [2] and Ref. [9]). This yields the Bessel function $J_0(qb)$. An additional factor $W[\delta]$, which multiplies $S(b)$ in a complete formal derivation [9], is unity in this limit. The approach followed by Wallace, with higher energies in mind, is to develop expansions of both the $W[\delta]$ term and $S(b)$ in inverse powers of k^2 and to collect terms of equal order. At the low energies of interest here and particularly for light projectiles, i.e., for small k, such an expansion scheme is not particularly useful. This will be seen below in the context of the phase shift expansion [7].

The eikonal approximation f_0 to the scattering amplitude f has the same form [1],

$$f_0(\theta)=-ik\int_0^{\infty}b\,db\,J_0(qb)[S_0(b)-1], \tag{4}$$

but $S_0(b)$ is now determined by the eikonal approximation to the phase shift $\mathcal{X}_0(b)$ the integral of the assumed interaction along a straight line path at impact parameter b

$$S_0(b)=\exp[i\mathcal{X}_0(b)]=\exp\left[-\frac{i}{\hbar v}\int_{-\infty}^{\infty}V(\sqrt{b^2+z^2})dz\right]. \tag{5}$$

Here $v=\hbar k/\mu$ is the asymptotic relative velocity and μ is the reduced mass of the projectile and target.

In this work we perform calculations based directly on the amplitude of Eq. (2), in which we make the small angle $W[\delta]\equiv 1$ approximation only. This can also be viewed as replacing the eikonal profile function $S_0(b)$ by an improved description, a viewpoint helpful in its generalization to composite projectiles. This scheme was used in Ref. [7] for composite systems. There, however, only an approximate description of $S(b)$ and $\overline{S}(b)$ of the next subsection, was used. Specifically, the power series expansion of the phase shift $\mathcal{X}(b)$ to third order in $\epsilon=1/\hbar kv$ about the eikonal phase was used which included the correction terms detailed by Rosen and Yennie [9,10]. The accuracy of the $S(b)$ arising from this expansion is a related but different issue to the accuracy of the approximate amplitude f of Eq. (2). Below we compare the $S(b)$ from this expansion with those of the exact continuation (radial equation solution) and from the eikonal model. We also require these two-body S matrices for the three-body scattering calculations considered in the next section where they appear as inputs.

To assess the accuracy with which the approximate amplitudes f of Eq. (2) reproduce observables calculated using the exact partial wave amplitude F, Eq. (1), and f_0, Eq. (4), we first perform calculations for neutron and ^{10}Be$+^{12}$C scattering at low and medium energies. For each system we consider energies of 10, 25, and 50 MeV/nucleon assuming, for simplicity, the same interaction parameters at each energy.

For $n+^{12}$C we assume a complex volume Woods-Saxon neutron potential with parameters $V=37.4$ MeV, $r_V=1.2$ fm, $a_V=0.75$ fm, $W=10.0$ MeV, $r_W=1.3$ fm, $a_W=0.6$ fm [7]. Figure 1 compares the moduli of the $n+^{12}$C S matrices as a function of impact parameter calculated using the eikonal (dashed curves) and EC (solid curves) phase shifts at the

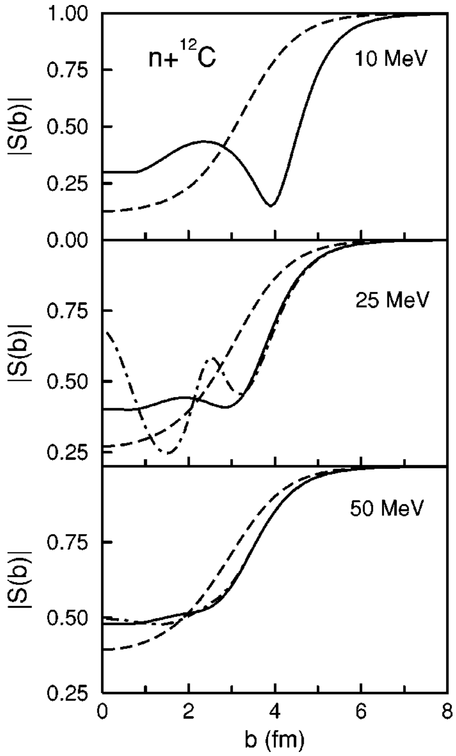

FIG. 1. Moduli of the elastic S matrices for $n+{}^{12}\mathrm{C}$ scattering at 10, 25, and 50 MeV calculated using the eikonal (dashed curves) and EC (solid curves) phase shifts. The dot-dashed curves at 25 and 50 MeV result from the expansion of the phase shift used in Ref. [7] (see text).

three energies. Even at the higher energy there are significant noneikonal corrections. These make the target appear larger and also more transparent to the neutron at small impact parameters. The dot-dashed curves at 25 and 50 MeV are the results when including terms to third order in the power series expansion of the phase shift [7]. To the same order this expansion is unstable at the lowest energy. The imaginary part of the approximate phase shift becomes positive for a range of impact parameters and so does not yield useful results. Direct use of the EC S matrix avoids such instabilities, is much simpler, and also avoids the slow convergence of the phase shift expansion, manifest in the 25 MeV calculation in Fig. 1.

Figure 2 shows the calculated $n+{}^{12}\mathrm{C}$ elastic differential cross section angular distributions at 10, 25, and 50 MeV. The solid curves are the exact partial wave calculations using Eq. (1), the dashed curves use the approximate (impact parameter integral) amplitude of Eq. (2), and the dotted curves use the eikonal amplitude of Eq. (4). The improvements resulting from the use of $S(b)$ rather than $S_0(b)$ are clear and extend to reasonably large scattering angles. The small deviations from the exact calculations suggest that corrections to the $W[\delta]=1$ approximation are indeed small.

B. Charged point particle scattering

To consider $^{10}\mathrm{Be}$ scattering we need to generalize the formalism to include the Coulomb interaction. The exact partial wave amplitude is now

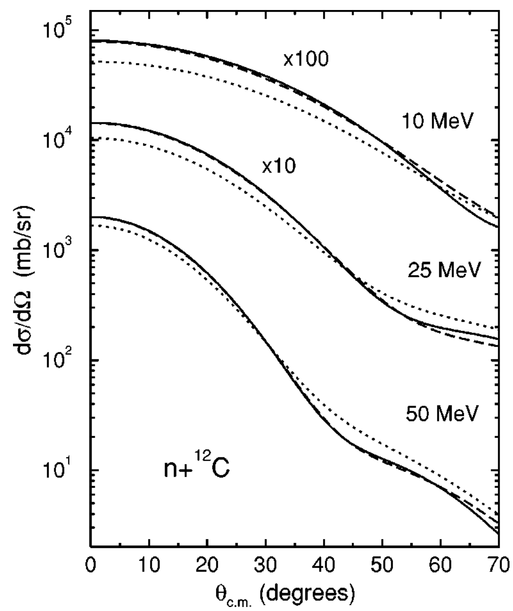

FIG. 2. Exact (solid curves), approximate impact parameter integral (dashed curves), and eikonal model (dotted curves) calculations of the elastic differential cross section angular distributions for $n+{}^{12}\mathrm{C}$ scattering at 10, 25, and 50 MeV.

$$\overline{F}(\theta)=f_{pt}(\theta)+\frac{1}{2ik}\sum_{l=0}^{\infty}(2l+1)P_l(\cos\theta)e^{2i\sigma_l}[\overline{S_l}-1], \tag{6}$$

where $f_{pt}(\theta)$ is the amplitude for point charge (Rutherford) scattering and σ_l is the Coulomb phase shift. The $\overline{S_l}$ here, obtained by matching to Coulomb functions the solution of the radial Schrödinger equation in the presence of both nuclear and Coulomb interactions, characterize only the deviations from point Coulomb scattering.

The eikonal approximation to $\overline{F}$ is obtained by including both Coulomb and nuclear interactions, $V(r)\equiv V_N(r)+V_C(r)$, in the eikonal phase of Eq. (5), yielding a sum $\overline{\mathcal{X}}_0(b)=\mathcal{X}_{0N}(b)+\mathcal{X}_{0C}(b)$ of nuclear and Coulomb terms. The Coulomb interaction is taken to be that of a uniformly charged sphere. The logarithmic divergence of the Coulomb phase requires screening arguments to be used, see, e.g., Ref. [11]. The result is that $\mathcal{X}_{0C}(b)=\mathcal{X}_{0\rho}(b)+\mathcal{X}_a$ where the first term is the phase due to the assumed Coulomb interaction and $\mathcal{X}_a=-2\eta\ln(2ka)$ is a constant (screening) phase, a denoting the screening radius. The eikonal amplitude analogous to Eq. (6) can be written

$$\overline{f}_0(\theta)=e^{i\mathcal{X}_a}\left\{f_{pt}(\theta)-ik\int_0^{\infty}b\,db\,J_0(qb)e^{i\mathcal{X}_{pt}(b)}\times[\overline{S}_0(b)-1]\right\}, \tag{7}$$

where $\mathcal{X}_{pt}(b)=2\eta\ln(kb)$ is a point Coulomb interaction eikonal phase. The effect of screening appears only as an overall real phase on the elastic amplitude and has no consequences for observables. We will not show it explicitly in subsequent expressions. $\overline{S}_0(b)$ here characterizes the deviations from point Coulomb scattering and includes the phase

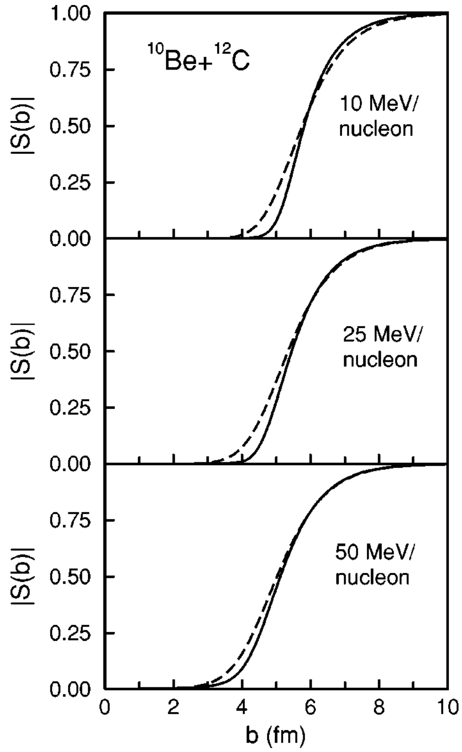

FIG. 3. Moduli of the elastic S matrices for $^{10}Be+{}^{12}C$ scattering at 10, 25, and 50 MeV/nucleon calculated using the eikonal (dashed curves) and EC (solid curves) phase shifts.

shifts due to both the nuclear interaction and the short-range (uniform sphere) deviations from a point Coulomb interaction, i.e.,

$$\overline{S_0}(b)=\exp[i\mathcal{X}_{0N}(b)+i\mathcal{X}_{0\rho}(b)-i\mathcal{X}_{pt}(b)]. \quad (8)$$

As for uncharged projectiles we replace the eikonal $\overline{S_0}(b)$ with $\overline{S}(b)=\overline{S}_\lambda=\exp[i\overline{\mathcal{X}}(b)]$, obtained by solution of the appropriate radial Schrödinger equation for noninteger λ. This yields an approximate (nuclear+Coulomb) impact parameter integral amplitude

$$\overline{f}(\theta)=f_{pt}(\theta)-ik\int_0^\infty b\,db J_0(qb)e^{i\mathcal{X}_{pt}(b)}[\overline{S}(b)-1]. \quad (9)$$

For the $^{10}Be+{}^{12}C$ potential we assume Woods-Saxon parameters $V=123.0$ MeV, $r_V=0.75$ fm, $a_V=0.8$ fm, $W=65.0$ MeV, $r_W=0.78$ fm, and $a_W=0.8$ fm. This potential, consistent with the available $^{10}Be+{}^{12}C$ data at 59.4 MeV/nucleon [12], is used for all three energies. The Coulomb interaction is taken as due to a uniformly charged sphere of radius parameter $r_c=1.20$ fm. All ^{10}Be radius parameters are multiplied by $10^{1/3}+12^{1/3}$. Figure 3 compares the calculated moduli of the S matrices as a function of impact parameter using the eikonal (dashed curves) and EC (solid curves) phase shifts for the $^{10}Be+{}^{12}C$ system at 10, 25, and 50 MeV/nucleon incident energy. Due to the larger k for this heavy fragment the deviations from the eikonal model are smaller but nevertheless still significant.

Figure 4 shows the calculated elastic differential cross section angular distributions (as a ratio to the Rutherford cross section) for $^{10}Be+{}^{12}C$ scattering at 10, 25, and 50 MeV/nucleon. The solid curves are the exact partial wave calculations resulting from Eq. (6). The dashed curves result from the impact parameter integral amplitude $\overline{f}$ while the dotted curves are the results of the eikonal amplitude of Eq. (7). The improvements in the calculated cross sections which result from the use of $\overline{S}(b)$ are significant and the small deviations from the exact calculations suggest that errors introduced by the $W[\delta]=1$ approximation are also small in this heavier charged particle case.

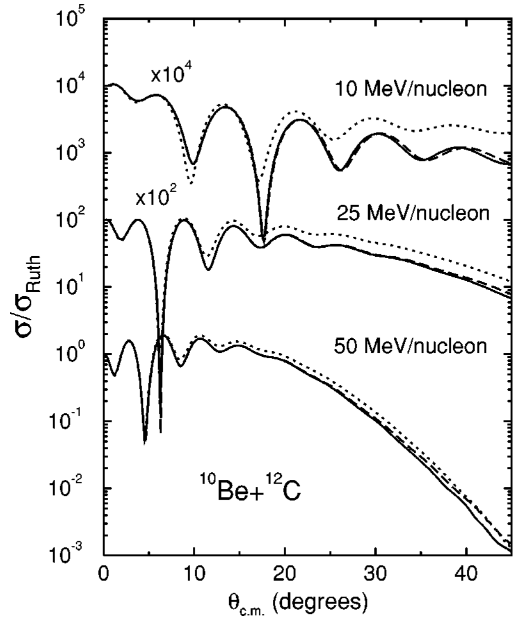

FIG. 4. Exact (solid curves), approximate impact parameter integral (dashed curves), and eikonal model (dotted curves) calculations of the elastic differential cross section angular distributions (ratio to Rutherford) for $^{10}Be+{}^{12}C$ scattering at 10, 25, and 50 MeV/nucleon.

III. COMPOSITE PROJECTILE SCATTERING

We now consider the scattering of a bound n-body system from a target. In addition to the considerations already discussed, for composite projectiles one also makes an adiabatic approximation. The positions of the clusters within the projectile are thus fixed for the calculation of their scattering phase shifts with the target. In the eikonal limit each of these phase shifts is computed, as in the point projectile case, assuming a straight line path through the interaction region.

However, having made *only* the adiabatic approximation, it is also possible to solve the Schrödinger equation without the use of the eikonal or additivity of phases approximations. Such scattering calculations can be carried out for both two-body [13,14] and three-body [15] projectiles. However, even for two-body projectiles, such calculations involve large coupled channels sets and are time consuming. For three-body projectiles [15] they are at the limit of what is computationally feasible. In the following the results of calculations for deuteron+^{12}C and $^{11}Be+{}^{12}C$ scattering, which solve the three-body adiabatic equation without further approximation [14,7], are compared with those of the approximate procedure discussed here.

Consider first the eikonal model elastic scattering amplitude for an n-body projectile, comprising uncharged clusters and with a ground state relative motion wave function $\Phi_0^{(n)}$. The amplitude, the n-body equivalent of f_0 of Eq. (4), is

$$f_0^{(n)}(\theta)=-ik\int_0^\infty b\,db J_0(qb)[S_0^{(n)}(b)-1], \tag{10}$$

where $S_0^{(n)}(b)$ is now the n-cluster projectile eikonal S matrix

$$S_0^{(n)}(b)=\langle\Phi_0^{(n)}|\prod_{j=1}^{n} S_0^j(b_j)|\Phi_0^{(n)}\rangle. \tag{11}$$

In these equations b and k are the impact parameter and wave number of the projectile center of mass. Each $S_0^j(b_j)$ in Eq. (11) is a point particle eikonal S matrix for cluster j evaluated at its own impact parameter b_j, as defined in Eq. (5).

When one or more of the clusters is charged we must follow the Coulomb screening arguments used above. Now, for each charged cluster, $\mathcal{X}_{0C}^j(b_j)=\mathcal{X}_{0\rho}^j(b_j)+\mathcal{X}_a^j$. Since the screening phases $\mathcal{X}_a^j=-2\eta_j\ln(2ka)$ depend linearly on the Sommerfeld parameter of each cluster η_j and $\eta=\Sigma_j\eta_j$, these phases add to give the screening phase appropriate to the projectile $\mathcal{X}_a$. The few-body eikonal amplitude in the presence of nuclear and Coulomb forces, analogous to $\overline{f_0}$ of Eq. (7), is therefore (omitting the overall screening phase)

$$\overline{f_0^{(n)}}(\theta)=f_{pt}(\theta)-ik\int_0^\infty b\,db J_0(qb)e^{i\mathcal{X}_{pt}(b)}[\overline{S_0^{(n)}}(b)-1], \tag{12}$$

where

$$\overline{S_0^{(n)}}(b)=\langle\Phi_0^{(n)}|\exp\left\{\sum_{j=1}^{n}[i\mathcal{X}_{0N}^j(b_j)+i\mathcal{X}_{0\rho}^j(b_j)] -i\mathcal{X}_{pt}(b)\right\}|\Phi_0^{(n)}\rangle. \tag{13}$$

In the charged point particle discussion above, the association is made between the EC S matrix $\overline{S}(b)$ and the Coulomb modified eikonal S matrix $\overline{S_0}(b)$ given by Eq. (8). In the few-body case we replace, for each cluster j,

$$\overline{S_0^j}(b_j)\equiv\exp[i\mathcal{X}_{0N}^j(b_j)+i\mathcal{X}_{0\rho}^j(b_j)-\mathcal{X}_{pt}^j(b_j)]\rightarrow\overline{S^j}(b_j), \tag{14}$$

with $\mathcal{X}_{pt}^j(b_j)=2\eta_j\ln(kb_j)$. That is we replace each cluster S matrix by the exact continued one. With these replacements $\overline{S_0^{(n)}}(b)$ of Eq. (13) is renamed $\overline{S^{(n)}}(b)$, consistent with earlier notation, where

$$\overline{S^{(n)}}(b)=\langle\Phi_0^{(n)}|\left[\prod_{j=1}^{n}\overline{S^j}(b_j)\right]\times\exp\left[\sum_{j'=1}^{n}\mathcal{X}_{pt}^{j'}(b_{j'})-i\mathcal{X}_{pt}(b)\right]|\Phi_0^{(n)}\rangle. \tag{15}$$

Each $\overline{S^j}(b_j)$ is obtained by solution of the appropriate two-body radial equation for all required b_j.

These EC S matrices include noneikonal corrections to each cluster-target phase shift to all orders. The resulting calculation retains the efficient computational structure of the few-body Glauber model, involving a product of each cluster S matrix. This approximation is expected to be good for weakly bound halo systems where the valence nucleon(s) spend most of their time far from the core. The full adiabatic calculations referred to earlier do not make the additivity of phases approximation and so provide an assessment of such effects.

A. Application to deuteron+ ^{12}C scattering

For deuteron scattering the inputs required are the deuteron wave function and the proton- and neutron-target S matrices $\overline{S^p}(b_p)$ and $S^n(b_n)$ obtained by solution of the free p- and n-target scattering problems at half the incident deuteron energy. The three-body elastic amplitude is

$$\overline{f^{(2)}}(\theta)=f_{pt}(\theta)-ik\int_0^\infty b\,db J_0(qb)e^{i\mathcal{X}_{pt}(b)}[\overline{S^{(2)}}(b)-1], \tag{16}$$

where b, b_p, and b_n are the deuteron, proton, and neutron impact parameters, and

$$\overline{S^{(2)}}(b)=\langle\Phi_0^{(2)}|\overline{S^p}(b_p)S^n(b_n)\exp[2i\eta\ln(b_p/b)]|\Phi_0^{(2)}\rangle. \tag{17}$$

We consider deuteron scattering at the three incident energies per nucleon of the previous section. The neutron and proton optical potential parameters at the three energies were calculated from the global parametrization of Ref. [16]. The deuteron ground state wave function was assumed to be a pure S wave and was calculated using a central Wood-Saxon interaction with depth 83.37 MeV, a radius of 0.95 fm, and a diffuseness of 0.65 fm. These parameters gave a deuteron binding energy of 2.224 MeV and an rms np separation of 4 fm.

Figure 5 shows the calculated elastic differential cross section angular distributions (as a ratio to Rutherford) for $d + ^{12}$C scattering at 10, 25, and 50 MeV/nucleon. The dotted curves are obtained using the eikonal model and the dashed curves using the EC phases. The solid curves are obtained from a full quantum-mechanical (coupled channels) calculation which makes only the adiabatic approximation. The latter calculation is time consuming and includes s, p, d, and f wave np breakup states. The eikonal and EC calculations, which include all breakup states through closure, are extremely fast. Even for this very light projectile system, tightly bound in comparison with halo nuclei, the agreement between the EC phase shifts and exact adiabatic calculations is rather good, even down to energies as low as 10 MeV/nucleon. There is no indication that corrections to the additivity of phases approximation, included in the full adiabatic calculation, are significant even for this light-ion and weakly absorptive system. Of greater current interest is the application of such ideas to halo nuclei with weaker binding, enhanced breakup channels, and larger radial extent.

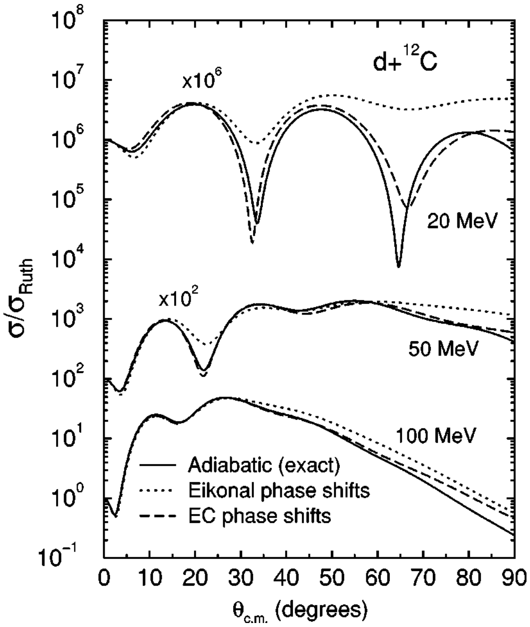

FIG. 5. Calculated elastic differential cross section angular distributions (ratio to Rutherford) for $d+{}^{12}$C scattering at 10, 25, and 50 MeV/nucleon using the eikonal (dotted curves) and EC (dashed curves) phase shifts. The solid curves are the results of the exact adiabatic model calculations.

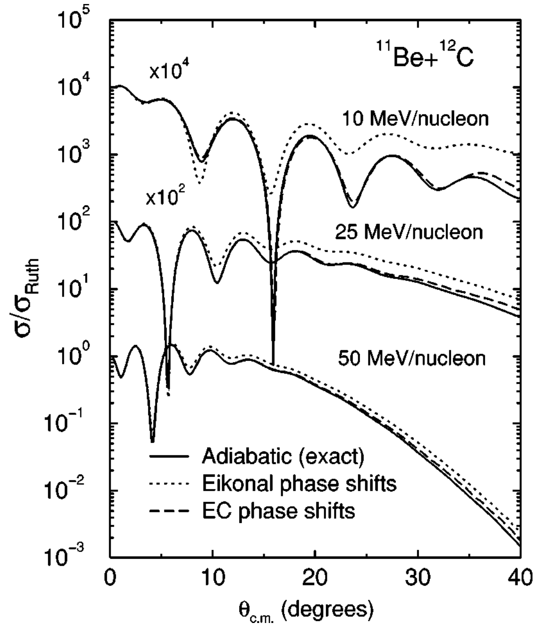

FIG. 6. Calculated elastic differential cross section angular distributions (ratio to Rutherford) for ^{11}Be+ ^{12}C scattering at 10, 25, and 50 MeV/nucleon. The curves have the same meanings as in Fig. 5.

B. Application to ^{11}Be+ ^{12}C scattering

^{11}Be is a good example of a two-body halo nucleus composed of a ^{10}Be core c and a valence neutron. We consider ^{11}Be+ ^{12}C scattering using the neutron and ^{10}Be+target S matrices already computed and shown in Figs. 1 and 3 of Sec. II. The scattering amplitude is given by Eq. (16). The two-body projectile S matrix $\overline{S^{(2)}}(b)$ appropriate to ^{11}Be is

$$\overline{S^{(2)}}(b)=\langle\Phi_0^{(2)}|\overline{S^c}(b_c)S^n(b_n)\exp[2i\eta\ln(b_c/b)]|\Phi_0^{(2)}\rangle, \tag{18}$$

with b_c and b_n the core and neutron impact parameters. The^{11}Be ground state wave function $\Phi_0^{(2)}$ was taken to be a pure $2s_{1/2}$ neutron single particle state, with separation energy 0.504 MeV, calculated in a central Wood-Saxon potential of geometry $r_0=1.00$ fm and $a_0=0.53$ fm. With a ^{10}Be root mean squared (rms) matter radius of 2.28 fm this generates a ^{11}Be composite with rms radius of 2.90 fm, in agreement with recent few-body analyses [17].

Figure 6 shows the calculated elastic differential cross section angular distributions (as a ratio to Rutherford) for ^{11}Be+ ^{12}C scattering at 10, 25, and 50 MeV/nucleon. The curves have the same meanings as those in Fig. 5. The agreement with the full adiabatic calculations in this case is excellent. We attribute the improved agreement in this halo nucleus case to the weaker binding and the probable further reduction in correlated scattering or overlapping potential contributions. As the figure shows, for practical purposes using the EC phases provides a reliable method for adiabatic model calculations of the scattering of one-nucleon halo systems.

C. Applications to many-body projectiles

The scheme presented here is readily applied to three- or more-cluster projectiles. A short report of an application to scattering of the two-neutron halo nucleus ^{6}He was recently presented elsewhere [18]. Comparisons, in the case with four-body adiabatic model calculations [15], showed a very similar quality of agreement to that reported here. Whereas the four-body calculations with EC phase shifts are rather straightforward, those adiabatic calculations are at the limit of computational feasibility. Use of the EC phase shifts for projectiles such as ^{8}He, modeled as a five-body $(\alpha+4n)$ structure, and where $\overline{S^{(5)}}(b)$ can be calculated using random sampling techniques, e.g., Ref. [6], is also straightforward. In that case the larger number of clusters occupy a relatively smaller volume of space. The probability that pairs of cluster-target interactions will overlap will therefore be greater and the additivity of phases approximation which underpins the current discussion may need to be reexamined. This possibility remains to be fully investigated.

IV. SUMMARY AND CONCLUSIONS

We have assessed calculations of the scattering of loosely bound n-cluster composite projectiles within the framework of a few-body Glauber model. Each cluster-target eikonal phase shift is replaced by the continuation of the exact partial wave phase shift for noninteger orbital angular momenta λ. The latter are computed, for each cluster(j)-target pair with wave number k_j, by numerical solution of the radial Schrödinger equation for the required impact parameters b_j, or angular momenta $\lambda_j=b_jk_j-1/2$. The calculations retain the simplicity which arises from the additivity of phases in Glauber's diffraction theory.

The accuracy of results using these phase shift continuations was first assessed in the cases of neutral and charged point projectile-target scattering. Results were then compared with full three-body adiabatic model calculations for two-cluster projectiles. The calculated cross sections in the two cases are found to be in good agreement, even at projectile energies as low as 10 MeV/nucleon.

We have assumed an adiabatic treatment of the projectile's cluster coordinates. At the lowest incident energy considered, 10 MeV/nucleon, there will almost certainly be corrections needed to the adiabatic approximation. This paper does not address these effects. We have shown, however, that the use of the continued exact phases may provide an alternative approximate starting point from which to consider such effects, as it provides an efficient means for performing such calculations. The procedure used is readily applied to three- and more-cluster projectiles for which full adiabatic model calculations are either extremely difficult or, in the latter case, not yet practical.

ACKNOWLEDGMENT

The financial support of the Engineering and Physical Sciences Research Council (U.K.) through Grant No. GR/J95867 is gratefully acknowledged.

[1] R.J. Glauber, in *Lectures in Theoretical Physics*, edited by W.E. Brittin (Interscience, New York, 1959), Vol. 1, p. 315.
[2] V. Franco and R.J. Glauber, Phys. Rev. **142**, 1195 (1966).
[3] K. Yabana, Y. Ogawa, and Y. Suzuki, Phys. Rev. C **45**, 2909 (1992); Nucl. Phys. **A539**, 295 (1992).
[4] J.S. Al-Khalili, I.J. Thompson, and J.A. Tostevin, Nucl. Phys. **A581**, 331 (1995).
[5] See, for example, I.J. Thompson, J.S. Al-Khalili, J.M. Bang, B.V. Danilin, V.D. Efros, F.M. Nunes, J.S. Vaagen, and M.V. Zhukov, Nucl. Phys. **A588**, 59c (1995); F. Barranco, E. Vigezzi, and R.A. Broglia, Z. Phys. A **356**, 45 (1996); K. Henken, G. Bertsch, and H. Esbensen, Phys. Rev. C **54**, 3043 (1996); L.V. Chulkov, C.A. Bertulani, and A.A. Korsheninnikov, Nucl. Phys. **A587**, 291 (1995); N. Takigawa, M. Ueda, M. Kuratani, and H. Sagawa, Phys. Lett. B **288**, 244 (1992), and references therein.
[6] J.A. Tostevin, J.S. Al-Khalili, M. Zahar, M. Belbot, J.J. Kolata, K. Lamkin, D.J. Morrissey, B.M. Sherrill, M. Lewitowicz, and A.H. Wuosmaa, Phys. Rev. C **56**, R2929 (1997).
[7] J.S. Al-Khalili, J.A. Tostevin, and J.M. Brooke, Phys. Rev. C **55**, R1018 (1997).
[8] H. Feshbach, in *Interaction of High Energy Particles with Nuclei*, edited by T.E.O. Ericson (Academic, New York, 1967); in *In Honour of Philip M. Morse*, edited by H. Feshbach and U. Ingard (MIT, Cambridge, MA., 1969).
[9] S.J. Wallace, Phys. Rev. D **8**, 1846 (1973).
[10] M. Rosen and D.R. Yennie, J. Math. Phys. **5**, 1505 (1964).
[11] R.J. Glauber, Proceedings of the 3rd International Conference on High Energy Physics and Nuclear Structure, Columbia, 1969, edited by S. Devons (unpublished); R.J. Glauber and G. Matthiae, Nucl. Phys. **B21**, 135 (1970).
[12] M. Delores Cortina-Gil, Ph.D. thesis, Université de Caen, France, 1996.
[13] H. Amakawa, S. Yamaji, A. Mori, and K. Yazaki, Phys. Lett. **82B**, 13 (1979).
[14] I.J. Thompson, Computer program ADIA, Daresbury Laboratory report, 1984 (unpublished).
[15] J.A. Christley, J.S. Al-Khalili, J.A. Tostevin, and R.C. Johnson, Nucl. Phys. **A624**, 275 (1997).
[16] F.D. Becchetti and G.W. Greenlees, Phys. Rev. **182**, 1190 (1969).
[17] J.S. Al-Khalili and J.A. Tostevin, Phys. Rev. Lett. **76**, 3903 (1996).
[18] J.A. Tostevin, J.S. Al-Khalili, J.M. Brooke, and J.A. Christley, J. Phys. G **24**, 1589 (1998).