INSTANTANEOUS PHASE TRACKING OF OSCILLATORY SIGNALS USING EMD AND RAO-BLACKWELLISED PARTICLE FILTERING

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ABSTRACT

A new method for instantaneous phase tracking of oscillatory signals in a narrow band frequency range is proposed. Empirical mode decomposition (EMD), as an adaptive and data-driven method for analyzing non-linear and non-stationary time series, is applied to a mixture of signals. Then, one of the resulted intrinsic mode functions (IMFs) is used for estimating the instantaneous phase of the signal in a certain frequency band. Since by applying EMD to the noisy signal the noise is distributed over the IMFs, the Rao-Blackwellised particle filtering (RBPF) is used to track the actual instantaneous phase from the noisy IMF. The formulated RBPF operates based on smoothing the instantaneous frequency traces in Hilbert domain and denoising the signal in time domain. Finally, the method is able to track the instantaneous phases across consecutive time points. The method is applied to both simulated and real data. As an application, it can be used for mental fatigue analysis based on the changes in phase synchronization of different brain rhythms in different brain regions before and during the fatigue state.

Index Terms— Empirical mode decomposition (EMD), Rao-Blackwellised particle filter (RBPF), intrinsic mode function (IMF), instantaneous phase.

1. INTRODUCTION

Decomposition of signals into their constituent oscillatory components has wide applications in various fields. This may be efficiently performed using empirical mode decomposition (EMD) [1] and often restoring the resulted intrinsic mode functions (IMFs) from noise. In some previous research, conventional filtering is suggested for removing the noise from the IMF. This usually results in loss of the phase information. In [2] a number of EMD-based denoising inspired by the standard wavelet thresholding is proposed. In [3] adaptive line enhancement (ALE) is applied to the IMF in order to enhance and de-noise it. Hilbert transform (HT) is then used for estimation of instantaneous phase. In this paper we propose a new method based on RBPF in order to directly estimate the instantaneous phase of the IMF. The method can be used as a general denoising algorithm but the focus in this paper is to estimate the instantaneous phase from the noisy IMF.

Electroencephalography (EEG) is the electrical activity of the brain that is used for studying brain functions with a high time resolution while there is a relatively modest spatial resolution [4]. Synchronous electrical activities in these regions is generally associated with the functional relationship between different brain regions. Recorded EEGs can be used for measuring synchronization of different brain regions. These measures can be obtained by first decomposing EEG into its corresponding oscillatory signals and then computing the linear/non-linear synchronization (coherency and phase synchronization) of different EEG rhythms in various regions of the brain. One problem is that Beta rhythm which is an oscillation usually around 20Hz is buried in noise and it is difficult to extract the actual Beta rhythm. In this paper we aim to estimate and track the instantaneous phase of the Beta rhythm (or any other noisy IMF) which later can be used effectively for phase synchronization analysis. EMD as an adaptive method is applied to the EEG signal in order to extract its different oscillations. Our proposed method in the first step uses the EMD in order to decompose the EEG into a number of oscillatory waveforms called IMF. The method tries to smooth the frequency traces in Hilbert domain and estimate the phase of signal. The remainder of the paper is structured as follows. In section 2 problem formulation using RBPF is described. Then, in section 3 the new phase tracking system based on RBPF is explained. The results of applying the method to the simulated and real data are provided in section 4. Finally, section 5 concludes the paper.

2. PROBLEM FORMULATION USING RBPF

Suppose that EMD is applied to the mixture signal $\mathbf{X}$ to decompose it to its corresponding IMFs. Then the HT is used to compute the analytic signal for the IMFs (having the IMF as the real part and its HT as the imaginary part) as:

$$imf_t(i) + J\mathbf{H}[imf_t(i)] = a_t(i)e^{J\theta_t(i)} \quad i = 1, ..., M \qquad (1)$$

where $J = \sqrt{-1}$ and $a_t(i)$ and $\theta_t(i)$ are the instantaneous amplitude and phase of the i^{th} IMF respectively, $\mathbf{H}[.]$ denotes the HT and M is the number of IMFs. Then, the mixture signal can be reconstructed as:

$$\mathbf{X}_t = \sum_{i=1}^{M} imf_t(i) = Real(\sum_{i=1}^{M} a_t(i)e^{J\theta_t(i)}) \qquad (2)$$

Now, it is possible to formulate the instantaneous amplitude and phase of each IMF as the state variables of the particle filtering (PF) [5] and the mixture signal as the observation. Therefore, the

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state transition and observation equations of the PF can be expressed as:

$$\mathbf{R}_t = [\theta_t(1) \quad \mathbf{a}_t(1) \quad \ldots \quad \theta_t(M) \quad \mathbf{a}_t(M)] \tag{3}$$

$$\mathbf{R}_t = f(\mathbf{R}_{t-1}) + \mathbf{w}_{t-1} \tag{4}$$

$$\mathbf{X}_t = G(\mathbf{R}_t) + \mathbf{v}_t \tag{5}$$

where $\mathbf{X}_t$ is the mixture signal, $\mathbf{v}_t, \mathbf{w}_{t-1}$ are Gaussian white noise (GWN) with known covariance matrices, M is the number of IMFs and

$$G(\mathbf{R}_t) = Real(\sum_{i=1}^{M} \mathbf{a}_t(i) e^{J\theta_t(i)}) \tag{6}$$

From the above equations it can be seen that it is possible to partition the state variables into linear and non-linear parts. Therefore, we formulate the problem using the concept of Rao-Blackwellied particle filtering (RBPF) [6]. The RBPF marginalizes out the linear state variables in order to reduce the size of the state space. Therefore, a reduced number of particles are required for the RBPF to achieve the same performance as for the PF. We rewrite eq. (3) as:

$$\mathbf{R}_t = [\mathbf{R}_t^1 \quad \mathbf{R}_t^2] \tag{7}$$

$$\mathbf{R}_t^1 = [\theta_t(1) \quad \ldots \quad \theta_t(M)] \tag{8}$$

$$\mathbf{R}_t^2 = [\mathbf{a}_t(1) \quad \ldots \quad \mathbf{a}_t(M)] \tag{9}$$

where $\mathbf{R}_t^1$ and $\mathbf{R}_t^2$ are respectively nonlinear and linear state variables. Then, the state transition and observation equations can be written in vector form as:

$$\begin{aligned} \mathbf{R}_t &= f(\mathbf{R}_{t-1}) + \mathbf{w}_{t-1} \\ \mathbf{X}_t &= G(\mathbf{R}_t) + \mathbf{v}_t \\ G(\mathbf{R}_t) &= \mathbf{R}_t^2 G'(\mathbf{R}_t^1) \\ &= [\mathbf{a}_t(1) \quad \ldots \quad \mathbf{a}_t(M)] \begin{bmatrix} Real(e^{J\theta_t(1)}) \\ \ldots \\ Real(e^{J\theta_t(M)}) \end{bmatrix} \end{aligned} \tag{10}$$

$\mathbf{R}_t^2$ is estimated using Kalman filtering (KF) [5] and $\mathbf{R}_t^1$ is estimated by PF. Considering the above equations, if the state transition function f were available, it would be possible to track the instantaneous amplitude and phase of each IMF and de-noise the mixture signal using RBPF. However, the state transition function is not known and especially for phase it can not be modelled simply using a Markov process. Therefore, in this paper first we exploit the RBPF in order to formulate the problem and then several equations and constraints are employed to the RBPF algorithm in order to track the instantaneous phase effectively. In the next section the new method for instantaneous phase estimation based on RBPF is described.

3. NEW PHASE TRACKING SYSTEM BASED ON RBPF

In this paper we focus on the phase tracking from one IMF. In future, the method can be extended to track the instantaneous phase and amplitude of multiple IMFs at each time point. Suppose that we applied EMD to one channel of EEG signal as $\mathbf{X}_t$. Usually the first and second IMFs are noisy since they contain the highest frequency available in the signal. If we compute the analytic signal using eq. (1), the instantaneous frequency (IF) of the IMF can be estimated using derivative of the phase as:

$$w_t(i) = \frac{d\theta_t(i)}{dt} \quad for \quad i = 1, ..., M \tag{11}$$

In the next section it will be shown that for the noisy IMF, the frequency trace across time points is not smooth and the effect of wide band noise can be clearly seen when there is a jump in the estimated IF of the time samples. We select the i^{th} IMF which is noisy. Therefore, the state variables, state transition, and observation function can be considered as:

$$\mathbf{R}_t(i) = [\theta_t(i) \quad \mathbf{a}_t(i)],\ \mathbf{R}_t^1(i) = \theta_t(i),\ \mathbf{R}_t^2(i) = \mathbf{a}_t(i) \tag{12}$$

$$\begin{aligned} \mathbf{R}_t(i) &= \widetilde{f}(\mathbf{R}_{t-1}(i)) + \mathbf{w}_{t-1} \\ imf_t(i) &= \widetilde{G}(\mathbf{R}_t(i)) + \mathbf{v}_t \\ \widetilde{G}(\mathbf{R}_t(i)) &= \mathbf{a}_t(i) Real(e^{J\theta_t(i)}) \end{aligned} \tag{13}$$

Our proposed method tries to smooth the IF traces in Hilbert domain and then estimates the instantaneous phase using the information provided by the IF. Each IMF belongs to a specified frequency band. So, it is possible to determine the minimum and maximum frequencies for the band in which IMF belongs to. By generating some simulated signals, applying the EMD, and evaluating the instantaneous phase of the IMF using HT, it is evident that the phase sign suddenly changes. However, if we consider the absolute value of the phase, the phase change is not significant. The instantaneous amplitude is smooth across all time samples. As shown in eq. (12), we formulated the phase as the nonlinear state variable and the amplitude as the linear state variable of the RBPF. Both of them are considered as the Markov process. We also save one variable as the estimated IF. This variable does not have direct role in estimating the phase, however it is useful in updating the weight of some particles and deciding on the phase transition. When we generate new particles corresponding to the instantaneous phase from the previous time point, we need to generate two phases. One phase is obtained from the phase in the previous time sample plus the state noise. Another phase is obtained from the negative of the phase in the previous time sample plus noise. Then, we calculate IFs for both generated phases. We select the generated phase in which the estimated IF smooths the frequency traces across time samples (using the stored IF estimate in the previous time point). When we tried to test and develop the method using simulated signal, there was a situation that the phase in one time sample was close to zero and the phase in the next time sample was also close to zero. In this case the calculated frequency went out of the frequency range and became negative in the later time samples. Although in consecutive time points, the IF traces return to the actual track before becoming negative or going beyond the frequency range. In this situation the method is not able to distinguish the correct generated phase based on the frequency traces. We used some if-then-rules and detected the situation that the estimated phase is close to zero. Therefore, in this case the decision on selection of the phase is based on considering the observation and the estimated amplitude by KF. Before deciding on the generated phase (from positive/negative of the phase in previous time sample) we estimate the instantaneous amplitude for both generated phases using KF. We update the weight of each particle using two scales. One scale relates to the frequency transition, the other scale relates to the weight given by using the observation and the amplitude estimated by KF. In the case where the generated frequency is out of band but still is a valid frequency and not related to noise (i.e. due to the phase transition around zero), we estimate another frequency from previous time point in order not to lose the frequency track. Therefore, we cannot have an equivalent scale for frequency transition like the case that frequency is inside the specific band. However, we set the first scale equal to 1 to have better phase transition. Then, we select the generated phase in which the estimated weight is higher. We do not store the generated frequency which is negative or out of the frequency band. Instead, we generate a new frequency which is equal to the stored frequency in the previous time sample plus a Gaussian random generated number. In this case we do not lose the frequency track inside the band in later time points and have better phase transition near zero.

Algorithm 1 Pseudo-code for phase tracking of one IMF

select i^{th} IMF as $imf(i)$, $n = 1, ..., N$, N *is the number of particles.*
set $t = 0$ *and generate random numbers* $\theta_0^{(n)}, freq_0^{(n)}$
according to random uniform distribution considering the HT of the IMF.
Initialize $\sigma, \sigma_1, min_f, max_f, Ts$ (*Sampling period*), τ
for $\{t = 1$ *to* $t_{max}\}$ $\{t_{max}$ *is the number of all time samples*$\}$
- ***generate*** *random numbers* $\mathbf{w}_t^{\rho(n)1}, \mathbf{w}_t^{\rho(n)2}$ *and* $\mathbf{z}_t^{\rho(n)}$
- ***set*** $\hat{\theta}_t^{(n)} = \theta_{t-1}^{(n)} + \mathbf{w}_t^{\rho(n)1}$
- ***set*** $\breve{\theta}_t^{(n)} = -\theta_{t-1}^{(n)} + \mathbf{w}_t^{\rho(n)2}$
- ***Calculate*** $freq1 = diff(unwrap\,[\theta_{t-1}^{(n)} \quad \hat{\theta}])/Ts/(2 \times \pi)$
- ***Calculate*** $freq2 = diff(unwrap\,[\theta_{t-1}^{(n)} \quad \breve{\theta}])/Ts/(2 \times \pi)$
- ***Estimate frequency*** $freq = freq_{t-1}^{(n)} + \mathbf{z}_t^{\rho(n)}$
- ***Set*** $\hat{\theta}_t^{(n)}$ *as the phase, estimate amplitude* $\mathbf{a}_1$ *by Kalman filtering*
- ***Set*** $\mathbf{F}_1 = \mathbf{a}_1 \times Real(e^{J\hat{\theta}_t^{(n)}})$,
  $W_{s1} = exp\,(-(imf_t(i) - \mathbf{F}_1) \times (imf_t(i) - \mathbf{F}_1)/(2 \times \sigma_1^2))$
- ***Set*** $\breve{\theta}_t^{(n)}$ *as the phase, estimate amplitude* $\mathbf{a}_2$ *by Kalman filtering*
- ***Set*** $\mathbf{F}_2 = \mathbf{a}_2 \times Real(e^{J\breve{\theta}_t^{(n)}})$,
  $W_{s2} = exp\,(-(imf_t(i) - \mathbf{F}_2) \times (imf_t(i) - \mathbf{F}_2)/(2 \times \sigma_1^2))$
- ***if*** $(|\hat{\theta}_t^{(n)}| < \tau$ *and* $|\theta_{t-1}^{(n)}| < \tau)$ *or* $(|\breve{\theta}_t^{(n)}| < \tau$ *and* $|\theta_{t-1}^{(n)}| < \tau)$
  $weight1 = W_{s1}, weight2 = W_{s2}$
  $freq1 = freq\,,\ freq2 = freq, W_{s1} = 1, W_{s2} = 1$
- ***else***
  -$weight1 = exp\,(-(freq - freq1) \times (freq - freq1)/(2 \times \sigma^2))$
  -$weight2 = exp\,(-(freq - freq2) \times (freq - freq2)/(2 \times \sigma^2))$
- ***end if***
- ***if*** $weight1 > weight2$
  $\theta_t^{(n)} = \hat{\theta}_t^{(n)}, freq_t^{(n)} = freq1, w_t^{(n)} = w_{t-1}^{(n)} \times W_{s1} \times weight1$
- ***else***
  $\theta_t^{(n)} = \breve{\theta}_t^{(n)}, freq_t^{(n)} = freq2, w_t^{(n)} = w_{t-1}^{(n)} \times W_{s2} \times weight2$
- ***end if***
- ***if*** $(\theta_t^{(n)} > \pi)$ *or* $(\theta_t^{(n)} < -\pi)$ $\quad w_t^{(n)} = 0$
- ***if*** $(freq_t^{(n)} > max_f)$ *or* $(freq_t^{(n)} < min_f)$ $\quad w_t^{(n)} = 0$
- ***Normalize particle weights*** $w_t^{(n)} = w_t^{(n)}/\sum_{n=1}^{N}(w_t^{(n)})$
- ***Resample***

- ***end for***

In addition, our method involves some constraints. When the selected phase is bigger than π or less than $-\pi$ the weight of the particle will be set to zero in order not to have a contribution to the estimation of the posterior density. In addition, when the estimated frequency is out of the frequency band range, we set the weight of that particle to zero. The pseudo-code of the proposed method for instantaneous phase tracking of one IMF is shown in Algorithm 1. τ is the threshold to determine when the phase transition is near zero. In our simulation this was set to 0.15. We consider a window of length $T + 1$ and apply the EMD. After one noisy IMF is selected, the HT is applied to that. We use the estimated IF, instantaneous phase, and instantaneous amplitude of the noisy IMF at the first time point and initialize the particles. This initialization moves the particles into the right part of posterior density and is helpful to speed up the convergence of the RBPF. The method is applied to the rest of T time samples.

4. RESULTS

We applied the method to both simulated and real data. What follows is the description of the simulated data and real data and the results of applying the method to both datasets.

4.1. Simulated Data

Four frequency and amplitude modulated sine waves which belong to four different frequency bands are generated and Gaussian white noise (GWN) is added to the sum of sine waves. The available signal to noise power is measured by SNR in dB unit which is defined as:

$$\mathbf{SNR} = 10log(\frac{P_{signal}}{P_{noise}}) \tag{14}$$

We generated two simulated signals in which the SNR for the first and second signal obtained as 3.0445dB and 7.2167dB respectively. Next, the EMD method is applied to decompose the generated signals into a number of IMFs. In both simulated signals, the second IMF belonged to the frequency range around 20Hz. Since, the resulted IMF was noisy, the proposed method in this paper is applied to the IMF in order to track the actual instantaneous phase of the signal in the frequency range around 20Hz. We computed the instantaneous phase, amplitude, and frequency of the actual generated sine wave using HT. Then, we compared the results of tracking the instantaneous phase, amplitude, and frequency using our method with the results obtained by the HT applied to the noisy IMF. The results are provided in Fig. 1 and Fig. 2. The mean square error (MSE) of the phase is calculated using the following equation and is shown in Table 1 for both SNR levels.

$$\mathbf{MSE} = 1/T\sum_{t=1}^{T}(\theta_{estimate}(t) - \theta_{actual}(t))^2 \tag{15}$$

where T is the number of time samples. It can be seen from Fig. 2 that around time sample number 10 (the corresponding phase in Fig. 1 is around 0) the actual frequency decreases, becomes negative then it returns to the previous track. However, our method can detect these changes in frequency which is not related to the noise and smooths the frequency such that it does not much affect the phase estimation. Therefore, in this case although we cannot estimate the actual frequency we keep the frequency in a track that won't be lost inside the specified frequency band and it can remove the wide-band noise in other time points as well. In addition, since the estimated frequency does not have direct impact on phase estimation, still the phase estimation around zero is very promising. It can be seen from the figures that when the SNR is lower, the error in estimation is higher. However the overall performance of the method depends also on the operation of EMD.

Table 1. MSE of the phase in two SNR levels

	SNR=3.0445dB	SNR=7.2167dB
Proposed method	3.7634	2.6952
HT of the noisy IMF	5.7522	3.5871

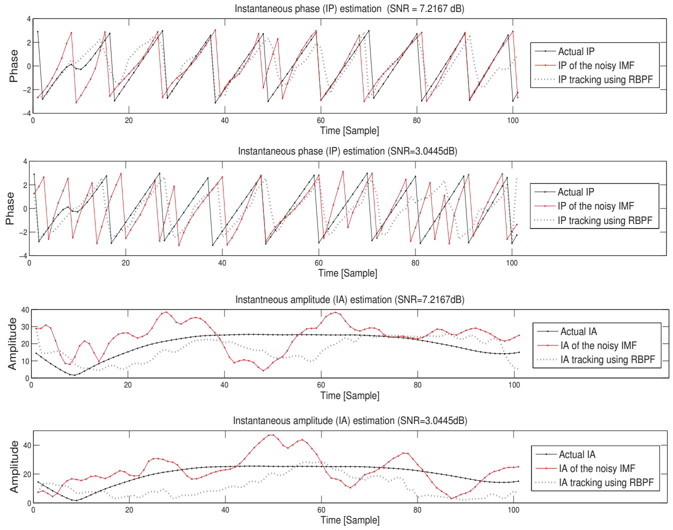

Fig. 1. IP (top two rows) and IA (bottom two rows) estimation using the proposed and HT methods in two SNR levels

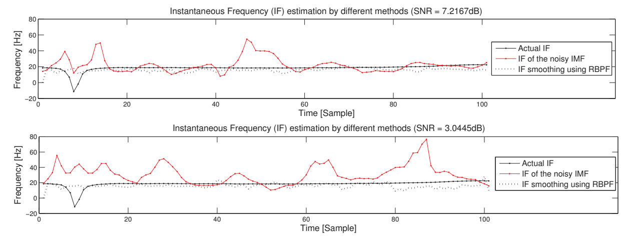

Fig. 2. IF estimation using the proposed and HT methods in two SNR levels

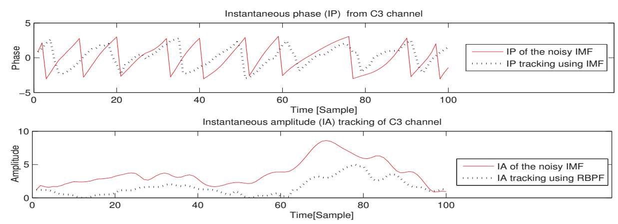

Fig. 3. IP (top) and IA (bottom) estimation of C3 channel using the proposed method

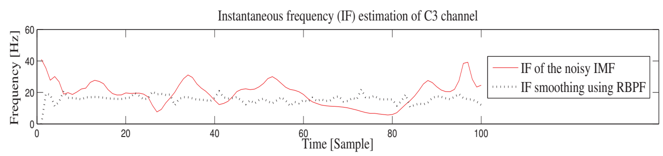

Fig. 4. IF estimation of C3 channel using the proposed method

4.2. Real Data

The real EEG data used here belongs to one of the subjects that participated in a continuous visual experiment. EEG was recorded from 22 scalp sites, using Sn electrodes attached to an electrode cap (ElectroCap International). Standard 10-20 sites were F7, F3, Fz, F4, F8, T7, C3, Cz, C4, T8, P7, P3, Pz, P4, P8, O1, Oz, and O2. Additional intermediate sites were FC5, FC1, FC2, and FC6. The electrodes were referenced to electronically linked earlobes. Electro-oculograms were recorded bipolarly with Sn electrodes from the outer canthi of both eyes and from above and below the left eye. An Ag/AgCl ground electrode was placed on the sternum. Electrode impedance was reduced to less than $5k\Omega$. The signals were amplified with a bandpass set at 30 Hz and a time constant of 10 s and digitized at a rate of 100 Hz. Using interpolation the sampling frequency was increased to 200Hz. We selected the C3 channel and applied the EMD algorithm. The first IMF belonging to Beta frequency range was selected and the method applied. The results are provided in Fig. 3 and Fig. 4.

5. CONCLUSION

A new phase tracking method is proposed in this paper. The method uses the IMF given by the EMD algorithm in order to remove the wide-band noise by applying a constrained version of RBPF. As a result, the frequency traces of the IMF becomes more smooth and results in a better estimation for the instantaneous phase. The proposed method here is very demanding in order to estimate the instantaneous phase of the Beta rhythm of the EEG signal, which is usually noisy and contains wide-band noise. The changes in phase synchronization of Beta rhythm among different brain regions before and during fatigue state, can play an important role in order to analyze mental fatigue. Another application of the method will be in speech analysis. When the EMD is applied to the speech signal, the noise at each time point will be distributed to the IMFs. Therefore, for enhancing several IMFs simultaneously and de-noising the speech signal at each time point, the method should be extended and effectively applied. Further, the method can be developed more in order to solve the mode mixing problem in the EMD algorithm.

6. REFERENCES

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