EPJ manuscript No.
(will be inserted by the editor)

Continuum effects: Structure and reactions of ^{6}He

M. Rodríguez-Gallardo[1,2,3 a], J. M. Arias[2], J. Gómez-Camacho[2], R. C. Johnson[3], A. M. Moro[2], I. J. Thompson[3], and J. A. Tostevin[3]

[1] Centro de Física Nuclear, Universidade de Lisboa, Av. Prof. Gama Pinto 2, 1649-003 Lisboa, Portugal
[2] Departamento de Física Atómica, Molecular y Nuclear, Universidad de Sevilla, Apartado 1065, 41080 Sevilla, Spain
[3] Department of Physics, University of Surrey, Guildford GU2 7XH, United Kingdom

Received: date / Revised version: date

Abstract. A description of three-body weakly bound systems using the Transformed Harmonic Oscillator (THO) method is addressed. First, relevant structure observables are presented for the Borromean nucleus ^{6}He. Then, the THO method is applied to the study of ^{6}He scattering within the Continuum-Discretized Coupled Channels (CDCC) framework.

PACS. 21.45.+v – 21.10.-k – 27.20.+n – 24.10.-i – 24.10.Eq – 25.60.-t – 25.60.Bx

1 Introduction

A particularly interesting example of weakly bound systems is that of Borromean nuclei, that is, three-body composite systems with no binary bound subsystems. For an appropriate description of the reactions induced by a Borromean nucleus, like ^{6}He, the introduction of the continuum part of the spectrum is necessary. For this purpose it is required to discretize the three-body continuum. This work has been developed by the group of M. Kamimura [1, 2] using a discretization method based on Gaussian functions. Here, an alternative discretization method, based on Hyperspherical Harmonics and orthogonal polynomials, is presented.

2 Three-body continuum discretization method: Application to ^{6}He

This formalism is a generalization of the method presented in Ref. [4] for a two-body system. For a three-body case the hyperspherical coordinates $\{\rho, \alpha, \widehat{x}, \widehat{y}\}$ will be used. They are obtained from the Jacobi coordinates $\{\boldsymbol{x}, \boldsymbol{y}\}$ (see, for instance, Ref. [8]). The Jacobi coordinates are presented in Fig. 1.

First, the wave functions of the system are expanded in Hyperspherical Harmonics (HH) as

$$\Psi_{\beta j\mu}(\rho, \Omega) = R_{\beta j}(\rho) \sum_{m\sigma} \langle lmS_x\sigma | j\mu\rangle \Upsilon^{l_x l_y}_{Klm}(\Omega)\chi^{\sigma}_{S_x} \qquad (1)$$

where $R_{\beta j}(\rho)$ is the hyperradial wave function, $\chi^{\sigma}_{S_x}$ the spin wave function of the two particles related by the coordinate $\boldsymbol{x}$, and $\Upsilon^{l_x l_y}_{Klm}(\Omega)$ are the hyperspherical harmonics that depend on the angular variables $\Omega \equiv \{\alpha, \widehat{x}, \widehat{y}\}$. The set of quantum numbers $\beta \equiv \{K, l_x, l_y, l, S_x\}$ defines each channel. Here, K is the hypermomentum, l_x and l_y are the orbital angular momenta associated with the Jacobi coordinates $\boldsymbol{x}$ and $\boldsymbol{y}$, $\boldsymbol{l} = \boldsymbol{l}_x + \boldsymbol{l}_y$ is the total orbital angular momentum, S_x is the spin of the particles related by the coordinate $\boldsymbol{x}$ and $\boldsymbol{j} = \boldsymbol{l} + \boldsymbol{S}_x$ is the total angular momentum.

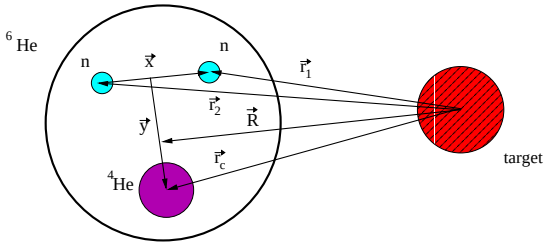

Fig. 1. Jacobi coordinates; ^{6}He scattered by a target.

Then the Transformed Harmonic Oscillator method (THO) [3] is used to obtain $R_{\beta j}(\rho)$. The basic idea of the THO method [4–7] is to convert the bound ground-state wave function of the system into the ground-state wave function of the Harmonic Oscillator (HO), defining a Local Scale Transformation (LST). Considering that the ground-state wave function can be written as a linear combination of the basis functions given in Eq. (1), the equation that defines the LST for every channel β is

$$\int_0^{\rho} d\rho' \rho'^5 |R_{B\beta}(\rho')|^2 = \int_0^{s} ds' s'^5 |R^{HO}_{0K}(s')|^2, \qquad (2)$$

where $R^{HO}_{0K}(s)$ is the hyperradial wave function of the HO. Finally, the THO basis is constructed for each channel

[a] *Present address:* Centro de Física Nuclear, Universidade de Lisboa

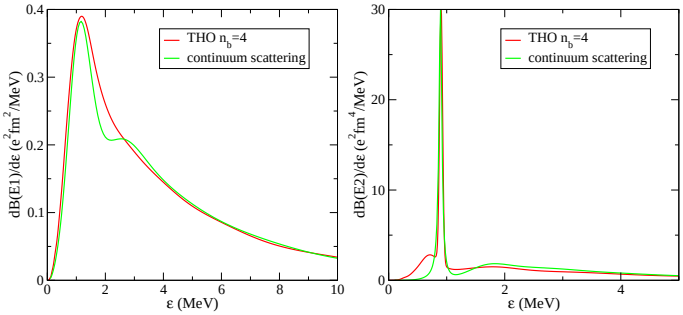

Fig. 2. (left) $B(E1)$ distribution for $n_b = 4$. (right) $B(E2)$ distribution for $n_b = 4$.

applying the LST, $s_\beta(\rho)$, to the HO basis

$$R^{THO}_{i\beta}(\rho) = R_{B\beta}(\rho) L_i^{K+2}(s_\beta(\rho)^2), \qquad (3)$$

where $L_i^\lambda(t)$ are Laguerre polynomials. In this way, a complete and orthonormal basis is obtained. Here the quantum number i denotes the hyperradial excitations. Note that as i increases, the functions $R^{THO}_{i\beta}(\rho)$ become more oscillatory and explore larger distances.

Next we apply the method to the Borromean nucleus ^{6}He. In that case the Hamiltonian (H) is detailed in [8] and includes two-body interactions plus an effective three-body potential and a treatment of Pauli principle using a pseudopotential. H is diagonalized in a finite THO basis with N states (including n_b hyperradial excitations in every channel so $N = (n_b + 1) \times nchan$, where $nchan$ is the number of channels for each state).

For $j = 0^+$ we get almost exactly the ground-state of the system. For $j = 2^+$ and $n_b < 5$, we obtain a very stable state around the experimental energy of the 2^+ resonance for ^{6}He.

In Fig. 2 we present the $B(E1)$ and $B(E2)$ distributions for $n_b = 4$ as a function of the excitation energy on the continuum. For comparison, the distribution calculated with the continuum scattering wave functions, taken from Ref. [9], is also shown. For both cases, $B(E1)$ and $B(E2)$, the two distributions are in good agreement with the same total strength. For the $B(E2)$ distribution the most relevant feature is the presence of a narrow low-lying resonance.

3 Reactions induced by ^{6}He

In Fig. 1 we show a scheme of the scattering of ^{6}He by a target, where $\boldsymbol{R}$ is the position of the center of mass of ^{6}He from the target. In order to perform CDCC [10] calculations, we take as the projectile basis, the eigenstates obtained from the diagonalization of H in a THO basis with a given numbers of states (N). Then it is necessary to calculate the coupling potentials $V^J_{Lnj,L'n'j'}(R) = \langle LnjJM|V_{1t}(\boldsymbol{r}_1)+V_{2t}(\boldsymbol{r}_2)+V_{ct}(\boldsymbol{r}_c)|L'n'j'JM\rangle$. These potentials are read externally in the program FRESCO [11], that solves the system of coupled equations as in the conventional CDCC method. As a preliminary result we show

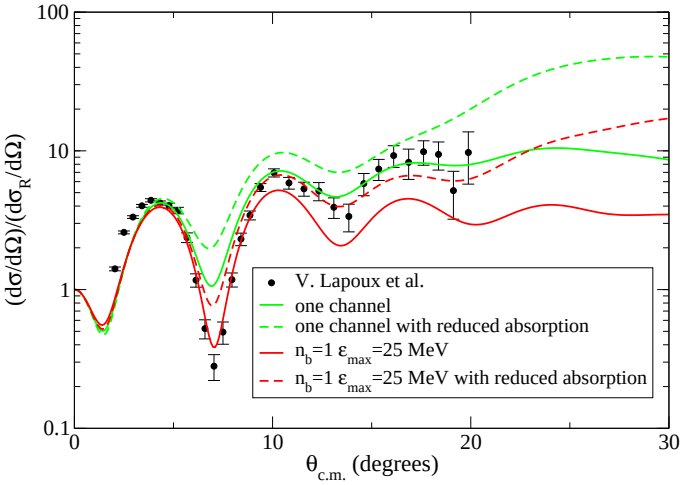

Fig. 3. Differential elastic cross section relative to Rutherford cross section for the reaction ^{6}He+^{12}C at 229.8 MeV.

in Fig. 3 the differential elastic cross section relative to Rutherford cross section for the reaction ^{6}He+^{12}C at 229.8 MeV. We can see how the calculation with a very small THO basis reproduces quite well the experimental data of V. Lapoux et al. [12]. In order to reproduce satisfactory the normalization of the data for angles beyond 10° we had to reduce the imaginary part of the coupling potentials by around 40%. A similar reduction was also required in the work [1].

The purpose, in the next future, is the application of this procedure to analyze scattering cross sections of halo nuclei.

Acknowledgments

This work was supported in part by the DGICYT under projects FIS2005-01105 and FPA 2005-04460. M.R.G. acknowledges a FCT grant POCTI/ISFL/2/275 and FCT under the grant POCTI/FIS/43421/2001.

References

1. T. Matsumoto et al., Phys. Rev. C **70** (2004) 061601 .
2. T. Matsumoto et al., Phys. Rev. C **73** (2006) 051602(R).
3. I. Zh. Petkov and M. V. Stoitsov, Compt. Rend. Bulg. Acad. Sci. **34** (1981) 1651.
4. F. Pérez-Bernal et al., Phys. Rev. A **63** (2001) 052111.
5. I. Martel et al., Phys. Rev. A **65** (2002) 052708.
6. M. Rodríguez-Gallardo et al., Phys. Rev. C **69** (2004) 034308.
7. M. Rodríguez-Gallardo et al., Phys. Rev. C **72** (2005) 024007.
8. I. J. Thompson et al., Comput. Phys. Commun.**161** (2004) 87.
9. I. J. Thompson et al., Phys. Rev. C **61** (2000) 24318.
10. N. Austern et al., Phys. Rep. **154** (1987) 125.
11. I. J. Thompson, Comput. Phys. Rep. **7** (1988) 167.
12. V. Lapoux et al., Phys. Rev. C **66** (2002) 034608.