Energy Efficiency and Optimal Power Allocation in Virtual-MIMO Systems

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***Abstract*—This paper investigates energy efficiency (EE) performance of a virtual multiple-input multiple-output (MIMO) wireless system using the receiver-side cooperation with the compress-and-forward protocol. We derive a linear approximation of EE as a function of spectral efficiency (SE) in the low SE operation regime. In addition, we obtain a closed-form lower bound for EE which is valid for both low and high SE regions. This lower bound can be used for optimizing the power allocation between the transmitter and the relay in order to minimize the overall energy per bit consumption in the system. Both analytical and simulation results demonstrate that the virtual MIMO system using the receiver-side cooperation outperforms the multiple-input single-output (MISO) case in terms of energy efficiency. Finally we show that, with the optimal power allocation, the virtual-MIMO system achieves an EE performance close to that of an ideal MIMO system.**

I. Introduction

Virtual multiple-input multiple-output (MIMO) systems, where the transmitter has multiple antennas and each of the receivers has a single antenna, have recently emerged as one effective technique that can improve spectral efficiency of wireless communications [1] [2]. The idea is that when channel state information (CSI) is available at the receivers only, multiple closely spaced mobile stations, each equipped with a single antenna, can cooperate to form a virtual antenna array and reap some benefits of MIMO systems [3]. Virtual-MIMO systems are practically appealing since base stations can be equipped with multiple antennas, but the mobile stations may not due to the physical constraints. Most of the previous work on virtual-MIMO systems focused on spectral efficiency (SE) and bit error ratio performance, such as [2]–[4]. However, compared to the multiple-input and single-output (MISO) case, the cooperation among the receivers accordingly consume more energy. To the best of our knowledge, energy efficiency (EE) of the virtual-MIMO systems has not been properly addressed so far. This problem is becoming increasingly important due to surging concerns about reducing carbon footprint of communication systems [5].

Some aspects of energy efficiency of cooperative communications have been studied in recent years. Most previous work concentrated on the classical three-terminal relay channel, such as [6]–[8]. In [6], the minimum achievable energy per bit, required for relay communications, was derived for ergodic fading channels. This study was then extended in [7], where the EE performance of two relay protocols including amplify-and-forward (AF) and decode-and-forward (DF) was compared. In [8], the synchronization between the transmitter and the relay were considered for practical scenarios. An initial study of virtual-MIMO based cooperative communications for distributed wireless sensor networks was given in [9] and [10]. The dependence of EE on transmission distances was analyzed in [9]. And, [10] investigated other influences such as constellation size and training overhead. Both [9] and [10] only take into account local energy cost for cooperation, without considering specific cooperation protocols. In addition, the potential power allocation problem for the transmitter and the relay is neglected.

Different from the existing work, we investigate energy efficiency of a virtual-MIMO system using the receiver-side cooperation with the compress-and-forward (CF) protocol. Since the relays are closer to the destination in our scenario, compared to AF and DF, the CF protocol provides superior performance [11] and, is therefore considered in this paper. The main contributions of this paper are summarized as follows. First, we analyze the EE performance of the virtual-MIMO system and derive a linear approximation of EE as a function of SE in the low SE regime. Next, a closed-form lower bound on EE, which is valid for both low and high SE regions, is obtained. Based on this lower bound, we formulate an energy per bit minimization problem. Our results indicate the optimal power allocation between the transmitter and the relay can minimize the total energy consumption.

The rest of this paper is organized as follows. Section II specifies the system model. The EE analysis of the virtual-MIMO system, including linear approximation of EE, lower bound on EE, and optimal power allocation are given in Section III. Some simulation results and discussions are provided in Section IV. And, Section V concludes the paper.

II. System Model

As shown in Fig. 1, we consider a remote transmitter with N_t antennas transmitting to one destination with a single antenna. There are $N_r - 1$ single-antenna relays in the proximity of the destination. We refer to the destination and relays as the receiver group, which together with the transmitter form a virtual-MIMO system [3]. For the sake of demonstrating performance of EE with more tractable mathematical expressions, we consider $N_t = N_r = 2$ in this paper. In our system model, we denote the transmitted signals from the two antennas by x_1 and x_2. The data channels from the transmitter to the receivers are represented by $h_i (i \in [1, ..., 4])$, and the received signals at the relay and the destination are denoted by y_r and y_d, respectively. The average transmit power from the two antennas are denoted by P_{s1} and P_{s2}, where $P_{s1} + P_{s2} = P_s$ is the total transmit power. There is a short-range cooperation channel between the relay and the destination which is orthogonal to the data channels. We consider the data and cooperation channels have equal unit bandwidth, i.e. $W = W_r = 1$ Hz. As the separate band used for short-range cooperation can be spatially reused across all cooperating nodes in a network, the bandwidth cost for a particular cooperating pair is neglected here [2]. We assume the relay can operate in full-duplex mode. The relay transmit power is denoted by P_r. In addition, without loss of generality, we model the data channels as Rayleigh fading with unit power gain, i.e. $\mathbb{E}[|h_i|^2] = 1$. The cooperation channel is modeled as additive white Gaussian noise (AWGN) channel with power gain G. As the receivers are close together, the case of interest is when G is high. We assume that perfect CSI is available at the receivers only.

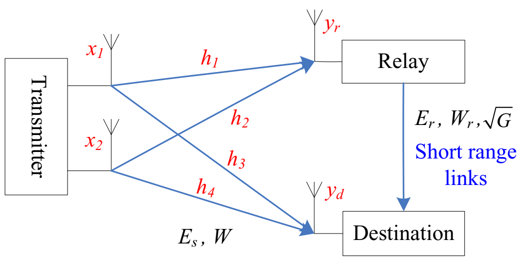

Fig. 1. System model ($N_t = N_r = 2$.)

Suppose that the relay sends its observation to the destination via CF cooperation, where a standard source coding technique [4] is implemented by the relay. That is, the relay is equipped with a vector quantizer. The key tasks of the relay thus include constructing a codebook, and forwarding a compressed version of the received signals to the destination. Alternatively, the relay could use the Wyner-Ziv (WZ) coding technique [2]; however, we do not consider it because that due to multiple transmit antennas in our system model, y_r and y_d are not highly correlated [12]. The WZ coding technique compresses y_r by treating y_d as the side information [11]; therefore, it does not result in enough improvement of the system capacity [13] to justify extra complexity and power used for implementation.

Let $\boldsymbol{x} = [x_1, x_2]^{\mathrm{T}}$ denote the transmit vector. In matrix form, the received vector $[y_r, y_d]^{\mathrm{T}}$ is given by:

$$\begin{bmatrix} y_r \\ y_d \end{bmatrix} = \boldsymbol{Hx} + \boldsymbol{n}; \quad \boldsymbol{H} = \begin{bmatrix} h_1 & h_2 \\ h_3 & h_4 \end{bmatrix}, \tag{1}$$

where $\boldsymbol{H}$ denotes the channel matrix. We define the noise vector $\boldsymbol{n} = [n_1, n_2]^{\mathrm{T}}$, where $n_1, n_2 \sim \mathcal{CN}(0, N_0)$ are i.i.d. zero-mean complex Gaussian noise with $N_0 = 1$ Watts/Hz. The noise on the cooperation channel is also modeled as $\mathcal{CN}(0, 1)$. With the CF protocol, the system is equivalent to a system where destination has two antennas that receive the signals $[y_r + n_c, y_d]^{\mathrm{T}}$, where $n_c \sim \mathcal{CN}(0, \sigma_c^2)$ is the compression noise [14]. If we employ a standard source coding technique at the relay, the variance of the compression noise is given by [4], [15]

$$\begin{aligned} \sigma_c^2 &= \frac{\mathbb{E}[|y_r|^2]}{2^{R_c/W} - 1} = \frac{\mathbb{E}[|y_r|^2]}{[1 + GP_r/(N_0 W_r)]^{W_r/W} - 1} \\ &= \frac{|h_1|^2 P_{s1} + |h_2|^2 P_{s2} + 1}{GP_r}, \end{aligned} \tag{2}$$

where R_c is the coding rate at the relay which is (smaller than but arbitrarily close to) [15]

$$R_c = W_r \log_2(1 + GP_r/(N_0 W_r)). \tag{3}$$

The destination scales $y_r + n_c$ using the degradation factor η, such that $\sqrt{\eta}(y_r + n_c)$ and y_d have the same power of additive Gaussian noise:

$$\tilde{\boldsymbol{y}} = [\sqrt{\eta}(y_r + n_c)\ ,\ y_d]^{\mathrm{T}} = \widetilde{\boldsymbol{H}}\boldsymbol{x} + [\tilde{n}_1\ ,\ n_2]^{\mathrm{T}}, \tag{4}$$

where

$$\widetilde{\boldsymbol{H}} \triangleq \begin{bmatrix} \sqrt{\eta} h_1 & \sqrt{\eta} h_2 \\ h_3 & h_4 \end{bmatrix}; \ \eta \triangleq \frac{1}{1 + \sigma_c^2}; \tag{5}$$

and $\tilde{n}_1 \sim$ i.i.d. $\mathcal{CN}(0, 1)$. The sum capacity of the system using the CF protocol is then given by [4], [15]

$$C_{\mathrm{CF}} = \mathbb{E}\left\{\log_2 \det\left[\boldsymbol{I} + \widetilde{\boldsymbol{H}}\left(\frac{P_s}{2}\boldsymbol{I}\right)\widetilde{\boldsymbol{H}}^{\dagger}\right]\right\} \text{ bits/s/Hz}, \tag{6}$$

where C_{CF} is maximized at $P_{s1} = P_{s2} = P_s/2$, as C_{CF} is symmetric and concave in P_{s1} and P_{s2}.

III. Energy Efficiency Analysis

We use the well known definition of the system achievable energy efficiency as the transmit energy consumption per information bit [8], i.e. $E_b = P/C_{\mathrm{CF}}$, where P is the total consumed power in the system: $P = P_s + P_r$. To account for the power allocation at the transmitter and the relay, we define $\gamma = P_s/P$. Then, we have $P_r = (1 - \gamma)P$. Using the notations P and γ, equation (6) can be reorganized as

$$C_{\mathrm{CF}} = \mathbb{E}\left\{\log_2 \det\left[\boldsymbol{I} + \boldsymbol{H}\begin{pmatrix} P\gamma\eta/2 & 0 \\ 0 & P\gamma/2 \end{pmatrix}\boldsymbol{H}^{\dagger}\right]\right\} \text{bits/s/Hz}, \tag{7}$$

where

$$\eta = \frac{G(1 - \gamma)P}{G(1 - \gamma)P + \frac{\gamma P}{2}(|h_1|^2 + |h_2|^2) + 1}. \tag{8}$$

To investigate the EE performance of the virtual-MIMO system, we aim to obtain the minimum E_b for a certain level of SE. However, the explicit solution for E_b is not feasible from (7). Thus, a closed-form approximation of E_b as a function of SE, which could give us significant insight into EE of the system, is a non-trivial problem and will be discussed in Section III-A and Section III-B. An EE optimization problem will be formulated and solved in Section III-C.

A. Linear Approximation of EE

We firstly use the analysis developed in [16] to find a linear approximation for the relation between EE and SE of the virtual-MIMO system in the low SE regime. We use notation suggested in [16], where C_{CF} denotes SE as a function of E_b. That is, for specific γ and G, $\mathsf{C}_{\text{CF}}(E_b) = C_{\text{CF}}(P)$. The choice of C_{CF} and C_{CF} helps avoid the abuse of notations that correspond to the capacity functions of E_b and P [8]. For low values of SE, EE can be approximated by [16]:

$$\frac{E_b}{N_0}\Big|_{\text{dB}} \approx \frac{E_b}{N_{0\,\min}}\Big|_{\text{dB}} + \mathsf{C}_{\text{CF}}\frac{10\log_{10}2}{S_0}, \tag{9}$$

where,

$$\frac{E_b}{N_{0\,\min}} = \frac{\ln(2)}{(C_{\text{CF}})'_P\Big|_{P=0}};\;\; S_0 = \frac{2\left[\,(C_{\text{CF}})'_P\Big|_{P=0}\right]^2}{-\,(C_{\text{CF}})''_P\Big|_{P=0}}. \tag{10}$$

Here $(C_{\text{CF}})'_P$ and $(C_{\text{CF}})''_P$ denote the first-order and second-order derivatives of the function C_{CF} (which is computed in nats/s/Hz) with respect to P.

Proposition 1: Consider a virtual-MIMO system ($N_t = N_r = 2$) with CF cooperation and Rayleigh fading channels from the transmitter to the receivers. When CSI is only available at the receiver, and in the low SE regime, we obtain

$$\frac{E_b}{N_0}\Big|_{\text{dB}} \approx \frac{\ln(2)}{\gamma}\Big|_{\text{dB}} + \mathsf{C}_{\text{CF}}\frac{\gamma - G(1-\gamma)}{\gamma/(10\log_{10}2)}. \tag{11}$$

Proof: According to Sylvester's determinant theorem [17], from the spectrum efficiency given by (7), we have

$$C_{\text{CF}} = \mathbb{E}\left\{\log_e \det\left[\boldsymbol{I} + \begin{pmatrix} P\gamma\eta/2 & 0 \\ 0 & P\gamma/2 \end{pmatrix}\boldsymbol{H}\boldsymbol{H}^\dagger\right]\right\} \text{nats/s/Hz}. \tag{12}$$

We define

$$\boldsymbol{A} = \boldsymbol{I} + \begin{pmatrix} P\gamma\eta/2 & 0 \\ 0 & P\gamma/2 \end{pmatrix}\boldsymbol{H}\boldsymbol{H}^\dagger. \tag{13}$$

Since $(\log_e|\boldsymbol{A}|)'_P = \text{trace}(\boldsymbol{A}^{-1}\boldsymbol{A}'_P)$ according to [18], and $\boldsymbol{A}^{-1}|_{P=0} = \boldsymbol{I}$, we have

$$(C_{\text{CF}})'_P\Big|_{P=0} = \mathbb{E}\left\{\text{trace}\left(\begin{bmatrix} (\frac{\gamma\eta}{2} + \frac{P}{2}\gamma\eta'_P)|_{P=0} & 0 \\ 0 & \gamma/2 \end{bmatrix}\boldsymbol{H}\boldsymbol{H}^\dagger\right)\right\}, \tag{14}$$

From (8), we get $\eta|_{P=0} = 0$ and

$$\eta'_P = \frac{4G(1-\gamma)}{[-2GP + 2GP\gamma - \gamma P(|h_1|^2 + |h_2|^2) - 2]^2} \tag{15}$$

Thus, $\eta'_P|_{P=0} = G(1-\gamma)$. Since $\eta|_{P=0}$ and $\eta'_P|_{P=0}$ are independent of the channel conditions, from (14) we get

$$(C_{\text{CF}})'_P\Big|_{P=0} = \text{trace}\left(\begin{bmatrix} 0 & 0 \\ 0 & \gamma/2 \end{bmatrix}\mathbb{E}[\boldsymbol{H}\boldsymbol{H}^\dagger]\right) = \gamma, \tag{16}$$

where Rayleigh fading channels with $\mathbb{E}[|h_i|^2] = 1$ are considered. Inserting (16) in (10), we obtain

$$\frac{E_b}{N_{0\,\min}} = \frac{\ln(2)}{\gamma}. \tag{17}$$

In addition,

$$\begin{aligned}(\log_e|\boldsymbol{A}|)''_P &= \left[\text{trace}(\boldsymbol{A}^{-1}\boldsymbol{A}'_P)\right]'_P \\ &= \text{trace}\left[(\boldsymbol{A}^{-1})'_P\,\boldsymbol{A}'_P + \boldsymbol{A}^{-1}\boldsymbol{A}''_P\right] \\ &= -\text{trace}\left[\boldsymbol{A}^{-1}(\boldsymbol{A}'_P)\boldsymbol{A}^{-1}\boldsymbol{A}'_P\right] + \text{trace}\left[\boldsymbol{A}^{-1}\boldsymbol{A}''_P\right].\end{aligned} \tag{18}$$

Then, we have

$$\begin{aligned}&(C_{\text{CF}})''_P\Big|_{P=0} \\ &= -\mathbb{E}\left\{\left[\text{trace}\left(\begin{bmatrix} (P\gamma\eta/2)'_P|_{P=0} & 0 \\ 0 & \gamma/2 \end{bmatrix}\boldsymbol{H}\boldsymbol{H}^\dagger\right)\right]^2\right\} \\ &\quad + \mathbb{E}\left\{\text{trace}\left(\begin{bmatrix} (P\gamma\eta/2)''_P|_{P=0} & 0 \\ 0 & 0 \end{bmatrix}\boldsymbol{H}\boldsymbol{H}^\dagger\right)\right\} \\ &= -\text{trace}\left(\begin{bmatrix} 0 & 0 \\ 0 & (\gamma/2)^2 \end{bmatrix}\mathbb{E}\left[(\boldsymbol{H}\boldsymbol{H}^\dagger)^2\right]\right) \\ &\quad + \text{trace}\left(\begin{bmatrix} (\gamma\eta'_P + \frac{P}{2}\gamma\eta''_P)|_{P=0} & 0 \\ 0 & 0 \end{bmatrix}\mathbb{E}[\boldsymbol{H}\boldsymbol{H}^\dagger]\right).\end{aligned} \tag{19}$$

As the magnitude of the channel coefficient, $|h_i|$, follows Rayleigh distribution with its Kurtosis equals 2 [16], we define $\kappa(|h_i|) = 2$. With $N_t = N_r = 2$, and $\eta'_P|_{P=0} = G(1-\gamma)$, we then obtain

$$\begin{aligned}&(C_{\text{CF}})''_P\Big|_{P=0} \\ &= -(\gamma/2)^2 N_t N_r[\kappa(|h_i|) + N_t + N_r - 2]/2 + N_r\gamma\eta'_P|_{P=0} \\ &= -2\gamma^2 + 2\gamma G(1-\gamma).\end{aligned} \tag{20}$$

Substituting (16) and (20) into (10), we get

$$S_0 = \frac{2\gamma^2}{2\gamma^2 - 2\gamma G(1-\gamma)} = \frac{\gamma}{\gamma - G(1-\gamma)}. \tag{21}$$

Inserting (17) and (21) in (9), we finally get the linear approximation of EE as a function of SE as shown in (11). □

Note that Rayleigh fading channels are considered in (11). For other types of channel distribution, $\mathbb{E}\left[\boldsymbol{H}\boldsymbol{H}^\dagger\right]$ and $\mathbb{E}\left[(\boldsymbol{H}\boldsymbol{H}^\dagger)^2\right]$ will be accordingly determined, which will result in different forms of the EE approximation.

B. A Closed-form Lower Bound on EE

The basic motivation for deriving the EE lower bound is to study the impact of varying the value of γ on the EE performance, as well as, the limitation of the linear approximation of EE. Given certain value of P_s at the transmitter, a large value of γ guarantees a small value of P_r, i.e., the total power consumption is low. But, under a specific G, small P_r will result in a large σ_c^2 in (2) which will degrade C_{CF} as shown in (7). Thus, different values of γ represent different levels of EE performance. However, the linear approximation derived in Section III-A can provide good insights into the EE performance in the low SE region. Outside this region, a linear approximation will not be suitable according to our numerical analysis in Section IV. In this subsection, we therefore derive a closed-form lower bound for EE, which is valid for both low and high SE regions. Based on this lower bound, we formulate the EE optimization problem, such that the total energy per bit consumption for a given effective transmission rate will be minimized.

When transmitters do not know CSI, the upper bound of capacity for the virtual-MIMO system (coming from (7)) is given by Jensen's inequality [19]:

$$\begin{aligned} C_{\mathrm{CF}} &= \sum_{j=1}^{N_{\min}} \mathbb{E}\left\{\log_2\left(1+\frac{P\gamma}{2}\hat{\lambda}_j^2\right)\right\} \\ &\leq N_{\min}\log_2\left\{1+\frac{P\gamma}{2N_{\min}}\mathbb{E}\left[\sum_{j=1}^{N_{\min}}\hat{\lambda}_j^2\right]\right\}, \end{aligned} \quad (22)$$

where $N_{\min} = \min(N_t, N_r)$ represents the channel rank for the virtual-MIMO system. And $\hat{\lambda}_1 \geq \hat{\lambda}_j$ are the ordered singular values of the scaled channel matrix $\widetilde{\boldsymbol{H}}$. The inequality (22) becomes equality if and only if the singular values are all equal [19]. Hence, we could expect a high upper bound as the channel matrix $\widetilde{\boldsymbol{H}}$ is sufficiently random. Moreover, the expectation of $\sum_{j=1}^{N_{\min}}\hat{\lambda}_j^2$ can be expressed as

$$\begin{aligned} \mathbb{E}\left[\sum_{j=1}^{N_{\min}}\hat{\lambda}_j^2\right] &= \mathbb{E}\left[\mathrm{trace}\left(\begin{bmatrix}\eta & 0\\ 0 & 1\end{bmatrix}\boldsymbol{H}\boldsymbol{H}^{\dagger}\right)\right] \\ &= (\bar{\eta}+1)N_r, \end{aligned} \quad (23)$$

where Rayleigh fading channels with unit power gain are considered. The scalar $\bar{\eta}$ denotes the expectation of η. From (8), we obtain

$$\bar{\eta} = \frac{G(1-\gamma)P}{G(1-\gamma)P+\gamma P+1}. \quad (24)$$

Using (23) and (24), equation (22) is further simplified

$$C_{\mathrm{CF}} \leq 2\log_2\left\{1+\frac{P\gamma}{2}\cdot\frac{(2G-2G\gamma+\gamma)P+1}{G(1-\gamma)P+\gamma P+1}\right\}. \quad (25)$$

According to the definition of energy efficiency, we can obtain a closed-form lower bound for the system EE as shown in the following.

Proposition 2: Consider the proposed virtual-MIMO system with CF cooperation under Rayleigh fading channels. When CSI is only available at the receiver, the energy consumption per information bit can be lower bounded as

$$E_b \geq \frac{-b(\mathsf{C}_{\mathrm{CF}},\gamma,G)+\sqrt{b^2(\mathsf{C}_{\mathrm{CF}},\gamma,G)-4a(\gamma,G)f(\mathsf{C}_{\mathrm{CF}},\gamma)}}{2\,\mathsf{C}_{\mathrm{CF}}\,a(\gamma,G)}, \quad (26)$$

where

$$\begin{cases} a(\gamma,G) = 2G-2G\gamma+\gamma,\\ b(\mathsf{C}_{\mathrm{CF}},\gamma,G) = 1-\frac{2}{\gamma}\left(2^{\mathsf{C}_{\mathrm{CF}}/2}-1\right)(G-G\gamma+\gamma),\\ f(\mathsf{C}_{\mathrm{CF}},\gamma) = -\frac{2}{\gamma}\left(2^{\mathsf{C}_{\mathrm{CF}}/2}-1\right). \end{cases} \quad (27)$$

A brief proof of the proposition is as follows: from (25), the expression of P as a function of C_{CF}, γ and G, can be arranged as

$$a(\gamma,G)\;P^2+b(C_{\mathrm{CF}},\gamma,G)\;P+f(C_{\mathrm{CF}},\gamma)\geq 0, \quad (28)$$

where the functions $a(\gamma,G)$, $b(C_{\mathrm{CF}},\gamma,G)$, and $f(\mathsf{C}_{\mathrm{CF}},\gamma)$, with respect to different variables, are defined in (27). When energy efficiency is defined as $E_b = P/C_{\mathrm{CF}}$, the upper bound of C_{CF} in (25) results in a lower bound on E_b. Substituting $P = E_b\,\mathsf{C}_{\mathrm{CF}}$ into (28), and solving the inequality (28), we thus obtain (26).

C. Optimal Power Allocation

For a given capacity-achieving transmission rate and a specific cooperation channel power gain G, varying the power allocation between the transmitter and the relay will result in different levels of EE performance. Therefore, the optimization problem for energy efficiency can be formulated as follows:

$$\begin{cases} \min\limits_{\gamma} E_b(\mathsf{C}_{\mathrm{CF}},\gamma,G)\\ \text{Subject to given values of } \mathsf{C}_{\mathrm{CF}} \text{ and } G. \end{cases} \quad (29)$$

One can apply the method for convex optimization to find the optimal solution for the above problem. If C_{CF} and G are given, our simulation results show that $E_b(\gamma)$ is a convex up function with respect to γ. That is, the local extremum of $E_b(\gamma)$ is also a global extremum. In addition, the local minimum of $E_b(\gamma)$ can be found by using Fermat's theorem. The first-order derivative of E_b with respect to γ is given by

$$\begin{aligned} (E_b)'_{\gamma} &= \frac{-b'_{\gamma}}{2a\mathsf{C}_{\mathrm{CF}}}+\frac{b\,b'_{\gamma}-2a'_{\gamma}f-2af'_{\gamma}}{2a\mathsf{C}_{\mathrm{CF}}\sqrt{b^2-4af}} \\ &\quad -\frac{(-b+\sqrt{b^2-4af})a'_{\gamma}}{2a^2\mathsf{C}_{\mathrm{CF}}}, \end{aligned} \quad (30)$$

where $(\cdot)'_{\gamma}$ denotes the derivative of a function with respect to the variable γ. From (27), we obtain

$$\begin{cases} a'_{\gamma}(\gamma,G) = -2G+1,\\ b'_{\gamma}(\mathsf{C}_{\mathrm{CF}},\gamma,G) = \frac{2G}{\gamma^2}\left(2^{\mathsf{C}_{\mathrm{CF}}/2}-1\right),\\ f'_{\gamma}(\mathsf{C}_{\mathrm{CF}},\gamma) = \frac{2}{\gamma^2}\left(2^{\mathsf{C}_{\mathrm{CF}}/2}-1\right). \end{cases} \quad (31)$$

Inserting (27) and (31) to (30), we get the first-order derivative of E_b. For given values of C_{CF} and G, by setting $[E_b(\gamma)]'_{\gamma} = 0$, one can obtain a unique solution $\gamma*$ which represents the optimal power allocation between the transmitter and the relay. With $\gamma*$, the whole energy consumption per information bit for the virtual-MIMO system will be minimized.

IV. Simulation Results and Discussion

In this section, we present the performance analysis of the virtual-MIMO system specified in Section II, by using both simulation and analytical results. The simulation results are computed via the Monte Carlo method for random channel realizations.

Firstly, we provide insight into energy efficiency of the virtual-MIMO system and verify the linear approximation given by (11). For this analysis, we consider $G = 10$ dB and

$\gamma = 0.95$. These values are selected because of the assumption of short-range cooperation channel: we consider a scenario where the average attenuation over cooperation channel is 10 times less than that of the data channels, i.e. $G = 10$ dB. In addition, the power consumed by the transmitter will be much greater than that of the relay which justifies the chosen value for γ. The results from this scenario are shown in Fig. 2. In this figure, the EE performance of the virtual-MIMO system against SE is compared with those of the non-cooperative MISO system and the ideal MIMO system as if the receivers were connected via a wire. Fig. 2 shows that for a specific SE with the help from the relay, the virtual-MIMO system always demonstrates a better EE performance than the MISO system. But, compared to the ideal MIMO case, the virtual-MIMO system consumes slightly more energy per bit due to the presence of compression noise at the relay. In addition, Fig. 2 shows how the linear approximations of EE performance for virtual-MIMO, MISO, and MIMO compare to the simulation results. For MIMO and MISO, we use the linear approximations given by equations (28), (213), and (215) in [16]. As the results demonstrate, for the given value of γ, the proposed approximation by (11) matches well to the simulation results and provides good insight into the EE behavior of the virtual-MIMO system.

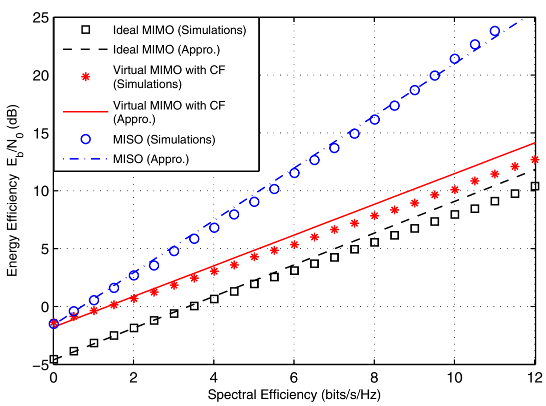

Fig. 2. EE performance of the virtual-MIMO system, compared with those of MISO and MIMO systems (For virtual MIMO, $G = 10$ dB and $\gamma = 0.95$ are considered.)

Next, we investigate the limitations of linear approximation of EE for the virtual-MIMO system. For this scenario, we consider different values of G and γ and demonstrate their impacts on the accuracy of the linear approximation. Different from the scenario of the previous figure, Fig. 3 shows the EE performance of virtual-MIMO for G=10, 15 dB and the corresponding *optimal value of* γ. This optimal value is found through exhaustive search over $\gamma \in (0, 1]$, such that the overall energy per bit consumption in this system is minimized. Fig. 3 shows that the linear approximation of EE given by (11) is sensitive to the value of γ and is valid only when the SE values are very low. It is because that in (9), S_0 represents the slope of $\mathrm{C_{CF}}$ in bits/s/Hz/3dB at $\frac{E_b}{N_0}_{\min}$. When SE is very small, the EE performance of the virtual-MIMO system is quite close to the MISO case. But with SE increasing, an optimal choice of γ causes a steep change of EE which is rapidly getting closer to and finally bounded by the MIMO case. However, the slope S_0 can not represent the changing behavior precisely. As shown in (21), an optimal value of γ which is smaller than $G/(1+G)$ will cause $S_0 < 0$, and thus result in EE decreasing in the very low SE regime. In this case, S_0 is only able to represent the initial decreasing behavior of EE, but is helpless for the following increasing and bounded behavior. Therefore, the linear approximation relying on S_0 is valid only when SE is very low. This analysis demonstrates the significance of the proposed lower bound given by (26), which can be used for general scenarios where the linear approximation is not valid.

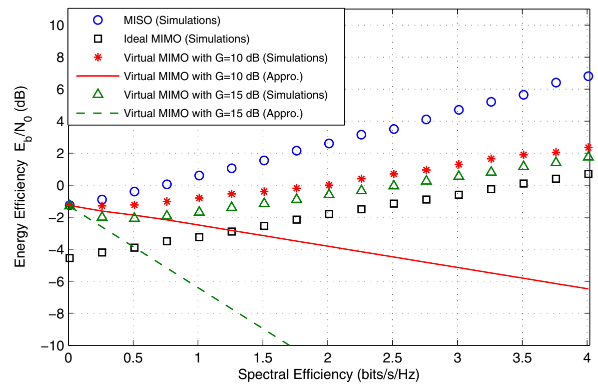

Fig. 3. EE performance of the virtual-MIMO system, compared with those of MISO and MIMO systems (For virtual MIMO, $G = 10, 15$ dB and the corresponding optimal γ are considered.)

Our next analysis demonstrates the impact of different power allocation choices, defined by different values of γ, on the EE performance of virtual-MIMO. We choose a specific capacity-achieving transmission rate $C_{\mathrm{CF}} = 8$ bits/s/Hz for example, and consider $G = 5, 10, 15$ dB. The results from this scenario are shown in Fig. 4, where the simulation results of EE against γ are presented in (a), and the lower bounds on EE given by (26) are in (b). It is shown that different values of γ result in different levels of EE performance. The lower bound is smaller than the simulated EE because that sufficiently random singular values in (22) cause a high upper bound on C_{CF}. Even though it is loose, the lower bound follows the same trend with the simulated EE (as shown in the figure), and therefore could be used to predict the practical EE performance. Thus, it is appropriate to implement the optimal $\gamma*$ which is a solution for the optimization problem described in (29), so that the total energy consumption per information bit will be minimized. Taking the assumptions $C_{\mathrm{CF}} = 8$ bits/s/Hz and $G = 10$ dB as an example, the optimal $\gamma*$ computed from $[E_b(\gamma)]'_\gamma = 0$ is 0.8549.

Finally, with the optimal choices of γ computed from setting $[E_b(\gamma)]'_\gamma$ in (30) to zero, the EE performance of the virtual-MIMO system (with $C_{\mathrm{CF}} = 8$ bits/s/Hz) against the cooperation channel power gain G is shown in Fig. 5. The EE performance of MISO and that of MIMO for various values of G are also given here for comparison. As the results demonstrate, when G is small, EE of the virtual-MIMO system is impaired by power consumption at the relay and also unstable transmission over the weak cooperation channel. As G increases, the helping relay enables the virtual-MIMO system to achieve an EE performance quite close to that of the ideal MIMO system.

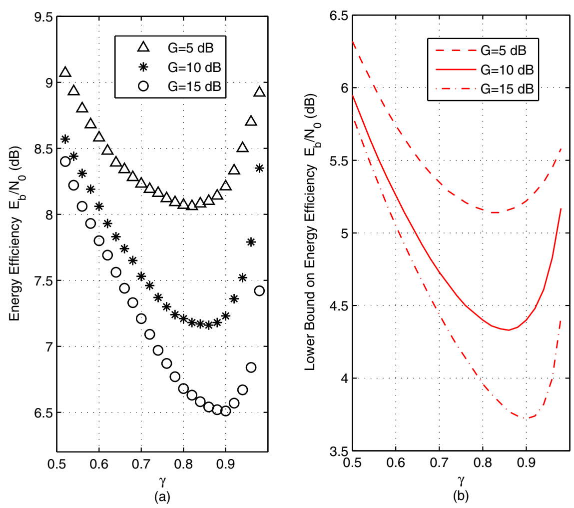

Fig. 4. The effects of varying the values of γ on the EE performance of the virtual-MIMO system with setting $C_{\text{CF}} = 8$ bits/s/Hz and $G = 5, 10, 15$ dB (The simulation results are shown in (a), and lower bounds are in (b).)

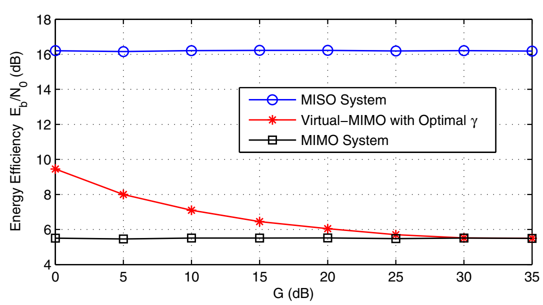

Fig. 5. EE Performance of MISO, virtual-MIMO, and MIMO systems for various values of G ($C_{\text{CF}} = 8$ bits/s/Hz is considered. For virtual MIMO, the optimal γ is computed from setting $[E_b(\gamma)]'_\gamma$ in (30) to zero.)

V. Conclusion

This paper investigated EE analysis of a virtual-MIMO system with a single remote two-antenna wireless transmitter sending information to two closely spaced single-antenna receivers. Virtual-MIMO operation was realized via receiver-side local communication using the CF cooperation. We firstly derived a linear approximation of EE as a function of SE in the low SE regime. We demonstrated the limitation of the linear approximation and showed that it provided good insights into the EE behavior, but only when SE was very low. The approximation could not be used in the optimization of power allocation between the transmitter and the relay. Furthermore, we derived the closed-form lower bound on EE, which is valid for both low and high SE regions. Based on this lower bound, the optimal power allocation was determined. The impact of varying the power allocation on the EE performance was also demonstrated. It was shown that, for a given capacity-achieving transmission rate and a specific cooperation channel condition, with the optimal power allocation, the overall energy consumption per information bit would be minimized, and the EE performance could get very close to the ideal MIMO case.

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