# Energy Efficiency of Transmit Diversity Systems Under a Realistic Power Consumption Model

Marcos Tomio Kakitani, Glauber Brante, Richard Demo Souza, and Muhammad Ali Imran

***Abstract*—We compare the downlink energy efficiency of spatial diversity multiple transmit antenna schemes. We determine the minimum required transmit power for a given outage probability. Our analysis shows that antenna selection is in general the most energy efficient option as it requires a single radio-frequency chain. We also investigate the limiting distances up to which the antenna selection technique outperforms the transmit beamforming scheme for different numbers of transmit antennas.**

***Index Terms*—Energy efficiency, transmit diversity.**

## I. INTRODUCTION

The information and communication technology (ICT) industry represents about 2% of the global $CO_2$ emissions, with the mobile networks operation representing around 10% of the ICT industry emissions [1]. Estimates indicate that the demand for data traffic will increase between a hundredfold to thousandfold before 2020 [2], therefore reflecting in a potential significant energy consumption increase.

Multiple antennas (MIMO) systems can present a considerable signal-to-noise ratio (SNR) improvement if compared to single antenna (SISO) systems, thus potentially increasing the energy efficiency. The SNR gains of MIMO schemes for a given target data rate for cellular networks were analyzed in [3]. However, the authors consider only the transmit power, other BS consumption factors as circuitry and cooling are not considered. In [4] it is shown that if more realistic power models are considered, the advantage of MIMO over SISO is not always evident for short-range communication systems, as wireless sensor networks. However, power models for large scale wireless systems [5] are considerably different than the power models for wireless sensor nodes [4]. The work in [6] shows that for realistic BS power consumption models, MIMO can be less efficient than SISO. However, the authors consider only the case of a spatial multiplexing, where the multiple antennas are used for increased spectral efficiency and not for spatial diversity. A comparative analysis in terms of the symbol error rate of different multi-antenna schemes, as transmit antenna selection and beamforming, is presented in [7] and [8]. However, the authors focus on optimizing the allocation of the total transmit power and the circuitry consumption is not considered. The authors of [9] perform an energy efficiency analysis of MIMO systems considering an appropriate power allocation. Although the circuitry consumption is considered, the analysis focuses on the optimum number of antennas, and does not investigate the transmit antenna selection scheme.

We investigate the energy efficiency of transmit diversity schemes for a target outage probability considering a realistic BS power consumption model [5]. Two spatial diversity schemes are considered: transmit beamforming (TBF), which is the best performing scheme in terms of outage probability; and transmit antenna selection (TAS), which is the simplest solution in terms of required hardware. Our results show that although TBF presents the best performance in terms of the outage probability, the TAS scheme is the most energy efficient option for most transmit distances, considerably outperforming SISO transmission. Moreover the TAS scheme can outperform TBF even with a smaller number of available transmit antennas. Such advantage of TAS comes from the fact that only a single radio-frequency (RF) chain is used at the transmitter, while other multiple antenna schemes use an RF chain per transmit antenna, compromising their energy efficiency. To the best of our knowledge, there are no similar works that compare the energy efficiency of TBF and TAS, including an analysis of the limiting distances from which a scheme is more energy efficient than the other.

## II. SYSTEM MODEL

The BS power consumption model follows [5], with the total energy consumption per bit being defined as:

$$\mathcal{E} = \left(N_{TRX} \cdot P_0 + \Delta_p \cdot \mathcal{P}\right)/R_b, \tag{1}$$

where $N_{TRX}$ is the number of transceivers (TRXs), or RF chains of the BS, $P_0$ is the non-load-dependent power consumption at the minimum non-zero output power, $\Delta_p$ is the slope of the load-dependent power consumption, $\mathcal{P}$ is the total RF output power at the antenna elements, and $R_b$ is the bit rate in bits/s. Furthermore, $R_b = \delta \cdot B$, where $\delta$ is the spectral efficiency and $B$ the system bandwidth. As the power consumption of the mobile station (MS) is not relevant compared to the power consumption of the BS, it is not included in the analysis. We consider that the BS is equipped with $M$ transmit antennas and the MS has one receive antenna, which is the most usual configuration for the MS. The path loss between the BS and the MS is defined as [10]

$$\gamma = \lambda^2/\left[(4\pi)^2 d^{\alpha}\right], \tag{2}$$

where $\lambda$ is the wavelength, $d$ is the BS to MS distance, and $\alpha$ is the path loss exponent. We consider the path loss after the power amplifier, thus the antennas consumption is already included in the model. Moreover, the unity energy fading coefficient $h_i$, between the BS $i$-th transmit antenna and the

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M. T. Kakitani and M. A. Imran, CCSR, University of Surrey, Guildford, UK, e-mails: mtkakitani@ieee.org and m.imran@surrey.ac.uk

M. T. Kakitani, G. Brante and R. D. Souza, CPGEI, Federal University of Technology - Paraná (UTFPR), Curitiba, Paraná, Brazil, e-mails: mtkakitani@ieee.org, gbrante@ieee.org and richard@utfpr.edu.br

MS receive antenna, is of the Rayleigh quasi-static type, the average SNR is $\rho = \frac{\gamma \cdot \mathcal{P}}{N}$, $N = N_0 \cdot B$ is the noise power, and $N_0$ is the thermal noise power spectral density per Hertz.

The following analysis is based on the outage probability, which is defined as the probability that the instantaneous SNR falls below the threshold $\beta = 2^{\delta} - 1$ at the MS [10].

## III. TRANSMISSION SCHEMES

Note that as in (1) both $P_0$ and $\Delta_p$ are fixed, and $R_b$ is also often a fixed design parameter, the minimum energy consumption is obtained by minimizing the transmit power $\mathcal{P}$. In this section we determine the minimum required power $\mathcal{P}^{\star}$, for a given target outage probability $\mathcal{O}^{\star}$, for both TBF and TAS.

In a system employing the TAS scheme, as only the selected transmit antenna is active, a single RF chain is required ($N_{TRX} = 1$). The outage probability is [11]

$$\mathcal{O}_{\text{TAS}}(M) = \left[1 - \exp\left(-\beta/\rho\right)\right]^{M}. \tag{3}$$

From (3) we can derive the minimum required transmit power by TAS for a given outage probability $\mathcal{O}^{\star}$ as

$$\mathcal{P}^{\star}_{\text{TAS}}(M) = \left(-\beta N\right) / \left[\gamma \ln\left(1 - \mathcal{O}^{\star \frac{1}{M}}\right)\right]. \tag{4}$$

For a given number of transmit antennas $M$ and target outage probability $\mathcal{O}^{\star}$, the total energy consumption per bit for TAS is

$$\mathcal{E}_{\text{TAS}}(M) = \left[P_0 + \Delta_p \cdot \mathcal{P}^{\star}_{\text{TAS}}(M)\right] / R_b. \tag{5}$$

Note that $\mathcal{E}_{TAS}(M') \leq \mathcal{E}_{TAS}(M'')$ if $M' \geq M''$, as $(\mathcal{O}^{\star})^{\frac{1}{M''}} \leq (\mathcal{O}^{\star})^{\frac{1}{M'}}$ with $0 < \mathcal{O}^{\star} < 1$. Therefore, with respect to the energy efficiency, the number of available transmit antennas in TAS, from which the best one is selected, should be made as large as possible.

The best performing transmit diversity scheme is the TBF, for which $N_{TRX} = M$, and the outage probability is [10]:

$$\mathcal{O}_{\text{TBF}}(M) = 1 - \exp\left(\frac{-\beta}{\rho}\right) \sum_{m=0}^{M-1} \frac{1}{m!} \left(\frac{\beta}{\rho}\right)^{m}, \tag{6}$$

which can be rewritten as

$$\mathcal{O}_{\text{TBF}}(M) = \Gamma\left(M, \frac{\beta}{\rho}\right) / \,\Gamma\left(M\right), \tag{7}$$

where $\Gamma\left(a, z\right) = \int_0^z t^{a-1} e^{-t} dt$ is the lower incomplete Gamma function and $\Gamma\left(a\right) = \int_0^{\infty} t^{a-1} e^{-t} dt$ is the complete Gamma function. At sufficiently high SNR or small outage probability, as considered in this paper, the incomplete Gamma function can be well approximated by $\Gamma(a, z) = \frac{1}{a} \cdot z^{a}$, thus

$$\mathcal{O}_{\text{TBF}}(M) \approx \left(\frac{\beta}{\rho}\right)^{M} \frac{1}{\Gamma\left(M + 1\right)}. \tag{8}$$

The minimum required transmit power for a target outage probability can be found from (8) as

$$\mathcal{P}^{\star}_{\text{TBF}}(M) = \left(\beta N\right) / \left(\gamma \left[\Gamma(M+1)\mathcal{O}^{\star}\right]^{\frac{1}{M}}\right), \tag{9}$$

while the total energy consumption is

$$\mathcal{E}_{\text{TBF}}(M) = \left[M \cdot P_0 + \Delta_p \cdot \mathcal{P}^{\star}_{\text{TBF}}(M)\right] / R_b, \tag{10}$$

which, opposed to (5), is not a monotonically decreasing function with $M$. In another words, in this case it is not always true that $\mathcal{E}_{TBF}(M'') \leq \mathcal{E}_{TBF}(M')$ if $M'' \geq M'$. However, for the initial condition of $\mathcal{E}_{TBF}(M') = \mathcal{E}_{TBF}(M'')$, and after replacing $\mathcal{E}_{TBF}$, $\mathcal{P}^{\star}_{TBF}$, and $\gamma$ by (10), (9) and (2), we can determine a sufficient distance $d_{TBF,TBF}(M', M'')$ from which using $M''$ transmit antennas is more energy efficient than using $M'$ antennas ($M'' > M'$), as shown in (11). Thus, using more transmit antennas is not always a more energy efficient solution in TBF.

Note that if the required transmit power is small enough (equivalently, the distance between the BS and the MS is small enough) so that the circuitry power is more relevant than the transmit power, then TAS is more energy efficient than TBF for any number of transmit antennas. Thus, in principle, TAS is a more energy efficient solution. However, due to its better outage performance, there might be a distance from which TBF with a given number of transmit antennas is more energy efficient than TAS with another number of antennas. Thus, let us consider that TAS operates with $M'$ antennas and TBF with $M''$ antennas. Then, similarly to the derivation of equation (11), and based on (2), (4), (5), (9) and (10), it is possible to find a limiting distance $d_{TAS,TBF}(M', M'')$, up to which TAS with $M'$ antennas is more energy efficient than TBF with $M''$ antennas, as shown in (12), even if $M' \leq M''$. Note from equation (12) that the numerator can never assume negative values. Thus, the denominator must be greater than zero. Then we have the condition

$$M' < \left(\ln \mathcal{O}^{\star}\right) / \left[\ln\left(1 - \exp\left(-\left(M''! \mathcal{O}^{\star}\right)^{\frac{1}{M''}}\right)\right)\right], \tag{13}$$

which has to be fulfilled, otherwise TBF never outperforms TAS.

Therefore, and which is our main finding, even though TBF performs better than TAS in terms of outage probability, TAS can outperform TBF in terms of energy efficiency up to some practical BS to MS distances, even if using less transmit antennas. For instance, if $\mathcal{O}^{\star} = 10^{-2}$ and $M' = 2$, then TAS is always more energy efficient than TBF no matter the choice of $M''$. Moreover, using (12) we can show that, for a set of realistic parameters, TAS with only $M' = 2$ antennas outperforms TBF with $M'' = 5$ antennas up to the reasonably large distance of $d_{TAS,TBF} = 1.6$ km, which we consider to be a quite surprising result.

Finally, in the case of SISO transmission ($N_{TRX} = 1$)

$$\mathcal{O}_{\text{SISO}} = 1 - \exp\left(-\beta/\rho\right), \tag{14}$$

so that the minimum required transmit power is

$$\mathcal{P}^{\star}_{\text{SISO}} = \left(-\beta N\right) / \left[\gamma \ln(1 - \mathcal{O}^{\star})\right], \tag{15}$$

and the total consumed energy per bit becomes

$$\mathcal{E}_{\text{SISO}} = \left(P_0 + \Delta_p \cdot \mathcal{P}^{\star}_{\text{SISO}}\right) / R_b. \tag{16}$$

Let us compare the energy efficiency of SISO and TAS. For $\mathcal{E}_{SISO} \leq \mathcal{E}_{TAS}$ we must have $\mathcal{O}^{\star \frac{1}{M}} \geq \mathcal{O}^{\star}$, which is only valid (with equality) for $M = 1$. Thus, the SISO transmission is always outperformed by TAS for $M \geq 2$. However, if SISO is compared to TBF, then similarly to (11) and (12), we can

$$d_{TBF,TBF}(M',M'') = \left( \frac{\lambda^2 P_0 (M''-M')[(M'')!\mathcal{O}^\star]^{\frac{1}{M''}}[(M')!\mathcal{O}^\star]^{\frac{1}{M'}}}{(4\pi)^2 \Delta_p \beta N \left\{[(M'')!\mathcal{O}^\star]^{\frac{1}{M''}} - [(M')!\mathcal{O}^\star]^{\frac{1}{M'}}\right\}} \right)^{\frac{1}{\alpha}}, \qquad (11)$$

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$$d_{TAS,TBF}(M',M'') = \left( \frac{-\lambda^2 \ln\left(1-\mathcal{O}^{\star\frac{1}{M'}}\right)[(M'')!\mathcal{O}^\star]^{\frac{1}{M''}} P_0\left(M''-1\right)}{\left(4\pi\right)^2 \Delta_p \beta N \left\{[(M'')!\mathcal{O}^\star]^{\frac{1}{M''}} + \ln\left(1-\mathcal{O}^{\star\frac{1}{M'}}\right)\right\}} \right)^{\frac{1}{\alpha}}, \qquad (12)$$

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$$d_{SISO,TBF}(M) = \left( \frac{-\lambda^2 \ln\left(1-\mathcal{O}^\star\right)[(M)!\mathcal{O}^\star]^{\frac{1}{M}} P_0\left(M-1\right)}{\left(4\pi\right)^2 \Delta_p \beta N \left\{[(M)!\mathcal{O}^\star]^{\frac{1}{M}} + \ln\left(1-\mathcal{O}^\star\right)\right\}} \right)^{\frac{1}{\alpha}}. \qquad (17)$$

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obtain the limiting distance $d_{SISO,TBF}$ up to which SISO outperforms TBF with $M$ antennas, as shown in (17).

## IV. RESULTS

The system parameters are: $N_0 = -174$ dBm/Hz and $\alpha = 3$. We analyze a scenario with bandwidth $B = 10$ MHz, as in [5]. The parameters of the macro power model also follow [5], with $P_0 = 84$ W and $\Delta_p = 2.8$. For increased efficiency, we consider that the macro BS uses a remote radio head, so that the power amplifier module is mounted at the same physical location as the transmit antenna.

Figure 1 presents the consumed energy per bit for an outage probability of $\mathcal{O}^\star = 10^{-2}$ and $\delta = 3$ b/s/Hz. We can see that the TAS schemes are the most energy efficient up to a given distance. Considering the same number of transmit antennas for both schemes, TAS is only outperformed by TBF for large distances. For example, for $M = 2$ TAS is outperformed by TBF when $d > 1595$ m, for $M = 3$ the distance is $d > 2323$ m, for $M = 4$, TBF outperforms TAS for $d > 2905$ m, and for $M = 5$, TAS is only outperformed by TBF for $d > 3393$ m. Moreover, with only two transmit antennas, TAS $(M = 2)$ is outperformed by TBF $(M = 3)$ when $d > 1415$ m, by TBF $(M = 4)$ when $d > 1540$ m, and by TBF $(M = 5)$ for $d > 1664$ m. It must be mentioned that the presented limiting distances obtained through simulations and the values obtained from equations (11), (12), and (17) show a close match. Moreover, as the required transmit power is inversely proportional to the scheme's diversity order, note that the slopes of the curves are also inversely proportional to the diversity order. Then, although the TBF schemes have the best performance in terms of outage, when the total energy consumption is considered they are outperformed by TAS for most distances, as the latter consumes much less circuitry power. In addition, for short distances, while the TAS schemes are still the most energy efficient, the TBF schemes are also outperformed by SISO transmission, which has the worst outage performance, but a smaller circuitry consumption. Finally, note that when the TBF schemes are compared, TBF $(M = 5)$ is the least energy efficient for short distances due to the larger circuitry consumption, but for greater distances, as the transmit power gets more relevant, TBF $(M = 5)$ becomes the most energy efficient among the TBF schemes in Figure 1.

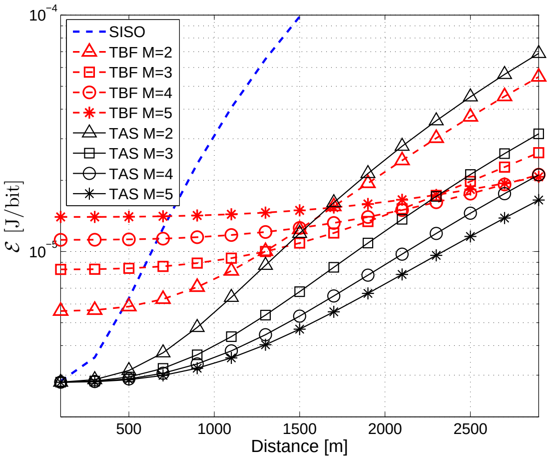

Figure 1. Total consumed energy per bit considering the transmission from a macro BS to a MS for $\mathcal{O}^\star = 10^{-2}$ and $\delta = 3$ b/s/Hz.

Similar conclusions are obtained for other spectral efficiencies and the same $\mathcal{O}^\star = 10^{-2}$, as is shown in Figure 2 for a BS-MS distance $d = 1000$ m. Note that the TAS schemes are the most energy efficient for most of the spectral efficiencies, with TAS only being outperformed by TBF if the spectral efficiency is significantly increased, resulting in scenarios where the required transmit transmit power has a larger impact in the energy efficiency than the circuitry consumption. Moreover, for a given number of antennas $M$, both schemes TAS and TBF have an optimal energy efficiency for a given distance, which can be obtained by the differentiation of equations (5) and (10) with respect to the spectral efficiency $\delta$. However, for brevity, these equations are not included in this paper.

Figure 3 shows that, for $\delta = 3$ b/s/Hz and $d = 1000$ m, in the case of a milder target outage as $\mathcal{O}^\star = 10^{-1}$ the transmitting circuitry consumption becomes even more relevant, and the advantage of TAS increases. Moreover, although the SISO scheme is still the least energy efficient for most distances, its performance becomes more competitive. In opposition, for a very strong outage requirement as $\mathcal{O}^\star = 10^{-4}$, although the relevancy of the transmitting circuitry consumption decreases, TAS $(M = 5)$ is still the most energy efficient scheme. Table I provides more detailed information for the case of $\mathcal{O}^\star = 10^{-4}$. Note that the TAS $(M = 5)$ scheme is outperformed by

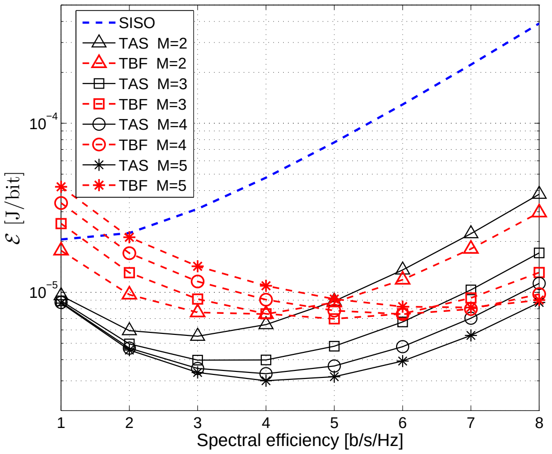

Figure 2. Total consumed energy per bit considering the transmission from a macro BS to a MS for $\mathcal{O}^{\star} = 10^{-2}$ and $d = 1000$ m.

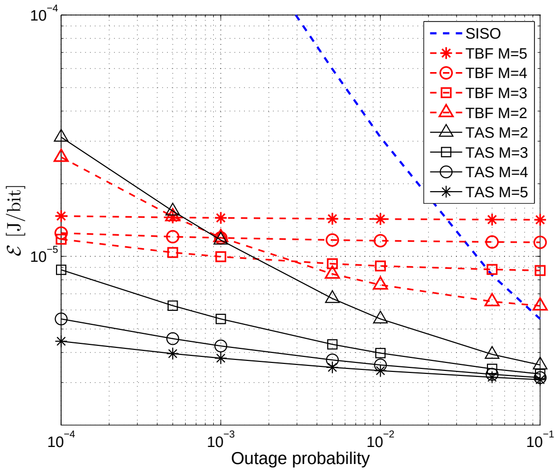

Figure 3. Total consumed energy per bit considering the transmission from a macro BS to a MS for $\delta = 3$ b/s/Hz and $d = 1000$ m.

TBF $(M = 5)$ only when $d > 2267$ m. Moreover, when compared to the scenario from Figure 1, the advantage of TAS over the TBF schemes slightly decreases.

We have investigated the case of $M > 5$ transmit antennas, but the qualitative conclusions are the same. Furthermore, if the number of antennas at the MS increases, the required transmit power decreases, and the circuitry consumption gets even more relevant in the energy efficiency analysis, making TAS still a more efficient solution than TBF. Moreover, transmit diversity schemes based on space-time coding perform worse than TBF in terms of outage and use the same number of RF chains. Therefore, such techniques are less energy efficient than TBF. As we show that TAS already outperforms TBF in terms of energy efficiency, we did not include results for space-time coding techniques for the sake of brevity.

## V. CONCLUSIONS

We investigate the energy efficiency of transmit diversity systems for a given target outage probability, considering a realistic power consumption model. The simulation results show the liming distance from which one transmit scheme start to outperform the other scheme. We provide an analytical formula that calculates the limiting distance directly (without need for simulations). Both simulation and analytical calculation of the limiting distance show a close match. We show that TAS, even though not the best in terms of outage probability, can be a very energy efficient solution. That is a consequence of TAS requiring a single RF chain, while TBF requires an RF chain per transmit antenna, compromising its energy efficiency. It is only in the case of considerably large distances that the required transmit power prevails over the circuitry consumption, and TBF can outperform TAS in terms of energy efficiency.

Table I
LIMITING DISTANCES FOR WHICH TAS IS OUTPERFORMED BY TBF, FOR $\mathcal{O}^{\star} = 10^{-4}$ AND $\delta = 3$ B/S/HZ.

| | | $d$ [m] |
|---|---|---|
| $\mathcal{E}_{TAS}(M') < \mathcal{E}_{TBF}(M'')$ | $M' = 2,\ M'' = 2$ | 699 |
| | $M' = 2,\ M'' = 3$ | 608 |
| | $M' = 2,\ M'' = 4$ | 677 |
| | $M' = 2,\ M'' = 5$ | 740 |
| | $M' = 3,\ M'' = 3$ | 1289 |
| | $M' = 4,\ M'' = 4$ | 1810 |
| | $M' = 5,\ M'' = 5$ | 2267 |

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