Liquid-gas phase transition in nuclear matter in the mean-field approximation

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Abstract. The liquid gas phase transition in nuclear systems is a unique phenomenon, at the frontier of nuclear, many-body and statistical physics. We use self-consistent mean-field calculations to quantify the properties of the transition in symmetric nuclear matter. We explore the available parameter space of critical properties by analyzing the mean-field dependence of the phase transition. The latent heat of the transition is computed and we find that it exhibits a model independent temperature dependence due to basic physical principles.

The experimental determination of the thermodynamical properties of nuclei within a heavy-ion reaction is a major challenge [1]. Significant advances in this direction will come from the next generation of radioactive beam facilities, especially due to the advances in instrumentation [2]. The access to isospin asymmetric nuclei will allow for a more detailed study of isospin diffusion and equilibration [3]. Theoretical approaches that can deal with finite temperature nuclear matter are therefore called for. A controlled and consistent approach in this direction is given by the self-consistent mean-field approximation, which is based on sound theoretical grounds (statistical mechanics together with quantum many-body theory) [4] and yields predictions that are equally valid at zero and finite temperature. This can also be casted as a finite temperature extension of density functional theory. So far, calculations for hot nuclei have been generally performed on selected spherical isotopes, mostly aimed at understanding the structural changes induced by temperature [5].

Before extensive finite nuclei calculations are implemented, however, the model dependence of the calculations should be assessed. Among the possible sources of uncertainty, the dependence associated with the underlying NN effective interaction has been scarcely investigated. Homogeneous nuclear matter within a mean-field picture provides the simplest testing ground for such an analysis, since it lies within the thermodynamical limit. It is well-known that different effective interactions predict different bulk properties for nuclear matter [6]. This statement also applies at finite temperature and, as a result, different liquid-gas critical points are found for different forces. One would also expect that a reliable determination of the critical point could be helpful in constraining the properties of the effective interaction [7].

The mean-field description of nuclear matter should not be taken as a meaningful description of the liquid-gas transition in nuclear systems, though. Clustering as well as fluctuations

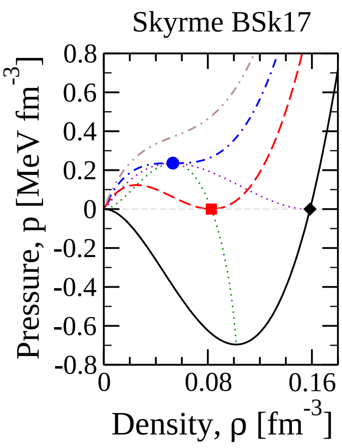

Figure 1. Pressure isotherms of symmetric nuclear matter for different temperatures: $T = 0$ (solid line), T_f (dashed line), T_c (dash-dotted line) and 18 MeV (dash-double-dotted line). The green and purple dotted lines represent the corresponding spinodal and coexistence regions.

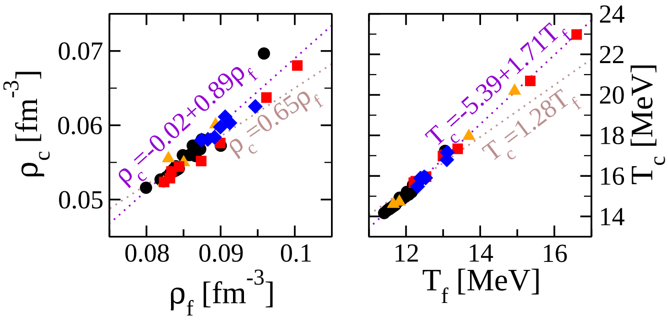

Figure 2. Left panel: critical density versus flashing density for different effective interactions. Right panel: critical temperature versus flashing temperature for different effective interactions.

dominate close to the critical point and none of these is taken into account within the mean-field approximation. As a matter of fact, here we do not try to provide a quantitatively accurate picture for such critical properties. Instead, we aim at investigating the effective interaction dependence of the liquid-gas transition as well as the overall behavior of its latent heat. As such, this can be taken as a first methodological contribution to test what information, if any, can be obtained from such analysis.

The density and temperature dependence of the thermodynamical properties obtained within the Hartree-Fock approximation are non-trivial. These can only be obtained numerically and they cannot be parametrized easily in any analytical form [8]. The mean-field solution describes *simultaneously* two different physical phases: a gas at low densities and a liquid at densities close to saturation [5]. This behavior is more easily visualized in the pressure isotherms of nuclear matter, as show in Fig. 1. This corresponds to results obtained with a typical Skyrme effective interaction (BSk17). Four characteristic temperatures have been chosen for illustrative purposes.

At $T = 0$ (solid lines), the pressure is a decreasing function of the density at low densities until a minimum at $\sim \rho_0/2$ (with ρ_0, the saturation density) is reached. This regime, for which $\frac{\partial p}{\partial \rho} < 0$, corresponds to the spinodal region, where the system is mechanically unstable. Beyond the minimum, the pressure increases with density until it changes sign at the saturation point (black diamond). As the temperature increases, the spinodal region, which is shown in green dotted lines in Fig. 1, shrinks. The isotherm at which the pressure becomes a completely positive function defines the so-called *flashing* temperature. The flashing point (red square) corresponds to the crossing of the spinodal with the zero pressure point, $p = \frac{\partial p}{\partial \rho} = 0$. For an isolated system, without an external gas to stabilize it, this would correspond to the maximum temperature at which the system could still be self-bound. For the infinite system, however, the system can exist above the flashing temperature because the gas and the liquid phase exert the same pressure on each other.

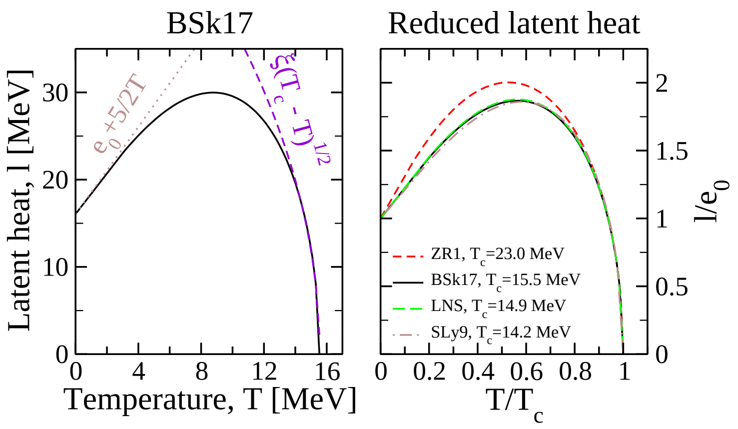

Figure 3. Left panel: temperature dependence of the latent heat of symmetric nuclear matter for the BSk17 interaction. The low- and high-temperature dependences correspond to the dotted and dashed lines, respectively. Right panel: reduced latent heat vs. reduced temperature for different Skyrme interactions.

According to the Maxwell criterion, the condition that the gas and liquid phases have the same pressure, together with the requirement that their chemical potentials are the same, set the coexistence points [9]. The coexistence region is shown with a purple dotted line in Fig. 1. Note that, at $T = 0$, there is a coexistence between a zero-density gas and a liquid at saturation density. Increasing the temperature results into larger gas and smaller liquid densities. Eventually, the two densities meet at the critical point (blue circle), which occurs at the critical temperature, $T_c = 15.5$ MeV for this particular Skyrme force. The critical isotherm is shown with a dashed-dotted line in Fig. 1. Above the critical temperature, the system only exists in the gas phase and the pressure isotherms become a monotonically increasing function of density. This behavior is observed in the dashed-double-dotted line, which has been computed at $T = 18$ MeV. Modern Skyrme and Gogny forces seem to favor a critical and flashing temperature in the regions $T_c \sim 14 - 17$ MeV and $T_f \sim 11 - 13$ MeV [8].

By computing the liquid-gas critical properties with a large set of Skyrme force parametrizations, one might also be able to identify behaviors which are independent of the effective interaction. An examples of such correlation is given in Fig. 2. A tight proportionality between the critical and the flashing densities (right panel) and temperatures (left panel) is found for a wide range of Skyrme and Gogny forces (blue circles). Different generations of Skyrme forces are represented by squares (old forces with no effective mass), triangles (old forces with $m^* \neq m$) and new forces (black circles). The flashing-critical correlation can be modeled by a single slope parameter or, for better quality, with a linear regression with an offset. Proportionality constants agree with those predicted by analytical models and the empirical relation $T_c/T_f = 2\rho_c/\rho_f$ is fulfilled. Finally, in the context of critical exponents, these have also been observed to be independent of the underlying mean-field [8].

The latent heat of nuclear matter is a very appealing quantity that provides a further characterization of the liquid-gas phase transition [10]. More interestingly, it can be potentially extracted from experiments, which suggests values of $l \sim 4 - 8$ MeV [1, 7, 11]. In the case of infinite nuclear matter, the latent heat per particle accounts for the amount of heat needed to take a nucleon from the liquid to the gas phase. One can describe its basic temperature dependence from basic quantum statistical mechanics arguments [10].

The Clausius-Clapeyron formula gives the latent heat per particle in terms of the product of the temperature, the difference between volume per particle of the two phases and the derivative of the pressure with respect to the temperature along the coexistence curve [9]. Alternatively, the latent heat can also be computed as the heat exchange between two phases that only involves the difference in entropies, $l = T(s_g - s_l)$. For nuclear matter, this formula is numerically more stable than the Clausius-Clapeyron one. One can however check the thermodynamic consistency of the theory by comparing both values [10]. Values of the vaporization specific latent heat for common liquids and gases are in the region of $10-5000$ kJ/kg when measured at the normal boiling point. In contrast, nuclear matter has a maximum specific latent heat of the order of ~ 30 MeV, *i.e.* $\sim 3\times 10^{12}$ kJ/kg, which is orders of magnitude higher and among the highest in nature. The origin of this extremely large value can be traced back to the strong force, which binds the nucleons tightly.

The latent heat, l, obtained with the BSk17 Skyrme parametrization is reported in the left panel of Fig. 3 (solid line). One finds that l has a characteristic bump shape as a function of temperature. We have observed that, for all other Skyrme parametrizations, the qualitative behavior of l is very similar [10]. The latent heat matches the value of the binding energy, e_0, at $T=0$, and then rises for small temperatures. The initial rise is linear and the slope is independent of the Skyrme parametrization. As a matter of fact, the low-temperature behavior can be extracted from a careful analysis of the thermodynamics of the phase transition, yielding $l \sim e_0 + 5/2T$. This analysis, valid for any fermionic system undergoing a liquid-gas phase transition, confirms that the latent heat grows linearly with temperature and that the linear growth is controlled by a model-independent slope [10].

Further up in temperature, l reaches a maximum and then drops to zero at the critical point, where the difference between the liquid and gas phases disappears. For BSk17, the latent heat maximum lies at $T=8.7$ MeV with $l_{\rm max}=29.9$ MeV. In the large temperature limit, critical exponents characterize the properties of the phase transition. The latent heat can be described, in this regime, in terms of a single critical exponent [10]. As shown in Ref. [12], the critical exponent for l is the same as that of the order parameter. Since in the Hartree-Fock approximation the latter is given by $\beta=1/2$, one finds that $l\sim (T_c-T)^{1/2}$.

The $T=0$ limit of the latent heat suggests that, if one wants to obtain interaction-independent results, l might be normalized to e_0. A plot of the latent heat in reduced units, l/e_0, as a function of the reduced temperature, T/T_c, is presented in the right panel of Fig. 3 for a selected subset of four Skyrme forces. These represent a wide range of critical temperatures and maximum latent heats, ranging from $l_{max}=34$ MeV for ZR1 to $l_{max}=28$ MeV for LNS. In genera, we observe that the dependence on the mean-field is eliminated to a large extent in the dimensionless plot. For all Skyrme forces, the latent heat tends to peak within a limited region of temperatures, $T/T_c\sim 0.5-0.6$. Moreover, the peak is also quite narrowly distributed around the value $l_{max}/e_0 \sim 1.7-2$. This indicates that the latent heat is more determined by thermal correlations than by the effective force itself.

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