Abstract

The failure of woven fabric glass/epoxy laminates manufactured from plain weave and eight harness satin weave reinforcements has been investigated.

Initially, damage development in unnotched plain weave laminates assembled from two layers of fabric was characterised. It was found that the matrix cracking damage morphology was influenced strongly by the tow structure, weave architecture and the relative shift of adjacent fabric layers. The effect of this damage on laminate stiffness was also investigated and modelled using a shear-lag analysis based on an equivalent cross-ply laminate. The models developed were able to predict the general trend of the data, but could not account for the initial reduction in Young's modulus at low damage densities.

An extensive data-base of unnotched and notched laminate properties has been established for laminates assembled from 2, 4, 6 and 8 fabric layers. A volume fraction normalisation technique was developed to account for volume fraction variations present between laminates. There was little effect of the variation of the number of fabric layers on the notched to unnotched strength ratios of the laminates investigated. Laminates containing circular notches had higher notched to unnotched strength ratios than laminates containing elliptical notches. Also, the notched to unnotched strength ratios of eight harness satin weave laminates were higher than those of plain weave laminates.

A comprehensive notch edge damage analysis was performed using plan view photography and two deply techniques. It was found that the notch edge damage initiation and propagation, which comprised matrix cracking, tow fracture, delamination and longitudinal splitting, was strongly influenced by the crimp regions in the fabrics. The notch size and shape, reinforcement type and relative shift of adjacent fabric layers were also seen to influence notch edge damage. Damage observations have been used to account for trends in the notched strengths of the laminate and notch configurations investigated.

The well known semi-empirical Whitney and Nuismer models resulted in accurate notched strength predictions. The average stress criterion was found to be more accurate than the point stress criterion. A fracture mechanics based model developed by Hitchen et al. yielded accurate notched strength predictions, while being very easy to implement. It was found that the Hitchen model also provides a satisfactory physical description of failure, especially for laminates containing elliptical notches. Predicted damage zone lengths generally agreed well with experimental observations for all models.
Acknowledgements

Throughout this project I have received support and encouragement from many different sources. I would like to take this opportunity to thank some of the individuals and organisations involved, without whom this project would not have been possible.

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Finally, I would like to say a very big “thank you” to my parents for their endless support and encouragement throughout my education.
Nomenclature

\( a \)  
Half length of major ellipse axis

\( a_r \)  
Tan model reference half length of major ellipse axis

\( a_c \)  
Average length of notch edge intense energy region

\( a_0 \)  
Characteristic distance for Average Stress Criterion

\( a_1 \)  
Length of notch edge intense energy region

\( A, B, D \)  
Components of the stiffness matrices

\( b \)  
Half length of minor ellipse axis

\( b_0 \)  
Thickness of longitudinal \((0^\circ)\) ply

\( b_1 \)  
Point strength model characteristic distance

\( b_2 \)  
Minimum strength model characteristic distance

\( B \)  
Single edge notch specimen thickness

\( c \)  
Crack length

\( \Delta c \)  
Crack increment

\( C \)  
Compliance

\( C_0 \)  
Notch sensitivity factor

\( d \)  
Thickness of transverse\((90^\circ)\) ply

\( d_0 \)  
Characteristic distance for Point Stress Criterion

\( d^* \)  
Critical DZC damage zone length

\( E_1 \)  
Young’s modulus of longitudinal lamina in the longitudinal direction

\( E_2 \)  
Young’s modulus of transverse lamina in the longitudinal direction

\( E_m \)  
Young’s modulus of matrix material

\( E_f \)  
Young’s modulus of fibres

\( E_x \)  
Young’s modulus along \( x \) co-ordinate axis

\( E_y \)  
Young’s modulus along \( y \) co-ordinate axis

\( F \)  
Force acting on damage increment

\( G_{xy} \)  
Laminate shear modulus

\( G_{23} \)  
Shear modulus of transverse lamina in the longitudinal direction
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G_c$</td>
<td>Critical energy release rate</td>
</tr>
<tr>
<td>$H_c$</td>
<td>Mar-Lin fracture toughness parameter</td>
</tr>
<tr>
<td>$k_0$</td>
<td>Karlak fracture model constant</td>
</tr>
<tr>
<td>$K$</td>
<td>Kelvin</td>
</tr>
<tr>
<td>$K_c$</td>
<td>Critical stress intensity factor</td>
</tr>
<tr>
<td>$K_T$</td>
<td>Stress concentration factor for a plate of finite width</td>
</tr>
<tr>
<td>$K_T^\infty$</td>
<td>Stress concentration factor for a plate of infinite width</td>
</tr>
<tr>
<td>$K_I$</td>
<td>Strain gauge transverse sensitivity coefficient</td>
</tr>
<tr>
<td>$m$</td>
<td>PWG model exponential parameter</td>
</tr>
<tr>
<td>$m_0$, $m_1$, $s$, $n$, $e$, $f$</td>
<td>Constants for Tan notched strength model</td>
</tr>
<tr>
<td>$M$</td>
<td>Membrane moment resultant</td>
</tr>
<tr>
<td>$M_0$</td>
<td>Magnification factor for finite width correction factor</td>
</tr>
<tr>
<td>$M_f$</td>
<td>Mass fraction of fibre material</td>
</tr>
<tr>
<td>$M_m$</td>
<td>Mass fraction of matrix material</td>
</tr>
<tr>
<td>$M_s$</td>
<td>Mass of sample</td>
</tr>
<tr>
<td>$n_g$</td>
<td>Number of tows between crimps in both orthogonal directions in a balanced fabric</td>
</tr>
<tr>
<td>$n_{fg}$</td>
<td>Number of tows between crimps in the weft direction</td>
</tr>
<tr>
<td>$n_{wg}$</td>
<td>Number of tows between crimps in the warp direction</td>
</tr>
<tr>
<td>$N$</td>
<td>Membrane stress resultant</td>
</tr>
<tr>
<td>$P$</td>
<td>Load</td>
</tr>
<tr>
<td>$P_{\text{max}}$</td>
<td>Maximum load</td>
</tr>
<tr>
<td>$P_Q$</td>
<td>Failure load</td>
</tr>
<tr>
<td>$Q_c$</td>
<td>Poe and Sova toughness parameter</td>
</tr>
<tr>
<td>$R$</td>
<td>Radius</td>
</tr>
<tr>
<td>$S$</td>
<td>Ratio of crack length ($c$) to notch half length ($R$ or $a$) plus crack length for Hitchen model</td>
</tr>
<tr>
<td>$t$</td>
<td>Laminate thickness</td>
</tr>
<tr>
<td>$V_f$</td>
<td>Volume fraction of fibre material</td>
</tr>
<tr>
<td>$V_m$</td>
<td>Volume fraction of matrix material</td>
</tr>
<tr>
<td>$W$</td>
<td>Sample width</td>
</tr>
<tr>
<td>$x$</td>
<td>Distance from laminate centre</td>
</tr>
<tr>
<td>Symbol</td>
<td>Definition</td>
</tr>
<tr>
<td>--------</td>
<td>------------</td>
</tr>
<tr>
<td>$Y$</td>
<td>Finite width correction factor</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Poe and Sova non-dimensionalised constant</td>
</tr>
<tr>
<td>$\Delta \sigma_y$</td>
<td>Difference between the modified normal stress ahead of notch ($\sigma_y^*$) and the normal stress ahead of the notch ($\sigma_y$)</td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>Strain</td>
</tr>
<tr>
<td>$\varepsilon_i$</td>
<td>Damage initiation strain</td>
</tr>
<tr>
<td>$\varepsilon_f$</td>
<td>Failure strain</td>
</tr>
<tr>
<td>$\varepsilon_j^0$</td>
<td>Strain at the geometrical mid-plane of a laminate</td>
</tr>
<tr>
<td>$\hat{\varepsilon}_x$</td>
<td>Uncorrected strain along $x$ axis obtained from a strain gauge</td>
</tr>
<tr>
<td>$\hat{\varepsilon}_y$</td>
<td>Uncorrected strain along $y$ axis obtained from a strain gauge</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Ratio of distance ahead of notch to major ellipse half length</td>
</tr>
<tr>
<td>$\eta$</td>
<td>Poe and Sova multiplication factor</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>Ellipse opening aspect ratio</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>Curvature about mid-plane of laminate axis</td>
</tr>
<tr>
<td>$\nu$</td>
<td>Crack opening displacement</td>
</tr>
<tr>
<td>$\nu_0$</td>
<td>Poisson’s ratio of material on which strain gauge is calibrated</td>
</tr>
<tr>
<td>$\nu_{xy}$</td>
<td>Poisson’s ratio</td>
</tr>
<tr>
<td>$\nu_c$</td>
<td>Critical crack opening displacement</td>
</tr>
<tr>
<td>$\pi$</td>
<td>Pi</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Notch root radius</td>
</tr>
<tr>
<td>$\rho_f$</td>
<td>Density of fibres</td>
</tr>
<tr>
<td>$\rho_m$</td>
<td>Density of matrix</td>
</tr>
<tr>
<td>$\sigma_y$</td>
<td>Normal stress ahead of notch</td>
</tr>
<tr>
<td>$\sigma_y^\infty$</td>
<td>Normal stress ahead of notch for a plate of infinite width</td>
</tr>
<tr>
<td>$\sigma_y^*$</td>
<td>Modified normal stress ahead of notch</td>
</tr>
<tr>
<td>$\bar{\sigma}_y$</td>
<td>Remote applied stress</td>
</tr>
<tr>
<td>$\sigma_0$</td>
<td>Unnotched laminate strength</td>
</tr>
<tr>
<td>$\sigma_N$</td>
<td>Notched laminate strength</td>
</tr>
<tr>
<td>Symbol</td>
<td>Definition</td>
</tr>
<tr>
<td>--------</td>
<td>------------</td>
</tr>
<tr>
<td>$\Sigma Sq$</td>
<td>Sum of square of errors</td>
</tr>
<tr>
<td>$\xi_1$</td>
<td>Ratio of notch half length (R or a) to notch half length plus the point stress criterion characteristic distance ($d_0$)</td>
</tr>
<tr>
<td>$\xi_2$</td>
<td>Ratio of notch half length (R or a) to notch half length plus the average stress criterion characteristic distance ($a_0$)</td>
</tr>
<tr>
<td>$2s$</td>
<td>Average crack spacing of a uniform array of cracks</td>
</tr>
<tr>
<td>$\infty$</td>
<td>Infinity</td>
</tr>
</tbody>
</table>
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Chapter 1: Introduction
1.1 Background

The term composite material was originally used to describe a structure in which a second material was used to improve the performance of a component. For example, cannon barrels were defined as composite materials as the primary wooden structures were bound with brass so that they were able to stand the internal pressure (Kelly, 1989). However, modern engineering composite materials are generally homogeneous on a macroscopic scale, appearing as one continuous structure to the naked eye. Only on a finer scale do they consist of two or more distinct phases. One definition of a composite material is, “a multi-phase material in which the phase distribution and geometry have been controlled in order to optimise one or more properties”, (Bader, 1996).

Although generally considered man-made, the concept of composite materials can be found in numerous examples of nature. Wood, an excellent engineering material, is a composite material, made up from crystalline cellulose fibres in a matrix of amorphous lignin and semi-crystalline hemicellulose (Ashby, 1986). Wood-based structures can be used to illustrate some of the properties and characteristics of modern composite materials. The fibrous cellulose provides the stiffness and strength in the structure. Hence, it is usually aligned parallel to the trunk and branches. This results in a highly anisotropic structure in which the high beam bending stresses along the axial directions are efficiently accommodated. The anisotropy of wood can be minimised by bonding the unidirectional layers together at various orientations to form plywood, a lamellar composite. Other examples of naturally occurring composite materials include bone, teeth and some geological formations.

An early example of a man-made composite is the mud and straw brick, dating back to around 1000 BC. Laminated archers' bows and wattle and daub buildings are further examples of man's early understanding of the benefits of multi-phase materials.

The development of modern engineering composites began around the 1940s with glass fibre reinforced polyester resins. Fibre reinforced composite materials are now found in many engineering applications ranging from consumer sports goods, such as golf
club shafts, to primary structures in the aerospace industry. The most common type of reinforcement used nowadays is based on layers of continuous unidirectional fibres stacked at various orientations to produce laminated structures. There are, however, many forms of reinforcement currently available, such as discontinuous short fibre mats, conventionally woven fabrics and non-crimp, braided, knitted and three-dimensional woven fabrics. The materials investigated in this project are two-dimensional woven fabric reinforced polymer matrix composites.

Resin impregnated fabric reinforcements woven from stiff fibres are now utilised in many modern load bearing engineering structures such as aeroplane fuselage components. Woven fabric structures go back to the very beginning of the aerospace industry. The wooden frames of the Wright brothers' machines were covered with a linen fabric and painted with a 'dope' to provide a light, aerodynamic material (Bailie, 1989). Doped fabric remained common in the aerospace industry up until the Second World War. The next major use of fabric composites were glass/polyester radomes, which were transparent to radar, enabling the operation of Second World War radar sets. The transition to primary composite structures was facilitated by the development of stiff engineering fibres, such as carbon, boron and Kevlar.

Most composite literature is based upon unidirectional tape structures as they undoubtedly offer higher strength and stiffness, when compared to fabric composites. However, fabric composites do offer some specific advantages. They have very good drapability, especially the common eight harness satin weave. This allows them to conform to complex geometries such as doubly curved surfaces. Manufacturing costs can be reduced as one layer of cloth replaces two unidirectional plies, resulting in easier handling and shorter lay-up times. Woven fabric composites have better impact resistance as the nature of the weave localises the damage, by arresting cracks and delaminations at sites of fibre-fibre interaction. Also, a major advantage of woven fabrics is their design flexibility, which allows them to be tailored to specific applications by choice of weave style and yarn and fibre characteristics.

In order for woven fabric reinforced composites to be fully exploited as primary structures, their behaviour must be more thoroughly understood. This thesis aims to
address some aspects of woven composite behaviour that require further investigation, as discussed below.

1.2 Scope of Thesis

The type of damage accumulated and the ability of a structure to perform after the onset of damage must be clearly understood by designers of composite components. It has been shown that the accumulation of damage in woven composite structures can be very complicated, differing greatly from that observed in non-woven composites (Marsden, 1996). However, this requires further characterisation for the range of weave styles and material systems available. The prediction of laminate properties, such as stiffness and strength, considering the effect of progressive damage, is complicated by the complex fibre architectures and damage morphologies found in woven composites. A more detailed understanding of the failure processes involved will facilitate the development of models with the ability to take these effects into account. Also, in order to have confidence in the structural integrity of woven composite components, the designer must possess a good understanding of the effect of stress concentrations around design features such as cut-outs. This should involve not only the ability to predict the strength of such structures, but also a good understanding of the role of the fabric structure on damage development and subsequent failure.

Aspects of the behaviour of woven composite materials relevant to the current project are discussed in the literature review presented in Chapter 2 of this thesis. The terms used to describe and characterise woven fabric architectures are initially discussed, followed by the definition of several common weave styles. One problem associated with woven composites is the effect of variations of the fibre architecture on the mechanical response of the structure. The literature review presents work in which the thermo-elastic behaviour of woven composites has been modelled. The development of damage and effect of this damage on mechanical properties are also discussed. The literature review then discusses the notched failure of composite laminates. A large range of predictive models are presented, along with experimental
programmes in which the notched strength of a variety of composite materials have been examined.

This project investigates the failure of notched woven composites. In particular, this thesis aims to provide a comprehensive study of notched laminate properties, notch edge damage and predictive modelling for a range of woven composite laminates and notch geometries. The styles of woven glass fibre reinforcements investigated are presented in Chapter 3, together with the experimental techniques employed during this project.

The damage development in an unnotched plain weave (PW) laminate is characterised in Chapter 4. This work, together with a similar study for an eight harness satin weave composite performed by Marsden (1996), enabled the damage found adjacent to notches to be understood more clearly. A shear-lag model, based on an equivalent cross-ply laminate, has been used predict the effect of this damage on laminate stiffness.

Chapter 5 presents unnotched and notched laminate properties for both PW and eight harness satin (8HS) weave laminates. The effect of the number of fabric layers within a laminate and notch size and shape are investigated. Using a range of techniques, the damage observed adjacent to the various notch configurations in the current woven composites is discussed in Chapter 6. The comprehensive study performed allows the influence of the notch edge damage on the laminate properties to be discussed. In Chapter 7, four notched failure criteria are applied to the current woven laminates. The way in which the different models represent the failure of the notched laminates are compared to the experimental observations of the damage zones. Also, the extent of the experimentally observed notch edge damage is compared to the size of the damage zones predicted by the notched failure criteria.

Finally, Chapter 8 presents the conclusions reached during this project and suggestions of further work for subsequent research in this area.
Chapter 2: Literature Review
2.1 Introduction

In recent years there has been a revival in the interest of textile composites for structural engineering applications. The selection of woven fabric composites for applications in areas such as the aerospace industry has often been based upon pragmatic factors such as manufacturing economics, geometrical considerations and damage tolerance. Hence, the volume of literature surrounding woven composites is not in line with their considerable usage. In order to exploit fully the advantages of woven fabric-based composites there is a strong need for all aspects of laminate behaviour to be understood and sound engineering design methodologies to be established. This has been the impetus behind much of the work concerning the elastic behaviour of woven composites (e.g. Ishikawa and Chou, 1982a). Over recent years more detailed analyses have been developed to predict woven composite laminate behaviour, including models capable of considering the effects of damage. Some of the failure criteria employed for woven composites, such as notched strength models, use existing criteria developed for non-woven composites (e.g., Whitney and Nuismer, 1974). In many cases these models ignore the details of laminate reinforcement and can be applied directly to woven composites. However, the validity of these approaches for use with woven composites must still be verified.

In order to be able to describe the literature relevant to the scope of this thesis this review begins by defining the terms used to characterise woven fabrics. Some of the common types of weave architectures used in modern engineering composite materials are subsequently discussed. The remainder of the review consists of two main areas of discussion. The first of these covers the behaviour of woven fabric composites under quasi-static loading. The various attempts that have been made to model the thermoelastic behaviour of woven fabric composites are considered. Studies investigating the damage development in woven composites and its effect on the elastic properties of laminates are then covered. The second topic is a review of work concerning the notched strength of composite laminates. This involves reviewing many of the notched strength failure models as well as discussing experimental investigations into notched composite laminate behaviour. Throughout this chapter the theoretical modelling approaches reviewed are compared to experimental data whenever possible.
2.2 Woven Fabric Reinforcements

There are many types of fabric reinforcements available for use in composite materials. These include two-dimensional (2D) woven, three-dimensional (3D) woven, knitted, braided and stitched non-crimp fabrics. The materials investigated in this study are 2D woven E-glass/epoxy composites and hence this discussion is based upon 2D woven fabrics. Before the various styles of weave are discussed, the terms used to describe fibre and weave architectures should be defined.

A 2D woven fabric consists of 2 sets of mutually orthogonal yarns or tows. The tows aligned in the weave, or roll, direction are termed warp tows. Conversely, the tows aligned across the weave direction are termed weft or fill tows. Care must be taken when describing a woven fabric composite laminate as the warp direction is not always defined as the principal axis. The term crimp refers to regions in a tow where curvature is present to enable it to pass under or over another tow orientated in the opposing orthogonal direction.

A woven cloth can be characterised by several parameters and definitions. The weight, normally expressed in g m⁻², and the thickness, quoted under a specified compressive load, are relatively crude cloth measurements. The yarns from which the cloth is woven can be described by a tex value which is the weight in grams of 1000 m of the tow. The terms ends and picks describe the number of warp tows and weft tows, respectively, per unit length. A cloth is described as balanced if the number of ends is equal to the picks. Cloths are often marginally unbalanced due to different levels of deformation of the 2 sets of yarns during the weaving process. The degree of imbalance of a fabric can be tailored to suit desired laminate properties. For example, if a laminate requires twice the stiffness in the longitudinal direction as opposed to the transverse direction then the number of ends required would be approximately twice the number of picks. Another method of obtaining superior laminate properties in one of the orthogonal directions is to use different fibres in the warp and weft tows. Such a fabric is described as hybrid. A hybrid fabric may also contain different types of fibre in the same orthogonal direction or even co-mingled into the same yarn. A woven
fabric may be identified by the repeating pattern formed by the interlaced or crimp regions. The common weave types are described below. They are described using the notation developed by Chou and Ishikawa (1989). The term $n_{fg}$ denotes that a warp thread is interlaced with every $n_{fg}^{th}$ fill thread. Similarly, the term $n_{wg}$ denotes that a fill thread is interlaced with every $n_{wg}^{th}$ warp thread. In most cases $n_{fg} = n_{wg} = n_{g}$.

In a plain weave (PW) cloth $n_{g} = 2$. A fill yarn is interlaced with every other warp yarn. Twill weaves have $n_{g} = 3$, i.e. a fill yarn is interlaced with every third warp yarn. In a satin weave $n_{g} \geq 4$ and the interlaced regions are isolated from one another. Common satin weaves are 5 harness satin (5HS) and eight harness satin (8HS), where $n_{g} = 5$ and 8, respectively. Some of the common weave styles are illustrated in figure 2.1.

These different weave styles result in fabrics with varying physical characteristics. An 8HS weave cloth produces a slightly stiffer laminate when compared to a PW cloth assuming the same volume fraction. This is due to the lower degree of crimp present in an 8HS fabric. An 8HS weave cloth is also more drapable than an equivalent PW cloth. Drapability is defined as the ability of a cloth to conform to complex shapes, such as doubly curved surfaces. The lower degree of interlacing in an 8HS weave cloth results in more freedom of movement in the orthogonal tows compared to a PW cloth. However, a PW cloth may result in a laminate with some advantageous properties, for example, superior dropped weight impact resistance. The high density of interlaced regions localise impact damage as cracking is arrested at the warp/weft crossover points.

2.3 Elastic Behaviour and Damage Development in Woven Composite Materials

2.3.1 Introduction

Much of the literature on composite materials is concerned with continuous (non-woven) laminates. Many aspects of the behaviour of these materials are now well understood, with appropriate models available to predict their thermo-mechanical behaviour. However, the literature concerning woven fabric composites is more
limited. This section initially discusses the prediction of undamaged woven laminate properties, followed by the effect of relative shift of adjacent laminae on laminate mechanical behaviour. Finally, the damage development under quasi-static loading and the modelling of subsequent property degradation is discussed.

2.3.2 Elastic Property Modelling

The theoretical modelling of the initial undamaged elastic behaviour of woven composite laminates has been investigated quite extensively. The published work ranges from simplified one-dimensional representations to computationally extensive three-dimensional models. It is convenient to consider the models discussed in two sections; closed form analyses and finite element (FE) methods.

2.3.2.1 Closed Form Analyses

Notable early closed form analyses are those of Ishikawa and Chou (1982a) with their one-dimensional (1D) models. Ignoring temperature effects, the starting point for the analysis is based upon classical laminate plate theory (LPT) (Chou and Ishikawa, 1989), as follows:

$$
\begin{pmatrix}
N_i \\
M_i
\end{pmatrix} =
\begin{bmatrix}
A_{ij} & B_{ij} \\
B_{ij} & D_{ij}
\end{bmatrix}
\begin{pmatrix}
\varepsilon_j^0 \\
\kappa_j
\end{pmatrix}
$$

(2.1)

where $N_i$ and $M_i$ are membrane stress and moment resultants, $\varepsilon_j^0$ and $\kappa_j$ indicate strain and curvature at the geometrical mid-plane of the laminate. The terms $A_{ij}$, $B_{ij}$ and $D_{ij}$ are the components of the stiffness matrix. The upper and lower bounds of the predicted thermo-elastic constants can be found by assuming iso-strain and iso-stress conditions in the laminate, respectively, in the LPT analysis.

The first model is the mosaic model which considers the laminate as an assemblage of cross-ply laminae ignoring fibre undulation and continuity (figure 2.2). The two dimensional form of the laminae were reduced to a one dimensional structure in one of
two ways. The parallel model assumes a constant strain in the laminate and this results in the upper bound of the in-plane stiffness constants. The series model assumes a constant stress in the laminate and this results in the lower bound for the in-plane stiffness constants. In the words of the author, the mosaic model "provides a convenient and rough estimate of the thermo-elastic properties" of woven fabric composite materials.

In a slightly more advanced approach the crimp (or fibre undulation) model considers fibre undulation and continuity in the direction of loading with the use of a sinusoidal function and is particularly suitable for plain weave fabrics. Figure 2.3 shows this one dimensional model consisting of the sinusoidally undulating fill thread, the lenticular warp thread and surrounding resin rich region. The model is an extension of the series model. Laminate theory is applied to each infinitesimal slice, of length $dx$, perpendicular to the loading direction, $x$. The in-plane elastic properties predicted by this model are lower than those predicted by the mosaic model as crimp is taken into account. Hence, it is reported that the thermo-elastic properties are in better agreement with experimental data.

The mosaic model and crimp model both consider a simplified one-dimensional fabric strip, resulting in relatively inaccurate predictions. The bridging model, developed for satin weaves, considers that the isolated interlaced region in the unit cell has a lower stiffness than the surrounding straight thread regions. These surrounding areas act as bridges for load transfer around the interlaced region. The hexagonal repeating unit in an 8HS weave is shown in figure 2.4a, which is simplified to a square for the idealised unit cell in figure 2.4b. The bridging model combines both the series and parallel models. The concept of the load transfer is shown in Figure 2.4c. The isolated crimp region C has a lower in-plane modulus and, therefore, carries less load than regions A, B, D and E. Regions B, C and D are assembled assuming an iso-strain condition in the laminate. This model takes fibre undulation into account in section C using the approach developed for the crimp model. Regions A, B-C-D and E are then assembled using an iso-stress condition.
Figure 2.5 shows the relationship between non-dimensionalised in-plane stiffness and \(1/n_g\) for the above-mentioned models. The non-dimensionalised stiffnesses were obtained by normalising the in-plane stiffnesses to those of the cross-ply laminate, in which \(1/n_g\) is zero. The upper and lower bounds of the predictions shown correspond to the parallel and series versions of the mosaic model. This illustrates the very large discrepancy in these models at low \(n_g\) values. The crimp model also results in a large range of experimental predictions. Good agreement was reported between the bridging model and experimental data (Ishikawa et al., 1985). The abbreviations LWC and LWA in figure 2.5 represent the cases where local warping is constrained and allowed, respectively. Local warping refers to the local out-of-plane deformation that that may be induced in a woven fabric by an in-plane force. The LPT analysis performed does not suppress these local deformations. However, in reality this warping is often constrained to some extent by transverse yarns and adjacent fabric layers. It can be seen that the limited experimental results fall within these limits, although the difference between the predictions is large for fabrics with low \(n_g\) values. The presented models have also been developed to predict the thermo-elastic properties of composites made from hybrid woven fabrics (Ishikawa and Chou, 1982b).

The 1D models presented so far do not take into account the elliptical cross section of the longitudinal yarns, the undulation of transverse yarns or the presence of a gap between adjacent yarns, as often seen in fabrics with low \(n_g\) values. This results in an unrealistic representation of the fabric structure. Raju and Wang (1994) reported improved classical LPT models for PW, 5HS and 8HS weaves based on the crimp and fibre undulation models of Ishikawa and Chou (1982a). Although still essentially 1D, the models are able to take into account the cross-sectional shape and undulation of yarns in the transverse y-direction and the presence of a gap between adjacent yarns. The rectangular repeating units of the different weaves are simplified to unit cells. These, in turn, are divided into regions and each region is further divided into sub-regions. These sub-regions now consist of 1D cells with fibre undulation in only one direction. The resulting unit cell is then able to be used to calculate the laminate thermo-mechanical properties in both orthogonal directions without the structural simplifications involved in the previously discussed analyses. The plain weave laminate is split into 4 regions, each containing 4 sub-regions. The laminate is then
reassembled and, assuming that classical LPT is valid, the thermo-elastic properties are predicted using a methodology similar to those of Ishikawa and Chou (1982a). Although not extensively investigated, the modulus and Poisson’s ratio predictions were said to be in general agreement with other available models and experimental data. The performance of this model is difficult to assess as the comparisons with experimental data are not shown. However, the coefficient of thermal expansion data was reported to be unreliable.

Naik and Shembekar have developed two 2D models for plain weave composites (Naik and Shembekar, 1992a, 1992b and Shembekar and Naik, 1992), also based upon laminate theory. They are an extension of the previously discussed 1D models. They represent the actual fabric structure more accurately as they consider fibre undulation and yarn continuity in both the warp and weft directions without sub-dividing the unit cell as in the model of Raju and Wang (1994). They can also take into account the effect of shifting the cloth layers relative to each other and the presence of a gap between adjacent yarns. Figure 2.6 shows the unit cell of a plain weave lamina. The models use similar shape functions to those used by Ishikawa and Chou (1982a) to describe yarn undulation.

The models are characterised by the way in which the unit cell is discretised and considered in the analysis. The lamina is subjected to an in-plane tensile stress along the x-axis. The infinitesimal pieces of a section parallel to the loading direction are in series with respect to the loading axis and are assumed to be under an iso-stress condition. This assembly is termed a series model. The infinitesimal pieces of a section perpendicular to the loading direction are in parallel with respect to the loading axis and are considered to be under an iso-strain condition. This assembly is termed a parallel model.

The analysis was approached in one of two ways. Firstly, in the Series-Parallel (SP) model all the infinitesimal pieces of sections parallel to the loading direction are assembled with an iso-stress condition. Then all of these sections parallel to the loading direction are assembled with an iso-strain condition. In the Parallel-Series (PS) model all the infinitesimal pieces of sections perpendicular to the loading direction are
assembled with an iso-strain condition. These sections perpendicular to the loading direction are then assembled with an iso-stress condition. This is achieved numerically by integrating the in-plane stiffness constants across the loading direction and integrating the compliance constants perpendicular to the loading direction, and assembling the models as described above. The SP model and PS model give the lower and upper bounds of the in-plane stiffness constants, respectively.

The elastic constants predicted by the SP and PS models showed good agreement with each other. Good agreement with experimental results was also reported by the authors. However, on closer inspection the data given for the prediction of Young’s modulus of carbon/epoxy and E-glass/epoxy plain weave laminates show errors ranging from approximately 7 to 25 %. The analysis showed that the optimum inter-yarn gap with respect to laminate modulus depends on the material system and fabric structure. However, a closed weave, with no gap, often gives better results. The effect of relative lamina shift with respect to the models will be discussed later.

Naik and Ganesh (1992) presented two, 2D LPT based models for the prediction of on-axes elastic properties of plain weave fabric composites. The unit cell is represented by one quarter of the interlaced repeating unit of a plain weave fabric, possible due to its symmetry.

The slice array model (SAM) discretises the unit cell into a number of slices parallel to the loading direction and analyses them separately by idealising them into asymmetric cross-ply laminates. It was found that dividing the unit cell into 50 slices was satisfactory. Considering undulation, the elastic properties of the individual layers are used to obtain the elastic constants of the idealised laminate slices by using appropriate trigonometric functions. The unit cell is then re-assembled with an iso-strain condition to obtain the elastic properties. In the element array model (EAM) the unit cell is discretised either parallel to or perpendicular to the loading direction into 50 slices. These slices are then subdivided into 50 further elements. These are re-assembled in one of two ways by a similar method to the one used by Naik and Shembekar (1992a) to create both series-parallel (SP) and parallel-series (PS) models.
Of the three material systems investigated, experimental Young's modulus data was reported for carbon/epoxy and E-glass/epoxy plain weave laminates. Although the experimental investigation was limited, good correlation was reported between predicted and experimental results, with agreement being within approximately 5%. The SAM predicted slightly higher elastic moduli than the slightly more complex EAM for all material systems investigated. The EAM-PS configuration was preferred as it does not include local bending deformations which are constrained in plain weave laminates. Modifications to the simple mosaic parallel model (Chou, 1982a) were also suggested.

The 2D analyses presented so far discretise the unit cell into elements and slices, and involve substantial computation. Naik and Ganesh (1995) proposed a simple, accurate closed form analysis for thermo-elastic property prediction of plain weave composites. The unit cell considered consists of one quarter of the repeating interlaced region. This unit cell was modelled as an asymmetric three layer cross-ply laminate, consisting of pure matrix, warp yarn and fill yarn. This approach is similar to the SAM but does not involve sub-dividing the unit cell into smaller elements. The thermo-elastic properties of the idealised cross-ply laminate need to be determined to obtain the thermo-elastic properties of the woven fabric lamina. The thermo-elastic properties of the strands forming the woven laminate are required to determine the thermo-elastic properties of the idealised cross-ply laminate. The warp and fill strand cross section and undulation are described by suitable sinusoidal shape functions. This enables strand thickness and undulation angle to be determined. The local reduced compliance constants can then be calculated, which are averaged over the length of the strand to give the effective strand compliance. The variation in inclination angle is considered to be sinusoidal or, as an approximation, linear, resulting in a circular path. The thermo-elastic properties are calculated for the idealised cross-ply laminate, based on the effective thermo-elastic properties of the undulated strands, using classical LPT.

For the twelve woven fabric systems investigated there was a good correlation between predicted and experimental data, comparing favourably with the EAM (Naik and Ganesh, 1992). The circular path generally gave slightly higher modulus predictions
than the sinusoidal path, especially at higher strand thickness/strand width ratios. As the sinusoidal path represents the yarn shape more accurately, it was thought more realistic.

Although relatively simple compared to finite element analyses the previously described models are not always particularly easy to implement. Vandeurzen et al. (1996a, 1996b) proposed a three-dimensional analysis for woven fabric composites based upon a library of 108 cells from which a wide variety of woven laminae unit cells can be built. The geometric analysis uses several parameters to describe a woven fabric which are implemented in a Microsoft Excel® application called TEXCOMP. The authors claimed that this provides a user-friendly, versatile design tool, although this was not demonstrated in the literature. The prediction of laminate properties is based upon the fabric geometry model (FGM) (Chou and Ko, 1989). This involves treating each yarn in the unit cell as a unidirectional lamina. The contributions from each of these laminae are combined using an iso-strain condition. A classical LPT analysis is used to predict laminate properties.

An improved FGM was developed for the TEXCOMP application. This involves modelling each yarn system with a simplified separate fibre and matrix layer and is termed the combi-cell model (CCM). The models have only been compared to limited Young’s modulus experimental data, hence, it is difficult to assess the performance of this approach. They have also been compared to a FE approach based upon the above analysis. It was reported that the models agreed well, except that the FGM approach over-predicts shear moduli.

Another micro-mechanical model has been proposed by Scida et al. (1997) called MESOTEX. This approach considers plain, twill and satin weaves as well as hybrid composites. It is based upon LPT and takes fibre undulation in both directions into account. The unit cell of the weave to be considered is identified and discretised into an assemblage of infinitely small pieces of uni-directional laminae. The model does not state the way in which the unit cell is re-assembled, as in the PS and SP models of Naik and Shembekar (1992a). It is stated that classical thin laminate theory is applied to each element producing the elastic properties of the lamina. Of the four material
systems investigated, good agreement with experimental elastic property data was reported. It is claimed that this was achieved in very fast computation times.

The closed form analyses presented vary in complexity. Although good correlation with experimental data is reported there is not currently a single approach available that is able to predict the thermo-elastic properties accurately for all of the common 2D woven composites. The models generally become more accurate with increase in complexity, predominantly due to a more accurate representation of the weave architecture.

2.3.2.2 Finite Element Methods

The closed form models discussed in the previous section may predict overall thermo-elastic properties satisfactorily but they are too crude for detailed local stress analysis. The analysis of textile composites using finite elements is able to provide this detailed stress analysis. However, FE methods are complicated due to the complex fibre architecture and require large computer memory and processing time. A number of FE models for fabric composites have been developed. Some of these are discussed below.

Three dimensional modelling of woven composites requires very large computational effort but still involves many simplifications and assumptions. Whitcomb (1991) used a three dimensional (3D) FE analysis to study plain weave composites. Using a large number of twenty noded isoparametric brick elements, the strain concentration, elastic modulus and Poisson's ratio were reportedly predicted successfully.

In practice, a woven composite structure will be several plies thick and the number of elements required soon becomes unfeasibly large. Whitcomb et al. (1994) proposed a displacement based finite element which accounts for the spatial variation in properties within the element. The stresses calculated inside the macro element are not accurate. However, the modelling of the global deformation behaviour is successful, with a reduction in the number of elements required when compared to traditional FE models. This analysis used only 2D models, but the theory is valid for 3D modelling.
Developing the use of macro elements Woo and Whitcomb (1994) proposed a global/local FE analysis for textile composites. Macro elements were used in the form of a relatively crude global mesh to obtain the overall response of the structure. Conventional finite elements were used as refined local meshes for local analysis at areas of interest where large stress variations were thought to exist. It was reported that this method results in a detailed local analysis where required with a dramatic reduction in computational time and only a relatively small loss in accuracy.

None of the models previously discussed consider the effect of a free surface boundary on the elastic properties and stress distributions within a unit cell of a woven composite laminate. The qualitative 2D analysis performed by Whitcomb *et al.* (1995) suggested that both stiffness and stress distributions were affected by boundary effects. A specimen thickness of twelve fabric layers was required to obtain a comparable Young’s modulus to thick specimens. The boundary effects seen were local to the surface and insensitive to the specimen thickness.

Glaessgen and Griffin (1994) proposed a FE technique for the modelling of mechanical and thermal loading of textile composites. Local details of stress, strain and failure parameters were determined. The textile geometry model (TGM) of Pastore *et al.* (1995) was used to represent the unit cell of a plain weave fabric. The yarns were assumed to be circular in cross section but were able to deform due to the crimp present in woven preforms. The fabric was represented by hexahedral elements. This approach resulted in a large model size and long computational times. The effect of geometrical and material parameters on the response of the composite were investigated. However, no mention was made of the performance of the model with respect to experimental data.

Marrey and Sankar (1997) proposed a FE based micro-mechanical model for textile composite plates. Plate stiffness constants and thermal expansion coefficients were predicted by modelling the composite as a homogeneous plate as opposed to a three-dimensional structure consisting of multiple unit cells in three dimensions. It was proposed that thin composite plate properties cannot be inferred from elastic constants
derived from larger 3D structures. This was shown to be valid with the plate stiffness coefficients computed from direct micro-mechanics being more accurate.

It has been shown that there are many different ways to model the thermo-elastic response of woven composites using FE techniques. Some of these models are able to reduce the complexity of the analysis while still retaining acceptable accuracy. The analysis of the thermo-elastic response of woven composites using finite elements is a powerful tool if a detailed stress analysis is required. However, if the macroscopic response is required the closed form models are much more attractive due to their relative simplicity.

2.3.3 Effect of Relative Lamina Shift

The term "relative lamina shift" is used to describe the horizontal movement in the plane of the laminate of one lamina layer with respect to another. When fabricating a multi-layer woven composite laminate there are countless combinations of relative lamina shifts possible. Modern tape laying machines are capable of positioning laminae accurately. However, the fine structure of some weaves and the tendency for others with low $n_e$ numbers to nest upon each other makes it very difficult, if not impossible, to lay up laminates precisely as required. Several authors have studied the stacking configuration of woven composites and its effect on laminate properties.

The models developed by Shembekar and Naik (1992) discussed earlier showed that elastic properties may be tailored by variation of the relative shift of the fabric laminae. Four different 4 layer laminate configurations with varying relative lamina shifts were investigated, as shown in figure 2.7. As in the case of the analysis of the woven fabric laminae discussed in section 2.3.2.1, the laminate analysis was also based upon the assumption of the validity of classical LPT. Optimum elastic properties were predicted for laminae configurations shifted with respect to each other. The extent of the elastic property variation between different lamina shift configurations was found to depend on the lamina thickness to yarn width ratio and undulation to yarn width ratio. For a constant undulation to yarn width ratio the elastic moduli were found to decrease as lamina thickness increased. For higher lamina thicknesses certain shifted
configurations were found to give 10-60% higher elastic moduli depending on undulation to yarn width ratio when compared to other configurations. It was also found that elastic properties could be further enhanced by increasing the number of fabric layers with appropriate shifts until a basic building block was reached. It was proposed that this block can then be repeated to build thicker laminates with optimum elastic properties. However, it should be considered that the fabrication of such precise aligned laminates is, in practice, very difficult to achieve.

Aboura et al. (1993) modelled the superposition of two plain weave fabric layers using an analysis based upon the models of Ishikawa and Chou (1982a). The model showed satisfactory correlation between predicted and experimental stiffness data. Three different relative shifts of the fabric layers were studied and the elastic moduli at 12 points within the unit cell along both the x and y axes were investigated. The limited model results presented suggested that variations in predicted local stiffness properties of up to 25% existed throughout the laminate.

It is not only laminate stiffness that has been investigated with respect to relative lamina shift. The simplified two-dimensional analysis of Yurgartis and Maurer (1993) investigated the effect of weave configuration, stacking sequence and relative lamina shift on interlaminar delamination shear strength. The two fabric layer model developed specifies the weave geometries of the top and bottom plies, their orientation and their relative shift with respect to each other. The principle of the model is shown in figure 2.8. The interlaminar shear stresses were estimated by applying a fixed displacement to the top layer of the model. The transverse yarns were divided into slices perpendicular to the direction of shear displacement. The shear strain of each slice was then estimated from the longitudinal yarn displacement and the local transverse layer thickness. The model suggested that stacking like faces of a cloth together reduced delamination resistance. Certain lamina shifts appeared to reduce and distribute the shear stress concentrations, present due to the interaction of crimp regions, more evenly than others. The results of the modelling also suggested that stacking of different weave styles within the same laminate resulted in poor shear delamination resistance.
The effect of layer nesting on the compressive strength of 5 harness satin woven composites has been investigated by Breiling and Adams (1996). Three idealised nesting configurations were investigated, termed stacked, split-span and diagonal. These are shown schematically in figure 2.9. Randomly nested specimens were also investigated. It was found that nesting configuration had little effect on compressive laminate stiffness. However, all configurations showed a reduction in compressive strength compared to the randomly nested laminate, especially the diagonal nesting configuration. A FE analysis predicted similar trends, although compressive strength was over predicted. The FE analysis predicted longitudinal compressive fibre failure for all cases, although this was unable to be determined experimentally due the catastrophic nature of the failure.

As discussed above, yarn shape and relative lamina shift influence woven composite properties and are important parameters for elastic property modelling and quality control. Measuring these parameters can be time consuming and tedious to perform. Yurgartis et al. (1993) used computer aided image analysis to efficiently quantify yarn shape and cloth nesting in plain weave composites, resulting in reproducible data in a reasonable time-scale. Yarn shape was quantified by inclination angle, the angle of the mean yarn direction to the horizontal and by crimp angle, the steepest angle between peak and trough. Nesting was quantified by angle match, the difference in inclination angles between adjacent yarns. It was shown that there is a statistical variability of yarn shape, that sinusoidal functions may not fit yarn shape accurately and that yarn shape may change during consolidation.

It has been shown in this section that relative lamina shift can affect a range of laminate properties. Although difficult to control during laminate fabrication, this lamina shift is a factor that should be taken into account when considering the mechanical response of a woven composite.

2.3.4 Damage Development in Woven Composite Materials

Gaining an understanding of the damage evolution in composite materials is an important pre-cursor to modelling the effect of this damage on laminate properties and
the subsequent failure of laminates. The development of damage in cross-ply laminates has been extensively investigated, as discussed by Crocker (1998). However, the literature concerning damage development in woven fabric composite materials is somewhat less extensive.

Roy (1994) studied the in-situ damage development in 4 layer 8HS weave carbon/epoxy laminates. The sample edges were polished to enable the damage to be monitored during incremental loading. The cloths were laid up in 2 different configurations. In the first of these the 4 layers were stacked one on top of the other represented by (0)_{4T}. The second configuration consisted of the 4 layers of cloth stacked symmetrically, represented by (0)_{2s}. The initial damage development was not discussed in great detail. It was noted that the damage was not consistent throughout the laminate cross-sections. This was attributed to the variation in local stress fields due to yarn nesting. The authors postulated that in order to predict damage development and failure mechanisms in woven fabric composites accurately the stress analysis of the full laminate thickness is required. This is perhaps an over-ambitious concept considering the complexity of some the current approaches for modelling damage in woven composites that consider only the fabric unit cell.

Marsden (1996) characterised the damage in an 8HS weave glass/epoxy composite under both quasi-static and fatigue loading. Damage development was found to be a function of tow and weave architecture. Fabrics with both twisted and untwisted tows were investigated. The tows termed untwisted actually contained a small degree of twist, whereas the tows termed twisted consist of three finer bundles twisted fairly tightly together. Under quasi-static loading, the damage in laminates fabricated from cloths woven from untwisted tows was found to initiate at approximately 1.2 % strain, in the form of transverse cracks. The cracks extended to several millimetres in length, often extending across the full laminate width. Edge sections revealed that the damage grew through both the 90° bundles and the neighbouring resin rich region. The initial damage was seen to pass through a region adjacent to the interface of the tow end and pure matrix region for each case studied. The damage found in the laminates fabricated from cloths woven from twisted tows was quite different. The transverse cracking initiated at approximately 0.6 % strain. The damage was found to be diverted by the
twist within the tows resulting in short cracks of approximately 2-3 millimetres in length aligned at approximately 7° to the transverse tow. These short transverse cracks were seen to form a staircase-type pattern, corresponding to the repeat pattern of the interlocking crimp regions of the fabric. Edge sections revealed that the transverse cracking was contained within the tows. Again, the initial damage was found adjacent to the interface of the tow ends and the pure matrix region. Damage under fatigue loading was similar with the addition of delaminations located at the interlocking crimp regions at higher numbers of cycles.

Gao et al. (1999a) investigated the development of damage in 8HS woven carbon fabric composites. Damage was observed in the form of transverse matrix cracking, delamination and longitudinal splitting. Transverse cracking was the first damage type to initiate. Short cracks initiated at low strain levels, spanning the entire fibre tow width. At higher strains the transverse cracks spanned the full sample width. The transverse cracking was similar in nature to that seen in non-woven composites. At high levels of strain delaminations were also seen to initiate at the intersection of transverse cracks and crimp regions of longitudinal tows. In fractured coupons approximately half of crimp regions had an associated delamination. Longitudinal splitting was seen to initiate soon after the onset of transverse cracking only in the surface layer of laminates. These splits remained fairly constant in length, approximately equivalent to the length of the inter-crimp distance of the cloth.

Another in-situ damage development and failure study was performed by Roy (1996). Carbon/epoxy unidirectional pre-pregs were laid up in a novel configuration producing model laminates with varying degrees of yarn crimp in one of the orthogonal directions only. The crimp in the laminates was achieved by accurately positioning strips of unidirectional plies either side of a uniaxial fibre array. Conventional cross-ply laminates were also fabricated as a reference material. Damage development and failure mode was observed in-situ using a loading stage attached to an optical microscope. Laminate edges were polished to enable the damage to be clearly seen.

The initial form of damage seen in all laminates was transverse ply cracking. This initiated at lower levels of strain in the model laminates compared to the cross-ply
laminate. The authors did not expect crack initiation in the form of transverse cracks in the model laminates. They described the expected initial damage in 8HS laminates as yarn interface cracks. This is not consistent with the findings of Gao et al. (1999a) and Marsden (1996). The two laminates with crimp regions similar in magnitude to 8HS laminates both failed within these regions, with strengths approximately 20 % lower than that of the cross-ply laminate. The two laminates with intermediate levels of crimp failed in longitudinal tow sections at similar strength levels to the cross-ply laminate.

It has been shown that the damage development in woven fabric composites can be very complex and is dependent on the architecture of the weave. The combination of weave styles, tow architectures and laminate configurations possible result in a vast range of conceivable damage morphologies.

2.3.5 Elastic Property Modelling After the Onset of Damage

2.3.5.1 Introduction

Elastic property degradation models taking into account the effect of damage are well established for cross-ply laminates, for example Ogin et al., 1984. However, work predicting woven composite laminate properties after damage accumulation is more recent. The fibre architecture of woven fabric structures is complex and highly variable from one fabric to another. The associated damage morphology is also complex and not always fully understood. The models discussed are presented in two sections; closed form models and FE methods.

2.3.5.2 Closed Form Models

Ishikawa and Chou (1983) produced early models for the effect of weft cracking on laminate behaviour based upon their earlier crimp and bridging models. Multiple transverse cracking was assumed to occur progressively at strain levels well below the laminate failure strain, contributing to non-linear stress-strain behaviour. Other factors considered in the model were shear deformation of longitudinal threads and extensional
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dehoration of pure matrix regions. The transverse cracking was modelled by assuming that the warp thread (transverse direction) fractures when the local strain reaches the failure strain of the transverse layer. The effective reduced stiffness constants of the weakened transverse warp material were divided by a factor of 100 to represent a reduction in elastic properties of this region. The transverse cracking process was assumed to continue until the lowest strain of the warp regions reached the fracture strain of the transverse warp thread. This model assumes the validity of applying LPT to the cracked warp region. Excellent agreement was shown for the prediction of the stress-strain response for a glass/polyimide 8HS weave laminate. However, comparisons for other laminate configurations in which more significant non-linear behaviour might be expected, such as a PW laminate, were not reported.

Marsden (1996) investigated the reduction in Young’s modulus with damage accumulation under quasi-static tensile loading. Property reduction due to damage was modelled using a shear lag analysis based on an equivalent cross-ply laminate. Before this could be performed the damage density had to be quantified. This was done by correlating the plan view crack count with the total length of cracking seen in edge section, per unit length. Elastic property degradation was found to be similar under both quasi-static and fatigue loading and the trends of the data compared well with the model (figure 2.10), although only transverse cracking was taken into account and fibre undulation was ignored. The effect of any delaminations on stiffness was also ignored for the modelling.

Gao et al. (1999b) investigated the effect of damage on laminate properties for 2, 4 and 6 layer woven carbon/epoxy laminates. It was seen that matrix cracking and delamination resulted in a stiffness reduction of approximately 5 % prior to failure of the 2 layer laminate. However, in the thicker laminates the stiffness reduction was only around 2 %. This difference was attributed mainly to the surface layer dominated delaminations as seen by Gao et al. (1999a). The reduction in stiffness was modelled using a shear-lag approach based on an equivalent cross-ply laminate, using a similar method to Marsden (1996). For the 2 layer laminate the crimp region was modelled and an associated delamination was simulated by decoupling the interface between the
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warp and weft tows in that region. Delaminations were not considered in the analysis of thicker laminates as it was shown that their location was irregular (Gao et al., 1999a) and they were thought not to have the same influence on mechanical properties. Prediction of Young’s modulus reduction for the 2 layer laminate was satisfactory. However, the models for the thicker laminates tended to over-predict the reduction in Young’s modulus. It was suggested that of the total Young’s modulus reduction of 5% prior to failure in the 2 layer laminate approximately 2% of this was due to the effect of the delaminations. Poisson’s ratio and residual strain in the laminates were also investigated. The change in Poisson’s ratio with increasing damage density was modelled using the shear-lag analysis, with general trends being satisfactorily described.

Kuo and Chou (1995) modelled the elastic property reduction in plain weave SiC/SiC ceramic matrix composites due to transverse cracking in the weft tows. The simple unit cell used to model the PW laminate is shown in figure 2.11. Both classical fracture mechanics (CFM) and a total energy balance (TEB) approach were used to study the crack initiation and growth. The CFM approach requires that the energy release rate due to infinitesimal crack increments is equal to the material fracture toughness. Alternatively, the TEB approach requires that the energy release rate due a transverse crack is equal to the energy required to create the new crack surfaces. The change in stress distribution in the transverse yarns due to transverse cracking was modelled in one of two ways. In the first method a shear-lag analysis was used. This was perhaps the first time that a shear-lag approach has been applied to a non-uniform cross section. In the second method the composite unit cell was modelled using a two-dimensional FE analysis. The shear lag approach gave satisfactory predictions for the transverse crack initiation strain and stiffness reduction, within 5 to 10% of those of the FE method. The shear-lag method clearly indicated the redistribution of stresses from the transverse plies to the longitudinal plies due to transverse cracking. The subsequent introduction of stress back into the transverse plies away from the transverse cracking was also well illustrated. The closed form expressions of the shear-lag approach obviously require much less computational effort than the FE method. However, neither model was compared to any experimental data.
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The closed form models presented are able to crudely predict the reduction in laminate properties with the introduction of damage. However, they involve many assumptions, such as, the representation of a woven fabric as a cross-ply material in the case of Marsden (1996).

2.3.5.3 Finite Element Methods

Finite element analyses allows the fabric structure to be represented accurately and detailed stress analyses to be performed. Obviously these approaches involve a much more complex analysis than the previously mentioned closed form models.

Kriz (1985) investigated the mechanical properties of glass/epoxy woven composites for use in magnetic fusion energy structures at low temperatures. The effect of weft cracking was modelled using FE methods. The model considered a unit cell with a single matrix crack in the transverse weft tow or without matrix damage. No attempt was made to study the effect of progressive damage accumulation on laminate properties. The plain weave unit cell was characterised by four identical planes. The elastic response of the unit cell is predicted by studying the 3D load deformation response within a single plane in the loading direction. The model crudely assumes a linear undulation path for the yarns. This approach should be considered qualitative as it only approximates crack tip stress states. Thermal (76 K) and mechanical loading was applied to a unit cell containing a transverse fill crack and the mechanical and internal stress response was studied. The model predicted that warp thread curvature and thermal stresses at low temperature reduces the transverse crack tip stresses. However, it was also shown that transverse cracks reduce stiffness more for fabrics with a high degree of warp thread curvature. The author suggested that a trade-off between stiffness and strength is necessary when considering warp thread curvature.

Kriz and Muster (1985) also used the FE model of Kriz (1985) to predict the influence of damage on the mechanical properties of glass/epoxy woven composites at low temperatures. An experimental investigation was performed to validate the model. Investigations were performed at 76 K and 295 K. Both modelling and experiments
suggested that fracture strength was increased at lower temperatures. Transverse cracking resulted in Young's modulus reduction for both experimental and FE modelling investigations. However, the density of damage in the model was not equivalent to that seen in experimental studies so comparisons are inconclusive. Delamination stresses were also found to decrease with the presence of transverse cracks.

Shindo and Ueda (1995) proposed another basic FE analysis to study the thermo-mechanical behaviour of damaged glass/epoxy woven composites at low temperatures. This model also assumed a linear yarn undulation path. In agreement with Kriz (1985), stiffness was found to increase with a reduction in temperature. However, contrary to the findings of Kriz (1985), crack tip stresses were said to increase at low temperatures.

In a more advanced study Whitcomb and Srirengan (1994) simulated the progressive failure of a plain weave composite using a 3D FE analysis. The fibre architecture was represented by a lenticular cross-section following a sinusoidal path. The effect of various approximations in the formulation of the model were investigated. Peak stress, damage initiation strain and stiffness reduction were all affected by varying the number of quadrature points from 8 to 64. Mesh refinement was also seen to have an effect on results. A coarse four element mesh was found to be quite inaccurate in predicting peak stress and corresponding strain, initial damage and size of damage zones. As the mesh was refined (32, 108 and 192 elements) there were indications that results were converging, but this required further numerical investigation. Three elastic property material degradation models were considered, all of which modified the constitutive matrix of the FE model to account for the damage. The first method assumed the material totally failed when any stress variable was exceeded, reducing the constitutive matrix to zero. This was termed the non-selective discount method. The second method selectively reduced the rows and columns of the constitutive matrix to zero depending on which particular stress allowable was exceeded. This was termed the selective RC method. The third method selectively reduced the engineering moduli according to the particular stress allowable that was exceeded. This was termed the Blackketter method and was the preferred technique. The three different material degradation methods investigated resulted in peak stress predictions varying by
approximately 20%. The effect of tow waviness on mechanical property and damage development was also investigated. The results suggested that tow waviness affected the stress at which damage initiates as well as the type of damage that occurs. As seen in previous FE studies, the complexity of the analysis meant that only an approximate treatment was possible.

A failure analysis of woven and braided composites was proposed by Naik (1995) using a micromechanics analysis implemented in a personal computer-based code called TEXCAD. Yarn architecture was modelled using suitable sinusoidal shape functions for the undulation. These were discretised into $n$ linear slices perpendicular to the yarn direction. The straight sections of the yarns were modelled as larger single sections. Fibre dominated failure was predicted using a maximum strain criterion. The composite was assumed to fail when all of the yarn discretised slices failed in a transverse tensile mode or when a single axial yarn failure was detected within the unit cell. Under shear loading failure was predicted when all yarn slices failed in an in-plane shear mode. The stiffness reduction was modelled using the Blackketter method described by Whitcomb and Srirengan (1994). The calculated mechanical properties compared well with the available test data. For the plain weave laminate strength predictions were within 5% of experimental data, with the exception of one shear strength prediction which showed a 10% deviation. The predicted compressive and tensile stress-strain relationship for the plain weave laminate also compares well with experimental data.

The FE techniques discussed are able to satisfactorily predict the reduction in laminate properties after the onset of damage. The advantage of the FE methods over the closed form models is the ability to predict the type and location of damage under quasi-static loading. As previously discussed the FE methods are complex and the current models still involve many simplifications and require further work before they may be utilised as useful design tools.
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2.4 Notched Strength of Composite Materials

2.4.1 Introduction

Basic joining methods for composite structures are mechanical or bonded joints. Mechanical joints require no surface preparation and are easily disassembled for repair and inspection. However, the cut-outs required result in high local stress concentrations and strength degradation. Also, composite structures can be subjected to impact damage, such as dropped tools and runway stone impact in the case of aerospace applications. Hence, the study of the fracture of composite materials containing through thickness holes or notches is of great interest to the designer. Analyses of stress concentrations and associated failure criteria around notches and cut-outs have been fairly extensively reported in continuous fibre (non-woven) composites. Some of these models have been applied to woven composites, but further investigation is required for both non-woven and woven composites.

Standard composite failure criteria are not able to predict notched failure strength. There is much literature published concerning the fracture behaviour of composite materials containing these artificial holes and cracks. The behaviour of such structures is complex due to the variation in damage progression and failure modes found in different composite systems. Consequently, there have been many different approaches used to predict the notched strength of composite materials. The use of Linear Elastic Fracture Mechanics (LEFM) in composites is limited. According to Wu (1968), fracture mechanics may only be applied directly to composites under specific conditions:

1. The orientation of the flaw with respect to the principal axis of symmetry must be fixed.
2. The stress intensity factors defined for anisotropic cases must be consistent with the isotropic case in stress distribution and in crack displacement modes.
3. The critical orientation coincides with one of the principal directions of elastic symmetry.
Perhaps more importantly, LEFM assumes self-similar crack growth, without the extensive damage zones often found in composite laminates.

Some non-woven composite materials satisfy these conditions and have confirmed the validity of a fracture mechanics approach (Wu, 1968). However, this is unsuitable for many practical laminates and notch configurations. Multi-directional laminates show complex damage development at the crack tip and fracture is strongly dependant on many material and loading variables.

This has led to the development of a diverse range of notched failure analyses, varying widely in complexity and accuracy. The following sections discuss such models in varying amounts of detail depending on their relative popularity as design criteria and relevance to the current project. The popular semi-empirical fracture models are initially presented. The remainder of the analyses are discussed under the heading of fracture models considering notch edge damage. Finally experimental studies investigating the effects of different laminate configuration on the notched strength of composite plates are discussed.

2.4.2 Fracture Mechanics-Based Models

As discussed above the application of LEFM to the notched strength prediction of composite laminates is limited. Some of the models based upon these principles are presented below, illustrating their limitations.

A notched failure model for centre cracked laminates was presented by Poe and Sova (1980) and Poe (1983). After experimental observations it was postulated that laminate failure occurs when the strain in the principal load carrying laminae reaches a critical value. By using a LEFM expression to obtain crack tip strains and the maximum strain failure criterion a constant toughness parameter, $Q_c$, was defined:
where $K_c$ is the stress intensity factor at fracture, $\gamma$ is the loading direction and the term $(\delta_1)_i$ is a non-dimensional function of the laminate elastic properties and the angle of the principal load carrying laminae with respect to the loading direction.

It was found that when the toughness parameter was divided by the failure strain of the fibres the resulting ratio returned an approximately constant value of 1.5 $\sqrt{mm}$. This was based upon a statistical analysis of all available fracture toughness data and was termed the general toughness parameter. Laminate notched strength was predicted from the following expression:

$$\sigma_N = \sigma_0 \left(1 + \pi a \left[\frac{\delta_1}{\sigma_0/Q_c E_y}\right]^2\right)^{-1/2} \quad (2.3)$$

It was found that for laminates exhibiting extensive notch edge damage zones the general toughness parameter deviated from the supposedly constant value of 1.5 $\sqrt{mm}$. For such laminates the author suggested re-evaluating the toughness parameter. The model then becomes a semi-empirical fracture model, similar to those presented in section 2.4.3. Excellent agreement with experiment was reported if $Q_c$ was determined for each laminate investigated.

Using a similar approach, Vaidya and Sun (1997) proposed a notched fracture criterion for composite laminates based upon the observation that final fracture is dominated by fibre failure in the $0^\circ$ plies. This analysis applies the principles of LEFM and considers laminates containing centre cracks. It was proposed that the fracture toughness of the $0^\circ$ plies, $K_c^0$, is constant for all laminate geometries. This fracture toughness cannot be measured directly from unidirectional laminates due to the influence of axial splitting. Instead $K_c^0$ was estimated for different laminate types using classical laminate plate theory. Values of $K_c^0$ for all laminate types were found to be within 10% of one another. The failure criterion states that failure
occurs when the load in the $0^\circ$ plies reaches the critical value governed by $K^0_c$, where:

$$K^0_c = \eta K_c$$  \hspace{1cm} (2.4)

where $\eta$ is a factor relating the stress in the laminate to the stress in the $0^\circ$ plies. This is dependent on laminate configuration and material properties and was found using classical laminate plate theory. The stress ratio, $\eta$, does not account for stress redistribution due to localised damage. Therefore, this approach may not be feasible for laminates exhibiting large damage zones. Once $K^0_c$ is determined fracture can be predicted for laminates within the same material system using the following expression:

$$\sigma_N = \frac{K^0_c}{\eta Y \sqrt{\pi a}}$$  \hspace{1cm} (2.5)

where $\sigma_0$ is the laminate fracture strength, $Y$ is a finite width correction factor and $a$ is the centre crack half length. Model predictions appeared to show very good agreement with experimental data for the laminates investigated.

Both models presented in this section are limited by the fact that they are only applicable to centre-cracked laminates that exhibit relatively small damage zones. This severely limits their usefulness as notched failure models.

### 2.4.3 Semi-Empirical Fracture Models

Many of the approaches developed overlook the actual details of the crack tip damage zone. Several simple semi-empirical fracture models have been proposed which are easy to utilise. These have been categorised into analyses based on LEFM and those based upon other principles.
2.4.3.1 LEFM-Based Fracture Models

Waddoups, Eisenmann and Kaminski (WEK) (1971) proposed a macroscopic LEFM approach for the notched strength prediction of composite laminates. Two semi-empirical models were developed, one for laminates containing circular holes, the other for laminates containing straight centre cracks.

For the case of the circular hole the model is based upon the relationship between the energy release rate, $G_c$, and the stress intensity factor, $K_c$, developed by Irwin (1948) for isotropic materials:

$$G_c = \frac{(1-\nu)^2 \pi}{2G} K_c^2 = \frac{(1-\nu)^2 \pi}{E} K_c^2$$  \hspace{1cm} (2.6)

where $G$ is the shear modulus and $\nu$ is the Poisson’s ratio. The model assumes that an intense energy region of length $a_1$ exists at the hole edge perpendicular to the applied stress (figure 2.12). The actual details of the damage in this region are unknown. Paris and Sih (1965) have shown that when this characteristic distance is small the stress intensity factor at the tip of the intense energy region (treated as a crack) is given by:

$$K_c = \bar{\sigma} \sqrt{\pi a_1} f\left(\frac{a_1}{R}\right)$$  \hspace{1cm} (2.7)

where $R$ is the hole radius and $\bar{\sigma}$ is the remote applied stress. Values of the function $f(a_1/R)$ can be found in Paris and Sih (1965). By combining Equations (2.6) and (2.7) and with the assumption that $R \gg a_1$ the following expression was obtained:

$$\frac{\sigma_0}{\bar{\sigma}} = f\left(\frac{a_1}{R}\right)$$  \hspace{1cm} (2.8)
Hence, by determining one notched strength and knowing the unnotched laminate strength the characteristic distance, $a_1$, can be found. The notched strength for other hole sizes can then be predicted.

It has been shown that $a_1$ varies with hole radius (Awerbuch and Madhukar, 1985) but generally a constant average value, $a_c$, results in good agreement between predicted and experimental notched strength values. Data has shown that $a_c$ is strongly dependent on laminate configuration and material system. Fabrication procedure, fibre volume fraction and test environment were also found to affect $a_c$. Hence, the effect of laminate constituents and configuration is not clear.

For the case of the straight cracks of length $2c$ the assumption of an intense energy region of length $a_c$ was also adopted. At failure, the Griffith stress intensity factor becomes:

$$K_{IC} = \sigma N^* \sqrt{\pi (c + a_c)} \quad (2.9)$$

For the case of an unnotched specimen with no crack:

$$K_{IC} = \sigma \sqrt{\pi a_c} \quad (2.10)$$

Thus, $a_c$ is considered to be the size of an inherent flaw. Accordingly this model is often referred to as the inherent flaw model. Combining Equations (2.9) and (2.10) gives:

$$\frac{\sigma N^*}{\sigma_0} = \sqrt{\frac{a_c}{c + a_c}} \quad (2.11)$$

Again, $a_c$ can be obtained with a notched strength and the unnotched laminate strength. The notched strength for other crack lengths can then be predicted. Test results indicated that $a_c$ may be considered independent of crack length. However, as in the case of circular notches $a_c$ is strongly dependant on laminate configuration and
material system. For the carbon/epoxy laminates investigated \(a_c\) values of approximately 1 mm were reported. However, no comparisons were made between the circular and centre-crack notch geometries investigated.

The WEK models are semi-empirical and if implemented correctly are found to agree well with experimental data. However, the characteristic distance, \(a_c\), is not constant for all hole sizes and no satisfactory physical interpretation was given to this parameter.

Another use of LEFM in the prediction of the notched strength of composites is the Mar-Lin (ML) fracture model (Mar and Lin, 1977). This model does not make use of a characteristic distance as seen in other models. The basis of this model is taken from a classical LEFM approach for isotropic metals. It is proposed that fracture is governed by:

\[
\sigma_n^* = H_c (2c)^n 
\]

(2.12)

The parameter \(H_c\) is analogous to fracture toughness, \(K_c\), in the isotropic case. The exponent \(n\) is "the order of the singularity of a crack tip at the interface of two materials" (in the isotropic case \(n = -1/2\)). If the strength ratios and \(2a/W\) are plotted on a log-log scale, \(H_c\) is the intercept and \(n\) is the slope of the line, fitted using linear regression.

The type of discontinuity, for example circular or slit, has been shown to have little effect on \(H_c\) or \(n\). However, the exponent \(n\) was seen to vary with laminate configuration. Therefore, this two parameter model is semi-empirical. The two constants, \(H_c\) and \(n\), must be determined by testing two specimens containing notches of different sizes. Good agreement between prediction and experiment has been reported. This is as expected considering the curve fitting procedure used in the determination of the parameters \(H_c\) and \(n\). However, one author (Awerbuch and Madhukar, 1985) expressed concern in the quantitative accuracy of this model due to the logarithmic nature of the curve fitting.
2.4.3.2 Non-LEFM-Based Fracture Models

Whitney and Nuismer (WN) (1974) proposed two stress criteria for predicting the notched strength of composite laminates. Neither of these two criteria apply the principles of LEFM directly. Both criteria assume laminate failure when the stress over some characteristic distance from the notch reaches the unnotched laminate strength. These two parameter, semi-empirical models, requiring the unnotched strength and a characteristic distance, are based upon the stress distribution adjacent to the discontinuity. These models are probably the most widely applied notched failure criteria over recent years. The models can be applied to any discontinuity geometry. However, circular holes and centre cracks were initially considered.

Consider the case of a circular hole of radius \( R \) in an isotropic, infinite plate. If a uniform tensile stress, \( \sigma_y \), is applied along the \( y \) direction remote from the notch then the normal stress adjacent to the notch, \( \sigma_y \), along the \( x \) axis is given by Timoschenko and Goodier (1951):

\[
\frac{\sigma_y}{\sigma_y} = 1 + \frac{1}{2} \left( \frac{R}{x} \right)^2 + \frac{3}{2} \left( \frac{R}{x} \right)^4 \tag{2.13}
\]

where \( x \) is the distance measured from the notch centre.

If this stress ratio is plotted against \( x-R \) (the distance ahead of the notch edge) then different hole sizes show very different stress distributions. This is demonstrated in figure 2.13. The stress concentration, \( K_T^\infty \), at the hole edge is 3 for all notch sizes. However, the stress concentration is more localised for smaller holes. It was suggested that a plate with a larger hole will have a lower strength as the stress concentration is distributed over a larger area and it is more likely that an inherent flaw will be present within this stress concentration. It was also postulated that a smaller notch allows the stress to be redistributed more effectively, resulting in a higher average strength.
The first of the WN fracture models is the Point Stress Criterion (PSC). It is assumed that failure occurs when the stress at some distance, $d_0$, away from the notch edge reaches the unnotched strength of the laminate. This is shown schematically in figure 2.14. This dimension was said to represent the distance over which the material must be critically stressed to encounter a sufficiently large flaw to initiate failure. Together with Equation (2.13) the above assumptions lead to the PSC expression for the notched strength of an isotropic plate:

$$\frac{\sigma_{N}}{\sigma_{0}} = \frac{2}{(2 + \xi_1^2 + 3\xi_1^4)}$$  \hspace{1cm} (2.14)

where

$$\xi_1 = \frac{R}{R + d_0}$$  \hspace{1cm} (2.15)

and $\sigma_N^*$ and $\sigma_0$ are the notched infinite plate and unnotched laminate strengths, respectively.

In the case of an infinite orthotropic plate containing a circular hole of radius $R$ subjected to a uniform stress, $\overline{\sigma_y}$, applied parallel to the $y$ axis the normal stress $\sigma_x$ along the $x$ axis can be expressed approximately as (Konish and Whitney, 1975):

$$\sigma_x(x,0) = \frac{\sigma_y}{\sigma_0} \left[ 2 + \left( \frac{R}{x} \right)^2 + 3 \left( \frac{R}{x} \right)^4 - (Kr^*)^3 \left( 5 \left( \frac{R}{x} \right)^6 - 7 \left( \frac{R}{x} \right)^8 \right) \right]$$  \hspace{1cm} (2.16)

where $Kr^*$ is the stress concentration factor of the infinite orthotropic plate and is given by:

$$Kr^* = 1 + \left[ 2 \left( \frac{E_y}{E_x} \right)^{\frac{1}{2}} - \nu_{yx} \right]^{\frac{1}{2}} + \frac{E_y}{G_{yx}} \right]^{\frac{1}{2}}$$  \hspace{1cm} (2.17)
In this case, for an orthotropic plate, the PSC becomes:

\[
\frac{\sigma_N^\infty}{\sigma_0} = \frac{2}{(2 + \xi_1^2 + 3\xi_2^4 - (\overline{K}^\infty - 3(5\xi_1^4 - 7\xi_2^8))}
\]  

(2.18)

The second of the WN (Whitney and Nuismer, 1974) fracture models is the Average Stress Criterion (ASC). It is assumed that failure occurs when the average stress over some distance, \(a_0\), reaches the unnotched laminate strength. This is shown schematically in figure 2.15. The physical argument for this criterion lies in the assumption that the material is able to redistribute local stress concentrations. The characteristic distance was considered as an approximation of the distance ahead of the discontinuity across which failure takes place. The criterion is of the form:

\[
\sigma_0 = \frac{1}{a_0} \int_{R+x_0}^{R+a_0} \sigma_0(x,0)dx
\]  

(2.19)

For the case of the isotropic plate, substituting Equation (2.13) into Equation (2.19) gives the ASC:

\[
\frac{\sigma_N^\infty}{\sigma_0} = \frac{2(1 - \xi_2)}{(2 - \xi_2^2 - \xi_2^4)}
\]  

(2.20)

where

\[
\xi_2 = \frac{R}{R + a_0}
\]  

(2.21)

For the case of an orthotropic plate the ASC is obtained by combining Equation (2.16) with Equation (2.19) (Nuismer and Whitney, 1975):


\[
\frac{\sigma_{N}^*}{\sigma_{n}} = \frac{2(1 - \xi_2)}{(2 - \xi_2^2 - \xi_2^4 + (Kr_{\infty} - 3)(\xi_2^{*4} - \xi_2^{*8}))}
\]

(2.22)

For both the PSC and ASC for large notch radii \( \sigma_{N}^*/\sigma_{0} \) reduces to the reciprocal of the stress concentration factor and for smaller notch radii \( \sigma_{N}^*/\sigma_{0} \) approaches unity. The characteristic distances, \( d_0 \) and \( a_0 \), were initially assumed to be material properties, independent of laminate construction and stress distribution.

Figure 2.16 shows (a) the PSC and (b) the ASC applied to experimental data for a quasi-isotropic glass/epoxy laminate. The actual values of the characteristic distance were selected so that the models result in a good fit for all hole sizes. The characteristic distances can also be determined using a classical error minimisation technique, such as minimisation of the least square errors. This is effectively another curve fitting technique, resulting in good agreement between prediction and experiment. The two WN criteria were also applied to straight centre cracked specimens (Nuismer and Whitney, 1975). Only one laminate containing a circular notch was investigated in the original paper (Whitney and Nuismer, 1974). Both criteria resulted in good correlation between predicted and experimental data. The WN failure criteria have also been applied to woven fabric reinforced composites (Naik and Shembekar, 1992c). Good correlation with experimental data for a range of stacking sequences was reported, especially for the ASC.

Awerbuch and Madhukar (1985) reported that \( d_0 \) and \( a_0 \) were not constant for different notch shapes and laminate configurations. It was suggested that the characteristic distances must be determined for each material system and laminate configuration. It has also been well documented that the WN stress criteria do not give characteristic distances that remain constant for different hole sizes, for example (Awerbuch and Madhukar et al. 1985). This change in characteristic distance with hole size was normally small. However, their simplicity and versatility, together with their excellent agreement with experimental data make them very attractive to the designer.
In models such as the WEK and WN it is assumed that the characteristic distances are material constants independent of discontinuity size. Karlak (1977) proposed a modification of the WN PSC called the Karlak (K) fracture model. This two parameter model is based on the assumption that $d_0$ is related to the square root of the notch radius. The realisation that the WN characteristic distances are not constant for all hole sizes led to Karlak studying the relationship between characteristic length and hole radius. The best fit to the data was obtained using an expression of the following form:

$$d_0 = k_0 R^\frac{1}{2}$$  \hspace{1cm} (2.23)

where $k_0$ has units of $\sqrt{\text{inch}}$. The data have been fitted using Equation (2.23) with error analysis. It was shown that $k_0$ is strongly dependent on stacking sequence and it was suggested that laminate fabrication and constituents may also affect $k_0$. Consequently, $k_0$ must be determined for each material system investigated.

The K fracture model can be expressed as:

$$\frac{\sigma_N}{\sigma_0} = 2\left[2 + \left(1 + k_0 R^2 \right)^{-2} + 3\left(1 + k_0 R^2 \right)^{-1}\right]^{-\frac{1}{2}}$$  \hspace{1cm} (2.24)

Karlak originally formulated the model for the case of $K_T = 3$. However, for orthotropic laminates the K fracture model has been reformulated for cases where $K_T \neq 3$ (Awerbuch and Madhukar, 1985). The model was reported to be valid for selected laminates only. For laminates where the characteristic distance is independent of hole size the K fracture model may result in a better fit with experimental data. However, for laminates that show that $d_0$ is independent of hole size the K fracture model results in less favourable agreement. A disadvantage of this approach is that it requires additional testing to determine the relationship between $d_0$ and notch size.
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Pipes, Wetherhold and Gillespie (PWG) (1979) further extended the idea that the characteristic distances in the WN fracture models are dependent on hole size. Two separate models were produced, one for circular holes and one for straight centre cracks, both based on the WN PSC. Similar to the approach of Karlak (1977), the characteristic distance, $d_0$, is assumed to be a function of hole radius:

$$d_0 \propto R^m$$  \hspace{1cm} (2.25)

Another parameter was introduced in this model, the notch sensitivity factor, $C_0$. The above expression can be written as:

$$d_0 = \left(\frac{R}{R_0}\right)^m C_0$$  \hspace{1cm} (2.26)

where $R_0$ is a reference radius so that the term $(R/R_0)$ is dimensionless. A value of $R_0 = 1$ inch was chosen for simplicity. The term $m$ is an exponential parameter. The WN PSC becomes:

$$\frac{\sigma_N}{\sigma_0} = \frac{2}{\left(2 + \lambda_0^2 + 3\lambda_0^4 - \left(K_T^{\infty} - 3\right)(5\lambda_0^6 - 7\lambda_0^8)\right)}$$  \hspace{1cm} (2.27)

where

$$\lambda_0 = \left[1 + R^{m-1} R_0^{-m} C_0^{-1}\right]^{-1}$$  \hspace{1cm} (2.28)

This is a three parameter fracture model. Using Equation (2.28) the notched laminate strength can be predicted if $\sigma_0$, $m$ and $C_0$ are initially determined experimentally. The higher the notch sensitivity factor, $C_0$, the more notch sensitive the material. A value of $C_0 = 0$ shows complete notch insensitivity, whereas a value of $C_0 = \infty$ results in $\sigma_N/\sigma_0 = 1/K_T^{\infty}$. The exponential parameter, $m$, ranges from zero to 1. When $m = 0$, the WN PSC is recovered, whereas when $m = 1$ the notched strength is independent of hole radius. The selection of the $R_0$ value affects the notch sensitivity.
curves. All three parameters are linked, hence different values of $R_0$ result in variation of $m$ and $C_0$. As with other models, the parameters concerned are dependent on laminate configuration and material system. As stated by Karlak (1977) the PWG approach is reported to yield more accurate predictions than the WN models. However, it is more detailed, requiring that the three parameters involved are carefully determined before proceeding with predictions.

The PWG fracture model is able to superimpose all notched strength data for materials with a similar stress concentration factor onto a single master curve of notched strength versus hole radius. This is achieved by the definition of radius shift parameters to account for variations in notch sensitivity parameter, $C_0$, and exponential exponent, $m$. This method allows relative comparisons between laminates of different stacking sequence, material system and orthotropy.

The majority of the studies on the residual strength of composite materials containing discontinuities concern circular notches and centre cracks. However, it is also of interest to the designer to consider other discontinuity shapes. Tan (1987a) extended the Whitney Nuismer fracture criteria to consider orthotropic composite laminates containing elliptical discontinuities. The approximate solution of the stress distribution ahead of the notch was assumed to be given by a polynomial function in addition to the isotropic solution. Two approaches were considered in evaluating this polynomial function. The second approach, in which the ellipse aspect ratio was considered, was shown to have significantly more accuracy than the first approach when compared to the exact elasticity solution. Hence, the stress distribution was given by:

\[
\frac{\sigma_y}{\sigma_y} = \frac{\lambda^2}{(1-\lambda)^2} + \frac{(1-2\lambda)\gamma}{(1-\lambda)^2 (\gamma^2 - 1 + \lambda^2)^{\gamma/2}} + \frac{\lambda^2 \gamma}{(1-\lambda)(\gamma^2 - 1 + \lambda^2)^{\gamma/2}}
\]

\[
-\frac{\lambda^2}{2} \left( K_{II}^e - 1 - \frac{2}{\lambda^2} \left( \frac{\gamma}{(\gamma^2 - 1 + \lambda^2)^{\gamma/2}} - \frac{7\lambda^2 \gamma}{(\gamma^2 - 1 + \lambda^2)^{\gamma/2}} \right) \right)
\]

(2.30)

where $\lambda$ is the ellipse opening aspect ratio:
\[ \lambda = \frac{b}{a} \quad (2.31) \]

and

\[ \gamma = \frac{x}{a} \quad (2.32) \]

where \( a \) and \( b \) are the major and minor ellipse half lengths, respectively.

The stress concentration factor for an infinite orthotropic plate, \( K_r^\alpha \), was given by:

\[ K_r^\alpha = 1 + \frac{1}{\lambda} \left( \frac{1}{2} \left( \frac{E_y}{E_x} \right)^{\frac{1}{2}} - \nu_{xy} \right)^{\frac{1}{2}} + \frac{E_{xy}}{G_{xy}} \right)^{\frac{1}{2}} \quad (2.33) \]

This stress distribution ahead of an elliptical discontinuity was then incorporated into the Whitney and Nuismer (1974) failure criteria. The extended Point Stress Criterion (PSC) is given by:

\[ \frac{\sigma_{\lambda}^o}{\sigma_0} = \left\{ 1 + \frac{1}{\sqrt{\xi_1^2 - 1 + \lambda^2}} \left[ \frac{\lambda^2 (1 + \lambda) \left( \xi_1^{-1} + 2 \sqrt{\xi_1^{-2} - 1 + \lambda^2} \right)}{\xi_1^{-2} - 1 + \lambda^2} \right] \right\}^{-1} \quad (2.34) \]

where

\[ \xi_1 = \frac{a}{a + d_0} \quad (2.35) \]
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The extended Average Stress Criterion (ASC) is given by:

\[
\frac{\sigma_{N}}{\sigma_{0}} = \left(\xi_{2}^{-1} - 1\right) \left[ \frac{\lambda^{2}}{(1-\lambda)^{2}} \xi_{2}^{-1} + \frac{1-2\lambda}{(1-\lambda)^{2}} \sqrt{\xi_{2}^{-2} - 1 + \lambda^{2}} - \frac{\lambda^{2}}{(1-\lambda)\sqrt{\xi_{2}^{-2} - 1 + \lambda^{2}}} \right]^{-1} + \frac{\lambda^{7}}{2} \left( K_{T}^{-1} - 1 - \frac{\lambda}{2} \right) \left[ \left( \xi_{2}^{-2} - 1 + \lambda^{2} \right)^{3/2} - \lambda^{2} \left( \xi_{2}^{-2} - 1 + \lambda^{2} \right)^{3/2} \right]^{-1}
\]  

(2.36)

where

\[
\xi_{2} = \frac{a}{a+a_{0}}
\]  

(2.37)

In both cases when \( \lambda = 1 \) the stress criteria reduces to the familiar WN expressions for circular discontinuities. Similarly, when \( \lambda = 0 \) the familiar WN expression for centre cracks is obtained.

An experimental programme was carried out using carbon/epoxy laminates in [0/90/\( \pm 45 \)]s and [0/902/0]s configurations. Discontinuities of \( \lambda = 1, 0.5 \) and 0.2 were investigated. The modified WN models resulted in good predictions for the laminates investigated. As previously reported for the WN criteria, the ASC performed better than the PSC. It was shown also that there is an effect of the opening aspect ratio, \( \lambda \), on the residual strength and that this more pronounced at larger openings. A suggested potential application of this model was the analysis of laminate damage that appears to be elliptical in shape, rather than circular.

In a subsequent paper Tan (1987b) performed a further experimental programme to investigate the modified WN failure criteria. It was found that the characteristic distances were dependent on the opening aspect ratio. In order to develop a closed form expression to predict the notched strength of laminates containing circular, elliptical and centre cracked notches the previously proposed extended Whitney Nuismer models were modified. It was assumed that centre cracks may be treated alongside circular notches as they must have a finite radius of curvature at the notch
The characteristic distances are considered as a power function of the opening length and aspect ratio as follows:

\[ d_o = m_o \left( a/a_r \right)^n \left( b/a \right)^f \]  
\[ a_o = m_1 \left( a/a_r \right)^e \left( b/a \right)^f \]

where \( m_0, n, s, m_1, e \) and \( f \) are constants and \( a_r \) is the reference half opening length so that \( a/a_r \) is dimensionless. This method requires at least three notched strengths to determine the characteristic distances, a minimum of two circular holed coupons and one centre cracked specimen.

The predictions obtained from this modified approach were reported to be very good. This model offers a closed form expression that is able to predict the residual strength of composite laminates containing notches varying from circles to centre cracks without the need to re-evaluate characteristic distances. This was achieved at the expense of losing some of the simplicity of the Whitney Nuismer models.

Tan also proposed two semi-empirical notched failure criteria based upon the first ply failure (FPF) stresses of notched and unnotched laminates (1987c, 1987d). The stress distribution of a laminate containing a discontinuity was analysed using the same method as Tan (1987a). The quadratic FPF criterion (Tsai and Hahn, 1980) was preferred over other criteria.

The point strength model (PSM) says that the notched to unnotched strength ratio of an infinite plate is given by the notched to unnotched FPF ratio at a characteristic distance, \( b_1 \), away from the notch edge:

\[ \frac{\sigma_{N}}{\sigma_0} = \frac{\text{FPF}_{\text{notched}}(r=0,y=b_1)}{\text{FPF}_{\text{unnotched}}} \]
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The minimum strength model (MSM) considers the strength distribution along a curve parallel to the opening at a characteristic distance, $b_2$, from the opening. The minimum FPF ratio along this curve was proposed to give the notched to unnotched strength ratio of an infinite plate:

$$\frac{\sigma_{N}}{\sigma_{0}} = \frac{\text{FPF}_{\text{notched}}}{\text{FPF}_{\text{unnotched}}} \left(\frac{x^2}{(a+b_h)^2} + \frac{y^2}{(b+h)^2}\right)^{-1}$$

(2.41)

where $x$ and $y$ refer to the co-ordinate axes and $a$ and $b$ are the ellipse major and minor opening lengths, respectively.

The current models were applied to carbon/epoxy and E-glass/epoxy orthotropic and quasi-isotropic non-woven laminates. Circular, elliptical and centre-cracked discontinuities were considered aligned perpendicular and at an angle to the applied load. The characteristic distances $b_1$ and $b_2$ were empirically determined.

The PSM predictions were found to be similar to the WN PSC for circular notches when the same characteristic distances were used. This was not reported for other notch geometries. The MSM resulted in good predictions for circular notches with a constant characteristic distance. The characteristic distance, however, did vary from laminate to laminate. It was attempted to predict notched strength for other notch geometries using a single value for the characteristic distance. This gave satisfactory results in some cases but appeared to be unreliable.

The developed models satisfactorily predict laminate notched strength. However, they do not appear to perform any better than the previously discussed WN failure criteria (Whitney and Nuismer, 1974) while being more complex and still requiring experimentally determined input parameters.

This approach was further developed by Xiao and Bathias (1993, 1994a and 1994b) who produced a series of papers investigating the notched strength of non-woven and
woven fabric laminates. The PSM and MSM of Tan (1987c) were modified by recalculating local and global stress distributions and determining characteristic dimensions for each individual ply. A numerical stress field was used around the discontinuity to consider the influence of finite width.

The models were compared to the WN PSC and ASC. The MSM was reported to give the most accurate predictions for the non-woven laminates. There appears to be some disagreement in which model best predicts the woven laminate notched strength. Xiao and Bathias (1994a) rated the WN ASC the most accurate, whereas Xiao and Bathias (1994b) quoted the improved MSM as the most precise.

The PSM and MSM are also able to provide some information concerning damage in notched laminates. The stage at which the first damage occurs and the particular ply and position within that ply that the damage occurs can be determined. The principal damage mechanism in non-woven laminates was predicted and verified experimentally. The damage in woven laminates was also predicted but could not be experimentally verified. The authors attributed this to a general lack of understanding of damage mechanisms in woven composite laminates.

In summary, all of the semi-empirical models reviewed show very good agreement with experimental data. This is as expected due to their semi-empirical "curve-fit" nature. All of the models require preliminary experiments to determine the relevant parameters. Most of the models discussed involve a characteristic distance ahead of the discontinuity edge. Whether or not this dimension is a material constant has been the subject of much debate. It can certainly be shown, in some cases at least, that the characteristic distances vary with hole size, laminate configuration and other parameters such as fabrication technique. It is generally accepted that these characteristic dimensions must be determined for different laminate configurations and material systems. However, there appears to be some disagreement between authors on whether the characteristic distances can be considered to be independent of discontinuity size. Experimental results indicate that sufficient accuracy is obtained by assuming that the characteristic distances are independent of discontinuity size. Most of the models have been applied to non-woven composites.
However, as details of the fibre architecture are ignored by many of the presented models they can be applied to woven composites.

2.4.4 Fracture Models Incorporating the Evolution of Notch Edge Damage

The majority of the semi-empirical models discussed in the previous section tend to ignore the notch edge damage accumulation prior to failure. Those that do consider some form of notch edge damage treat it as a general damage region such as an inherent flaw in the WEK model (Waddoups et al., 1971). The PSM and MSM of Xiao and Bathias (1993, 1994a, 1994b) considered the damage in more detail. However, their approach involved a complex numerical analysis and, by the authors own admission, this damage analysis was “tentative”. Some of the more recent models incorporate the damage zone into the notched strength analysis. This allows some measure of the critical size of the damage zone to be estimated. These approaches are generally more complex than the previously discussed semi-empirical models.

The models discussed have been separated into three sections. The first of these is based upon general fracture mechanics. The second section discusses methodologies in which the damage zones are described by an effective crack whose opening displacements are controlled by a fracture mechanics parameter. The final section describes some of the FE models applied to predict the notched strength and damage development of notched composite laminates.

2.4.4.1 Fracture Mechanics-Based Models

In a straightforward fracture mechanics-based approach Hitchen et al. (1994) proposed a model to predict the notched strength of short carbon fibre/epoxy laminates. The analysis is based upon the stable growth and subsequent catastrophic failure of a damage zone, or effective crack, at the notch edge. This is conceptually similar to an approach used by Soutis et al. (1991) to predict the compressive failure of carbon/epoxy laminates.
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An ASC (Whitney and Nuismer, 1974) was used to predict the growth of the damage zone across the laminate with increase of the remote applied load:

$$\sigma_0 = \frac{1}{c} \int_0^c Y \sigma_y \, dx \quad (2.42)$$

where $c$ is the damage zone length, $Y$ is a finite width correction factor and $\sigma_y$ is the stress adjacent to the notch edge. Hence, the length of the damage zone can be predicted from the remote applied load.

A fracture mechanics approach was used to predict the point at which the crack becomes unstable and catastrophic fracture occurs. The stress required for catastrophic failure was given by:

$$K_c = \bar{\sigma}_y \sqrt{\pi c F_0 Y_2} \quad (2.43)$$

where $K_c$ is the critical stress intensity factor and $\bar{\sigma}_y$ is the remote applied stress. The parameter $F_0$ is a circular hole correction factor for cracks growing from a notch from Tada et al. (1985). The term $Y_2$ is a finite width correction factor for a crack emanating from a hole (Soutis et al., 1991). Fracture was assumed to occur when the stress required to advance the damage zone was equal to the stress required for catastrophic failure. This point also gives a critical damage zone size. The principles upon which the model is based are shown schematically in figure 2.17.

The effect of notch root radius was investigated using circular and elliptical discontinuities of constant major axis length. Fracture toughness data were obtained using single edge notch specimens. The model notched strength predictions were satisfactory, but tended to under-predict the experimental data. No attempt was made to observe the notch edge damage zone in the laminates investigated. This approach is attractive as it is based upon physically meaningful parameters while being very easy to implement. However, the limited results presented suggest that further investigation is required to ascertain the validity of this model.
2.4.4.2 Effective Crack Growth Notched Fracture Models

This section describes models in which the damage zone is represented by a fictitious crack extending from the notch edge perpendicular to the applied load. The crack opening displacements of these cracks are controlled by traditional fracture mechanics parameters to simulate stress redistribution within the notch edge damage zone.

Bäcklund and Aronsson (1986) and Aronsson and Bäcklund (1986a) proposed a damage zone analysis for the prediction of the tensile strength of composite laminates containing circular, non-circular and through-thickness crack discontinuities. The method employed was originally developed for the fracture analysis of concrete beams (Hillerborg, 1976). Unlike semi-empirical models such as those of Whitney and Nuismer (1974) the current analysis is based upon more traditional concepts. This general method requires the basic properties of the laminate such as the unnotched strength, laminate stiffness and the apparent fracture energy. This approach, known as the damage zone model (DZM), represents the damage zone in the stress intense region ahead of the notch as a crack extending perpendicular to the applied load. This effective crack represents the delamination, matrix and fibre failure, matrix yielding and fibre-matrix debonding found in these materials. Only half of the laminate was considered in the analysis. The damage was represented on two adjacent crack surfaces and treated using a Dugdale-Barenblatt type of analysis (Barenblatt, 1962) (figure 2.18). This involves the assumption of cohesive stresses, $\sigma_{coh}$, acting on the crack faces. As the damage propagates the crack opening, $v$, increases, resulting in a reduction in the cohesive stresses. A linear decreasing relationship is assumed between the cohesive stresses and the crack opening, as shown in figure 2.19.

The area below the $\sigma_{coh}-v$ curve is equivalent to the apparent fracture energy, $G_c^*$, representing the energy dissipated by the fracture processes described above. This apparent fracture energy was determined using a numerical method. The strength of a particular notched coupon was experimentally determined and also modelled by a
FE approach using a linear relationship between the stress and crack opening (figure 2.19). This was performed using various $G'_c$ values. The value of $G'_c$ that gave the same experimental fracture load was taken as the appropriate value. When the undamaged laminate is loaded a crack is assumed to form when the stress at the discontinuity edge reaches $\sigma_0$, the unnotched laminate strength. At this point the crack opening is zero according to figure 2.19. On further loading the crack extends into the laminate and the crack opening increases. As the crack opens the cohesive stress follows the relationship described by figure 2.19. The stresses along the predetermined crack path are also assumed to follow the same linear relationship. This approach accounts for the stress redistribution and reduction in stiffness due to damage growth. Using a two-dimensional FE analysis, crack nucleation, stable growth and unstable growth was predicted. The critical damage zone size occurs when the crack becomes unstable and corresponds to the maximum applied load, shown for carbon/epoxy laminates in figure 2.20. The DZM assumes linear elastic behaviour and uses the cohesive stress along the crack surfaces to represent the two-dimensional nature of the damage zone.

The results of the DZM were compared to experimental data and also some of the semi-empirical failure criteria. For the circular and non-circular (rectangular) discontinuities in carbon/epoxy quasi-isotropic laminates (Bäcklund and Aronsson, 1986) the DZM and the Inherent Flaw Model (IFM) proposed by Waddoups et al. (1971) both showed very good agreement with experimental data. However, it was reported that the DZM is generally more accurate than the IFM. Centre cracked quasi-isotropic carbon/epoxy laminates and short glass fibre/polyester laminates also showed that the DZM yields accurate strength predictions (Aronsson and Bäcklund, 1986a). The accuracy of the IFM and the WN PSC (1974) varied with sample geometry and discontinuity size. This has been shown previously (Awerbuch and Madhukar, 1985). However, this was not the case for the DZM as it involves parameters that are independent of crack length and specimen geometry that can be regarded as material parameters. For all cases the critical size of the damage zone predicted by the DZM was of a similar magnitude to experimental observations. This critical damage zone was not considered to be a material constant as assumed in the IFM and PSC and was seen to vary with specimen geometry although the apparent
fracture energy remained constant. Regardless of specimen or discontinuity geometry it was seen that cohesive stresses act on the crack faces until failure occurs. This suggested that complete separation of the crack faces does not occur and was in agreement with experimental observations of the damage zone.

In a later study Aronsson and Backlund (1986b) investigated the sensitivity of the DZM. The sensitivity of the predicted notched strength with variation of the unnotched strength and apparent fracture energy was seen to vary with discontinuity shape. The shape of the cohesive stress-crack opening plot was also investigated (figure 2.21).

The DZM was found to be more sensitive to variations in unnotched laminate strength than apparent fracture energy for the case of circular notches. The opposite effect was seen in laminates containing rectangular and edge crack notch configurations. It was seen that the notched strength of laminates containing smooth notches was more sensitive than laminates containing sharp notches to variations in the shape of the $\sigma_{coh}$ vs $v$ plot. It was also shown that the linearly decreasing $\sigma_{coh} - v$ relationship effectively takes into account the stress redistribution in the damage zone as the external load increases. No comment was made concerning the relative sensitivities of the shape of the $\sigma_{coh} - v$ plot compared to the magnitude of the apparent fracture energy on the DZM predictions.

Although the DZM reported excellent accuracy in its notched strength predictions it was unattractive to the designer due to the numerical nature of the FE technique employed. Eriksson and Aronsson (1990) proposed the Damage Zone Criterion (DZC), claiming the accuracy of the DZM with the simplicity of the PSC. The DZC uses the same basic principles as the DZM with the damage zone being represented as a fictitious crack with cohesive stresses on the crack faces. As discussed in the description of the DZM damage initiates and grows from the notch edge with the assumption that a stress relaxation is present within this damage zone. This results in a stress within the damage zone at the notch edge which is now lower than $\sigma_0$. For simplicity a constant relationship between the cohesive stress, $\sigma_{coh}$, and the crack opening, $v$, was assumed, as shown in figure 2.21.
The stress distribution ahead of the damage zone, $\sigma_y^*$, was assumed to have the same form as the linear elastic stress distribution, $\sigma_y$. However, for equilibrium to be maintained $\sigma_y^*$ is in a different position to $\sigma_y$. The difference between $\sigma_y^*$ and $\sigma_y$ is $\Delta \sigma_y^*$, given by the difference between the unnotched laminate strength, $\sigma_0$, and $\sigma_y$ at the critical length of the damage zone, $d_1^*$, i.e.:

$$\sigma_y^* = \sigma_y + \Delta \sigma_y^* = \sigma_y + \left(\sigma_0 - \sigma_y \bigg|_{z = c + d_1^*}\right) \quad (2.44)$$

The basis of the DZC is the following closed form expression based on the equilibrium between the applied load and the axial force acting on the net section plane of the laminate:

$$\sigma_0 d_1^* t + \int_{c + d_1^*}^{d_1^*} \sigma_y^* t dx = \sigma_N \frac{W}{2} t \quad (2.45)$$

The assumption of a constant relationship between the cohesive stress, $\sigma_{coh}$, and the crack opening, $v$, does not produce the stress relaxation previously described. This results in a constant stress of $\sigma_0$ within the damage zone. The model combines Equations (2.44) and (2.45) with the linear elastic stress distribution to produce an expression for the notched to unnotched strength ratio of the laminate. This was done for both centre cracked and circular notched configurations. The DZC is effectively another two parameter semi-empirical model.

The DZC was compared to experimental results, the DZM and the PSC. The experimental programme investigated carbon/epoxy laminates in three laminate configurations. Centre cracked and circular notches were investigated, each at three different notch sizes. The critical length of the damage zone, $d_1^*$, was calculated for the smallest notch size for each of the laminate types from the expression for the notched strength of the centre cracked specimens. This parameter, $d_1^*$, was then used to predict the notched strength of each laminate configuration at each notch size and notch geometry. The PSC characteristic distance, $d_0$, and the apparent fracture
energy, $G_c^*$, required for the DZM were also determined from the same notched strength as used to determine $d_1^*$. It was reported that the DZC offers significant improvement in accuracy compared to the PSC and is at least as accurate as the DZM.

In summary, the authors reported excellent accuracy using a relatively simple approach. However, the predictions were based upon the unnotched strength of one specimen geometry and one material system. No attempt was made in this initial investigation to study the effect of notch size, laminate configuration or material system on the critical length of the damage zone, $d_1^*$, and overall performance of the model. As with other semi-empirical fracture models, such as the PSC, the fundamental parameters of the DZC must be determined for each individual laminate configuration investigated.

The majority of the previously discussed models are generally formulated to predict the size, or some form of measure, of the damage zone adjacent to the notch at a given applied load. Afaghi-Khatibi et al. (1996a) considered the problem using a reverse approach, i.e., the applied load was predicted for a given measure of damage. This method, known as the effective crack growth model (ECGM), uses an iterative technique and is based on the principles and closed form expressions used in the DZC (Eriksson and Aronsson, 1990). As in the DZC the ECGM approximates the damage zone adjacent to the notch using an effective crack with cohesive stresses acting on the crack faces. However, the ECGM assumes a linear decreasing relationship between the cohesive stress and the crack opening (figure 2.19), resulting in a stress relaxation within the damage zone. The apparent fracture energy, $G_c^*$, was determined using a similar method to that of Bäcklund and Aronsson (1986). This involves fitting the fracture energy to a notched strength using a numerical analysis. Hence, the ECGM is another semi-empirical fracture model. For simplicity, the ECGM analysis considers only half of a notched coupon.

The first step in the ECGM is to divide the laminate ahead of the notch edge into increments of $\Delta c$. It is then assumed that the normal stress, $\sigma_n$, reaches the unnotched
lamine strength, $\sigma_0$, at a distance $c_1 = \Delta c$ from the notch edge. The applied load, $P_1$, is then calculated from the following equilibrium between the applied load and the resultant axial force acting on the net section of the plane:

$$F_1 + \int_{R+c_1}^{R/2} \sigma_y^* t dx = P_{(1)} (2.46)$$

where

$$\sigma_y^* = \sigma_y + \Delta \sigma_y (2.47)$$

$$\Delta \sigma_y = \sigma_0 - \sigma_y \bigg|_{x=R+c_1} (2.48)$$

This modification of the stress distribution ahead of the notch is necessary for equilibrium as the stress is assumed to reach the unnotched laminate strength at the tip of the damage zone. This is shown schematically in figure 2.22. The term $F_1$ is the force acting on the tip of the first damage increment, given by $\sigma_0 \Delta c t$. This represents the stress redistribution within the damage zone. The integral of the modified stress distribution ahead of the notch multiplied by the laminate thickness completes the expression for the resultant axial force acting on the notched laminate section.

The next step is to introduce a new damage increment, $c_2 = 2\Delta c$, and calculate the new applied load, $P_2$. The second step of the fictitious crack growth is shown schematically in figure 2.23. Now the equilibrium is given by the following expression:

$$F_1 + F_2 + \int_{R+c_2}^{R/2} \sigma_y^* t dx = P_{(2)} (2.49)$$

The term $F_2 = \sigma_0 \Delta c t$, which corresponds to the normal stress over the distance from $c_2$ to $c_1$ reaching the unnotched laminate strength, $\sigma_0$. 
The term $F_1 = \sigma_1 \Delta c t$, where $\sigma_1$ must be carefully determined as a non-zero crack opening, $v$, was induced at the tip of the previous damage increment, $c_t$. The crack opening at the tip of the damage increment, $v_{(n)}$, is made up of a contribution from the crack opening due to the applied load, $v_{p(n)}$, and the crack opening due to the cohesive stress, $v_{c(n)}$. The term $v_{c(n)}$ is negative indicating crack closure under the cohesive stress. This results in the following expression:

$$v_{(n)} = v_{p(n)} + v_{c(n)}$$  \hspace{1cm} (2.50)

If the ratio of the fictitious crack length to the notch radius is less than 1.8 then it was assumed that the crack opening displacement formulations for edge cracks extending from a straight edge are valid (Tada et al., 1985):

$$v = \frac{4\sigma}{E_{11}} \sqrt{\frac{c^2 - x^2}{\epsilon}} D\left(\frac{x}{c}\right)$$  \hspace{1cm} (2.51)

where

$$D\left(\frac{x}{c}\right) = 1.454 - 0.727\left(\frac{x}{c}\right) + 0.618\left(\frac{x}{c}\right)^2 - 0.224\left(\frac{x}{c}\right)^3$$  \hspace{1cm} (2.52)

This is not strictly valid as cracks extending from a circular notch will not behave identically to cracks extending from a straight edge, especially for smaller notch radii.

For the determination of the crack closure due to the cohesive stress acting on the crack faces, $v_{c(n)}$, the term $\sigma$ in Equation (2.51) and is given by:

$$\sigma = \sigma_0 \left(1 - \frac{v_{(n)}}{v_c}\right)$$  \hspace{1cm} (2.53)
This expression is derived from the linear relationship between the cohesive stress and the crack opening (figure 2.19). The contribution to the crack opening displacement due to the applied load also uses Equation (2.51). However, in this case the $\sigma$ term is given by the expression for $\sigma_y$ (equation 2.47), the normal stress distribution ahead of the notch. The total crack opening is then obtained from Equation (2.50).

This total crack opening displacement is then used in Equation (2.53) to determine the stress acting on the particular damage increment under consideration. Hence, the term $\sigma_1$ in the expression for $F_1$ in Equation (2.49) can be obtained from Equations (2.50) through to (2.53). However, these equations cannot be solved directly and an iterative approach must be used. The previously obtained values of the applied load and crack opening are used as initial estimates in the iteration. In this case these values are $P_2 = P_1$ and $v_1 = 0$. The crack opening due to the applied load is calculated from Equation (2.51), using a stress on the damage increment calculated from $P_1$ together with the expression for the stress distribution ahead of the notch. The crack opening due to the cohesive stress is determined from Equations (2.51) and (2.53) using the initial crack opening of zero. These equations are then solved using the most recent values of the applied load and crack opening until convergent values are obtained.

The next step is to introduce a new damage increment resulting in a new crack opening profile. As the crack grows an initial estimate of the crack opening due to the cohesive stress, $v_{\alpha, (\alpha)}$, must be determined. The cohesive stress on the crack surfaces is described by a group of equivalent forces acting on the tip of each damage increment. This can be expressed by an equivalent cohesive stress by assuming that both crack surfaces behave as cantilever beams. From the equilibrium of bending moments acting on the fictitious crack tip the equivalent cohesive stress is given by:

$$\sigma_{coh} = \frac{2(nF_1 + (n-1)F_2 + \ldots + F_n)}{in^2\Delta c} \quad (2.54)$$
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This cohesive stress may then be used to estimate the initial crack opening displacement using Equation (2.51).

In summary, the ECGM is based upon a balance of forces given by the general expression:

\[ \sum_{n=1}^{i} F_n + \int_{R+c(n)}^{R+c(i)} \sigma_i dx = P_0 \]  

(2.55)

where the term \( \sum_{n=1}^{i} F_n \) describes the redistribution of stress within the damage zone which is controlled by the apparent fracture energy, \( G_c^* \), a fracture mechanics parameter. By repeating the above procedure for each damage increment the applied load corresponding to a range of damage zone sizes can be determined. This results in a maximum load whereupon the crack becomes unstable and fracture is assumed to occur. This fracture load corresponds to a critical damage zone length, assumed in the model to be an indication of the size of the damage zone at failure.

Carbon/epoxy [0/90], and quasi-isotropic laminates (Afaghi-Khatibi et al., 1996a) and plain weave glass/epoxy, satin weave glass/polyester and plain weave carbon/epoxy woven laminates (Afaghi-Khatibi and Ye, 1996) containing circular notches were experimentally investigated. The authors reported very good correlation of the model with experimental results. Results were compared to those obtained by the DZC (Eriksson and Aronsson, 1990) and the PSC (Whitney and Nuismer, 1974). Although varying from laminate to laminate, the ECGM generally appeared to perform better than the other models used. An example of the notched strength prediction using the ECGM is given in figure 2.24. However, the way in which the characteristic distances required for the DZC and PSC were determined may not yield as accurate results as possible. The characteristic distances for the non-woven laminates investigated were calculated on only one notch size and one specimen width (Afaghi-Khatibi et al., 1996). Hence, for some PSC predictions, the characteristic distance used did not relate to the sample width on which it was
applied. The characteristic distances for the woven laminates were obtained by averaging over the range of notch diameters (Afagh-Khatibi and Ye, 1997). It has been well documented for the Whitney Nuismer models that the characteristic distance should be fitted using error analysis over a range of geometries (Awerbuch and Madhukar, 1985). It appears that the ECGM requires further experimental verification in order to assess its performance compared to the other available models.

The authors investigated the effect of the size of damage increment used in the model. It was found that varying $\Delta c$ from 0.01 to 0.1 mm resulted in a deviation of the predicted applied stresses of approximately 1%. Therefore damage increments of 0.1 mm were thought to be sufficiently accurate. The stress profiles within the damage zone at progressive steps of the modelling were also investigated. The results showed that the model redistributes stress within the damage zone with growth of the fictitious crack. However, the stress distributions determined in by Afaghi-Khatibi et al. (1996a) appear different to those performed by Afagh-Khatibi and Ye (1996). There appears to be no satisfactory explanation for this discrepancy. It was found that the critical damage zone length increased with increasing hole diameter suggesting that this critical length is not a laminate constant.

The ECGM is a model developed to predict the residual strength of notched composite laminates which requires the basic elastic properties of the laminate, unnotched laminate strength and the apparent fracture energy. Once these properties are determined the ECGM does not require the experimental determination of notched strengths as in the Whitney and Nuismer models (1974). However, the above-mentioned properties of each laminate investigated are required and the model itself is significantly more complex than the WN models. The fitting method used to determine the apparent fracture energy is also contentious. No attempt was made to compare these data with measured critical energy release rate values. The main features of the ECGM compared to other models are the stress redistribution and damage growth predicted adjacent to the discontinuity.
Afaghi-Khatibi, et al. (1996b) modified the ECGM to investigate the residual strength of laminates containing sharp notches. A virtually identical approach to the case of the circular notches was used, substituting the relevant expressions for the stress distributions and crack opening profiles. Again, excellent agreement with experimental data was reported. The ECGM has also been applied to metal matrix composites (Afaghi-Khatibi et al., 1996c; Afaghi-Khatibi and Ye, 1997 and Ye et al. 1998).

2.4.4.3 Finite Element Methods

The models in section 2.4.4.2 describe the damage zone in the laminates as an effective crack. This is often an inaccurate description of notch edge damage as many laminates exhibit more complex damage morphologies. The use of FE analyses enable a more detailed failure analysis to be performed. The extent and type of damage developed at the notch edge can be predicted more accurately.

Chang and Chang (1987) proposed a FE based model capable of assessing damage and notched strength of laminates with arbitrary ply orientations. The stress analysis of the laminate was performed by a FE analysis based upon classical laminate plate theory. A set of failure criteria (Chang et al., 1984) were used to predict in-plane failure with the corresponding damage failure mode. The failure modes considered were matrix cracking, fibre-matrix shearing and fibre fracture. Once local failure occurred the stress analysis redistributed stresses and strains due to reduction in local mechanical properties. The local property degradation was performed on the basis of the predicted failure mode. For example, for the case of matrix cracking the transverse Young’s modulus and Poisson’s ratio were reduced to zero in the failed region.

The model predictions were compared to experimental data for a range of carbon/epoxy prepreg laminates. All predictions of notched strength were reported to be within 20 % of experimental values. This is not particularly accurate considering the complexity of the analysis. This approach predicts the type, extent, position and load at which damage is initiated around the notch. However, little effort was made
to compare these predictions to experimental observations. This model is able to provide a detailed analysis of the notched failure of composite laminates. However, the complex nature of the FE method means that the speed and simplicity with which the semi-empirical methods may be applied is lost.

Another FE approach was proposed by Kamiya and Sekine (1996). Notched fracture was predicted by considering interlaminar cracks which form the delaminated zone at notch tips in composite laminates. A FE method based upon classical LPT predicted the growth of this delamination zone at a constant energy release rate. Fibre fracture within this zone was predicted using a probabilistic approach. Predictions of notched strength were compared to experimental investigations of E-glass/epoxy angle-ply laminates. Compact tension specimens were used for the fracture tests. The predicted and experimental values appear to agree fairly well, being within approximately 10% of each other. This analysis requires the determination of three unknown parameters which are difficult to measure. The first of these was the critical energy release rate. The other two were Weibull parameters used in the probabilistic fibre fracture predictions. These were obtained by comparing the simulation with experimental results for one laminate type. The simulation results for other laminates were in good agreement using these parameters. The authors claim that these parameters are independent of laminate geometry but this does not appear to have been extensively investigated. Considering the complexity of the FE analysis, the fact that this model still requires experimental determination of unknown parameters is an added complication.

Kennedy and Nahan (1996) reported that the damage growth in composite materials in the vicinity of crack tips often results in strain softening. In other words, a damage zone develops in which the load carrying capability of the material deteriorates. This is analogous to the methodology used in the ECGM (Afaghi-Khatibi et al., 1996a) described in section 2.4.4.2. One of the problems associated with the FE analysis of such damage adjacent to notches in composite laminates is the tendency of the damage zone to localise to zero volume as the mesh is refined. The authors presented a non-local damage model that prevents excessive localisation of the damage. The basic idea of this non-local continuum is that the stress at a given point
is dependant, not only on the strains at that point, but also on neighbouring strains. This relatively simple approach was incorporated into a two-dimensional FE model.

The analysis was performed by considering the laminate as a homogeneous material. When damage was predicted the effective load carrying area of that region was reduced by a factor to simulate the occurrence of stress relief and redistribution. Calculations were performed for 13 and 15 layer angle-ply graphite/epoxy laminates containing centre cracked discontinuities. The stress-strain response of the material was predicted and seen to merge with mesh refinement. Of the two laminates investigated the notched strength prediction for various notch sizes was excellent. The growth of the damage zone was also monitored. This damage was predicted to grow primarily perpendicularly to the applied load.

The FE models discussed are capable of predicting the growth of notch edge damage and the notched strength of composite laminates. However, they do not appear to perform significantly better than the models discussed in previous sections in their notched strength predictive accuracy. Hence, the complex analysis involved does not make them particularly attractive design tools, unless a detailed damage analysis is required.

2.4.5 Laminate Configuration Effects on the Notched Strength of Composite Materials

Many of the notched failure criteria discussed recognise the need to re-determine certain parameters when variables such as material system, laminate lay-up and even processing route are changed. This section discusses experimental programmes in which some of these issues have been addressed.

2.4.5.1 Laminate Lay-Up Effects

Naik et al. (1990) and Shembekar and Naik (1990) investigated the effect of ply lay-up on the notched strength of PW E-glass/epoxy laminates. Eight different lay-ups were studied: $(0)_{4s}$, $(45,0)_{2s}$, $(45,0_{2,45})_{2s}$, $(0_{2,45})_{2s}$, $(45_2,0_2)_{2s}$, $(0,45)_{2s}$, $(0,45_2,0)_{2s}$ and $(45)_{4s}$. 
Holes ranging from 5 to 20 mm in diameter were introduced into 50 mm wide specimens.

Obviously, the differently stacked laminates produced a range of unnotched strengths. In summary, the \((0)_{4s}\) laminate showed the highest strength and the \((45)_{4s}\) laminate showed the lowest strength. The other laminates exhibited strengths approximately midway between the 2 unidirectional specimens. The \((0,45)_{2s}\) and \((45,0)_{2s}\) laminates, in which the plies are thinly dispersed, exhibited the highest notched strengths. The \((0)_{4s}\) laminate showed a similar notched strength. The notched strengths of the other laminates were approximately 15\% lower, except for the \((45)_{4s}\) laminate which was approximately 40\% lower than the stronger materials. The \((45)_{4s}\) and \((0,45)_{2s}\) laminates produced the highest notched to unnotched strength ratios. The lowest strength ratio was given by the \((0)_{4s}\) laminate, at approximately 20-30\% lower than the \((0,45)_{4s}\) lay-up. The remainder of the laminates were all within 10\% of the least notch sensitive lay-ups. The characteristic distances of the WN fracture models were used as a measure of laminate notch sensitivity. The characteristic distance of the ASC, \(a_0\), was shown to vary from approximately 2 to 11 mm for the laminates investigated.

Generally it was seen that dispersing the 0° plies between 45° plies, and clustering 45° plies near the laminate mid-plane improved the notched strength of the materials investigated. This experimental investigation illustrated the difference in notched strength behaviour of laminates made up from the same constituent laminae with different lay-up sequences. The inadequacy of many of the current notched failure criteria to account for this without re-establishing the parameters upon which the models are based was also highlighted.

**2.4.5.2 Laminate Thickness Effects**

The majority of the experimental investigations into the theoretical modelling of notched composite materials has been carried out on thin laminates of up to a few millimetres in thickness. In reality many composite components are significantly thicker. The role of thickness effects on fracture mechanics parameters and notched
strength data has not been well established for composite laminates. Current models do not have any built in thickness parameters.

Harris and Morris (1984) studied the effect of laminate thickness on the fracture of carbon/epoxy non-woven composites. Three laminate configurations were investigated; \([0/\pm45/90]_{ns}\), \([0/90]_{ns}\) and \([0/\pm45]_{ns}\) with thicknesses ranging from 6 to 120 plies. This corresponds to thicknesses of approximately 0.75 to 15 mm. Laminate fracture toughness was measured using centre-cracked tension, compact tension and three point bend methods. The \([0/\pm45/90]_{ns}\), and \([0/90]_{ns}\) laminates showed a decreasing fracture toughness with increasing number of plies. However, the \([0/\pm45]_{ns}\) laminate showed increasing fracture toughness with increasing thickness. In all cases the fracture toughness asymptotically reached a maximum/minimum value in the region of 30-60 plies. The three different methods of fracture toughness measurement gave similar results within each laminate type. It was seen that thin laminate fracture was dominated by delamination and matrix splitting. This was also evident in the surface plies of thicker laminates. However, the interior region of thicker laminates exhibited a self similar fracture, virtually free of delamination. This observation was used to explain the decrease in fracture toughness of the \([0/\pm45/90]_{ns}\), and \([0/90]_{ns}\) laminates with increasing number of plies. The increase in fracture toughness with increasing number of plies shown by the \([0/\pm45]_{ns}\) laminate was explained by fracture and uncoupling of the outer 2 plies (0° and +45°) from the remainder of the laminate.

The centre-cracked tension data were used to investigate the notched strength of the laminates. The notched strength data showed the same trends as the fracture toughness data. The inherent flaw model (IFM) of Waddoups Eisenmann and Kaminski (1971), the PSC and ASC of Whitney and Nuismer (1974), the general toughness parameter of Poe et al. (1980) and the Mar-Lin curve fit model (1977) were used to predict the notched strength. Of the above-mentioned models only the general toughness parameter of Poe et al. was able to use thin laminate data to predict thick laminate behaviour. This might be expected as the toughness parameter of Poe et al. (1980) has been shown to be constant for a range of laminates within the same material system, as discussed in section 2.4.2. However, it was also shown that this
approach was only suitable for centre-cracked notches in laminates exhibiting localised damage zones.

Vaidya et al. (1998) studied the effect of ply thickness on the notched strength of composite laminates. Non-woven carbon/epoxy cross-ply and quasi-isotropic laminates were investigated. The ply thicknesses were varied between 1 and 4 layers of pre-preg corresponding to 0.127 mm and 0.508 mm, respectively. A significant ply thickness effect was seen in cross-ply laminates. The laminates with thicker plies exhibited larger damage zones than the thinner laminates. This produced higher notched strengths as a result of stress relief due to higher levels of damage at the notch tip. As ply thickness was increased a transition of failure mode from crack dominated to delamination induced was observed. Very little increase in notched strength was seen in the quasi-isotropic laminates investigated. However, if the ply thickness was considerably increased the failure mode was seen to change in the same manner as in the cross-ply laminate. The crack tip damage in the materials under investigation was modelled using a two-dimensional FE analysis. The strain energy release rates for the quasi-isotropic laminates were small compared to the cross-ply laminates. This was in agreement with experimental observations of notch edge damage zones. This study effectively illustrates the effect of ply thickness and laminate lay up on the evolution of damage and subsequent failure. The authors identified the need for alternative or modified criteria to be able to take this into account.

No literature was found concerning thickness effects or the effect of the number of fabric layers on the notched tensile strength of woven composites.

2.5 Concluding Remarks

It has been shown in the current literature review that there is a good understanding of the theoretical modelling of the initial undamaged woven fabric laminate properties, with methods varying in the level of structural simplification and computational effort. In general, good agreement was reported with experimental results. Some of these models were also able to be modified to account for the effect of shifting adjacent
layers on laminate properties. The ideal woven fabric composite analysis would be able to predict damage development and the degradation of laminate properties for specified lay-ups and relative lamina shifts. However, the effect of damage on laminate properties is not well understood. Present models are often complex and involve a high level of approximation due to the complex weave architecture and associated damage morphology. It is clear from the work of Marsden (1996) that cracking damage in woven composite materials has a complex morphology. This morphology needs to be fully understood before the effect of damage can be modelled more accurately.

There are many different models available to predict the notched strength of composite laminates. The majority of these models have been applied to non-woven composites, although some have been shown to perform well for woven composite materials. The semi-empirical models discussed generally result in excellent notched strength predictions due to the curve fitting nature of the techniques used. However, these approaches require preliminary testing for each laminate and notch geometry considered. Some of the more recent models incorporating damage effects into the strength analyses are more attractive to the designer as they provide a much more realistic physical concept of progressive failure ahead of the notch. However, these approaches require further investigation across a range of laminate and notch configurations. The FE methods available provide detailed analyses of the notched failure of composite laminates, but are too complex to be useful design tools.

The effect of laminate and notch configuration on the notched strength of composites has been well documented. However, none of the semi-empirical models are able to account for effects such as laminate thickness or stacking sequence. Hence, the model parameters must be pre-determined in each case. The notched fracture models incorporating notch edge damage do not have any built-in criteria to account for thickness or stacking sequence. However, the damage zones are controlled by fracture mechanics-based parameters which may account for the above-mentioned effects.

This thesis aims to investigate further the damage accumulation and failure in woven composites. The development of damage morphologies in plain weave glass/epoxy
laminates and associated elastic property degradation are investigated. It has been shown in section 2.3.3 that the relative shift of fabric layers in laminates influences elastic and strength properties. The change in damage morphology for these various relative lamina shifts in 2 layer laminates is investigated in this study. The notched strength study undertaken investigates the effect of the number of fabric layers with variation of fabric reinforcement, notch size and notch shape. Four of the notched fracture models discussed in this chapter have been applied to the materials investigated. The ability of these models to account for the variation in number of fabric layers, reinforcement type and notch configuration is investigated. Most of the notched strength analyses presented in this chapter ignore details of the laminate structure and notch edge damage zones. A detailed study of the damage developed adjacent to notches is presented in this thesis. This enables a discussion between laminate properties, notched model results and notch edge damage to be presented.
Figure 2.1. Common weave styles, (a) \( n_w = 2 \), plain weave, (b) \( n_w = 3 \), twill weave, (c) \( n_w = 4 \), crow foot satin weave, (d) \( n_w = 8 \), eight harness satin weave (Ishikawa and Chou, 1982).

Figure 2.2. Principle of the mosaic model (a) repeating unit of an 8HS fabric, (b) basic cross-ply, (c) parallel model, (d) series model (Ishikawa and Chou, 1982).
Figure 2.3. Crimp model unit cell (Ishikawa and Chou, 1982).

Figure 2.4. Bridging model (a) repeat unit of 8HS weave fabric, (b) simplified repeat unit of 8HS weave fabric, (c) assembly of unit cell (Ishikawa and Chou, 1982).
Figure 2.5. Comparison of one dimensional closed form models (Ishikawa et al., 1985).

Figure 2.6. Plain weave lamina unit cell used by Naik and Shembekar (1992a).
Figure 2.7. Four plain weave laminate configurations with varying relative lamina shifts (Shembekar and Naik, 1992).

Figure 2.8. Representation of two layer woven fabric laminate for interlaminar shear stress model (Yurgartis and Maurer, 1993).
Figure 2.9. Idealised Nesting configurations investigated by Breiling and Adams (1996); (a) stacked, (b) split-span and (c) diagonal.

Figure 2.10. Graph showing experimental data and model predictions of normalised modulus as a function of damage density for a 2 layer 8HS GFRP laminate; (a) with untwisted tows and (b) with twisted tows (Marsden, 1996).
Figure 2.11. Simplified unit cell used to model effect of damage in PW laminates (Kuo and Chou, 1995).

Figure 2.12. Principle of damage region in WEK fracture model (Waddoups, Eisenmann and Kaminski, 1971).
Figure 2.13. Stress distributions for two circular holes in an infinite plate (Whitney and Nuismer, 1974).

Figure 2.14. Schematic representation of the point stress criterion for a laminate containing a circular hole (Whitney and Nuismer, 1974).
Figure 2.15. Schematic representation of the average stress criterion for a laminate containing a circular hole (Whitney and Nuismer, 1974).
Figure 2.16. Strength reduction in quasi-isotropic glass/epoxy laminates predicted by (a) the point stress criterion and (b) the average stress criterion (Whitney and Nuismer, 1974).
Figure 2.17. Schematic diagram illustrating the principle of the notched strength prediction technique of Hitchen et al. (1994).

Figure 2.18. Dugdale/Barenblatt cohesive zone (Barenblatt, 1962).
Figure 2.19. Assumed linear decreasing relationship between cohesive stresses and the crack opening for Dugdale/Barenblatt-type analysis.

Figure 2.20. Normalised stress versus fictitious crack length for carbon/epoxy specimens, (Aronsson and Backlund, 1986a).
Figure 2.21. Stress-crack opening relationships for (1) constant, (2) linearly decreasing and (3) general models (Aronsson and Bäcklund, 1986b).

Figure 2.22. Modified elastic stress distribution in a notched composite laminate (Afaghi-Khatibi and Ye, 1996).
Figure 2.23. Second step of fictitious crack growth (Afaghi-Khatibi et al., 1996a).

Figure 2.24. Effect of hole diameter on notched laminate strength for a cross-ply carbon/epoxy laminate showing experimental data and ECGM model predictions (Afaghi-Khatibi et al., 1996a).
Chapter 3: Materials and Experimental Methods
3.1 Introduction

This chapter describes the materials investigated and the practical work undertaken in this project. After introducing the composite materials investigated, their manufacture and sample preparation are discussed. The methods used to determine the various laminate properties investigated are then described, including the relevant testing standards followed. Finally, the various techniques used to observe and quantify the damage accumulated in the composite laminates are presented.

3.2 Materials selection

This section describes the materials investigated in this study and the reasons behind their choice. The two woven fabrics selected are discussed initially, followed by the resin system employed.

3.2.1 Laminate Reinforcement

Two types of E-glass fabric reinforcements have been used in this study; a plain weave (PW) fabric and an eight harness satin (8HS) weave fabric. These weave styles represent the two extremes of common two-dimensional woven fabrics with respect to the degree of crimp in their structures. Styles of woven fabric architecture have been discussed in section 2.2.

The reinforcement used in manufacturing the PW glass reinforced epoxy laminates was a Fothergill Engineered Fabrics Ltd., YO94 E-glass PW fabric. This fabric is woven from similar warp and weft tows consisting of 3 finer bundles twisted together. Each of the finer bundles has a tex value of 22. The cloth has a weight of 182 g/m² and thickness of 0.15 mm. It is marginally unbalanced with 142 and 126 ends and picks per 100 mm, respectively. Figure 3.1 shows a scanning electron microscopy (SEM) photomicrograph of the cloth. Measurements of the cloth confirmed that the warp and weft tow repeat, i.e. the average distance between the midpoint of warp tows, is approximately 0.8 mm. It can be seen from figure 3.1 that the weave is open in nature.
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The tow width is approximately 0.6 mm and the gap between adjacent tows is approximately 0.2 mm.

Similarly, the reinforcement used in manufacturing the 8HS weave glass reinforced epoxy laminates was a Fothergill Engineered Fabrics Ltd., Y0227 E-glass 8HS weave fabric. This cloth is woven from the same twisted tows as the PW fabric. The cloth has a weight of approximately 297 g/m² and thickness of 0.23 mm. Again, it is marginally unbalanced with 224 and 213 ends and picks per 100 mm, respectively. Figure 3.2 shows a SEM photomicrograph of the 8HS weave cloth (note that the magnification of the SEM images of the two cloths are different). Measurements of the cloth confirmed that the warp and weft tow repeat, i.e. the average distance between the midpoint of warp tows is approximately 0.45 mm. The 8HS weave cloth has a denser structure than the PW cloth and narrower tows with smaller warp and weft tow repeat distances. These differences in dimensions are probably due to the deformation of tows necessary to obtain the highly crimped PW structure.

3.2.2 Matrix

The matrix used was a Shell Epikote 828 (Bisphenol-A) epoxy resin with a Shell epicure nadic methyl anhydride (NMA) curing agent and Ancamine K61B accelerator. This resin system was chosen as it has a refractive index similar to that of E-glass fibres and forms a good interfacial bond with the fibres, resulting in a transparent material. It also readily wets glass fibres, aiding laminate fabrication.

The resin was mixed in the following proportions:

100 g of resin
60 g of curing agent
4 ml of accelerator.
3.3 Laminate Fabrication

Laminates were fabricated using a modified wet lay up process. Laminates of 2, 4, 6 and 8 layers were prepared. The fabric layers were marked using a fine permanent marker pen along three tows in each orthogonal direction. This produced a grid of 200 mm by 200 mm to enable any shear in the cloths to be identified and eliminated during fabrication, ensuring alignment in the orthogonal directions. The roll side and direction were also identified. The fabric layers were stacked so that the warp, or roll, direction was aligned in the direction of loading. In the case of the 8HS weave fabric the $0^\circ$ dominated side was positioned as the surface layer of the laminates. This $0^\circ$ dominated side was the internal roll side and was labelled side 1. For consistency the same procedure was applied to the PW fabric.

The resin was thoroughly mixed and degassed in a vacuum oven at 50°C prior to laminating. The laminates were laid-up between 2 flat glass plates of approximately 260 mm square. Prior to use these two plates were cleaned thoroughly using a razor blade and methanol. A thin layer of mould release wax was then applied to the surface of the plates prior to being preheated in an oven at 50°C. One of these plates was placed on a preheated aluminium plate, also at 50°C, to ensure that this temperature was maintained for as long as possible during the laminating process to assist resin flow.

A square of pink silicone release agent impregnated melanex was then placed onto the glass plate. The melanex covered plate was wet with resin before a layer of glass cloth was carefully laid down in the correct orientation. This was allowed to wet for thirty seconds before a small reservoir of resin was applied to the centre of the fabric and a layer of melanex without release material was applied. The cloth was aligned with a square grid marked on a transparent polymer film, similar to that on the fabric reinforcement. Once aligned, any remaining air pockets were carefully forced out with some of the excess resin using a straight edged tool. This procedure was repeated with another cloth using a third preheated glass plate. In this case it was not necessary to wax the glass plate or use release material impregnated melanex as neither surface would be the final laminate surface. The top melanex layers were then removed from
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each cloth and the two cloths were brought together, with more excess resin added between them, ensuring like sides and directions were stacked together to maintain overall symmetry about the mid-plane. Once aligned, any remaining air pockets were carefully forced out as before.

Additional layers of fabric reinforcement were stacked onto the original glass plate following the procedure described above until the required number of layers was reached. The top melanex layer was then removed and more resin was introduced to the centre of the laminate. A square of pink release material impregnated melanex was then applied to the laminate. A final check on alignment was performed before expelling any remaining air. The second preheated wax plate was then placed onto the laminate. The laminate was placed in the centre of a level plate in the oven. Cast iron weights were then applied at a pressure of 4.92 kN m\(^{-2}\) for the two layer laminates and up to 8.61 kN m\(^{-2}\) for the 8 layer laminates. The laminates were cured at 100°C for 3 hours.

The method described above produced excellent quality 4, 6 and 8 layer laminates. However, initial problems were encountered producing the 2 layer laminates, especially the very thin plain weave laminates. The resulting laminates had a rippled surface layer, attributed to the thin nature of the reinforcements. The top layer of release material impregnated melanex would not remain in contact with the waxed glass plate during the curing process. To overcome this problem a modification of the previously described procedure was devised. Using silicone grease, a square of pink release material impregnated melanex was adhered onto each of the two glass plates to ensure a flat surface profile. The excess silicone grease was removed by carefully forcing it out using a straight edged tool. The 2 glass plates were not pre-heated in this case. The first plate was allowed to warm through on a pre-heated aluminium plate before the resin was applied. The laminating procedure was then identical to the previously described method until the final step. Instead of applying the final sheet of release material impregnated melanex, the second glass plate with release material impregnated melanex was lowered directly onto the reservoir of excess resin at the laminate centre. A little pressure was applied to the top glass plate to begin to force out the excess resin.
and any remaining air. The laminate was then placed into the oven and the weights were applied prior to curing.

After inspection the cured laminate edges were trimmed parallel to the orthogonal fabric axes using a water-cooled diamond saw. Test coupons of 20 x 200 mm were prepared for the unnotched PW laminate quasi-static investigations. Coupons of 25 mm x 230 mm were used for the unnotched 8HS weave samples, all notched samples and also the single edge notch (SEN) tests. The samples were post-cured at 150°C for 3 hours held between glass plates to prevent warping.

3.4 Volume Fraction Determination

Volume fractions were obtained by the matrix burn off method. Representative laminate samples were cut into squares of approximately 15 mm x 15 mm and weighed with a balance accurate to 10^-4 g. The laminate samples were placed in pre-weighed porcelain crucibles and held at 600°C for three hours. After cooling, the crucibles containing the laminate reinforcement were re-weighed. The laminate fibre volume fractions were calculated from the relative masses and the densities of resin and reinforcement:

\[
V_f = \frac{(M_f / \rho_f)}{(M_f / \rho_f) + (M_m / \rho_m)}
\]

(3.1)

where \( V_f \) is the volume fraction of glass fibres in the laminate, \( M_f \) and \( M_m \) are the masses of fibre and matrix, respectively, and \( \rho_f \) and \( \rho_m \) are the densities of fibre and matrix, respectively. The density of the glass fibres, \( \rho_f \), is 2.56 g cm\(^{-3}\) (from Fothergill Engineered Fabrics Ltd.). The density of the epoxy resin matrix, \( \rho_m \), was measured using the Archimedes' immersion principle. A post-cured cube of resin was weighed in air and then re-weighed suspended in water. The density was then calculated from the following expression:
where $\rho_s$ is the density of the specimen, $M_s$ is the mass of the specimen, $\rho_w$ is the density of water and $\Delta M_s$ is the difference in the mass of the specimen when weighed in air and in water. The calculated density of the epoxy resin was 1.21 g cm$^{-3}$, in agreement with Marsden (1996).

3.5 Sample preparation

This section describes the methods used to prepare the post-cured coupons for mechanical testing. This involves end tabbing the samples and the introduction of discontinuities for notched investigations.

3.5.1 End tabbing

In order to transfer the load effectively from the grips of the mechanical testing apparatus to the samples, suitable end tabs must be affixed. Using grit paper, a 50 mm length at each end of the coupons was abraded to produce a roughened surface suitable for bonding. This was cleaned with a light wipe of a water based alkaline surface cleaner. Aluminium end tabs of 20 mm x 50 mm were etched for 30 minutes at 65°C in a concentrated sulphuric acid/sodium dichromate solution. After drying, the aluminium tabs were bonded to the coupon ends using Permabond F241 two part acrylic adhesive, leaving a 100 mm or 130 mm gauge length depending on the size of the original coupons.

3.5.2 Machining of Notches

Circular centre holes of 2.5, 5 and 10 mm in diameter were drilled in each of the laminate types. This was achieved using a hardened steel drilling jig to ensure that the holes were centred and that the exit of the drill bit was as clean as possible. A similar jig was used to introduce effective elliptical notches. Two 1 mm diameter circular notches were drilled and joined using a jewellers saw producing effective elliptical
notches of 2.5, 5 and 10 mm in width. The effective ellipse minor axis half length, \( b \), is given by:

\[
b = \sqrt{ap} \quad (3.3)
\]

where \( a \) is the major axis half length and \( p \) is the notch root radius. The drill bits used were low helix, high speed steel bits for use with thermosetting plastics. The acute point angle reduces end pressure and minimises burning of the matrix and break away of the material as the drill exits the laminate. The notched samples were inspected after drilling to ensure that excess damage was not introduced.

Notches, or cracks, for the single edge notch (SEN) toughness samples were introduced using a jewellers saw with a blade width of approximately 0.35 mm. The edge notches were sharpened using a fresh scalpel blade. The blade was inserted in the SEN and tapped with a metal rule, using a similar force for each sample.

3.6 Quasi-Static Mechanical Testing

The materials in this study have been investigated under quasi-static tensile loading. This section discusses the apparatus and techniques used to monitor the response of the laminates investigated under these conditions.

3.6.1 Mechanical Testing Techniques

All coupons were tested using either an Instron 1175 or 1196 machine at a constant crosshead speed of 0.5 mm per minute. Both 5 kN and 100 kN load cells were used. Load and strain data were recorded at one second intervals using a PC-based data-logging package.

The in-plane tensile properties of the laminates investigated in this study were determined following ASTM standard D 3039 whenever possible. The quasi-static tests investigating Young’s modulus reduction involved loading up the coupons to...
progressively higher levels of strain and recording the level of damage and Young's modulus at each stage.

The in-plane shear stress-strain response of the laminates was determined according to ASTM Standard D 3518/D 3518M. This is based on the uniaxial tensile stress-strain response of a ±45° laminate. This was performed for 2, 4, 6 and 8 layer 8HS weave laminates and 2 and 8 layer PW laminates.

The critical energy release rate, $G_c$, of the laminates was determined using SEN specimen configuration. Compact tension specimens were originally considered, however, the thin nature of the 2 layer laminates caused concern over the possibility of compression buckling. The methods recommended for the toughness determination of metallic materials in ASTM E 399-90 were followed where appropriate.

Before toughness measurements were made, it was necessary to establish the relationship between the specimen compliance and SEN length. The compliance calibration was achieved by recording load-extension profiles at incrementally increased SEN lengths from 0 mm to 14 mm, at 1 mm intervals. The edge notches were advanced using a jewellers saw. This data was averaged over 2 or 3 samples before a fourth order polynomial was fitted. This equation was then differentiated to obtain an expression for the relationship between the rate of change of compliance with SEN length ($dC/da$) and SEN, or crack, length ($a$).

The toughness measurements were performed at three SEN lengths of 4, 8 and 12 mm. Again, the edge notches were cut using a jewellers saw after which they were sharpened using a scalpel blade. The initial set of toughness tests were performed without recording extension. In this case the maximum load, $P_{max}$, was used as a fracture load, $P_Q$. The second batch of testing recorded load and extension data. This was done to investigate the degree of non-linearity present in the load-extension responses of the laminates. In this case the fracture load, $P_Q$, was defined as the point at which the compliance of the sample had increased by 5%. This corresponds to the intersection of the load-extension profile with a straight line drawn with a
compliance of 5% more than the original elastic response. If the compliance of the sample did not increase by 5% before fracture, then the maximum load was taken as $P_0$. The toughness of the laminates, of thickness $B$, were then found using the following expression:

$$G_c = \frac{P_0^2}{2B} \frac{dC}{da} \quad (3.4)$$

### 3.6.2 Strain Measurement

Both strain gauges and an extensometer were used to monitor the applied strain in the materials investigated in this project.

Measurements Group Inc. CEA-06-240UZ-120 alloy strain gauges were bonded at the centre of samples using a cyanoacrylate adhesive. The gauges had an active gauge length of 6 mm. The centre of one side of the sample was lightly abraded using a glass fibre abrasive pen. Centre lines were marked on the coupon using a sharp pencil before the surface was cleaned using a water based alkaline surface cleaner. The coupon was taped to a piece of 1 mm square graph paper to aid alignment. The strain gauge was laid down on a clean surface and carefully picked up with adhesive tape. The gauge was positioned accurately, parallel to the required direction of strain measurement, and lightly applied to the surface of the sample with the adhesive tape. The tape was then carefully peeled back to expose the bonding surface. A small bead of cyanoacrylate adhesive was applied to the coupon just below the gauge and the adhesive tape was immediately replaced onto the coupon using one firm stroke from the bottom of the gauge ensuring complete wetting of the gauge surface. Firm finger pressure was applied to the strain gauge for one minute.

For the quasi-static testing of unnotched samples longitudinal and transverse strain gauges were bonded at the centre of coupons. For the strain measurement of notched strength samples, strain gauges were positioned 30 mm from the end tabs. This location was selected to ensure that the gauge was remote from stress concentrations due to notches and end tab effects. The response of a gauge in this position was
compared to a centre mounted gauge on an unnotched sample. This was also compared to a gauge mounted 30 mm from the end tabs of a sample with a 10 mm diameter notch. This analysis was performed for both 2 and 8 layer laminates. All stress-strain responses were found to be in good agreement. The Young’s modulus data for the two gauge positions were found to be within 2% of each other.

The applied strain in quasi-static tensile samples was measured using bonded resistance strain gauges in conjunction with strain indicator boxes. After inspection of the stress-strain response of the laminates it was decided to perform a linear regression between 1000 and 4000 microstrain to obtain moduli and Poisson’s ratio data.

The effect of transverse sensitivity of the gauges used in this study was considered. Transverse sensitivity refers to the behaviour of a strain gauge in a direction perpendicular to the primary sensing direction of the gauge (Measurements Group Tech. Note, 1982). Manufacturers design their strain gauges to compensate for transverse sensitivity for materials with a Poisson’s ratio of 0.285. However, a material with a different Poisson’s ratio will always give some degree of transverse sensitivity error which may require correction.

Considering a two gauge 0°/90° configuration the corrected strains are given by the following expressions:

\[
\varepsilon_x = \frac{(1-v_0 K_t) (\hat{\varepsilon}_x - K_t \hat{\varepsilon}_y)}{1 - K_t^2} \tag{3.5}
\]

\[
\varepsilon_y = \frac{(1-v_0 K_t) (\hat{\varepsilon}_y - K_t \hat{\varepsilon}_x)}{1 - K_t^2} \tag{3.6}
\]

where \( \varepsilon_x \) and \( \varepsilon_y \) are the corrected strains along the x and y directions, respectively, and \( \hat{\varepsilon}_x \) and \( \hat{\varepsilon}_y \) are the uncorrected strains along the x and y directions, respectively. The term \( v_0 \) is the Poisson’s ratio of the material on which the manufacturer measured the gauge factor and \( K_t \) is the gauge transverse sensitivity coefficient.
The transverse sensitivity errors were calculated for gauges mounted both longitudinally and transversely with respect to the specimen axis. Longitudinal gauges showed errors of approximately 0.1 %. However, the transverse gauges showed errors in the region of 2.5 %. Hence, the data for the transverse gauges were corrected for transverse sensitivity, whereas the errors for the longitudinal gauges were considered negligible.

In the case of the SEN toughness samples a 75 mm gauge length extensometer was used to measure the displacement. This gauge length was chosen to ensure that the extension measured was remote from the SEN and also far enough away from the sample ends to eliminate the possibility of end tab effects. Epoxy glue spots were adhered to the specimen surface to aid grip of the knife-edges of the extensometer. The extensometer was held tightly in place using rubber bands. Linear regression was performed between 500 and 1000 micro-strain to obtain the compliance of the laminates investigated.

The sample dimensions and strain gauge and extensometer positions are shown schematically in figure 3.3.

3.7 Damage Observation

Observations of damage accumulation were facilitated by the transparent nature of the laminates. Conventional light photography was used to observe damage, as opposed to the more complex techniques, such as dye-penetrant enhanced X-radiography, necessary when working with carbon fibre composites.

3.7.1 In-situ Plan View Damage Observation

The term plan view damage has been used to describe the damage seen when viewed perpendicular to the plane of the laminate. The plan view damage development was observed during testing by in-situ photography. The camera was aligned carefully to ensure it was perpendicular with respect to the sample. A light source was positioned
behind the sample to produce contrast between damaged and undamaged regions of
the laminate. Samples were unclamped from the grips of the testing machine prior to
being photographed.

3.7.2 Polished Edge Sections

The through thickness profile of the damage was observed in polished edge sections
along the plane of the loading direction. Coupons were sectioned using a water-
cooled diamond saw and mounted in a two part Epofix transparent resin. The resin
mounts were polished in two stages using a Struers Planopol-2 rig fitted with a
pedemax-2 rotating stage according to the schedule shown in table 3.1. An ultrasonic
bath and methanol wash was used between each polish to ensure no cross
contamination of abrasives occurred.

Polished edge sections were observed using a Zeiss Axiophot light microscope in
transmitted light mode. This clearly showed the weave structure and damage in the
samples, as cracks and delaminations appeared dark in contrast to the transparent fibre
and matrix regions.

3.7.3 Scanning Electron Microscopy

Plan view photographs show matrix damage and give an indication of the extent of
delamination and fibre failure at notch edges. However, to observe fibre failure
properly it is necessary to burn off the matrix, exposing the reinforcement.
Conventional light microscopy does not have sufficient depth of focus to image the
undulating glass fibre reinforcement. Therefore, a scanning electron microscope
(SEM) was chosen to study the damaged reinforcements. A variable pressure Hitachi
environmental S300N SEM was used, eliminating the need to gold coat the samples.
Imaging was performed in back-scattered electron mode, giving some atomic number
contrast information.
3.7.4 Deply technique

Plan view micrographs show the extent of the matrix damage in the plane of the surface of the transparent laminates. However, very little information can be gained concerning the profile of this damage layer by layer throughout the thickness of the laminate. Polishing longitudinal edge sections parallel to the loading direction does reveal the location of damage layer by layer. However, the exact position of the polished plane cannot be predetermined and the damage shown may not be representative of the laminate.

As an additional tool for damage investigation, the damage developed adjacent to the notches in the laminates was also studied using the deply technique, as described by Freeman (1982). This method involves separating the plies so that the plan view damage profile may be studied layer by layer. In order to identify damage sites, a marker must be applied to the laminates prior to separation of the plies. A 9.2 weight \% solution of gold (III) chloride in diethyl ether was used to penetrate the damaged region. This was carefully deposited in the notches of the damaged laminates, backed by adhesive tape to contain the solution. However, due to the thin nature of some of the laminates and small notch sizes the solvent was prone to rapid evaporation. Hence, the solution was continuously applied to laminates so that it was in contact with the damaged region for approximately 30 minutes, in accordance with the procedure described by Freeman (1982). As a measure to ensure that the penetrant infiltrated all of the fine damage sites connected to the notch, the solution was also applied to the samples under pressure. This was achieved using a Struers Epovac vacuum penetration unit. A rubber O-ring was adhered around the notch using silicone rubber to produce a reservoir in which the solvent could sit. The laminates were then placed in the vacuum chamber and partially evacuated. The marker solution was then deposited, drop by drop, into the reservoir using a screw tap fitted to the top of the infiltration unit. The chamber was then slowly let up to atmospheric pressure, forcing the solution into damage sites connected to the notch. The surfaces of the laminates were then wiped to remove any excess penetrant. The samples were heated to approximately 60°C for 30 minutes to evaporate any ether solvent remaining in the damage sites, leaving the gold chloride marker deposit.
The laminates were then placed in a furnace at 450° C to partially pyrolyse the epoxy resin so that the layers were able to be separated. As a guide, the 2 layer laminates were left in the furnace for 15 minutes. For thicker laminates an extra 5 minutes was allowed for every extra 2 layers, i.e. 30 minutes for the 8 layer laminates. In practice the laminates were continually checked in the furnace to monitor their progress. When sufficiently pyrolysed the layers were carefully separated using tweezers and placed in labelled containers. Freeman (1982) proposed that the epoxy resin was only partially pyrolysed so that the layers of a cross-ply laminate were able to be separated and still remain intact. A thorough investigation of this technique for the laminates of interest here found that for the woven reinforcements the gold chloride marker was best seen when the majority of the resin was burnt off. Hence, laminates were left in the furnace for a further 10 minutes after separation of the plies. The resin was not totally burnt off to ensure the structural integrity of the reinforcement and to avoid potential deterioration of the gold chloride marker.

The damage throughout the laminates was then be observed using a low magnification stereo-microscope. The gold chloride marker was revealed by illuminating the sample from the side. The damage was recorded, layer by layer, using colour photography.

3.8 Damage Quantification

In order for the damage observed in the woven laminates to be incorporated into elastic property reduction modelling it must be quantified. Damage in the form of transverse cracking in cross-ply laminates may be expressed as a function of crack spacing as each transverse crack often extends the full width and depth of the transverse ply. In a transparent glass/epoxy system, this spacing may be obtained by a linear plan view crack density count. This is simply the number of cracks intersecting a line on the laminate surface per unit length of sample. However, in a woven laminate the through thickness crack length is variable due to the elliptical tow cross section and the cracks do not span the entire sample width. Therefore, a linear plan view crack density count is not an accurate measure of damage density. In order to accurately quantify this
damage the total through-thickness length of crack per unit sample length must be measured. That is the cumulative length of transverse cracks in the xz-plane, with the x-direction being the loading direction and the z-direction being the through-thickness dimension of the laminate. This is a time consuming process involving sectioning the laminate, polishing and measuring each individual crack length. In order to eliminate the need to do this for every sample investigated it is possible to calibrate the linear plan view crack density with the more accurate through thickness crack length per unit sample length. This is shown schematically in figure 3.4. With accurate measurements during sample preparation, a plan view array of cracks may be viewed in cross section. Following the method of Marsden (1996), a section of approximately 20 mm x 8 mm was cut from the damaged laminate using a water-cooled diamond saw. The sample was marked and photographed, an example of which can be seen in figure 3.5, before being encapsulated in a transparent resin mount of known dimensions. The dimensions were again recorded after polishing enabling the sectioned plane to be identified accurately on the plan view photograph. This is shown schematically in figure 3.6. Performing these measurements at several levels of strain enabled the relationship between the above-mentioned damage densities to be determined. This enabled a plan view crack density, simply the number of transverse cracks intersecting a line perpendicular to the loading direction per unit length (mm\(^{-1}\)), to be calibrated against the total through-thickness length of transverse crack in the equivalent polished edge section per unit sample length (dimensionless).

3.9 Concluding Remarks

This chapter has described the fabric reinforcement and matrix materials used to manufacture the composite specimens investigated in detail. The techniques used to determine the laminate properties and the extent of damage in the laminates have also been described. In the next chapter, some of these techniques for damage monitoring are applied to the measurement and characterisation of damage accumulation in a PW laminate.
Chapter 3: Materials and Experimental Methods

<table>
<thead>
<tr>
<th>Paper grade</th>
<th>Load</th>
<th>Time (sec.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>320</td>
<td>4</td>
<td>90 (or until flat)</td>
</tr>
<tr>
<td>500</td>
<td>4</td>
<td>90</td>
</tr>
<tr>
<td>800</td>
<td>3</td>
<td>90</td>
</tr>
<tr>
<td>1200</td>
<td>3</td>
<td>90</td>
</tr>
<tr>
<td>2400</td>
<td>3</td>
<td>90</td>
</tr>
<tr>
<td>4000</td>
<td>3</td>
<td>60</td>
</tr>
</tbody>
</table>

Table 3.1a. Water lubricated silicon carbide grit paper grinding stage.

<table>
<thead>
<tr>
<th>Diamond spray</th>
<th>Load</th>
<th>Time (sec.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>6 µm</td>
<td>3</td>
<td>180</td>
</tr>
<tr>
<td>3 µm</td>
<td>3</td>
<td>180</td>
</tr>
<tr>
<td>1 µm</td>
<td>3</td>
<td>180</td>
</tr>
</tbody>
</table>

*Repeated 3 times.*

Table 3.1b. DUR cloth diamond polishing stage with alcohol-based lubricant.
Figure 3.1. Scanning electron micrograph of plain weave fabric reinforcement

Figure 3.2. Scanning electron micrograph of eight harness satin weave fabric reinforcement.
Figure 3.3. Schematic edge view of specimen geometry for (a) unnotched plain weave quasi-static tests and (b) notched strength and single edge notch tests.

Figure 3.4. Schematic representation of plan view damage and longitudinal edge section damage morphologies.
Figure 3.5. Photograph of a damaged laminate section used for crack density quantification.

Figure 3.6. Schematic representation of method used to locate sectioned plane in polished edge section.
Chapter 4: Damage Accumulation in Plain Weave Laminates
4.1 Introduction

The damage development in woven fabric composite materials is often complex and highly dependent on weave architecture (Marsden, 1996). As discussed in Chapter 2, an understanding of damage mechanisms is required if subsequent laminate failure and failure analyses are to be studied in detail. This chapter investigates the damage development and the associated effect on Young’s modulus in 2 layer laminates fabricated using the PW reinforcement. A similar detailed damage study has been performed by Marsden (1996) on the 8HS weave laminates also of interest to this study.

The PW laminate properties are presented initially, followed by observations on the development of damage under quasi-static loading. This was studied using both plan view and polished edge section photomicrographs. The damage quantification is then presented and incorporated into simple Young’s modulus reduction modelling. Throughout this chapter, the damage and its subsequent effects on laminate properties in the PW and 8HS weave laminates are compared.

4.2 Laminate Properties

Laminate properties for the 2 layer PW laminates are given in table 4.1. The measured laminate Young’s modulus of 18.7 GPa is comparable to data reported for similar materials (Naik and Shembekar, 1992c; Marsden, 1996). The variation in Young’s modulus has been attributed mainly to local variations in laminate thickness due to nesting and the thin nature of the cloth. The laminate strength is approximately 20% lower than that reported by Marsden (1996) for an equivalent 8HS weave composite. This is probably partially due to the slightly lower fibre volume fraction reported for the current material. However, the lower strength may also be explained by the findings of Roy (1996) who found that crimp regions in a composite laminate act as sites for the initiation of fibre fracture, significantly reducing laminate strength. The current PW laminates obviously contain a much higher degree of crimp than the 8HS weave laminates investigated by Marsden (1996). Crack initiation and failure strains are comparable to those found by Marsden (1996) on a similar 8HS weave material. The
relatively low fibre volume fraction of 36% is attributed to the open nature of the weave and low pressure used in fabrication. A two layer plain weave fabric composite will also contain relatively large pure resin pockets at the surface due to the undulation of the tows.

4.3 Damage Development

Under quasi-static loading the initial form of damage seen was transverse cracking in the weft tows at approximately 0.6% strain. These transverse cracks are short in length (approximately 0.2 mm) and appear to initiate randomly throughout the laminate. This can be seen in the plan view photograph in figure 4.1. Photomicrographs of polished edge sections show the initial damage occurs near the tow centre (Fig.4.2). This is in contrast to the work of Marsden (1996) where transverse cracking was seen to initiate at the tow ends. This difference is believed to be due to the shape of the tow cross section. The PW cloth used in this work has an elongated tow cross section that tapers off at either end. In contrast, the 8HS weave cloth used by Marsden (1996) has a lower aspect ratio and is more rectangular in shape. It is proposed that in the case of the 8HS weave laminates, the damage initiates at the tow ends due to stress concentrations and that in the PW laminates this stress concentration effect is not as pronounced due to the tapering of the tow ends.

At higher applied strains, plan view photomicrographs showed that the transverse cracking was aligned at approximately 5-10° to the horizontal weft tows, as shown in figure 4.3. This was also found in the work of Marsden (1996) and was thought to be due to the cracks following the line of the twist within the tows. Polished edge sections revealed that the transverse cracking also initiated towards the tow ends at the higher applied strains (figure 4.4). The crack path was often complicated and was diverted at the interfaces of the 3 twisted bundles within each tow which was also observed by Marsden (1996). The average crack length at saturation was approximately 0.6 mm, roughly equivalent to the inter-crimp distance, as indicated in the plan view photograph in figure 4.5. This saturation crack length is shorter than that seen by Marsden (1996) in the 8HS weave laminates. However, the inter-crimp distance is larger in the 8HS weave fabric. Polished edge sections revealed that, at saturation, a reasonably regular
crack spacing within the tows of approximately two tow thicknesses was found (figure 4.6). The damage was confined to the tows and did not extend into the matrix, again similar to the findings of Marsden (1996).

Interestingly, there were three types of crack morphology that could be found in any one laminate. The use of edge sections has shown the crack morphologies to be dependent on the local relative position of the cloths as follows.

A. Banded transverse cracking in weft tows.

The plan view image here shows damage contained in horizontal bands perpendicular to the loading direction (figure 4.7a). These damage bands are approximately 0.5 mm wide (in the loading direction), with damage free bands of approximately 0.3 mm. This damage occurs when the two cloths lie in phase with one another, as shown in figure 4.7b and indicated schematically in figure 4.7c. The two weft tows are superimposed resulting in damage free bands due to the open nature of the weave. Measurements of the PW fabric geometry in Chapter 3 confirm that these damage bands are a result of the superposition of fabric layers. The tow repeat distance of approximately 0.8 mm is equivalent to the combined width of the damage and damage-free bands.

B. Banded transverse cracking in weft tows with a regular array of delaminations.

The plan view damage in this case is similar to case A, but with an additional array of damage features orientated in the loading direction (Fig. 4.8a). These damage features are approximately 0.5 mm in length and were seen to initiate at approximately 1.5 % strain. Polished edge sections revealed that the two cloths were 180° out of phase (figure 4.8b and figure 4.8c). The transverse cracking is similar to case A since the two sets of weft tows are still superimposed. The damage features orientated in the loading direction are in fact delaminations which occur at sites where two warp tows touch, i.e. every 360° phase difference. This leads to a delamination at each alternate warp/weft cross over point. The delaminations were dependent on laminate consolidation and were only found if the two warp tows were touching.
C. Regularly distributed plan view transverse cracking in weft tows.
The plan view damage in this case appears to be similar to the damage reported by Marsden (1996) for similar 8HS weave laminates. This is shown in figure 4.9a. This damage morphology results when the cloths are at some intermediate phase difference between the above two cases as shown by polished edge section photomicrographs and schematically in figure 4.9b and figure 4.9c, respectively. The weft tows in adjacent fabric layers are no longer superimposed and, when viewed in plan view, overlap, resulting in no through-thickness damage free regions.

Similarities in cracking morphology can be found for all three cases. The transverse cracking appears to be dominant at cross over points between the warp and weft tows. This can clearly be seen for both cases A and B in figures 4.7a and 4.8a, respectively. The delaminations seen clearly in case B may be found in cases A and C at high damage densities, although they do not occur with the regularity seen in case B. Such a delamination can be seen in figure 4.7b. However, this delamination is located at the interface of a warp and a weft tow. This was also observed by Marsden (1996) at higher levels of applied strain. Of the three crack morphologies, the banded plan view damage morphology (case A) is dominant, suggesting that the cloths tend to nest during consolidation. It was shown in chapter two that the relative cloth shift influences laminate properties. It has now been shown that it can also influence the damage morphology, and even the type of damage found in the PW laminates. The effect of these different damage morphologies on laminate properties are unknown because it is not possible to manufacture a coupon which contains one of these morphologies throughout its length and width.

4.4 Crack Density Quantification

In the current woven system, a plan view crack count is not sufficient to accurately determine damage densities. Cracks may be shielded when two tows are superimposed on one another and the through thickness crack length is variable due to the elliptical tow architecture. Marsden (1996) correlated the linear plan view, crack density with the through-thickness crack length per unit length of sample, for a 2 layer 8HS weave laminate which had identical glass tows to the current PW material. A similar
quantification of damage has been performed in this study (the experimental details are
discussed in Chapter 3). Figure 4.10 shows the data for the current PW material and
that of the 8HS weave material investigated by Marsden (1996). The data for the PW
laminates superimpose on those of Marsden (1996), with only a small amount of
scatter. This suggests that, as in the case of the 8HS weave material, the plan view
measurement is a valid method for obtaining the more accurate total through thickness
 crack length if the two measurements are carefully determined beforehand. It also
suggests that the matrix cracking damage in materials with similar tows but differing
weave architectures is comparable. It is also possible that the curve of Marsden (1996)
is a master curve for matrix damage accumulation for this particular range of woven
material (PW to 8HS weave). Such a conclusion would suggest that damage
accumulation is independent of tow cross section, since the fibre undulation required in
the plain weave results in a more elongated tow, with a higher aspect ratio (6.5),
compared to the 8HS weave material (4.2).

The 8HS weave material has a higher saturation crack density as it is a denser weave,
with significantly more ends and picks per unit length and can physically accommodate
more damage.

4.5 Elastic Property Degradation Modelling

4.5.1 Shear-Lag Theory

The damage developed during quasi-static loading leads to an associated reduction in
Young’s modulus. Modelling of this behaviour has been attempted by reducing the
woven system to an equivalent cross-ply material and applying a shear lag analysis, as
performed by Marsden (1996). This model allows transfer of stress back into the
cracked transverse plies via shear across the 90° ply thickness from the interface of the
transverse and longitudinal plies. The shear lag analysis developed by Steif (appendix
to Ogin et al., 1984) has been applied here. This assumes a parabolic variation of the
longitudinal displacements in the transverse ply, as opposed to linear variations
assumed by other workers (Garret and Bailey, 1977).
The reduction in longitudinal modulus is expressed as a function of crack spacing in an idealised cross-ply laminate, by:

\[
\frac{E}{E_0} = \frac{1}{1 + \left( \frac{E_0}{E_1} \right) \left( b_0 + d \right) b_0 E_0 \left( \frac{E_1}{E_0} \right) \left( \tanh(\lambda s) \right) \lambda s} \quad (4.1)
\]

where

\[
\lambda^2 = \frac{3G_{23}(b_0 + d)E_0}{d^2b_0E_2E_1} \quad (4.2)
\]

and, $E$, $E_0$, $E_1$ and $E_2$ are the moduli of the damaged laminate, the undamaged laminate, the outer shear-lag ply and the transverse inner shear-lag ply, respectively. $G_{23}$ is the shear modulus of the transverse inner shear-lag ply in the longitudinal direction. The half crack spacing is denoted by $s$, and $b_0$ and $d$ are the half thicknesses of the outer ($0^\circ$) and inner (cracked $90^\circ$) shear-lag plies, respectively.

4.5.2 Model Dimension Determination

The equivalent cross-ply laminates are made up from $0^\circ$ ply, $90^\circ$ ply and pure matrix regions. These regions are assumed to be crimp free and are in the same proportions as found in the woven laminates. Hence, before the woven laminates can be represented as equivalent cross-ply laminates the volume fraction of pure matrix, $0^\circ$ ply and $90^\circ$ ply must be determined. This was achieved by taking area fraction measurements from polished edge sections. The volume fraction of a phase in a multi-phase structure is independent of particle shape and can be expressed as area fractions in a random planar section (Pickering, 1976), summarised by the following expression developed for particulate composite materials:
where $V_f$ is the volume fraction of $\alpha$ phase, $V_\alpha$ is the volume of $\alpha$ phase in the specimen, $V$ is the total volume of the specimen, $A_f$ is the area fraction of a phase, $A_\alpha$ is the area of $\alpha$ phase in the random section and $A$ is the total area in the random section. The “particulates” in this study are the continuous infiltrated warp tows and continuous infiltrated weft tows. The area fraction analysis was applied to the tows perpendicular to the plane of the polished section in both the $0^\circ$ and $90^\circ$ specimens giving the volume fraction of warp tow, weft tow and pure matrix regions. Area fractions of the tows perpendicular to the plane of the micrographs can be considered to represent volume fractions, since the tow cross-section is constant in this plane. However, the volume fraction of tows running parallel to the plane of the paper was not considered, as the area fraction obtained would be dependent on the particular plane through which the laminate was sectioned due to the elliptical tow cross-section and the presence of gaps between adjacent tows. The thickness of the sample was averaged over the $0^\circ$ and $90^\circ$ edge sections to account for thickness variations.

Volume fractions of infiltrated warp tow, infiltrated weft tow and pure matrix were found by taking area fractions from a total length of 60 mm of laminate cross sections at both $0^\circ$ and $90^\circ$ to the loading direction. The area fraction data were taken from weight measurements of edge section photomicrographs in which the longitudinal tow, transverse tow and pure matrix regions had been carefully been removed using a scalpel. The weight of the photographic paper was characterised and found to be $2.66 \times 10^{-2}$ gcm$^{-2}$. The volume fraction of fibre within the tows was calculated from the overall volume fraction obtained by the burn off method and the pure matrix volume fraction (table 4.2). The slightly higher proportion of warp tows to weft tows is consistent with the unbalanced nature of the cloth.

**4.5.3 Model Ply Property Determination**

The Young's modulus data for the $0^\circ$ and $90^\circ$ tows were derived in two different ways. In the first case, the $90^\circ$ tow Young's modulus, $E_2$, was assumed from literature.
(Marsden, 1996). Using a rule of mixtures expression and the model constituent volume fractions, the effective $0^\circ$ tow Young's modulus, $E_1$, was then calculated so that the overall laminate Young's modulus was equivalent to the measured undamaged laminate Young's modulus, $E_0$. This was termed the assumed $E_2$ model. In the second case, assuming straight, untwisted tows, the $0^\circ$ tow Young's modulus was calculated from a rule of mixtures expression. The $90^\circ$ tow Young's modulus was then calculated from a rule of mixtures expression using the model constituent volume fractions to match the measured $E_0$ of the woven composite. This was termed the assumed $E_1$ model. The Young's modulus of the pure matrix region, $E_m$, was taken from literature (Marsden, 1996). The shear modulus of the $90^\circ$ ply, $G_{23}$, was given by the following expression, assuming transverse isotropy:

$$G_{23} = \frac{E_2}{2(1 + v_{23})} \quad (4.4)$$

where $v_{23}$ was assumed to be 0.4 (from Marsden, 1996).

In a woven composite material the transverse crack length varies due to the elliptical tow cross section. However, in the model cross-ply system the transverse ply is continuous and of a constant thickness. The crack density quantification method proposed allows the derived through-thickness crack length per unit sample length for the woven material to be expressed as an equivalent damage density ($1/2s$) in the cross-ply model. This was achieved by normalising the data to the equivalent cross-ply model $90^\circ$ ply width by assuming regularly spaced cracks spanning the full $90^\circ$ ply width and thickness.

Laminate volume fractions obtained by area fraction measurements and derived model constituent properties are shown in table 4.2. The fibre volume fraction within the tows of 59.8 % is consistent with previous work (Marsden, 1996). The tow Young’s moduli for the assumed $E_1$ model agree closely with those of the assumed $E_2$ model. If the laminate modulus is calculated using the assumed $E_1$ (longitudinal) and assumed $E_2$ (transverse) Young’s moduli, together with the constituent volume fractions using a
rule of mixtures expression then the laminate Young’s modulus is 18.5 GPa. This is only marginally lower than the measured experimental value of 18.7 GPa (table 4.1).

4.5.4 Equivalent Cross-Ply Models

Four models were considered, all variations of a cross ply laminate based upon regions of pure matrix, longitudinal continuous fibre ply and transverse continuous fibre ply. These were constructed in four different ways in an attempt to simulate the structure of the woven composite using cross-ply “building blocks”.

Model 1 (figure 4.11a) consists of two shear lag regions, one being matrix/90° ply/matrix and the other 0° ply/90° ply/0° ply. This configuration can be compared to case A where the 2 cloths are aligned in-phase with one another as the transverse 90° plies are separated from each other. The behaviour of the two regions were considered separately for the shear lag analysis and the stiffness contribution of each was summed using a rule of mixtures expression.

Model 2 (figure 4.11b) consists of one 0° ply/90° ply/0° ply shear lag region surrounded on either side by pure matrix regions. The transverse ply thickness in this case is twice the size of the two transverse plies considered separately in model 1. The stiffness reduction of the central 0° ply/90° ply/0° ply region was found using the shear lag analysis and summed with the pure matrix stiffness contribution using a rule of mixtures expression.

Model 3 (figure 4.11c) consists of a similar configuration to model 1. However, the 90° tows are not continuous. There are pure resin regions separating the 90° tows in the direction of the applied load. This model was devised to simulate the gap between tows in the woven fabric, a result of the open nature of the weave. The size of this gap relative to the cross-sectional length of the transverse tows is consistent with measurements of the cloth. The 0° tows are assumed to be continuous. The laminate is effectively split into 2 regions as indicated by the dotted line in figure 4.11c. The stiffness contribution from these 2 sections is combined using a Reuss-type model.
Model 4 is a combination of model 1 and model 2 (figure 4.11d). Considering the limitations of the current approach it quite closely simulates the case when the 2 cloths are 180° out of phase with one another (figure 4.8). In half of the laminate adjacent 90° tows are separated and in the other half they are next to one another. Again, the stiffness contribution from each separate section (separated by the dotted line in figure 4.11d) is combined using a Reuss-type model.

In all models the damage was assumed to be contained in the 90° plies as revealed by edge section microscopy in section 4.3. The dimensions of the cross-ply models are given in table 4.3 as ply thicknesses. The height of the models in the direction of loading is taken as unity. In model 3 the height of the 90° ply is taken as ¾ of the total height to simulate the size of the gap measured in the PW fabric (see section 3.2.1). The heights of the separate regions in model 4 are equal, to simulate an equal Young's modulus contribution from each region. The dimensions of model 4 are not given as it is combination of models 1 and 2. The modelling is performed based upon a measured average laminate thickness of 407 µm.

### 4.5.5 Shear-Lag Modelling Results

Figure 4.12 shows the measured reduction in normalised modulus as a function of damage density, $1/2s$. The predictions with the assumed 90° tow Young's modulus lie just above those for the assumed 0° tow Young's modulus for each model. None of the models accurately predict the Young's modulus reduction very well for this PW laminate. The data show a more rapid stiffness reduction at low damage densities. A similar, but much smaller effect was also seen in the 8HS weave laminates studied by Marsden (1996).

Model 1 predicts approximately the slope of the experimental data for the higher crack densities, i.e., the rate of Young’s modulus reduction with increasing damage density, but lies above the experimental data for lower damage densities.
Model 2 predicts a rapid reduction in Young's modulus with increasing damage density. This configuration is thought unrealistic as the transverse 90° ply is much larger than those found in the woven laminate. This configuration alone does not accurately represent the fabric structures seen in edge section photomicrographs (figures 7b, 8b and 9b).

Model 3, in which the gap between adjacent tows is considered, shows a marginally larger reduction in Young's modulus compared to model 1. This is due to the slightly thinner 0° and pure matrix plies adjacent to the transverse 90° plies.

Model 4 is a combination of model 1 and model 2. Hence, it predicts a reduction in Young's modulus approximately midway between the predictions of model 1 and model 2.

In summary, the general trend of the data has been modelled here, but some modifications to the modelling are required to take into account the initial reduction in Young's modulus at low damage densities.

It is suspected that this initial reduction in Young's modulus at low damage densities is associated with the crimp present in fabric composites. In a woven composite, transverse cracking in 90° tows adjacent to crimp regions in longitudinal tows may have a larger influence on laminate stiffness than transverse cracking in other regions. A possible explanation of this effect is the relaxation of constraint provided by a 90° tow on the neighbouring crimp region in a 0° tow when transverse cracking is present. This would enable the crimped 0° tows to straighten more easily, resulting in a reduction in laminate stiffness. The smaller initial reduction in stiffness for the 8HS weave laminates is consistent with the smaller amount of crimp compared to the current PW laminates.

The lack of accuracy in this approach is not surprising due to the assumptions made in simplifying the woven structure as an equivalent cross-ply material. The presence of crimps has been shown to strongly influence the elastic properties of woven composites, as demonstrated by the work of Roy (1996). The presented models only
consider damage in the form of transverse cracking. They ignore the delaminations seen to occur in this material and their effect on the Young's modulus. It has been shown in this chapter that when the two fabric layers are shifted at 180° with respect to each other that a regular array of delaminations occur. The shear lag analysis used by Gao et al. (1999b) to study the effect of damage in woven composites suggested that approximately 40% of the stiffness reduction was due to delaminations. This may account for some of the inaccuracy of this approach. However, it cannot explain the rapid initial reduction in laminate stiffness at low damage densities as the delaminations do not initiate until an applied strain of approximately 1.5%.

4.6 Concluding Remarks

The PW laminates investigated in this chapter exhibit similar laminate properties to the 8HS weave laminates investigated by Marsden (1996). The damage found in the PW laminates has been shown to be influenced by tow characteristics, weave architecture and the relative shift of the fabric layers. The transverse crack morphologies within the tows are similar for both the PW and 8HS laminates. However, the open nature of the PW cloth combined with the tendency of the cloths to shift with respect to each other produces a range of possible damage morphologies not seen in the 8HS weave laminates.

The simple shear lag models used to predict the reduction in laminate stiffness with increasing damage density are not able to account for the initial rapid reduction in Young's modulus seen at low damage densities. The current approach, in which the woven composite is represented using a cross-ply laminate, is able to predict the general trend of the data, but cannot predict the Young's modulus/crack density behaviour of the PW laminates in detail.
## Chapter 4: Damage Accumulation in Plain Weave Laminates

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_0$ (GPa)</td>
<td>18.7 ± 1.4 (12)</td>
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<tr>
<td>$\sigma_{TS}$ (MPa)</td>
<td>260</td>
</tr>
<tr>
<td>$\varepsilon_i$ (%)</td>
<td>0.6</td>
</tr>
<tr>
<td>$\varepsilon_f$ (%)</td>
<td>2.1</td>
</tr>
<tr>
<td>$V_f$ (%)</td>
<td>36</td>
</tr>
<tr>
<td>Thickness (mm)</td>
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</tbody>
</table>

Table 4.1. Plain weave laminate properties.

<table>
<thead>
<tr>
<th>Property</th>
<th>Assumed $E_2$ ($90^\circ$)</th>
<th>Assumed $E_1$ ($0^\circ$)</th>
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</thead>
<tbody>
<tr>
<td>$V_m$ pure matrix (%)</td>
<td>42.6</td>
<td></td>
</tr>
<tr>
<td>$V_{warp}$ infiltrated warp ($0^\circ$) tows (%)</td>
<td>29.1</td>
<td></td>
</tr>
<tr>
<td>$V_{weft}$ infiltrated weft ($90^\circ$) tows (%)</td>
<td>28.3</td>
<td></td>
</tr>
<tr>
<td>$V_f$ within tows (%)</td>
<td>59.8</td>
<td></td>
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</table>

Table 4.2. Cross-ply model properties.

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_1$ (GPa)</td>
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</tr>
<tr>
<td>$E_2$ (GPa)</td>
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<td>$G_{23}$ (GPa)</td>
<td>4.64</td>
</tr>
<tr>
<td>$E_m$ (GPa)</td>
<td>3.56</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Property</th>
<th>Assumed $E_2$ ($90^\circ$)</th>
<th>Assumed $E_1$ ($0^\circ$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_1$ (GPa)</td>
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</tr>
<tr>
<td>$E_2$ (GPa)</td>
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</tr>
<tr>
<td>$G_{23}$ (GPa)</td>
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<td></td>
</tr>
<tr>
<td>$E_m$ (GPa)</td>
<td>3.56</td>
<td></td>
</tr>
</tbody>
</table>
### Table 4.3. Cross-ply model dimensions.

<table>
<thead>
<tr>
<th>Model</th>
<th>Thickness of pure matrix region (µm)</th>
<th>Thickness of 0° plies (µm)</th>
<th>Thickness of 90° plies (µm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>86.7</td>
<td>59.1</td>
<td>57.6</td>
</tr>
<tr>
<td>2</td>
<td>86.7</td>
<td>59.1</td>
<td>115.2</td>
</tr>
<tr>
<td>3</td>
<td>52.0</td>
<td>44.3</td>
<td>57.6</td>
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</table>
Figure 4.1. *In situ* plan view photograph of damage initiation at 0.6 % strain.

Figure 4.2. Polished edge section photomicrograph of damage initiation at 1.0 % strain.
Figure 4.3. *In situ* plan view photograph of damage developed at 1.2% strain.

Figure 4.4. Polished edge section photomicrograph of damage developed at 1.2% strain.
Figure 4.5. *In situ* plan view photograph of damage developed at 1.6 % strain with marker indicating a typical transverse crack length.

Figure 4.6. Polished edge section photomicrograph of damage developed at 1.6 % strain.
Figure 4.7. Damage morphology of case A, (a) plan view photomicrograph, (b) polished edge section photomicrograph and (c) schematic representation of edge section.
Figure 4.8. Damage morphology of case B, (a) plan view photomicrograph, (b) polished edge section photomicrograph and (c) schematic representation of edge section.
Figure 4.9. Damage morphology of case C, (a) plan view photomicrograph, (b) polished edge section photomicrograph and (c) schematic representation of edge section.
Figure 4.10 Linear plan view crack density versus total through thickness crack length per unit sample length for current plain weave laminate and similar eight harness satin laminate.
Figure 4.11. Equivalent cross-ply models for shear-lag modelling.
Chapter 4: Damage Accumulation in Plain Weave Laminates

Figure 4.12 Shear-lag modelling results. Normalised Young’s modulus versus damage density.
Chapter 5: Unnotched and Notched Laminate Properties
5.1 Introduction

This chapter presents the unnotched and notched laminate property data obtained from uniaxial quasi-static tensile tests. After outlining the experimental programme performed, the unnotched laminate properties for both 8HS weave and PW laminates, with varying numbers of fabric layers, are discussed. These data are required to be able to determine the notched to unnotched laminate strength ratios and also to provide the input parameters for the notched strength modelling performed in Chapter 7. This is followed by a section discussing the notched strengths of 8HS weave and PW laminates with varying numbers of fabric layers, for a range of notch sizes and shapes. These will be compared to the notch strength predictions from various models in Chapter 7. The next section discusses the critical energy release rate, or toughness, of both 8HS weave and PW laminates. Some of the models investigated in Chapter 7 require a fracture mechanics parameter, since they consider the notch edge damage as an effective crack emanating from the discontinuity, and require laminate toughness either to model the stress redistribution associated with damage growth (Afaghi-Khatibi et al., 1996) or to model catastrophic failure (Hitchen et al., 1994).

In a subsequent section the unnotched and notched laminate properties are corrected for variations in fibre volume fraction. The normalisation method proposed is presented initially, followed by the results obtained from applying this normalisation procedure to tensile strength and toughness data from all the laminate types investigated.

5.2 Experimental Programme

This section describes the combination of laminate types and notch geometries used in the unnotched and notched strength investigation in this study. The sample configurations and number of samples tested are listed in table 5.1.

The main study investigated 8HS weave laminates of 2, 4, 6 and 8 fabric layers. The longitudinal unnotched laminate properties were determined for all four laminate
Chapter 5: Unnotched And Notched Laminate Properties

thicknesses. The transverse and shear unnotched laminate properties were determined for the 2 and 8 layer laminates. Samples containing circular notches of 2.5 mm, 5 mm and 10 mm in diameter were investigated for laminates of 2, 4, 6 and 8 fabric layers. This was followed by a similar study for elliptical notches with major ellipse axes of the same dimensions as the circular notches investigated. The notch root radius was kept constant at 0.5 mm, resulting in elliptical notches with opening aspect ratios, $\lambda$, given by the ratio of the minor to the major ellipse half length, of 0.63, 0.45 and 0.32. This study was performed for 2, 4, 6 and 8 layer laminates.

A smaller study was performed to investigate the notched strength of PW laminates. Materials containing 2 and 8 fabric layers were used to determine the longitudinal, transverse and shear unnotched laminate properties. The notched investigation also used 2 and 8 layer laminates containing circular notches of 2.5 mm, 5 mm and 10 mm in diameter.

5.3 Unnotched Laminate Properties

5.3.1 Introduction

The data presented in this section are an average of 5 samples for the 8HS weave laminates and 3 samples for the PW laminates. The 8HS unnotched laminate properties are shown in table 5.2 and the PW unnotched laminate properties are shown in table 5.3.

5.3.2 Eight Harness Satin Weave Laminates

The variation in longitudinal Young's modulus with increasing number of fabric layers is shown in figure 5.1. It can be seen that the longitudinal Young's modulus remains relatively constant (within experimental variations) for the 2, 4 and 6 layer laminates at approximately 19.3 GPa. The Young's modulus of the 8 layer laminate is 20.4 GPa, an increase of approximately 5 % compared to the thinner materials. This increase in stiffness could possibly be explained by a higher degree of constraint of transverse contraction in the thicker laminates, but this is perhaps too large an
increase to be due to this effect alone. The error bars in figure 5.1 show the standard deviations of the data. The scatter of the data is relatively small, except for the 4 layer laminate.

The transverse Young’s modulus was determined for the 2 and 8 layer laminates. The data are also shown in figure 5.1. The average values were 18.3 GPa and 19.6 GPa for the 2 and 8 layer laminates, respectively. This represents an increase of approximately 7% from the 2 to the 8 layer laminate. In each case the transverse Young’s modulus was approximately 5% lower than the longitudinal Young’s modulus. This can be explained by the slightly unbalanced nature of the cloth (see section 3.2.1). Hence, the transverse Young’s moduli for the 4 and 6 layer laminates were estimated by assuming that they were equal to 95% of the corresponding longitudinal Young’s moduli. This gives estimated transverse Young’s moduli of 18.24 GPa and 18.43 GPa for the 4 and 6 layer laminates, respectively.

The shear moduli were also determined for the 2 and 8 layer laminates. Values of 3.37 and 3.55 GPa were measured for the 2 and 8 layer laminates, respectively, an increase of approximately 5% between the 2 and 8 layer materials. This is consistent with the increase in Young’s modulus between the 2 and 8 layer laminates. The shear moduli for the 4 and 6 layer laminates were estimated by assuming a linear increasing relationship between the 2 and 8 layer laminates, resulting in values of 3.43 GPa and 3.49 GPa, respectively.

The Poisson’s ratio remains fairly constant with laminate thickness as shown in figure 5.2. The 2 layer laminate shows the highest Poisson’s ratio, approximately 6% higher than the lowest value exhibited by the 6 layer laminate. However, the error bars indicating the standard deviations of the data in figure 5.2 suggest that it is reasonable to assume a constant value of Poisson’s ratio.

The laminate strengths for the longitudinal samples are presented in figure 5.3 as a function of the number of fabric layers, and show similar trends to those shown by the Young’s modulus data. The 2, 4 and 6 layer laminate strengths are all fairly constant, in the range of approximately 320-340 MPa. The 8 layer laminate is
Chapter 5: Unnotched And Notched Laminate Properties

stronger at 382 MPa, approximately 20 % higher than the weakest laminate. The error bars of the strength data in figure 5.3 are fairly large and indicate that a constant laminate strength with increasing number of layers is feasible. However, in looking at the unnotched property data further it is necessary first to consider the laminate fibre volume fractions.

Experimental data for laminate fibre volume fractions are given in table 5.4 and are displayed in figure 5.4, labelled as 8HS laminate volume fractions. The values given are an average of several sections taken from regions across laminates prior to testing. The data can be considered as average laminate fibre volume fractions. The fibre volume fraction increases in an approximately linear fashion, from 36.7 % for the 2 layer laminate to 39.5 % for the 8 layer laminate, a relative increase of approximately 7.5 %. This could be attributed to the diminishing proportion of resin rich regions at the laminate surface as the number of fabric layers is increased.

It was observed from measurements taken during the determination of sample cross-sectional area that for laminates of a given number of layers variations in thickness existed across laminates and from laminate to laminate. Hence, fibre volume fraction as a function of laminate thickness was investigated in detail to ascertain whether thickness variations could be related quantitatively to local changes in fibre volume fractions. Eight laminate sections of varying thickness were investigated for each laminate type. The data, shown in figure 5.5, demonstrates that there is a linear relationship between fibre volume fraction and laminate thickness. A trend-line was fitted to each data set using a linear regression. The equation of each line is displayed in figure 5.5. These equations were then used to predict average fibre volume fractions from average sample thicknesses for the set of samples used to obtain the unnotched laminate properties. The change in predicted average fibre volume fraction of the samples used to determine the unnotched laminate properties with varying number of layers is shown in figure 5.4. It is believed that these data reflect the trends in fibre volume fraction of the samples from which the unnotched property data were determined. It can be seen that the predicted fibre volume fractions for the unnotched samples are lower than the average laminate data, especially for the 4 and 6 layer laminates. The trends of the predicted unnotched
sample fibre volume fraction data appear to be very similar to those of the Young’s modulus and laminate strength data as the number of fabric layers is increased (see figures 5.1 and 5.3, respectively). The predicted fibre volume fraction of the 2 layer laminate is approximately 10% relatively lower than that of the 8 layer laminate. The corresponding increases in Young’s modulus and laminate strength are 5% and 20%, respectively. A 10% relative increase in fibre volume fraction relating to a 5% increase in Young’s modulus is feasible. However, the 20% increase in laminate strength is too high to be solely due to the fibre volume fraction effect. Hence, this laminate thickness-fibre volume fraction effect may account for the variation in Young’s modulus and account, at least partially, for the variation in laminate strength with increasing number of fabric layers.

5.3.3 Plain Weave Laminates

In a less extensive investigation, 2 and 8 layer PW laminates were studied. The variation in Young’s modulus with the number of fabric layers is shown in figure 5.1. An increase of approximately 5% can be seen from the 2 layer to the 8 layer laminate. This is a similar effect to that seen in the 8HS weave laminates.

The effect of varying numbers of fabric layers on the transverse Young’s modulus is also shown in figure 5.1. The transverse Young’s moduli for the 2 layer and 8 layer laminates are 10% and 5% lower, respectively, than their longitudinal Young’s moduli. This slightly higher discrepancy between longitudinal and transverse Young’s moduli for the PW laminates compared to the 8HS weave laminates is consistent with the PW fabric being slightly more unbalanced than the 8HS weave fabric.

The shear modulus for the 8 layer laminate is approximately 7.5% higher than that of the 2 layer laminate. This difference is slightly higher than that seen in the 8HS laminates.

The variation of Poisson’s ratio with the number of fabric layers is shown in figure 5.2. The Poisson’s ratio for the 2 layer laminate is 0.19, approximately 15% higher
than that of the thicker 8 layer laminate. The Poisson’s ratios reported in literature for PW glass/epoxy laminates are generally in the range of 0.11-0.16 (Kim et al., 1995). The value of 0.19 for the PW 2 layer laminate is slightly higher than other reported values. However, the trend of the results are supported by the findings of Naik and Shembekar (1992b), who predicted higher Poisson’s ratios for laminates with fewer fabric layers compared to thicker laminates. Also, Raju and Wang (1994) predicted higher Poisson’s ratios for PW laminates compared to 8HS weave laminates, as found in this study.

The change in laminate strength with the number of fabric layers is shown in figure 5.3. The 8 layer laminate is approximately 15% stronger than the 2 layer laminate. This is similar to the trend seen in the 8HS weave laminates.

The volume fractions of the PW laminates are given in table 5.4. A volume fraction-thickness investigation was performed for the PW laminates, as for the 8HS weave laminates. Four laminate sections were investigated for each laminate type. The data for the PW laminates together with the equations fitted to the data are shown in figure 5.5. The predicted fibre volume fractions for the unnotched samples (figure 5.4) show that the 8 layer laminate has a fibre volume fraction approximately 6% higher (expressed as a percentage of the difference in volume fractions) than that of the 2 layer laminate. As with the 8HS weave laminates, this volume fraction effect can be compared to the trends seen in the Young’s modulus and laminate strength data when varying the number of fabric layers. Again, the increase in Young’s modulus from the 2 layer to the 8 layer laminate is consistent with the increase in predicted fibre volume fraction. Similar to the 8HS weave laminates, the increase in laminate strength (15%) is probably not attributable solely to the increase in fibre volume fraction (6%).
5.3.4 Comparison of Unnotched Laminate Properties for Plain Weave and Eight Harness Satin Weave Laminates

The smaller experimental programme performed with the PW laminates meant that only 2 and 8 layer laminate properties may be compared between the two fabric reinforcements.

The PW laminates show a higher Young’s modulus for both the 2 and 8 layer laminates with approximately 5% and 4% increases, respectively, compared to the 8HS weave laminates. A similar effect can also be seen for the transverse Young’s modulus data. This is shown in figure 5.1. This is perhaps surprising, as an 8HS weave laminate is generally considered to be stiffer than an equivalent PW laminate, a result of the lower degree of crimp in a 8HS weave fabric (see section 2.2). However, there are two factors which must be taken into account in this case. The first of these is the fibre volume fraction of the laminates. The PW unnotched samples show higher predicted fibre volume fractions than the 8HS weave unnotched samples, as seen in figure 5.4. This effect has been attributed to improved laminating techniques acquired with experience during the duration of this project and possibly the ability of the PW fabric to nest more tightly. The second factor is the degree of imbalance in the fabrics. The PW fabric is slightly more unbalanced than the 8HS weave fabric. Therefore, the PW fabric will have a slightly higher proportion of its tows aligned in the longitudinal orthogonal direction. This effect is supported by the fact that there is a larger discrepancy between the PW longitudinal and transverse Young’s moduli data than those of the 8HS weave laminate (figure 5.1).

In contrast to the stiffness data, the 8HS weave laminates have higher unnotched strengths than the PW laminates for both 2 and 8 layer laminates. This is shown in figure 5.3. This appears to contradict the reasons given above for the higher stiffness of the PW laminate. However, it has been reported that crimp regions in woven fabric composites are prone to longitudinal fibre failure and that laminates with higher degrees of crimp exhibit lower unnotched strengths (Roy, 1994). Both the PW and 8HS weave materials have higher unnotched strengths for the 8 layer
Chapter 5: Unnotched And Notched Laminate Properties

Laminates compared to the thinner laminates. This could not be accounted for by considering the predicted fibre volume fractions for the various laminate types.

5.4 Notched Laminate Properties

5.4.1 Introduction

The strength data presented in this section are an average of three samples. The notched to unnotched strength ratios presented have not been corrected for finite width. Such corrections will be incorporated into the subsequent modelling presented in Chapter 7. The 8HS weave notched laminate property data are given in tables 5.5 and 5.6 for the circular and elliptical notches, respectively. The PW laminate notched property data are given in table 5.7 for circular notches.

5.4.2 Eight Harness Satin Weave Laminate

5.4.2.1 Circular Notches

The notched to unnotched strength ratios for 8HS weave laminates containing circular notches is plotted as a function of notch opening to plate width ratios in figure 5.6. It can be seen that the 4, 6 and 8 layer laminates show similar notched strength ratios at all normalised notch sizes. However, the 2 layer laminate appears to show significantly lower strength ratios for the 2.5 mm and 5 mm diameter notches. The notched strength ratios are similar for all laminates for 10 mm diameter holes. The scatter of the three data points for each notch size is generally small (see table 5.5).

The fibre volume fraction-thickness investigation described in section 5.3.1 was also applied to the samples on which the notched laminate strengths were measured. The predicted fibre volume fractions for the 8HS weave laminates containing circular notches are shown in figure 5.7 as a function of normalised notch size. It can be seen that the fibre volume fractions generally increase with increasing numbers of fabric layers. This increase in fibre volume fraction between the laminate types would not
affect the notched strength ratios significantly so long as the fibre volume fractions within a particular laminate type were relatively constant with respect to the value of the unnotched laminate strength \((2a/W = 0)\). This is the case for the 4 and 8 layer samples. The 6 layer samples show small (i.e. < 10 \%) variations in predicted fibre volume fraction. However, the predicted fibre volume fractions of the 2 layer samples vary significantly. The 2.5 mm and 5 mm diameter notched samples have predicted fibre volume fractions of up to 15 \% less than the unnotched and 10 mm diameter notched samples. This is reflected in the lower notched strength ratios for the 2 layer 8HS weave samples with 2.5 mm and 5 mm diameter circular notches compared to the thicker laminates, as shown in figure 5.6.

5.4.2.2 Elliptical Notches

The notched to unnotched strength ratio for 8HS weave laminates containing elliptical notches is plotted as a function of notch opening to plate width ratio in figure 5.8. The notched strength ratio data for laminates with varying numbers of layers show variations of up to approximately 15 \% at similar notch sizes, but there does not appear to be any pattern to these differences. The 8 layer laminate shows the lowest strength ratios while the 4 layer laminate shows the highest strength ratios. The strength ratios of all laminates are very close at the largest notch size, as observed for the circular notches.

The predicted fibre volume fractions for the elliptically notched samples (based on their thickness) are shown in figure 5.9 as a function of normalised notch size. This graph is plotted using a similar scale to the equivalent graph for the circular notches (figure 5.7). It can be seen that the variations in predicted fibre volume fractions are smaller for the elliptically notched samples. This is probably due to improvements in laminating technique gained during this project. The higher notched strength ratios seen for the 4 layer samples at the 2.5 mm and 5 mm notch diameters correspond to increased predicted fibre volume fractions for these notched samples compared to the unnotched samples, shown in figure 5.9. An opposite effect can be used to explain the lower notched strength ratios seen for the 8 layer elliptically notched samples. The predicted fibre volume fractions for the 8 layer laminates show a reduction for
the 2.5 mm and 5 mm elliptically notched samples compared to the unnotched samples.

5.4.3 Plain Weave Laminates

The notched to unnotched strength ratio is plotted as a function of the notch opening to plate width ratios (for PW laminates containing circular notches) in figure 5.10. The 2 layer laminate has higher strength ratios than the 8 layer laminate at all notch sizes and the effect is more pronounced at larger notch sizes.

The predicted fibre volume fraction for the PW notched samples is shown in figure 5.11 as a function of normalised notch size. Again, this graph is plotted on a similar scale to the equivalent graphs for the 8HS weave notched samples. The predicted volume fractions are approximately constant at all notch sizes for both laminate types. The predicted fibre volume fractions between the laminate types are also quite similar. Again, this is probably a result of improved laminating techniques. As there is little variation in predicted fibre volume fraction for the unnotched and notched samples within both laminate types this data set may be considered to be virtually free of volume fraction effects. This suggests that for the PW laminates containing circular notches there is some effect of the number of layers on the notched to unnotched strength ratio with variation of notch size.

5.4.4 Discussion

It is difficult to compare the trends shown in notched strength ratios for the 8HS weave and PW laminates due to the presence of the fibre volume fraction effects discussed in section 5.4.2 and 5.4.3. Figure 5.12 shows the variation in notched strength ratio with increasing notch size for all laminate types and notch geometries. This gives an indication of the spread of the data and the initial impression of the random nature of the trends seen. It is proposed that the fibre volume fraction effects discussed previously obscure any possible trends in the notched strength ratios reported.
Figure 5.13 shows the notched strength ratios for circular notches in both 8HS weave and PW laminates. Ignoring the behaviour of the 2 layer 8HS weave samples, attributed in section 5.4.2.1 to volume fraction effects, the only major deviation of notched strength ratios are those of the 8 layer PW laminate. This shows the lowest notched strength ratio for all notch sizes, especially at the larger notch sizes. This cannot be accounted for by thickness effects as the PW laminates have predicted fibre volume fractions similar in magnitude to the thicker 8HS weave laminates.

Figure 5.14 shows the notched strength ratios for all 8HS weave samples. This appears to show the general trend of the elliptically notched samples having slightly lower notched strength ratios than samples with circular notches. This is in contrast to the findings of Awerbuch and Madhukar (1985) who stated that the notch root radius had very little, if any, effect on notched strength. However, it must be considered that their work concerned non-woven composites where notch edge damage (splitting and delamination in particular) is often more extensive than in the woven composites investigated in the present study. The extent of this notch edge damage can have a pronounced effect on the stress field adjacent to the discontinuity and, hence, the notched laminate strength.

Figure 5.15 shows the notched strength ratios for only 2 and 8 layer notched samples so that the extremes of laminate thickness may be more easily compared. Also, the PW laminates were only investigated at 2 and 8 layers so comparisons may only be made using these laminate types. The 8HS weave samples with elliptical notches and the 8 layer PW samples with circular notches generally have lower notched strength ratios than the other samples, especially at the largest notch size. As previously discussed, the 2 layer 8HS weave laminate has low notched strength ratios for circular holes of 2.5 mm and 5mm in diameter.

The notched strength ratios for the various laminate and notch geometries investigated in this study are presented in a different way in figure 5.16. The strength ratios are plotted against the number of fabric layers for each notch size, notch type and laminate reinforcement. It has been suggested that any trends seen with the
variation of the number of fabric layers can be accounted for largely by the proposed volume fraction effects. The elliptically notched 8HS weave samples have lower notched strength ratios for all but the 2 layer laminates compared to the 8HS weave samples with circular notches. It can also be seen clearly that the 2 layer PW notched laminates have higher notched strength ratios than the 8 layer PW notched laminates. The error bars for the different laminate and notch configurations, indicating standard deviations of the data, overlap in most cases. This may be due to fibre volume fraction variations between the laminates and is discussed further in section 5.6.

5.5 Critical Energy Release Rate Determination

5.5.1 Introduction

This section discusses the results of the critical energy release rate, or toughness, determination. The methods used in this determination are discussed in section 3.6.

5.5.2 Eight Harness Satin Weave Laminates

A compliance calibration was initially performed, that is the change in specimen compliance with SEN length was determined. The results are shown for all 8HS weave laminates in figure 5.17. The fourth order polynomial fitted to the average of each data set is displayed on the plots. In all cases it can be seen that the fitted polynomial agrees well with the experimental data. The compliance versus SEN length data for all laminate types are shown plotted together in figure 5.18. As expected, the compliance of the samples decreases as the number of fabric layers is increased.

In order to investigate the consistency of the compliance calibration between the laminate types investigated, the data was normalised to the number of fabric layers in the laminates. This involved plotting the specimen compliance multiplied by the number of fabric layers against the SEN length, as shown in figure 5.19. It can be seen that the data for all laminate types is superimposed, giving confidence in the compliance calibration performed.
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The rate of change of compliance, \( \frac{dC}{da} \), with SEN length was obtained by differentiating the fourth order polynomial expressions obtained from figure 5.17. The resulting data are presented in figure 5.20. This plot enables the rate of change of compliance at any SEN length to be determined for all 8HS weave laminate types.

The individual results of the 8HS weave toughness determination are given in table 5.8. In this case, the maximum load was used as the fracture load. This may result in slight over-predictions in toughness due to any non-linearity in the load-extension plots, a result of crack tip damage. The measured toughness of 2, 4, 6 and 8 layer 8HS weave laminates is plotted as a function of SEN size in figure 5.21. It appears that the laminate toughness is dependent on SEN length, especially for the 4 layer laminate, showing an increase in toughness with increasing SEN length. The error bars show the standard deviations and, although some error bars are large, it is apparent that the thinnest laminate appears to be generally tougher than the thicker laminates. Figure 5.22 shows the average toughness over the three SEN lengths investigated plotted against the number of fabric layers. The 2 layer laminate appears to be approximately 30% tougher than the thicker laminates which all show similar toughness values. The error bars show the standard deviations for the data. The 4 layer laminate shows a broad spread of data, due to the dependence of toughness on SEN length. The other laminates have smaller standard deviations. The general level of toughness values are of a similar magnitude values quoted for other woven glass based composites (Kim et al., 1995; Harris, 1986).

5.5.3 Plain Weave Laminates

The change in specimen compliance with SEN length for the 2 and 8 layer PW laminates is shown in figure 5.23. The fourth order polynomial fitted to each data set is displayed on the plot. Again, it can be seen that the fitted polynomials agree well with the experimental data. The compliance versus SEN length data for both laminate types are shown together in figure 5.24. Similar to the case of the 8HS weave laminates, the 2 layer laminates have the higher compliance and the 8 layer laminates have the lower compliance.
The specimen compliance versus SEN length data were normalised to the number of fabric layers in the laminates. The plot of specimen compliance multiplied by the number of fabric layers against the SEN length is shown in figure 5.25. Again, it can be seen that the data for both the 2 and 8 layer laminates are superimposed, giving confidence in the compliance calibration performed. The differentiated fourth order polynomial expressions give the rate of change of compliance, dC/da, with increasing SEN length. The resulting data are presented in figure 5.26.

The results of the PW toughness determination are given in table 5.9. Two methods were used to define the sample fracture load. In the first method the laminate toughness was calculated using the maximum recorded load. In the second method the assumed fracture load, P_Q, was given by the intersection of a line with a compliance of 5% less than the original specimen compliance with the load extension trace. An example of a load extension plot from a SEN fracture test is given in figure 5.27. This shows both methods of fracture load determination. The change in toughness with increasing SEN size is shown for 2 and 8 layer PW laminates in figure 5.21. The laminate toughness, based on either the maximum load or on the data corrected for the effect of a non-linear load-extension response, is more dependent on SEN length for the PW laminates than for the 8HS weave laminates. Due to non-linearity in the load-extension plots, the toughness determined using the maximum recorded load is higher in all cases than that determined using the 5% reduction in compliance secant load. Although the standard deviations for the data, shown by the error bars, are quite large it appears that the 2 layer laminates are tougher than the 8 layer laminates, as seen for the 8HS weave material.

Figure 5.22 shows the average toughness over the three SEN lengths investigated plotted against the number of fabric layers for both methods of fracture load determination. The 2 layer laminate appears to be approximately 14% and 28% tougher than the 8 layer laminates for the maximum load and 5% reduction in compliance secant load methods, respectively. Due to the apparent dependence of toughness on SEN length, the average toughness data has large variations for the PW laminates. This could be used to argue against the findings of the 2 layer laminates
being tougher than the 8 layer laminates. However, this number of fabric layers-toughness effect was also seen in the 8HS weave laminates. The toughness data obtained using the maximum load as the fracture load shows toughness values of 10 % and 25 % higher for the 2 and 8 layer laminates, respectively, compared to the 5 % reduction in compliance secant method. This suggests that the load-extension profiles do not conform to the guidelines defined for the validity of LEFM.

5.5.4 Comparison of Critical Energy Release Rate Determination for Eight Harness Satin Weave and Plain Weave Laminates

It was shown in section 5.5.3 that the PW laminates exhibited non-linear behaviour, which was accounted for by using the 5 % reduction in compliance secant method to obtain a fracture load. However, when comparing the 8HS weave and PW laminates the data acquired using the maximum fracture load has been used so that the methods of toughness measurement are similar.

The variation of toughness with SEN length is shown for all laminates in figure 5.21. It can be seen that toughness appears to be more dependent on SEN length for the PW laminates. An explanation for the dependence of toughness on SEN length is not immediately apparent. It is possible that the dimensions of the fabric reinforcements influence the toughness of the laminates over the three SEN lengths. The smaller SEN lengths (4 mm) are not significantly larger than the dimensions of the weave architecture, such as the tow width, gap between tows and the distance between crimp regions.

The average toughness values for all SEN sizes are shown plotted against the number of fabric layers in figure 5.22. For both reinforcement types there is a reduction in laminate toughness with increasing number of fabric layers. The 2 layer 8HS weave laminate is approximately 12 % tougher than the equivalent PW laminate. The 8 layer 8HS weave and PW laminates show similar toughness values. The reasons for these variations in toughness with reinforcement type and number of fabric layers are unclear. Due to the previously described SEN length dependence the error bars for
the average toughness data are large. This must be considered when comparing the relative toughness of different laminates.

A laminate thickness-fibre volume fraction investigation was applied to the SEN specimens used to measure the toughness. The predicted fibre volume fractions are shown plotted against SEN length for the 8HS weave and PW laminates in figures 5.28 and 5.29, respectively. It can be seen that there are significant variations in predicted fibre volume fractions for almost all data sets. These variations in fibre volume fraction will influence the toughness data. For example, the 4 layer 8HS weave samples show an increasing predicted fibre volume fraction as the SEN length increases (figure 5.28). This may explain the significant increase in toughness with increasing SEN length seen for the 4 layer samples in figure 5.21. Again, the thickness tolerance in the PW laminates is better than in the 8HS weave laminates.

5.6 Normalisation of Laminate Properties

5.6.1 Introduction

It has been proposed in sections 5.3-5.5 that the unnotched and notched laminate properties are influenced by variations in fibre volume fraction between samples. This section suggests a method of eliminating these effects from the strength and toughness data.

It can be seen from figures 5.7, 5.9 and 5.11 that variations in predicted fibre volume fractions exist between the unnotched and notched samples within each laminate and notch type. This is especially apparent for the 8HS weave laminates containing circular notches (figure 5.7).

The proposed method for normalising the strength data is based on converting the unnotched laminate strength to a value at the average predicted fibre volume fraction of the notched samples for each laminate type and notch shape. This approach was chosen as the largest variations in predicted fibre volume fractions were generally seen for the unnotched samples. Also, this method required that only the unnotched
strength data were modified. It is not necessary to normalise the data between laminates with varying numbers of fabric layers as the resulting strength properties are expressed as notched strength ratios. It should be noted that the normalised unnotched strength for a particular laminate type may vary when considering different notch geometries.

The normalisation was performed using the ratio of the predicted fibre volume fractions of the samples:

\[ \sigma_0^{\text{norm}} = \sigma_0 \frac{V_f^{\text{notched}}}{V_f^{\text{unnotched}}} \]  

(5.1)

where \( \sigma_0^{\text{norm}} \) is the normalised unnotched laminate strength, \( \sigma_0 \) is the unnotched laminate strength, \( V_f^{\text{notched}} \) is the average predicted fibre volume fraction of the notched samples and \( V_f^{\text{unnotched}} \) is the average predicted fibre volume fraction of the unnotched samples.

The toughness results have also been corrected for variations in volume fraction that exist between the samples (figures 5.28 and 5.29). The proposed method normalises the fracture load, \( P_Q \), to the average volume fraction of the SEN specimens for each laminate type.

\[ P_Q^{\text{norm}} = P_Q \frac{V_f^n}{V_f^{\text{avg}}} \]  

(5.2)

where \( P_Q^{\text{norm}} \) is the normalised SEN specimen fracture load, \( V_f^n \) is the predicted fibre volume fraction of the SEN specimen with notch length \( n \) and \( V_f^{\text{avg}} \) is the average predicted fibre volume fraction for all three notch lengths.

This normalisation is not intended to account for variations in fibre volume fraction between laminates containing various numbers of fabric layers. As the toughness
data is required for the notched strength modelling, the values obtained should be relevant to the fibre volume fractions of the laminates investigated. Hence, the average toughness for each laminate type is only marginally affected by the normalisation. The normalisation intends to investigate the variation in toughness with increase of edge notch length due to volume fraction effects. It should be noted that this normalisation method will also modify the fracture toughness, \( K_{IC} \), data, required for the model of Hitchen et al. (1994) investigated in Chapter 7.

The normalised unnotched strength data are given in tables 5.10-5.12 and the normalised SEN specimen fracture load data are given in tables 5.13-5.14. The normalised strength data are expressed as notched to unnotched strength ratios in tables 5.15-5.17. Similarly, the normalised SEN specimen fracture loads are expressed as toughness data in tables 5.18-5.19.

The normalised data reported in this section are compared to the equivalent raw, or un-normalised, data presented earlier in this chapter. When referring to figures in this section, the corresponding figure displaying the un-normalised data is given in parentheses. Note that the term “normalised notched to unnotched strength ratio” used in subsequent sections refers to the ratio of the notched strength of a laminate which has been non-dimensionalised by dividing by the normalised unnotched laminate strength.

5.6.2 Notched Strength Normalisation

5.6.2.1 Eight Harness Satin Weave Laminates

The relationship between the normalised notched to unnotched strength ratio with increasing notch opening to plate width ratios for circular notches is shown in figure 5.30 (5.6). The spread of the normalised data is much less compared to the un-normalised data. The normalisation has brought the 2 layer laminate data back into line with the data for thicker laminates, suggesting that the proposed fibre volume fraction effects have been successfully accounted for. At the largest notch size the 2 layer laminate shows the highest strength ratio for this 10 mm notch. There appear to
be no obvious trends, with respect to the number of fabric layers, in the small variations in strength ratios seen.

The relationship between the normalised notched to unnotched strength ratio with increasing notch opening to plate width ratio for elliptical notches is shown in figure 5.31 (5.8). The spread of the normalised data is very small between the laminates containing varying numbers of fabric layers, compared to the un-normalised data. The data for 5 mm notches show that the strength ratios decrease as the number of fabric layers is increased. However, this is not shown at the other notch sizes.

5.6.2.2 Plain Weave Laminates

The relationship between the normalised notched to unnotched strength ratio with increasing notch opening to plate width ratios for circular notches in the PW laminates is shown in figure 5.32 (5.10). In this case the normalised data is very similar to the un-normalised data as the volume fraction variations between samples are negligible (see figure 5.11). The 2 layer laminate has higher strength ratios than the 8 layer laminate at all notch sizes.

5.6.2.3 Comparison of Normalised Notched Laminate Properties for Eight Harness Satin Weave and Plain Weave Laminates

Figure 5.33 (5.12) shows the variation in normalised notched strength ratio with increasing notch size for all laminate types and notch geometries. It can be seen that even after the proposed fibre volume fraction correction there is quite a large spread of the data, although it is not as pronounced as shown by the un-normalised data.

Figure 5.34 (5.13) shows the notched strength ratios for circular notches in both 8HS weave and PW laminates. As proposed for the un-normalised data the only major deviation of notched strength ratios are those of the 8 layer PW laminate. This shows the lowest notched strength ratio for all notch sizes. The scatter of the data appears to be smaller than for the un-normalised data.
Figure 5.35 (5.14) shows the notched strength ratios for all 8HS weave samples. The elliptically notched samples generally have slightly lower notched strength ratios than samples with circular notches, as observed for the un-normalised data. Again, the scatter of the data is smaller than for the un-normalised data.

Figure 5.36 (5.15) shows the notched strength ratios for only 2 and 8 layer notched samples so that the extremes of laminate thickness may be compared more easily. The 8 layer 8HS weave samples with elliptical notches and the 8 layer PW samples with circular notches have lower notched strength ratios than the other samples, as seen for the un-normalised data. This effect appears to be more pronounced for the normalised data presented here.

The strength ratios are plotted against the number of fabric layers for each notch size, notch type and laminate reinforcement in figure 5.37 (5.16). The general trend seen in the data is that the 8HS weave laminates with circular notches have higher notched strength ratios than the 8HS weave laminates with elliptical notches. Also, the 8HS weave laminates containing circular notches generally have higher notched strength ratios than the PW laminates containing circular notches. The 8HS weave laminates have approximately constant notched strength ratios for both notch shapes and all notch sizes with variation of the number of fabric layers. The one exception to this are the 5 mm elliptically notched samples which show decreasing notched strength ratios with increasing number of fabric layers. This was also observed in figure 5.31. It can also be seen that the 2 layer PW notched laminates have higher notched strength ratios than the 8 layer PW notched laminates for all notch sizes.

5.6.3 Normalised Critical Energy Release Rate

5.6.3.1 Eight Harness Satin Weave Laminates

The change in corrected toughness with increasing SEN length is shown for 2, 4, 6 and 8 layer laminates in figure 5.38 (5.21). Only the values obtained using the maximum fracture load is presented for the normalised toughness data. The laminate
toughness still appears to be dependent on SEN length after the normalisation for
volume fraction effects. However, the large increase in toughness with increasing
SEN length shown for the raw data for the 4 layer laminate has been accounted for.
The data for the 4, 6 and 8 layer laminates are very similar. The toughness values for
the 2 layer laminates are significantly higher than the thicker laminates at all SEN
lengths. All 8HS weave laminates show a similar rate of increase in toughness with
increasing SEN length.

Figure 5.39 (5.22) shows the average toughness over the three SEN lengths
investigated plotted against the number of fabric layers. The normalised laminate
toughness values are similar to those presented in section 5.5.2. The 2 layer laminate
appears to be approximately 30 % tougher than the thicker laminates, which all show
a similar toughness.

5.6.3.2 Plain Weave Laminates

The change in normalised toughness with increasing SEN length is shown for 2 and 8
layer PW laminates in figure 5.38. The normalised toughness data shows a similar
dependence on SEN length as seen for the un-normalised data. Again, it appears that
the 2 layer laminates are tougher than the 8 layer laminates.

Figure 5.39 shows the average toughness over the three SEN lengths investigated plotted against the number of fabric layers for both methods of fracture load
determination. As for the case of the 8HS weave laminates, the average normalised
toughness data are similar to the data presented in section 5.5.3.

5.6.2.3 Comparison of Normalised Critical Energy Release Rate for Eight
Harness Satin Weave and Plain Weave Laminates

When comparing the 8HS weave and PW laminates the data acquired using the
maximum fracture load has been used so that the methods of toughness measurement
are similar.
Chapter 5: Unnotched And Notched Laminate Properties

The variation of toughness with SEN length is shown for all laminates in figure 5.38. It can be seen clearly that the toughness appears to be more dependent on SEN length for the PW laminates after the proposed fibre volume fraction effects have been considered. The reasons for this are unclear, as discussed in section 5.5.4. The differences between the two reinforcement types concern the distribution of crimp regions (and associated tow curvature) and the gap between adjacent tows. As previously speculated, the fabric architecture may influence the relationship between toughness and SEN length.

The average toughness values across all SEN lengths are shown plotted against the number of fabric layers in the laminates in figure 5.39. The data is almost identical to that presented in Chapter 5. For both reinforcement types there is a reduction in laminate toughness with increasing number of fabric layers. As previously discussed, the reasons for this effect are not clear.

5.7 Concluding Remarks

The laminate properties presented in this chapter provide a comprehensive data-base of elastic properties and unnotched and notched strength properties of 8HS weave and PW composites with varying numbers of fabric layers. It has been shown that the fibre volume fraction variations present in the materials studied have a strong influence on laminate properties.

The Young's modulus and unnotched strength increase with the number of fabric layers for both reinforcement types. It is proposed that this is, at least partly, due to an increase in fibre volume fraction with increasing number of fabric layers.

There is also a strong influence of predicted fibre volume fraction variations on the notched strengths of the laminates investigated in this study. It is proposed that these volume fraction variations initially tend to obscure any trends in the notched strengths present for the various laminate and notch configurations investigated. The notched strength normalisation method proposed appears to have accounted for the
fibre volume fraction variations present. There is little difference between the normalised notched strength ratios for 8HS weave laminates with varying numbers of fabric layers. Perhaps the strongest indication of a notched strength-number of fabric layers effect can be seen for the PW laminates in which the notched strength ratios for the 8 layer samples are lower than those of the 2 layer samples for all notch sizes. The normalised results show that, in general, the notched strength of the 8HS weave laminates containing circular notches are higher than those of the 8HS weave laminates containing elliptical notches and the PW laminates containing circular notches.

Although the load-extension responses of PW laminates were shown to be non-linear, the maximum recorded load was used as a fracture load in the toughness determination, enabling comparisons between the two laminate reinforcements to be made. All laminate types showed an apparent increase in toughness with increasing SEN length for both the un-normalised and normalised data. The critical energy release rate, or toughness, results suggest that the thinner laminates are tougher than the thicker laminates and that for the 2 layer materials the 8HS weave laminates are tougher than the PW laminates. Again, this was found for both the un-normalised and normalised data.

The data presented in this chapter is required for the notched strength modelling performed in Chapter 7. The models investigated will be compared to both the un-normalised and normalised notched strengths of the laminates investigated. The unnotched strength, elastic property and toughness data are required as input parameters in the models.
### Chapter 5: Notched and Unnotched Laminate Properties

#### Table 5.1. Mechanical property experimental programme.

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**Key:**

- **8HS** - Eight harness satin weave laminate
- **PW** - Plain weave laminate
- **t** - Transversely orientated
- **s** - For shear properties, aligned at ±45°
- **c** - Circular notch
- **e** - Elliptical notch
- **(n)** - Number of samples tested

---

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Table 5.2. Unnotched 8HS weave laminate properties.
### Table 5.3. Unnotched PW laminate properties.

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<th>Number of layers</th>
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<th>E (trans.) (GPa)</th>
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<th>$\sigma_0$ (MPa)</th>
<th>P ratio (%)</th>
<th>max load (kN)</th>
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### Table 5.4. Average laminate fibre volume fractions.

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<th>Fibre volume fraction of PW laminate (%)</th>
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<td>-</td>
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<tr>
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<td>39.5</td>
<td>40.7</td>
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<tr>
<td></td>
<td>2.5 mm (circular)</td>
<td>5 mm (circular)</td>
</tr>
<tr>
<td>----------------</td>
<td>-------------------</td>
<td>----------------</td>
</tr>
<tr>
<td>E (GPa)</td>
<td>σ_n (MPa)</td>
<td>σ_n/δ_n %</td>
</tr>
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<td>17.47</td>
<td>181</td>
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<tr>
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<tr>
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<tr>
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<td>17.62</td>
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Table 5.5. Notched 8HS weave laminate properties for circular notches.
### Table 5.6. Notched 8HS weave laminate properties for elliptical notches.

<table>
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<tr>
<th>notch size</th>
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<th>10 mm (elliptical)</th>
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<td>σ_0/σ_f</td>
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Notched and Unnotched Laminate Properties
### Table 5.7: Notched PW laminate properties.

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<td>---------</td>
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</tr>
<tr>
<td>a (m)</td>
<td>B (m)</td>
<td>dC/da (m/N)</td>
<td>P (N)</td>
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| 0.008   | 0.0007  | 1.99E-05   | 1.36E+03 | 27884   | 0.008   | 0.0013  | 8.40E-06   | 2.45E+03 | 20168    |
| 0.008   | 0.0005  | 1.99E-05   | 1.36E+03 | 27884   | 0.008   | 0.0013  | 8.40E-06   | 2.56E+03 | 21845    |
| 0.008   | 0.0007  | 1.99E-05   | 1.29E+03 | 25088   | 0.008   | 0.0013  | 8.40E-06   | 2.55E+03 | 21675    |

| 0.012   | 0.0006  | 4.07E-05   | 9.30E+02 | 27501   | 0.012   | 0.0013  | 1.83E-05   | 1.75E+03 | 22240    |
| 0.012   | 0.0006  | 4.07E-05   | 9.70E+02 | 30883   | 0.012   | 0.0012  | 1.83E-05   | 1.88E+03 | 26292    |
| 0.012   | 0.0007  | 4.07E-05   | 9.70E+02 | 28158   | 0.012   | 0.0012  | 1.83E-05   | 1.92E+03 | 28109    |

| Mean over all SEN lengths | 26610 | 21049 |

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</tbody>
</table>

| 0.008   | 0.0019  | 5.97E-06   | 3.56E+03 | 20339   | 0.008   | 0.0024  | 3.91E-06   | 4.86E+03 | 19402    |
| 0.008   | 0.0018  | 5.97E-06   | 3.55E+03 | 20445   | 0.008   | 0.0024  | 3.91E-06   | 4.93E+03 | 19716    |
| 0.008   | 0.0018  | 5.97E-06   | 3.57E+03 | 20789   | 0.008   | 0.0024  | 3.91E-06   | 4.86E+03 | 18925    |

| 0.012   | 0.0018  | 1.30E-05   | 2.40E+03 | 20459   | 0.012   | 0.0024  | 9.49E-06   | 3.18E+03 | 19665    |
| 0.012   | 0.0018  | 1.30E-05   | 2.67E+03 | 26479   | 0.012   | 0.0024  | 9.49E-06   | 3.19E+03 | 19871    |
| 0.012   | 0.0018  | 1.30E-05   | 2.51E+03 | 22500   | 0.012   | 0.0023  | 9.49E-06   | 3.37E+03 | 23328    |

| Mean over all SEN lengths | 21036 | 20994 |

Table 5.8. 8HS weave laminate toughness data.
Table 5.9. PW laminate toughness data calculated using (a) maximum load and (b) 5% reduction in compliance secant load.
### Table 5.10. Normalised unnotched strengths for 8HS weave laminates containing circular notches

<table>
<thead>
<tr>
<th>Layers</th>
<th>$\sigma_0$ (MPa)</th>
<th>$V_f^{\text{unnotched}}$ (%)</th>
<th>$V_f^{\text{notched}}$ (%)</th>
<th>$\sigma_0^{\text{norm}}$ (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>328</td>
<td>36.1</td>
<td>32.8</td>
<td>298.4</td>
</tr>
<tr>
<td>4</td>
<td>322</td>
<td>34.8</td>
<td>35.7</td>
<td>330.3</td>
</tr>
<tr>
<td>6</td>
<td>338</td>
<td>36.1</td>
<td>38.5</td>
<td>360.1</td>
</tr>
<tr>
<td>8</td>
<td>382</td>
<td>39.6</td>
<td>40.2</td>
<td>387.0</td>
</tr>
</tbody>
</table>

### Table 5.11. Normalised unnotched strengths for 8HS weave laminates containing elliptical notches

<table>
<thead>
<tr>
<th>Layers</th>
<th>$\sigma_0$ (MPa)</th>
<th>$V_f^{\text{unnotched}}$ (%)</th>
<th>$V_f^{\text{notched}}$ (%)</th>
<th>$\sigma_0^{\text{norm}}$ (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>328</td>
<td>36.1</td>
<td>35.6</td>
<td>323.6</td>
</tr>
<tr>
<td>4</td>
<td>322</td>
<td>34.8</td>
<td>37.3</td>
<td>345.0</td>
</tr>
<tr>
<td>6</td>
<td>338</td>
<td>36.1</td>
<td>37.7</td>
<td>353.1</td>
</tr>
<tr>
<td>8</td>
<td>382</td>
<td>39.6</td>
<td>38.8</td>
<td>373.5</td>
</tr>
</tbody>
</table>

### Table 5.12. Normalised unnotched strengths for PW laminates containing circular notches

<table>
<thead>
<tr>
<th>Layers</th>
<th>$\sigma_0$ (MPa)</th>
<th>$V_f^{\text{unnotched}}$ (%)</th>
<th>$V_f^{\text{notched}}$ (%)</th>
<th>$\sigma_0^{\text{norm}}$ (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>312</td>
<td>38.0</td>
<td>38.7</td>
<td>317.9</td>
</tr>
<tr>
<td>8</td>
<td>362</td>
<td>40.1</td>
<td>40.6</td>
<td>366.7</td>
</tr>
</tbody>
</table>
### Table 5.13. Normalised SEN specimen fracture loads for 8HS weave laminates containing circular notches

<table>
<thead>
<tr>
<th>Layers</th>
<th>a (mm)</th>
<th>$P_Q$ (kN)</th>
<th>$V_i$ (%)</th>
<th>$V_i^{av}$ (%)</th>
<th>$P_Q^{norm}$ (kN)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>4</td>
<td>1950</td>
<td>35.5</td>
<td>35.8</td>
<td>1907</td>
</tr>
<tr>
<td>2</td>
<td>8</td>
<td>1340</td>
<td>35.2</td>
<td>35.8</td>
<td>1365</td>
</tr>
<tr>
<td>2</td>
<td>12</td>
<td>957</td>
<td>35.7</td>
<td>35.8</td>
<td>951</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>3980</td>
<td>34.6</td>
<td>36.0</td>
<td>4148</td>
</tr>
<tr>
<td>4</td>
<td>8</td>
<td>2520</td>
<td>35.4</td>
<td>36.0</td>
<td>2497</td>
</tr>
<tr>
<td>4</td>
<td>12</td>
<td>1850</td>
<td>37.2</td>
<td>36.0</td>
<td>1794</td>
</tr>
<tr>
<td>6</td>
<td>4</td>
<td>6080</td>
<td>37.9</td>
<td>37.6</td>
<td>6070</td>
</tr>
<tr>
<td>6</td>
<td>8</td>
<td>3560</td>
<td>37.4</td>
<td>37.8</td>
<td>3605</td>
</tr>
<tr>
<td>6</td>
<td>12</td>
<td>2530</td>
<td>38.2</td>
<td>37.8</td>
<td>2503</td>
</tr>
<tr>
<td>8</td>
<td>4</td>
<td>7500</td>
<td>39.7</td>
<td>38.5</td>
<td>7268</td>
</tr>
<tr>
<td>8</td>
<td>8</td>
<td>4880</td>
<td>37.7</td>
<td>38.5</td>
<td>4979</td>
</tr>
<tr>
<td>8</td>
<td>12</td>
<td>3250</td>
<td>38.0</td>
<td>38.5</td>
<td>3290</td>
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</table>

### Table 5.14. Normalised SEN specimen fracture loads for PW laminates containing circular notches

<table>
<thead>
<tr>
<th>Layers</th>
<th>a (mm)</th>
<th>$P_Q$ (kN)</th>
<th>$V_i$ (%)</th>
<th>$V_i^{av}$ (%)</th>
<th>$P_Q^{norm}$ (kN)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>4</td>
<td>1130</td>
<td>38.0</td>
<td>38.4</td>
<td>1144</td>
</tr>
<tr>
<td>2</td>
<td>8</td>
<td>762</td>
<td>39.4</td>
<td>38.4</td>
<td>743</td>
</tr>
<tr>
<td>2</td>
<td>12</td>
<td>479</td>
<td>38.0</td>
<td>38.4</td>
<td>485</td>
</tr>
<tr>
<td>8</td>
<td>4</td>
<td>4390</td>
<td>40.7</td>
<td>40.8</td>
<td>4401</td>
</tr>
<tr>
<td>8</td>
<td>8</td>
<td>2480</td>
<td>40.7</td>
<td>40.8</td>
<td>2486</td>
</tr>
<tr>
<td>8</td>
<td>12</td>
<td>1880</td>
<td>41.0</td>
<td>40.8</td>
<td>1871</td>
</tr>
<tr>
<td>d (mm)</td>
<td>2.5</td>
<td>5</td>
<td>10</td>
<td></td>
<td></td>
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<tr>
<td>-------</td>
<td>-----</td>
<td>---</td>
<td>----</td>
<td></td>
<td></td>
</tr>
<tr>
<td>d/W</td>
<td>0.1</td>
<td>0.2</td>
<td>0.4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\sigma_n/\sigma_0)</td>
<td>(\sigma_u/\sigma_0)</td>
<td>(\sigma_s/\sigma_0)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2 layer</td>
<td>0.627</td>
<td>0.523</td>
<td>0.429</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4 layer</td>
<td>0.669</td>
<td>0.548</td>
<td>0.378</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6 layer</td>
<td>0.647</td>
<td>0.508</td>
<td>0.378</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8 layer</td>
<td>0.646</td>
<td>0.535</td>
<td>0.388</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 5.15. Normalised notched to unnotched strength ratios for 8HS weave laminates containing circular notches.

<table>
<thead>
<tr>
<th>a (mm)</th>
<th>2.5</th>
<th>5</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>2a/W</td>
<td>0.1</td>
<td>0.2</td>
<td>0.4</td>
</tr>
<tr>
<td>(\sigma_n/\sigma_0)</td>
<td>(\sigma_u/\sigma_0)</td>
<td>(\sigma_s/\sigma_0)</td>
<td></td>
</tr>
<tr>
<td>2 layer</td>
<td>0.606</td>
<td>0.535</td>
<td>0.364</td>
</tr>
<tr>
<td>4 layer</td>
<td>0.626</td>
<td>0.516</td>
<td>0.339</td>
</tr>
<tr>
<td>6 layer</td>
<td>0.615</td>
<td>0.496</td>
<td>0.336</td>
</tr>
<tr>
<td>8 layer</td>
<td>0.584</td>
<td>0.468</td>
<td>0.351</td>
</tr>
</tbody>
</table>

Table 5.16. Normalised notched to unnotched strength ratios for 8HS weave laminates containing elliptical notches.

<table>
<thead>
<tr>
<th>d (mm)</th>
<th>2.5</th>
<th>5</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>d/W</td>
<td>0.1</td>
<td>0.2</td>
<td>0.4</td>
</tr>
<tr>
<td>(\sigma_n/\sigma_0)</td>
<td>(\sigma_u/\sigma_0)</td>
<td>(\sigma_s/\sigma_0)</td>
<td></td>
</tr>
<tr>
<td>2 layer</td>
<td>0.601</td>
<td>0.541</td>
<td>0.396</td>
</tr>
<tr>
<td>8 layer</td>
<td>0.586</td>
<td>0.472</td>
<td>0.335</td>
</tr>
</tbody>
</table>

Table 5.17. Normalised notched to unnotched strength ratios for PW laminates containing circular notches.
### Table 5.18. Normalised toughness for 8HS weave laminates.

<table>
<thead>
<tr>
<th>Layer</th>
<th>$G_c$ (J/m²)</th>
<th>Layer</th>
<th>$G_c$ (J/m²)</th>
<th>Layer</th>
<th>$G_c$ (J/m²)</th>
<th>Layer</th>
<th>$G_c$ (J/m²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 layer</td>
<td>23293</td>
<td>4 layer</td>
<td>17765</td>
<td>6 layer</td>
<td>19357</td>
<td>8 layer</td>
<td>18745</td>
</tr>
<tr>
<td>0.004</td>
<td></td>
<td>0.004</td>
<td></td>
<td>0.004</td>
<td></td>
<td>0.004</td>
<td></td>
</tr>
<tr>
<td>0.008</td>
<td>28097</td>
<td>0.008</td>
<td>20839</td>
<td>0.008</td>
<td>21044</td>
<td>0.008</td>
<td>20107</td>
</tr>
<tr>
<td>0.012</td>
<td>29041</td>
<td>0.012</td>
<td>23936</td>
<td>0.012</td>
<td>22625</td>
<td>0.012</td>
<td>21458</td>
</tr>
</tbody>
</table>

Average: 26810.4, Standard Deviation: 3083

### Table 5.19. Normalised toughness for PW laminates.

<table>
<thead>
<tr>
<th>Layer</th>
<th>$G_c$ (J/m²)</th>
<th>Layer</th>
<th>$G_c$ (J/m²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 layer</td>
<td>14690</td>
<td>8 layer</td>
<td>14245</td>
</tr>
<tr>
<td>0.004</td>
<td></td>
<td>0.004</td>
<td></td>
</tr>
<tr>
<td>0.008</td>
<td>23984</td>
<td>0.008</td>
<td>15710</td>
</tr>
<tr>
<td>0.012</td>
<td>31135</td>
<td>0.012</td>
<td>29855</td>
</tr>
</tbody>
</table>

Average: 23269.9, Standard Deviation: 8246

Average: 19937.1, Standard Deviation: 8821
Figure 5.1. Young’s modulus versus number of fabric layers for 8HS weave and PW laminates.

Figure 5.2. Poisson’s ratio, \( \nu \), versus number of fabric layers for 8HS weave and PW laminates.
Figure 5.3. Unnotched tensile strength versus number of fabric layers for 8HS weave and PW laminates.

Figure 5.4. Average fibre volume fraction versus number of fabric layers for 8HS weave and PW laminates.
Figure 5.5: Fibre volume fraction versus laminate thickness for 8HS weave and PW laminates.
Figure 5.6. Notched to unnotched strength ratio versus notch diameter to plate width ratio for 8HS weave laminates containing circular notches.

Figure 5.7. Predicted fibre volume fraction versus notch diameter to plate width ratio for 8HS weave laminates containing circular notches.
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Figure 5.8. Notched to unnotched strength ratio versus notch size to plate width ratio for 8HS weave laminates containing elliptical notches.

Figure 5.9. Predicted fibre volume fraction versus notch size to plate width ratio for 8HS weave laminates containing elliptical notches.
Figure 5.10. Notched to unnotched strength ratio versus notch diameter to plate width ratio for PW laminates containing circular notches.

Figure 5.11. Predicted fibre volume fraction versus notch diameter to plate width ratio for PW laminates containing circular notches.
Figure 5.12. Notched to unnotched strength ratio versus notch size to plate width ratio for all notched laminate configurations.

Figure 5.13. Notched to unnotched strength ratio versus notch diameter to plate width ratio for 8HS weave and PW laminates containing circular notches.
Figure 5.14. Notched to unnotched strength ratio versus notch size to plate width ratio for 8HS weave laminates containing circular and elliptical notches.

Figure 5.15. Notched to unnotched strength ratio versus notch size to plate width ratio for all 2 and 8 layer laminates.
Figure 5.16. Notched to unnotched strength ratio versus number of fabric layers for all laminate configurations.
Figure 5.17. Compliance versus edge notch length for 8HS weave laminates containing (a) 2 layers, (b) 4 layers, (c) 6 layers and (d) 8 layers.
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Figure 5.18 Compliance versus edge notch length for all 8HS weave laminates.

Figure 5.19 Compliance multiplied by number of fabric layers versus edge notch length for all 8HS weave laminates.
Figure 5.20. Rate of change of compliance with edge notch length versus edge notch length for all 8HS weave laminates.

Figure 5.21. Critical energy release rate versus edge notch length for 8HS and PW laminates.
Figure 5.22. Average critical energy release rate versus number of fabric layers for 8HS and PW laminates.
Figure 5.23. Compliance versus edge notch length for PW laminates containing (a) 2 layers and (b) 8 layers.
Figure 5.24. Compliance versus edge notch length for all PW laminates.

Figure 5.25. Compliance multiplied by number of fabric layers versus edge notch length for all PW laminates.
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Figure 5.26. Rate of change of compliance with edge notch length versus edge notch length for all PW laminates.

Figure 5.27. Load versus extension for 2 layer PW laminate containing a 4 mm edge notch.
Figure 5.28. Predicted fibre volume fraction versus edge crack length for 8HS weave laminates.

Figure 5.29. Predicted fibre volume fraction versus edge crack length for PW laminates.
Figure 5.30. Normalised notched to unnotched strength ratio versus notch diameter to plate width ratio for 8HS weave laminates containing circular notches.

Figure 5.31. Normalised notched to unnotched strength ratio versus notch size to plate width ratio for 8HS weave laminates containing elliptical notches.
Figure 5.32. Normalised notched to unnotched strength ratio versus notch diameter to plate width ratio for PW laminates containing circular notches.

Figure 5.33. Normalised notched to unnotched strength ratio versus notch size to plate width ratio for all laminates configurations.
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Figure 5.34. Normalised notched to unnotched strength ratio versus notch diameter to plate width ratio for 8HS weave and PW laminates containing circular notches.

Figure 5.35. Normalised notched to unnotched strength ratio versus notch size to plate width ratio for 8HS weave laminates containing circular and elliptical notches.
Figure 5.36. Normalised notched to unnotched strength ratio versus notch size to plate width ratio for all 2 and 8 layer laminates.
Figure 5.37. Normalised notched to unnotched strength ratio versus number of fabric layers for all laminate configurations.
Figure 5.38. Normalised critical energy release rate versus edge notch length for 8HS weave and PW laminates.

Figure 5.39. Average normalised critical energy release rate versus number of fabric layers for 8HS and PW laminates.
Chapter 6: Notch Edge Damage
6.1 **Introduction**

This chapter discusses the type and extent of damage found adjacent to the various notch geometries investigated in this study. The three techniques used to observe the notch edge damage are described in detail in section 3.7.

The first technique discussed is *in-situ* photography, performed during mechanical testing. This was carried out for all laminate types and notch geometries during the acquisition of the notched laminate properties. This is followed by the examination of damaged 8HS weave fabric reinforcements containing circular and elliptical notches using a scanning electron microscope (SEM). Finally, the notch edge damage in 8HS weave laminates is observed using a deply technique, where damage sites are identified using a gold chloride marker. This combination of damage observation techniques enables a comprehensive damage analysis to be performed.

The plan view and deply micrographs presented in this chapter do not contain scale bars. The notch diameter is given in the figure caption in each case. Also, in the plan view images the sample width of 25 mm is constant throughout all samples. The constant notch root radius of 0.5 mm is another feature of reference for the elliptically notched samples. The SEM images contain scale bars.

6.2 **In-Situ Plan View Damage**

6.2.1 **Eight Harness Satin Weave Laminates**

6.2.1.1 **Overview**

In this section the general sequence of notch edge damage found in the 8HS weave laminates is initially described qualitatively, before observations relating to individual laminates and notch geometries are discussed in more detail in subsequent sections.
Chapter 6: Notch Edge Damage

The matrix cracking damage developed under quasi-static loading in unnotched 8HS weave laminates has been described by Marsden (1996), as discussed in Chapter 2. The morphology of the damage adjacent to notches in the 8HS weave laminates is consistent with his observations.

The notch edge damage was seen to initiate at a remote applied strain lower than the crack initiation strain of 0.6 % measured for unnotched samples (Marsden, 1996), consistent with there being a stress concentration at the hole edge. The initial damage seen was matrix cracking, as observed by Marsden (1996) in unnotched specimens. As the remote applied strain was increased this transverse cracking increased in density in the vicinity of the notch.

The transverse cracking notch edge damage and sequence of events in the 2, 4, 6 and 8 layer laminates containing both circular and elliptical notches were similar. As the number of fabric layers was increased the apparent plan view transverse cracking density increased as a result of the superposition of the cloths. This can be seen by comparing figure 6.1, a photograph of the 2 layer laminate containing a 2.5 mm circular notch at an applied strain of 0.8 % with figure 6.2, a photograph of the notch edge damage for the 8 layer laminate containing a 2.5 mm circular notch, also at an applied strain of 0.8 %. The damage morphology and sequence of events for laminates containing the 5 mm and 10 mm diameter notches were also similar. The damage in 2 layer and 8 layer laminates adjacent to 10 mm diameter circular notches at 0.6 % strain can be seen in figures 6.3 and 6.4, respectively. The increased apparent crack density in the 8 layer laminate is again clear.

For the elliptical notches, the damage morphology and sequence of events for the different notch sizes are very similar due to the constant notch root radius. The damage in a 2 layer laminate containing a 10 mm elliptical notch at an applied strain of 0.3 % is shown in figure 6.5. Similarly, the notch edge damage in a 8 layer laminate containing a 10 mm elliptical notch at an applied strain of 0.3 % is shown in figure 6.6.
Just prior to failure, another form of damage was seen to initiate at the notch edge. It was thought that this damage comprised of longitudinal fibre breaks and associated splits and delaminations. During this stage of the loading regime, there were acoustic events, which were thought to be a result of the proposed fibre breaks. The damage zones associated with the supposed fibre breaks and delaminations were seen to propagate sporadically from the notch edge, often rapidly advancing a small distance before stopping. It was thought that the advance of this damage zone paused between adjacent fibre tow failures, although this could not be confirmed. For laminates containing circular notches, this damage zone initiated almost immediately prior to failure. The damage zones in laminates containing elliptical notches initiated earlier in the loading regime with respect to catastrophic failure compared to laminates with circular notches. In general, the larger the critical damage zone length at the point of catastrophic failure, then the sooner this damage was seen to initiate with respect to final failure. The critical length of these dense damage zones were measured from photomicrographs taken immediately prior to failure. These critical damage zone lengths must be considered approximate due to the way in which they were measured and the small number of samples investigated. The camera was set in continuous shooting mode when failure was imminent. The catastrophic nature of the failure and the time required for the camera film to wind on suggests that the critical length measured may not be strictly correct. Hence, the maximum, not average, size of the damage zone observed in the three notched coupons for each sample configuration was taken as representative. It is proposed that these experimental observations should be regarded as an approximate measure of the extent of notch edge damage at failure.

The critical size of the dense notch edge damage zones varied for the different samples investigated. The critical notch edge damage zones are shown in the photomicrographs in figures 6.7-6.30 for all 8HS weave laminate types and notch geometries. The measured damage zone lengths are given in table 6.1.
6.2.1.2 Critical Damage Zone Observations - Circular Notches

(a) \(D = 2.5\) mm

The critical notch edge damage zone for the 2 layer laminate containing a 2.5 mm notch is shown in figure 6.7. This extends 0.9 mm from the notch edge and is approximately 0.5 mm in depth (in the direction of loading). The damage zone is not situated at the centre of the notch edge, nominally the point of highest stress concentration, but is slightly offset. There are two longitudinal splits either side of this damage zone, approximately 1 mm in length. The separation of these splits is approximately 0.5 mm, similar to the width of the tows from which the 8HS weave fabric is constructed. There are also longitudinal splits associated with the notch edge and smaller dense damage zones in these areas.

The equivalent 4 layer laminate shows a larger critical notch edge damage zone, measured at 1.2 mm in length (figure 6.8). This damage zone is also larger in the direction of loading, at approximately 1 mm in depth. Again the damage zone is not positioned at the centre of the notch. There are also more longitudinal splits, but without the regular spacing seen for the 2 layer laminates. This is probably due to a staggering effect from the superposition of multiple fabric layers. These longitudinal splits are up to 2 mm in length. Smaller damage zones have also initiated at the notch edge in similar positions to the larger, dominant damage zone.

The damage zone measured for the 6 layer laminate is 0.9 mm in length, similar to that observed in the 2 layer laminate, although it is larger in the direction of loading, at approximately 2-3 mm in depth (figure 6.9). Similar to the thinner laminates, the dominant damage zone is not situated at the centre of the notch edge. Again, the splitting damage extends approximately 2 mm from the notch.

The dominant damage zone in the 8 layer laminate measures 1.2 mm in length and 2-3 mm in depth (figure 6.10). The dense damage zones situated on both sides of the notch cover a larger area than those seen in the thinner laminates. Again, the splitting damage extends approximately 2 mm from the notch.
Chapter 6: Notch Edge Damage

(b) \( D = 5 \text{ mm} \)

The critical damage zone length seen in the 2 layer laminate shown in figure 6.11 is relatively long, at 2.1 mm, but also quite narrow, at less than 1 mm in depth. Longitudinal splits can be seen extending approximately 1 mm from this damage zone.

The 4 layer laminate shows a considerably smaller critical damage zone length at 1.2 mm in length but, at approximately 2 mm in depth, is twice as deep as that seen in the 2 layer laminate (figure 6.12).

The critical damage zone length in the 6 layer laminate (figure 6.13) is slightly more extensive at 1.5 mm in length and 2 mm in depth. Again, longitudinal fibre splitting can be seen extending from the damage zone.

The 8 layer laminate shows a critical damage zone covering a considerable area, with a length of 2 mm (figure 6.14). This damage zone is also extensive in the loading direction with a depth of approximately 3-4 mm. Longitudinal splits of approximately 2 mm can be seen extending from the dense damage zone.

(c) \( D = 10 \text{ mm} \)

The critical damage zone length in the 2 layer laminate is very small at 0.3 mm in length and 1 mm in depth (figure 6.15). There are longitudinal splits of approximately 1-2 mm in length at the notch edges.

The 4 layer laminate shows a slightly larger critical damage zone length at 0.9 mm in length, with a similar depth (figure 6.16). Two of these damage zones can be seen initiating from the same notch edge.
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The critical damage zone length in the 6 layer laminate is more extensive again (figure 6.17). It measures 1.7 mm in length and approximately 2 mm in depth. Longitudinal fibre splits of up to 1 mm extend from the notch edge damage zone.

The 8 layer laminate shows a critical damage zone length of 2 mm (figure 6.18). This damage zone is extensive in the loading direction with a depth of approximately 6-7 mm. Longitudinal splits of approximately 2-3 mm extend from the dense damage zone.

(d) Summary of Damage Around Circular Notches

The measured critical damage zone length is shown in figure 6.31 as a function of notch size to sample width ratio for 8HS weave laminates containing circular notches. It can be seen that the critical damage zone length increases with increasing notch size for the thicker 6 and 8 layer laminates. However, this is not the case for the 2 and 4 layer laminates. The critical damage zone length can be seen to generally increase as the number of fabric layers increases, with the exception of the 2 layer laminate containing a 5 mm diameter notch, which shows a large damage zone size.

For all hole sizes, as the number of fabric layers was increased, the depth of the damage zones increased. The reason for this behaviour cannot be deduced from in-situ photography. At this stage, it is speculated that the superposition of the damage in multi-layered laminates is responsible for this effect.

6.2.1.3 Critical Damage Zone Observations - Elliptical Notches

(a) \(2a = 2.5\) mm

The critical damage zone length in the 2 layer laminate was measured at 3.4 mm (figure 6.19). The damage extends from either side of the notch and appears to resemble an array of fibre breaks and delaminations forming a crack-like damage feature. This damage zone is narrow, at approximately 1 mm in depth. The crack-like damage zone does not grow precisely along the ellipse major axis. It can be seen
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to deviate slightly in either direction. The longitudinal splits are more prominent than those seen for the circular notches, extending 2-3 mm from the damage zone. They are regularly spaced at approximately 0.5 mm apart, which corresponds to the width of the tows in the 8HS weave cloth. This suggests that they form at the interface of fractured longitudinal tows.

The 4 layer laminate shows a critical damage zone length of 2.6 mm (figure 6.20). In this case the damage zone is slightly larger in the direction of loading, extending up to approximately 2 mm in depth. The longitudinal splits are similar to those seen in the 2 layer laminate.

The critical damage zone length in the 6 layer laminate is shorter at 1.3 mm (figure 6.21). This damage zone extends up to approximately 2 mm in depth. Longitudinal splits are present, extending up to 2 mm from the damage zone.

The critical damage zone length in the 8 layer laminate is 1.4 mm (figure 6.22). It is also relatively deep, measuring approximately 2 mm. Longitudinal splitting is present although it is more difficult to identify due to the superposition of damage throughout the fabric layers.

(b) 2a = 5 mm

Figure 6.23 shows the critical damage zone length extending 3.6 mm from the notch edge in the 2 layer laminate. The extent of the damage zone in the loading direction is of similar magnitude to the notch root diameter of 1 mm.

The critical damage zone length for the 4 layer laminate is shorter, at 2.5 mm (figure 6.24). However, the damage zone appears to be slightly more extensive in the loading direction than for the 2 layer laminate.

The critical damage zone length for the 6 layer laminate is shorter still, at 2.3 mm (figure 6.25). The depth of this damage zone is approximately 2 mm.
The 8 layer laminate has the shortest critical damage zone length of 2 mm, shown in figure 6.26. This damage zone is also approximately 2 mm in depth.

The longitudinal splitting damage in these laminates containing elliptical notches is similar to that seen in the laminates containing 2.5 mm notches.

(c) $2a = 10$ mm

The critical damage zone length for the 2 layer laminate is 2.4 mm (figure 6.27). The damage zone is approximately 1 mm in depth, similar to the 2 layer laminates containing 2.5 mm and 5 mm elliptical notches.

Figure 6.28 shows the critical damage zone length extending 3.1 mm from the notch edge in the 4 layer laminate. The extent of the damage zone in the loading direction is larger than that of the 2 layer laminate, at approximately 2 mm.

The 6 layer laminate has a critical damage zone length of 1.9 mm, shown in figure 6.29. This damage zone is approximately 3 mm in depth.

The critical damage zone length for the 8 layer laminate is 2.4 mm (figure 6.30). The damage zone is the most extensive in the loading direction at approximately 3-4 mm.

The longitudinal splitting damage in these laminates containing elliptical notches is similar to that seen in the laminates containing 2.5 mm and 5 mm notches.

(d) Summary of Damage Around Elliptical Notches

Figure 6.32 shows the change in critical damage zone length with notch size to plate width ratio for the 8HS weave laminates containing elliptical notches. The critical damage zone lengths increase with increasing notch size for the 4, 6 and 8 layer laminates. However, the damage zone length decreases slightly for the 2 layer laminate as the notch size is increased. The critical damage zone length decreases
with increasing number of fabric layers for the 2.5 mm and 5 mm size notches. However, no such trends were observed for the 10 mm size notches.

As the number of fabric layers increased the damage zone depth was also seen to increase for all notch sizes. As speculated for the circular notches, the superposition of the damage in adjacent fabric layers is thought to be responsible for this effect. This increase in damage zone depth with the number of fabric layers was smaller than for the circular notches. This may be due to a more localised stress concentration for the case of the elliptical notches.

6.2.2 Plain Weave Laminates

6.2.2.1 Overview

The damage developed under quasi-static loading in unnotched PW laminates has been described in Chapter 4. This can be related to the damage observed adjacent to the circular notches investigated in these laminates.

In this section the general sequence of notch edge damage found in the PW laminates is initially described qualitatively, before observations relating to individual laminates and notch geometries are discussed in more detail in subsequent sections.

The notch edge damage was seen to initiate at a remote applied strain lower than the crack initiation strain of 0.6 % measured for the unnotched samples discussed in Chapter 4, consistent with there being a stress concentration at the hole edge. The initial damage seen was matrix cracking, as observed in Chapter 4 for unnotched specimens. As the remote applied strain was increased, this transverse cracking increased in density in the vicinity of the notch. The damage morphologies resulting from different relative cloth shifts described in Chapter 4 were identified across the range of samples investigated.

The transverse cracking notch edge damage and sequence of events in the 2 and 8 layer laminates were similar. This can be seen for the 2 layer laminate containing a
2.5 mm notch at an applied strain of 0.8% in figure 6.33. Similarly, the notch edge damage for the 8 layer laminate containing a 2.5 mm notch at an applied strain of 0.8% is shown in figure 6.34. The damage morphology and sequence of events for laminates containing the 5 mm and 10 mm diameter notches were also similar. The damage in a 2 layer laminate adjacent to a 10 mm diameter notch at 0.4% strain can be seen in figure 6.35. Similarly, the damage in an 8 layer laminate adjacent to a 10 mm diameter notch at 0.4% strain can be seen in figure 6.36. As seen for the 8HS weave laminates, the damage density in the 8 layer laminates appears to be significantly denser than that in the 2 layer laminates at all notch sizes.

Similar to observations of the 8HS weave laminates, another form of damage was seen to initiate at the notch edge prior to failure. Again, this was thought to consist of longitudinal fibre breaks and delaminations. The critical lengths of these dense damage zones were also measured from photomicrographs taken immediately prior to failure.

As observed in the 8HS weave laminates, the critical size of the dense notch edge damage zone varied for the different samples investigated. The critical notch edge damage zones are shown in the photomicrographs in figures 6.37-6.44 for all PW laminate types and notch geometries. The measured damage zone lengths are given in table 6.1.

6.2.2.2 Critical Damage Zone Observations - Circular Notches

(a) \(D = 2.5 \text{ mm}\)

The critical damage zone length for the 2 layer laminate was measured at 3.7 mm (figure 6.37). The damage zone is crack-like in appearance, extending approximately 1 mm in depth. The transverse cracking damage in this case is banded. There is also a regular array of delaminations surrounding the notch edge damage, indicating that the 2 cloths are shifted 180° with respect to each other. This corresponds to case B described in section 4.3. The dominant dense damage zone appears to have propagated through two of the transverse cracking bands. Three smaller dense
damage zones can also be seen on the other side of the notch. Each of these damage zones initiate and propagate through the transverse cracking bands. These transverse cracking bands appear to be the favoured sites for the initiation of the dense damage regions, irrespective of their position with respect to the centre of the notch. Almost identical behaviour was observed for the other similar samples, one of which is shown in figure 6.38. In this case the damage zone has propagated through the transverse cracking band, forming a crack-like feature less than 1 mm in depth. A regular array of delaminations is also present, indicating that the two cloths are shifted 180° with respect to each other.

The critical damage zone length for the 8 layer laminate was measured at 1.4 mm (figure 6.39). The damage zone is approximately 1.5 mm in depth and does not appear as crack-like as in the 2 layer laminate. This may be the result of the superposition of damage in multiple fabric layers shifted with respect to each other. The transverse cracking is at least partially banded adjacent to the dominant damage zone. On the other side of the notch the transverse cracking appears homogeneous. There do not appear to be any delaminations present in this case. This suggests that the 8 fabric layers are predominantly arranged at an intermediate shift with respect to each other. On one side of the sample at least some of the layers are nested, resulting in the banded transverse cracking termed case A in section 4.3. It would not be expected for all 8 layers to nest perfectly upon each other, hence, this damage morphology might be expected. Similar behaviour was also observed for the other 8 layer samples containing 2.5 mm diameter notches.

(b)  $D = 5 \text{ mm}$

The critical damage zone length in the 2 layer laminate was measured at 1.6 mm (figure 6.40). The non-banded notch edge transverse cracking in the vicinity of the damage zone indicates that the 2 cloths are at some intermediate shift with respect to each other. This damage zone is approximately 1.5 mm in depth but does not appear to be as crack-like as seen for the 2.5 mm notch. Similar damage zones have also initiated on the other side of the notch. Figure 6.41 shows a similar sample just prior
to failure. In this case the transverse cracking is banded and two damage zones are initiating within these damage bands, forming crack-like features.

Figure 6.42 shows the critical notch edge damage in the 8 layer laminate. This measures 1.7 mm in length and approximately 1.5 mm in depth. The transverse cracking for this case is banded, without the presence of delaminations, suggesting that the cloths are nested. The dense damage zone follows one of these damage bands at the centre of the notch.

(c) \( D = 10 \text{ mm} \)

Similar to the 8HS weave laminates, the critical damage zone in the 2 layer PW laminate containing a 10 mm notch is relatively small at 0.7 mm in length and approximately 1 mm in depth (figure 6.43). The transverse cracking is homogeneous in the vicinity of the largest damage zone and banded on the other side of the notch, where another damage zone is initiating.

The 8 layer laminate shows a similar critical damage zone length of 0.8 mm and a depth of approximately 2 mm (figure 6.44). In this case the transverse cracking is homogeneous on the left side of the notch and partially banded on the right side. No delaminations can be seen. This suggests that at least some of the fabric layers are nested to the right of the notch.

(d) Summary of Notch Edge Damage in Plain Weave Laminates

The change in critical damage zone length with notch size to sample width ratio is shown for the PW laminates containing circular notches in figure 6.45. The critical damage zone length decreases sharply with increasing notch size for the 2 layer laminates. However, this is not the case for the 8 layer laminates, where the critical damage zone length is approximately constant, with perhaps a slight decrease in damage zone length with increasing notch size. The critical damage zone length of the 2 layer laminate is significantly larger than that of the 8 layer laminate at the
smallest notch size. However, at the larger notch sizes the critical damage zone lengths are similar for 2 and 8 layer laminates.

The damage zone depths are smaller than those seen in the 8HS weave laminates. The damage zone depths generally increased with increasing fabric layers, but this effect was small. It was observed that when the transverse cracking damage was banded, as discussed in section 4.3, the notch edge damage zones were narrow and crack-like. The path of the damage zone was seen to propagate through this banded transverse cracking. When the cloths were at an intermediate shift the damage zones were generally more extensive in the direction of loading. This was often the case for the thicker laminates, in which it is more unlikely for all 8 layers to nest.

6.2.3 Summary of Notch Edge Damage in Eight Harness Satin Weave and Plain Weave Laminates

The change in critical damage zone length with notch size to plate width ratio is shown in figure 6.46 for all 2 and 8 layer laminates. The 2 layer 8HS weave laminates containing elliptical notches generally show the longest critical damage zone lengths, considerably longer than the 8HS weave laminates containing circular notches. The PW laminates show similar damage zone lengths to the 8HS weave laminates containing circular notches, except for at the smallest notch size. All 8 layer laminate configurations show similar trends in damage zone length with increasing notch size, except for the PW laminate at the largest notch size. As stated in section 6.2.1.1, these critical damage zone lengths should be considered as general observations of the extent of the damage zone at failure, rather than accurate measurements. Hence, the trends shown by the plots of critical damage zone length as a function of notch size should not be inferred to be representative of the current laminates. It may be that the method of critical damage zone length measurement selected does not properly represent the extent of the damage at failure. A more accurate measure may have been the average of the length of dense damage extending from either sides of the notch.
The transverse cracking damage adjacent to notches in 8HS weave and PW laminates is similar to that seen in unnotched samples. The different damage morphologies identified in Chapter 4 can be found adjacent to notch edges. In all cases, the damage zone depth, in the direction of loading, was seen to increase with increasing number of fabric layers. The 8HS weave laminates containing circular notches exhibited deeper damage zones compared to the elliptically notched samples. This is probably due to the more localised stress concentration adjacent to the elliptical notches. The PW laminates containing circular notches exhibited the narrowest, most crack-like damage zones. The reason for these differences in the damage zone morphologies between the 8HS weave and PW laminates containing circular notches is not immediately apparent using in-situ photography.

6.3 Scanning Electron Microscopy of Notch Edge Damage

6.3.1 Introduction

In order to characterise further the notch edge damage zones discussed in section 6.2, the fabric reinforcements were studied using a scanning electron microscope (SEM). The damage zones observed in section 6.2 were photographed during tensile tests to failure. In order to observe the notch edge damage in fabric reinforcements using the SEM, the tensile test must be stopped prior to catastrophic failure. The matrix material must then be burnt off. This was achieved by very carefully observing tensile tests of notched samples. When the notch edge damage zones were seen to initiate, the test was stopped and immediately reversed at a cross-head speed of 5 mm per minute in order to unload the sample rapidly. This resulted in notched samples containing transverse cracking and the dense notch edge damage zones thought to consist of fibre breaks and delaminations. These damage zones were not considered to be of a critical length as the extent of failure at which the tests were stopped at was not known. Only the 8HS weave reinforcements were studied using the SEM.

In section 6.2.1.1 it was observed that laminate failure occurred very soon after the notch edge damage zone initiated. Hence, stopping the tensile tests prior to failure often proved difficult. After several attempts it was found that it was not possible to
successfully stop a 2 layer laminate containing a circular notch, after the initiation of a notch edge damage zone, prior to failure. However, damage zones in 8 layer laminates containing circular notches were obtained successfully. In section 6.2.1.1 it was observed also that the damage zones around elliptical notches initiated slightly earlier with respect to laminate failure. Hence, damage zones in laminates of 2 and 8 layers containing elliptical notches were successfully obtained. Plan view photographs of the notch edge damage were taken prior to the resin being burnt off. Only fibre fracture could be identified in the woven fabrics, hence, the scanning electron micrographs were taken of the 0° tow dominated side of the 8HS weave cloth. The layers were numbered from the top layer down through the laminates. As the fabric layers were stacked back-to-back during fabrication, the left and right sides are reversed for even layer numbers. Hence, figures labelled left side correspond to the left side with respect to the plan view image, irrespective of what side the notch appears on, and vice versa.

6.3.2 Circular Notches

Figure 6.47 shows the notch edge plan view damage in an 8 layer laminate containing a 2.5 mm diameter notch. The dominant damage zone is located to the top right of the notch, extending approximately 1 mm in length. Other areas around the notch show smaller damage zones.

The SEM images show that tow failures and individual fibre breaks are present to the top right of the notch throughout the fabric layers. Some partial tow failures are present on the left side of the notch. The nature, or extent, of the damage does not appear to change throughout the laminate. The tow and fibre fractures extend approximately 1-1½ tow widths from the notch edge, a distance of approximately 0.75 mm. This suggest that the damage zones observed in plan view photographs consist of fibre and tow failures. Such a tow failure is shown in layer 1 in figure 6.48, partially hidden by the notch edge crimp region. Similar tow failures are shown in layers 6 and 8 in figures 6.49 and 6.50, respectively. In each of the above cases the tow failures have occurred in the region of the dominant damage zone identified by the plan view photograph (figure 6.47). It can be seen that the tow fractures appear
to be found adjacent to crimp regions and, therefore, are not directly superimposed
due to the random relative shifts of the 8HS weave cloths. This would account for
the observations in section 6.2 that the damage zones adjacent to circular notches in
the thicker laminates were relatively large in the direction of loading.

6.3.3 Elliptical Notches

The plan view damage in a 2 layer laminate containing a 10 mm elliptical notch is
shown in figure 6.51. A large damage zone, extending approximately 2-3 mm, can
be seen on the right side, initiating from the top half of the notch root. This damage
zone is very narrow, at less than 1 mm in depth. A much smaller damage zone is
also present on the left side of the notch.

Figures 6.52 and 6.53 show SEM images of layer 1 at the left and right sides of the
notch edge, respectively. There is a small amount of damage on the left side of the
notch, consisting of a few fibre breaks in the first two tows. This damage appears in
the region of a crimp situated at the bottom left of the notch root. In contrast, the
fabric on the right side of the notch is heavily damaged. Five tows are completely
fractured, forming a very narrow crack-like feature, as suggested by the plan view
photograph (figure 6.51). These tow failures occur over a distance of approximately
2.5 mm, equivalent to the length of the damage zone in the plan view photograph.
Again, this series of fractures appears to have initiated at a notch edge crimp region.
This supports the hypothesis that the damage zones observed in plan view
photographs consist of fibre and tow fractures. Figures 6.54 and 6.55 show SEM
images of layer 2 on the left and right sides of the notch edge, respectively. Again,
the damage on the left side of the notch is minimal. There are a few fibre failures
within the notch edge tows. The damage in layer 2 on the right side of the notch is
similar to that in layer 1. There are 4 fully fractured tows. However, the fractures are
not as planar as in layer 1. These fractures appear to follow a path, meandering
between the crimp regions in the fabric.

The plan view damage in an 8 layer laminate containing a 10 mm elliptical notch is
shown in figure 6.56. Damage zones are present on both sides of the notch. The
damage zone on the left side is approximately 1 mm in length, whereas the damage zone on the right side is approximately 2 mm in length. Both damage zones are approximately 2 mm in depth.

The SEM images show that the damage is relatively uniform throughout the laminate. On the left side of the notch the number of tow fractures present are 1, 0, 1, 0, 2, 1 and 1 in layers 1 to 8, respectively. An example of these tow failures is shown in layer 6 in figure 6.57. The fracture of the second tow from the notch edge is thought to be hidden under the crimp region. The damage on the right side of the notch is more extensive. The number of tow fractures present are 3, 4, 4, 4, 4, 3, 4 and 3 in layers 1 to 8, respectively. These tow fractures are shown layer by layer in the eight micrographs in figures 6.58-6.65, corresponding to layers 1 to 8, respectively. Again, the tow fractures can be seen to meander between the crimp regions in the fabric. In layer 3 (figure 6.60), the supposed fracture in the second tow appears to be hidden under the crimp region. In order to investigate this further a SEM image was obtained of the reverse side of the fabric. This is shown in figure 6.66 and confirms that the longitudinal tow is fractured in this crimp region.

6.3.4 Summary of SEM Investigation

The SEM investigation confirmed that the notch edge damage zones observed by plan view photography contain fibre and tow fractures. These fractures appear to be relatively uniform, with respect to their length and propagation paths, throughout the fabric layers in laminates containing both circular and elliptical notches. The presence of crimp regions at the notch edge appear to be favoured sites for the initiation of tow fractures. The paths of these fractures were seen to meander between the notch edge crimp regions, especially for laminates containing circular notches. It is thought that the additional stress concentration of the crimp regions promote fracture. This is supported by the findings of Roy (1996), where failure was seen to initiate at crimp regions in model laminates. The superposition of these meandering paths of tow fractures in a multi-layered laminate is thought to produce the deep damage zones observed in the thicker laminates.
6.4 Deply Technique

6.4.1 Introduction

The SEM investigation performed in section 6.3 confirmed the presence of tow fractures in the notch edge damage zones. However, no information concerning the position of longitudinal splitting, delaminations or transverse damage was able to be obtained. The deply method described in Chapter 3 was used to investigate this notch edge matrix damage in more detail.

Samples containing notch edge damage were obtained using the same method as described in section 6.3. Samples containing 2.5 mm and 10 mm circular and elliptical notches were investigated from both 2 and 8 layer laminates. The stereomicroscope used to view the damage in the fabrics was sufficient to observe the position of the gold chloride marker with respect to the notch and general fabric structure. However, the stereomicroscope does not have sufficient depth of focus to study the fibre architectures in detail. Hence, scanning electron micrographs were taken for selected samples to confirm the presence of fibre damage. When referring to SEM images, the corresponding deply image figure number is also given in parentheses. The stereomicroscope images of the notched fabric reinforcements were taken of both sides of the cloth. Images of the 0° tow dominated side enabled information concerning fibre fracture, longitudinal splits and delamination to be obtained. The 90° tow dominated side of the cloth gave information on notch edge transverse cracking and delamination. The fabric layers are numbered from the top of the laminate down. Side 1 is defined as the top of the fabric with respect to the plan view, whereas side 2 refers to the bottom of the fabric with respect to the plan view. As the fabric layers were stacked back to back during fabrication, the left and right sides are reversed for side 2 of a fabric layer. Hence, for images of side 2, figures labelled left side correspond to the left side with respect to the plan view, irrespective of what side the notch appears on, and vice versa.
6.4.2 Circular Notches

6.4.2.1 2.5 mm Notches

The deply images of the 2 layer laminate containing a 2.5 mm notch do not provide a great deal of extra information beyond that apparent from *in-situ* photography. The 0° tow dominated side 1 of layer 1 (figure 6.67) shows a small amount of notch edge damage on the left side. This only extends a half tow width from the notch edge and is thought to consist of fibre breaks and associated delaminations. The other damage sites present around the circumference of the notch, indicated by the gold chloride marker, are thought to be due to drilling damage. The 90° tow dominated side 1 of layer 2 of this cloth (figure 6.68) shows transverse cracking damage extending approximately 1-1.5 mm from the notch edge.

The equivalent 8 layer laminate also shows little notch edge damage. A damage feature, thought to be tow fracture, is shown on the right side of the notch on side 1 in layer 5 in figure 6.69. An associated delamination can be seen extending approximately 0.5 mm from this damage feature. The facing fabric surface (layer 4, side 2), shown in figure 6.70, shows a region of damage thought to be delamination and longitudinal splitting, of approximately 1 mm in length directly opposite the proposed tow fracture in layer 5. The splitting and delamination damage in these two layers is thought to be associated with the tow fracture. The 90° tow dominated sides of the cloths show transverse cracking damage extending approximately 1 mm from the notch edge as shown on side 2 of layer 7 in figure 6.71. The transverse cracking and longitudinal splitting damage seen appears to be consistent throughout all fabric layers in the laminate.

6.4.2.2 10 mm Notches

The damage shown by the deply images for laminates containing 10 mm notches is almost non-existent. For both the 2 and 8 layer laminates the small amount of gold chloride marker present around the notch circumference is thought to be due to drilling damage.
6.4.3 Elliptical Notches

6.4.3.1 2.5 mm Notches

The damage shown in the dply images of the 2 layer laminates containing 2.5 mm elliptical notches is more extensive than that found adjacent to the equivalent circular notches. The damage zones were easier to capture in the elliptically notched samples due to their earlier initiation with respect to laminate failure. Figure 6.72 shows the damage on side 1 of layer 1. The black debris scattered across the fabric is residual resin that was not totally burnt off. The main damage feature to the right of the notch appears to be tow fracture. The fracture is difficult to resolve but appears to follow a complicated path. A significant amount of gold chloride has been deposited around this fracture, suggesting that there is also delamination damage present. Longitudinal splitting damage is difficult to identify in the surface layers due to the debris scattered across the fabric. The reverse side of this cloth shows an area of significant damage on the right side of the notch, shown on the left side of figure 6.73. This appears to be associated with the presence of a fractured notch edge crimp region, associated with the damage shown on the opposite side of the cloth (figure 6.72). This damage is thought to consist predominantly of delamination and fibre fracture. The damage on the 0° tow dominated side 2 of layer 2 (figure 6.74) is similar to that in layer 1. A tow fracture appears to be present on the same side of the notch as seen in layer 1. This fracture appears to be more planar than the one seen in layer 1. The reverse side of this cloth (figure 6.75) does not show any significant damage, apart from transverse cracking damage, extending approximately 1 mm on both sides of the notch.

The subsequent SEM investigation of these fabric layers confirms the presence of the proposed tow fractures. Figure 6.76 (6.72) shows a SEM image of the right side of the notch on side 1 of layer 1. The complex path of the tow fracture can be seen clearly. It can also be seen that the notch root is situated adjacent to a crimp region in the fabric. No damage was found on the left side of the notch on side 1 in layer 1. Figure 6.77 (6.74) shows a SEM image of the right side notch edge damage on side 2
in layer 2. A relatively planar fracture, extending 1½ tows from the notch edge, can be seen. The randomly orientated single fibres seen in this image are debris deposited during sample preparation and should be ignored.

The deply images of the 8 layer laminate containing a 2.5 mm elliptical notch show that the damage is similar to that seen in the 2 layer laminate. Although the most extensive fibre fracture is situated on the surface in layer 8, the tow fractures in this laminate are relatively consistent throughout the fabric layers. The deply and SEM images show that 1, 0, ½, ¼, 1, 1, 1 and 2 notch edge tows are fractured on at least one side of the notch in layers 1 to 8, respectively. This can be seen in the 0° dominated sides of layers 1 to 8, shown in the 8 colour deply images in figures 6.78-6.85, respectively. The areas surrounding the tow fractures in these images are marked by gold chloride, suggesting that associated delaminations are present. The longitudinal splitting damage in the 0° tow dominated sides of the fabrics are typically 1 mm long. The more extensive longitudinal splitting appears to occur adjacent to tow failures, as seen on side 2 of layer 8 (figure 6.85).

These tow fractures are confirmed by SEM images. Figure 6.86 (6.78) shows a SEM image of the right side of the notch on side 1 in layer 1. The tow fracture can be seen at the bottom right of the notch, in close proximity to the crimp region situated below the notch root. The gold chloride deposited around this fracture can also be seen as a lighter colour, due to the atomic number contrast effect highlighted by the SEM. The tow fracture observed on the left side of the notch in layer 7 is shown in the SEM image in figure 6.87 (6.84). The largest tow fracture, in layer 8, is shown in the SEM image in figure 6.88 (6.85). The complicated path of the fracture can be seen more clearly here than in the deply image. The damage initiated at the top of the notch root, adjacent to a crimp region. The fracture then moved down towards the crimp in the third tow from the notch edge. The extent of the gold chloride either side of this fracture can also be seen in this case due to the atomic number contrast effect.

The deply images also show that the notch edge transverse cracking damage is relatively uniform throughout the laminate. All layers have transverse cracking damage extending approximately 1-1.5 mm from the notch edge. The areas of
heavier transverse damage appear to be found adjacent to other damage features. For example, the transverse cracking on the left of the notch on side 2 of layer 7 (figure 6.89) is on the reverse side of the cloth, immediately adjacent to the tow failure shown in figure 6.84. The deply images can also be used to identify areas of delamination damage. Three such regions were identified in this sample. These are shown in layers 2, 4 and 5 in figures 6.90-6.92, respectively. In each case the delaminated area corresponds to a notch edge crimp region.

6.4.3.2 10 mm Notches

The damage shown in deply images in the 2 layer laminate containing a 10 mm notch is similar to that around the equivalent laminate containing a 2.5 mm notch. A small damage zone, consisting of what appears to be a tow fracture, can be seen in the deply image on the right side of the notch on side 1 of layer 1 in figure 6.93, extending one tow width from the notch edge. Tow fracture is also present on the right side of the notch on side 2 of layer 2, as shown by the deply image in figure 6.94. The corresponding SEM image in figure 6.95 (6.94) shows that this tow failure initiated at the notch root centre and climbed up towards the crimp region in the third tow. The gold chloride in the deply image indicates a significant amount of delamination around the tow fracture and transverse damage in the crimp region adjacent to the tow fracture. The deply image of the 90° tow dominated side of this cloth (figure 6.96) suggests that the tow fracture has not propagated through the tow under this crimp region. However, fairly heavy transverse damage can be seen in the vicinity of the tow fracture on the opposite side of the cloth.

The deply and SEM images of the 8 layer laminate show that the damage is consistent throughout the fabric layers. The damage also is similar on both sides of the notch. On the left side of the notch there are 1½, 1, 1, 1½, 0, ½, ½ and ½ tow fractures in layers 1 to 8, respectively. Similarly, on the right side of the notch there are 1½, 1, 1, 2, 1½, 0, 1 and 0 tow fractures in layers 1 to 8, respectively. The deply image in figure 6.97 shows what is thought to be a tow fracture on the left side of the notch on side 1 in layer 1. It appears that the damage initiated at the notch edge crimp region. The deply image of the reverse side of this cloth (figure 6.98) shows
that the tow fracture passes through the longitudinal crimp region on side 2 of layer 1. This image shows clearly the delamination surrounding the tow fracture. The corresponding SEM image of the 0° tow dominated side 1 of layer 1 is shown in figure 6.99 (6.97). This shows the notch edge crimp region and confirms the fracture of the second tow from the notch edge. The deply image of another example of a tow fracture on the left side of side 1 of layer 3 is given in figure 6.100. The corresponding SEM image in figure 6.101 confirms the presence of the tow fracture.

The deply images were also used to identify areas of delamination damage. Three delaminations were identified in this sample. These are shown in the deply images of the 90° dominated side of the cloth in layers 1, 6 and 7 in figures 6.98, 6.102 and 6.103, respectively. As seen for the 8 layer laminate containing a 2.5 mm elliptical notch, the delaminated areas correspond to notch edge crimp regions. The deply images also show that the notch edge transverse cracking damage is similar throughout the fabric layers. Transverse cracking damage of approximately 1-2 mm in length was measured throughout the fabric layers on both sides of the notch. The larger transverse damage regions appear to occur in the vicinity of other damage sites. For example, there is heavy transverse damage on the left side of the notch on side 1 of layer 2 (figure 6.104). This is situated opposite to the delamination damage observed on the left of the notch on side 2 of layer 1 (figure 6.98). The longitudinal splitting damage extends up to approximately 1 mm from the notch root throughout the fabric layers. The splitting damage appears to be consistent throughout the fabric layers. However, the longitudinal splitting damage may be associated with other damage sites. The splitting damage shown on the right side of side 1 of layer 3 in the deply image in figure 6.105 appears adjacent to the partial tow failure and delamination at the bottom right of the notch root. The corresponding SEM image (figure 6.106) shows this tow fracture and also some fibre breaks above the notch, thought to be associated with the splitting damage.

6.4.4 Summary of Deply investigation

The damage in 8HS weave fabric reinforcements containing circular notches was uniform throughout the laminate thickness. The damage observed was minimal and
consisted of matrix cracking, longitudinal splitting and limited fibre fracture. Hence, the deepy investigation undertaken for laminates containing circular notches was not followed up by scanning electron microscopy. The SEM investigation performed in section 6.3 showed that the notch edge damage zones obtained around circular notches were relatively small, especially for the thinner laminates. This has been reflected in the deepy investigation, although larger damage zones were achieved for the thicker laminates in the SEM investigation.

The type of damage observed in deepy images in 8HS weave fabric reinforcements containing elliptical notches was similar in nature to that adjacent to circular notches, although the damage was much more extensive. As observed in the SEM investigation, the tow fractures appeared to be similar in extent throughout the fabric layers. Crimp regions were seen to be preferred sites for tow fracture initiation and propagation routes. Delaminations were found to occur at notch edge crimp regions. The position and extent of transverse cracking and longitudinal splitting damage appeared to be encouraged by other local damage features, such as fibre failures and delaminations.

6.5 Influence of Damage on Notched Laminate Properties

This section discusses the possible relationship between the notch edge damage morphology and the notched laminate properties presented in Chapter 5. The damage investigation suggested that the propagation of notch edge damage was strongly influenced by the reinforcement type and, to a lesser extent, the notch geometry. It is proposed that these different damage processes affect the notched properties of the laminates, as discussed below.

The higher notched to unnotched strength ratios of the 8HS weave laminates compared to the PW laminates (figures 5.16 and 5.37) may possibly be explained by the observation that the notch edge damage propagation path is more tortuous for the 8HS weave laminates. The propensity of the fracture path in the 8HS weave laminates to meander between the crimp regions results in a damage zone of greater area with significant associated delamination. It is proposed that the energy required
to propagate this damage is larger than that associated with the more crack-like damage zones found in the PW laminates, resulting in higher notched strength ratios for the 8HS weave laminates.

Similarly, the higher notched to unnotched strength ratios of the 8HS weave laminates containing circular notches compared to those containing elliptical notches (figures 5.16 and 5.37) may be explained by considering the notch edge fracture path. It was seen that the notch edge damage zones for the circular notches were deeper in the direction of loading than for the elliptical notches. This was explained by the presence of a higher, more localised, stress concentration for the case of the elliptical notches. Again, it is proposed that the more complicated crack path and associated delamination requires more energy, resulting in higher notched strengths. Also, after the initial damage propagation, the deeper damage zones seen adjacent to the circular notches produce a lower stress concentration compared to the more crack-like damage zones adjacent to the elliptical notches. This may result in a "blunting effect" with respect to the stress concentration ahead of the circular notches and explain the longer damage zone lengths observed adjacent to the elliptical notches.

The effect of the fabric structure may also be discussed with respect to the laminate toughness. The toughness of the 2 layer 8HS weave laminate was slightly higher than that of the 2 layer PW laminate. The crack propagation path in the single edge notch (SEN) samples is planar for both reinforcement types due to the sharp notch and resulting high stress concentration factor. Hence, the crack propagation paths cannot explain the difference in toughness. The small difference in toughness between the 2 laminate types may possibly be explained by the distribution of crimp regions in the laminates. The PW laminates have a regular array of crimp regions compared to the 8HS weave laminates. The 2 layer PW laminates have been shown to nest readily (section 4.3), resulting in alignment of the crimp regions. In contrast, the crimp regions in the 8HS weave laminates are not aligned along a planar fracture path. It is proposed that the stress concentration associated with these crimp regions results in a lower fracture stress, as suggested by Roy (1996). However, the toughness data measured for 8 layer 8HS weave and PW laminates were similar.
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It was observed that as the number of fabric layers increased, the damage zone depth in notched samples (in the direction of loading) increased, especially for the 8HS weave laminates containing circular notches. Initially, it was thought that the energy associated with the more extensive damage zones for the thicker samples would be larger than that in the thinner laminates, resulting in higher notched strength ratios for the thicker laminates. This is clearly not the case considering the normalised notched strength data. The notched strength ratios remain approximately constant with increasing number of fabric layers, showing a decrease in some cases (figure 5.37).

After more careful consideration, it is proposed that the damage zones appear to be more extensive due to the superposition effect. Considering each layer individually, the energy required to propagate fracture in a 2 layer laminate is similar to that for an 8 layer laminate. The higher toughness measured for 2 layer laminates compared to 8 layer laminates, for both reinforcement types, cannot be easily explained by considering fibre architecture or damage propagation effects.

Considering the proposed relationship between the fabric architecture, the notch edge damage and the notched laminate properties, there are other factors that should also be considered when determining the notched strength and toughness of multi-layered woven laminates. The size and position of the notch with respect to the fibre architecture should be taken into account. Ideally, the minimum notch size should be considerably larger than the unit cell of the fabric, in order to minimise any effects arising from the random position at which the notch is introduced. This may be the case for large notches in current PW fabric in which the unit cell measures approximately 1.5 mm. However, the 8HS weave fabric has a repeat distance of approximately 3.5 mm. Hence, in the 8HS weave laminates the random nature of the position of the notch with respect to the weave architecture may affect damage development, considering the role of crimp regions in the initiation and propagation of damage. Another variable is the combination of relative shifts possible in a multi-layered laminate. A combination of these effects may contribute to the scatter in the notched strength data obtained.
In summary, both the notched strength results and the notch edge damage analysis suggest that the effect of weave architecture and notch geometry is more significant than the variation of the number of fabric layers on notched failure. The normalised notched strength data suggested that the number of fabric layers had little effect on notched laminate failure. Also, the damage was found to be similar in extent and morphology throughout the fabric layers. The differences in plan view damage for laminates of varying number of fabric layers has been attributed to a superposition effect.

6.6 Concluding Remarks

The three damage observation techniques employed have enabled a comprehensive damage analysis to be performed on the woven fabric composites investigated. The dense notch edge damage zones seen to initiate prior to failure in the woven laminates investigated comprise fibre failure with associated delamination, transverse cracking and longitudinal splitting.

The simple plan view photography method has proved to be a powerful tool in the damage investigation of the transparent glass/epoxy laminates. The extent and shape of the damage zones at the point of catastrophic failure were seen to be influenced by the reinforcement type, relative shift of the fabric layers, notch shape and size, and the number of fabric layers. In 8HS weave laminates the damage zones adjacent to elliptical notches were generally longer and narrower than those observed adjacent to circular notches. Similarly, the damage zones in PW laminates containing circular notches were longer and narrower than those seen in the equivalent 8HS weave laminates. These planar damage zones are associated with the localisation of stress and the likelihood of finding crimp regions in the path of a tow fracture.

The SEM and deply techniques enabled the damage observed during mechanical testing to be understood in more detail, using a layer-by-layer damage analysis. The damage initiation and propagation path was found to be strongly influenced by the crimp regions present in the woven reinforcements. This has been used to account for the greater extent of damage zones in the direction of loading in the 8HS weave.
laminates compared to the PW laminates. In the 8HS weave laminates, the tow fractures meandered between the crimp regions in the fabric, especially for laminates containing circular notches. It is thought that the more localised stress concentration adjacent to the elliptical notches reduces this effect. In the PW laminates the damage zones appeared to be narrow and more crack-like. Although the SEM and deply investigations were not performed for the PW laminates, it is assumed that the tow fractures propagate directly along the banded transverse cracking regions due to the presence of the regular array of tow crimps in the fabric. The damage appeared to be similar in extent and morphology throughout the fabric layers in all laminates investigated.

The damage analysis has allowed the trends in notched strength to be discussed with respect to the fabric architecture, number of fabric layers and notch geometry. It is proposed that the effect of weave architecture and notch geometry is more significant than the variation of the number of fabric layers on notched failure of the woven laminates.

The damage investigation performed has enabled the notch edge damage processes to be understood more clearly. In Chapter 7, these experimental observations are discussed with respect to the principles and results of the notched strength models investigated.
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Table 6.1. Critical notch edge damage zone lengths for all laminate types and notch geometries.
Figure 6.1. *In-situ* photograph of the notch edge damage in a 2 layer 8HS weave laminate containing a 2.5 mm circular notch at an applied strain of 0.8 %.

Figure 6.2. *In-situ* photograph of the notch edge damage in an 8 layer 8HS weave laminate containing a 2.5 mm circular notch at an applied strain of 0.8 %.
Figure 6.3. *In-situ* photograph of the notch edge damage in a 2 layer 8HS weave laminate containing a 10 mm circular notch at an applied strain of 0.6 %.

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Figure 6.7. *In-situ* photograph showing the critical notch edge damage zone for a 2 layer 8HS weave laminate containing a 2.5 mm circular notch.
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Figure 6.9. *In-situ* photograph showing the critical notch edge damage zone for a 6 layer 8HS weave laminate containing a 2.5 mm circular notch.

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Figure 6.17. *In-situ* photograph showing the critical notch edge damage zone for a 6 layer 8HS weave laminate containing a 10 mm circular notch.
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Figure 6.22. *In-situ* photograph showing the critical notch edge damage zone for an 8 layer 8HS weave laminate containing a 2.5 mm elliptical notch.

Figure 6.23. *In-situ* photograph showing the critical notch edge damage zone for a 2 layer 8HS weave laminate containing a 5 mm elliptical notch.
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Figure 6.35. *In-situ* photograph of the notch edge damage in a 2 layer PW laminate containing a 10 mm circular notch at an applied strain of 0.4 %.
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Figure 6.55. SEM image showing the fibre damage on the right side of the notch in layer 2 of a 2 layer 8HS weave laminate containing a 10 mm elliptical notch.

Figure 6.56. *In-situ* photograph showing the notch edge plan view damage in an 8 layer 8HS weave laminate containing a 10 mm elliptical notch.
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Figure 6.57. SEM image showing the fibre damage on the left side of the notch in layer 6 of an 8 layer 8HS weave laminate containing a 10 mm elliptical notch.

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Figure 6.65. SEM image showing the fibre damage on the right side of the notch in layer 8 of an 8 layer 8HS weave laminate containing a 10 mm elliptical notch.

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Figure 6.80. Deply image of side 1 of layer 3 of an 8 layer 8HS weave laminate containing a 2.5 mm elliptical notch.
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Figure 6.96. Deply image of the right side of the notch on side 1 of layer 2 of a 2 layer 8HS weave laminate containing a 10 mm elliptical notch.
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Figure 6.97. Deply image of the left side of the notch on side 1 of layer 1 of an 8 layer 8HS weave laminate containing a 10 mm elliptical notch.

Figure 6.98. Deply image of the right side of the notch on side 2 of layer 1 of an 8 layer 8HS weave laminate containing a 10 mm elliptical notch.
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Figure 6.99. SEM image showing the fibre damage on the left side of the notch on side 1 of layer 1 of an 8 layer 8HS weave laminate containing a 10 mm elliptical notch.

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Figure 6.106. SEM image showing the fibre damage on the right side of the notch on side 1 of layer 3 of an 8 layer 8HS weave laminate containing a 10 mm elliptical notch.
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Chapter 7: Notched Strength Modelling

7.1 Introduction

As discussed in Chapter 2, there have been many different failure criteria proposed for the prediction of the notched strength of composite laminates, varying widely in complexity and accuracy. This chapter investigates four models for the notched strength prediction of the current woven laminates. These models have been presented in detail in Chapter 2 and are introduced briefly below. In subsequent sections the methods used to implement these approaches in the current study are presented. This is followed by the results of the notched strength modelling for both the un-normalised and normalised data.

Whitney and Nuismer Point (PSC) and Average Stress Criteria (ASC)
The WN failure criteria are relatively simple semi-empirical models. Previous work has shown that they are accurate across a range of composite material systems and notch geometries. However, they do not have a clear physical concept of failure upon which the models are based.

Hitchen Model
This is based upon the evolution and subsequent catastrophic propagation of a crack from the notch edge. The Hitchen model requires as input parameters the laminate fracture toughness, unnotched laminate strength and elastic properties to predict catastrophic failure of the laminate.

Effective Crack Growth Model (ECGM)
The ECGM is also based on the propagation of a crack from the notch edge. The key to the model is the more complex redistribution of stress within the damage zone at the notch edge. This approach requires similar laminate properties to the Hitchen model. The principles behind the model are simple. However, this approach requires fairly extensive computation.
7.2 Stress Distributions

7.2.1 Stress Distributions in Infinite Orthotropic Plates

The common factor in each of these models is that they are based upon the elastic stress distribution ahead of the notch edge. For an infinite orthotropic plate containing a circular hole of radius \( R \), subjected to a uniform stress, \( \bar{\sigma}_y \), applied parallel to the \( y \) axis, the normal stress, \( \sigma_y \), along the \( x \) axis can be expressed approximately as (Konish and Whitney, 1975):

\[
\sigma_y(x, 0) = \frac{\bar{\sigma}_y}{2} \left[ 2 + \left( \frac{R}{x} \right)^2 + 3 \left( \frac{R}{x} \right)^4 - \left( K_{r}^\infty \right)^2 - 3 \left( 5 \left( \frac{R}{x} \right)^6 - 7 \left( \frac{R}{x} \right)^8 \right) \right] \quad (7.1)
\]

where \( K_{r}^\infty \) is the stress concentration factor for an infinite orthotropic plate containing a circular notch, given by (Lekhnitskii, 1968):

\[
K_{r}^\infty = 1 + \left[ 2 \left( \frac{E_x}{E_y} \right)^\frac{1}{2} - \nu_{yx} \right] + \left( \frac{E_y}{G_{yx}} \right)^\frac{1}{2} \quad (7.2)
\]

Similarly, the approximate stress distribution ahead of an elliptical notch is given by (Tan, 1987a):

\[
\frac{\sigma_y}{\sigma_y} = \frac{\lambda^2}{(1 - \lambda)^2} + \frac{(1 - 2\lambda)\gamma}{(1 - \lambda)^3 (\gamma^2 - 1 + \lambda^2)^{\frac{3}{2}}} + \frac{\lambda^2 \gamma}{(1 - \lambda)(\gamma^2 - 1 + \lambda^2)^{\frac{3}{2}}} - \frac{\lambda^2}{2} \left( K_{r}^\infty - 1 - \frac{2}{\lambda} \left( \frac{5\gamma}{(\gamma^2 - 1 + \lambda^2)^{\frac{3}{2}}} - \frac{7\lambda^2 \gamma}{(\gamma^2 - 1 + \lambda^2)^{\frac{3}{2}}} \right) \right) \quad (7.3)
\]

where \( \lambda \) is the notch opening aspect ratio, given by the minor ellipse half length divided by the major ellipse half length \( (b/a) \). Also, \( \gamma = x/a \), the ratio of the distance
ahead of the notch to the major ellipse half length. The stress concentration factor for an infinite orthotropic plate containing an elliptical notch is given by (Tan, 1987a):

\[
K_T^* = 1 + \frac{1}{\lambda} \left\{ 2 \left[ \left( \frac{E_y}{E_x} \right)^{\frac{1}{2}} - \nu_{yx} \right] + \frac{E_y}{G_{yx}} \right\}^{\frac{1}{2}}
\]  

(7.4)

The stress concentration factors for infinite orthotropic plates, \( K_T^* \), calculated for the laminate and notch configurations investigated in this study are given in Table 7.1.

7.2.2 Finite Width Correction Factors (FWCF)

The notched failure criteria investigated in this study incorporate finite width correction factors (FWCF), so that the fracture behaviour of plates of a finite width may be predicted from analyses based upon infinite plate solutions.

The definition of a FWCF is a scale factor which is multiplied by the notched infinite width solution to obtain the notched finite plate result (Tan, 1988). Assuming that the normal stress profile for a plate of infinite width is identical to that of a plate of finite width, except for the FWCF, then:

\[
Y = \frac{K_T}{K_T^*} = \frac{\sigma_y(x,0)}{\sigma_y^*(x,0)}
\]  

(7.5)

where \( Y \) is the FWCF, \( K_T \) and \( K_T^* \) are the stress concentration factors at the notch edge for the finite width plate and infinite width plate, respectively. The terms \( \sigma_y \) and \( \sigma_y^* \) are the normal stress profiles ahead of the notch for the plates of finite width and infinite width, respectively.

In this study the notched strength data are presented in their finite width form. Therefore, since the failure criteria investigated are based upon the stress distribution ahead of the notch, the relevant FWCF is incorporated into the models. The
alternative approach of correcting the notched strength to a value for an infinite width plate (e.g. Naik and Shembekar, 1992) seems cumbersome.

The woven fabric based materials investigated in this study are orthotropic in nature. Tan (1988) derived a compact solution for FWCF's of anisotropic and orthotropic plates containing both circular and elliptical openings. In a previous study, Tan (1987a) proposed an approximate solution for the stress distribution ahead of an elliptical notch in an orthotropic plate (equation 7.3). This has been used to derive an expression for the FWCF of an elliptical opening in an orthotropic plate:

\[
\frac{1}{Y^*} \frac{K^*}{K_r} = \frac{\lambda^2}{(1-\lambda)^2} + \frac{(1-2\lambda)}{(1-\lambda)}\left[1+\left(\frac{3}{2}\frac{2a}{W}M\right)^2 - \frac{\lambda^2}{(1-\lambda)}\left(\frac{2a}{W}M\right)^2\right]^{1/2}
\]

where \( M \) is a magnification factor incorporated to improve the accuracy of the FWCF expression, especially at low values of \( a/b \). This was obtained by equating equation 7.6 with a known isotropic FWCF formula under isotropic conditions and solving for \( M \). This magnification factor is given by:

\[
M = \left[1 - 8\left(\frac{3(1-2a/W)}{2 + (1-2a/W)^3} - 1\right) \right]^{1/2} / 2(2a/W)^2
\]  

(7.7)

If an opening aspect ratio, \( \lambda \), of 1 is assumed, the following expression for a circular notch is obtained:

\[
\frac{K^*}{K_r} = \frac{3(1-2R/W)}{2 + (1-2R/W)^3} + \frac{1}{2} \left(\frac{2R}{W}M\right)^6 \left[K^* - 3\left(\frac{2R}{W}M\right)^2\right]
\]  

(7.8)
The finite width correction factors calculated for the laminate and notch configurations investigated in this study are also presented in Table 7.1.

7.3 Notched Strength Models

7.3.1 Whitney and Nuismer Failure Criteria

The notched strengths of the laminates in this study have been predicted using the semi-empirical Whitney and Nuismer (WN) failure criteria. Whitney and Nuismer (1974) proposed two stress criteria for predicting the notched strength of composite laminates. These criteria assume laminate failure when the stress either at (point) or averaged (average) over some characteristic distance from the notch reaches the unnotched laminate strength. The WN models were chosen since they are well established and have been successfully applied to many composite material systems. The WN notched failure criteria are described in detail in section 2.4.3.2.

The characteristic distances for the various laminate types, notch geometries and notch sizes have been calculated for each individual notched strength. This gives an indication of the effect of laminate type and discontinuity size on characteristic distances. The WN model predictions have been fitted to the data using an error analysis method. A minimisation of least-square errors, based on all notch sizes, was performed. This can be described as the smallest value returned by evaluating the following expression over a range of characteristic distances from $0 \rightarrow \infty$:

$$
\sum \Delta Q = \left[ \left( \frac{\sigma^{\text{pred}}_{N(2.5)}}{\sigma_0} \right) - \left( \frac{\sigma^{\text{exp}}_{N(2.5)}}{\sigma_0} \right) \right]^2 + \left[ \left( \frac{\sigma^{\text{pred}}_{N(5)}}{\sigma_0} \right) - \left( \frac{\sigma^{\text{exp}}_{N(5)}}{\sigma_0} \right) \right]^2 + \left[ \left( \frac{\sigma^{\text{pred}}_{N(10)}}{\sigma_0} \right) - \left( \frac{\sigma^{\text{exp}}_{N(10)}}{\sigma_0} \right) \right]^2
$$

(7.9)

where $\sum \Delta Q$ is the sum of the squares of the errors for the various notch sizes. The superscript "pred" denotes the predictions of the WN models and the superscript "expt" denotes the experimental data. The subscripts in brackets denote the 3 notch sizes in millimetres used in this study.
The variation of fitted characteristic distance from laminate to laminate was also investigated. This has been illustrated by comparing the upper and lower limits of the strength predictions of the WN models.

### 7.3.2 Hitchen Model

The notched strength of the laminates investigated in this study have also been predicted using the model developed by Hitchen et al. (1994). The model is based upon the development and catastrophic propagation of a crack at the notch tip and is described in detail in section 2.4.4.1. This approach was chosen for its ease of implementation and also for the physical representation of notch edge damage in the form of a crack. This may not be an entirely satisfactory explanation of the damage adjacent to notches in non-woven composites. However, the notch edge damage zones presented in Chapter 6 suggest that this approach is more realistic for the current woven composites. Another attractive feature of this approach is the fact that it is not semi-empirical in nature. Once the relevant parameters have been established, no further testing is required. It is proposed that the fracture toughness parameter required for the model will account for the variations in notched strength when varying the number of fabric layers in a laminate. The current analysis is based upon orthotropic materials as opposed to the isotropic (short fibre composite) laminates in the original paper. Hence, the relevant expressions describing stress distributions ahead of notches have been incorporated into the Hitchen model, as described below.

When the stress at the notch tip reaches the unnotched strength of the laminate, $\sigma_0$, a damage zone in the form of a crack is assumed to form. The growth of this crack with increasing stress is assumed to be given by a WN type average stress criterion (Whitney and Nuismer, 1974):

$$\sigma_0 = -\frac{1}{c} \int_{\Gamma} \sigma_{x} \, dx$$ (7.10)
where $c$ is the length of the damage zone and $a$ is the notch radius or half ellipse length. The term $Y$ is the finite width correction factor given by equation 7.8 for a circular notch and equation 7.6 for an elliptical notch. The term $\sigma_y$ represents the stress distribution ahead of the notch and is given by equations 7.1 and 7.3 for circular and elliptical notches, respectively.

This average stress criterion approach predicts that the stress necessary for damage growth increases with crack length. By expanding and rearranging equation 7.10 the stress necessary to advance the damage zone adjacent to a circular notch in an orthotropic laminate of finite width is given by the following expression:

$$
\bar{\sigma}_y = \frac{\sigma_0 2 \left(1 - \xi_2\right)}{Y \left[2 - \xi_2^2 - \xi_4^4 - \left(K_r^\infty - 3\right) \xi_2^8 - \xi_2^6\right]} \quad (7.11)
$$

Similarly, the stress necessary to advance the damage zone adjacent to an elliptical notch in an orthotropic laminate of finite width is given by the following expression:

$$
\bar{\sigma}_y = \frac{\sigma_0 \left(\xi_2^{-1} - 1\right)}{Y \left[\frac{\lambda^2}{(1-\lambda)^2} \xi_2^{-1} + \frac{1 - 2\lambda}{(1-\lambda)^2} \sqrt{\xi_2^{-2} - 1 + \lambda^2} - \frac{\lambda^2}{(1-\lambda) \sqrt{\xi_2^{-2} - 1 + \lambda^2}}\right]} \frac{\lambda^7}{2} \left(K_r^\infty - 1 - \frac{\lambda}{2}\right) \left[\xi_2^{-2} - 1 + \lambda^2\right]^{-\gamma_2} - \left(\xi_2^{-2} - 1 + \lambda^2\right)^{-\gamma_2} \quad (7.12)
$$

Equations 7.11 and 7.12 enable the damage zone size to be found as a function of the applied stress. A fracture mechanics approach is used to predict the stress at which the crack becomes unstable and catastrophic fracture occurs. The stress required for catastrophic failure is given by:

$$
\bar{\sigma}_y = \frac{K_c}{\sqrt{\pi c F_0 Y_0}} \quad (7.13)
$$
where $K_c$ is the critical stress intensity factor and $\sigma_y$, the remote applied stress. Ignoring orthotropy, the critical stress intensity factor is related to the energy release rate by:

$$K_c = \frac{EG_c}{1 - \nu_{xy}^2} \sqrt{\gamma} \quad (7.14)$$

The parameter $F_0$ in equation 7.13 is a circular hole correction factor for cracks growing from a discontinuity. This is given by Tada et al. (1985) for circular and elliptical holes in an infinite isotropic plate. These analyses are not available for orthotropic plates. Hence, the stress required to catastrophically propagate the cracks was calculated using the appropriate isotropic expressions. This will be discussed in further detail later.

For a circular plate in uniaxial tension this correction factor is given by:

$$F_0 = \frac{(3 - S)}{2} \left[1 + 1.243(1 - S)^{1/3}\right] \quad (7.15)$$

where $S = \frac{c}{c + R}$ and $c$ is the length of the crack emanating from the hole of radius $R$.

The case for an elliptical notch is more complex. There is not a closed form expression available to determine this correction factor, $F_0$. In Tada et al. (1985) the correction factor was evaluated numerically for ellipse aspect ratios of 0, 0.25, 0.5, 1, 2, 4 and $\infty$ and is shown graphically, plotted against $S$. For the elliptical notches, the term $S = \frac{c}{c + a}$.

The correction factor was calculated at $S = 0$, at which point $F_0 = 1.22 K_T$. The isotropic stress concentration factor, $K_T$, is given by:
where \( K_T = 1 + 2(a/b) \) \hspace{1cm} (7.16)

The relationship between \( F_0 \) and \( S \) was then assumed by using the calculated value of \( F_0 \) at \( S = 0 \) and following the form of the curves given by Tada et al. (1985). This is shown in figure 7.1. This plot was then used to obtain the variation in correction factor with crack length to calculate the stress required for catastrophic propagation.

The term \( Y_2 \) in equation 7.13 is a finite width correction factor for a crack emanating from a hole in an isotropic material (Soutis et al., 1991):

\[
Y_2 = \sqrt{\sec(\pi a/2W)\sec[\pi(a+c)/2W]} \hspace{1cm} (7.17)
\]

Fracture is assumed to occur when the stress required to form the damage zone is equal to the stress required for catastrophic failure.

The stress required for crack growth was also calculated for an isotropic material. This was done in order to observe the differences between the isotropic and orthotropic cases, since the stress required for catastrophic crack propagation was only able to be calculated for the isotropic case.

7.3.3 The Effective Crack Growth Model (ECGM)

The effective crack growth model (ECGM) proposed by Afaghi-Khatibi et al. (1996a) has also been investigated in this study. In the previous applications of the ECGM an apparent fracture energy, \( G^*_c \), was used. This was obtained by fitting the value to notched strength data using a numerical analysis. Hence, the ECGM as presented is effectively a semi-empirical approach. The current ECGM approach proposes to use experimentally measured toughness data in an attempt to assess the model without the "curve-fitting" required in the determination of the apparent fracture energy. It is proposed that the toughness parameter required for the model
will account for the variations in notched strength when varying the number of fabric layers in a laminate. The ECGM is similar to the Hitchen model in that damage is represented by a crack propagating from the notch edge, analogous to the experimentally observed damage zones presented in Chapter 6. The model has been implemented in a Microsoft Excel® spreadsheet for the current study. The ECGM is described in detail in section 2.4.4.2. The ECGM has already been applied to woven composites containing circular notches (Afaghi-Khatibi and Ye, 1996). The current work also proposes to apply this model to woven composites containing elliptical discontinuities. Hence, the formulation of the ECGM for circular and elliptical notches are described below.

The ECGM approximates the redistribution of load carried in the damage zone adjacent to the notch by assuming an effective crack with cohesive stresses acting on the crack faces. The load carried by the net section of the laminate ahead of the damage zone is assumed to be given by a modified normal stress distribution, $\sigma_y^*$, necessary for equilibrium. The applied load on half of the laminate, $P_i$, is given by the general expression describing the equilibrium between the applied load and the resultant axial force acting on the net section of the plane:

$$\sum_{n=1}^{i} F_n + \int_{R+c_i}^{\frac{W}{2}} \sigma_y^* t dx = P_i \quad (7.18)$$

where $t$ is the laminate thickness, $W$ is the laminate width, $R$ is the notch radius (or the major half ellipse length, $a$) and $c_i$ is the length of the damage zone. The term $\sum_{n=1}^{i} F_n$ represents the sum of the forces assumed to act on the tip of each damage increment, $\Delta c$, within the damage zone. The modified stress distribution ahead of the damage zone is given by:

$$\sigma_y^* = \sigma_y + \Delta \sigma_y \quad (7.19)$$

where
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\[ \Delta \sigma_y = \sigma_0 - \sigma_y \bigg|_{z=R+c_i} \]  \hspace{1cm} (7.20)

and \( \sigma_0 \) is the unnotched laminate strength. Only half of the laminate is considered in the analysis.

For the case of a circular discontinuity the stress distribution ahead of the notch, \( \sigma_y \) is given by equation 7.1. The integral of the modified stress distribution ahead of the notch multiplied by the laminate thickness represents the load carried ahead of the notch. On performing the integration the following expression for the applied load is obtained:

\[
P_i = \sum_{n=1}^{i} F_n + \left[ \sigma_0 c_i \right]_{R+c_i}^{w/2} + \frac{\bar{\sigma}_y}{2} \left[ 2x - \frac{R^3}{x} - \frac{R^4}{x^3} - \left( K_T^\infty - 3 \left( \frac{R^8}{x^7} - \frac{R^6}{x^5} \right) \right) - x \left( \frac{R}{R+c_i} \right)^2 + 3 \left( \frac{R}{R+c_i} \right)^4 \right]_{R+c_i}^{w/2} \]  \hspace{1cm} (7.21)

The FWCF was not incorporated at this stage, as discussed later. The term \( \bar{\sigma}_y \) in equation (7.21) is the remote applied stress. This can be expressed as the applied load divided by the cross-sectional area of half of the laminate, given by \( \bar{\sigma}_y = \frac{2P_i}{Wt} \).

By substituting this into equation 7.21 and rearranging, the following expression for the applied load is given:

\[
P_i = \frac{\sum_{n=1}^{i} F_n + \left[ \sigma_0 c_i \right]_{R+c_i}^{w/2}}{1 - \frac{1}{W} \left[ 2x - \frac{R^3}{x} - \left( K_T^\infty - 3 \left( \frac{R^8}{x^7} - \frac{R^6}{x^5} \right) \right) - \left( \frac{R}{R+c_i} \right)^2 + 3 \left( \frac{R}{R+c_i} \right)^4 \right]_{R+c_i}^{w/2}} \]  \hspace{1cm} (7.22)

With the exception of the term \( \sum_{n=1}^{i} F_n \), all of the parameters in the above expression are geometrical dimensions or can be easily derived from laminate properties.
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The term \( \sum_{n=1}^{i} F_n \) is obtained by determining the stress acting on the tip of each damage increment within the effective crack. This is achieved by considering the relationship between the effective crack opening displacement and the stresses assumed to act on the crack faces which is controlled by the laminate toughness. As explained in detail in section 2.4.4.2, these calculations cannot be solved directly and an iterative approach is required until convergent values are obtained. If the opening displacement of the effective crack exceeds the critical value then it is assumed that no load is being carried across the crack at that point. The methodology used for the calculation of the stress redistribution in the damage zone for the current ECGM approach is illustrated in the flowchart in figure 7.2.

The ECGM has been modified so that it may be applied to the elliptical discontinuities investigated. The basis of the model is identical to the case of the circular notches, i.e. the applied load is assumed to be in equilibrium with the forces acting on the net section of the laminate. The same principles of redistribution of stress within the damage zone apply. The differences between the case of the elliptical notch and the circular notch concern the stress field around the discontinuity.

Due to the different notation describing the elliptical notch geometry the term for the circular notch radius, \( R \), is replaced by the term for the major ellipse half length, \( a \).

Hence, the general expression for the model is given by:

\[
\sum_{n=1}^{i} F_n + \int_{-e_i/2}^{e_i/2} \sigma_y^* tdx = P(i) \quad (7.23)
\]

and

\[
\Delta \sigma_y^* = \sigma_0 - \sigma_y \bigg|_{x=a+e_i} \quad (7.24)
\]
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By integrating the expression for the modified stress distribution ahead of the notch multiplied by the laminate thickness, the following expression for the load on the laminate is obtained:

\[
P = \sum_{n=1}^{4} F_n \left[ \sigma_T \right]_{\omega_n}^2 + \frac{\sigma_T}{2} \left[ \frac{\lambda^2}{1-\lambda^2} x + \frac{1-2\lambda}{(1-\lambda)^2} \left( \frac{x}{a} \right)^2 - 1 + \lambda^2 \right] \frac{\lambda^2}{1-\lambda^2} \left( \frac{x}{a} \right)^2 - 1 + \lambda^2 \right]^{7/2}
\]

\[
\times \frac{\lambda^2}{2} \left( K_f - \frac{2}{\lambda} \right) \left[ \left( \frac{x}{a} \right)^2 - 1 + \lambda^2 \right]^{5/2} + 2\lambda^2 \left[ \left( \frac{x}{a} \right)^2 - 1 + \lambda^2 \right]^{3/2}
\]

\[
-\lambda^2 \left[ \frac{1-2\lambda}{(1-\lambda)^2} \left( \frac{a+c}{a} \right)^2 - 1 + \lambda^2 \right]^{7/2} + \lambda^2 \left( a+c \right) \left( \frac{a+c}{a} \right)^2 - 1 + \lambda^2 \right]^{3/2}
\]

\[
-\frac{\lambda^2}{2} \left( K_f - \frac{2}{\lambda} \right) \left[ \left( a+c \right) \left( \frac{a+c}{a} \right)^2 - 1 + \lambda^2 \right]^{7/2} - \lambda^2 \left( a+c \right) \left( \frac{a+c}{a} \right)^2 - 1 + \lambda^2 \right]^{9/2}
\]

(7.25)

The term \( \bar{\sigma}_y \) in equation 7.25 is the remote applied stress. Similar to the expression for the circular notch, this can be expressed as the applied load divided by the cross-sectional area of half of the laminate, given by \( \bar{\sigma}_y = \frac{2P}{Wt} \). By substituting this into equation (7.25) and rearranging, the following expression for the applied load is given:

\[
P = \sum_{n=1}^{4} F_n \left[ \sigma_T \right]_{\omega_n}^2 \left[ \frac{1-2\lambda}{(1-\lambda)^2} \left( \frac{x}{a} \right)^2 - 1 + \lambda^2 \right]^{2} - \frac{\lambda^2}{1-\lambda^2} \left( \frac{x}{a} \right)^2 - 1 + \lambda^2 \right]^{7/2}
\]

\[
\times \frac{\lambda^2}{2} \left( K_f - \frac{2}{\lambda} \right) \left[ \left( \frac{x}{a} \right)^2 - 1 + \lambda^2 \right]^{5/2} + 2\lambda^2 \left[ \left( \frac{x}{a} \right)^2 - 1 + \lambda^2 \right]^{3/2}
\]

\[
-\lambda^2 \left[ \frac{1-2\lambda}{(1-\lambda)^2} \left( \frac{a+c}{a} \right)^2 - 1 + \lambda^2 \right]^{7/2} + \lambda^2 \left( a+c \right) \left( \frac{a+c}{a} \right)^2 - 1 + \lambda^2 \right]^{3/2}
\]

\[
-\frac{\lambda^2}{2} \left( K_f - \frac{2}{\lambda} \right) \left[ \left( a+c \right) \left( \frac{a+c}{a} \right)^2 - 1 + \lambda^2 \right]^{7/2} - \lambda^2 \left( a+c \right) \left( \frac{a+c}{a} \right)^2 - 1 + \lambda^2 \right]^{9/2}
\]

(7.26)
The formulation of the model is now identical to the case of the circular notch and the methodology used for the calculation of the stress redistribution in the damage zone described in figure 7.2 can be applied.

It was initially attempted to incorporate the appropriate FWCF (equation 7.6 or 7.8) into the expression for the applied load (equation 7.21 or 7.25). However, the FWCF is not compatible with the modified stress distribution ahead of the damage zone (equation 7.19). The form in which the FWCF is used must be carefully considered. The WN and Hitchen models involve a condition of failure, based upon the elastic stress distributions ahead of the notch. If the elastic stress distribution is multiplied by the appropriate FWCF then the condition of failure is reached at a lower applied stress. However, if the FWCF is used in the same way in the ECGM, then the applied load and, hence, the notched strength, increases, since the strength is based upon the stress profile across the laminate. This would also result in a stress level higher than the unnotched laminate strength being carried in the region of the fictitious crack tip, which is clearly not possible. In the current version of the ECGM, the calculated load on the laminate is divided by the FWCF. This lowers the remote applied stress on the laminate to a level that would produce the required stress profile for a laminate of finite width. This method assumes that the FWCF is valid in the region of the fictitious crack.

7.4 Whitney-Nuismer Model Results

7.4.1 Eight Harness Satin Weave Laminates Containing Circular Notches

The characteristic distances calculated for the WN models give an indication of the relative notch sensitivity of laminates. The lower the characteristic distance, the more notch sensitive the laminate. The characteristic distances calculated for all laminates and notch sizes are given for the PSC and the ASC in tables 7.2 and 7.3, respectively. Using these characteristic distances with the corresponding WN model results in the exact prediction of notched laminate strength for each laminate type and hole size. These data are shown graphically in figure 7.3 as characteristic distance
versus the notch size to plate width ratio. The most striking feature of the data is the increase in characteristic distance with hole size for all laminates. Similar behaviour has been well documented in other studies (Karlak, 1977; Awerbuch and Madhukar, 1985). The trends seen for the notched strength data in Chapter 5 can also be seen clearly here. The most obvious of these is the previously reported fact that the 2 layer laminate has a lower notched to unnotched strength ratio (hence, lower characteristic distance) than the thicker laminates for the smaller notch sizes. This is most obvious for the ASC characteristic distance, $a_0$, data. The characteristic distances range from 0.39 to 1.02 mm for the PSC and 1.16 to 2.74 mm for the ASC.

The best fit characteristic distances for the PSC and the ASC over the range of laminates and notch types investigated are given in tables 7.2 and 7.3, respectively. These were obtained from the error analysis method described in section 7.3.1. The data are shown graphically in figure 7.4 as characteristic distance versus number of fabric layers. The average of the individual characteristic distances for each notch size are also shown for comparison. It is clear that the 2 layer laminates have lower characteristic distances than the thicker laminates, indicating that they are more notch sensitive. The 4, 6 and 8 layer laminates all have very similar characteristic distances. In each case the characteristic distances fitted using error analysis are slightly lower than the average data. This difference is more pronounced for the 2 layer laminate. The characteristic distances show considerable variation, ranging from 0.44 to 0.72 mm for the PSC and 1.33 to 2.29 mm for the ASC.

The predicted notched to unnotched strength ratios for the PSC and the ASC are given in tables 7.4 and 7.5, respectively, for all laminates using the characteristic distances fitted using error analysis. The experimental data and the magnitude of the errors of the predictions are also given. The fit of the PSC and ASC are shown in figure 7.5 for the 2, 4, 6 and 8 layer laminates. Both WN models satisfactorily predict the notched strengths for all laminates investigated. The predictions for the 2 layer laminates show larger errors than the predictions for the thicker laminates. For all laminates the ASC results in more accurate predictions than the PSC. This has also been reported by other workers (Awerbuch and Madhukar, 1985). For the 2 layer laminates, the PSC has an average error of approximately 13 % over the three
notch sizes. For the thicker laminates the errors are in the range of approximately 7 to 9%. The ASC shows an average error of approximately 9% for the 2 layer laminates and errors in the range of approximately 2.5 to 4.5% for the thicker laminates.

Figure 7.6 shows the predicted notched to unnotched strength ratios using the highest and lowest characteristic distances for both the PSC and ASC, together with all experimental data. This shows the large range of predictions for both the PSC and the ASC. The need to evaluate the characteristic distances for each laminate variation investigated is illustrated by the spread of the data. For example, if the characteristic distances obtained for the 2 layer laminate were used to predict the strength of the thicker laminates then the errors would be significant.

7.4.2 Eight Harness Satin Weave Laminates Containing Elliptical Notches

The characteristic distances calculated for all laminates and elliptical notch sizes are given for the PSC and the ASC in tables 7.2 and 7.3, respectively. These data are shown graphically in figure 7.7 as characteristic distance versus the notch size to plate width ratio. Again, the characteristic distance generally increases with hole size for all laminates. The trends seen for the notched strength data in Chapter 5 can also be seen here. The 4 layer laminate has the highest notched to unnotched strength ratios and, therefore, also the highest characteristic distances. The characteristic distances range from 0.39 to 0.69 mm for the PSC and 1.38 to 2.47 mm for the ASC.

The fitted characteristic distances obtained from the error analysis are given for the PSC and the ASC in tables 7.2 and 7.3, respectively. These data are shown graphically in figure 7.8 as characteristic distance versus number of layers. Again, the average of the individual characteristic distances for each notch size are also shown for comparison. In this case the characteristic distances are fairly constant with increasing number of fabric layers. There is also very little discrepancy between the average and fitted characteristic distances. The characteristic distances range from 0.46 to 0.63 mm for the PSC and 1.58 to 2.26 mm for the ASC.
The predicted notched to unnotched strength ratios for the PSC and the ASC are given in tables 7.4 and 7.5, respectively, for all laminates using the characteristic distances fitted by error analysis. The fit of the PSC and ASC are shown in figure 7.9 for the 2, 4, 6 and 8 layer laminates. Again, the predictions for the 2 layer laminates generally show larger errors than the predictions for the thicker laminates. The predictions for the 8 layer laminate also show quite large discrepancies. The ASC results in more accurate predictions than the PSC for all laminates. The PSC shows average errors of approximately 8 to 10 % for the 2 and 8 layer laminates and approximately 5 % for the 4 and 6 layer laminates. The ASC shows average errors of approximately 5 to 7 % for the 2 and 8 layer laminates and less than 2 % for the 4 and 6 layer laminates.

The notched strength analysis of Tan (1987a), based upon the WN models, predicts the notched strength of composite laminates containing elliptical notches in which the ellipse opening aspect ratio, \( \lambda \), is kept constant. However, in the current work the notch root radius is kept constant as the major ellipse axis length is increased. Both the PSC and ASC result in good predictions for these notches of varying aspect ratio, hence, varying stress concentration factor. This illustrates the versatility of the semi-empirical WN models and their ability to yield accurate predictions for a wide range of laminate configurations, if implemented properly.

Figure 7.10 shows the predicted notched to unnotched strength ratios using the highest and lowest characteristic distances for both the PSC and ASC, together with all experimental data. The range of predictions of the WN models for the 8HS weave laminates containing elliptical notches is smaller than that for the 8HS weave laminates containing circular notches. This smaller spread of data is not surprising, considering the smaller scatter of strength data for the 8HS weave laminates containing elliptical notches. However, there is still a considerable discrepancy between the predictions arising from the upper and lower values of characteristic distance. Again, this emphasises the need to establish the characteristic distances for each laminate investigated.
7.4.3 Plain Weave Laminates Containing Circular Notches

The characteristic distances calculated for all laminates and notch sizes are given for the PSC and the ASC in tables 7.2 and 7.3, respectively. These data are shown graphically in figure 7.11 as characteristic distance versus the notch size to plate width ratio. The characteristic distances increase with hole size for all laminates. The trends seen for the notched strength data in Chapter 5 can also be seen here. The lower notched to unnotched strength ratio for the 8 layer laminate compared to the 2 layer laminate is reflected in the characteristic distance data. The characteristic distances range from 0.43 to 1.03 mm for the PSC and 1.33 to 2.79 mm for the ASC.

The fitted characteristic distances obtained from the error analysis are given for the PSC and the ASC in tables 7.2 and 7.3, respectively. These data are shown graphically in figure 7.12 as characteristic distance versus number of fabric layers. The average of the individual characteristic distances for each notch size are also shown. It is clear that the 2 layer laminates have higher characteristic distances than the thicker laminates. The characteristic distances fitted using error analysis are slightly lower than the average data, especially for the 2 layer laminate. The characteristic distances range from 0.49 to 0.63 mm for the PSC and 1.4 to 1.99 mm for the ASC.

The predicted notched to unnotched strength ratios for the PSC and the ASC are given in tables 7.4 and 7.5, respectively, for all laminates. The fit of the PSC and ASC to the experimental data are shown in figure 7.13 for the 2 and 8 layer laminates. Similar to the data for the 8HS weave laminates, the predictions for the 2 layer laminates show larger errors than the predictions for the 8 layer laminate. Again, the ASC results in more accurate predictions than the PSC for both laminates. The PSC results in average errors of approximately 14 and 8 % for the 2 and 8 layer laminates, respectively. The ASC results in average errors of approximately 9 and 4 % for the 2 and 8 layer laminates, respectively.

Figure 7.14 shows the predicted notched to unnotched strength ratios using the characteristic distances for the 2 and 8 layer laminates for both the PSC and ASC,
together with all experimental data. The spread of the data is similar to that of the 8HS weave laminates containing elliptical notches.

7.4.4 **Summary of Whitney-Nuismer Modelling for All Laminate Types and Notch Geometries**

The variation of the fitted PSC characteristic distance, $d_0$, with the number of fabric layers is shown for all laminates in figure 7.15. It should be noted that the data point for the 8 layer PW laminate containing a circular notch is hidden by that of the 8HS weave laminate containing an elliptical notch. Similarly, the variation in the fitted ASC characteristic distance, $a_0$, with the number of fabric layers is shown for all laminates in figure 7.16. Both data sets show almost identical trends due to the similar nature of the two models. The 8HS weave laminates containing circular notches show the largest variations in characteristic distance. This reflects the experimental notched strength data, discussed in Chapter 5. The 4, 6 and 8 layer 8HS weave laminates containing circular notches have higher characteristic distances than the other laminate configurations. This suggests that these 4, 6 and 8 layer laminates are less notch sensitive than the other laminates. In contrast, the 2 layer 8HS weave laminate containing circular notches has one of the lowest characteristic distances, suggesting it is one of the most notch sensitive materials.

The WN models result in good notched strength predictions for all laminates investigated, as reported by other workers (e.g. Awerbuch and Madhukar, 1985). For all laminate types and notch geometries, the ASC results in more accurate predictions than the PSC. The predictions for the ASC were excellent, generally within 5% of the experimental data. The best predictions were obtained for the 8HS weave laminates containing elliptical notches. For all laminate configurations it was found necessary to re-establish the model characteristic distances as the number of fabric layers within the laminates was varied.

It is proposed that the variation in characteristic distances for different laminate configurations and the resulting large range of strength predictions are, at least partly, due to volume fraction effects. For example, the relatively low characteristic
distances calculated for the 2 layer 8HS weave laminates containing circular notches (figures 7.15 and 7.16) may possibly be explained by the low fibre volume fractions predicted for these samples (figure 5.7). Hence, it is proposed that any trends in the behaviour of the laminate configurations investigated may be hidden by the volume fraction variations identified in Chapter 5.

The accurate notched strength predictions for all laminates and notches investigated were expected, due to the curve-fitting nature of the WN models. The proposed fibre volume fraction effects do not affect the accuracy or usefulness of the WN models, since it is well documented that the characteristic distances should be re-established for each laminate type investigated. The modelling is performed using the strength data corrected for variations in volume fraction in section 7.7.

7.5 Hitchen Model Results

7.5.1 Eight Harness Satin Weave Laminates Containing Circular Notches

As indicated earlier, the Hitchen model predicts a critical damage zone size as well as giving notched strength predictions. The variation in predicted critical damage zone length with hole size to sample width ratio is shown in table 7.6 and figure 7.17 for 2, 4, 6 and 8 layer 8HS weave laminates containing circular notches. The predicted critical damage zone length increases as the number of fabric layers is decreased for each notch size. For each laminate thickness there is also a slight increase in predicted critical damage zone length as the notch size increases. The extent of this damage zone at the point of predicted failure was not compared to experimental observations in the original analysis (Hitchen et al., 1994). Therefore, no comparisons can be made with the data presented here.

The model predictions, experimental data and percentage error of the predictions for 8HS weave laminates containing circular notches are given in table 7.7. The predictions of notched to unnotched strength ratios with increasing hole size to
sample width ratio for these 2, 4, 6 and 8 layer laminates are shown in figure 7.18, together with the experimental data.

The 2 layer laminate predictions show the largest discrepancies with respect to the experimental data, with an average error of approximately 11.5 % over the three notch sizes. The predictions for the thicker 4, 6 and 8 layer laminates are more accurate, with average errors of approximately 6, 8 and 5.5 %, respectively. The 2 and 8 layer laminate predictions are higher than the experimental data, whereas the opposite is true of the 4 and 6 layer laminate predictions.

As discussed in section 7.3.2, the stress necessary for catastrophic failure was calculated for the isotropic case. In the analysis presented above, the stress necessary to advance the crack was calculated for an orthotropic material. In order to investigate the effect of considering the anisotropy of the laminates, the analysis was also performed by calculating the stress necessary to advance the crack in an isotropic material. The results of the Hitchen model for the fully isotropic case are given in table 7.8 for the 8HS weave and PW laminates containing circular notches. It can be seen that the two approaches result in very similar notched strength predictions. Figures 7.19 and 7.20 show the stress necessary for the advance of the crack (for both orthotropic and isotropic laminates) and the stress necessary for catastrophic failure versus crack length for a 2 layer 8HS weave laminate containing a 2.5 mm and 10 mm diameter circular notch, respectively. Again, the similarity in the different approaches can be seen. Hence, the use of orthotropic expressions for the prediction of the stress necessary to advance the crack and isotropic expressions for the prediction of catastrophic failure was considered acceptable.

7.5.2 Eight Harness Satin Weave Laminates Containing Elliptical Notches

The variation in predicted critical damage zone length with hole size to sample width ratio is shown in table 7.6 and figure 7.21 for 2, 4, 6 and 8 layer 8HS weave laminates containing elliptical notches. The trends in predicted damage zones lengths for the 8HS weave laminates containing elliptical notches are similar to those
seen for the circular notches. Again, the predicted critical damage zone length increases as the number of fabric layers is decreased for each notch size. There is also an increase in predicted critical damage zone length as the notch size increases for each laminate thickness, especially for the 2 layer laminate.

The model predictions, experimental data and percentage error of the predictions for 8HS weave laminates containing elliptical notches are given in table 7.7. The predictions of notched to unnotched strength ratio with increasing hole size to sample width ratio for these 2, 4, 6 and 8 layer laminates are shown in figure 7.22, together with the experimental data.

The 2 layer laminate predictions show the largest discrepancies with respect to the experimental data, with an average error of approximately 16 % over the three notch sizes. The predictions for the thicker 2, 4 and 6 layer laminates are more accurate, with average errors of approximately 7, 5 and 3 %, respectively. Again, the 2 and 8 layer laminate predictions are higher than the experimental data, whereas the opposite is true of the 4 and 6 layer laminate predictions.

7.5.3 Plain Weave Laminates Containing Circular Notches

The variation in predicted critical damage zone length with hole size to sample width ratio is shown in table 7.6 and figure 7.23 for 2 and 8 layer PW laminates containing circular notches. The predicted critical damage zone lengths for the 2 layer laminates are higher than those for the 8 layer laminates. In this case, the predicted critical damage zone length remains approximately constant as the notch size increases for each laminate thickness.

The model predictions, experimental data and percentage error of the predictions for PW laminates containing circular notches are given in table 7.7. The predictions of notched to unnotched strength ratios with increasing hole size to sample width ratio for these 2 and 8 layer laminates are shown in figure 7.24, together with the experimental data.
The 2 layer laminate predictions show an average error of approximately 4% over the three notch sizes. The predictions for the 8 layer laminates show average errors of approximately 6.5%. Both the 2 and 8 layer laminate predictions are higher than the experimental data.

**7.5.4 Summary of Hitchen Model Results for All Laminate Types and Notch Geometries**

The predicted critical damage zone length decreases with the number of fabric layers for all sample configurations (figures 7.17, 7.21 and 7.23). This effect can be explained by considering the equations on which the model is based. As the number of layers increases, the unnotched laminate strength increases. Hence, the stress required for crack growth (equation 7.11 or 7.12) increases. Also, as the number of layers are increased the laminate fracture toughness decreases. This results in a lower catastrophic crack propagation stress (equation 7.13). This has the effect of prediction of failure (when the stress required to grow the crack is equal to the stress required to catastrophically propagate the crack) at shorter critical damage zone lengths as the number of fabric layers is increased. The predicted critical damage zone lengths for the 8HS weave laminates are similar for the circular and elliptical notches at the smallest notch size. However, as the notch size increases the previously reported increase in predicted critical damage zone lengths are larger for the elliptically notched samples. For all sample configurations, the predicted critical damage zone lengths of 8 layer laminates are in the region of 1-1.5 mm. The predicted critical damage zone lengths of the 2 layer laminates are approximately in the range of 2-2.5 mm for laminates containing circular notches. However, the 8HS weave laminates containing elliptical notches show predicted critical damage zone lengths of approximately 2-3.5 mm.

The prediction of notched strength for all sample configurations is satisfactory, with a maximum average error of approximately 16% over the three notch sizes, compared to the experimental data. The majority of the predictions are within 10% of the experimental data. The accuracy of the predictions do not appear to be better
for any particular laminate type or notch geometry. In general, there appear to be larger discrepancies between predicted and experimental data for the smaller hole sizes.

7.6 Effective Crack Growth Model Results

During the implementation of the ECGM several problems were experienced and certain aspects of the model have been critically reviewed, as discussed below.

Initially, the current ECGM model was used to attempt to reproduce one of the original authors predictions for a woven composite material (Afaghi-Khatibi and Ye, 1996). A plain weave glass/epoxy laminate containing a 1.2 mm diameter circular notch in a 40 mm wide specimen was investigated. The measured experimental notched to unnotched strength ratio for this sample was 0.896. The ECGM prediction of Afaghi-Khatibi and Ye (1996) was shown graphically to be approximately 0.88. The current ECGM prediction is 0.916. The discrepancy between the ECGM predictions has been explained by comparing the redistribution of normal stresses across the laminate for Afaghi-Khatibi and Ye (1996) and the current approach, shown in figures 7.25 and 7.26, respectively. The stress distributions ahead of the damage zone are virtually identical. However, the stress profiles along the length of the effective crack are very different in the two approaches. The example from Afaghi-Khatibi and Ye (1996) shows the stress rapidly increasing towards the crack tip, resulting in a sharp peak of stress at the tip of the damage zone (figure 7.25). The current example shows the stress gradually approaching the unnotched laminate strength towards the crack tip, resulting in a plateau-like stress profile at the tip of the damage zone (figure 7.26). The differences in these stress profiles are consistent with a slightly higher prediction for the current version of the ECGM.

The shape of stress profile obtained for the current approach is supported by several relevant examples from literature. These include the schematic representations of stress redistribution in the various papers concerning the ECGM (Afaghi-Khatibi et al., 1996a, 1996b; Afaghi-Khatibi and Ye, 1996). Also, using a finite element
approach, the damage zone model of Aronsson and Backlund (1986b) predicts a similar notch edge stress profile to the current ECGM analysis, shown in figure 7.27a.

The stress profiles within the damage zone are determined from the calculated crack opening profiles along the fictitious crack length. The crack opening profiles for the original and current ECGM analyses are shown in figure 7.28 for a 1 mm fictitious crack adjacent to a 1.2 mm diameter circular notch in the woven laminate discussed above. The two approaches show very different crack opening profiles. The original ECGM crack opening displacement (COD) profile (Afaghi-Khatibi, 1999) suggests that the crack opens up relatively quickly at the crack tip and has an approximately constant COD along its length. Conversely, the COD profile resulting from the current approach suggests that the crack remains shut towards the tip, before gradually opening up at an increasing rate towards the notch edge. The COD contributions due to the applied load and the cohesive stress for the current approach are also given in figure 7.28. The COD due to the applied load, based upon the elastic stress distribution adjacent to the notch (equation 7.1), rapidly increases towards the notch edge due to the effect of the stress concentration. The COD due to the cohesive stress shows a maximum towards the centre of the fictitious crack and tapers off to zero at either end of the fictitious crack. The combination of these two COD profiles produces the total COD profile shown in figure 7.28. The COD profile predicted by the current ECGM approach is consistent with that predicted by Aronsson and Backlund (1986b), shown in figure 7.27b. The form of the current COD profile also agrees with an ECGM COD profile for a cross ply carbon/epoxy laminate determined by the original authors of the model (Ye et al., 1998), shown in figure 7.29. This work references the original methodology of the ECGM calculation (Afaghi-Khatibi et al., 1996) and obviously contradicts the previously presented fictitious crack stress and COD profiles presented by Afaghi-Khatibi and Ye (1996) and Afaghi-Khatibi (1999).

The next step in the evaluation of the ECGM was to apply it to the current notched strength investigation. Figure 7.30 shows the normalised applied stress versus the fictitious crack length for the 2 layer 8HS weave laminate containing a 2.5 mm
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diameter circular notch. Failure is predicted at the maximum applied stress, giving a notched to unnotched strength ratio of 0.78. This over-predicts the experimental notched to unnotched strength ratio of 0.57 by approximately 37%. However, it should be noted that after considering fibre volume fraction effects, the experimental notched to unnotched strength ratio was normalised to 0.63. The critical value of the fictitious crack length was 1.9 mm, of similar magnitude to the equivalent WN and Hitchen model parameters. The redistribution of stress across the laminate with damage growth is shown in figure 7.31. At short fictitious crack lengths (c = 0.1, 0.2 and 0.3 mm) the stress is redistributed along the fictitious crack, showing a reduced load-carrying capability in this region. When the fictitious crack reaches 1 mm in length it can be seen that the cohesive stress holds the crack shut towards the crack tip, resulting in the unnotched laminate strength being carried in this region (a negative crack opening is actually obtained, which is assumed to be zero). The crack then opens up, producing a stress relaxation towards the notch edge. The region of the fictitious crack immediately adjacent to the notch edge carries no load, since the COD has exceeded the critical value. At the critical fictitious crack length of 1.9 mm approximately half of the crack remains shut. It then opens up rapidly and soon exceeds the critical COD. This results in very little stress redistribution within the fictitious crack length. The stress distribution profiles of Afaghi-Khatibi et al. (1996) also show that the crack remains shut towards the tip. Similarly, the COD profiles of Ye et al. (1998) show that the crack remains shut towards the crack tip. Also, Afaghi-Khatibi (1999) reported that negative crack openings were experienced in some cases, which were converted to zero.

It was found that if the applied stress on the laminate was calculated for increasing fictitious crack lengths beyond the previously determined maximum failure load, the iterative COD calculations became unstable. This resulted in an ever-increasing load carrying capability within the fictitious crack and an ever-increasing predicted applied stress with increasing fictitious crack length. During the ECGM calculation for other laminate and notch configurations, the COD calculations often became unstable before a failure load had been determined. The reason for the instability of the model at longer fictitious crack lengths is unclear. Possible influencing factors are discussed below.
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The original ECGM analysis was implemented in a FORTRAN program, where the iterative process was allowed to run until complete convergence was obtained. In the current approach it was assumed that 100 iterations were sufficient for convergence. The refinement of the analysis is another possible factor in the stability of the model. The original ECGM analysis reported that damage increments of 0.1 mm resulted in satisfactory results. Hence, this damage increment was used in the current analysis. It is possible that a smaller damage increment would result in more stability at the longer fictitious crack lengths. The equations used to determine the COD's in the ECGM were devised for edge cracks in plates (Tada et al., 1985). The ECGM assumes that they are also applicable to cracks emanating from notches. The validity of the COD equations at longer crack lengths must also be considered. Afaghi-Khatibi et al. (1996) stated that these COD equations are valid “if the length of the fictitious crack is much smaller than the hole diameter”. Afaghi-Khatibi and Ye (1996) state that the equations are valid if the length of the fictitious crack divided by the hole radius (c/R) is less than 1.8. After stating the above conditions for which the model is valid, Afaghi-Khatibi et al. (1996) applied the ECGM to a laminate where the critical c/R was 3.6. Similarly, Afaghi-Khatibi and Ye (1996) considered a case where the critical c/R was 2.5. In the current analysis, the COD calculations for the ECGM became unstable when c/R was in the region of 1-2.

Other issues were highlighted during the implementation of the ECGM. The current analysis considered laminate orthotropy, whereas the original analysis assumed an isotropic material with $K_r^* = 3$ (Afaghi-Khatibi, 1999), although the orthotropic elastic stress distributions were used. The way in which finite width correction factors were incorporated into the ECGM was not clearly explained in the literature. Also, a finite size correction factor was introduced into the COD calculations by Afaghi-Khatibi and Ye (1997) without explanation. The semi-empirical nature of the original ECGM should also be considered. Afaghi-Khatibi et al. (1996) fitted apparent fracture energies of 120, 70 and 35 kJ/m² for non-woven carbon/epoxy laminates. This is a significant variation in apparent toughness, suggesting that experimentally determined toughness data may not yield as accurate notched strength predictions compared to the fitted toughness data.
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The problems experienced concerning the instability of the COD calculations, combined with their questionable validity, and the general lack of clarity in the explanation of the methodology used for the ECGM led to the modelling for the current laminates to be concluded without notched strength predictions being made. An ECGM type model is still considered to be a potentially valid approach for the notched strength prediction of the laminates investigated. It is believed that a more detailed explanation of the methodology used in the model calculations is required before this can be achieved.

7.7 Whitney-Nuismer Model Normalised Results

7.7.1 Introduction

This section presents the results of the WN modelling using the strength data corrected for fibre volume fraction variations between laminates in section 5.6. This normalisation technique affects the unnotched laminate strength, a WN model input parameter. Perhaps of more significance to the model predictions is the effect of the normalised unnotched strengths on the notched to unnotched strength ratios, since the WN models are effectively "curve-fitting" techniques.

7.7.2 Eight Harness Satin Weave Laminates Containing Circular Notches

The characteristic distances calculated for all laminates and notch sizes are given for the PSC and the ASC in tables 7.9 and 7.10, respectively. These data are shown graphically in figure 7.32 as characteristic distance versus the notch size to plate width ratio. As seen for the WN modelling based upon the un-normalised data in section 7.4, there is an increase in characteristic distance with hole size for all laminates. The characteristic distances for the laminates of varying number of fabric layers are similar, except for that of the 2 layer laminate containing a 10 mm notch. This reflects the notched strength ratios discussed in section 5.6.2. The characteristic distances range from 0.50 to 1.21 mm for the PSC and 1.61 to 3.38 mm for the ASC.
The fitted characteristic distances given in tables 7.9 and 7.10 were obtained from the error analysis method discussed in section 7.3.1. These data are shown graphically in figure 7.33 as characteristic distance versus number of layers. The average of the individual characteristic distances for each notch size are also shown for comparison. The characteristic distances are approximately constant for all laminates, possibly showing a slight decrease as the number of fabric layers is increased. This is more obvious for the ASC data. The characteristic distances range from 0.61 to 0.70 mm for the PSC and 1.86 to 2.18 mm for the ASC.

The predicted notched to unnotched strength ratios for the PSC and the ASC are given in tables 7.11 and 7.12, respectively, for all laminates using the characteristic distances fitted by error analysis. The normalised experimental data and the magnitude of the errors of the predictions are also given. The fit of the PSC and ASC are shown in figure 7.34 for the 2, 4, 6 and 8 layer laminates. Both WN models satisfactorily predict the notched strengths for all laminates investigated. As seen for the WN modelling in section 7.4, the models do not fit the 2 layer laminate notched data as well as for the thicker laminates. Again, for all laminates the ASC results in more accurate predictions than the PSC. For the 2 layer laminates the PSC has an average error of approximately 11 %. For the thicker laminates the errors are in the range of approximately 5 to 7.5 %. The ASC shows an average error of approximately 7 % for the 2 layer laminates and errors in the range of approximately 1 to 3.5 % for the thicker laminates.

Figure 7.35 shows the predicted notched to unnotched strength ratios using the highest and lowest characteristic distances for both the PSC and ASC together with all experimental data. The range of strength predictions for both the PSC and ASC are small, reflecting the small range of the normalised notched strength ratios.
7.7.3 Eight Harness Satin Weave Laminates Containing Elliptical Notches

The characteristic distances calculated for all laminates for each elliptical notch size are given for the PSC and ASC in tables 7.9 and 7.10, respectively. These data are shown graphically in figure 7.36 as characteristic distance versus the notch size to plate width ratio. Again, the characteristic distance generally increases with hole size for all laminates. The trends seen for the normalised notched strength data in Chapter 5 can also be seen here. The characteristic distances for the normalised data are very similar to those observed for the equivalent un-normalised data in section 7.4. This reflects the minor effect of the normalisation on the strength of 8HS weave laminates containing elliptical notches. The characteristic distances range from 0.37 to 0.63 mm for the PSC and 1.32 to 2.23 mm for the ASC.

The fitted characteristic distances obtained from the error analysis are also given in tables 7.9 and 7.10 for the PSC and ASC, respectively. These data are shown graphically in figure 7.37 as characteristic distance versus number of layers. Again, the average of the individual characteristic distances for each notch size are also shown for comparison. In this case the characteristic distances show a slight decrease with increasing number of fabric layers. This suggests that the thicker laminates containing elliptical notches are slightly more notch sensitive than the thinner laminates. The characteristic distances range from 0.49 to 0.54 mm for the PSC and 1.69 to 1.90 mm for the ASC.

The predicted notched to unnotched strength ratios for the PSC and the ASC are given in tables 7.11 and 7.12 for all laminates, together with the normalised experimental data and the errors of the predictions. The fit of the PSC and ASC are shown in figure 7.38 for the 2, 4, 6 and 8 layer laminates. Again, the predictions for the 2 layer laminates generally show larger errors than the predictions for the thicker laminates. Similar to the WN modelling based upon the equivalent un-normalised data presented in section 7.4, the predictions for the 8 layer laminate also show quite large discrepancies. The ASC results in more accurate predictions than the PSC for all laminates. The PSC shows average errors of approximately 9 % for the 2 and 8
layer laminates and less than 6% for the 4 and 6 layer laminates. The ASC shows average errors of approximately 6% for the 2 and 8 layer laminates and less than 3% for the 4 and 6 layer laminates.

Figure 7.39 shows the predicted notched to unnotched strength ratios using the highest and lowest characteristic distances for both the PSC and ASC, together with all experimental data. The range of predictions of the WN models for the 8HS weave laminates containing elliptical notches very small. This reflects the small scatter of normalised strength data for this range of laminates.

7.7.4 Plain Weave Laminates Containing Circular Notches

The characteristic distances calculated for all laminates and notch sizes for the PSC and ASC are given in tables 7.9 and 7.10, respectively. These data are shown graphically in figure 7.40 as characteristic distance versus the notch size to plate width ratio. The characteristic distances increase with hole size for both 2 and 8 layer laminates, ranging from 0.41 to 0.98 mm for the PSC and 1.27 to 2.62 mm for the ASC.

The fitted characteristic distances obtained from the error analysis are also given in tables 7.9 and 7.10. These data are shown graphically in figure 7.41 as characteristic distance versus number of fabric layers. The average of the individual characteristic distances for each notch size are also shown. The 2 layer laminates have higher characteristic distances than the thicker laminates, suggesting that the thicker laminates are more notch sensitive. The characteristic distances fitted using error analysis are slightly lower than the average data. This difference is more pronounced for the 2 layer laminate. The characteristic distances range from 0.44 to 0.56 mm for the PSC and 1.26 to 1.76 mm for the ASC.

The predicted notched to unnotched strength ratios for the PSC and the ASC are given in tables 7.11 and 7.12 for all laminates, respectively. The fit of the PSC and ASC to the experimental data are shown in figure 7.42 for the 2 and 8 layer laminates. The performance of the WN models is very similar to that for the
equivalent un-normalised data in section 7.4, since the normalisation had a very small effect on the strength properties of the PW laminates. Similar to the data for the 8HS weave laminates, the predictions for the 2 layer laminates show larger errors than the predictions for the 8 layer laminate. Again, the ASC results in more accurate predictions than the PSC for both laminates. The PSC results in average errors of approximately 14 and 8% for the 2 and 8 layer laminates, respectively. The ASC results in average errors of approximately 9 and 3.5% for the 2 and 8 layer laminates, respectively.

Figure 7.43 shows the predicted notched to unnotched strength ratios using the characteristic distances for the 2 and 8 layer laminates for both the PSC and ASC, together with all experimental data. The spread of the data is similar to that for the equivalent un-normalised data, presented in section 7.4.

7.7.5 Summary of Whitney-Nuismer Model Normalised Results for All Laminate Types and Notch Geometries

The variation in the PSC characteristic distance, $d_0$, with the number of fabric layers is shown for all laminates in figure 7.44. Similarly, the variation in the ASC characteristic distance, $a_0$, with the number of fabric layers is shown for all laminates in figure 7.45. Both data sets show almost identical trends due to the similar nature of the two models. For both models, the characteristic distances show slight decreases with increase in the number of fabric layers for all laminate and notch configurations. This suggests that the thinner laminates are less notch sensitive than the thicker laminates. The normalised notched strength ratios are reflected by the characteristic distances being highest for the 8HS weave laminates containing circular notches. Similarly, the smallest characteristic distances are shown for the PW laminates containing circular notches, which show the lowest normalised notched strength ratios.

The WN models resulted in good notched strength predictions for all laminates investigated. The magnitude of the errors are similar for all laminate and notch configurations. These accurate predictions were expected due to the curve fitting
nature of the WN models. For all laminate types and notch geometries the ASC results in more accurate predictions than the PSC. The predictions for the ASC were excellent, generally within 5% of the experimental data.

7.8 Hitchen Model Normalised Results

7.8.1 Introduction

This section presents the results of the Hitchen model using the strength and fracture toughness data corrected for fibre volume fraction variations between laminates in section 5.6. The unnotched laminate strength and fracture toughness are input parameters in the Hitchen model. Hence, the normalisation technique affects the model predictions.

7.8.2 Eight Harness Satin Weave Laminates Containing Circular Notches

The variation in predicted critical damage zone length with hole size to sample width ratio is shown in table 7.13 and figure 7.46 for 2, 4, 6 and 8 layer 8HS weave laminates containing circular notches. The predicted critical damage zone length increases slightly as the number of fabric layers is decreased for each notch size for the 4, 6 and 8 layer laminates. The critical damage zone lengths for the 2 layer laminate are significantly higher than those of the thicker laminates. For each laminate thickness there is also a slight increase in predicted critical damage zone length as the notch size increases.

The model predictions, normalised experimental data and percentage error of the predictions for 8HS weave laminates containing circular notches are given in table 7.14. The predictions of notched to unnotched strength ratios with increasing hole size to sample width ratio for these 2, 4, 6 and 8 layer laminates are shown in figure 7.47, together with the experimental data. The 8 layer laminate predictions show the largest discrepancies with respect to the experimental data, with an average error of approximately 11.5% over the three notch sizes. This relatively large average error
is greater than the 5.5 % average error for the equivalent un-normalised model prediction (table 7.7). The predictions for the 2, 4 and 6 layer laminates are more accurate, with average errors of approximately 8, 5 and 6 %, respectively.

7.8.3 Eight Harness Satin Weave Laminates Containing Elliptical Notches

The variation in predicted critical damage zone length with hole size to sample width ratio is shown in table 7.13 and figure 7.48 for 2, 4, 6 and 8 layer 8HS weave laminates containing elliptical notches. The predicted critical damage zone length increases as the number of fabric layers decreases for each notch size, especially for the 2 layer laminate. There is also an increase in predicted critical damage zone length as the notch size increases for each laminate thickness, especially for the 2 layer laminate.

The model predictions, normalised experimental data and percentage error of the predictions for 8HS weave laminates containing elliptical notches are given in table 7.14. The predictions of notched to unnotched strength ratios with increasing hole size to sample width ratio for these 2, 4, 6 and 8 layer laminates are shown in figure 7.49, together with the experimental data. The 2 layer laminate predictions show the largest discrepancies with respect to the experimental data, with an average error of approximately 7 % over the three notch sizes. The predictions for the thicker 4, 6 and 8 layer laminates are more accurate, with average errors within 4 % of the normalised experimental data. The prediction for the 8 layer laminate shows a significant improvement on the equivalent un-normalised notched strength prediction which had an error of almost 16 % (table 7.7).

7.8.4 Plain Weave Laminates Containing Circular Notches

The variation in predicted critical damage zone length with hole size to sample width ratio is shown in table 7.13 and figure 7.50 for 2 and 8 layer PW laminates containing circular notches. The predicted critical damage zone lengths for the 2 layer laminates are higher than those for the 8 layer laminates. The predicted critical damage zone
length remains approximately constant as the notch size increases for each laminate thickness.

The model predictions, experimental data and percentage error of the predictions for PW laminates containing circular notches are given in table 7.14. The predictions of notched to unnotched strength ratios with increasing hole size to sample width ratio for these 2 and 8 layer laminates are shown in figure 7.51, together with the experimental data. Both laminates show predictions with an average error of approximately 6% for the three notch sizes.

### 7.8.5 Summary of Hitchen Model Normalised Results for All Laminate Types and Notch Geometries

The predicted critical damage zone length increases as the number of fabric layers decreases for all sample configurations (figures 7.46, 7.48 and 7.50), as discussed in section 7.5.4.1. As the number of fabric layers decreases, the increase in critical damage zone length is similar for the 8HS weave and PW laminates containing circular notches. However, for the 8HS weave laminates containing elliptical notches, this increase in critical damage zone length with decreasing number of fabric layers is more pronounced. For all sample configurations, the predicted critical damage zone lengths of 2 layer laminates are in the region of 1-2 mm. The predicted critical damage zone lengths of the 8 layer laminates are approximately in the range of 2-3.5 mm.

The prediction of notched strength for all sample configurations is good, with a maximum average error of 11.5% over the three notch sizes, compared to the normalised experimental data. The majority of the predictions are within 6% of the normalised experimental data. The accuracy of the predictions appear to be better for the 8HS weave laminates containing elliptical notches compared to the other laminate configurations. Possible reasons for this are discussed later.
7.9 Comparison of Notched Strength Models

This section initially discusses the characteristic distances, or critical damage zone lengths, predicted by the various models and compares them to the experimental observations from Chapter 6. The variation in experimental and model damage zone lengths, or characteristic distances, with the notch size to sample width ratio are shown in figures 7.52 to 7.54. Similarly, the variation in experimental and normalised data model damage zone lengths, or characteristic distances, with the notch size to sample width ratio are shown in figures 7.55 to 7.57. The predicted damage zone lengths, or characteristic distances, are similar for the un-normalised and normalised data. Hence, both data sets are discussed simultaneously. As discussed in Chapter 6, the experimental critical damage zone lengths should be considered approximate due to the way in which they were measured. They should be treated as an indication of the extent of the notch edge damage zone at failure.

The initial impression gained from the data is that all models predict distances of the same magnitude to the experimentally observed damage zone lengths. The WN ASC and Hitchen models generally predict similar distances. The WN PSC predicts distances which are approximately half the size of those from the other two models.

The predictions for the 8HS weave laminates containing circular notches are shown in figures 7.52 and 7.55 for the un-normalised and normalised data, respectively. None of the models show particularly good agreement with the experimentally observed damage zone lengths for all laminates. The Hitchen model shows very good agreement with the experimental data for the 6 layer laminates and acceptable agreement for the 4 and 8 layer laminates. Similarly, the WN PSC shows good agreement for the 4 layer laminates and the WN ASC shows good agreement for the 8 layer laminates.

The predictions for the 8HS weave laminates containing elliptical notches are shown in figures 7.53 and 7.56 for the un-normalised and normalised data, respectively. In this case, the Hitchen model and WN ASC show excellent agreement with the experimental data for all but the 2 layer laminates. The agreement of these models
with the experimental data for the 2 layer laminates is reasonable. The WN PSC underestimates the damage zones lengths in all cases.

The predictions for the PW laminates containing circular notches are shown in figures 7.54 and 7.57 for the un-normalised and normalised data, respectively. None of the models are able to predict the experimental damage zone lengths for the 2 layer laminate. However, it should be considered that the experimental damage zone lengths vary considerably with notch size for this 2 layer laminate. The Hitchen model results in good predictions compared to the experimental data for the 8 layer laminate, performing slightly better than the WN ASC.

The remainder of this section discusses the relative performance of the WN and Hitchen models for both the un-normalised and normalised notched strength data. The WN ASC results in the most accurate un-normalised notched strength predictions, with an average error over all notch sizes and laminate configurations of 4.7%. The WN PSC and the Hitchen model show similar accuracy in their un-normalised notched strength predictions, with average errors over all notch sizes and laminate configurations of 8.5% and 7.3%, respectively. The WN ASC also results in the most accurate normalised notched strength predictions, with an average error over all notch sizes and laminate configurations of 4.5%. This is slightly more accurate than the ASC predictions for the un-normalised data. The Hitchen model normalised notched strength predictions are also very good, with an average error over all notch sizes and laminate configurations of 5.9%. The WN PSC model normalised notched strength predictions are similar to the equivalent un-normalised predictions, with an average error over all notch sizes and laminate configurations of 8.1%.

In summary, both WN models show similar accuracy for the un-normalised and normalised notched strength predictions. This was expected for the WN models, due to their curve-fitting nature. In contrast, the Hitchen model results show a considerable improvement in accuracy for the normalised data. In Chapter 5 it was suggested that the normalisation method employed accounted for the fibre volume
fraction effects present. It also appears that the Hitchen model is able to predict this normalised data more accurately than the un-normalised data.

7.10 Suitability of Notched Fracture Models

This section discusses the suitability of the notched failure models investigated, in terms of the way in which they represent notch edge damage compared to the experimentally observed notch edge damage presented in Chapter 6.

The WN models do not have a clear physical concept of failure upon which the models are based. They predict that failure occurs over a characteristic distance from the notch edge, without discussing any form of damage progression. Hence, they lend themselves to many laminate types and damage morphologies.

The Hitchen model considers the propagation of a crack from the notch edge, using a fracture mechanics approach. The notch edge damage analysis performed suggests that this approach may be appropriate for the current woven laminates. It was shown that the damage zones adjacent to all notch types in the current study consisted of multiple tow failures, forming crack-like features. The damage zones adjacent to the elliptical notches in the current study consisted of relatively planar arrays of multiple tow failures, forming crack-like features, although these tow fractures were not true “through-thickness” cracks as they were never totally planar and did not directly superimpose throughout the fabric layers. The damage zones in 8HS weave laminates containing circular notches were more extensive in the direction of loading. The damage zones in PW laminates containing circular notches were also seen to be relatively planar.

The Hitchen model resulted in the most accurate notched strength predictions for 8HS weave laminates containing elliptical notches, where the damage closely resembled the idealised notch edge crack assumed by the model. The predictions for 8HS weave laminates containing circular notches were the least accurate, showing similar predictions to the WN PSC. Also, the Hitchen model resulted in very good agreement in critical damage zone length predictions compared to experimental
observations for 8HS weave laminates containing elliptical notches. The critical
damage zone length predictions for 8HS weave laminates containing circular notches
appeared to show the least correlation with experimental observations. Hence, the
results of the notched strength modelling and the notch edge damage analysis suggest
that the Hitchen model is applicable to laminates in which the notch edge damage
zones are crack-like in morphology. The current work suggests that the model
satisfactorily describes the failure of such laminates in both a quantitative and
qualitative manner.

The ECGM also represents the notch edge damage as a crack, extending from the
notch edge. However, only limited results were obtained using this model.

7.11 Concluding Remarks

The WN and Hitchen model notched failure criteria have been successfully applied to
the un-normalised and normalised notched strength data from Chapter 5.

The WN models have been confirmed to be straightforward and accurate notched
failure criteria. The variations in notched strength from laminate to laminate have
emphasised the need to carefully establish the model parameters for each laminate
investigated. The ASC was found to be more accurate than the PSC. The
characteristic distances predicted by the WN PSC were generally lower than the
experimentally observed damage zone lengths. However, the characteristic distances
predicted by the WN ASC generally showed good agreement with the experimental
observations.

The Hitchen model has been shown to yield accurate notched strength predictions,
especially for elliptically notched samples, while being very easy to implement. The
predictions for the normalised notched strength were only marginally less accurate
than those of the ASC and significantly better than those of the PSC. The fracture
toughness parameter utilised in this model appears to account effectively for the
differences in notched strength for the various laminates investigated. The Hitchen
model critical damage zone length predictions generally showed good agreement
with the experimentally observed values. The current results suggest that the Hitchen model is particularly apt for laminates in which the notch edge damage zones are crack-like in nature. For such laminates, the Hitchen model appears to provide a satisfactory physical explanation of failure. An advantage of the Hitchen model compared to the WN criteria is that it is not semi-empirical in nature, requiring only the laminate elastic properties, unnotched strength and fracture toughness.
### 8HS weave laminates containing circular notches

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### PW laminates containing circular notches

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Table 7.1. Stress concentration factors and finite width correction factors for infinite orthotropic plates calculated for the laminate and notch configurations investigated in this study.
### Table 7.2. PSC characteristic distances calculated for all laminate types and notch sizes and geometries.

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### Table 7.3. ASC characteristic distances calculated for all laminate types and notch sizes and geometries.

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306
### Table 7.4: PSC model predictions and experimental results for all laminate types and notch sizes and geometries.

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Table 7.5. ASC model predictions and experimental results for all laminate types and notch sizes and geometries.
Table 7.6. Hitchen model predicted critical damage zone lengths for all laminate types and notch sizes and geometries.
### Table 7.7. Hitchen model predicted notched strength ratios and experimental results for all laminate types and notch sizes and geometries.

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### Table 7.8. Hitchen model predicted notched strength ratios, assuming laminate isotropy, and experimental results for all laminate types and notch sizes and geometries.

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Table 7.9. Normalised data PSC characteristic distances calculated for all laminate types and notch sizes and geometries.

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Table 7.10. Normalised data ASC characteristic distances calculated for all laminate types and notch sizes and geometries.

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Table 7.11. Normalised data PSC model predictions and experimental results for all laminate types and notch sizes and geometries.
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Table 7.12. Normalised data ASC model predictions and experimental results for all laminate types and notch sizes and geometries.
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Table 7.13. Normalised data Hitchen model predicted critical damage zone lengths for all laminate types and notch sizes and geometries.
Table 7.14. Normalised data Hitchen model predicted notched strength ratios and experimental results for all laminate types and notch sizes and geometries.
Figure 7.1. Circular hole correction factor, \( F_0 \), versus the ratio of the crack length to the crack length plus the hole radius, \( S \), for cracks extending from a circular hole (Tada et al., 1985). Note that the notation used to describe the geometry of the notch is different to the current work.
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START

INPUT DATA (Δc, R or a, t, W, Gc, E, σ0)

\[ c = Δc \]
\[ x = c - 1 \]
\[ v_1 = 0 \]
\[ P_1 = 0 \]

\[ v_{σ_{app}n} = \frac{4σ_y}{E_{11}} \sqrt{c^2 - x^2} D\left(\frac{x}{c}\right) \]

\[ v_{σ_{col}n} = \frac{4σ_y(1 - v_n/v_c)}{E_{11}} \sqrt{c^2 - x^2} D\left(\frac{x}{c}\right) \]

\[ v_n = v_{σ_{app}n} + v_{σ_{col}n} \]

\[ σ_n = σ_0 \left(1 - \frac{v_n}{v_c}\right) \]

\[ ΣF_n = ΣF_{n-1} + σ_n Δc t \]

\[ P_n = P_{n-1} \]
\[ v_n = v_{n-1} \]

\[ x = 0 \]

\[ x = x - 1 \]

\[ i = i + 1 \]
\[ c = c + Δc \]
\[ x = c - 1 \]

Figure 7.2. Flowchart showing the methodology used for the ECGM calculations.
Figure 7.3. WN characteristic distances versus notch diameter to sample width ratio for 8HS weave laminates containing circular notches.

Figure 7.4. WN characteristic distances fitted using error analysis and average values versus the number of fabric layers for 8HS weave laminates containing circular notches.
Figure 7.5. Notched to unnotched strength ratio versus hole diameter to sample width ratio showing the PSC, ASC and experimental data for (a) 2 layer, (b) 4 layer, (c) 6 layer and (d) 8 layer 8H1S weave laminates containing circular notches.

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Figure 7.6. Notched to unnotched strength ratio versus hole diameter to sample width ratio showing the PSC, ASC and experimental data for the minimum and maximum calculated characteristic distances for 8HS weave laminates containing circular notches.

Figure 7.7. WN characteristic distances versus notch diameter to sample width ratio for 8HS weave laminates containing elliptical notches.
Figure 7.8. WN characteristic distances fitted using error analysis and average values versus the number of fabric layers for 8HS weave laminates containing elliptical notches.
Figure 7.9. Notched to unnotched strength ratio versus hole diameter to sample width ratio showing the PSC, ASC and experimental data for (a) 2 layer, (b) 4 layer, (c) 6 layer and (d) 8 layer 8HS weave laminates containing elliptical notches.
Figure 7.10. Notched to unnotched strength ratio versus hole diameter to sample width ratio showing the PSC, ASC and experimental data for the minimum and maximum calculated characteristic distances for 8HS weave laminates containing elliptical notches.

Figure 7.11. WN characteristic distances versus notch diameter to sample width ratio for PW laminates containing circular notches.
Figure 7.12. WN characteristic distances fitted using error analysis versus the number of fabric layers for PW laminates containing circular notches.
Figure 7.13. Notched to unnotched strength ratio versus hole diameter to sample width ratio showing the PSC, ASC and experimental data for (a) 2 layer, (b) 8 layer PW laminates containing circular notches.
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Figure 7.14. Notched to unnotched strength ratio versus hole diameter to sample width ratio showing the PSC, ASC and experimental data for the minimum and maximum calculated characteristic distances for PW laminates containing circular notches.

Figure 7.15. PSC characteristic distance, $d_0$, fitted using error analysis, versus the number of fabric layers for all laminate configurations.
Figure 7.16. ASC characteristic distance, \( a_0 \), fitted using error analysis, versus the number of fabric layers for all laminate configurations.

Figure 7.17. Hitchen model critical damage zone length versus hole size to sample width ratio for 2, 4, 6 and 8 layer 8HS weave laminates containing circular notches.
Figure 7.18. Notched strength ratio versus hole size to sample width ratio for (a) 2, (b) 4, (c) 6 and (d) 8 layer 8HS weave laminates containing circular notches. Hitchen model notched strength predictions and experimental data are given.
Figure 7.19. Applied stress versus crack length for a 2 layer 8HS weave laminate containing a 2.5 mm diameter circular notch. The Hitchen model strength prediction is shown for the stress necessary for the advance of the crack for both orthotropic and isotropic laminates.

Figure 7.20. Applied stress versus crack length for a 2 layer 8HS weave laminate containing a 10 mm diameter circular notch. The Hitchen model strength prediction is shown for the stress necessary for the advance of the crack for both orthotropic and isotropic laminates.
Figure 7.21. Hitchen model critical damage zone length versus hole size to sample width ratio for 2, 4, 6 and 8 layer 8HS weave laminates containing elliptical notches.
Figure 7.22. Notched strength ratio versus hole size to sample width ratio for (a) 2, (b) 4, (c) 6 and (d) 8 layer 8HS weave laminates containing elliptical notches. Hitchen model notched strength predictions and experimental data are given.
Figure 7.23. Hitchen model critical damage zone length versus hole size to sample width ratio for 2 and 8 layer PW laminates containing circular notches.
Figure 7.24. Notched strength ratio versus hole size to sample width ratio for (a) 2 and (b) 8 layer PW laminates containing circular notches. Hitchin model notched strength predictions and experimental data are given.
Figure 7.25. Redistribution of normal stress versus distance from the laminate centre, $x$, for a 40 mm wide woven glass/epoxy laminate containing a 1.2 mm diameter circular notch (Afaghi-Khatibi and Ye, 1996).

Figure 7.26. Redistribution of normal stress versus distance from the laminate centre, $x$, for a 40 mm wide woven glass/epoxy laminate containing a 1.2 mm diameter circular notch for the current ECGM.
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Figure 7.27. (a) Normalised stress and (b) crack opening displacement versus distance ahead of the notch tip at different remote applied loads for a quasi-isotropic carbon epoxy laminate (Aronsson and Bäcklund, 1986b).

Figure 7.28. Fictitious crack opening displacement versus number of damage increments (of $\Delta c = 0.1 \text{ mm}$) for a 1 mm fictitious crack in a 40 mm wide woven glass/epoxy laminate containing a 1.2 mm diameter circular notch.
Figure 7.29. Crack opening displacement versus distance ahead of the notch for two cross ply carbon/epoxy laminates (Ye et al., 1998).

Figure 7.30. Normalised applied stress versus fictitious crack length for a 2 layer 8HS weave laminate containing a 2.5 mm diameter circular notch.
Figure 7.31. Redistribution of normal stress versus distance from the laminate centre, $x$, for a 2 layer 8HS weave laminate containing a 2.5 mm diameter circular notch.

Figure 7.32. Normalised data WN characteristic distances versus notch diameter to sample width ratio for 8HS weave laminates containing circular notches.
Figure 7.33. Normalised data WN characteristic distances fitted using error analysis and average values versus the number of fabric layers for 8HS weave laminates containing circular notches.
Figure 7.34. Normalised notched to unnotched strength ratio versus hole diameter to sample width ratio showing the PSC, ASC and experimental data for (a) 2 layer, (b) 4 layer, (c) 6 layer and (d) 8 layer 8HS weave laminates containing circular notches.
Figure 7.35. Normalised notched to unnotched strength ratio versus hole diameter to sample width ratio showing the PSC, ASC and experimental data for the minimum and maximum calculated characteristic distances for 8HS weave laminates containing circular notches.

Figure 7.36. Normalised data WN characteristic distances versus notch diameter to sample width ratio for 8HS weave laminates containing elliptical notches.
Figure 7.37. Normalised data WN characteristic distances fitted using error analysis and average values versus the number of fabric layers for 8HS weave laminates containing elliptical notches.
Figure 7.38. Normalised notched to unnotched strength ratio versus hole diameter to sample width ratio showing the PSC, ASC and experimental data for (a) 2 layer, (b) 4 layer, (c) 6 layer and (d) 8 layer 8HS weave laminates containing elliptical notches.
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Figure 7.39. Normalised notched to unnotched strength ratio versus hole diameter to sample width ratio showing the PSC, ASC and experimental data for the minimum and maximum calculated characteristic distances for 8HS weave laminates containing elliptical notches.

Figure 7.40. Normalised data WN characteristic distances versus notch diameter to sample width ratio for PW laminates containing circular notches.
Figure 7.41. Normalised data WN characteristic distances fitted using error analysis and average values versus the number of fabric layers for PW laminates containing circular notches.
Figure 7.42. Normalised notched to unnotched strength ratio versus hole diameter to sample width ratio showing the PSC, ASC and experimental data for (a) 2 layer and (b) 8 layer PW laminates containing circular notches.
Figure 7.43. Normalised notched to unnotched strength ratio versus hole diameter to sample width ratio showing the PSC, ASC and experimental data for the minimum and maximum calculated characteristic distances for PW laminates containing circular notches.

Figure 7.44. Normalised data PSC characteristic distance, \(d_0\), fitted using error analysis, versus the number of fabric layers for all laminate configurations.
Figure 7.45. Normalised data ASC characteristic distance, $a_0$, fitted using error analysis, versus the number of fabric layers for all laminate configurations.

Figure 7.46. Normalised Hitchen model critical damage zone length versus hole size to sample width ratio for 2, 4, 6 and 8 layer 8HS weave laminates containing circular notches.
Figure 7.47. Normalised notched strength ratio versus hole size to sample width ratio for (a) 2, (b) 4, (c) 6 and (d) 8 layer 8HS weave laminates containing circular notches, showing Hitchen model notched strength predictions and experimental data.
Figure 7.48. Normalised Hitchen model critical damage zone length versus hole size to sample width ratio for 2, 4, 6 and 8 layer 8HS weave laminates containing elliptical notches.
Figure 7.49. Normalised notched strength ratio versus hole size to sample width ratio for (a) 2, (b) 4, (c) 6 and (d) 8 layer 8HS weave laminates containing elliptical notches. Hitchen model notched strength predictions and experimental data are given.
Figure 7.50. Normalised Hitchen model critical damage zone length versus hole size to sample width ratio for 2, 4, 6 and 8 layer PW laminates containing circular notches.
Figure 7.51. Normalised notched strength ratio versus hole size to sample width ratio for (a) 2 and (b) 8 layer PW laminates containing circular notches, showing Hitchen model notched strength predictions and experimental data.
Figure 7.52. Notch edge damage zone length versus notch size to plate width ratio for (a) 2 layer, (b) 4 layer, (c) 6 layer and (d) 8 layer 8HS weave laminates containing circular notches, showing experimental data and model predictions.
Figure 7.53: Notch edge damage zone length versus notch size to plate width ratio for (a) 2 layer, (b) 4 layer, (c) 6 layer and (d) 8 layer 8HS weave laminates containing elliptical notches, showing experimental data and model predictions.
Figure 7.54. Notch edge damage zone length versus notch size to plate width ratio for (a) 2 layer and (b) 8 layer PW laminates containing circular notches, showing experimental data and model predictions.
Figure 7.55. Normalised data notch edge damage zone length versus notch size to plate width ratio for (a) 2 layer, (b) 4 layer, (c) 6 layer and (d) 8 layer 81/3S weave laminates containing circular notches, showing experimental data and model predictions.
Figure 7.56. Normalised data notch edge damage zone length versus notch size to plate width ratio for (a) 2 layer, (b) 4 layer, (c) 6 layer and (d) 8 layer 8HS weave laminates containing elliptical notches, showing experimental data and model predictions.
Figure 7.57. Normalised data notch edge damage zone length versus notch size to plate width ratio for (a) 2 layer and (b) 8 layer PW laminates containing circular notches, showing experimental data and model predictions.
Chapter 8: Conclusions and Further Work
8.1 Conclusions

The aim of this project was to build upon previous work studying the damage accumulation, and effect of this damage on laminate properties, in 8HS weave composites, extending the work to other reinforcement types. Once the damage development had been characterised, it was intended to focus attention on the notched failure of the woven laminates, looking at both laminate properties and damage development. The final area of interest was the investigation and development of notched failure predictive models. These aims have been achieved, as outlined below.

The range of damage observation techniques employed has enabled a comprehensive damage analysis to be performed for both unnotched and notched laminates. The damage observed in notched and unnotched samples was seen to be strongly influenced by tow characteristics and weave architecture for both 8HS weave and PW laminates.

The unnotched laminate damage analysis showed that the transverse crack morphologies within the tows were similar for the different reinforcement types. However, the open nature of the PW cloth, combined with the tendency of cloths in adjacent layers to shift with respect to each other, resulted in a range of possible damage morphologies not seen in the 8HS weave laminates. The proposed shear lag models used to predict the reduction in PW laminate stiffness with increasing damage density were able to predict the general trend of the data. However, these models, in which the woven composite is represented by an equivalent cross-ply laminate, were not able to account for the initial rapid reduction in Young’s modulus seen at low damage densities.

The dense damage zones observed adjacent to notches initiated just prior to failure and comprised of predominantly fibre failure, with associated delamination, transverse cracking and longitudinal splitting. These damage zones were seen to be influenced by the reinforcement type, notch shape and size, and the number of fabric layers. In 8HS weave laminates the damage zones adjacent to elliptical notches were generally longer and narrower than those observed adjacent to circular notches.
Similarly, the damage zones in PW laminates containing circular notches were longer and narrower than those seen in the equivalent 8HS weave laminates. The range of damage morphologies due to the relative shift of adjacent fabric layers in the unnotched PW laminates were also observed adjacent to the notches investigated.

The SEM and deply techniques showed that the damage initiation and propagation path was strongly influenced by the crimp regions present in the woven reinforcements. In the 8HS weave laminates, the tow fractures meandered between the crimp regions in the fabric, especially for laminates containing circular notches. The more planar damage zones observed adjacent to the elliptical notches has been attributed to a more localised stress concentration associated with these notches. In the PW laminates, the crack-like damage zones were attributed to the propensity of tow fractures to propagate directly along the regular array of tow crimps in the fabric. The damage appeared to be similar in extent and morphology throughout the fabric layers in all laminates investigated.

In association with the damage analysis a comprehensive data-base of elastic and unnotched and notched strength properties of woven composites has been established.

Variations in fibre volume fractions between laminates have been shown to have a strong influence on laminate properties. The proposed fibre volume fraction normalisation method appears to have accounted for this effect. There was little difference between the normalised notched strength ratios for 8HS weave laminates containing varying numbers of fabric layers. In the PW laminates the notched strength ratios for the 8 layer samples were lower than those of the 2 layer samples for all notch sizes. The normalised notched strengths of the 8HS weave laminates containing circular notches were higher than those of the 8HS weave laminates containing elliptical notches and the PW laminates containing circular notches.

All laminate types showed an increase in critical energy release rate, or toughness, with increasing notch length for both the un-normalised and normalised data. Also, the thinner laminates were tougher than the thicker laminates. The 8HS weave laminates were tougher than the PW laminates for the 2 layer materials.
The Whitney and Nuismer (WN) point (PSC) and average stress criteria (ASC) and the Hitchen model have been shown to yield accurate notched strength predictions for the current woven laminates. The ASC was found to be more accurate than the PSC. The normalised data Hitchen model predictions were only marginally less accurate than those of the ASC and significantly better than those of the PSC. The Hitchen model predictions were excellent for elliptically notched samples.

The characteristic distance, or critical damage zone length, predictions generally showed good agreement with the experimentally observed values. In particular, the data predicted for elliptically notched laminates by the Hitchen model and WN ASC were in good agreement with experimental observations. The Hitchen model is not semi-empirical in nature like the WN models, and is very easy to implement.

The observations of damage adjacent to the various notch and laminate configurations investigated have been used to explain the trends in notched strength. The higher normalised notched strengths for 8HS weave laminates containing circular notches have been explained by the higher energy required to propagate the meandering notch edge fracture paths observed. The effect of weave architecture and notch geometry appears to be more significant than the variation of the number of fabric layers on notched failure of the woven laminates.

The Hitchen model predicted critical damage zone lengths and notched strength predictions were in good agreement with experimental observations and strength data for elliptically notched samples. Also, the shape of the notch edge damage zones in the elliptically notched laminates were similar to the idealised crack assumed by the model. This suggests that the Hitchen model is particularly apt for laminates in which the notch edge damage zones are crack-like in nature.
8.2 Further Work

The current project, combined with that of Marsden (1996), has studied the damage observed in both unnotched and notched laminates in PW and 8HS weave laminates. The stiffness reduction due to this damage and the notched laminate strength have been predicted using suitable models. A similar investigation on an intermediate reinforcement type, such as a five harness satin weave, would provide a more comprehensive study on the role of the fabric structure on the fracture of woven composites and the prediction of their behaviour. It would be useful to supplement this modelling by a more sophisticated approach, such as a finite element analysis, which is able to take into account the complex weave architecture and damage morphology observed for these materials.

The notch edge damage investigation for PW laminates involved only in-situ plan view photgraphy. Further SEM and deply damage analyses for PW laminates would allow the role of the fabric crimp regions to be discussed in more detail.

The current project investigated the notched failure of orthotropic laminates, with the fabric tows aligned at 0° and 90°. However, in practice, a typical composite structure would contain some fibre reinforcement at 45°. It would be of interest to investigate the notched strength, notch edge damage and performance of the current notched failure criteria in quasi-isotropic woven laminates.

An assessment of the performance of the current notched failure criteria for woven carbon/epoxy laminates would be of interest. A notch edge damage analysis would also allow comparisons with the damage in the current woven fabric glass/epoxy laminates.
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