Adaptive Modulation for Cooperative Communications with Noisy Feedback

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***Abstract*—Cooperative communication is an alternative way to generate spatial diversity. However due to the extra time slot required for relay transmission, the spectral efficiency (SE) is reduced. Adaptive modulation can optimize the spectral efficiency by adopting the transmission parameters to the channel conditions. This requires the perfect channel state information (CSI) feedback at the transmitter. However, in real life this is not practical. The feedback channel is noisy and fading which causes feedback errors. In this contribution, first we investigate the performance of adaptive modulation schemes for cooperative communications with perfect feedback, the adaptive modulation is applied both by the source and the relay. Then the impact of noisy feedback channels on the performance is analysed. The simulation results demonstrate the spectral efficiency improvement, and illustrates that the effect of the noisy feedback can be ignored when the feedback channel signal-to-noise ratio (SNR) is relatively high i.e. 10 dB.**

***Index Terms*—Adaptive modulation, cooperative communications, noisy feedback, relay channel**

I. Introduction

The future generations of wireless communication systems are required to offer a variety of multimedia services with reliable transmissions at high data rates. However, due to the effect of multi-path fading, multiple access interference (MAI), this task becomes a real challenge. The advantages of multiple-input-multiple-output (MIMO) systems have been widely acknowledged, the use of *diversity* – transmission of independent copies of signals, have been investigated to combat the channel effects and support such high rates. However using MIMO at mobile devices is difficult because the correlation of the signals due to the limited size of mobile devices, as well as the cost and hardware complexity. To solve this problem, cooperative communication is proposed. Cooperative communication can generate spatial diversity by using a collection of distributed antennas belonging to multiple users. The basic idea of cooperative communication is that one user may receive other users' signals due to the broadcasting nature of wireless communications, in which case it can forward some version of the "overheard" signal along with its own information, which emulates a virtual Multiple-Input-Multiple-Output (MIMO) system. The basic ideas of cooperative communications can be traced back to Cover and El Gamal's work on the information theoretic analysis of the relay channel[1]. The basic cooperative protocols include Amplify-and-Forward(AF) and Decode-and-Forward(DF) [2]-[5]. A simple cooperative scheme is AF method. Each user in this method receives a noisy version of its partner's signal then amplifies and retransmits the noisy version. Although the noise is also amplified with the information signal, this method can provide diversity. For DF protocol, the relay decodes the information signal from the source then re-encode and forward it to destination. The method has the advantage of adaptability to channel conditions. However, this method can lead to error propagation if the decoding at the relay fails.

Because an extra time slot will be needed for the relay to transmit, cooperation will lead to a loss in spectral efficiency. In order to compensate this problem, adaptive modulation schemes[6] [7] can be employed in cooperative communications. In adaptive modulation schemes, a higher order modulation will be chosen from a few candidate modulation schemes, when the instantaneous channel quality is good in order to increase the spectral efficiency in term of bit per second per Hz ($bit/s/Hz$). In the other hand, a more robust low-order modulation scheme will be employed, when the instantaneous channel condition is bad, in order to improve the bit error rate (BER) performance. The adaptive modulation is performed based on the feedback of the channel conditions. In most of the published papers dealing with adaptive modulations, it is assumed that the channel feedback is error-free, however this is not practical in real-life scenarios especially when the sufficient and powerful error control method cannot be implemented over the feedback channel. More and more attention has been paid to the imperfect feedback channel, in [8] and [9], the impact of imperfect feedback channels on the performance of adaptive modulation schemes is investigated and two feedback detection strategies are proposed to mitigate the degradation in performance due to the errors caused by the imperfect feedback channel.

In this paper, we consider a relay network with noisy and fading feedback channel as shown in Fig.1, where z and h are the Additive White Gaussian Noisy (AWGN) and fading channel coefficient respectively. An adaptive modulation scheme for relay network where both the source and the relay employ adaptive modulation is proposed. We investigated the impact of noisy feedback on the performance of the proposed relay network in a slow time-varying channel. It is shown that with erroneous feedback the proposed scheme can still improve the performance of fixed DF relaying in terms of spectral efficiency.

The rest of the paper is organized as follows. Section II

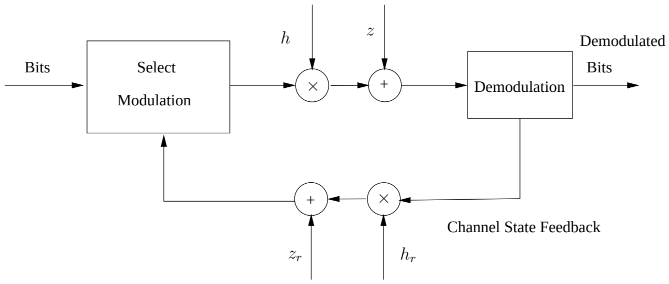

Fig. 1. Adaptive Modulation with Noisy Feedback

introduces the system model and channel model. In Section III the adaptive modulation scheme for cooperative communications is investigated. Section VI presents the performance analysis of the proposed scheme with noisy feedback. Section V includes the simulation results. At last, the conclusion is drawn in Section VI.

II. System and Channel Model

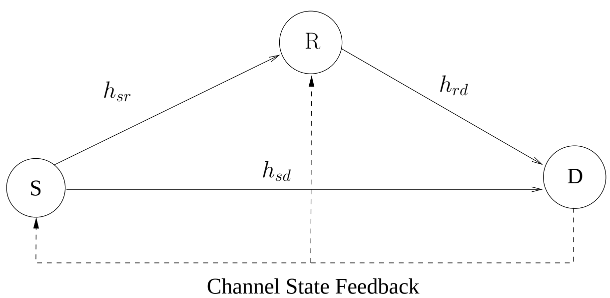

Fig. 2. System Model

Consider a Decode-and-Forward (DF) relay network including one source node (S), one relay node (R) and one destination node (D) with noisy feedback channels as shown in Fig.2, where $h_{\rm sd}$, $h_{\rm sr}$ and $h_{\rm rd}$ are the instantaneous channel coefficients of SD, SR and RD links, respectively and all the nodes are equipped with a single antenna. We assume that the node transmit in time-orthogonal channels. In the proposed scheme, Cyclic-redundancy-check (CRC) code is employed to check whether the decoding at the relay and destination nodes is successful or not. The transmission in the relay network can be divided into 2 time slots:

Time Slot 1: The source transmits the information signal with $M_{\rm s}$ modulation based on the source-destination channel feedback, where $M_{\rm s}$ is the constellation size. We assume that there are 4 candidate constellations–BPSK, QPSK, 16-QAM and 64-QAM. Both the relay and the destination try to decode their received signals. If the decoding at the destination node is successful, checked by CRC code, then the second time slot won't be needed. Otherwise, one retransmission will be carried out in the second time slot.

Time Slot 2: If the relay can correctly decode the data from the source, it will re-encode the message with $M_{\rm r}$ modulation based on the channel conditions of relay-destination link and transmit it to the destination. If the relay node fails, the source will retransmit the data to the destination itself.

Let $d_{\rm ab}$ denote the geometrical distance between node a and b. Then the path loss between these two nodes can be written as:

$$PL({\rm ab}) = K/d_{\rm ab}^{\alpha} \tag{1}$$

where K is a constant varies with the environment and α is the path loss exponent. For the case of free-space path loss model, $\alpha = 2$. Thus the relationship of energy received at the relay node $E_{\rm sr}$ and that at the destination node $E_{\rm sd}$ can be represented as :

$$E_{\rm sr} = \frac{PL({\rm sr})}{PL({\rm sd})} E_{\rm sd} = G_{\rm sr} E_{\rm sd} \tag{2}$$

where $G_{\rm sr}$ is the geometrical gain[10] experienced by the link between the source and relay nodes due to the reduced distance for source-relay link compared to the source-destination link, which can be calculated as:

$$G_{\rm sr} = \left(\frac{d_{\rm sd}}{d_{\rm sr}}\right)^2 \tag{3}$$

Similarly, the geometrical gain of the relay-destination link with respect to the source-destination link can be written as:

$$G_{\rm rd} = \left(\frac{d_{\rm sd}}{d_{\rm rd}}\right)^2 \tag{4}$$

Naturally, the geometrical gain of the source-destination link $G_{\rm sd} = 1$, with respect to itself. Thus, if the relay is in the middle of source and destination i.e. $d_{\rm sr} = \frac{1}{2} d_{\rm sd}$, we have $G_{\rm sr}$ and $G_{\rm rd}$ of 6 dB.

After the first time slot, the received signal at the relay and destination, $y_{\rm sr}$ and $y_{\rm rd}$, can be written as:

$$y_{\rm sr} = \sqrt{G_{\rm sr}} h_{\rm sr} x_{\rm s} + n_{\rm sr} \tag{5}$$

$$y_{\rm sd} = h_{\rm sd} x_{\rm s} + n_{\rm sd} \tag{6}$$

where $n_{\rm sr}$ and $n_{\rm sd}$ are the Additive White Gaussian Noise (AWGN) having a variance of $N_0/2$ per dimension. During the second time slot, if the relay successful in decoding the data from the source, the signal received at the destination $y_{\rm rd}$ can be expressed as:

$$y_{\rm rd} = \sqrt{G_{\rm rd}} h_{\rm rd} x_{\rm r} + n_{\rm rd} \tag{7}$$

Note that the mapping constellation for $x_{\rm s}$ and $x_{\rm r}$ are adjusted based on the instantaneous CSI of SD and RD links respectively, so they might be different. Maximum A-posteriori Probability (MAP) detection is employed at the destination to combine $y_{\rm sd}$ and $y_{\rm rd}$ with different modulation constellations.

We consider a block fading model in this paper, where the fading coefficients are assumed to remain constant for one frame duration. Moreover, the fading coefficients $h_{\rm sd}$, $h_{\rm sr}$ and $h_{\rm rd}$ are assumed to follow the Rayleigh model and to be independent and identically distributed (i.i.d) between different frame durations. Hence if let $\gamma_{\rm ab}$ denote the instantaneous received SNR of the a-b link, then the faded SNR $\gamma_{\rm ab}$ follows the same exponential distribution, with common probability density function (PDF) and cumulative distribution function (CDF) give as:

$$f_{\gamma_{\rm ab}}(x) = \frac{1}{\bar{\gamma}_{\rm ab}} \exp(-\frac{x}{\bar{\gamma}_{\rm ab}}) \quad (8)$$

and

$$F_{\gamma_{\rm ab}}(x) = 1 - \exp(-\frac{x}{\bar{\gamma}_{\rm ab}}) \quad (9)$$

where $\bar{\gamma}_{\rm ab}$ is the average SNR of a-b link.

III. Adaptive Modulation for Cooperative Communications

In our scheme, both the source and the relay adopt constant-power variable-rate M-ary quadrature amplitude modulation (M-QAM), where the constellation size, M, is restricted to a power of 2, 2^n. For adaptive modulation, the switching of modulation schemes are subject to certain constraint condition. In this paper, we choose modulation schemes to make sure that the link is reliable, in other words, the error probability should be kept below certain level.

The instantaneous bit error rate (BER) of 2^n-QAM transmission over an AWGN channel with SNR of γ can be well approximated by [7]:

$$BER_n(\gamma) = 0.2 \exp(-g_n \gamma) \quad (10)$$

where g_n is a constellation size related constant, $g_n = \frac{1.5}{2^n - 1}$ for $M_n \geq 4$ and $g_n = 1$ for BPSK and γ is the instantaneous received SNR of the channel.

The modulation sizes at source node and relay node are determined locally to make sure that the BER performance of SD and RD link can meet the system target BER $BER_{\rm t}$. First, the source uses the maximum $M_n^{\rm s}$ to ensure that SD link can achieve the target BER:

$$P_{\rm sd}(M_n^{\rm s}) \leq BER_{\rm t} \quad (11)$$

where $P_{\rm sd}(M_n^{\rm s})$ is the instantaneous BER of SD link when modulation $M_n^{\rm s}$ is used. If the SD link is reliable, checked by CRC code, then only one time slot is used. As a result, the spectral efficiency is improved. The destination chooses the most suitable modulation size, $M_n^{\rm s}$, based solely on the fading channel conditions of SD link. This is done by estimating the faded received SNR and then find the region it falls. If the estimated SNR is in the nth region, the destination will inform the source to used modulation n for transmission. In the case that no modulation size can achieve the target BER, the lowest modulation size will be used for transmission. Similarly, the modulation size at the relay is chosen to ensure that the instantaneous BER of RD link to achieve the target BER:

$$P_{\rm rd}(M_n^{\rm r}) \leq BER_{\rm t} \quad (12)$$

where $P_{\rm rd}(M_n^{\rm r})$ is the instantaneous BER of RD link when modulation $M_n^{\rm r}$ is used. Therefore, the boundary threshold for our modulation scheme can be calculated in terms of a target BER, $\mathrm{BER_t}$, as

$$\gamma_n = \begin{cases} -\frac{2}{3}\ln(5BER_t)(2^n-1), & n = 1,2,4,6 \\ 0, & n = 0 \\ \infty, & n > 6 \end{cases} \quad (13)$$

IV. Feedback Error and its Quantification

In this section, we consider a noisy feedback channel for our adaptive modulation systems which will cause feedback errors. To focus on feedback errors, we assume that the perfect CSI is obtained at the destination node.

To evaluate the effect of feedback errors, we need to determine probability that constellation i is used while constellation j is selected at the destination due to the feedback errors. This transition probability can be modelled as a matrix $\mathcal{Q} = [q_{i,j}]$, where $q_{i,j}$ is the constellation size transition probability. Note that $q_{i,j}$ depends on the quality of the feedback channel as well as the signalling scheme used over the feedback channel. In this paper, we assume that Phase Shift Keying (PSK) based signalling scheme is employed over the feedback channel. We use N PSK symbols to denote N modulation sizes, when the jth PSK symbol is received at the transmitter, the jth modulation size will be chosen for transmission, thus only one symbol is needed for the feedback which significantly reduce the channel overhead. Assuming that each constellation size is mapped to a corresponding PSK symbol, then the decision region of th jth symbol representing the jth constellation size will be the wedge shaped area in the PSK signal circle. As a result, the constellation size transition probability $q_{i,j}$ equals to the average probability that the decision variable of the feedback channel detection erroneously falls into the ith wedge shape area instead of jth wedge shaped area. Assume that the wedge shaped area for jth symbol is between the lower and upper phase angles θ_j^- and θ_j^+, similarly, θ_i^- and θ_i^+ for the ith wedge shaped area. Thus the difference of the phase angels can be calculated in a circular term as [11]:

$$\theta_1 = \min\{|\theta_i^- - \theta_j^-|, 2\pi - |\theta_i^- - \theta_j^-|\} \quad (14)$$

and

$$\theta_2 = \min\{|\theta_i^+ - \theta_j^+|, 2\pi - |\theta_i^+ - \theta_j^+|\} \quad (15)$$

the instantaneous transition probability $\Pi(\gamma;\theta_1,\theta_2)$ can be calculated using the results from [12], γ is the instantaneous received SNR for the feedback channel. Noting that the instantaneous received SNR γ is a random variable over the feedback channel which follows the distribution as shown in 8 and 9, then we can obtain the constellation size transition probability, $q_{i,j}$, by averaging the instantaneous transition probability over the channel as:

$$q_{i,j} = \int_0^\infty \Pi(\gamma;\theta_1,\theta_2) f_\gamma(\gamma) d\gamma \quad (16)$$

where $f_\gamma(\cdot)$ is the PDF of the instantaneous received SNR of the feedback channel and $\Pi(\cdot;\cdot,\cdot)$ is the instantaneous constellation size transition probability.

Note that in our scheme the symbol for the highest constellation order is in the opposite position to the symbol of the lowest order. Thus the transition probability between the highest and lowest constellation order is the lowest in $\mathcal{Q}$, then we can in general expect a higher spectral efficiency than the case that those two are adjacent to each other.

V. Performance Analysis

In this section, we analyze the performance of the modulation-adaptive cooperative communication schemes with feedback error over Rayleigh fading channels. More specifically, we obtain its average spectral efficiency and average BER.

In our proposed DF relay network, the whole transmission may end up with three different situations: a) the destination decode the information from the source correctly at the end of the first time slot, then the second time slot is omitted; b) the destination fails in the first time slot as well as the relay, the source re-transmits the information in the second time slot; c) the destination fails but the relay succeeds during the first time slot, then the relay carries out the re-transmission. Let P_{a}, P_{b} and P_{c} denote the probability of the three situations as:

$$P_{\mathrm{a}} = 1 - P_{\mathrm{sd}}^{b} \tag{17}$$

$$P_{\mathrm{b}} = P_{\mathrm{sd}}^{b} P_{\mathrm{sr}}^{b} \tag{18}$$

$$P_{\mathrm{c}} = 1 - P_{\mathrm{a}} - P_{\mathrm{b}} \tag{19}$$

where P_{sd}^{b} and P_{sr}^{b} are the block error probability of SD and SR links respectively.

A. Spectral Efficiency

In the case without feedback error, the spectral efficiency of a point-to-point adaptive modulation system, η, can be calculated as [7]:

$$\eta = \sum_{n=1}^{N} \log_2^{(M_n)} Pr_n \tag{20}$$

where M_n is the constellation size of nth modulation scheme and Pr_n is the probability that nth modulation scheme is chosen for transmission, which can be expressed as:

$$Pr_n = F_\gamma(\gamma_{n+1}) - F_\gamma(\gamma_n) \tag{21}$$

where $F_\gamma(\cdot)$ is the CDF of the received SNR and γ_n is the threshold for adaptive modulation obtained from (13).

When we consider about the feedback errors, the spectral efficiency can be obtained as:

$$\eta = \sum_{i=1}^{N} \sum_{j=1}^{N} \log_2^{M_j} q_{i,j} Pr_j \tag{22}$$

When the relay comes to help, the overall spectral efficiency of the system can be expressed as:

$$\eta_{\mathrm{srd}} = \frac{\eta_{\mathrm{s}} \eta_{\mathrm{r}}}{\eta_{\mathrm{s}} + \eta_{\mathrm{r}}} \tag{23}$$

where η_{s} and η_{r} are the spectral efficiency for the source and the relay respectively, which are given by (22) above. Finally, the overall spectral efficiency can be written as:

$$\eta_{\mathrm{overall}} = P_a \cdot \eta_{\mathrm{s}} + P_b \cdot \frac{\eta_{\mathrm{s}}}{2} + P_c \cdot \eta_{\mathrm{srd}} \tag{24}$$

B. Error Probability

The average Bit Error Rate (BER) of a point-to-point adaptive modulation system without feedback errors can be calculated as [7]:

$$BER = \frac{1}{\eta} \sum_{n=1}^{N} \log_2^{M_n} \overline{BER}_n \tag{25}$$

where $\overline{BER}_n$ is the average error rate when nth constellation size is used for transmission, in the case of error-free feedback channel, which can be expressed as:

$$\overline{BER}_n = \int_{\gamma_n}^{\gamma_{n+1}} BER_n(\gamma) f_\gamma(\gamma) d\gamma \tag{26}$$

where $BER_n(\gamma)$ is the instantaneous BER of nth modulation size given in (10). When there is feedback error, $\overline{BER}_n$ can be calculated as

$$\overline{BER}_i = \sum_{j=0}^{N} q_{i,j} \int_{\gamma_j}^{\gamma_{j+1}} BER_i(\gamma) f_\gamma(\gamma) d\gamma \tag{27}$$

With case a), we can easily obtain that the average BER for the system is 0.

For case b), the error probability of SD link P_{sd} can be written as:

$$P_{\mathrm{sd}} = \sum_{j=0}^{N} q_{i,j} \int_{\gamma_{\mathrm{sd}}^{j}}^{\gamma_{\mathrm{sd}}^{j+1}} 0.2 \exp(-2 g_{\mathrm{s}} \gamma_{\mathrm{sd}}) f_\gamma(\gamma_{\mathrm{sd}}) d\gamma_{\mathrm{sd}} \tag{28}$$

where γ_{sd} is the received SNR of the S-D link.

Similarly, the error probability for case c) P_{srd} for combined SR and SD links can be expressed as:

$$P_{\mathrm{srd}} = \sum_{j=0}^{N} q_{i,j} \int_{\gamma_{\mathrm{sd}}^{j}}^{\gamma_{\mathrm{sd}}^{j+1}} \sum_{l=0}^{N} q_{k,l} \int_{\gamma_{\mathrm{rd}}^{l}}^{\gamma_{\mathrm{rd}}^{l+1}} \overline{P}_{\mathrm{srd}} f_\gamma(\gamma_{\mathrm{rd}}) f_\gamma(\gamma_{\mathrm{sd}}) d\gamma_{\mathrm{rd}} d\gamma_{\mathrm{sd}} \tag{29}$$

where $q_{i,j}$ and $q_{k,l}$ are the transition probabilities for the source and the relay node respectively, $\overline{P}_{\mathrm{srd}} = 0.2 \exp(-g_{\mathrm{s}} \gamma_{\mathrm{sd}} - g_r \gamma_{\mathrm{rd}})$ is the instantaneous combined BER at the destination node.

The overall BER then can be expressed as

$$\begin{aligned} P_{\mathrm{overall}} &= P_{\mathrm{a}} \cdot 0 + P_{\mathrm{b}} \cdot P_{\mathrm{sd}} + P_{\mathrm{c}} \cdot P_{\mathrm{srd}} \\ &= P_{\mathrm{b}} \cdot P_{\mathrm{sd}} + P_{\mathrm{c}} \cdot P_{\mathrm{srd}} \end{aligned} \tag{30}$$

VI. Simulation Results

In this section, we illustrate the impact of the noisy feedback channel on the proposed adaptive modulation schemes with simulation results. We set 10^{-3} as the target BER of the system while BPSK, QPSK, 16-QAM and 64-QAM are used for adaptive modulations. The average SNR of SD and SR links ranges from 0 dB to 20 dB, the average SNR of RD link is 20 dB higher, which means that the relay is 10 times closer to the destination than it to the source.

Fig.3 and Fig.4 show the simulation results for spectral efficiency and BER performance of the adaptive modulation relay network with a single relay experiencing a noisy feedback channel for both the source and the relay. Different feedback channel conditions are considered, the SNR of feedback channel is set to be -10, 0, 5 and 10 dB respectively. The performance of perfect feedback channel is shown in the figures as benchmark and a fixed DF QPSK scheme is shown in the figures. The performance improvement of the proposed adaptive modulation scheme over fixed DF scheme can be seen from the simulation results. It can be observed that due to the feedback errors, wrong constellation sizes are used which results in a shift in transmission data rate while the BER performance is degraded. As the SNR of feedback channel increases, the impact of noisy feedback channel becomes neglectable. From the results shown in Fig.3 and Fig.4, we can observe that when the SNR of feedback channel is 10 dB, the performance is almost the same as the perfect feedback channel.

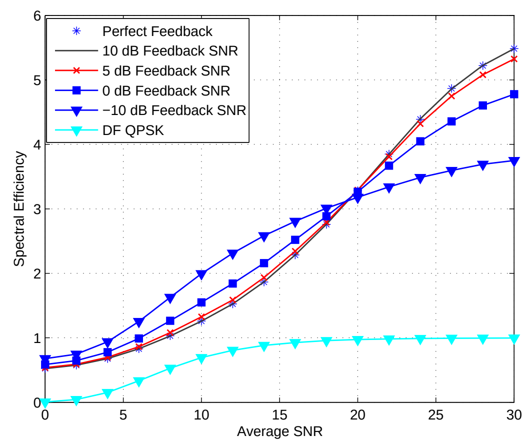

Fig. 3. Spectral Efficiency of Adaptive Modulation Cooperative Communication System with Noisy feedback Channel

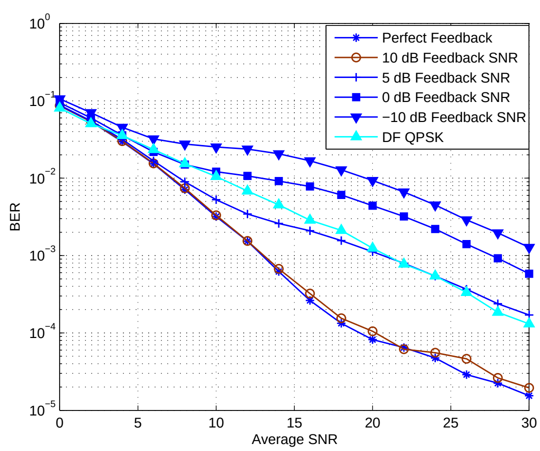

Fig. 4. BER Performance of Adaptive Modulation Cooperative Communication System with Noisy feedback Channel

VII. Conclusion

In this paper, we discuss the impact of noisy feedback channel on the adaptive modulation systems for cooperative communications. Given the noisy feedback at both the source and the relay, the adaptive modulation schemes are decided respectively. Simulation results show that when the SNR of the feedback channel is relatively high i.e. 10 dB, the impact of the noisy feedback is neglectable.

Acknowledgement

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