

Probing the shell model using nucleon knockout reactions

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Abstract. We assess the sensitivity of the intermediate energy, direct two-nucleon knockout reaction mechanism to details of the correlated motions of the two removed nucleons - as might be revealed by measurements of partial cross sections to different final states of the mass $A-2$ reaction residues. Comparisons of new reaction calculations with recent data for exotic nuclei in the sd -shell suggest that this sensitivity is significant and that the coherence of the two-nucleon configurations to the reaction allows small wave function components to be probed. The relative yields for the different J^π transitions are well described by the USD shell model calculations, however the measured cross sections require an overall suppression of the shell model two-nucleon spectroscopic strengths.

1. Introduction

One-nucleon knockout reactions from intermediate-energy (projectile fragmentation) beams are an increasingly important tool for mapping the single-particle structures of nuclei with exotic neutron- and proton-number combinations. The single-nucleon removal cross sections are large (several tens of mb) and both the efficiency and selectivity of the associated reaction measurements are high. This allows an examination of orbitals of both the weakly-bound (excess) and strongly-bound (deficient) nucleon species at their respective (and very displaced) Fermi surfaces [1, 2]. Exclusive (with respect to the knockout residue final states) single-nucleon knockout reaction analyses on light nuclear targets (usually ${}^9\text{Be}$ and ${}^{12}\text{C}$), with energies from 40 MeV - 1 GeV/nucleon, are now able to assess structure model predictions of level orderings and shell-gaps and of details of level occupations and spectroscopic strengths in some of the most rare, unstable species. Examples are discussed in [1, 3, 4] and in references therein.

In contrast, two-nucleon removal reactions - and what they can teach us - are less well understood. The intrinsic two-nucleon removal cross sections are now smaller (of order 1 mb) and their theoretical interpretation is also more complicated. Different $[j_1 \otimes j_2]$ configurations of the two-nucleons now contribute coherently to the cross section [5] resulting in a less transparent connection between data and the underlying structures. An added complication is that the reactions will, in general, proceed by both (i) a sudden two-nucleon knockout component with direct population of the mass $A-2$ residue, and (ii) indirect, two-step mechanisms - involving one-nucleon knockout to excited, intermediate fragments of mass $A-1$, followed by nucleon

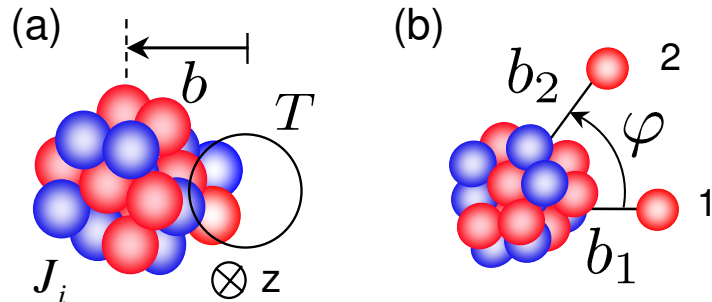


Figure 1. (a) Schematic representation of the cylindrical volume probed by a light nuclear target (T) in the direct two-nucleon knockout mechanism. (b) The coordinates (in the plane normal to the beam direction) used for presentation of the correlated two-nucleon position probabilities $P(b_1, b_2, \varphi)$.

evaporation. However, for the subset of reactions that involve the removal of two like-nucleons of the more deficient species - such as two-neutron removal from an already neutron-deficient system - the reactions are expected to populate the $A-2$ residues only directly [6, 5]: the result of the then much lower threshold for evaporation of a nucleon of the excess species.

Four such reactions on sd -shell nuclei have been measured recently [6, 7]. The availability of data on sd -shell nuclei, for which the predictive power of the shell-model is very high, thus allows a first assessment of the sensitivity of the reaction mechanism to the assumed two-nucleon structures in a rather clean, direct-reaction regime. In particular, we assess the likely sensitivity of the reaction mechanism to spatial correlations of the removed like-nucleon pair.

2. Reaction selectivity and two-nucleon wave functions

As was discussed in Reference [5], unlike for example the (p,t) reaction whose transfer reaction vertex is highly spin-selective (probing only $S = 0$ neutron pairs) [8], the sudden two-nucleon knockout mechanism has no reaction-driven spin-selectivity. As the projectile and the target pass at high-speed, Figure 1(a), the target will simply drill a cylindrical hole through the surface region of the projectile in the direction of the incident beam at an impact parameter b . One- and two-nucleon removal events involve only such grazing, surface collisions: trajectories at smaller b result in greater mass removal and strong absorption of mass $A-1$ and $A-2$ residues. Although not spin-selective, it is clear that the measured two-nucleon removal cross section will be very sensitive to the *spatial* proximity (position correlations) of pairs of like-nucleons. The correlations discussed here are these geometrical effects. We can gain understanding of the dependence of these spatial correlations on the assumed configurations and wave functions of the two nucleons by calculating the position probability, $P(b_1, b_2, \varphi)$, that two (point) nucleons (1,2) will be found with values of b_1 and b_2 (near the projectile surface) with an angular separation φ , see Figure 1(b). The relevant nucleon positions are those in the plane perpendicular to the beam direction and the relevant probabilities are those already integrated along the beam direction, the z -axis.

To be specific, we consider the two-proton removal reaction from neutron-rich $^{28}\text{Mg}(J_i^\pi=0^+)$ at 82.3 MeV/nucleon for which partial cross data are available to the bound $J^\pi = 0^+, 2_1^+, 4^+$ and 2_2^+ ^{26}Ne final states [6]. The reaction target was ^9Be . The dominant ^{28}Mg ground-state proton configuration is of course $[\pi d_{5/2}]^4$ and thus knockout is predominantly of a $[d_{5/2}]^2$ proton pair. More precisely however, the shell model two-nucleon overlap functions for each J^π final

state are [5]

$$F_{JM}(1, 2) = \sum_{I\mu\alpha} C_{\alpha}^{J_i J I} (I\mu J M | J_i M_i) \overline{[\phi_{j_1}(1) \otimes \phi_{j_2}(2)]_{I\mu}}, \quad (1)$$

where α labels pairs of active orbitals having normalized, antisymmetrized pair wave functions $[\phi_{j_1} \otimes \phi_{j_2}]$. All structure details are contained in the two-nucleon amplitudes (TNA) C_{α} , calculated here using the USD [9] *sd*-shell model: these were given in Table II of reference [5] for the ^{28}Mg transitions discussed here. The associated (correlated) two-nucleon position probabilities $P_J(b_1, b_2, \varphi)$ are therefore

$$P_J(b_1, b_2, \varphi) = \frac{1}{2J+1} \sum_M \int dz_1 \int dz_2 \langle F_{JM}(1, 2) | F_{JM}(1, 2) \rangle_{sp}, \quad (2)$$

where $\langle \dots \rangle_{sp}$ indicates integration over all spins. In the absence of explicit shell model correlations, i.e. assuming just a single $\beta \equiv (j_1, j_2)$ configuration, then $C_{\beta} \neq 0$ and will be determined by the two-particle coefficients of fractional parentage. For $\beta \equiv [d_{5/2}]^2$ removal from an assumed $[d_{5/2}]^4$ ground state, the C_{β} are $\sqrt{4/3}$, $\sqrt{5/3}$ and $\sqrt{3}$ for 0^+ , 2^+ and 4^+ transitions, respectively. In the limit that the removed nucleons are assumed completely uncorrelated then the position probability is

$$P(b_1, b_2) \equiv \frac{1}{(2j_1+1)(2j_2+1)} \sum_{m_1 m_2} \int dz_1 \int dz_2 \langle \phi_{j_1 m_1} \phi_{j_2 m_2} | \phi_{j_1 m_1} \phi_{j_2 m_2} \rangle_{sp}, \quad (3)$$

which is then independent of both φ and the final state J .

3. Effects on two-nucleon removal calculations

It is evident from the above that several *correlations* are at work and may affect the position probability density of the two nucleons, and hence the two-nucleon removal probability. In going from the fully uncorrelated, φ - and J -independent limit of Eq. 3 to having a single C_{β} configuration in Eq. 2 we include explicitly the (trivial) correlations from the proper angular momentum coupling/antisymmetrisation of the two nucleons. The effects on the resulting $P_J(b_1, b_2, \varphi)$ are already important, as is shown in Figure 2.

The φ -dependence of the $P_J(b, b, \varphi)$, $b=2.5$ fm, show two-nucleon spatial correlations that will enhance 0^+ , suppress 4^+ , and have little effect on the 2^+ final state populations, as compared to uncorrelated-limit estimates. As these changes arise only from angular momentum coupling/antisymmetrisation, they will be a very general feature. This density-based discussion is confirmed by the results of full calculations of the two-proton knockout cross sections, Figure 2(b), using the formalism of Refs. [5, 10]. The calculations extend those of Ref. [5] to include two-nucleon removal by the diffraction dissociation mechanism [10]. To aid the comparison with the data, the calculated inclusive cross sections (to all states) have been normalised to the experimental value, $\sigma_{\text{incl}}=1.50(10)$ mb [6]. The measured 2_1^+ and 2_2^+ cross sections have been summed, under 2^+ , as these simpler calculations have no mechanism to fragment transition strength for a given J^{π} . The scaling factors used in Figure 2(b) are 0.35 (uncorrelated) and 0.36 (assuming $[d_{5/2}]^2$). Both calculations use the C_{β} stated earlier, so the fully-uncorrelated 0^+ , 2^+ , 4^+ cross sections are in the ratio 4:5:9.

Beyond these baseline effects are those correlations specific to the many-body shell model. These will now (i) redistribute the strengths (and the phases) of the TNA, C_{α}^{0JJ} , between a number of active two-nucleon configurations and between the final states J^{π} , and (ii) introduce the coherence of the α configurations implicit in the full calculation of Eq. 2. The sensitivity of the two-nucleon knockout mechanism to these more subtle structural/residual interaction effects

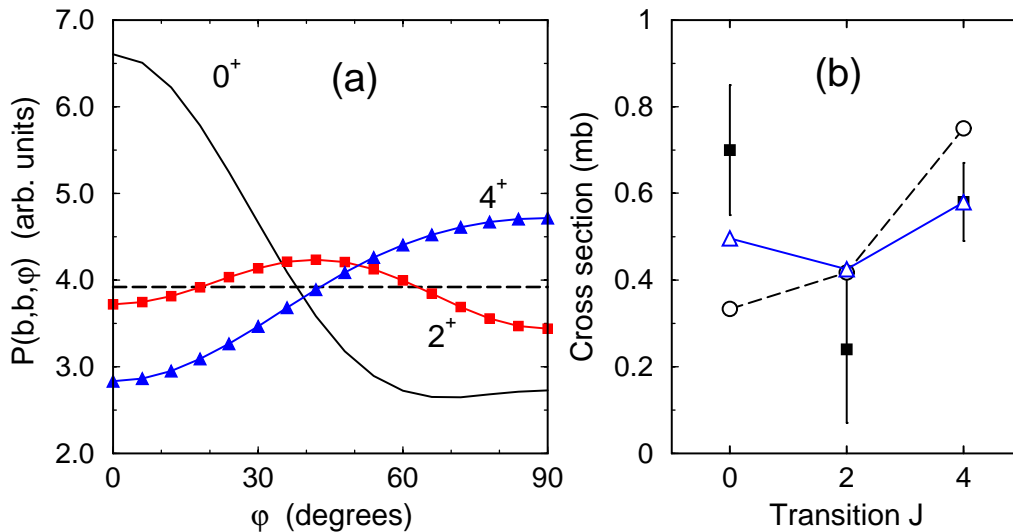


Figure 2. (a) The φ -dependence of the two nucleon position probabilities $P_J(b, b, \varphi)$, with $b=2.5$ fm, for $[d_{5/2}]^2$ two-proton removal from ^{28}Mg . The curves show the changes from the completely uncorrelated limit (the constant dashed line) due to angular momentum coupling/antisymmetrisation for the 0^+ , 2^+ and 4^+ state transitions. (b) The corresponding 0^+ , 2^+ and 4^+ partial cross sections (open triangles) are compared with the values for a completely uncorrelated pair (open circles) and with the experimental data (filled squares) of Ref. [6] at 82.3 MeV/nucleon.

is our primary interest here. Both will affect the $P_J(b_1, b_2, \varphi)$ and the inclusive and partial cross sections for two-nucleon knockout. In addition, the shell model makes predictions for the distribution and fragmentation of J^π strength - in the present example the strengths of the transitions to bound final states and the division of strength between the two bound 2^+ states - that can be compared with experiment. Figure 3(a) illustrates the coherence effect, (ii) above, for the case of the $^{28}\text{Mg}(0^+)$ to $^{26}\text{Ne}(0^+)$ ground-state to ground-state transition. The curves show calculations of both the (correct) fully-coherent and (for comparison) the incoherent combination of the $\alpha = [d_{5/2}]^2$, $[d_{3/2}]^2$ and $[s_{1/2}]^2$ shell model contributions to $P_0(b, b, \varphi)$. As is quite evident, although the incoherent contributions from the $d_{3/2}$ and $s_{1/2}$ orbitals are small, the coherent correlated shell model density has a significantly enhanced pair (small φ) probability that will further enhance the 0^+ ground-state to ground-state knockout. Such coherent (pairing) effects are found to be even more significant in projectiles with a greater mixing of orbital components in their ground state: these include the case of two-proton knockout from ^{44}S discussed in [3].

Although our reaction mechanism is not directly spin-selective, it is instructive to examine the $S=0,1$ two-nucleon spin contributions in the shell model wave function/probability. As a baseline, a pure $[d_{5/2}]^2$ two-proton knockout has total probabilities of spin $S=0(1)$ of 60(40)%, 48(52)%, 20(80)% for 0^+ , 2^+ , 4^+ transitions. Figure 3(b) shows that the fractions of the full shell model $P_J(b, b, \varphi)$ arising from spin-singlet configurations are enhanced: particularly those for the 0^+ transition, compared to the 60% expectation for $[d_{5/2}]^2$. It is clear that accurate partial cross section measurements will probe this underlying spin structure and test the consistency of the calculated singlet-pair content of the many-body wave function.

The partial cross sections resulting from the use of the shell model wave functions are presented in Figure 4. There is an excellent description of the measured distribution of partial

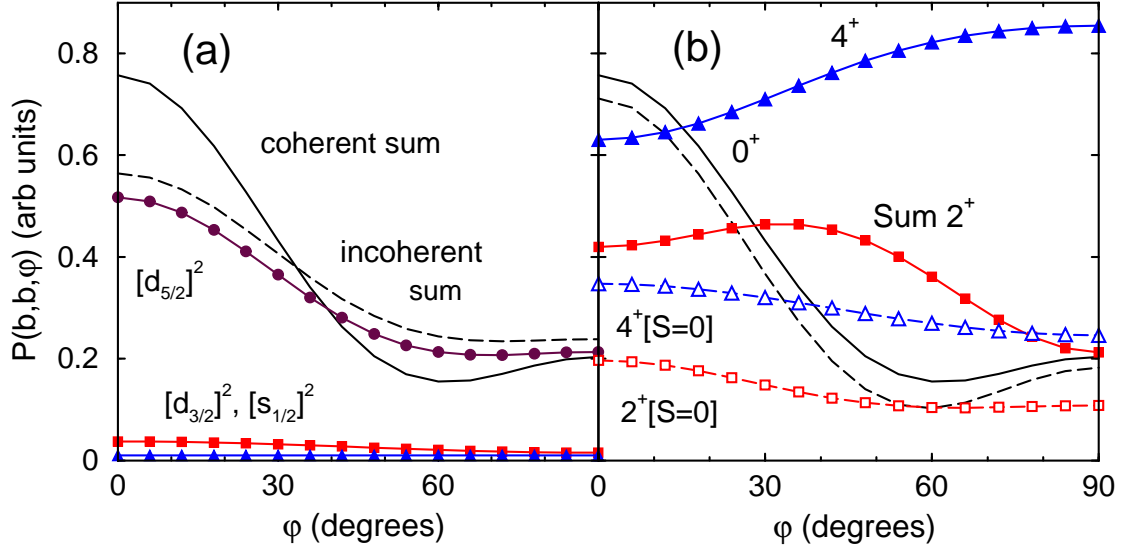


Figure 3. (a) The φ -dependence of the 0^+ state two-nucleon position probability $P_0(b, b, \varphi)$ (with $b=2.5$ fm) from the USD shell model wave function for neutron rich ^{28}Mg . The (correct) fully-coherent and (for comparison) the incoherent density is shown. (b) The full shell model $P_J(b, b, \varphi)$ (full lines and symbols) are compared with their $S=0$ spin-singlet pair components (dashed lines and open symbols) calculated using the formalism in Ref. [5].

cross section strength among the 0^+ , 2^+ and 4^+ final states, including the fragmentation of the 2^+ state strength, seen in part (b) of the figure. The coherence-induced enhancement of the

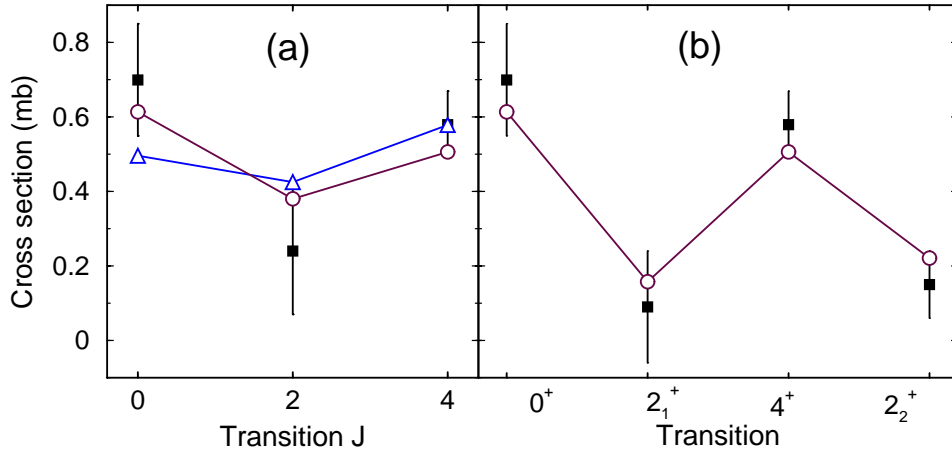


Figure 4. (a) The 0^+ , 2^+ and 4^+ partial cross sections resulting from the use of the full shell model wave functions (open circles) are compared with those for a pure $[\pi d_{5/2}]^2$ configuration (open triangles, as in Figure 2) for two-proton removal from ^{28}Mg at 82.3 MeV per nucleon. (b) The calculated and experimental exclusive cross sections to the 0^+ , 2_1^+ , 4^+ and 2_2^+ ^{26}Ne final states are compared. All USD shell model cross sections (open circles) have been scaled by a suppression factor $R_s=0.52$.

0^+ cross section is required by the data and is thus a sensitive probe of the smaller shell model components. Both the calculated inclusive and the partial cross sections in Figure 4 have been scaled by a common factor of $R_s=0.52$, indicating the need for *suppression*, or quenching, of the shell model two-nucleon strengths. Very similar suppression factors are required to describe the three measured two-neutron removal reaction cases also. These are discussed more fully elsewhere [10, 7].

4. Summary

We have examined the sensitivity of two-nucleon knockout reaction partial cross sections to details of the many-body shell model wave function. For the *sd*-shell, ^{28}Mg to ^{26}Ne two-proton removal case presented here, the reaction is dominated by $[\pi d_{5/2}]^2$ knockout. Nevertheless, coherent admixtures with smaller $[0d_{3/2}]$ and $[1s_{1/2}]$ components lead to enhanced pairing, an enhanced 0^+ cross section, and a good description of the observed J^π final state relative yields. Analogous results for the 0^+ and 2^+ final state yields for the three related *sd*-shell (two-neutron knockout) reactions are presented in [7]. The coherence of the shell model configurations means that although not nucleon spin-selective, the reaction mechanism, together with the J^π -selectivity, is nevertheless a sensitive probe of details of structure model wave function admixtures. The overall shell model strength is found to be about a factor of two larger than required by the experimental cross sections. Further discussion of this latter observation for other systems, and full details of the new stripping plus diffraction two-nucleon knockout reaction calculations, will be discussed elsewhere [10]. The potential to use such direct two-proton knockout reactions, with heavier primary and secondary beams, to selectively excite low-seniority high-spin isomeric states, has also recently been considered. A preliminary account is presented in [11].

Acknowledgments

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