Capacity of Cellular Uplink with Multiple Tiers of Users and Path Loss

D. Kaltakis, E. Katranaras, M. A. Imran, C. Tzaras
Centre for Communication Systems Research
University of Surrey, UK
email: d.kaltakis@surrey.ac.uk

***Abstract*—With the emergence and continuous growth of wireless data services, the value of a wireless network is not only defined by how many users it can support, but also by its ability to deliver higher data rates. Information theoretic capacity of cellular systems with fading is usually estimated using models originally inspired by Wyner's Gaussian Cellular Multiple Access Channel (GCMAC). In this paper we extend this model to study the cellular system with users distributed over the cellular coverage area. Based on the distance from the cell-site receiver, users are grouped as tiers, and received signals from each tier are scaled using a distance dependent attenuation factor. The optimum capacity in fading environment is then found by calculating the path-loss for users in each tier using a specific path-loss law and some interesting insights are derived. The results correspond to a more realistic model which boils down to Wyner's model with fading, with appropriate substitutions of parameter values. The results are verified using Wyner's model with fading and Monte-Carlo simulations. Insights are provided for the real world scenarios.**

I. Introduction

In the past, wireless systems were designed to accommodate a large number of voice and/or low data rate users. With the emergence and continuous growth of wireless data services, the value of a wireless network is not only defined by how many users it can support, but also by its ability to support high data capacity at localized spots.

Shannon's work in [1] gave birth to the field of information theory. In information theoretic literature different approaches have been reported to determine maximum data rate and the means to achieve this under various assumptions and constraints. Despite the work in this field, the first important attempt to study the capacity of a cellular system was carried out in the previous decade by Wyner [2]. Wyner's model studies the uplink channel and although it considers a very crude approximation of path loss with no path loss variability across the cell, it manages to provide an insight into the cooperation of the base stations and the benefits that can be achieved through that process.

Fading was incorporated in Wyner's model by Somekh and Shamai in [3]. They maintained the assumption of a hyper-receiver with delay-less access to all cell-site receivers and assumed the same interference pattern as Wyner's. They used a "raster-scan" method to transform the two-dimensional system into an equivalent linear system in order to arrange the fading coefficients and the system's path gains in the channel matrix. Their results showed that for a certain range of relatively high inter-cell interference, the fading improves the system performance as compared to the case when there is no fading.

Letzepis in [4] modified the one-dimensional version of Wyner's model to account for the free-space path loss. The optimum capacity is estimated by using the Shannon-transform of the Marcenko-Pastur law. To keep the problem tractable, all users are assumed to be collocated at the cell-site receiver's position. The major contribution of the work is that it accounts for the interference caused not only from the two neighboring cells but from all cells in the system enabling the study of the effects of changing cell-density in the system.

Further work has also been reported on the analysis of the achievable capacity in Distributed Antenna Systems (DAS) from an information theoretic standpoint (see [5], [6] and references within).

The available literature usually assumes a single path loss factor for all users in each cell. In this paper we extend the model by assuming that there are at least two classes of users in each cell: users close to the BS and users close to the cell edge. As a result each cell-site receiver is receiving signals from 5 tiers of users. Each tier is delineated based on the users' average distance from the center of the cell of interest. Each tier is considered to have a different path gain which corresponds to a specific path loss law.

This paper is organized as follows: In section II we present the mathematical model for the cellular uplink channel followed by the analysis for information theoretic capacity under any fading environment, based on the work presented in [3], [7], [8], in section III. The information theoretic results, with their interpretation for real-world systems, are presented in section IV and some conclusions are drawn in section V.

II. System Model

We consider a modified version of Wyner's [2] hexagonal cellular array model. Each cell (an approximate hexagon in shape, see Figure 1) is considered to be composed of 7 hexagonal *subcells*. The Base Stations (BSs) are positioned at the center of each cell (centre of central subcell). There are N^2 cells in the system with K users per cell. Each sub-cell contains $\acute{K} = K/7$ users placed at its center. The interference pattern follows the one proposed by Wyner in [2] and each subcell has a specific distance dependent interference factor (see Figure 2). Thus each BS receives signals from the user-transmitters in the six neighboring cells (i.e. 42 subcells), each

$$
\begin{aligned}
Y_{m,n} = {} & \sum_{k=1}^{\acute{K}} \tilde{X}^{0}_{m,n,k} + \alpha_1 \sum_{k=1}^{\acute{K}} \sum_{i=1}^{6} \tilde{X}^{i}_{m,n,k} + \\
& \alpha_2 \sum_{k=1}^{\acute{K}} \left(\sum_{i=3,4} \tilde{X}^{i}_{m-1,n-1,k} + \sum_{i=4,5} \tilde{X}^{i}_{m,n-1,k} + \sum_{i=5,6} \tilde{X}^{i}_{m+1,n,k} + \sum_{i=1,6} \tilde{X}^{i}_{m+1,n+1,k} + \sum_{i=1,2} \tilde{X}^{i}_{m,n+1,k} + \sum_{i=2,3} \tilde{X}^{i}_{m-1,n,k} \right) + \\
& \alpha_3 \sum_{k=1}^{\acute{K}} \left(\sum_{i=0,2,5} \tilde{X}^{i}_{m-1,n-1,k} + \sum_{i=0,3,6} \tilde{X}^{i}_{m,n-1,k} + \sum_{i=0,4,1} \tilde{X}^{i}_{m+1,n,k} + \sum_{i=0,5,2} \tilde{X}^{i}_{m+1,n+1,k} + \sum_{i=0,6,3} \tilde{X}^{i}_{m,n+1,k} + \sum_{i=0,1,4} \tilde{X}^{i}_{m-1,n,k} \right) + \\
& \alpha_4 \sum_{k=1}^{\acute{K}} \left(\sum_{i=6,1} \tilde{X}^{i}_{m-1,n-1,k} + \sum_{i=1,2} \tilde{X}^{i}_{m,n-1,k} + \sum_{i=2,3} \tilde{X}^{i}_{m+1,n,k} + \sum_{i=3,4} \tilde{X}^{i}_{m+1,n+1,k} + \sum_{i=4,5} \tilde{X}^{i}_{m,n+1,k} + \sum_{i=5,6} \tilde{X}^{i}_{m-1,n,k} \right) \quad \text{(A)}
\end{aligned}
$$

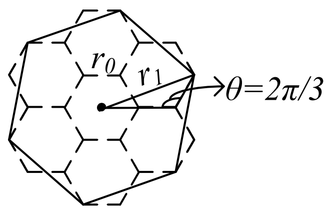

Fig. 1. A cell is represented as a cluster of seven hexagonal subcells. The boundary of the cell can be represented by an equivalent Hexagon, whose sides can be calculated using simple geometrical facts.

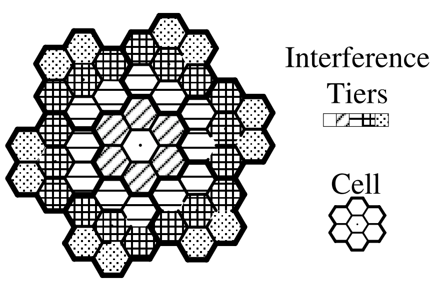

Fig. 2. System model with 5 tiers, each having a different interference factor. Central cell (cluster of 7 hexagons) and its six surrounding neighbors are shown. The last tier is incomplete as it only includes the subcells of the six neighbours of the central cell.

one multiplied by a fading coefficient (random variable) and the path gain corresponding to the specific sub-cell that the transmitter belongs to. By scaling and rotating the structure in Figure 2, a more tractable rectangular array representation of the system is obtained. The points of the rectangular array are indexed by (m, n) where m and n are the row and column numbers respectively [2].

In order to write the output at a BS receiver of cell (m, n) the 7 sub-cells are numbered from 0 to 6. By following an interference pattern, the resulting indexed system (with sub-cell indices of two cells shown, other subcells are numbered similarly) is shown in Figure 3. The received signal at the BS of cell (m, n) can be written as the sum of the received signals from the same-cell and neighboring-cell transmitters. Each transmitted signal is multiplied by the fading and path gain attenuation coefficients so as to obtain the received signal.

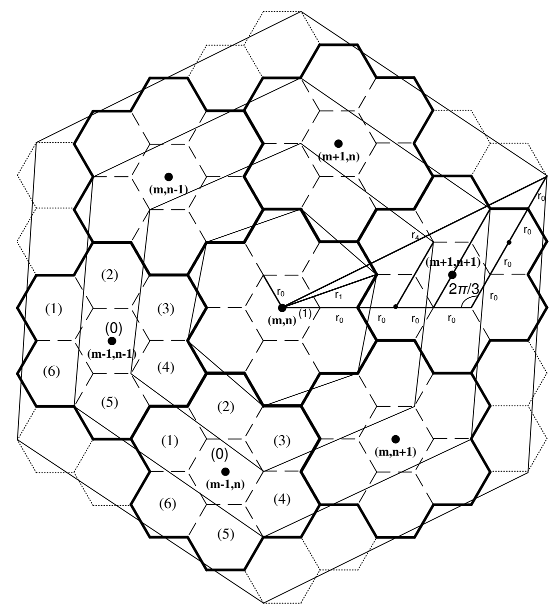

Fig. 3. Hexagons representing all tiers of interference and the triangles used to find each hexagon's side.

The received signal at cell (m, n) can be mathematically expressed by equation (A), where $\tilde{X}^{i}_{\acute{m},\acute{n},k} \triangleq X^{i}_{\acute{m},\acute{n},k} b^{i}_{\acute{m},\acute{n},k}$, $\acute{m}, \acute{n}$ may also represent the neighbour cell indices with offsets. In (A), $b^{i}_{\acute{m},\acute{n},k}$ is the fading coefficient corresponding to transmitter k of cell $(\acute{m}, \acute{n})$. The index i refers to the sub-cell the transmitter is located in. All the complex fading coefficients are normalized to unit power and are considered circular symmetric i.i.d. complex Gaussian, strictly stationary and ergodic complex random processes. Their mean value is defined as $\mathbb{E}[b^{i}_{\acute{m},\acute{n},k}] \triangleq \sqrt{\frac{\kappa}{\kappa+1}} \exp(j\phi^{i}_{\acute{m},\acute{n},k})$ with $\phi^{i}_{\acute{m},\acute{n},k}$ being the received phase for user k in cell $(\acute{m}, \acute{n})$ subcell i, and κ is the ratio of the LoS and NLoS components (Rician factor).

Each $X^{i}_{\acute{m},\acute{n},k}$ is the complex Gaussian input corresponding to transmitter k at sub-cell i of cell $(\acute{m}, \acute{n})$. A power constraint is considered for each input, $\mathbb{E}[(X^{i}_{\acute{m},\acute{n},k})^2] \leq P$. $Y_{m,n}$ is the complex Gaussian output at cell (m, n) and $Z_{m,n}$ is the noise at cell (m, n), normalized to unit power .

The output vector of the system can be written in the form:

$$\mathbf{y} = \mathbf{G}\mathbf{x} + \mathbf{z} \tag{1}$$

where $\mathbf{y} = [y_{1,1}, y_{1,2}, \cdots, y_{N,N}]^T$ is the $N^2 \times 1$ received signals vector, $\mathbf{x} = [\mathbf{x}^0_{1,1}, \mathbf{x}^1_{1,1}, \cdots, \mathbf{x}^5_{N,N}, \mathbf{x}^6_{N,N}]^T$ is the concatenation ($7N^2\acute{K} \times 1$ vector) of all the transmitted signal vectors – $1 \times \acute{K}$ row-vectors $\mathbf{x}^i_{m,n}$ for all cells (m, n) and subcells i – in the system and $\mathbf{z} = [z_{1,1}, z_{1,2}, \cdots, z_{N,N}]^T$ is the $N^2 \times 1$ noise vector. Based on the channel equation (A) and expanding the raster scan method presented in [3] the overall channel gain matrix $\mathbf{G}$ is a block circulant matrix. In order to represent this matrix in a compact form, some definitions are needed. First a $N^2 \times 1$ column vector $\mathbf{e}_i$ is defined with its i^{th} component one and all other components zero. Then a block circulant matrix $\mathbf{A}$ with N^2 rows is defined with its first row, $\mathbf{A}_{[1]}$, given by (2) and it's j^{th} row , $\mathbf{A}_{[j]}$, by (3):

$$\mathbf{A}_{[1]} \triangleq \left[\boldsymbol{\alpha}_0 \ \boldsymbol{\alpha}_5 \ \overbrace{\mathbf{0}\cdots\mathbf{0}}^{N-2} \ \boldsymbol{\alpha}_3 \ \boldsymbol{\alpha}_4 \ \overbrace{\mathbf{0}\cdots\mathbf{0}}^{N^2-2N-3} \ \boldsymbol{\alpha}_1 \ \boldsymbol{\alpha}_6 \ \overbrace{\mathbf{0}\cdots\mathbf{0}}^{N-2} \ \boldsymbol{\alpha}_2 \right] \tag{2}$$

$$\mathbf{A}_{[j]} = \mathbf{A}_{[j]}\mathbf{C}^{j-1}, \ j = 1 \cdots, N^2 \tag{3}$$

where $\mathbf{C} \triangleq [\mathbf{e}_{N^2}, \mathbf{e}_1, \cdots, \mathbf{e}_{N^2-1}]$ is a right-circular-shift matrix and the elements of $\mathbf{A}_{[1]}$ in (2) are 1×7 row vectors defined as:

$$\begin{aligned}
&\boldsymbol{\alpha}_0 \triangleq [1 \ \alpha_1 \ \alpha_1 \ \alpha_1 \ \alpha_1 \ \alpha_1 \ \alpha_1] \\
&\boldsymbol{\alpha}_j \triangleq [\alpha_3 \ \boldsymbol{\alpha_x}\mathbf{C}_x^{j-1}] \text{ for } j \in \{1, 2, 3, 4, 5, 6\} \\
&\quad \text{with } \boldsymbol{\alpha}_x \triangleq [\alpha_4 \ \alpha_3 \ \alpha_2 \ \alpha_2 \ \alpha_3 \ \alpha_4] \\
&\mathbf{0} \triangleq [0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0],
\end{aligned} \tag{4}$$

where $\mathbf{C}_x \triangleq [\mathbf{e}_6, \mathbf{e}_1, \cdots, \mathbf{e}_5]$ is the corresponding right-circular-shift matrix. Using definitions (2), (3) and (4) the $\mathbf{G}$ matrix is given by:

$$\mathbf{G} = \left(\mathbf{A} \otimes \mathbf{1}_{1\times\acute{K}}\right) \odot \mathbf{B} \tag{5}$$

where $\mathbf{B}$ is the $N^2 \times 7N^2\acute{K}$ fading matrix that contains all fading coefficients $b^i_{m,n,k}$, $\otimes$ is the Kronecker product and $\odot$ represents the Hadamard multiplication.

III. Cell Capacity Analysis

Applying the theorems proved in [3] to the model described above, it can be easily shown that for large number of users per cell the maximal achievable per-cell capacity is achieved when all users are allowed to transmit all the time at their maximum power (reported as the WB scheme in [3]). Using Jensen's inequality a tight upper bound on the maximum reliable uplink sum capacity can be found [3], and it is given by:

$$C(\alpha_1, \alpha_2, \alpha_3, \alpha_4) = \lim_{N\to\infty} \frac{1}{N^2} \log\det \mathbb{E}[\boldsymbol{\Lambda}] \tag{6}$$

where expectation is taken over all random fading realizations and $\boldsymbol{\Lambda}$ is the normalized covariance matrix of the output vector:

$$\boldsymbol{\Lambda} = P\mathbf{G}\mathbf{G}^\dagger + \mathbf{I}_{N^2\times N^2} \tag{7}$$

The expectation of the product of a complex fading coefficient with its complex conjugate, is equal to unity as the fading coefficients are assumed normalized to unit power. Furthermore the expectation of the product of a complex fading coefficient with the conjugate of a different one, following the same distribution, is equal to its expected value squared. let us define the two dimensional index: $\mathbf{n} \triangleq (u, v)$. The difference between two indices of this form is defined as $\mathbf{n}_1 - \mathbf{n}_2 \triangleq (u_1 - u_2, v_1 - v_2)$. As we are considering a planar cellular system, the cells need to be indexed using two dimensional indices as above. For the formation of an uplink channel matrix, a single index is more convenient to map each row of the channel matrix to one and only one cell receiver in the system. A raster scan is used to form a one-to-one mapping of the two dimensional indices (m, n) to the row indices c:

$$\mathbf{n}_c \mapsto (m, n), \ c = 1 \cdots N^2, m = 1 \cdots N, n = 1 \cdots N \tag{8}$$

When we multiply the channel matrix with its complex conjugate transpose (in order to find the covariance), each entry of the resulting matrix depends on the row (corresponding to a specific cell) in the channel matrix that is multiplied with the column (corresponding to another specific cell) in the conjugate matrix. The covariance of received vectors depends on the difference between the 2D indices defined as above. Thus each entry at position (i, j) of the resulting matrix is a function of the difference between the 2D index of the j^{th} column of the $\mathbf{G}^\dagger$ matrix and the 2D index of the i^{th} row of the $\mathbf{G}$ matrix. Considering the above, using the $\mathbf{G}$ matrix in (5) and substituting in (7), the expectation of $\boldsymbol{\Lambda}$ is:

$$\mathbf{R}_{(r,t)}(i, j) = \mathbb{E}[\boldsymbol{\Lambda}] = \begin{cases} 1 + \acute{\gamma}B & (t, t) \\ \acute{\gamma}\Gamma\,|m_b|^2 & (r, t) \in \mathcal{S}_0 \\ \acute{\gamma}\Delta\,|m_b|^2 & (r, t) \in \mathcal{S}_1 \\ \acute{\gamma}H\,|m_b|^2 & (r, t) \in \mathcal{S}_2 \\ 0 & \text{otherwise} \end{cases} \tag{9}$$

$$\begin{aligned}
&B = 1 + 6\alpha_1^2 + 12\alpha_2^2 + 18\alpha_3^2 + 12\alpha_4^2 \\
&\Gamma = 2\alpha_3 + 2\alpha_3^2 + 4\alpha_1(\alpha_2 + \alpha_3 + \alpha_4) + 2\alpha_2^2 \\
&\qquad + 2\alpha_4^2 + 4\alpha_2\alpha_3 + 4\alpha_3\alpha_4 \\
&\Delta = 2\alpha_3^2 + 4\alpha_2\alpha_3 + 4\alpha_2\alpha_4 + 4\alpha_3\alpha_4, H = 3\alpha_3^2 + 4\alpha_2\alpha_4
\end{aligned} \tag{10}$$

where (r, t) is defined as a function of $\mathbf{n}_j - \mathbf{n}_i \triangleq (r, t)$, $\acute{\gamma} \triangleq \gamma/7$, $\gamma \triangleq KP$ and the sets $\mathcal{S}_0, \mathcal{S}_1, \mathcal{S}_2$ are defined as:

$$\begin{aligned}
&\mathcal{S}_0 \triangleq \{(-1, -1), (0, -1), (1, 0), (1, 1), (0, 1), (-1, 0)\} \\
&\mathcal{S}_1 \triangleq \{(1, 2), (2, 1), (1, -1), (-1, -2), (-2, -1), (-1, 1)\} \\
&\mathcal{S}_2 \triangleq \{(0, 2), (2, 2), (2, 0), (0, -2), (-2, -2), (-2, 0)\}
\end{aligned} \tag{11}$$

The asymptotic expression for the maximum capacity can be found by applying the two dimensional extension of Szego's

$$C(\alpha_1,\alpha_2,\alpha_3,\alpha_4) = \int_0^1\int_0^1 \log\Big[1+\acute{\gamma}B+\gamma|m_b|^2\Big(2\Gamma\big(\cos(2\pi\theta_1+2\pi\theta_2)+\cos(2\pi\theta_1)+\cos(2\pi\theta_2)\big)$$
$$+\Delta\big(2\cos(4\pi\theta_1+2\pi\theta_2)+2\cos(2\pi\theta_1+4\pi\theta_1)+\cos(2\pi\theta_1-2\pi\theta_2)\big)+2H\big(\cos(4\pi\theta_1+4\pi\theta_2)+\cos(4\pi\theta_1)+2\cos(2\pi\theta_2)\big)\Big)\Big]d\theta_1 d\theta_2 \tag{B}$$

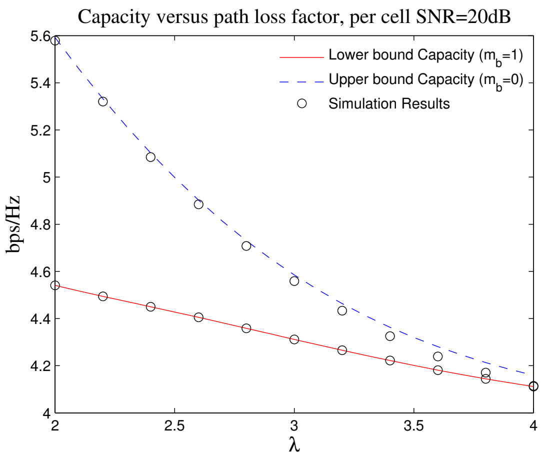

Fig. 4. Per-cell capacity vs. path loss exponent in deterministic (AWGN with constant received phase) environment (lower bound) and Rayleigh fading environment (upper bound). KP=20dB.

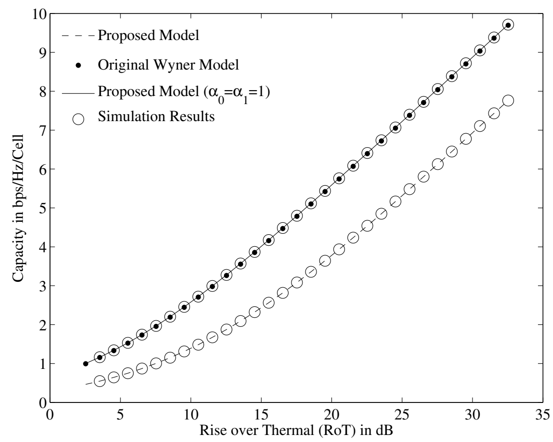

Fig. 5. Per-cell capacity vs. cell RoT for the proposed planar system model and comparison with the corresponding Wyner's model [2]. To compare the two models the α factor in Wyner's model is considered to be the average of the α factors of the neighboring cells in the proposed model: $\alpha_{\text{WYNER}} \Leftrightarrow [\alpha_2+\alpha_3+\alpha_4]/3$.

theorem [7], [8] to (6):

$$C(\alpha_1,\alpha_2,\alpha_3,\alpha_4) = \lim_{N\to\infty}\frac{1}{N^2}\log\det\mathbb{E}[\mathbf{\Lambda}_N] = \int_0^1\int_0^1 \log\left(\mathbf{\Phi}_Y(\theta_1,\theta_2)\ \right)d\theta_1 d\theta_2 \tag{12}$$

where $\mathbf{\Phi}_Y(\theta_1,\theta_2)$ is the two-dimensional Fourier transform of $\mathbf{R}$ given in (9):

$$\mathbf{\Phi}_Y(\theta_1,\theta_2) = \sum_{m,n=0}^{\infty}\mathbf{R}_{(r,t)}(m,n)\mathrm{e}^{-j2\pi\theta_1 m}\mathrm{e}^{-j2\pi\theta_2 n} \tag{13}$$

where $j\triangleq\sqrt{-1}$, m,n are dummy variables. Thus the per-cell capacity for this model is given by equation (B).

To incorporate path loss in this model we evaluate a *representative* distance for users in each tier and *map* it to a path loss using a specific path loss model. To do this, we delineate the hexagonal boundaries of the tiers as shown in Figure 3. The irregular boundary of each tier can be represented by a regular hexagon, as the average distance between the center of the cell of interest and the points on the perimeter of both shapes is the same. Assume that the side of each subcell hexagon is r_0. The side of the hexagonal boundary of each tier, r_l, can be found using the geometry of the shape and is given by the following general formula (using phasor notation)

$$r_l = r_0\left|1+l+l\ \mathrm{e}^{-j\pi/3}\right| \tag{14}$$

where $|\cdot|$ represents the magnitude of a complex phasor. It is easy to show (see Figure 1) that the average distance from the center of a regular hexagon (with its side r_l) to each point on its perimeter is given by:

$$d_l = \frac{6}{\pi}\int_0^{\frac{\pi}{6}}\frac{r_l}{\cos\theta}\cos\left(\frac{\pi}{6}\right)d\theta \tag{15}$$

Using this equation, we can find the average distance of the points on the boundary of each tier. We use the mean of the "average distance of points" on the inner and the outer boundary of the tier to find the *representative* distance for the l^{th} tier, i.e. $\bar{d}_l = (d_l+d_{l+1})/2$. Using the widely used modified path loss model, we *map* the *representative* distance $\bar{d}_l$ of the l^{th} tier to a path loss factor α_l [4]: $\alpha_l = \left(1+\bar{d}_l\right)^{-\eta/2}$ where $\bar{d}_l$ is the normalized representative distance (w.r.t. a reference distance) and $2\leq\eta\leq 4$ is the path loss factor.

IV. Results and Discussions

The following figures illustrate some interesting results for the per-cell capacity of the model presented in this paper. Figure 4 shows the lower and upper bounds of the capacity in our model. The lower bound capacity (perfectly synchronized phases of the received signals and thus mean equal to one) decays slowly with the path loss factor. The upper bound capacity (more realistically modelled independent and uniformly distributed phases of the received signals and thus mean equal to zero) tends to the lower bound as the path loss factor grows. For the lower bound the capacity loss between path loss factors 2 and 4 is only 0.43 bps/Hz. For the upper bound the corresponding capacity loss is 1.43 bps/Hz. This

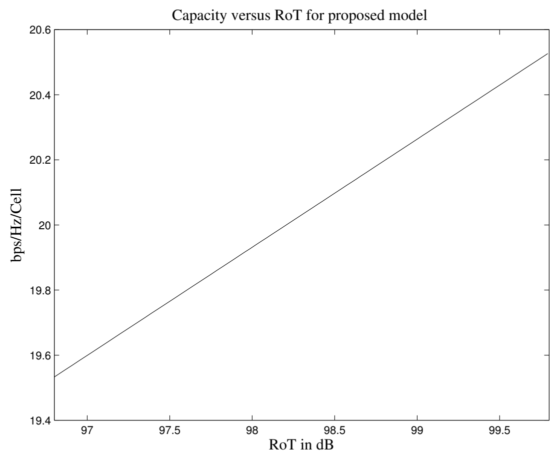

Fig. 6. Real-world System Per-cell capacity (upperbound) vs. RoT

happens due to the fact that the same user is received by a smaller number of BS antennas when the path loss factor is higher (limited multi-user diversity effect [8]).

In Figure 5 the per-cell capacities vs. Rise over Thermal (defined as the total received power-desired and interference-over the thermal noise power at each BS receiver.) are plotted for both Wyner and the lower bound of the proposed model. Here we have to mention that Wyner's capacity can be related to the lower bound of our model due to the fact that in his work he assumed a real channel model (mean value equal to one). The gap observed between our lower bound and Wyner's capacity is due to the fact that in his work [2], Wyner, assumed unit path gain for all users in the cell of interest while in this study only a fraction of the users are assumed to have unit gain. The other users' path gains are following the path loss law. Nevertheless in Figure 5 the lower bound capacity of the proposed model considering unit gain for all users in the cell of interest is also plotted. It coincides perfectly with Wyner's result for the 2D cellular array. The circle-points in figures 4, 5 are obtained by simulation. For the simulation we generated 100 random fading matrices, $\mathbf{B}$, distributed according to the channel model described in section II, in order to obtain $\mathbf{G}$ using (5). Then the maximum capacity was calculated using $\mathbb{E}[\log\det(\mathbf{I} + P\mathbf{G}\mathbf{G}^{\dagger})]$ for a given value of P (normalised SNR). As it is illustrated in figures 4 and 5 the simulation results obtained coincide with the theoretical results.

We calculate the information theoretic capacity for a real-world scenario. Consider a scenario [9] where cells have a radii of 1km and the path loss at a reference distance of 1m is -38 dB (for a carrier frequency of 1.9 GHz) and the path loss exponent is 2. The system has 21 UTs per cell with transmit power constraint of 100-200 mW and thermal noise density of -169 dBm/Hz with channel bandwidth of 5MHz. Random received phases are assumed to plot the capacity in Fig. 6 for a real-world scenario.

V. CONCLUSIONS

In this paper, we extend the planar Gaussian Cellular Multi Access Channel (GCMAC) model initially proposed by Wyner (and later extended by Shomekh and Shamai to incorporate fading). This extension is done using a more realistic distance-dependent path loss factor. The users with similar distance from the base station of interest are grouped in tiers of interference and their path loss is approximated using the average distance of the users from the base station of interest. It should be noted that the average path-loss is approximated using the average distance to simplify the analysis here. The capacity of the cellular system is then formulated for fading environments with different mean. It is found that the adjacent channel interference is quite accurately modelled by the single alpha factor but assuming same path loss within the cell of interest gives a loose upper bound in Wyner's model. It is also observed that the capacity is significantly higher in zero mean fading environments when path loss exponent is small. This is due to the fact that with a small path loss exponent, the transmission of any user is effectively "heard" by multiple base station receivers and joint decoding of the multiple received copies increases the capacity. The analytical results are verified by Monte-Carlo simulations and it has also been demonstrated that the proposed model boils down to original Wyner's model with appropriate substitutions of the model parameters. By appropriately de-normalizing system parameters we find the per-cell capacity of more practical systems.

ACKNOWLEDGMENT

The work reported in this paper has formed part of the "Fundamental Limits to Wireless Network Capacity" Elective Research Programme of the Virtual Centre of Excellence in Mobile & Personal Communications, Mobile VCE, www.mobilevce.com. This research has been funded by the following Industrial Companies who are Members of Mobile VCE - BBC, BT, Huawei, Nokia, Nokia Siemens Networks, Nortel, Vodafone. Fully detailed technical reports on this research are available to staff from these Industrial Members of Mobile VCE. The authors would like to thank Prof. G. Caire and Prof. D. Tse for the fruitful discussions.

REFERENCES

[1] C. E. Shannon, "A mathematical theory of communication," *Bell Systems Technical Journal*, vol. 27, pp. 379–423, 1948.

[2] A. Wyner, "Shannon-theoretic approach to a Gaussian cellular multiple-access channel," *IEEE Transactions on Information Theory*, vol. 40, no. 6, pp. 1713–1727, Nov. 1994.

[3] O. Somekh and S. Shamai, "Shannon-theoretic approach to a Gaussian cellular multiple-access channel with fading," *IEEE Transactions on Information Theory*, vol. 46, no. 4, pp. 1401–1425, July 2000.

[4] N. Letzepis and A. Grant, "Information capacity of multiple spot beam satellite channels," in *Proc. 6th Australian Communications Theory Workshop, 2005*, 2-4 Feb. 2005, pp. 168–174.

[5] P. Popescu and C. Rose, "Sum capacity and TSC bounds in collaborative multibase wireless systems," *IEEE Transactions on Information Theory*, vol. 50, no. 10, pp. 2433–2438, October 2004.

[6] W. Choi, A. J. G., and C. Yi, "The capacity of multicellular distributed antenna networks," in *International Conference on Wireless Networks, Communications and Mobile Computing*, 2005, pp. 1337–1342.

[7] H. J. Landau, "On Szego's eigenvalue distribution theorem and non-hermitian forms," *J. d'Analyse Math*, vol. 28, pp. pp. 335–357, 1975.

[8] U. Grenander and G. Szego, *Toeplitz Forms and their Applications*. Chelsea, New York, 1984.

[9] T. S. Rappaport, Ed., *Wireless Communications: Principles and Practice*. Upper Saddle River, NJ, USA: Prentice-Hall, Inc., 1995.