Liquidity uncertainty and intermediation
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Abstract

The paper performs a welfare comparison between demand deposit and equity contracts in the presence of intrinsic aggregate uncertainty. In this framework, the welfare dominance of deposit contracts emerges under corner preferences. It is shown that aggregate uncertainty creates high price volatility of ex-dividend equity claims traded in a secondary market and the resulting consumption allocations offer less risk-sharing opportunities to risk-averse consumers than tailor-made deposit contracts. The contingency of early payoffs on depositors’ withdrawal order reinforces the welfare performance of deposit contracts, whereas costly liquidation of productive long-term investments deteriorates their welfare performance relative to equity contracts.

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1. Introduction

Financial crises are an important reminder that fractional reserve banking systems are prone to liquidity problems that can lead to periods of financial instability. Recently, central banks and governments alike worldwide have followed both conventional and unconventional policies to enhance banks’ liquidity in order to restore public confidence. In performing the asset transformation function, depository intermediaries raise funds by issuing debt contracts which provide their holders the option to terminate the contract on demand at a pre-determined return, in order to finance high yielding yet illiquid long-term investments. Due to the liquidity mismatch in their balance sheet, depository intermediaries are prone to liquidity problems which could lead to insolvency. This has raised questions regarding the performance of demand deposit contracts in liquidity provision against alternative contractual forms.

It is well-established in the literature pioneered by Bryant (1980) and Diamond and Dybvig (1983) that demand deposit contracts improve on the competitive outcome by providing risk-sharing opportunities against consumers’ private consumption contingencies, but the resulting illiquidity of banks’ asset portfolio renders them vulnerable to runs. The present paper investigates the welfare performance of deposit contracts against the default-free equity contracts introduced by Jacklin (1987). Examining the characteristics of the two contractual arrangements in the presence of intrinsic aggregate uncertainty about the demand for liquidity, the paper identifies cases where this friction imposes tighter constraints in the design of optimal equity contracts which therefore can be dominated by a deposit banking structure.

The vulnerability of banks to default has attracted the attention of a strand of literature focusing on the design of deposit contracts that eliminate the bank run equilibrium when depositors are assumed to be sequentially served. An alternative run-proof contractual arrangement is developed by Jacklin (1987) where intermediaries are entirely financed by issuing equity rather than debt and permit the interim trading of equity claims. In a Diamond-Dybvig set-

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1See Gertler and Kiyotaki (2010).
2For a comprehensive literature survey on the effects of liquidity shortages, see Tirole (2011).
3Policies that prevent or mitigate the effects of bank runs have been widely discussed in the literature. For a review of the literature, see Gorton and Winton (2002).
ting where consumers have corner preferences such that they consume only once in their lifetime, tradable equity contracts are welfare dominant as they provide consumers with optimal risk-sharing opportunities against idiosyncratic consumption shocks without the possibility of default. However, when preferences are assumed to be smooth over time such that different types of consumers have different valuation of consumption at different dates, the restriction that characterises the design of equity contracts which imposes the same wealth to consumers prior to trade in the secondary market, results in a welfare loss in comparison to tailor-made deposit contracts’ allocations.\textsuperscript{5} Hence, the assumption of smooth preferences has been widely adopted to justify the evident prominence of depository intermediaries in liquidity provision relative to alternative arrangements.

The model developed in this paper highlights the role of intrinsic aggregate liquidity uncertainty in the design of optimal financial contracts. Contrary to existing literature that relies on the smooth preferences assumption to show the welfare dominance of deposit contracts, the model demonstrates that when the aggregate demand for liquidity is not ex ante known, deposit contracts can dominate in terms of welfare even under a more restrictive preference structure such as corner preferences since they offer more liquidity insurance to risk-averse consumers. This assumption on preferences is adopted to simplify the analysis. A more general preference structure will only reinforce the welfare dominance of deposit contracts in the presence of liquidity uncertainty.

In particular, it is assumed that the fraction of agents that want to consume early in their lifespan is random, so that intermediaries can only make conjectures about the actual realisation of the aggregate demand for liquidity from a known distribution when contracts are offered. Therefore, contracts are incomplete as their interim payoffs are independent of the prevailing state of the world and only second-best allocations are attainable. Specifically, deposit contracts are liquidated on demand, providing consumers with the option to receive a pre-determined interim payoff. In contrast, equity contracts provide a stream of payoffs and intermediaries are restricted ex ante to commit to a level of investment in the underlying technologies as specified by the contracts’ terms. Thus, equity contracts have a positive worth at the interim date that

facilitates trade among different types of consumers in an ex post secondary market. Market clearing ensures that the equilibrium market price reveals the state of the world so that interim consumption is contingent on the realised demand for liquidity. Depending on the prevailing state and the primitives of the model, two possible equilibrium configurations that can arise in the ex post secondary market are identified.

The findings suggest that when liquidation of investment in the productive technology is costless relative to storage, equity contracts could result in a greater welfare loss than deposit contracts because they offer less liquidity insurance in the state of the world that is mostly wanted by risk-averse consumers, and more liquidity insurance when it is least desirable. To facilitate the comparison with the literature that sees bank fragility emanating from extrinsic uncertainty when depositors are sequentially served, contingency of the deposit contracts’ interim payoff on depositors’ withdrawal order reinforces their welfare dominance. However, when investment in the productive technology is costly to liquidate, equity contracts can become welfare dominant as equilibrium consumption allocations are independent of the liquidation costs.

Although the paper focuses on the welfare analysis of financial intermediaries with different capital structure, the results can also be extended to welfare comparison between alternative channels of liquidity provision. The contractual arrangements considered in the paper can be interpreted as depository intermediaries with mutual ownership that raise funds by issuing deposit contracts, and non-depository intermediaries that issue tradable equity claims such as mutual funds. In contrast to the welfare evaluation in Qi (2003) who distinguishes between monitoring banks and non-monitoring mutual funds in the presence of borrowers’ moral hazard, the present paper does not impose any qualitative differences between these two types of intermediaries. Alternatively, it can be viewed that liquidity is supplied in the economy indirectly through depository intermediaries, or directly by trading of firms’ shares that have a pre-determined dividend policy and access to the productive investments in the economy. As

\[\text{Departing from the welfare comparison between the two contractual forms, Gorton and Pennacchi (1990) focus on intermediaries with mixed capital structure where both types of contracts are issued by the same intermediary (i.e. commercial banks). They demonstrate that, in equilibrium, informed consumers hold equity whereas uninformed consumers hold debt as deposit contracts provide a mechanism to protect the latter from being exploited in the secondary market by coalitions of the informed consumers.}\]
such, the results of the paper provide a welfare evaluation of consumption allocations attainable under different configurations of the financial system of an economy.\footnote{For more information, see Allen and Gale (1995, 2000).}

The rest of the paper is structured as follows. Section 2 describes the model and the benchmark case of full information. Section 3 analyses the optimal form of the demand deposit and equity contracts. The welfare comparison of these two contracts is described in Section 4, and conclusions are presented in Section 5. Proofs are provided in the Appendix.

2. The model

The banking environment in this model is based on Allen and Gale (2005) framework that introduces uncertainty about consumers’ demand for liquidity in the Diamond and Dybvig (1983) setup.

There is a single homogeneous commodity in the economy that can be used for consumption and investment, and three dates indexed by $t = 0, 1, 2$. There is a continuum of measure one of ex ante identical consumers born at date 0 with an endowment of one unit of the commodity, and nothing thereafter. Consumers receive a privately observed liquidity shock at date 1 and may become either impatient with probability $\pi \in (0, 1)$, or patient with probability $1 - \pi$. Consumers are assumed to have corner preferences such that impatient consumers derive utility only from the consumption of the commodity at date 1, whereas patient consumers only from consumption at date 2. Expected utility is given by

$$V(C_1, C_2; \pi) = \pi U(C_1) + (1 - \pi) U(C_2),$$

where $C_t$ denotes consumption at date $t = 1, 2$. The utility function $U(C_t)$ is twice continuously differentiable with $U''(C_t) < 0 < U'(C_t)$ and satisfies the Inada conditions.

Aggregate uncertainty is modelled by assuming that the preference shock $\pi$ is a random variable that takes two possible values $0 < \pi^L < \pi^H < 1$ with respective probabilities $q \in [0, 1]$ and $1 - q$. The distribution of the liquidity shock is common knowledge at date 0 and uncertainty is resolved after consumption decisions have been made at date 1. The liquidity shock is
independently and identically distributed across consumers so that, from the law of large numbers, \( \pi \) also represents the proportion of impatient consumers in the economy. Therefore, there is ex ante uncertainty about the aggregate demand for liquidity as the fraction of consumers who turn out to be either type is random.

There are two risk-free technologies available to all consumers in the economy. There is a storage technology with a return of 1 unit at date \( t + 1 \) for every unit of the commodity invested at date \( t = 0, 1 \), and a long-term technology with a certain return of \( R > 1 \) units for every unit of the commodity invested at date 0. If the long-term productive technology is interrupted at date 1, it yields a return equal to the return from storage.\(^8\)

As an alternative to the direct investment in the above technologies, consumers can use their endowment to participate in contracts offered by financial intermediaries which are assumed to have access to all the technologies described above. Free entry and competition among intermediaries force them to maximise consumers’ expected utility. Hence, without loss of generality, the analysis focuses on the contractual relationship between consumers and a representative financial intermediary. As the aggregate demand for liquidity is unknown when contracts are designed, the representative intermediary offers incomplete contracts that provide a payoff at date 1 that is independent of the state of the world, while date 2 payoffs exhaust its remaining resources.

The representative intermediary can offer either a demand deposit or an equity contract to the consumers at date 0 in return for their endowment,\(^9\) and is obliged to pay the amounts of the commodity specified in the contract. However, depository intermediaries that offer liquidity insurance to risk-averse consumers by issuing debt contracts are always subject to default when publicly observable, but not contractible, variables (sunspots) coordinate depositors’ actions to withdraw early. Focusing on the ex ante welfare performance of the two contracts in the presence of intrinsic uncertainty, the possibility of bank runs is ignored.

\(^8\)Note that the assumed weak dominance of the long-term investment’s return provides flexibility in the design of debt contracts. If early liquidation is costly relative to storage, this will impose additional restrictions in the optimal contract design and influence the welfare performance of the debt contract as discussed in the welfare analysis in Section 4.

\(^9\)This assumption removes the possibility of ex post arbitrage opportunities by trading in private markets so that risk-sharing allocations provided by deposit contracts are incentive compatible. For further discussion on the effects of side trades, see Haubrich and King (1990), Diamond (1997), Allen and Gale (2004b) and Farhi et al. (2009).
2.1. Full information

To facilitate the welfare comparison between the two contractual arrangements under incomplete information, the benchmark full-information case is examined first, where the only friction in the economy is the unobservability of consumers’ individual consumption preferences. Consider a social planner that invests consumers’ endowment in the underlying technologies on their behalf at date 0, and provides consumption allocations that maximise consumers’ expected utility. The social planner is assumed to observe the state of the world at date 1 and before any consumption decision is made. As such, payoffs are contingent on the state of the world and the social planner solves the following maximisation problem:

Problem 1.

\[
\max_{(C_1^S, C_2^S)} qV (C_1^H, C_2^H, \pi^H) + (1 - q) V (C_1^L, C_2^L, \pi^L) \tag{2}
\]

subject to the budget constraints:

\[
\pi^S C_1^S = X^S \quad \text{at } t = 1
\]

\[
(1 - \pi^S) C_2^S = R (1 - X^S) \quad \text{at } t = 2, \tag{3}
\]

where \( S = H, L \) is the state of the world.

As the return from early liquidation of the productive technology is equal to the return from storage, consumers’ endowment is fully invested long-term at date 0 and a proportion \( X^S \in (0, 1) \) is liquidated in order to meet the total demand for liquidity at date 1, while the remainder comes to maturity in the next period and finances consumption at date 2.

Let the payoffs \( C_1^{S^*} \) and \( C_2^{S^*} \) be the solutions to the social planner’s problem that satisfy the budget constraints and the first-order condition for each state

\[
U'(C_1^{S^*}) = RU'(C_2^{S^*}). \tag{4}
\]

Similar to relevant literature, the coefficient of relative risk aversion is assumed to be greater than 1. This convention is adopted for simplification. In general, optimality requires that the proportion of consumers’ endowment kept in storage does not exceed the aggregate demand for liquidity at date 1 for either state. This is discussed in Section 4 where the welfare effects of costly liquidation of long-term investment are examined.
than one as risk-averse consumers seek insurance against the event of becoming impatient and forego the higher return of the productive technology. This assumption implies that any feasible allocation which transfers consumption from date 2 to date 1 in relation to the autarkic outcome leads to a Pareto-improvement in welfare.\textsuperscript{11} As such, \(1 < C_1^{S^*} < C_2^{S^*} < R\) and consumers self-select the payoff that is designed for their consumption profile. In particular, the social optimum payoffs across the two states are related in the following way:

\[
1 < C_1^{H^*} < C_1^{L^*} \quad \text{at } t = 1 \\
C_2^{H^*} < C_2^{L^*} < R \quad \text{at } t = 2.
\]

Note that \(X^{L^*} < X^{H^*}\) for the optimal risk-sharing allocation to be attained as a larger proportion of the commodity needs to be liquidated to meet a greater demand for liquidity.

3. **Intermediation under incomplete information**

When the financial system is characterised by incomplete information, the state of the world remains unknown prior to early withdrawals. Consequently, consumers are offered incomplete contracts in the sense that date 1 payoffs are non-contingent on the realisation of \(\pi\). The objective function of the welfare-maximising intermediary is identical to the one in the full-information case, but depending on the contractual arrangement in question, different feasibility and incentive constraints need to be introduced as individual consumption preferences remain private information.

3.1. **Demand deposit contract**

Suppose that in return for consumers’ endowment at date 0, a bank is offering a demand deposit contract that has the form \(\{D_1; D_2^S\}\) and provides consumers with the option to withdraw a specified amount of the homogenous commodity at date 1 or date 2 denoted as \(D_1\) and \(D_2^S\), respectively. The maximisation problem that the representative intermediary solves is:

\textsuperscript{11}This assumption guarantees that \(CU''(C)\) is decreasing in \(C\), and therefore any feasible allocation such that \(1 < C_1\) and \(C_2 < R\) can attain a higher level of consumers’ expected utility than autarky.
Problem 2.

\[
\max_{\{D_1, D_2^S\}} qV(D_1, D_2^H; \pi^H) + (1 - q) V(D_1, D_2^L; \pi^L)
\]

subject to the budget constraints:

\[
\pi^S D_1 = x^S \quad \text{at } t = 1
\]
\[
(1 - \pi^S) D_2^S = R (1 - x^S) \quad \text{at } t = 2,
\]

and the incentive compatibility constraint

\[
U(D_1) \leq qU(D_2^H) + (1 - q)U(D_2^L).
\]

By committing at date 0 to a fixed date 1 payoff, depending on the realisation of the state, let \(x^S \in [0, 1]\) be the proportion of the investment in the long-term technology that is liquidated in order to meet the total demand for early withdrawals, while the rest remains invested until date 2 and is used to finance late withdrawals. The incentive compatibility constraint given in equation (6) ensures that patient consumers will not misrepresent their type and withdraw early since the utility that they derive from storing for one period and consuming \(D_1\) at date 2, does not exceed the expected utility they derive from withdrawing and consuming the payoff that is designed for their type.

Let \(D_1^*\) and \(D_2^S^*\) be the optimal payoffs that satisfy the constraints in the above maximisation problem\(^\text{12}\) and the first-order condition

\[
\frac{(q\pi^H + (1 - q)\pi^L) U'(D_1)}{q\pi^H U''(D_2^H) + (1 - q)\pi^L U''(D_2^L)} = R.
\]

The following property summarises the comparative static effects of \(q\).

**Property 1.** \(D_1^*\) is strictly decreasing in \(q\), whereas \(D_2^H^*\) and \(D_2^L^*\) are strictly increasing in \(q\).

The depositors’ expected utility is strictly decreasing and convex in \(q\).

\(^{12}\text{Clearly, the incentive constraint is satisfied when } D_1^* < D_2^H^* \text{ but this relationship depends on the functional form of the utility and the parameters of the model. For simplicity, focusing on incentive-efficient allocations, it is sufficient that the incentive constraint is monotonic in } q \text{ when evaluated at the optimal payoffs. Since the constraint is not binding when } q = 0 \text{ and } q = 1, \text{ this assumption guarantees that the constraint is satisfied with strict inequality for any } q.\)
Proof. See Appendix

Note that when the state of the world is known with certainty (i.e. \( q = 0 \) or \( q = 1 \)), equation (7) becomes identical to the first-order condition in the social planner’s case. Therefore, from the relationship between the social optimum and autarky allocations, the above property implies that \( D_1^* > 1 \) and \( D_2^H < D_2^L < R \).

3.2. Equity contract

An intermediary offers an equity contract to consumers at date 0 in return for their endowment which is invested in the underlying technologies. As a convention, it is implicitly assumed that the price of each contract is equal to one and also that contracts are infinitely divisible. The equity contract gives consumers the right to receive two payments and has the form \( \{ \delta_1, \delta_2 \} \), where \( \delta_1 = \delta \in (0, 1) \) and \( \delta_2 = R(1 - \delta) \) denote the dividend and the liquidating dividend payments that consumers receive at date 1 and date 2, respectively.

A secondary market opens at date 1 that allows trade of ex-dividend claims to take place. Having realised their individual consumption preferences, consumers have incentives to participate in the market as they are entitled to receive an additional payment at the date that they do not value consumption. Market forces determine the equilibrium market price which is, therefore, dependent on the prevailing state of the world. Consequently, the final consumption allocations attainable by the equity contract do not only depend on the terms of the contract, but also on the market-clearing price in the secondary market, which in turn depends on the realisation of \( \pi \). Anticipating the equilibrium market price for each state of the world, the intermediary selects \( \delta \) to maximise consumers’ welfare.

In an attempt to provide a full description of the market forces that determine the attainable allocations under an equity contract, consumers’ incentives to trade in the secondary market are examined first. The consumption of impatient and patient consumers, denoted as \( C_{1E}^S \) and \( C_{2E}^S \) respectively, will be:

\[
\begin{align*}
C_{1E}^S &= \delta + p^S \\
C_{2E}^S &= (1 + \delta/p^S) R(1 - \delta).
\end{align*}
\]
Impatient consumers are always willing to sell their ex-dividend equity contract at a price 
$p^S > 0$ since they can obtain additional utility of consumption at date 1. Therefore, the supply 
of ex-dividend equity in the secondary market is perfectly inelastic and equal to the number of 
impatient consumers, or $Q_S = \pi^S$. In contrast, the demand for ex-dividend equity derives from 
patient consumers who can use $\delta$ to buy additional $\delta/p^S$ equity contracts when this provides 
them with consumption at date 2 at least equal to the consumption that they could otherwise 
achieve if they do not participate in the secondary market. Therefore, the demand for ex-
dividend equity is given by 

$$Q_D = \begin{cases} 
(1 - \pi^S)\delta/p^S & \text{for } p^S \leq R(1 - \delta) \\
0 & \text{for } p^S > R(1 - \delta).
\end{cases}$$ 

That is, patient consumers are willing to buy additional ex-dividend equity contracts at date 1 
only if the price they have to pay for each contract does not exceed its discounted return.

Trade in the secondary market determines the equilibrium market price $p^{S*}$, given by 

$$p^{S*} = \begin{cases} 
(1 - \pi^S)\delta/\pi^S & \text{for } \delta \leq \tilde{\delta}_S \\
R(1 - \delta) & \text{for } \delta > \tilde{\delta}_S,
\end{cases}$$ 

(9)

where $\tilde{\delta}_S = \pi^S R/ (\pi^S R + 1 - \pi^S)$ (so that $\tilde{\delta}_L < \tilde{\delta}_H$ as $\pi^L < \pi^H$) denotes the threshold value 
of $\delta$ for which the liquidating dividend payment is equal to the market-clearing price. Thus, 
$p^{S*}$ depends on $\delta$ and on the parameters of the model. When $\delta \leq \tilde{\delta}_S$, the market-clearing price 
is equal to the ratio of the supply of the commodity by patient consumers to the supply of ex-
dividend equity by impatient consumers. When $\tilde{\delta}_S < \delta$, the market-clearing price reaches its 
ceiling value and is equal to the liquidating dividend.

The two possible equilibrium configurations that can arise in the secondary market are 
represented in Figure 1 where the quantity and price of the equity contracts traded are measured 
on the horizontal and vertical axis, respectively. The supply of equity is perfectly inelastic at $\pi^S$, 
while the demand is initially horizontal at the price for which patient consumers are indifferent 
to trade, up to the point where, given $\delta$ chosen by the intermediary, there are gains from trade
and the demand becomes strictly decreasing and convex thereafter. If the market equilibrium is located on the convex segment of the demand such as point A, there is a large number of impatient consumers selling their ex-dividend claims and liquidity is therefore limited. This puts a downward pressure on price and patient consumers receive a positive surplus as the cost of buying additional ex-dividend equity is less than the discounted return of this investment.\footnote{In this case, the market-clearing price depends on the size of the liquidity shock. A similar relationship between liquidity and asset prices, referred to as “cash-in-the-market pricing”, is identified by Allen and Gale (2005) in examining how price fluctuations of illiquid assets traded in an interbank market exacerbate liquidity shortfalls in the banking system.}

However, if the equilibrium is located on the horizontal segment of the demand, then liquidity is plentiful and patient consumers do not derive any additional benefit from trading as the cost of this investment opportunity is equal to its discounted reward.

Substituting for $p^{S^*}$ into the consumption of the two types of consumers, the latter becomes

\[
C_{1E}^S = \begin{cases} 
\frac{\delta}{\pi^S} & \text{for } \delta < \tilde{\delta}_S \\
\delta + R(1 - \delta) & \text{for } \delta \geq \tilde{\delta}_S
\end{cases}
\quad \text{and } C_{2E}^S = \begin{cases} 
\frac{R(1 - \delta)}{1 - \pi^S} & \text{for } \delta < \tilde{\delta}_S \\
\delta + R(1 - \delta) & \text{for } \delta \geq \tilde{\delta}_S.
\end{cases}
\quad (10)
\]

Anticipating $p^{S^*}$, the intermediary selects the dividend payment $\delta^*$ that maximises consumers’ expected utility. Let $C_{1E}^{S^*}$ and $C_{2E}^{S^*}$ denote the equilibrium consumption of impatient and patient consumers, respectively. Note from equation (10) that in a given state, $C_{1E}^{S^*} < C_{2E}^{S^*}$ for $\delta^* < \tilde{\delta}_S$ and $C_{1E}^{S^*} = C_{2E}^{S^*}$ for $\delta^* \geq \tilde{\delta}_S$.

From these two plausible scenarios that may occur in the secondary market and the two states of the world, the following lemma indicates that there are only two different regions where $\delta^*$ can lie, and therefore there are two possible configurations of the secondary market that can arise in equilibrium.

**Lemma 1.** Depending on the specifications of the model, the dividend payment chosen by a welfare-maximising intermediary will be either $\delta^* < \tilde{\delta}_L$, or $\tilde{\delta}_L \leq \delta^* < \tilde{\delta}_H$.

**Proof.** See Appendix

Maximisation of consumers’ expected utility yields the first-order condition

\[
\frac{qU'(C_{1E}^{H}) + (1 - q)U'(C_{1E}^{L})}{qU'(C_{2E}^{H}) + (1 - q)U'(C_{2E}^{L})} = R,
\]

\(11\)
where \( C_{1E}^{L^*} < C_{2E}^{L^*} \) for \( \delta^* < \tilde{\delta}_L \), and \( C_{1E}^{L^*} = C_{2E}^{L^*} \) for \( \tilde{\delta}_L \leq \delta^* < \tilde{\delta}_H \).

The relationship between consumers’ equilibrium consumption across the two states is such that
\[
C_{1E}^{H^*} < C_{1E}^{L^*} \leq C_{2E}^{L^*} < C_{2E}^{H^*} \tag{12}
\]
and indicates that the equity contract offers more liquidity insurance to risk-averse consumers in the low state as the dispersion between the equilibrium payoffs for the two types of consumers is greater in the high state. This is due to the negative effect of \( \pi^S \) on \( p^S^* \) which influences the equilibrium consumption allocations. A high number of impatient consumers implies that a greater quantity of ex-dividend equity contracts is supplied in the secondary market which results in a low equilibrium market price. The consumption of impatient consumers falls since they are forced to sell their equity claim at a low price, and the consumption of patient consumers increases as they can buy a greater number of equity contracts using their dividend payment to finance their consumption at date 2. In terms of Figure 1, an increase in the number of impatient consumers can be represented by a shift of the convex segment of the demand to the left and a rightward shift of the inelastic supply of equity contracts, resulting in a lower market-clearing price.

The following comparative static property with respect to \( q \) provides a greater insight on the welfare performance of the equity contract.

**Property 2.** For \( \delta^* < \tilde{\delta}_L \), \( C_{1E}^{S^*} \) is strictly increasing in \( q \) and \( C_{2E}^{S^*} \) is strictly decreasing in \( q \). For \( \tilde{\delta}_L \leq \delta^* < \tilde{\delta}_H \), \( C_{1E}^{L^*} \) becomes strictly decreasing in \( q \) (where \( C_{1E}^{L^*} = C_{2E}^{L^*} \)). The consumers’ expected utility is strictly decreasing and convex in \( q \), for any \( \delta^* \in [X^{L^*}, X^{H^*}] \).

**Proof.** See Appendix

Intuition behind the above property is provided in terms of \( \delta^* \) by noticing that \( \delta^* \) increases with \( q \). For \( \delta^* < \tilde{\delta}_L \), an increase in \( \delta^* \) has a positive direct effect on \( C_{1E}^{S^*} \), and a positive indirect effect through the resulting increase in the demand for ex-dividend claims which puts an upward pressure on the market-clearing price. Due to the feasibility constraints and the higher market-clearing price, both the direct and indirect effects on \( C_{2E}^{S^*} \) are negative. However, for \( \tilde{\delta}_L \leq \delta^* < \tilde{\delta}_H \), the positive direct effect of \( \delta^* \) on \( C_{1E}^{L^*} \) is outweighed by the negative indirect
effect on the market-clearing price. When the demand is perfectly elastic, a rise in $\delta^*$ results in a lower return that patient consumers receive from purchasing additional equity claims which puts a downward pressure on the equilibrium market price for patient consumers to participate in the secondary market.

A unique threshold value of $q$ can be identified from property 2, namely $\tilde{q}$, for which the optimal configuration of the secondary market changes in the low state. Since $\tilde{q}$ is the threshold value for which $\delta^*(\tilde{q}) = \tilde{\delta}_L$ and $\tilde{\delta}_L$ is independent of $q$, from the first-order condition it follows that

$$\tilde{q} = \frac{U'(C_{1E}^L(\tilde{\delta}_L))(R - 1)}{U'(C_{1E}^H(\tilde{\delta}_L)) - U'(C_{2E}^H(\tilde{\delta}_L)) + U'(C_{1E}^L(\tilde{\delta}_L))(R - 1)},$$

where $C_{1E}^L(\tilde{\delta}_L) = C_{2E}^L(\tilde{\delta}_L)$. Note that $\tilde{q} \in (0, 1]$ since $C_{1E}^H(\tilde{\delta}_L) \leq C_{1E}^H$ and $C_{2E}^H \leq C_{2E}^H(\tilde{\delta}_L)$ as is discussed in Section 4.

For $q \in [0, \tilde{q})$ such that $C_{1}^{S_S} < C_{2}^{S_S}$, property 2 suggests that the equity contract provides more liquidity insurance in both states for higher values of $q$. However, as the intermediary has to commit to a fixed dividend payment at date 0, this results in more liquidity insurance than what is socially optimal in the low state. Indeed, for $\tilde{q} \leq q$, the contract offers full insurance against the risk of being impatient in the low state.

4. Welfare evaluation

The welfare analysis focuses on the restrictions that characterise each contract design as in either case the intermediary maximises the same objective function but subject to different constraints. From the sequential budget constraints, simplifying for the proportion of the commodity liquidated at date 1 (i.e. $X^S$, $x^S$ or $d$), all the feasible allocations satisfy the intertemporal budget constraint

$$(1 - \pi^S)C_2^S \leq R(1 - \pi^S C_1^S).$$

The standard budget line-indifference curve approach provides a valuable insight on the welfare performance of the two contractual arrangements.

Consider firstly the case where $q \in [0, \tilde{q})$ as illustrated in Figure 2. The budget lines for the
two states of the world cross at the autarky allocation \((1, R)\). Note that in order to simplify the diagrammatic analysis, a homothetic utility function is considered such that income expansion paths are represented as rays from the origin.\(^\text{14}\) The social optimum allocations are located at the intersections between the two budget lines and the ray from the origin \(SO\) which captures the fixed proportionality of the marginal utilities between the two types of consumers across the two different states. For \(q = 0\), the income expansion paths for the equity and deposit contract coincide with the \(SO\) for the low state, and for the high state are represented by the rays \(EC_0\) and \(DC_0\), respectively. Therefore, welfare dominance for values of \(q\) in the region around zero, depends on which contract’s allocation lies on a higher indifference curve in the high state. As \(q\) increases, properties 1 and 2 suggest that the income expansion paths of the equity contract are rotating downwards, whereas the income expansion paths of the deposit contract rotate upwards. In particular, when \(q = \tilde{q}\), an additional condition is introduced in the design of the optimal equity contract such that \(C_{1E}^L = C_{2E}^L\). In terms of Figure 2, the income expansion path of the equity contract in the low state is the 45 degrees line and the optimal consumption allocation is now determined by the intersection of the 45 degrees line with the corresponding budget constraint, represented by point \(F_L\).

For higher values of \(q\) such that \(\tilde{q} < q \leq 1\), the allocation of the equity contract in the low state does not satisfy the intertemporal budget constraint described in equation (13) with equality. As \(C_{1E}^L\) is decreasing in \(q\) from property 2, higher values of \(q\) correspond to a movement along the 45 degrees line in Figure 2 which leads to inferior allocations in the low state inside the budget set. Finally, for \(q = 1\), both contracts attain the social optimum allocation for the high state as illustrated in Figure 3. The income expansion paths of both contracts coincide with that of the social optimum at \(SO\), whereas for the low state they are represented by the 45 degrees line and the ray from the origin \(DC_1\) for the equity and the deposit contract, respectively. The equilibrium allocation of the equity contract lies on the bold segment of the 45 degrees line between points \(F_L\) and \(F_H\).\(^\text{15}\)

\(^{14}\)This specification of the utility function is utilised only for illustrative purposes as the results of the model hold for any utility function that satisfies the standard neoclassical properties with a coefficient of relative risk aversion greater than one.

\(^{15}\)Note that Figure 3 is drawn such that \(C_{2H}^H\) exceeds the full-insurance payoffs in the low state indicated by the point \(F_L\). In the case where \(C_{2H}^H\) is lower that the full-insurance payoffs in the low state, the allocation that the equity contract can attain in the low state lies on the segment of the 45 degrees line above point \(F_H\) but below
Evaluating the welfare performance of the two contracts, the following Proposition summarises the main result of the paper.

**Proposition 1.** When \( q^* \in (0, 1) \), the demand deposit contract ex ante dominates the equity contract in terms of welfare for any \( q \in (q^*, 1) \), otherwise the deposit contract is welfare optimal for any \( q \in (0, 1) \).

**Proof.** See Appendix

From properties 1 and 2, the proof of the above statement focuses on the comparison of the slope of the optimal value function for each contractual arrangement at the limit cases where the state of the world is known with certainty and is depicted in Figure 4 where the horizontal and vertical axis measure \( q \) and consumers’ expected utility, respectively. It is shown that the expected utility of the deposit contract \( EV^*_D \) is steeper than that of the equity contract \( EV^*_E \) when evaluated at \( q = 1 \). In terms of Figure 3, this implies that the allocation attained by the deposit contract lies on a higher indifference curve than that of the equity contract for \( q = 1 \). However, the slope of \( EV^*_D \) relative to \( EV^*_E \) when evaluated at \( q = 0 \) depends on the parameters of the model and on the functional form of utility. Numerical examples provided in the Appendix show that both possibilities may arise. When \( EV^*_D \) is flatter than \( EV^*_E \) at \( q = 0 \), the deposit remains the dominant contract for any \( q \in (0, 1) \), where \( EV^*_D \) is represented by the dashed line and \( EV^*_E \) by the bold line. In contrast, when \( EV^*_D \) is steeper relative to \( EV^*_E \) at \( q = 0 \), a threshold value \( q^* \in (0, 1) \) exists for which \( EV^*_D \) and \( EV^*_E \) cross, so that the deposit contract dominates for any \( q \in (q^*, 1) \) as it is represented by the solid line.

Further intuition on welfare dominance can be provided by examining the restrictions that each contract imposes on the equilibrium allocations in relation to the benchmark allocations under full information. In contrast to the social planner, a depository intermediary is constrained to provide a fixed payoff to impatient depositors while the level of liquidation of the initial investment in the long-term technology is contingent on the prevailing state to meet the aggregate demand for liquidity. Imposing on the social planner’s budget constraints that date 1

\[
C^H_2 \text{ since } \pi C^H_1 + (1 - \pi) C^H_2 < C^H_2.
\]

\(^{16}\)In order to ensure that the optimal payoffs of the deposit contract are positive when evaluated at \( q = 0 \), the model’s specifications should satisfy \( C^L_1 < 1/\pi^H \).

\(^{17}\)The relationship between the threshold values \( \tilde{q} \) and \( q^* \) depends on the parameter values and the utility function.
payoffs are state-independent yields $x^{L^*} < X^{L^*} < X^{H^*} < x^{H^*}$. The relationship between the optimal payoffs for any $q \in (0, 1)$ is therefore:

$$\begin{align*}
C_1^{H^*} &< D_1^* < C_1^{L^*} \quad \text{at } t=1 \\
D_2^{H^*} &< C_2^{H^*} < C_2^{L^*} < D_2^{L^*} \quad \text{at } t=2.
\end{align*}$$

(14)

Although the contract eliminates the risk that impatient depositors face due to the uncertainty about the aggregate demand for liquidity, this risk is borne by patient depositors as the fixed date 1 payoff results in a higher dispersion between date 2 payoffs relative to the social optimum payoffs. In terms of the liquidity risk which can be captured by the dispersion between the payoffs designed for each type of consumer for a given state, from the relationships described in equation (14) it follows that the contract offers more risk-sharing in the high state than what is socially desirable as $D_2^{H^*} - D_1^* < C_2^{H^*} - C_1^{H^*}$ and less risk-sharing in the low state as $C_2^{L^*} - C_1^{L^*} < D_2^{L^*} - D_1^*$.

In contrast, an equity contract provides individual consumers with two payments that are independent of the state of the world; the dividend and the liquidated dividend available at date 1 and date 2, respectively. Contrary to the social planner, the intermediary must commit to a fixed level of investment in order to meet its contractual obligations. Imposing the restriction on the social planner’s budget constraints that the amount of the investment liquidated at date 1 is independent of the state yields $X^{L^*} \leq \delta^* \leq X^{H^*}$. The relationship between the optimal consumption across states and for any $q \in (0, 1)$ is therefore:

$$\begin{align*}
C_1^{H^*} < C_1^{H^*} < C_1^{L^*} < C_1^{L^*} \quad \text{at } t=1 \\
(C_1^{L^*} \leq) C_2^{L^*} < C_2^{L^*} \quad \text{and } C_2^{H^*} < C_2^{H^*} \quad \text{at } t=2.
\end{align*}$$

(15)

Depending on the realisation of the state, trade in the secondary market for ex-dividend equity determines the equilibrium consumption allocations. As indicated in equation (15), market forces in the secondary market create a high dispersion between the equilibrium con-

\footnote{18}This relationship holds even when the utility function and the parameters of the model are such that $D_2^{H^*} < D_1^*$.

\footnote{19}In the graphical representations in Figures 2 and 3 for a homothetic utility function, the risk-sharing performance of the deposit contract is illustrated by the steepness of the income expansion paths relative to the social optimum income expansion path where $D_2^{H^*} / D_1^* < C_2^{H^*} / C_1^{H^*}$ and $C_2^{L^*} / C_1^{L^*} < D_2^{L^*} / D_1^*$. 

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consumption for each type of consumer across states in relation to the corresponding social optimum consumption. In terms of liquidity risk, the equity contract offers less risk-sharing in the high state and more risk-sharing in the low state than the social optimum allocations as

\[ C^H - C^H < C^H - C^H \text{ and } C^L - C^L > C^L - C^L. \]

Comparing the two contracts in terms of liquidity insurance, it is apparent that providing consumers with state-independent consumption at date 1 is less restrictive than committing to an investment policy when uncertainty is not resolved before payments are made. The ex ante welfare dominance of the deposit contract arises from the fact that it offers more liquidity insurance in the ‘bad’ state of the world (i.e. high state) than the equity contract since \( D^H - D^H < C^H - C^H < C^H - C^H, \) which is more valuable ex ante to risk-averse consumers. However, the equity contract offers more liquidity insurance in the ‘good’ state of the world (i.e. low state) than the deposit contract since \( C^L - C^L > C^L - C^L, \) which is ex ante less valuable to risk-averse consumers. According to Proposition 1, this implies that if the deposit is not already the optimal contract, it becomes the optimal one when the state of the world is more likely to be high.

4.1. Sequential service constraint

Provision of liquidity insurance coupled with illiquid productive investments creates a liquidity mismatch between banks’ liabilities and assets that explains their vulnerability to default. Extrinsic uncertainty can result in an abnormal rise in early withdrawals and lead to liquidity problems due to the fixed nature of banks’ liabilities. A common assumption in the relevant literature that precludes the design of deposit contracts with contingent date 1 payoffs on the total withdrawal demand is that intermediaries face a sequential service constraint such that depositors are served on a first-come, first-served basis. Consequently, date 1 payoffs cannot be contingent on the actual cumulative withdrawal requests and therefore are independent of the order of depositors’ early withdrawals.\(^{21}\)

While maintaining the view of banks’ vulnerability to default arising from exogenous

\(^{20}\)Equivalently, in Figures 2 and 3, the steepness of equity contract’s income expansion paths for each state relative to the social optimum expansion path is such that \( C^H / C^H < C^H / C^H \) and \( C^L / C^L > C^L / C^L. \)

\(^{21}\)An extensive discussion on the role of the sequential service constraint to incomplete deposit contracts and banks’ subjectability to default is provided by Ennis and Keister (2010).
shocks, in the presence of aggregate liquidity uncertainty with a known prior distribution, the state of the world can be inferred when a sequential service constraint is in place and this information can be incorporated in the design of the deposit contract. When depositors are served sequentially, the intermediary observes the state of the world after π^L depositors are served as any further early withdrawals will indicate a high demand for liquidity. Thus, a contract can be designed such that date 1 payoffs are contingent on the random order in which depositors withdraw. According to Wallace (1988, 1990), such contingency improves the welfare performance of deposit contracts in the presence of aggregate uncertainty.

A representative bank offering such contingent deposit contract of the form \( \{D_{c1}^s, D_{c2}^s\} \) solves the following maximisation problem:

**Problem 3.**

\[
\max_{\{D_{c1}^s, D_{c2}^s\}} q \left[ \pi^L U(D_{c1}^L) + (\pi^H - \pi^L)U(D_{c1}^H) + (1 - \pi^H)U(D_{c2}^H) \right] + (1 - q)V(D_{c1}^L, D_{c2}^L; \pi^L)
\]

subject to the budget constraints:

\[
\begin{align*}
\pi^L D_{c1}^L &= x^L, \text{ for } S = L \quad \text{at } t = 1 \\
\pi^L D_{c1}^L + (\pi^H - \pi^L) D_{c1}^H &= x^H, \text{ for } S = H \\
(1 - \pi^S) D_{c2}^S &= R(1 - x^S) \quad \text{at } t = 2
\end{align*}
\]  

(16)

and the incentive compatibility constraint

\[
q \left( \pi^L U(D_{c1}^L) + (\pi^H - \pi^L)U(D_{c1}^H) \right) + (1 - q)U(D_{c1}^L) \leq qU(D_{c2}^H) + (1 - q)U(D_{c2}^L).
\]  

(17)

The contingent contract provides a payoff \( D_{c1}^L \) to the first \( \pi^L \) depositors who withdraw at date 1 and \( D_{c1}^H \) to the remaining \( \pi^H - \pi^L \) depositors when the state of the world turns out to be high. According to the budget constraints in equation (16), a sufficient amount of the commodity is liquidated to meet the demand for early withdrawals, while the remainder is used to finance late withdrawals. The incentive compatibility constraint in equation (17) ensures that patient depositors’ expected utility from withdrawing at date 1, storing their proceeds for one period and consuming at date 2, does not exceed the expected utility that they derive by waiting.
Let $D_{c1}^{S^*}$ and $D_{c2}^{S^*}$ denote the payoffs that solve the bank’s maximisation problem and satisfy the budget constraints and the first-order conditions

$$U'(D_{c1}^H) = RU'(D_{c2}^H),$$
$$U'(D_{c1}^L) = R \left( qU'(D_{c2}^H) + (1 - q)U'(D_{c2}^L) \right).$$

(18)

To facilitate the welfare comparison with the standard (non-contingent) deposit contract, the following property indicates that the optimal contract is incentive compatible and provides the comparative static effect of $q$.

**Property 3.** *The optimal contingent deposit contract is incentive compatible. $D_{c1}^L$ is strictly decreasing in $q$, whereas $D_{c2}^L$, $D_{c1}^H$ and $D_{c2}^H$ are strictly increasing in $q$. Depositors’ expected utility is strictly decreasing and convex in $q$.***

**Proof.** See Appendix

Note that when the state of the world is known with certainty, the contingent deposit contract attains the social optimum allocation. Thus, from the above property it follows that for any $q \in (0, 1)$:

$$D_{c1}^H < C_1^H < D_{c1}^L < C_1^L \quad \text{at } t=1$$
$$D_{c2}^H < C_2^H < D_{c2}^L < C_2^L \quad \text{at } t=2.$$
The above Proposition reproduces Wallace (1988, 1990)’s result that the contingency of date 1 payoffs on the order with which withdrawals are made improves the welfare performance of the deposit contract. This is because, contrary to the non-contingent contract, impatient depositors share some of the uncertainty about the future state of the world with impatient depositors, which results in a welfare improvement for the risk-averse consumers. As such, a sequential service constraint enhances the welfare dominance of the deposit over equity contracts.

4.2. Costly liquidation of productive technology

The assumed underlying technologies have an important role to play in the determination of the optimal contract. The weak dominance of the productive long-term technology’s return over storage makes investment decisions at date 0 trivial with regard to the uncertainty about consumers’ demand for liquidity. Costless liquidation provides flexibility in the design of the deposit contract as banks’ liquidity can be adjusted depending on the realisation of $\pi$. However, if liquidation is costly relative to storage, ex ante investment decisions impose additional restrictions on the feasible allocations by a deposit contract, and consequently erodes its welfare performance. In contrast, the welfare performance of the equity contract remains unaffected because investment decisions are independent of the state of the world.

Specifically, let $\tau \in [0, 1)$ be the return from early liquidation. Since $\tau < 1$ and $R > 1$, it is ex ante optimal for the depository intermediary to invest a proportion $y$ of the consumers’ endowment in storage to finance early withdrawals such that $\pi^L D_1 \leq y \leq \pi^H D_1$ and no additional investment in storage takes place at date 1. Hence, the following budget constraints should be satisfied:

\[
\begin{align*}
\pi^L D_1 & \leq y & \text{at } t = 1, \\
\pi^H D_1 & = y + x^H \tau \\
(1 - \pi^L) D_2^L & = R(1 - y) + y - \pi^L D_1 & \text{at } t = 2, \\
(1 - \pi^H) D_2^H & = R(1 - y - x^H).
\end{align*}
\]

Any excess amount of the commodity held at date 1 is reinvested for an additional period while patient depositors receive the residue of investments at date 2. In contrast, an intermediary offering an equity contract can satisfy its contractual obligations in each state without hold-
ing excess liquidity and therefore can attain a higher level of welfare for low values of \( \tau \) as described in the Proposition below.

**Proposition 3.** The welfare performance of the deposit contract deteriorates as \( \tau \) decreases relative to the equity contract, where the latter is the dominant contract for \( \tau = 0 \).

Clearly, the lower the return from liquidation, the greater the amount of commodity that has to be invested in storage to meet early withdrawals, which in turn results in a fall of depositors’ expected utility.\(^{22}\) In particular, if investment in the productive technology is irreversible as in Allen and Gale (2004a, 2005), an equity contract can attain superior allocations. Indeed, feasible allocations in the high state satisfy the same intertemporal budget constraint for either contract. In the low state, however, feasible allocations of the deposit contract satisfy the intertemporal budget constraint

\[
(1 - \pi^L)D^L_2 = R(1 - \pi^H D_1) + (\pi^H - \pi^L)D_1,
\]

which lies within the feasible set of allocations attainable by an equity contract for any \( q \).\(^{23}\) Thus, consumers’ expected utility is maximised subject to tighter constraints and therefore the equity contract is welfare dominant when \( \tau = 0 \). The monotonicity of consumers’ expected utility in terms of \( \tau \) implies that equity becomes the welfare dominant contract when \( \tau \) is low.

The intuition behind this result lies in the characteristics of the two contracts. Contrary to the deposit contract, by providing another payoff at date 2, the ex-dividend equity contract has a positive worth at date 1 as reflected by \( p' \). Trade in the secondary market creates another ‘liquid’ asset between dates 1 and 2 that makes consumption allocations contingent on the state of the world and independent of the return from liquidation.

Finally, it is important to highlight that the introduction of an interbank market that facilitates trade of claims in the irreversible productive technology at date 1 does not improve the

\(^{22}\)Analytically, applying the Envelope theorem when a welfare maximising depository intermediary is facing the budget constraints given in equation (19) yields

\[
\frac{\partial EV(V^*(\tau))}{\partial \tau} = qU'(D^H_2(\tau))R(\pi^H D^*_1(\tau) - y)/\tau > 0
\]

so that depositors’ expected utility is increasing in \( \tau \).

\(^{23}\)Contrary to the budget lines for the two states described in equation (13) that cross each other at \((1, R)\) as illustrated in Figures 2 and 3, the budget line described by (20) crosses that of the high state at the 45 degrees line. Hence, feasible allocations described by (20) also lie within the feasible set of allocations that can be attained in the high state and consumers’ expected utility maximisation problem indicates that \( D^L_2 < D^H_2 \).
welfare performance of the deposit contract in this framework. In the presence of intrinsic aggregate uncertainty, Allen and Gale (2004a) show that when the liquidity shock does not involve an idiosyncratic component which can be diversified through trading in the interbank asset market, welfare-maximising banks remain autarkic.

5. Conclusions

The aim of the paper is to highlight the role of intrinsic aggregate uncertainty in the design of the optimal financial contracts by intermediaries. Two contractual arrangements are compared in terms of ex ante welfare optimality; non-tradeable deposit contracts that can be liquidated on demand, and equity contracts with a pre-determined stream of payments that can be traded in an ex post secondary market. When uncertainty about the aggregate demand for liquidity is not resolved at the time period financial contracts are designed and liquidation of the productive technology is costless relative to storage, the results of the paper suggest that deposit contracts can outperform equity contracts as social welfare is maximised subject to less restrictive constraints. Specifically, it is shown that commitment to a pre-determined level of investment in underlying technologies results in a greater welfare loss relative to the full information case than restricting date 1 consumption to be independent of the state of the world. Liquidity uncertainty creates volatility on the equilibrium market price for ex-dividend equity claims such that the equilibrium consumption allocations provide less liquidity insurance to risk-averse consumers than deposit contracts. When a sequential service constraint is assumed to be in place, date 1 payoffs become contingent on the order with which depositors are served, improving the welfare performance of deposit contracts as aggregate risk is shared between the two types of depositors.

The welfare analysis performed in this model utilises the standard neoclassical properties of the utility function that represents consumers’ preferences for consumption at each date and therefore the findings of the paper are independent of its functional form. Moreover, although it is assumed that agents consume only once in their lifetime, the results obtained in this model do not depend on the assumed preference structure. A more general preference structure will only add to the complexity of the analysis without altering the main findings of the paper. In
particular, if consumers are assumed to have smooth preferences, this assumption will only reinforce the welfare dominance of the deposit contracts as shown in relevant literature.

However, extrinsic uncertainty and costly liquidation of the productive technology can have a crucial effect on the welfare results. When uncertainty about the early withdrawal demand is extrinsic, depository intermediaries that provide liquidity insurance are always prone to default since a bank run is another possible equilibrium in the post-deposit game between patient consumers. In the absence of a safety net, taking the possibility of runs into account when contracts are designed as in Cooper and Ross (1998), will create distortions on intermediaries’ asset portfolio that erode the welfare performance of deposit relative to equity contracts. This is because, independent of whether the optimal deposit contract allows for one or multiple equilibria in the post-deposit game, a greater amount of the commodity is kept in storage to finance early withdrawals as runs can occur with a positive probability. Moreover, costly liquidation of investment in the productive technology also deteriorates the welfare performance of deposit contracts by restricting the set of feasible allocations as intermediaries need to adjust their liquidity to meet the state-independent contractual obligations at date 1.

The model also provides an alternative approach to financial contracting when financial fragility is viewed as the outcome of aggregate shocks to the demand for liquidity as in Allen and Gale (2004a, 2005). When banks in liquidity distress can sell claims on irreversible productive investments in an interim asset market, small liquidity shocks can propagate liquidity shortages by dampening asset prices, causing isolated bank failures to become systemic. In contrast, intermediaries financed by equity rather than debt are not ‘fragile’ as contractual obligations are independent of the liquidity demand. The optimal configuration of the financial system in the Allen and Gale (2004a, 2005) setting is an interesting topic for future research.

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provided.
Figures

Figure 1: Equilibria in the Secondary Market

$R(1 - \delta)$

$\delta(1 - \pi^S)$

$\pi^S$

$\delta(1 - \pi^S)$

$\frac{R(1 - \delta)}{R(1 - \delta)}$

$Q$

$P$

Demand

Figure 2: Equilibrium Allocations for $0 \leq q \leq \tilde{q}$

$C^S_2, D^S_2, C^S_{2E}$

$EC_0$

$SO$

$DC_0$

$45^0$ line

$V^H_E$

$V^H_B$

$F_L$

$F_H$

$1/\pi^H$

$1/\pi^L$

$C^S_1, D_1, C^S_{1E}$
Figure 3: Equilibrium Allocations for $q = 1$

Figure 4: Welfare Comparison of the Contracts
A. Appendix

Proof of Property 1

Expressing the first-order condition in terms of $D_1^*$ using the sequential budget constraints, it follows that $\partial D_1^*/\partial q = -\partial^2 EV_D(V^*)/\partial q \partial D_1^*/(\partial^2 EV_D(V^*)/\partial D_1^{*2})$, where $EV_D(V^*) = qV_D^H + (1-q)V_D^L$ denotes the consumers’ ex ante expected utility at date 0 and $V_D^S$ denotes consumers’ expected utility in a given state as specified in equation (1). The denominator is negative from the concavity of the utility function, while the numerator

$$\partial^2 EV_D(V^*)/\partial q \partial D_1^* = \pi^H \pi^L R (U'(D_2^{L^*}) - U'(D_2^{H^*})) / \pi < 0 \quad (A.1)$$

is also negative since $D_2^{H^*} < D_2^{L^*}$ from $D_1^* > 1$, where $\pi = q\pi^H + (1-q)\pi^L$ is the expected value of $\pi$. Therefore $\partial D_1^*/\partial q < 0$, and from the feasibility constraints $\partial D_2^{S^*}/\partial q > 0$. Note also that the monotonicity of $D_1^*$ with respect to $q$ implies that $D_1^* \in (C_1^{H^*}, C_1^{L^*})$ for any $q \in (0, 1)$ and $D_2^{H^*} < C_2^{H^*} < C_2^{L^*} < D_2^{L^*}$ from the feasibility constraints. Thus, from equation (4) and the concavity of the utility function, the following relationships can be obtained:

$$U'(D_1^*)/U'(D_2^{H^*}) < U'(C_1^{H^*})/U'(C_2^{H^*}) = R,$$

$$U'(D_1^*)/U'(D_2^{L^*}) > U'(C_1^{H^*})/U'(C_2^{H^*}) = R. \quad (A.2)$$

Moreover, according to the Envelope theorem, the partial total derivative of $EV_D(V^*)$ with respect to $q$ is $dEV_D(V^*)/dq = \partial EV_D(V^*)/\partial q = V_D^{H^*} - V_D^{L^*}$, where $d^2 EV_D(V^*)/dq^2 = \partial D_1^*/\partial q (\partial^2 EV_D(V^*)/\partial D_1^*/\partial q) > 0$ from equation (A.1) and Young’s theorem. Thus, in order to prove that $dEV_D(V^*)/dq < 0$ for any $q \in [0, 1]$, it is sufficient to show that it is negative when evaluated at $q = 1$ where

$$\left. \frac{dEV_D(V^*)}{dq} \right|_{q=1} = (\pi^H - \pi^L)U(C_1^{H^*}) + (1-\pi^H)U(C_2^{H^*}) - (1-\pi^L)U \left( \frac{R(1-\pi^L/C_1^{H^*})}{1-\pi^L} \right) < (\pi^H - \pi^L) \left( U(C_1^{H^*}) - U(C_2^{H^*}) \right). \quad (A.3)$$

The inequality derives from the relationship $R(1-\pi^L/C_1^{H^*})/(1-\pi^L) > C_2^{H^*}$ since $C_1^{H^*} > 1$. This implies that $dEV_D(V^*)/dq|_{q=1} < 0$ as $C_1^{H^*} < C_2^{H^*}$. Thus, consumers’ expected utility is
strictly decreasing and convex in $q$ when the optimal deposit contract is offered. □

**Proof of Lemma 1**

The concavity of the utility function and linearity of consumption allocations with respect to $\delta$ guarantee the existence of a unique dividend payment for each configuration that maximises consumers’ expected utility. In particular, for the market configurations where $\delta < \tilde{\delta}_L$ and $\tilde{\delta}_L \leq \delta < \tilde{\delta}_H$, the optimal dividend payment in each case is an interior solution to the maximisation problem. However, for $\tilde{\delta}_H \leq \delta$, the dividend payment that maximises consumers’ utility, given in this case by $U_\delta + R(1 - \delta)$, is the boundary solution $\tilde{\delta}_H$ since $dU/d\delta < 0$ for any $\delta \in (0, 1)$. Hence, introducing the constraints on $\delta^*$ for which each case is defined, consumers’ expected utility is maximised either for $\delta^* < \tilde{\delta}_L$ or $\tilde{\delta}_L \leq \delta^* < \tilde{\delta}_H$. These cases constitute possible equilibrium configurations of the secondary market depending on the parameters of the model and the utility function. □

**Proof of Property 2**

Expressing the first-order condition in terms of $C^H_{1\bar{E}}$, differentiation with respect to $q$ for $\delta^* < \tilde{\delta}_L$ yields

$$\frac{\partial C^H_{1\bar{E}}}{\partial q} = -\frac{U'(C^H_{1\bar{E}}) - U'(C^L_{1\bar{E}}) - R (U'(C^H_{2\bar{E}}) - U'(C^L_{2\bar{E}}))}{qU''(C^H_{1\bar{E}}) + (1 - q) \pi^H U''(C^L_{1\bar{E}}) + \pi^H R^2 \left( q \frac{U''(C^H_{1\bar{E}})}{1 - \pi^H} + (1 - q) \frac{U''(C^L_{1\bar{E}})}{1 - \pi^H} \right)} > 0.$$ 

The above derivative is positive because the denominator is negative from the concavity of the utility function, while the numerator is positive from the relationship between the equilibrium payoffs given by equation (12) and the concavity of the utility function. Hence, for $\delta^* < \tilde{\delta}_L$, $\partial C^H_{1\bar{E}}/\partial q > 0$ and $\partial C^L_{2\bar{E}}/\partial q < 0$ from the feasibility constraints.

In a similar manner, for $\tilde{\delta}_L \leq \delta^* < \tilde{\delta}_H$, it follows that

$$\frac{\partial C^H_{1\bar{E}}}{\partial q} = -\frac{U'(C^H_{1\bar{E}}) - U'(C^L_{1\bar{E}}) - R (U'(C^H_{2\bar{E}}) - U'(C^L_{2\bar{E}}))}{qU''(C^H_{1\bar{E}}) + qR^2 \pi^H U''(C^H_{2\bar{E}}) + (1 - q) \pi^H (1 - R)^2 U''(C^L_{1\bar{E}})} > 0$$

which is positive following similar reasoning as above. Hence, $\partial C^H_{1\bar{E}}/\partial q > 0$ and $\partial C^H_{2\bar{E}}/\partial q < 0$, whereas $\partial C^L_{1\bar{E}}/\partial q = \pi^H (1 - R) \partial C^H_{1\bar{E}}/\partial q < 0$ when $\tilde{\delta}_L \leq \delta^* < \tilde{\delta}_H$. This also implies that
\( \partial^2 EV_E(V^*)/\partial q \partial C^{H^*}_{1E} > 0 \) for any \( q \in [0, 1] \).

According to the Envelope theorem \( dEV_E(V^*)/dq = \partial EV_E(V^*)/\partial q = V^H_E - V^L_E \), where \( d^2 EV_E(V^*)/dq^2 = (\partial^2 EV_E(V^*)/\partial C^{H^*}_{1E} \partial q) \partial C^{H^*}_{1E} / \partial q > 0 \) from Young’s theorem. Since \( d^2 EV_E(V^*)/dq^2 > 0 \), evaluation at \( \tilde{q} \) for \( \delta^* \leq \tilde{\delta}_L \) yields

\[
\frac{dEV_E(V^*)}{dq} \bigg|_{q=\tilde{q}} = \pi^H U(C^{H^*}_{1E}(\tilde{\delta}_L)) + (1 - \pi^H) U \left( \frac{R(1 - \pi^H C^{H^*}_{1E}(\tilde{\delta}_L))}{1 - \pi^H} \right) - U(\pi^H C^{H^*}_{1E}(\tilde{\delta}_L) + R(1 - \pi^H C^{H^*}_{1E}(\tilde{\delta}_L))) < 0
\]

which is negative from Jensen’s inequality due to the strict concavity of the utility function, and therefore, \( dEV_E(V^*)/dq < 0 \) for any \( q \in [0, \tilde{q}) \).

Similarly, \( dEV_E(V^*)/dq < 0 \) for \( \tilde{\delta}_L \leq \delta^* < \tilde{\delta}_H \) as

\[
\frac{dEV_E(V^*)}{dq} = \pi^H U(C^{H^*}_{1E}) + (1 - \pi^H) U \left( \frac{R(1 - \pi^H C^{H^*}_{1E})}{1 - \pi^H} \right) - U(\pi^H C^{H^*}_{1E} + R(1 - \pi^H C^{H^*}_{1E})) < 0
\]

from Jensen’s inequality due to the strict concavity of the utility function. Thus, the consumers’ expected utility is strictly decreasing and convex in \( q \in [0, 1] \) when the optimal equity contract is offered.

**Proof of Proposition 1**

Let \( \Delta^*_i = EV_E(V^*) - EV_D(V^*) \) be the difference between the expected utility attained under an equity and deposit contract, where \( d\Delta^*_i / dq = \partial EV_E(V^*)/\partial q - \partial EV_D(V^*)/\partial q \) the partial total derivative with respect to \( q \). From properties 1 and 2, \( \partial EV^*/\partial q = V^H - V^L < 0 \) for either contract.

Evaluating the above difference at \( q = 1 \) yields \( d\Delta^*_i / dq \bigg|_{q=1} = V^L_D(q = 1) - V^L_E(q = 1) \) as both contracts achieve the social optimum in state \( H \). In state \( L \) (where \( \tilde{q} < 1 \)), the equilibrium consumption of both types of consumers will be

\[
C^{L^*}_{2E}(q = 1) = C^{L^*}_{1E}(q = 1) = \pi^H C^{H^*}_{1E} + R(1 - \pi^H C^{H^*}_{1E}) \text{ since } \delta^* = \pi^H C^{H^*}_{1E},
\]

\[
D^*_1(q = 1) = C^{H^*}_{1E} \text{ and } D^*_2(q = 1) = \frac{R(1 - \pi^L C^{H^*}_{1E})}{1 - \pi^H}.
\]

To prove that the deposit is the welfare dominant contract at \( q = 1 \), from properties 1 and 2 it is
sufficient to show that \( d\Delta_{ED}/dq \big|_{q=1} > 0 \). Let \( \pi^L = \pi^H - \epsilon \), where \( \epsilon > 0 \) a parameter. It can be verified that \( d\Delta_{ED}/dq \big|_{q=1} \) is strictly increasing in \( \epsilon \). Since \( \tilde{q} < 1 \), the model’s primitives should satisfy \( C^L_{1E}(q = 1) \leq R/(\pi^L R + 1 - \pi^L) \), or \( \pi^L \leq \pi^H C^H_{1E}/C^L_{1E}(q = 1) \). Taking this condition into account, note that \( \lim_{\epsilon \to 0^+} d\Delta_{ED}/dq \big|_{q=1} = 0 \) and therefore, \( d\Delta_{ED}/dq \big|_{q=1} > 0 \) for any \( \epsilon > 0 \). Thus, \( EV_E(V^*) < EV_D(V^*) \) for values of \( q \) in the region around one as illustrated in Figure 4.

Similarly, evaluation at \( q = 0 \) yields \( d\Delta_{ED}/dq \big|_{q=0} = V^*_E(q = 0) - V^*_D(q = 0) \). In this case, the equilibrium consumption of both types of consumers for state \( H \) will be

\[
C^H_{1E}(q = 0) = \pi^L C^L_{1}/\pi^H, \quad C^H_{2E}(q = 0) = R(1 - \pi^L C^L_{1})/(1 - \pi^H),
\]

\[
D^*_1(q = 0) = C^L_{1} \quad \text{and} \quad D^*_2(q = 0) = R(1 - \pi^H C^L_{1})/(1 - \pi^H).
\]

From properties 1 and 2, for the deposit to be the welfare dominant contract for values of \( q \) in the region around \( 0 \) requires \( d\Delta_{ED}/dq \big|_{q=0} < 0 \). However, no positive conclusions can be drawn about the sign of \( d\Delta_{ED}/dq \big|_{q=0} \) as this depends on the parameters of the model and the functional form of the utility. This is illustrated with the use of numerical examples assuming a constant relative risk aversion utility function of the form \( U(C) = C^{1-\gamma}/(1 - \gamma) \), where \( \gamma > 1 \) the coefficient of relative risk aversion.

From the first-order condition and the intertemporal budget constraint (equations (13) and (4), respectively), it follows that \( C^L_{1} = (\pi^L + (1 - \pi^L)R \tilde{q}^{1-\gamma})^{-1} \). Suppose that \( \gamma = 2 \), \( \pi^H = 0.7 \) and \( \pi^L = 0.4 \). For \( R = 3.5 \), \( \tilde{q} \approx 0.47 \) and \( d\Delta_{ED}/dq \big|_{q=0} \approx 0.46 \) which implies that \( V^*_E(q = 0) > V^*_D(q = 0) \). In terms of Figure 4 and from properties 1 and 2, this implies that \( EV_E(V^*) \) is flatter than \( EV_D(V^*) \) at \( q = 0 \). In this case, \( EV_D(V^*) < EV_E(V^*) \) for \( q \in (0, q^*) \) and \( EV_E(V^*) < EV_D(V^*) \) for \( q \in (q^*, 1) \), where \( EV_D(V) \) is represented by the solid line. In terms of Figure 2, the equity contract’s allocation in state \( H \) lies on a higher indifference curve than that attained by the deposit contract. However, for \( R = 2 \), \( \tilde{q} \approx 0.26 \) and \( d\Delta_{ED}/dq \big|_{q=0} \approx -0.22 \) which implies \( V^*_E(q = 0) < V^*_D(q = 0) \). Hence, \( EV_E(V^*) \) is steeper than \( EV_D(V^*) \) at \( q = 0 \) in Figure 4 and therefore \( EV_E(V^*) < EV_D(V^*) \) for any \( q \in (0, 1) \), where \( EV_D(V^*) \) is represented by the dashed line. In this case, the equity contract’s allocation lies on a lower indifference curve in Figure 2.
Proof of Property 3

Using the sequential budget constraints to express the first-order conditions in terms of $D_{c_1}^{H*}$ and $D_{c_1}^{L*}$, differentiating with respect to $q$ yields

$$\frac{\partial D_{c_1}^{L*}}{\partial q} = \frac{R \left( U'(D_{c_2}^{H*}) - U'(D_{c_2}^{L*}) \right)}{U''(D_{c_2}^{L*}) + \pi^L R \left[ \frac{qU''(D_{c_2}^{H*})U''(D_{c_2}^{H*})}{(1-\pi^H)U''(D_{c_2}^{H*})+(\pi^H-\pi^L)RU''(D_{c_2}^{H*})} \right]} < 0,$$

since $D_{c_2}^{H*} < D_{c_2}^{L*}$ and from the concavity of the utility function.

To prove that the optimal contingent deposit contract is incentive compatible, it is sufficient to show that $IC_{cH} = \pi^L U(D_{c_1}^{H*}) + (\pi^H - \pi^L) U(D_{c_1}^{H*}) - U(D_{c_1}^{H*}) \leq 0$ since $D_{c_1}^{L*} < D_{c_1}^{H*}$. Given that $D_{c_1}^{H*} < D_{c_1}^{L*}$, let $D_{c_1}^{L*} = D_{c_1}^{L*} - \epsilon$ where $\epsilon > 0$ is a parameter. Differentiating $IC_{cH}(\epsilon)$ with respect to $\epsilon$ yields $\partial IC_{cH}(\epsilon)/\partial \epsilon = - (U'(D_{c_1}^{H*}) + RU'(D_{c_2}^{H*})) (\pi^H - \pi^L)/(1-\pi^H) < 0$. Therefore, as $\lim_{\epsilon \to 0} IC_{cH}(\epsilon) = \pi^H U(D_{c_1}^{H*}) - U(R(1 - \pi^H D_{c_1}^{H*})/(1 - \pi^H)) < 0$ from the first-order condition, it follows that $IC_{cH}(\epsilon) < 0$ for any $\epsilon > 0$. Hence, the optimal payoffs of a contingent deposit contract satisfy the incentive compatibility constraint given in equation (17).

Differentiating $dEV_C(V^*)$ twice with respect to $q$, where $dEV_C(V^*)/dq = V_C^{H*} - V_C^{L*}$ from the Envelope theorem, yields

$$\begin{align*}
dEV_C(V^*)/dq &= (\pi^H - \pi^L) U(D_{c_1}^{H*}) + (1 - \pi^H) U(D_{c_2}^{H*}) - (1 - \pi^L) U(D_{c_2}^{L*}), \\
d^2EV_C(V^*)/dq^2 &= \pi^L R \left( U'(D_{c_2}^{H*}) - U'(D_{c_2}^{L*}) \right) \partial D_{c_1}^{L*} / \partial q > 0.
\end{align*}$$

The sign of the second derivative is positive as $D_{c_2}^{H*} < D_{c_2}^{L*}$ and $\partial D_{c_1}^{L*} / \partial q < 0$. To prove that $dEV_C(V)/dq < 0$ for any $q \in [0, 1]$, it is sufficient to show that $dEV_C(V^*)/dq|_{q=1} < 0$. From the first-order condition in each state note that $D_{c_1}^{L}(q = 1) = D_{c_1}^{H*}(q = 1) = C_1^{H*}$ and $D_{c_2}^{H*}(q = 1) = C_2^{H*}$, whereas $D_{c_2}^{L*}(q = 1) = R(1 - \pi^L C_1^{H*})/(1 - \pi^L)$. Notice, however, that $dEV_C(V^*)/dq|_{q=1} = dEV_D(V^*)/dq|_{q=1}$ (equation (A.3)) which has been shown to be negative. Hence, consumers’ expected utility is strictly decreasing and convex in $q$ when the optimal deposit contract with contingent date 1 payoffs is offered. □

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Proof of Proposition 2

Similar to the proof of Proposition 1, let $\Delta_{CD}^* = EV_C(V^*) - EV_D(V^*)$ be the difference between the expected utility attained by deposit contracts with contingent and fixed date 1 payoffs.

In the proof of property 3, it has been shown that $dEV_C(V^*)/dq|_{q=1} = dEV_D(V^*)/dq|_{q=1}$. Hence, $d\Delta_{CD}^*/dq|_{q=1} = 0$ so that $EV_C(V^*)$ and $EV_D(V^*)$ are tangent at $q = 1$. Properties 1 and 3 imply that the welfare dominant contract for any $q \in (0, 1)$ is the one that dominates in the region of $q$ around zero. For $q = 0$, both contracts attain the social optimum allocation in state $L$, whereas the equilibrium consumption of both types of consumers in state $H$ are

$$D_{1L}^*(q = 0) = R \left(1 - \pi_L C_{1L}^* - (\pi_H - \pi_L) D_{1L}^*(q = 0)\right) / (1 - \pi_L),$$
$$D_{1H}^*(q = 0) = C_{1H}^*, \text{ and } D_{2H}^*(q = 0) = R(1 - \pi_H C_{1L}^*)/(1 - \pi_H).$$

Hence, $d\Delta_{CD}^*/dq|_{q=0}$ will be

$$\left.\frac{d\Delta_{CD}^*}{dq}\right|_{q=0} = \left(\pi^H - \pi^L\right) \left(U(D_{1L}^*(q = 0)) - U(C_{1L}^*)\right) + (1 - \pi^H) \left(U(D_{1H}^*(q = 0)) - U(D_{2H}^*(q = 0))\right).$$

Since $D_{1H}^*(q = 0) < C_{1L}^*$, let the parameter $\varepsilon > 0$ such that $C_{1L}^* = D_{1L}^*(q = 0) + \varepsilon$. Differentiation of $d\Delta_{CD}^*/dq|_{q=0}$ with respect to $\varepsilon$ provides

$$\frac{\partial}{\partial \varepsilon} \left.\frac{d\Delta_{CD}^*}{dq}\right|_{q=0} = - \left(\pi^H - \pi^L\right) U'(C_{1L}^*) - \pi^L R U'(D_{1L}^*(q = 0)) + \pi^H R U'(D_{2L}^*(q = 0))$$
$$> \left(\pi^H - \pi^L\right) R \left(U'(D_{2L}^*(q = 0)) - U'(C_{1L}^*)\right),$$

where the inequality derives from the relationship $D_{2L}^*(q = 0) < D_{2L}^*(q = 0)$, the first-order condition of the social planner’s problem for state $L$ and the concavity of the utility function. Hence, the above derivative is positive since $D_{2L}^*(q = 0) < C_{1L}^*$. Taking the limit as $\varepsilon$ tends to zero it follows that $\lim_{\varepsilon \to 0^+} d\Delta_{CD}^*/dq|_{q=0} = 0$, and therefore $d\Delta_{CD}^*/dq|_{q=0} > 0$ for any $\varepsilon > 0$. This implies that $EV_C(V^*)$ is flatter than $EV_D(V^*)$ for values of $q$ in the region around zero, and consequently $EV_D(V^*) < EV_C(V^*)$ for any $q \in (0, 1)$.  

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Although the welfare performance of the deposit contract with a sequential service constraint is improved, the equity contract can still dominate for low values of $q$ depending on the primitives of the model and the assumed utility function. From the discussion in the proof of Proposition 1, the contingent deposit contract dominates for high values of $q$ in the region around one. The monotonicity of $EV_E(V^*)$ and $EV_C(V^*)$ with respect to $q$ suggests that if $EV_C(V^*) < EV_E(V^*)$ for values of $q$ in the region around zero, then a threshold value of $q$ exists, namely $q^* \in (0, q^*)$, such that $EV_C(V^*) < EV_E(V^*)$ for any $q \in (0, q^*)$. Otherwise, $EV_E(V^*) < EV_C(V^*)$ for any $q \in (0, 1)$.

Consider the utility function and the parameters’ values in the example provided in the proof of Proposition 1. It has been shown that the equity contract initially dominates the deposit contract with fixed date 1 payoff for $\gamma = 2$, $\pi^H = 0.7$, $\pi^L = 0.4$ and $R = 3.5$. Let $\Delta^*_EC = EV_E(V^*) - EV_C(V^*)$ be the difference between the expected utility attained under an equity and contingent deposit contract. For the same specification of the model’s parameters, $d\Delta^*_EC/dq|_{q=0} \approx -0.18$ which implies that $EV_C(V^*)$ is flatter than $EV_E(V^*)$ at $q = 0$, and therefore $EV_E(V^*) < EV_C(V^*)$ for any $q \in (0, 1)$. Suppose now that $\gamma = 5$, $\pi^H = 0.7$, $\pi^L = 0.6$ and $R = 9$. For these values of the model’s parameters $\bar{q} \approx 0.8$ and $d\Delta^*_EC/dq|_{q=0} \approx 0.14$. In this case, $EV_C(V^*)$ is steeper than $EV_E(V^*)$ at $q = 0$ and $EV_C(V^*) < EV_E(V^*)$ for $q \in (0, q^*)$, whereas $EV_E(V^*) < EV_C(V^*)$ for $q \in (q^*, 1)$. \Box

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