Spatial correlation of heat release rate and sound emission from turbulent premixed flames

Y. Liu*, A.P. Dowling, N. Swaminathan*, T.D. Dunstan

*Department of Engineering, University of Cambridge, Trumpington Street, Cambridge CB2 1PZ, UK

Abstract

The two-point spatial correlation of the rate of change of fluctuating heat release rate is central to the sound emission from open turbulent flames, and a few attempts have been made to address this correlation in recent studies. In this paper, the two-point correlation and its role in combustion noise are studied by analysing direct numerical simulation (DNS) data of statistically multi-dimensional turbulent premixed flames. The results suggest that this correlation function depends on the separation distance and direction but, not on the positions inside the flame brush. This correlation can be modelled using a combination of Hermite-Gaussian functions of zero and second order, i.e. functions of the form \((1 - Ax^2)e^{-Bx^2}\) for constants \(A\) and \(B\), to include its possible negative values. The integral correlation volume obtained using this model is about \(0.2\delta_L^3\) with the length scale obtained from its cube root being about \(0.6\delta_L\), where \(\delta_L\) is the laminar flame thermal thickness. Both of the values are slightly larger than the values reported in an earlier study because...
of the anisotropy observed for the correlation. This model together with the turbulence-dependent parameter $K$, the ratio of the root-mean-square (RMS) value of the rate of change of reaction rate to the mean reaction rate, derived from the DNS data are applied to predict the far-field sound emitted from open flames. The calculated noise levels agree well with recently reported measurements and show a sensitivity to $K$ values.

Keywords: Combustion noise from open flames, Two-point correlation, Heat release rate fluctuation, Correlation volume and length scale, DNS V-flames

1. Introduction

Turbulent combustion produces sound emission due to its inherent unsteadiness. Many studies [1–6] have indicated that noise due to unsteady combustion may become a significant noise source particularly for lean burn gas turbines because lean burning generally involves highly unsteady flames. Combustion noise is also a signature of combustor health and performance because of its important role in the dynamics of the combustion system. Therefore, a thorough understanding of the source mechanisms of combustion noise is required at the design stage of gas turbines to help minimise the noise emissions. It is known that the combustion noise is generated by the fluctuating heat release rate, which causes unsteady expansion of reacting gases inside the turbulent flame brush and hence behaves locally as a distributed monopole source.

The prediction of combustion noise level for a practical burner is still a challenging issue [7] despite many efforts [2, 8–11] to develop a semi-empirical
correlation between the far-field acoustic power and burner geometry or operating flow conditions. It has been recently noted [5, 6] that the two-point spatial correlation of the rate of change of fluctuating heat release rate, and hence the associated correlation volume $v_{cor}$ and its length scale, is crucial to predicting combustion noise. In previous studies, different length scales have been suggested to estimate $v_{cor}$ empirically, ranging from the laminar flame thickness $\delta_L$ [12], turbulence integral length scale $\Lambda$ [13] to the turbulent flame-brush thickness [14, 15] or a combination of them. However, the two-point correlation of heat release rate has not received sufficient attention primarily owing to the lack of availability of reliable numerical or experimental data for the fluctuating heat release rate required to directly investigate the correlation length scale.

Recently, it has become feasible to obtain high-fidelity information on the correlation of fluctuating heat release rate because of the advancement in computing technologies and techniques, and laser metrology. Swaminathan et al. [5, 6] have attempted to analyse and model this two-point correlation using DNS [16–18] and laser diagnostics [19] data of turbulent premixed flames. They found that the two-point correlations of heat release rate, $\Omega$, and the rate of change of fluctuating heat release rate, $\Omega_1$, can be well represented by Gaussian-type functions commonly used in classical turbulence, and that the length scale of the correlation volume, $v_{cor}$, is $0.5\delta_L$. This model was then shown to give a good agreement with recent experimental measurements [20] of the far-field overall sound pressure level (OASPL) from open turbulent premixed flames.

The construction of $\Omega_1$ requires the rate of change of fluctuating heat
release rate, which was calculated in Refs. [5, 6] indirectly by using a balance
equation for a progress variable and taking the instantaneous reaction rate
to be a function of the progress variable. In this way the time derivative
is obtained using the spatial derivative of the progress variable field at one
single time step from the DNS data [16–18]. These numerical data are,
however, for statistically planar flames and the correlation length scale in
the mean flame propagation direction (normal to the flame brush) was then
used to estimate the correlation volume as $v_{\text{cor}} \sim \delta L^3/8$ by assuming isotropy
for the correlation. This value of $v_{\text{cor}}$ may be underestimated, as one shall
see in Section 4.3, since the heat release rate varies most dramatically in the
flame normal direction and hence the correlation length is relatively short
in this direction compared with the other directions in the turbulent flame.
Moreover, the parameter $K$ related to a time scale for the rate of change of
fluctuating heat release rate, required to predict the combustion noise level,
was estimated through combined analyses of the DNS and laser diagnostics
data. Furthermore, this estimate strongly depends on the approximation
used to obtain the time derivative of the fluctuating heat release rate.
Nevertheless, the work of Swaminathan et al. [5, 6] makes an important
contribution towards improved understanding of the two-point correlation
of the rate of change of heat release rate fluctuation and its role in noise
emission from open turbulent flames.

The prime objectives of this study are three folds, viz., (i) to assess
the isotropy of the two-point spatial correlation function, $\Omega_1$, assumed in
an earlier study [6], using DNS data [21] of statistically multi-dimensional
flames; (ii) to evaluate the time derivative of the fluctuating heat release
rate directly in the DNS and its influence on the value of the parameter \( K \); and (iii) to study the sensitivity of the far-field OASPL values to the newly obtained information from the first two objectives. The results of Reynolds-Averaged-Navier-Stokes (RANS) calculations in Ref. [6] are used to recompute the far-field OASPL to address the third objective.

This paper is organised as follows. The theoretical background on the two-point correlation and its relation to the OASPL is reviewed briefly in the next section. The salient features of DNS data and the analyses are discussed in Section 3. The results are discussed in Section 4 and the conclusions are summarised in the final section.

2. Theoretical formulation

2.1. Combustion noise

The sound emission from a turbulent reacting flow has been derived by Dowling [4] using Lighthill’s theory [22, 23] which includes a variety of sources associated with flow noise, viscous dissipation, heat and molecular transports, direct and indirect combustion noise. At low Mach number condition as in open flames, the direct noise from unsteady heat input is the dominant source and is about two orders of magnitude larger than other sources [24, 25]. This leads to a linear wave equation of the reduced form:

\[
\frac{1}{a_0^2} \frac{\partial^2 p'}{\partial t^2} - \nabla^2 p' = \frac{\partial}{\partial t} \left[ \rho_0 (\gamma - 1) \frac{\dot{Q}'}{\rho a^2} \delta(y, t) \right],
\]

where \( p' \) is the pressure perturbation, \( \gamma \) is the ratio of specific heat capacities, and \( \dot{Q}' \) is the fluctuating heat release rate per unit volume; \( \rho \) and \( a \) denote the fluid density and sound speed in the combustion zone and differ from the
ambient values $\rho_0$ and $a_0$. When the turbulent combustion occurs at ambient pressure and $\gamma$ is assumed to be independent of temperature,

$$\rho a^2 = \gamma \rho_0 = \rho_0 a_0^2,$$  \hspace{1cm} (2)

and thus the wave equation (1) simplifies to

$$\frac{1}{a_0^2} \frac{\partial^2 p'}{\partial t^2} - \nabla^2 p' = \frac{(\gamma - 1)}{a_0^2} \frac{\partial \dot{Q}'(y, t)}{\partial t}. \hspace{1cm} (3)$$

Readers are referred to Refs. [4, 6] for a complete derivation of the above expression starting from the Lighthill equation and a detailed analysis of the contributions of various source terms.

The solution of Eq. (3) can be written as

$$p'(x, t) = \frac{(\gamma - 1)}{4\pi ra_0^2} \frac{\partial}{\partial t} \int_{v_f} \dot{Q}'(y, t - r/a_0) \, d^3 y \hspace{1cm} (4)$$

by using a free-space Green’s function, where $x$ denotes the observer point in far field, $y$ is the source position inside the acoustically compact flame brush of volume $v_f$, and $r = |x|$ is the observer distance from an origin within $v_f$. The effects of convection and refraction of sound [26, 27] caused by the variations of $\gamma$ and $a$ within the flame brush due to temperature inhomogeneities are neglected as noted earlier in order to focus on the role of the dominant source from the unsteady heat addition. More importantly, it is clearly shown in Eq. (4) that the far-field sound pressure is expressed in terms of a volume integral of the unsteady heat release rate over the turbulent flame brush at a retarded time $t - r/a_0$. The source of combustion noise originates from the rate of change of this integral and behaves as an acoustic monopole. The far-field OASPL characterised by $\overline{p^2}$, a measurable
quantity in experiments, can be simply obtained from Eq. (4) as
\[ p''(x, t) = \frac{(\gamma - 1)^2}{16\pi^2 r^2 a_0^4} \int_{v_f} \int_{v_{cor}} \ddot{Q}'(y, t) \dddot{Q}'(y + \Delta, t) \, d^3 \Delta \, d^3 y, \]  
(5)
where \( \dddot{Q}' \) is the time derivative of the fluctuating heat release rate, \( \Delta \) is the separation vector, and the overbar refers to an averaging process. The volume over which \( \ddot{Q}' \) is correlated is denoted as the correlation volume \( v_{cor} \) that has originated from Bragg’s theory [12]. The two-point correlation \( \ddot{Q}'(y, t) \dddot{Q}'(y + \Delta, t) \) and its volume \( v_{cor} \) are the focus of this paper and will be discussed in detail subsequently.

### 2.2. Two-point correlation

Following the common practice in the analysis of turbulent premixed flames [5, 6], one can use a progress variable, \( c \), which varies from zero in reactants to unity in products and define it using the fuel mass fraction [28]. The instantaneous heat release rate \( \dot{Q} \) is then related to the instantaneous chemical reaction rate \( \dot{w} \) as \( \dot{Q} = Y_{f,u} H \dot{w} \), where \( Y_{f,u} \) is the fuel mass fraction in the unburnt reactants, and \( H \) is the lower heating value of the fuel. This approximation holds good even when a complex chemical kinetics is used to model combustion [28]. In doing so, \( \dot{Q} \) can be replaced by \( \dot{w} \) in the analysis. Noting the simple relation between the heat release rate and the reaction rate, the two-point correlation in Eq. (5) is expressed as

\[ \ddot{Q}'(y, t) \dddot{Q}'(y + \Delta, t) = Y_{f,u}^2 H^2 \ddot{w}'(y, t) \dddot{w}'(y + \Delta, t), \]  
(6a)
\[ \ddot{w}'(y - \Delta/2, t) \dddot{w}'(y + \Delta/2, t) = \Omega_1(y, \Delta) \ddot{w}'^2(y, t), \]  
(6b)
where \( \Omega_1 \) is the correlation function for the rate of change of heat release rate fluctuation. All terms with overbar are independent of time \( t \). By definition
the correlation function $\Omega_1$ depends on both the spatial location, $y$, and separation, $\Delta$, in the flame brush. In spite of this, the former dependence is nearly negligible as will be observed in Section 4.2, and hence the correlation function can be denoted as $\Omega_1(\Delta)$. Further discussion on the assumptions of Eq. (6) can be found in Ref. [6].

For the purpose of computing OASPL, it is helpful to express the two-point correlation $\overline{\dot{Q}(y, t) \dot{Q}(y + \Delta, t)}$ in Eq. (6) in terms of the square of the local mean heat release rate $\overline{\dot{Q}(y, t)}^2$. This is achieved by relating $\overline{\dot{w}''^2}$ to the mean reaction rate $\overline{\dot{w}}$ as

$$\overline{\dot{w}''^2(y, t)} = K^2 \overline{\dot{w}(y, t)}^2,$$

(7)

where the parameter $K$ was decomposed into two terms as in Refs. [5, 6]:

$$K = \left( \frac{\overline{\dot{w}''^2}}{\overline{\dot{w}'}^2} \right)^{1/2} \cdot \left( \frac{\overline{\dot{w}''^2}}{\overline{\dot{w}}^2} \right)^{1/2} = B_1 B$$

(8)

with $B_1$ as the inverse of an average time scale for the rate of change of the fluctuating reaction rate, and $B$ the ratio of the RMS of the reaction rate fluctuation to the mean value. Using the relationship (7) the two-point correlation in Eq. (6) becomes

$$\overline{\dot{Q}'(y, t) \dot{Q}'(y + \Delta, t)} = Y_{f,u}^2 H^2 \overline{\dot{w}(y, t)}^2 \Omega_1(\Delta) \overline{\dot{w}(y, t)}^2.$$

(9)

By substituting the above expression into Eq. (5), one obtains the far-field OASPL as:

$$p^2(x, t) = \frac{(\gamma - 1)^2}{16\pi^2 r^2 a_0^4} Y_{f,u}^2 H^2 \int_{v_f} K^2 \overline{\dot{w}(y, t)}^2 d^3y \int_{v_{cor}} \Omega_1(\Delta) d^3\Delta,$$

(10)

where the original double integral is split into two single integrals to divide up the respective contributions from the flame brush $v_f$. 
(turbulence, thermo-chemistry and their interaction) and correlation volume \( v_{\text{cor}} \) (thermo-chemistry). The first integral contains the turbulence-thermo-chemistry dependent terms \( K, \overline{\dot{w}} \) and \( v_f \). The spatially varying \( K \) can be derived from the V-flame DNS data of Dunstan et al. [21] as mentioned before. The second integral over the correlation volume is denoted as the integral correlation volume \( V_{\text{cor}} \) and its length scale is defined as the cube root of \( V_{\text{cor}} \). This correlation integral can be evaluated separately since the correlation function \( \Omega_1 \) is found to be insensitive to source positions inside the flame brush. Also, the spatial correlation of \( \ddot{Q} \) is dictated predominantly by the thermo-chemical processes and the influence of turbulence on \( \Omega_1 \) is negligibly small [6]. The DNS data from turbulent premixed V-flames [21] will be analysed to study the correlation function \( \Omega_1 \) and a length scale for \( V_{\text{cor}} \) to address the first two objectives of this study. The mean reaction rate \( \overline{\dot{w}(\mathbf{y}, t)} \) available from steady RANS calculations [6] is used to predict the far-field OASPL of open flames [20] to address the third objective.

3. DNS V-flames

The single V-flame, one of several canonical configurations for premixed turbulent flames, has been studied extensively in previous experimental work [29–33] and numerical simulations [34–36]. Recently, DNS for laboratory-scale V-flames has become available and much progress has been made [21, 35, 36]. Dunstan et al. [21] carried out three-dimensional (3D) fully compressible DNS of premixed turbulent V-flames with particular emphasis on evaluation of turbulent flame speed. They used a cubic computation domain with the Navier-Stokes Characteristic Boundary
Conditions (NSCBC) [37] in the streamwise, \( x \), direction and the transverse, \( y \), direction. The standard NSCBC has been modified to accommodate the steep thermal and compositional gradients generated when the flame crosses the boundary. The spanwise, \( z \), direction was specified to be periodic. The flame holder is aligned in the spanwise direction and the flame gradually evolves into two statistically symmetric branches that become progressively thicker as they propagate downstream of the flame holder. The general features of these V-flames will be discussed in more detail in Section 4.1.

3.1. Flame conditions

Three cases of weak, moderate and high intensity turbulence were used by Dunstan et al. [21] and all of these cases will be analysed in this paper. Their fluid dynamic conditions are summarised in Table 1 and a single-step chemical mechanism was used for preheated reactants with unity Lewis numbers. The unstrained laminar flame thermal thickness is \( \delta_L = 0.43 \text{ mm} \) and the flame speed is \( S_L = 0.60 \text{ m s}^{-1} \) giving a flame time of \( \tau_f = \delta_L / S_L = 0.71 \text{ ms} \). These thermo-chemical parameters are representative of a premixed methane-air flame with an equivalence ratio of \( \phi \approx 0.6 \).

<table>
<thead>
<tr>
<th>Case</th>
<th>( u_{rms}^+ )</th>
<th>( \bar{u}^+ )</th>
<th>( \Lambda^+ )</th>
<th>Re</th>
<th>Da</th>
<th>( \tau_f^+ )</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>1.0</td>
<td>16.57</td>
<td>3.57</td>
<td>18</td>
<td>3.57</td>
<td>1.79</td>
</tr>
<tr>
<td>II</td>
<td>2.0</td>
<td>16.57</td>
<td>3.62</td>
<td>37</td>
<td>1.81</td>
<td>1.79</td>
</tr>
<tr>
<td>III</td>
<td>6.0</td>
<td>24.86</td>
<td>3.43</td>
<td>92</td>
<td>0.57</td>
<td>1.19</td>
</tr>
</tbody>
</table>
quantities with a superscript + in the following discussion are normalised appropriately using the laminar flame thickness, its speed and the density of unburnt reactant $\rho_u$. In Table 1, $u_{\text{rms}}$ is the inlet turbulence intensity, i.e. the RMS value of turbulence velocity fluctuation, $\bar{u}$ is the inlet mean streamwise velocity, and $\Lambda$ is the inlet turbulence integral length scale. The turbulence Reynolds number $Re$ is based on $u_{\text{rms}}$ and $\Lambda$, and the Damköhler number is defined as $Da = \Lambda^+ / u_{\text{rms}}^+$. The flame conditions in Table 1 suggest that Cases I to III respectively correspond to the wrinkled flamelets, corrugated flamelets and thin reaction zones, regimes of turbulent combustion, according to the Borghi-Peters classification [38].

With respect to the spatial resolution, the computational domain has dimensions of 12.77 mm ($29.7\delta_L$) in each direction and is discretised using $512 \times 512 \times 512$ uniform grid, ensuring a minimum of about 10 grid points inside the laminar flame thickness $\delta_L$. The flame holder was positioned at $3.48\delta_L$ from the inlet to ensure stable combustion whilst minimising its effect on the downstream flame statistics. While running the DNS, data were collected consecutively during one flow-through time $\tau_D$ (the mean convection time from the inlet to outlet boundaries) after the turbulence had reached a fully developed state and one time length of $\tau_D$ had been run to ensure the decay of initial transients. The data of Cases I and II have 254 uniform time steps ($\Delta t = 5 \times 10^{-6}$ s), whereas for Case III only 85 time steps ($\Delta t = 10^{-5}$ s) were taken due to the shorter flow-through and turbulence time scales. More details of the DNS V-flames can be found in Ref. [21].
3.2. Data processing

The rate of change of fluctuating reaction rate, $\dot{w}'$, required to construct the two-point correlation, $\Omega_1(\Delta)$, is calculated using the $\dot{w}$ fields saved during the DNS. First, the mean reaction rate is obtained using

$$\overline{\dot{w}}(x, y) = \frac{1}{N_t N_z} \sum_{n=1}^{N_t} \sum_{k=1}^{N_z} \dot{w}(x, y; t_n, z_k),$$  \hspace{1cm} (11)$$

where $N_t$ is the number of sample fields collected over the second flow-through time in the DNS, and $N_z$ is the number of grid points in the spanwise, periodic direction. This type of averaging helps us to improve the statistical convergence for the mean values. The fluctuating reaction rate is then obtained using $\dot{w}' = \dot{w} - \overline{\dot{w}}$. Since the averaging is done over the time also one gets $\dot{w}' = \ddot{w}$.

The construction of the correlation function $\Omega_1$ is then straightforward following Eq. (6b). A second-order central difference scheme is used to obtain the time derivative, $\ddot{w}$, and the results have been verified using a higher-order numerical scheme and a finer time resolution. The mean flow field of the DNS flames is predominantly two-dimensional (2D) and thus the Favre-averaged (density weighted) progress variable $\tilde{c}$ is related to the $(x, y)$ plane only. The progress variable $\tilde{c}$ can be used to denote the transverse position, $y$, inside the flame brush for a given streamwise location. Figure 1 illustrates the 2D contours of $\tilde{c}$ for Cases I to III and this figure will be discussed further later.

The samples for analysis are collected at points located at least $10\delta_L$ downstream of the flame holder and at least $\delta_L$ apart from any boundaries to avoid any possible influences from the flame holder and boundary conditions. Moreover, sample points are restricted in the range $0.1 < \tilde{c} < 0.9$ to ensure
meaningful statistics by avoiding regions with $\bar{\dot{w}}$ close to zero [5, 6]. Three sample positions, $x_o^+ = 16.7, 20.4$ and $24.2$ (see Fig. 1) as in Ref. [21], are selected for analysis. For each $x_o^+$ position, a series of $\bar{c}$ values, $0.2, 0.3, \ldots, 0.8$, distributed along the transverse direction are chosen in each of the two flame branches. The separation distances in all three directions $\Delta_x, \Delta_y$ and $\Delta_z$ are taken from these sample positions and the correlation function $\Omega_1(\Delta)$ is finally averaged between the two flame branches.

4. Results and discussion

4.1. Flame features

First the contour plots of $\bar{c}$ in the $(x^+, y^+)$ plane are shown in Fig. 1 for the three cases, together with the three streamwise positions used in the analysis. As in typical V-flames, the orientation angle of the mean flame brush to the reactant stream increases with downstream position resulting in a slightly curved mean flame. The effects of the turbulence intensity, $u_{\text{rms}}^+$, and the mean inlet velocity, $\bar{u}^+$, can also be observed from Fig. 1. The thickness of the turbulent flame brush increases with $u_{\text{rms}}^+$ and the separation between the flame branches is further reduced by the larger value of $\bar{u}^+$ in Case III, which is in agreement with earlier observation [39].

Figures 2 and 3 illustrate typical contours of the instantaneous reaction rate, $\dot{\bar{w}}$, and its time derivative, $\ddot{\bar{w}}$, respectively in the mid $(x^+, y^+)$ plane. The values of $\dot{\bar{w}}$ and $\ddot{\bar{w}}$ are normalised by $\rho_u S_L / \delta_L$ and $\rho_u S_L^2 / \delta_L^2$, respectively. The $\dot{\bar{w}}$ contours are confined to thin regions and typical thickness of the reaction zone, $\delta_{\dot{\bar{w}}}$, is about one laminar flame thickness $\delta_L$. This thickness remains almost unchanged along the flame front except in regions with large
curvature where $\delta \dot{w}$ becomes larger up to $2\delta_L$ and this is clear in Case III. The influence of turbulence is also observed as the curvature of the contours increases from Case I to Case III with increasing turbulence intensities, generating gradually more sinuous contours.

The contours of $\ddot{w}$ in Fig. 3 retain the same overall wrinkling patterns as the $\dot{w}$ contours. Profiles through the local flame normal exhibit two adjacent positive and negative peaks due to the transit of the reaction zone. The instantaneous direction of displacement of the flame front is along a line going from the negative to positive peaks of $\ddot{w}$, and is predominantly in the streamwise direction due to the strong mean flow velocity. The thickness of the $\ddot{w}$ contours, $\delta \ddot{w}$, is similar between Cases I and II, and the single positive or negative peak of the $\ddot{w}$ contours is confined to a thinner region compared to the thickness $\delta \dot{w}$ of the $\dot{w}$ contours in Fig. 2. The thickness $\delta \ddot{w}$ for Case III, however, increases due to the higher mean streamwise velocity, resulting in the thicker $\delta \ddot{w}$ for a single peak than the thickness $\delta \dot{w}$.

4.2. Correlation function and model

The results of the two-point correlation function $\Omega_1(\Delta^+)$ for the rate of change of reaction rate fluctuation $\dot{w}$ are shown in Figs. 4–6 for Cases I to III, respectively. In these figures, the separation distances are taken in the streamwise, $\Delta_x$, transverse, $\Delta_y$, and spanwise, $\Delta_z$, directions; three $x_o^+$ positions and seven transverse positions, denoted by $\tilde{c}$ inside the flame brush, are shown giving 21 groups of data in total for each flame case and each direction. As can be seen from Figs. 4–6, the correlation function $\Omega_1$ is symmetric about $\Delta^+ = 0$ in all cases and directions. The value of $\Omega_1$ drops quickly from one within a short distance and negative values are observed,
i.e. $\Omega_1$ is as small as $-0.4$ at $|\Delta_y^+| \approx 0.6$ for Cases I and II and at $|\Delta_x^+| \approx 0.9$

for Cases III. For Cases I and II with low turbulence levels, the negative zones
for $\Omega_1(\Delta_y^+)$ arise due to the adjacent positive and negative peaks in the $\dot{w}$
contours (see Fig. 3) in which the flame normal is predominantly aligned in
the transverse, $y$, direction. The highly turbulent Case III is associated with
highly intermittent reaction rate signals in space as well as in time. The
large increase in the intermittency of the reaction zone makes it obvious a
strong convection in the streamwise direction which accounts for the negative
zones of $\Omega_1(\Delta_x^+)$, as will be explained below. The mean velocities in other
directions are much smaller and the respective convection effect is negligible.

The results from the V-flame DNS data enables the development of
a model for $\Omega_1$, plotted as solid lines in Figs. 4–6. Previous studies
on combustion noise [5, 6] and jet noise [40] have indicated that the
correlation of the noise sources can be represented reasonably well by a
Gaussian-type function. Small negative values of the correlation function
$\Omega_1$ were also observed by Swaminathan et al. [5, 6] but were ignored in
the model of a standard Gaussian function. In this work, a combination of
Hermite-Gaussian functions of zero and second order is used to model the
negative zones in the $\Delta_x$ and $\Delta_y$ directions as well as a standard Gaussian
function (zero-order Hermite-Gaussian function) in the $\Delta_z$ direction. These
models developed from the V-flame DNS data are:
\[ \Omega_1(\Delta^+) = \Omega_1(\Delta^+_x) \cdot \Omega_1(\Delta^+_y) \cdot \Omega_1(\Delta^+_z), \]  
\text{(12a)}

and  
\[ \Omega_1(\Delta^+_x) = \left(1 - \epsilon_x \pi \Delta^+_x \right) \exp \left( -\sigma_x \pi \Delta^+_x \right), \]  
\text{(12b)}

\[ \Omega_1(\Delta^+_y) = \left(1 - \epsilon_y \pi \Delta^+_y \right) \exp \left( -\sigma_y \pi \Delta^+_y \right), \]  
\text{(12c)}

\[ \Omega_1(\Delta^+_z) = \exp \left( -\sigma_z \pi \Delta^+_z \right). \]  
\text{(12d)}

The Hermite-Gaussian functions for \( \Omega_1(\Delta^+_x) \) and \( \Omega_1(\Delta^+_y) \) are inspired by a correlation model of the reaction rate fluctuation, \( \dot{w}' \), in an earlier study [41]. In that work, the convection effect in the streamwise direction led to a coupled two-point space-time correlation function for \( \dot{w}' \) as

\[ \Omega(\Delta^+_x, \tau^+) = \exp \left[ -\sigma_1 \pi (\Delta^+_x - \bar{u}^+ \tau^+) - \sigma_2 \pi (\Delta^+_x + \bar{u}^+ \tau^+) \right], \]  
\text{(13)}

where the constants \( \sigma_1 \gg \sigma_2 \). Taking a double time derivative, \( \partial^2 \Omega / \partial \tau^+^2 \), of Eq. (13) at \( \tau^+ = 0 \) and recognising that \( \Omega(0,0) = 1 \), one gets the form of Hermite-Gaussian functions as in Eq. (12b).

In the correlation model given in Eq. (12), the coefficients \( \sigma_x, \epsilon_x, \sigma_y, \epsilon_y, \sigma_z \) represent the rate of decay for \( \Omega_1 \) from unity to zero. These coefficients are given in Table 2. They take different values in different directions and directions.

<table>
<thead>
<tr>
<th>Case</th>
<th>( \sigma_x )</th>
<th>( \epsilon_x )</th>
<th>( \sigma_y )</th>
<th>( \epsilon_y )</th>
<th>( \sigma_z )</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>0.8</td>
<td>0.0</td>
<td>2.0</td>
<td>3.4</td>
<td>0.2</td>
</tr>
<tr>
<td>II</td>
<td>2.0</td>
<td>0.0</td>
<td>1.2</td>
<td>1.8</td>
<td>0.5</td>
</tr>
<tr>
<td>III</td>
<td>0.6</td>
<td>1.0</td>
<td>1.5</td>
<td>0.0</td>
<td>0.9</td>
</tr>
</tbody>
</table>

Table 2: Coefficient constants of decay rate in the correlation model (12) for \( \Omega_1 \).
vary from case to case. For the streamwise $\Delta_x$ direction, the value of the correlation function $\Omega_1$ drops rapidly from 1 to about 0.05 within about half to one laminar flame thickness $\delta_L$. The dynamics and fluctuation levels of the chemical reactions occurring in thin regions are predominantly controlled by the dynamics of small scale turbulence \[5, 6\], and this accounts for the sharp fall of the correlation of the fluctuating $\ddot{w}$ in space. The decay rate in the $\Delta_y$ direction is close to that in the $\Delta_x$ direction for Case III, but it becomes greatly faster for Cases I and II because of the large transverse gradients of fluctuating quantities within the relatively thinner flame brush of these two cases as shown in Fig. 1. The correlation in the spanwise direction $\Omega_1(\Delta_z^+)$, however, drops slowly in comparison with the other two directions. The smallest value of $\sigma_z = 0.2$ occurs for Case I and $\Omega_1(\Delta_z^+)$ reaches zero within about $3\delta_L$.

As can be seen from Figs. 4–6, the Hermite-Gaussian function given in Eq. (12) represents a reasonable approximation to the DNS results including the negative values. A very small variation of $\Omega_1$ with $x^+$ and $\bar{c}$ is seen within the flame brush, which confirms the independence of the correlation function on spatial position as has been claimed in Section 2.2. In addition, small oscillations of $\Omega_1$ at large values of $|\Delta_x^+|$ are seen in Cases II & III, and small negative values of $\Omega_1(\Delta_z^+)$ occur at some spatial positions for all cases. These are due to the limited statistical samples available for averaging since these oscillations increase when the sample size is halved. Nevertheless, the level of agreement between the DNS and modelled values of the correlation function $\Omega_1$ in Figs. 4–6 is acceptable.

Similar to the two-point correlation for $\ddot{w}$, one can write another
correlation function, \( \Omega \), for the reaction rate fluctuation as

\[
\dot{w}'(y - \Delta/2, t) \dot{w}'(y + \Delta/2, t) = \Omega(y, \Delta) \dot{w}'^2(y, t) .
\] (14)

This correlation has no explicit effect on the OASPL of combustion noise as it does not appear in Eq. (10). The results of \( \Omega \) are presented in this paper merely for comparison with the correlation function \( \Omega_1 \). Figure 7 shows the correlation function \( \Omega \) for Case III exhibiting very similar features as those of \( \Omega_1 \) in Figs. 4–6 except for the broader curves and less distinct negative zones. It can be seen that \( \Omega \) can be approximately modelled as the standard Gaussian function \( \exp(-\sigma \pi \Delta^+ \Delta^+) \) but with slower decay coefficients \( \sigma_x = 0.6, \sigma_y = 1.0 \) and \( \sigma_z = 0.5 \) as expected. The value of \( \Omega \) reaches zero around \( |\Delta_x^+| = 2, |\Delta_y^+| = 1 \) and \( |\Delta_z^+| = 3 \), larger than the respective values of 0.5, 0.5 and 1.5 for \( \Omega_1 \). This is because the time derivative of the fluctuating reaction rate, \( \ddot{w} \), is associated with stronger fluctuations and hence is correlated within a shorter length than the reaction rate fluctuation, \( \dot{w}' \), itself. Note that the decay rate \( \sigma_x \) remains the same for both correlation functions \( \Omega \) and \( \Omega_1 \) because \( \Omega_1 \) retains the exponential form of \( \Omega \) after the double time derivative.

4.3. Correlation volume and length scale

The integral length scale for \( \ddot{w} \), normalised by the laminar flame thickness \( \delta_L \), is defined as

\[
\ell_1^+ = \int_0^\infty \Omega_1(\Delta^+) \, d\Delta^+ .
\] (15)

By using the Hermite-Gaussian function model for \( \Omega_1 \) described by Eq. (12), the integral length scales in the streamwise, transverse and spanwise directions are calculated as
because

\[
\int_{-\infty}^{\infty} \exp\left(-\sigma \pi \Delta^{+2}\right) \, d\Delta^{+} = \sigma^{-1/2}
\]

and

\[
\int_{-\infty}^{\infty} \Delta^{+2} \exp\left(-\sigma \pi \Delta^{+2}\right) \, d\Delta^{+} = \frac{1}{2\pi} \sigma^{-3/2}
\]

through analytical integration. The calculated values of \(\ell_{1,x}^{+}, \ell_{1,y}^{+}, \ell_{1,z}^{+}\) are given in Table 3 by applying the constants of decay rate in Table 2, and they show more clearly the anisotropy of the correlation function \(\Omega_1(\Delta)\).

Therefore, the integral of \(\Omega_1\) over the correlation volume in Eq. (10) can be evaluated as

\[
\int_{V_{\text{cor}}} \Omega_1(\Delta) \, d^3\Delta = V_{\text{cor}}^{+} \, \ell_{L}^{3},
\]

Table 3: Integral length scales \(\ell_{1}^{+}\) in the \(x, y, z\)-directions, integral correlation volume \(V_{\text{cor}}^{+}\) and its length scale \(\ell_{\text{cor}}^{+}\).

<table>
<thead>
<tr>
<th>Case</th>
<th>(\ell_{1,x}^{+})</th>
<th>(\ell_{1,y}^{+})</th>
<th>(\ell_{1,z}^{+})</th>
<th>(V_{\text{cor}}^{+})</th>
<th>(\ell_{\text{cor}}^{+})</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>0.56</td>
<td>0.05</td>
<td>1.12</td>
<td>0.27</td>
<td>0.64</td>
</tr>
<tr>
<td>II</td>
<td>0.35</td>
<td>0.11</td>
<td>0.71</td>
<td>0.23</td>
<td>0.61</td>
</tr>
<tr>
<td>III</td>
<td>0.11</td>
<td>0.41</td>
<td>0.53</td>
<td>0.19</td>
<td>0.57</td>
</tr>
</tbody>
</table>
where the non-dimensional integral correlation volume

\[ V_{\text{cor}}^+ = 8 \ell_{1,x}^+ \ell_{1,y}^+ \ell_{1,z}^+ = \left( 1 - \frac{1}{2} \epsilon_x \sigma_x^{-1} \right) \left( 1 - \frac{1}{2} \epsilon_y \sigma_y^{-1} \right) \left( \sigma_x \sigma_y \sigma_z \right)^{-1/2}. \]  

(20)

The values of \( V_{\text{cor}}^+ \) are 0.27, 0.23 and 0.19 for Cases I, II and III respectively, and they are all around 0.2 despite the variation of \( \ell_1^+ \) in different directions and cases. This can be accounted for by the similar thermo-chemical processes of the three cases that dictate the correlation function \( \Omega_1 \) and hence the correlation integral of Eq. (19), and it confirms that the correlation length scale does not depend on turbulence [5, 6]. Note that this correlation volume evaluated in all three directions and with the negative zones considered is about 1.5 to 2 times larger than the value \( \delta_3^3/8 \) estimated in Refs. [5, 6] using the correlation in the mean flame propagation direction only which was modelled by a standard Gaussian function \( \exp(-4\pi \Delta^+^2) \).

The length scale of \( V_{\text{cor}}^+ \) is obtained as \( \ell_{\text{cor}}^+ = V_{\text{cor}}^+^{1/3} \) and its values are 0.64, 0.61 and 0.57, as given in Table 3, for Cases I, II and III respectively. This length scale is about 0.6\( \delta_L \) which is slightly larger than 0.5\( \delta_L \) reported in Refs. [5, 6]. Similarly, for the correlation function \( \Omega \) shown in Fig. 7 for Case III, the integral volume is 1.8\( \delta_L^3 \) and its length scale is 1.2\( \delta_L \), almost twice the length scale \( \ell_{\text{cor}} \) for \( \Omega_1 \). This relationship between the length scales of the correlation functions \( \Omega_1 \) and \( \Omega \) is in agreement with the finding in Refs. [5, 6] that the integral length scale of \( \Omega_1 \) is half the integral length scale of \( \Omega \).
4.4. Prediction of combustion noise level

With the second integral over $v_{\text{cor}}$ in Eq. (10) evaluated separately, the far-field OASPL of combustion noise can be rewritten as

$$p^2(x, t) = \frac{(\gamma - 1)^2}{16\pi^2\tau^2 a_0^4} Y_f^2 H^2 V_{\text{cor}}^+ \delta L S_L^2 \int_{v_f} \mathcal{K}^+ \overline{w(y, t)}^2 d^3 y,$$

where the non-dimensional parameter $\mathcal{K}^+$ has been normalised using the inverse of the laminar flame time, $1/\tau_f = S_L/\delta L$. The remaining integral represents the contribution from the turbulence through the parameter $\mathcal{K}$, mean reaction rate, $\overline{w}$, and flame-brush size, $v_f$.

First, the parameter $\mathcal{K}$ can be calculated directly from the DNS data of the V-flames using the definition in Eq. (7), i.e. $\mathcal{K} = (\overline{\omega^2})^{1/2}/\overline{\omega}$, for the three cases. This approach of obtaining $\mathcal{K}$ is inherently more consistent than that used by Swaminathan et al. [5, 6] in which the parameter $B_1$ in Eq. (8) was derived indirectly from the DNS flames [16–18] whereas $B$ was obtained from the laser diagnostics data [19]. In addition, both $\mathcal{K}$ and $B_1$ were treated in Refs. [5, 6] as a constant from all the numerical and experimental flames, and a direct measurement of the parameter $B_1^+$ was therefore suggested for a more rigorous modelling.

Figure 8 shows the variation of $\mathcal{K}^+$ and $B_1^+$ within the flame brushes for Cases I to III. It can be seen that both $\mathcal{K}^+$ and $B_1^+$ vary with the $x_o^+$ and $\tilde{c}$ positions and they increase at the lower bound of $\tilde{c}$ due to the decreases in the mean reaction rate $\overline{\omega}$ and the RMS value of reaction rate fluctuation $\sqrt{\overline{\omega^2}}$ in which $\overline{\omega}$ drops more rapidly than $\sqrt{\overline{\omega^2}}$. A rise of $\mathcal{K}^+$ at the upper bound of $\tilde{c}$ can also be seen for Case III. In order to simplify the prediction of combustion noise level, $\mathcal{K}^+$ is approximated to be constants of 18, 40 and
Table 4: Operating conditions of the experimental flames for OASPL prediction. The fuel type is natural gas for No. 1–8 and propane for the other flames.

<table>
<thead>
<tr>
<th>No.</th>
<th>$D$ (mm)</th>
<th>$U_b$ (m s$^{-1}$)</th>
<th>$\phi$</th>
<th>$u_{rms}^+$</th>
<th>$S_L$ (m s$^{-1}$)</th>
<th>$\delta_L$ (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10.9</td>
<td>21.8</td>
<td>1.02</td>
<td>1.80</td>
<td>0.40</td>
<td>0.42</td>
</tr>
<tr>
<td>2</td>
<td>10.9</td>
<td>19.0</td>
<td>0.82</td>
<td>1.77</td>
<td>0.31</td>
<td>0.48</td>
</tr>
<tr>
<td>3</td>
<td>10.9</td>
<td>21.8</td>
<td>1.02</td>
<td>1.31</td>
<td>0.40</td>
<td>0.42</td>
</tr>
<tr>
<td>4</td>
<td>6.4</td>
<td>24.1</td>
<td>0.90</td>
<td>0.54</td>
<td>0.36</td>
<td>0.44</td>
</tr>
<tr>
<td>5</td>
<td>6.4</td>
<td>24.1</td>
<td>1.08</td>
<td>0.48</td>
<td>0.40</td>
<td>0.41</td>
</tr>
<tr>
<td>6</td>
<td>17.3</td>
<td>17.4</td>
<td>1.02</td>
<td>1.74</td>
<td>0.40</td>
<td>0.42</td>
</tr>
<tr>
<td>7</td>
<td>34.8</td>
<td>8.6</td>
<td>1.02</td>
<td>2.69</td>
<td>0.40</td>
<td>0.42</td>
</tr>
<tr>
<td>8</td>
<td>10.9</td>
<td>21.8</td>
<td>0.95</td>
<td>0.81</td>
<td>0.38</td>
<td>0.43</td>
</tr>
<tr>
<td>9</td>
<td>10.9</td>
<td>16.3</td>
<td>0.67</td>
<td>1.84</td>
<td>0.19</td>
<td>0.61</td>
</tr>
<tr>
<td>10</td>
<td>6.4</td>
<td>32.2</td>
<td>0.80</td>
<td>0.74</td>
<td>0.30</td>
<td>0.45</td>
</tr>
<tr>
<td>11</td>
<td>17.3</td>
<td>17.4</td>
<td>1.03</td>
<td>4.67</td>
<td>0.43</td>
<td>0.37</td>
</tr>
<tr>
<td>12</td>
<td>17.3</td>
<td>17.4</td>
<td>1.03</td>
<td>0.97</td>
<td>0.43</td>
<td>0.37</td>
</tr>
<tr>
<td>13</td>
<td>17.3</td>
<td>8.7</td>
<td>0.99</td>
<td>0.83</td>
<td>0.43</td>
<td>0.37</td>
</tr>
</tbody>
</table>

68, marked as dashed lines in Fig. 8, for Cases I, II and III respectively. The increasing values of $K^+$ reflect the corresponding turbulence levels in the three cases (see Table 1).

Recently, Rajaram [20] and Rajaram & Lieuwen [42] reported combustion noise measurements of the far-field OASPL from statistically stationary, pilot stabilised, turbulent premixed flames. These axisymmetric flames with
burner diameter $D$ cover a wide range of turbulence and thermo-chemical conditions in terms of turbulence intensity, $u_{\text{rms}}$, mean burner exit velocity, $U_b$, fuel type, equivalence ratio, $\phi$, etc. Following Swaminathan et al. [5, 6], thirteen natural gas- and propane-air flames are selected arbitrarily out of the large database of flame conditions obtained by Rajaram [20], and their flame conditions as listed in Table 4 will be used for the OASPL prediction in this study. The combustion noise level was measured in the far field at $r = 1.02$ m in an anechoic facility with an estimate of the maximum error $\sim \pm 2$ dB. Further details of these experimental flames can be found in Ref. [20]. These flames are specifically chosen since one of the objectives of this study is to investigate the sensitivity of the far-field OASPL to the changes in $K^+$ values and correlation volume.

To predict the far-field OASPL for the thirteen flames in Table 4, RANS data of the mean reaction rate field, $\bar{\dot{w}}(y, t)$, is required for the volume integral in Eq. (21). Swaminathan et al. [5, 6] performed RANS calculations of the flames in Table 4 using a standard $k-\varepsilon$ turbulence model. The mean reaction rate, $\bar{\dot{w}}$, was calculated using an algebraic closure [43] and a recent scalar dissipation rate model [44]. The spatial distribution of $\bar{\dot{w}}(x, R)$ from the RANS results [5, 6] and the values of $K^+$ obtained from the DNS of V-flames [21] are employed to compute the integral over the flame brush in Eq. (21). The axisymmetry of the Bunsen flames allows the differential volume to be written as $\text{d}^3y = 2\pi R \text{d}R \text{d}x$ where $x$ and $R$ denote the axial and radial coordinates.

The integral correlation volume $V_{\text{cor}}^+$ obtained from Cases I to III in Section 4.2 and the laminar flame speed and its thickness given in Table 4 are
used in Eq. (21) to predict the far-field OASPL for all 13 flames. Figure 9 shows the comparison between the predicted OASPL, the calculations by Swaminathan et al. [5, 6] and the measured results of Rajaram [20] together with the error bar of ±2 dB. Note that the OASPL obtained by Swaminathan et al. [5, 6] would somewhat decrease if the negative values were considered in their correlation model. As shown in Fig. 9, the agreement between the predicted and measured noise levels are observed to be very good for Cases II and III. When compared to Swaminathan et al.’s calculations, the predictions of Case III exhibit even better degree of agreement with measurements in 5 flames (No. 1 & 8–11), while Case II overestimates the OASPL by about 0.3 dB resulting in slightly better agreement in most of the flames. For Case I, however, the predicted noise levels are too low and the underprediction is about 6 dB compared to the predictions of Case II.

The difference in the predictions from Cases I to III can be explained by their different turbulence levels and correlation volumes. As noted by Dunstan et al. [21], the turbulence intensities evaluated locally on the \( \tilde{c} = 0.05 \) isosurface, i.e. \( u_{\text{rms}}^+ = 0.6, 1.1 \) and 3.3 for the three cases, are considerably lower than the nominal inlet intensities and only decay slightly with downstream position. On the other hand, in the experimental flames the majority of the values of \( u_{\text{rms}}^+ \) vary between 0.5 to 2.0 (see Table 4) and their average is about 1.3 if flame No. 11 \( (u_{\text{rms}}^+ = 4.67) \) is excluded.

In terms of turbulence intensity, Case II is very close to the experimental flames and this explains its best agreement with the measured noise data among all predictions. The underprediction of 6 dB for Case I is due to its small turbulence intensity which is less than half the average \( u_{\text{rms}}^+ \) for
the experimental flames. With regard to Case III, its high turbulence level
gives the highest predicted noise levels among the three cases, which also
accounts for the closest prediction to measurement for flame No. 11 with the
highest $u_{\text{rms}}^+$ value. This very high turbulence level of Case III ($u_{\text{rms}}^+ = 3.3$),
however, is somewhat compensated by the relatively small correlation volume
$V_{\text{cor}}^+ = 0.19$. The resulting predictions of Case III are only 3.5 dB higher than
the Case II predictions, and still exhibit good agreement with measurements
particularly for the propane flames. The results of the predicted combustion
noise levels shown in Fig. 9 indicate that turbulence plays a significant role
as well as the two-point correlation of $\bar{w}$ in sound emission from turbulent
premixed flames.

5. Concluding remarks

It has been suggested in very recent studies [5, 6] that the sound emission
from open turbulent flames is dictated by the two-point spatial correlation of
the rate of change of heat release rate fluctuation. In this work, recent data
of 3D DNS of turbulent premixed V-flames [21] are analysed to investigate
this two-point correlation and its role in the production of combustion noise,
specifically to address the isotropy of the correlation volume suggested in
Refs. [5, 6] and its influence on the far-field OASPL prediction. The three
DNS cases of V-flames, with inlet turbulence intensities of $u_{\text{rms}}^+ = 1, 2$
and 6, represent turbulent combustion in the regimes of wrinkled flamelets,
corrugated flamelets and thin reaction zones. The correlations of the rate
of change of fluctuating heat release rate in the streamwise, transverse
and spanwise directions within the flame brush are studied, and the time
derivative of the instantaneous reaction rate is calculated using the DNS data stored at discrete time steps to directly construct the correlations. Furthermore, the turbulence controlled parameter $K$, which affects the noise prediction, is directly obtained from the V-flame DNS data.

The two-point correlation function, $\Omega_1$, constructed at a number of positions inside the flame brush has shown that the value of $\Omega_1$ drops quickly from one resulting in a short correlation length scale and negative values. It is also observed that the decay rates of the correlation functions vary in different directions, a conclusion that differs from Swaminathan et al. [6] who suggested that these correlation functions can be isotropic. Also, the correlation is observed to be different for the three flames studied. However, there is no spatial variation of this correlation function inside the flame brush. More importantly, the correlation function $\Omega_1$ including the negative values can be well approximated by a Hermite-Gaussian function, which gives the non-dimensional integral correlation volume to be $V_{\text{cor}}^+ \simeq 0.2$ for all three flames and it is about 1.5 to 2 times larger than the value reported in earlier studies [5, 6]. The length scale, $\ell_{\text{cor}}$, is about 60% of the laminar flame thickness, slightly longer than $0.5 \delta_L$ in Refs. [5, 6] as well. The correlation function of the reaction rate fluctuation is very similar to $\Omega_1$ but with slower decay rates leading to the enlarged correlation volume and length scale. Although an anisotropic behaviour of the correlation function, $\Omega_1$, is observed for statistically multi-dimensional flames, its influence on the correlation volume and its length scale is seen to be small.

The correlation model and the parameter $K^+$ have been applied to predict the far-field OASPL of open turbulent flames measured by Rajaram [20]. The
noise levels calculated using the information obtained from the Cases II and III flames agree very well with the measured values and show slightly better agreement than the calculations reported earlier [5, 6]. The predicted OASPL using Case I DNS data appears too low. The far-field OASPL is found to be sensitive to $K^{+}$, since it has a direct influence as shown by Eq. (21). Also, the value of $K^{+}$ is observed to strongly depend on turbulence and thermo-chemical conditions, and their interactions. Further investigations on DNS flames are therefore suggested: i) more conditions for a possible scaling of the parameter $K^{+}$; and ii) more time steps to inspect how an enlarged sample size would improve the correlation model. A sufficiently long DNS signal is needed to study the spectral characteristics of combustion noise which will be addressed in future.

Acknowledgements

The current research has been conducted under UK Technology Strategy Board contract SYMPHONY AB201K 100539 whose financial support is gratefully acknowledged. N. Swaminathan and T.D. Dunstan acknowledge the support of EPSRC through project EP/FO28741/1.

References


Figure 1: Contours of $\tilde{c}$ in the $(x^+, y^+)$ plane for Cases I to III. Streamwise sample positions 1, 2 and 3 are located at $x_o^+ = 16.7, 20.4$ and 24.2, respectively.
Figure 2: Contours of the instantaneous reaction rate, $\dot{w}^+$ (normalised by $\rho_u S_L/\delta_L$), in the mid $(x^+, y^+)$ plane for Cases I to III.
Figure 3: Contours of the instantaneous time derivative of reaction rate, $\ddot{\bar{w}}^+$ (normalised by $\rho_\infty S_L^2/\delta_L^3$), in the mid $(x^+, y^+)$ plane for Cases I to III.
Figure 4: Correlation function $\Omega_1$ for Case I. Separation distances in $\Delta x$, $\Delta y$, $\Delta z$ directions, streamwise sample positions at $x^+_c = 16.7$, $x^+_c = 20.4$, $x^+_c = 24.2$. 

$\tilde{c} = 0.5$, $\tilde{c} = 0.6$, $\tilde{c} = 0.7$, $\tilde{c} = 0.8$, model.
Figure 5: Correlation function $\Omega_1$ for Case II. Separation distances in $\Delta_x$, $\Delta_y$, $\Delta_z$ directions, streamwise sample positions at $x^*_o = 16.7, 20.4, 24.2$. 
Figure 6: Correlation function $\Omega_1$ for Case III. Separation distances in $\Delta_x, \Delta_y, \Delta_z$ directions, streamwise sample positions at $x^+_o = 16.7, 20.4, 24.2$. 

$\tilde{c} = 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8$. 

Model
Figure 7: Correlation function $\Omega$ for Case III. Separation distances in $\Delta_x$, $\Delta_y$, $\Delta_z$ directions, streamwise sample positions at $x^+ = 16.7, 20.4, 24.2$. 
Figure 8: Parameters $K^+$ and $B_1^+$ within the flame brushes for Cases I to III. Streamwise sample positions at $x_{o1}^+ = 16.7$, $x_{o2}^+ = 20.4$ and $x_{o3}^+ = 24.2$. 
Figure 9: Comparison of OASPL between the experimental data [20], previous calculations [5, 6] and current predictions from Cases I, II and III.