Probing the states of nucleons in exotic nuclei

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Abstract. Many of the most exotic neutron-proton asymmetric nuclei are produced in relatively small numbers in high-energy fragmentation reactions. They are produced as fast secondary beams with energies of 100 MeV per nucleon or more. Developments made and recent results that both exploit and assess fast one- and two-nucleon removal reactions from such secondary beams are reviewed. This includes very recent work that interfaces the sudden, eikonal reaction models used with more ab-initio nuclear structure inputs. The potential use of neutron pick-up reactions to study particle-like states in exotic nuclei is also outlined.

1. Introduction

The theoretical framework of nuclear shell structure, which explained the empirical nucleon magic numbers and their requirement (beyond \(N, Z = 20\)) for an attractive nucleon spin-orbit (\(\vec{\ell} \cdot \vec{s}\)) interaction, emerged less than 40 years after Rutherford’s 1911 paper. See e.g. Ref. [1]. These guiding ideas of nuclear structure and nuclear stability remain at the heart of today’s mean-field, shell- and microscopic models of light- and medium-mass nuclei. It is acknowledged that the actual nuclear many-body systems have strong two (and weaker three) nucleon correlations that induce coupling of independent particle motions to, and mixing with (i) high-lying shell configurations, and (ii) longer-range collective configurations. The precise role and the importance of three-nucleon (3N) forces is less well understood and remains to be clarified. So, in the absence of exact solutions, shell-model, mean-field and beyond mean-field solutions for nuclear wave functions use modified Hamiltonians, the nucleons being restricted to truncated model spaces where they interact through more benign effective interactions, with appropriate renormalisations of operators for calculations of observables. Detailed studies of the nature of these effective (single-particle) states of nucleons in the stable nuclei, e.g. by knocking-out bound protons using high-energy electron beams, reveal this interplay of independent-particle motion with short-, medium- and long-ranged correlations within the nuclear medium, e.g. [2] and references therein. An established result is the expectation that only \(\approx 60 – 65\%\) of the strength expected for an independent-particle will be observed when one removes valence nucleons (protons) from single-particle orbitals near the Fermi-surface of stable nuclei.

Existing and future experimental facilities, such as the RIBF at RIKEN, FRIB at Michigan State University, and FAIR at the GSI, can provide fast secondary beams of the most rare and neutron-rich nuclei. The near-future will bring higher intensities and also higher beam energies, typically in excess of 250 MeV per nucleon. These high energies allow quantitative theoretical calculations of nuclear reaction cross sections based on the sudden (fast adiabatic) and eikonal approximations. Probing the behavior of single nucleons and pairs of valence nucleons in states
Figure 1. Point neutron and proton densities (left) and the nucleon single-particle energies (right), for $^{38}\text{Si}$, computed using spherical (SkX interaction) Skyrme Hartree-Fock calculations.

near the Fermi-surfaces in such asymmetric nuclei has required new experimental and theoretical tools. In particular, fast, direct one- and two-nucleon removal reactions, that exploit fast surface-grazing collisions of the nuclei of the beam with a light nuclear target such as beryllium or carbon, have been shown to be particularly efficient techniques for this purpose [3, 4]. Data and calculations for these reactions are helping to expose the systematic behavior of valence nucleons and suggest novel correlation effects in the most asymmetric nuclei. As was stated above, the reaction methods are considerably simplified when exploiting fast exotic beams, due to the direct, sudden and forward focussed (eikonal) collision approximations [5, 6]. We will review these topics briefly and discuss the necessary interface of such dynamical reaction models with shell- or more microscopic structure models. Recent topical spectroscopic applications and predictions are used to illustrate current uses of these tools, that encourage further developments.

2. Features of neutron-proton asymmetric nuclei
Two novel and generic features which emerge in nuclei having large (abnormal $N:Z$ ratio) asymmetry are represented in Fig. 1 for the neutron-rich nucleus $^{38}\text{Si}$ (with $Z = 14$, $N = 24$). First (in the energy domain) is the development of two displaced Fermi surfaces. Already in $^{38}\text{Si}$, some distance from the neutron drip-line, the experimental neutron and proton separation energies are $S_n = 5.29$ MeV and $S_p = 20.64$ MeV, respectively, with a difference of $\Delta S = 15.35$ MeV. The figure also shows that this $\Delta S$ is rather well predicted by a spherical (SkX interaction) Skyrme Hartree-Fock calculation, shown in the right-hand panel. Second (in the spatial domain, left hand panel) is the presence of a more extended density for the neutrons at the nuclear surface and hence that the valence protons near the Fermi surface are now (spatially) embedded in nuclear matter that is neutron dominated. Thus, in one- or two-valence-proton removal reactions from such a system we might expect enhanced np-correlation effects due e.g. to the strong short-ranged and tensor np interaction. The physics and possible observation of such np-correlations is one motivation of these studies. Clear from the above is that, with large neutron or proton excess, the Fermi surface of the excess nucleon species will approach, couple with and be affected by the continuum. How this affects measurements and the importance and
treatment of correlations is of considerable interest in validating structure models.

3. Fast one- and two-nucleon removal reactions

Full details of the formalism used and the approximations made in the sudden, eikonal description of one- and two-nucleon removal can be found in Refs. [3, 5, 6] and references therein. Experimentally, for a fast beam of projectiles of mass number $A$, one observes the heavy projectile-like reaction residues with masses $(A - 1)$ and/or $(A - 2)$ travelling within a small cone about the forward (beam) direction. Measurements of the momentum distributions of these residues are also made with good statistics, those for two-nucleon removal having recently been shown to have high spectroscopic value [7, 8]; specific recent examples of which will be referenced later. Exclusive yields of the different final states of the fast moving residue, but not the target nucleus, are often detected by Doppler-reconstructed $\gamma$-ray spectroscopy. All such removal measurements are inclusive with respect to the final states of the target and, except for a small number of precision dedicated experiments designed to confront/validate the one-nucleon removal [9] and two-nucleon removal [10] reaction mechanism predictions, are also inclusive with respect to the final states of the removed nucleon(s). So, the measured cross sections include nucleon removal due to both elastic and inelastic interactions of the removed nucleon(s) and the target. These different contributions are calculated and summed in the theoretical calculations.

3.1. Nuclear structure and reactions interface

In the eikonal reaction treatment, the interactions of the residue ($r$) and the removed nucleons with the target nucleus are described by their elastic S-matrix elements, e.g. $S_r(b_r)$. These are expressed as functions of their individual impact parameter, $b_r$, for the residue, with the target nucleus. An important feature of the high-energy ion-ion interaction (the residue-target system at energies of $\approx 100$ MeV per nucleon and above) is its highly-absorptive nature, the S-matrix approximating that of an absorptive black disk of radius determined by the residue and target sizes. The reaction is thus highly geometrical. It is assumed that the residue is not inelastically excited to low-lying collective and/or single-particle states in the fast, grazing collisions of importance to removal events. The black disc of the residue-target interaction at small $b_r$ and the necessity to find and remove one or two nucleons thus localizes the removal reactions to grazing collisions of the nuclear surfaces. Since the target is light, it will sample the wave function(s) of the removed nucleons in a cylindrical volume, with axis in the beam direction, at the surface of the projectile. It is evident that:

(i) in two-nucleon removal, cross sections will be sensitive to the proximity (spatial correlations) of pairs of nucleons near the surface, and can therefore also provide a means to probe these [11],

(ii) the reaction-mechanism-sampled spatial volume invokes no spin-selection, e.g. of spin $S = 0$ over $S = 1$ pairs, and the reaction will sum these contributions,

(iii) the removal mechanism imposes no linear or angular momentum matching requirement, so the mechanism will see all nucleon configurations that have a non-vanishing probability amplitude in the sampled volume,

(iv) the reaction proceeds through the parentage/component of the residual nucleus state present in the entrance channel, i.e. the projectile ground-state.

More formally, in one-nucleon removal from the incident projectile ground-state $|A Y(J^\pi_i)\rangle$ to a given residue final state $|A - 1 X(J^\pi_f)\rangle$ one is sensitive to the nucleon radial overlap function $I_{\ell j}(r)$, whose norm is the spectroscopic factor for the transition, $S(J_i, J_f \ell j)$. Schematically,

$$\langle 1, A - 1 X(J^\pi_f) | A Y(J^\pi_i) \rangle \rightarrow I_{\ell j}(r)/r , \quad \int_0^\infty |I_{\ell j}(r)|^2 dr = S(J_i, J_f \ell j) . \quad (1)$$

The spectroscopic factors in the following are provided by: (a) the shell-model, or (b) microscopic calculations [12]. For case (a), the shell-model, the geometry of the potentials used
Figure 2. Ratios of measured to theoretical one-nucleon removal cross sections as a function of the Fermi-surface asymmetry $\Delta S$. As indicated by the inserts, $\Delta S$ is defined positive when a more strongly-bound nucleon is removed and negative for removal of a nucleon of the weakly-bound species. The figure is adapted from [13].

to generate the radial form factors is constrained using Hartree-Fock calculations; for details see Section III of [13]. As we have seen in Figure 1, the Hartree-Fock predicts the nuclear size and binding systematics reasonably well into the neutron-rich nuclei. In case (b), when using variational Monte Carlo (VMC) or no-core shell-model (NCSM) microscopic calculations, both the $I_{ij}(r)$ and their norms are then given by the structure models.

For two-nucleon removal, no microscopic calculations of the two-nucleon overlap functions are yet available, thus the shell-model is used. For a spin $J_1^π = 0^+$ projectile, as is the most usual and for simplicity, then for a transition to a given residue final state $|^{4-2}X(J^π)\rangle$ (that removes two nucleons with total angular momentum quantum numbers $J - M$) \[5, 6\]

$$
\langle 1, 2, ^{A-2}X(J^π) | ^A Y(0^+) \rangle \rightarrow \sum_{j_1j_2} (-)^{J+M} \frac{C(j_1j_2;J)}{J} \langle \phi_{j_1} \otimes \phi_{j_2} | J-M \rangle .
$$

The shell-model provides the amplitudes $C(j_1j_2;J)$ for each two-nucleon pair configuration $(j_1j_2)$, called the two-nucleon amplitudes (TNA). Computations of the radial form factors associated with each normalised nucleon shell-model wave function $\phi_{j_2}$ are constrained using Hartree-Fock calculations, as was discussed above for the one-nucleon removal case, see also [13].

3.2. One-nucleon removal: Dependence on neutron-proton asymmetry

The theoretical scheme outlined above permits systematic calculations of nucleon removal cross sections, without adjustable parameters, using shell-model structure input and with complex interactions (S-matrices) and nucleon orbital rms radii constrained by the experimental (reproduced by the Hartree-Fock) systematics. This has now been done in many cases for data sets with energies near and above 100 MeV per nucleon, including both stable [14] and very exotic nuclei. The summary of the cross section magnitudes, presented as the ratio $R_s = \sigma_{\text{exp}}/\sigma_{\text{th}}$, of the measured to the theoretical cross sections, is presented in Fig. 2, adapted from [13]. Additional data sets, e.g. for neutron and proton removal from $^{36}$Ca [15], having $\Delta S = \pm 16.55$ MeV, agree
in their preliminary analysis with these systematics. The points for stable nuclei, near \( \Delta S = 0 \) in the centre of the plot, are compared with the electron-induced knockout expectations with reasonable agreement. The departures of the systematics from this \( R_s \approx 0.5 - 0.7 \) value, and in particular the small values of \( R_s \) when removing well-bound valence nucleons with large positive \( \Delta S \), has been the subject of much interest and discussion. From a structural point of view, since these calculations use the shell-model, questions arise concerning (i) its use of a highly-truncated model space, (ii) its treatment of the continuum, and (iii) the potential role of 3N-forces, among others.

Very recent work [12] has attempted to make a first assessment of such questions by interfacing reaction measurements and calculations with microscopic nuclear structure models. Specifically, to use the one-nucleon overlap functions (and point nucleon densities for the optical interactions) from variational Monte-Carlo (VMC) and converged multi-shell (12\( h \Omega \)) no-core shell-model calculations (NCSM). These overlaps are available only for light nuclei and were used to compare with measurements for the neutron removal reactions from \(^{10}\text{Be}\) and \(^{10}\text{C}\), bound by 6.8 and 21.3 MeV, respectively. Both microscopic models use realistic two-nucleon (2N) forces, the Argonne \( v_{18} \) (VMC) and CDBonn (NCSM) interactions. In addition the VMC includes the Urbana IX 3N interaction. More significantly the NCSM, like the shell model, uses an oscillator basis whereas the VMC has a more flexible basis with which to incorporate continuum effects; e.g. the proton separation energy in the \(^9\text{C}\) residue is just 1.3 MeV. The results show [12] that the changes to the \( p \)-shell-model spectroscopic factors and overlap functions due to the 12\( h \Omega \) multi-shell model space are relatively modest and do not lead to significant changes. The VMC calculations on the other hand lead to significantly reduced spectroscopic factors and a closer agreement with experiment, with calculated [measured] cross sections of 72.8 [73(4)] mb and 30.8 [23.2(10)] mb for \(^{10}\text{Be}\) and \(^{10}\text{C}\), respectively. Further precision measurements to facilitate such testing against \textit{ab-initio} structure models is suggested by these results. These first results do not yet quantify the importance of 3N versus the improved treatment of near-continuum effects in the VMC approach, but are highly suggestive that oscillator-basis methods are severely challenged by the near-continuum Fermi-surface in the \(^{10}\text{C}\) to \(^9\text{C}\) neutron removal case.

Another very recent relevant theoretical study [16] calculates the one-nucleon overlaps and spectroscopic factors for the oxygen isotopes, \(^{14-28}\text{O}\), within the coupled-cluster method. The radial overlaps are thus also available but have not yet been used in reaction studies. An important conclusion of this work is that when including continuum-coupling effects, by the use of a Hartree-Fock (HF) basis built from a Woods-Saxon single-particle basis (that treats bound and continuum states on an equal footing), there is significant suppression of the spectroscopic factors for the removal of well-bound valence protons with increasing neutron excess; see Figure 3 of Ref. [16]. Moreover, the same effect could not be reproduced when trying to build the HF basis using an harmonic oscillator (HO) single-particle starting point, even when this HO basis spanned 17 major oscillator shells. These calculations suggest strongly, see also Fig. 1, that the proximity (in the energy domain) of the neutron continuum in the most neutron-rich isotopes has a significant coupling (correlation effect) on the well-bound but valence proton orbitals. Both new measurements and reaction calculations are needed to confirm these predictions.

### 3.3. One- and two-nucleon removal: momentum distributions and spectroscopy

The spectroscopic value of residue momentum distributions following one-nucleon removal has been recognized and used for some time. Here the momentum distribution has a shape and a width that are characteristic of the orbital angular momentum of the orbit from which the nucleon has been removed. When measured exclusively, using \( \gamma \)-spectroscopy to identify the final state populated, the location and ordering and states can be compared with nuclear structure predictions and provide a handle on the quality of the effective interactions used.

\( \gamma \)-spectroscopy is not possible in the case of unbound final states. So, in neutron removal from
Figure 3. Measured and calculated inclusive momentum distributions following two-proton removal from $^{36}$Mg (left) and two-neutron removal from $^{22}$Mg (right). The figure is adapted from Ref. [8].

neutron-rich systems leading to unbound mass $A - 1$ final states, neutron detection is needed. New data on neutron removal reactions from the most neutron-rich carbon isotopes, $^{19-22}$C, taken at the RIBF at RIKEN at 240 MeV per nucleon [17], suggests such measurements would be of value for e.g. the spectroscopy of the last particle-bound (Borromean) carbon isotope and heavy halo nucleus candidate, $^{22}$C. The data for $^{22}$C of Ref. [17] is for inclusive neutron-removal to unbound $^{21}$C with subsequent neutron evaporation. Measurements are therefore of the yield and the momentum distribution of the bound $^{20}$C residues. The reaction analysis of these data, of removal to the predicted shell-model states of $^{21}$C, and taking account of the additional broadening of the $^{20}$C residue momenta in the neutron evaporation step, are in excellent agreement with the measurements. They and the shell-model suggest a large 1/2$^+$ spectroscopic factor (of $\approx 1.4$) for neutron removal from the $^{22}$C ground state. The calculated cross section has almost equal contributions from $2s_{1/2}$ valence neutron and $1d_{5/2}$ neutron-hole configurations. This provides strong support for the halo character of the $^{22}$C ground state, as suggested previously (but less quantitatively) by a large measured interaction cross section [18]. The $R_\sigma$ values deduced from these new inclusive neutron-rich carbon data are also consistent with the systematics shown in Fig. 2.

The spectroscopic significance of exclusive two-nucleon momentum distributions has been clarified only more recently [7, 8] and few exclusive data sets are available to fully validate these detailed predictions. The inclusive data sets available, and the exclusive data set for two-proton removal from $^{28}$Mg, are for cases where one removes two nucleons of the minority species from already very asymmetric nuclei, which ensures that the reaction mechanism is direct [4, 5]. These available momentum distributions data are very well described [7, 8], as is seen e.g. in Fig. 3. The analysis of Ref. [8] uses an LS-decomposition of the $jj$-coupled two-nucleon overlap of Eq. (2). It shows that the sensitivity of the shape and width of the residue’s momentum distribution is now to the total orbital angular momentum ($L$) content of these overlaps. In cases where a single ($j_1 j_2$) two-nucleon configuration is dominant this translates simply into a strong dependence of the momentum distribution on the final state spin of the residue, $J$, with obvious spectroscopic significance. In more complex, highly-mixed ($j_1 j_2$) configurations the details of the momentum distribution are determined by, and so can also test, the magnitudes and phases of the shell-model TNA. Examples of these sensitivities, which can be large, are given in [8].

A very recent study of the spectroscopy of low-lying excited states in the neutron-rich $N = 28$
nucleus $^{44}\text{S}$, populated using fast two-proton removal from $^{46}\text{Ar}$, has already exploited this momentum distribution sensitivity [19]. The measured exclusive momentum distributions in coincidence with decay $\gamma$-rays were able to clearly characterize populated excited states in the $^{44}\text{S}$ residues as having $J = 2$ and $J = 4$, and so challenge shell model expectations. Such two-nucleon removal reactions, which populate residues that are even more exotic than the projectile, thus offer a rather unique tool to probe the spatial correlations of the removed nucleons [11], the assignment of final-state spins, and also to assess the TNA given by the shell model or alternative nuclear structure models.

4. Fast nucleon pickup reactions
The removal reactions discussed above preferentially populate states in the residual nuclei with a strong hole-like character. We now make brief mention of a recent experiment and reaction mechanism study to ascertain the applicability and potential of fast nucleon pickup reactions [20] for spectroscopic studies of particle-like states. Usually light-ion single-nucleon transfer reactions such as the $(d,p)$ reaction are used for this purpose, however, the overheads of such reactions are high (on both the required beam intensity and detection systems) and in many cases they are currently impractical, particularly for very exotic nuclei. The work of Ref. [20] considered test measurements and associated direct reaction model calculations for reaction events in which a single neutron is picked-up by a fast secondary beam. As in the removal studies, such measurements can employ thick targets and $\gamma$-ray spectroscopy of the pickup residues and thus take full advantage of the properties of fast fragmentation beams. The analysis used measurements for a 84 MeV per nucleon $^{22}\text{Mg}$ beam on beryllium and carbon targets. Related earlier studies considered reactions involving the pickup of a strongly-bound proton from a $^{9}\text{Be}$ target [21, 22].

In summary, it was shown that the pickup reaction on a carbon target (with four well-bound valence neutrons) proceeds predominantly by a (two-body) direct, single-particle pickup mechanism, with neutrons transferred between bound states in the target and the residue. For the beryllium target, the reaction also proceeds through pickup of the well-bound rather than the single weakly-bound valence neutron, leading predominantly to complex multi-particle final states. The carbon target single-particle cross sections, summed over the bound $^{11}\text{C}$ final states, were calculated assuming a unit spectroscopic factor and a separation energy of 12.70 MeV. For $2s_{1/2}$, $1p_{3/2}$, $1d_{5/2}$, and $1f_{7/2}$ $^{23}\text{Mg}$ final states the cross sections were $\sigma_{sp} = 0.04$, 0.58, 3.51, and 11.12 mb, respectively. Thus, high-$\ell$ pickups are considerably enhanced due to the inherent linear and angular momentum mismatch of the reaction. This suggests that future measurements could be used to identify high-$\ell$ neutron intruder components in the low energy spectra of the pickup residues. Such high-$\ell$ intruder configurations are typically angular momentum mismatched and so only weakly coupled in the lower-energy, well-matched $(d,p)$ transfer reactions.

Since fast-pickup cross section measurements do not provide an angular distribution or an $\ell$-dependent pickup residue momentum distribution, they provide insufficient information to determine both the dominant angular momentum of the transferred nucleon and its spectroscopic strength. Like the removal reactions approach, we envisage fast pickup could be used alongside theoretical predictions of level ordering and spectroscopic factors, to study the systematics of level migration predictions along isotopic chains - assessing e.g. different effective interaction predictions. This could provide both complementary information to transfer reaction studies, where these overlap, as well as extend the range of nuclei accessible to experiment.

5. Summary comments
The experimental data underpinning the Rutherford 1911 paper and its interpretation involved the observation of rare backscatter events. Similarly, the basic requirements for nuclear reactions to be useful at today’s fragmentation facilities, where the most exotic nuclei are produced as
fast secondary radioactive ion beams (of relatively low intensity), are high detection efficiency, measurable cross sections, and quantitative theoretical model descriptions. These criteria are satisfied by fast one- and two-nucleon removal and by nucleon-pickup reactions, all of which can and have been measured with good precision. We have shown that these reaction mechanisms offer effective probes of single-particle structure, are contributing to spectroscopic studies in some of the most exotic asymmetric nuclei, and are posing questions on the roles of the continuum, 3N forces, and their resulting correlations in asymmetric systems. Final-state-exclusive cross sections and their parallel momentum distributions can be predicted theoretically after one- and two-nucleon removal and further exclusive measurements can be used to validate, or reject, the one- and two-nucleon correlations predicted by theoretical structure models. It is clear that there will be many new opportunities at the future facilities that promise more intense radioactive beams and thus access to more exclusive measurements.

Acknowledgments
This work was supported by the UK Science and Technology Facilities Council (STFC) under Grant ST/F012012. The significant theoretical and experimental contributions of Ed Simpson, Alexandra Gade, Alex Brown, Daniel Bazin, and the numerous other collaborators to these ongoing reaction investigations is gratefully acknowledged.

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