# Low Density Spreading for Next Generation Multicarrier Cellular Systems

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***Abstract*—Multicarrier-Low Density Spreading Multiple Access (MC-LDSMA) is a promising technique for high data rate mobile communications. In this paper, the suitability of using MC-LDSMA in the uplink for next generation cellular systems is investigated. The performance of MC-LDSMA is evaluated and compared with current multiple access techniques, OFDMA and SC-FDMA. Specifically, Peak to Average Power Ratio (PAPR), Bit Error Rate (BER), spectral efficiency and fairness are considered as performance metrics. The link and system-level simulation results show that MC-LDSMA has significant performance improvements over SC-FDMA and OFDMA. It is shown that using MC-LDSMA can significantly improve the system performance in terms of required transmission power, spectral efficiency and fairness among the users.**

***Index Terms*—Multiple access technique, low density spreading, PAPR, link-level, spectral efficiency, fairness.**

## I. Introduction

Increasing interest in high data rate services in mobile communications demands high spectral efficiency multiple access techniques. Orthogonal Frequency Division Multiplexing (OFDM) is one of the prominent multicarrier transmission techniques. For wireless applications, an OFDM-based system offers greater immunity to multipath fading and impulsive noise [1]. Orthogonal Frequency Division Multiple Access (OFDMA) is the extension of OFDM transmission to a multiuser communication scenario. A major drawback of multicarrier communication systems is their high Peak to Average Power Ratio (PAPR) in comparison to single carrier systems [2]. This high PAPR will reduce the efficiency of high-power amplifiers as more back-off will be required to avoid non-linear effects on multi-carrier signal. Single Carrier Frequency Division Multiple Access (SC-FDMA) [3] is a modified form of OFDMA where Discrete Fourier Transform (DFT) pre-coding is implemented. SC-FDMA benefits from low PAPR comparing to OFDMA making it suitable for uplink transmission by user equipment. In the Third Generation Partnership Project Long Term Evolution (3GPP-LTE) standard [4], OFDMA has been accepted as the downlink scheme and SC-FDMA for uplink. However, other standards use OFDMA in both uplink and downlink, such as Worldwide Interoperability for Microwave Access (WiMAX) [5].

Low Density Spreading Multiple Access (LDSMA) concept is introduced in [6] to manage the multiuser interference and allow overloaded conditions with near single user performance [7]. To overcome the wideband channel disadvantages such as multipath fading channel, Multicarrier-LDSMA (MC-LDSMA) is proposed in [8]. MC-LDSMA combines the two concepts of low density spreading and OFDM to establish an efficient multiple access technique over wideband channels.

In MC-LDSMA, due to low density spreading structure, every data symbol will only be spread on a small subset of subcarriers (effective processing gain), and also every subcarrier will only be used by a small subset of data symbols that could belong to different users. Consequently, MC-LDSMA technique can exploit both the channel and the multiple access interference diversities over the frequency domain, which will improve the link-level performance in terms of Bit Error Rate (BER). Furthermore, as there is no exclusivity in the subcarrier allocation, there is plenty of room to exploit the high degree of flexibility of subcarrier allocation. This resource allocation flexibility can effectively improve the system-level performance such as the spectral efficiency, the supported number of users and the fairness among users. Owning these advantages, MC-LDSMA represents a strong candidate for next generation cellular system as a multiple access technique.

In this paper, we will investigate the suitability of using MC-LDSMA in the uplink for next generation cellular systems. We will carry out a detailed comparison between MC-LDSMA and the current multiple access techniques: SC-FDMA for 3GPP-LTE and OFDMA for WiMAX. The comparison will be done in PAPR, link-level and system-level performance through extensive Monte Carlo simulations. It will be shown that MC-LDSMA and OFDMA have the same PAPR, which is higher than SC-FDMA. However, the incurred loss due to high PAPR comparing to SC-FDMA is compensated by better link-level performance, which makes MC-LDSMA outperform SC-FDMA. Furthermore, the results indicate significant system-level performance improvements over SC-FDMA and OFDMA techniques in terms of spectral efficiency and fairness.

This paper is organized as follows: Section II presents the MC-LDSMA system model. The PAPR and link-level performance evaluation are provided in section III. In section IV, we evaluate and compare the system-level performance for the considered multiple access techniques. Finally, section V is devoted to concluding remarks.

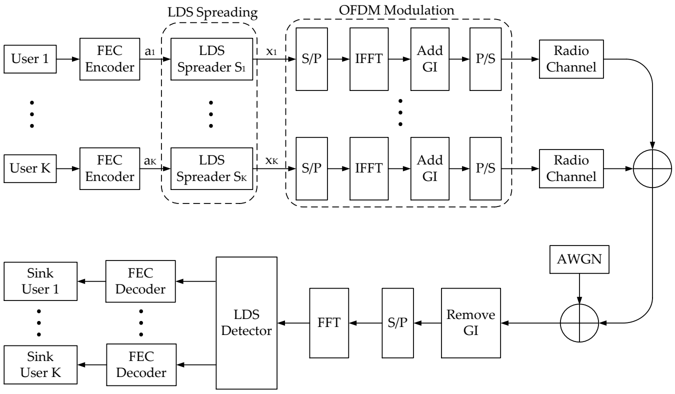


Fig. 1. Uplink MC-LDSMA block diagram.

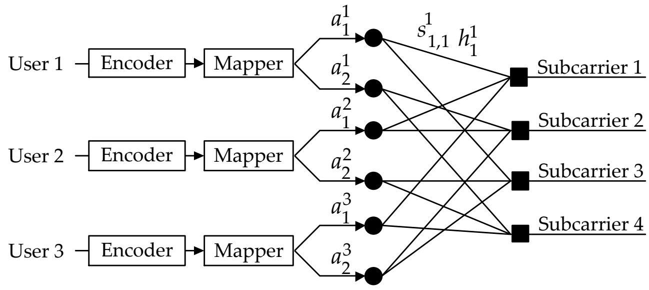


Fig. 2. Graphical representation of MC-LDSMA.

## II. MC-LDSMA System Model

In this section, a single cell uplink MC-LDSMA system model is presented. The conceptual block diagram of an uplink MC-LDSMA system is depicted in Fig. 1. Consider an uplink MC-LDSMA system with a set of users $\mathcal{K} = \{1, \cdots, K\}$ transmitting to the same base station where the base station and each user are equipped with a single antenna. The total frequency band is divided into a set of subchannels (subcarriers/tones) $\mathcal{N} = \{1, \cdots, N\}$. A user $k \in \mathcal{K}$ can transmit over a subset of the subcarriers, with transmission power $p_{k,n}$ over subcarrier $n \in \mathcal{N}$ subject to individual maximum power constraints $P_k : \sum_{n \in \mathcal{N}} p_{k,n} \leq P_k$.

Let $\mathbf{a}_k$ be a data vector of user $k$ consisting of $M_k$ modulated data symbols and denoted as;

$$\mathbf{a}_k = [a_{k,1}, a_{k,2}, \cdots, a_{k,M_k}]^T. \tag{1}$$

The signature matrix $\mathbf{S}_k$ assigned for the $k$th user consists of $M_k$ Low Density Signatures (LDS);

$$\mathbf{S}_k = [\mathbf{s}_{k,1}, \mathbf{s}_{k,2} \cdots, \mathbf{s}_{k,M_k}]. \tag{2}$$

Where each LDS signature, $\mathbf{s}_{k,m} \in \mathbb{C}^{N \times 1}$, is a sparse vector consisting of $N$ chips. Among these $N$ chips only $d_v$ chips have non-zero values, where $d_v$ is the effective spreading factor. Each data symbol $a_{k,m}$ will be spread using the $m$th spreading sequence. Let $\mathbf{x}_k = [x_{k,1}, x_{k,2}, \cdots, x_{k,N}]^T$ denotes the chips' vector belonging to user $k$ after the spreading process which is given by;

$$\mathbf{x}_k = \mathbf{S}_k \, \mathbf{a}_k. \tag{3}$$

So, the whole system's signature matrix has $N$ rows and $M$ columns each containing a unique spreading sequence, where $M$ can be calculated as follows;

$$M = \sum_{k=1}^{K} M_k. \tag{4}$$

The overloading will be $M/N$. Each user's chip will be transmitted over a subcarrier of the OFDM system. Fig. 2 illustrates the MC-LDSMA principle by an example of a system with four subcarriers ($N = 4$) serving three users ($K = 3$) with two data symbols per user ($M_1 = M_2 = M_3 = 2$), which means $150\%$ overloading. Here, the effective spreading factor is two ($d_v = 2$) with each three chips sharing one subcarrier ($d_c = 3$), where $d_c$ denotes the number of users interfere in each subcarrier. The figure shows in more details the process of low density spreading. As it can be observed that each chip represents a subcarrier of OFDM modulation and the data symbols using the same subcarrier will interfere with each other. The system's signatures matrix can be represented by an indicator matrix $\mathbf{I}_{LDS,4\times6}$ which represents the positions of the non-zero chips in each signature;

$$\mathbf{I}_{LDS,4\times6} = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \end{bmatrix}$$

As users are not bound to exclusively use the subcarriers, at the receiver side users' signals that are using the same subcarrier will be superimposed. However, the number of users interfere in each subcarrier is much less than the total number of users, $d_c \ll K$. At the receiver side, after performing OFDM demodulation operation, the received signal is given by;

$$\mathbf{y} = \sum_{k=1}^{K} \mathbf{H}_k \, \mathbf{x}_k + \mathbf{v}, \tag{5}$$

where $\mathbf{v}$ is the Additive White Gaussian Noise (AWGN) and $\mathbf{H}_k$ is the frequency domain channel transfer function of user $k$;

$$\mathbf{H}_k = \mathrm{diag}(h_{k,1}, h_{k,2}, \cdots, h_{k,N}), \tag{6}$$

where $h_{k,n}$ is the channel gain of user $k$ on subcarrier $n$. This signal $\mathbf{y}$ is passed to LDS multiuser detector (MUD) to separate users' symbols. The LDS structure can be captured by a low density graph, thus the detection of MC-LDSMA can be done using close to optimum multiuser detection based on Message Passing Algorithm (MPA) presented in [7]. The complexity of the multiuser detection for MC-LDSMA will turn out to be $\mathcal{O}(|\mathbb{X}|^{d_c})$, which is significantly reduced comparing to complexity of order $\mathcal{O}(|\mathbb{X}|^{K})$ for optimal multiuser detection, where $\mathbb{X}$ denotes the constellation alphabet. More details regarding the LDS receiver can be found in [7].

TABLE I
PAPR AND LINK-LEVEL SIMULATION PARAMETERS

| Parameter | Value |
|---|---|
| Number of users | 6 |
| FFT size | 512 |
| Subcarrier bandwidth | 15 KHz |
| Subcarriers per RB | 12 |
| Number of data RBs | 25 |
| LDS Scheme | $d_c = 3$, $d_v = 3$ |
| Multipath channel model | ITU Pedestrian Channel B |
| Channel coding | Half-rate convolutional code |

## III. PAPR AND LINK-LEVEL PERFORMANCE

In this section, the PAPR and the link-level performance of MC-LDSMA are evaluated and compared with OFDMA and SC-FDMA.

### A. PAPR Comparison

A major drawback of multicarrier communication systems is their high PAPR. This high PAPR will reduce the efficiency of high-power amplifiers as more back-off will be required to avoid non-linear effects on multicarrier signal. This problem is more critical for uplink due to the limited power of the user equipment. Low PAPR comparing to OFDMA was the major reason for choosing SC-FDMA as multiple access technique for 3GPP-LTE. So, it is crucial to evaluate the PAPR of MC-LDSMA and compare it with OFDMA and SC-FDMA. The complementary cumulative distribution function (CCDF) is frequently used as a measure of the PAPR. The CCDF of PAPR denotes the probability that PAPR exceeds a certain value $PAPR_0$, ($Pr\{PAPR > PAPR_0\}$). In addition to the CCDF curves, we will compare the PAPR values that are exceeded with probability less than $0.1\%$, 99.9-percentile PAPR, which is defined by $Pr(PAPR > PAPR_{99.9}) = 10^{-3}$ [9]. The CCDF of PAPR is calculated by Monte Carlo simulation. In the simulations, $10^5$ uniformly random symbols per user were generated to acquire the CCDF of PAPR. The simulation parameters used for PAPR and link-level performance are listed in Table I. In practical systems, the user allocated subcarriers that are grouped into basic units called Resource Blocks (RB) [10]. Considering that the number of resource blocks allocated to each user affects the PAPR [11], we generated the results for different number of resource blocks per user.

Figures 3 and 4 show the CCDF of PAPR for OFDMA, SC-FDMA and MC-LDSMA with one RB and three RBs per user, respectively. The PAPR of OFDMA and MC-LDSMA is the same for different modulation orders, hence, only the results of 16QAM modulation is represented in the figures. As MC-LDSMA uses spreading, it will use more resource blocks comparing to the other two techniques to transmit the same amount of data. So, when we mention 1RB (and 3RB), it is 3RB (and 9RB) for MC-LDSMA because effective spreading factor $d_v = 3$ is used. It can be seen from the figures that MC-LDSMA and OFDMA have the same PAPR values. However, SC-FDMA has lower PAPR, especially for the QPSK modulation. Table II lists the 99.9-percentile PAPR values that each signal experiences for SC-FDMA and MC-LDSMA. For the worst case (QPSK modulation), MC-LDSMA has an 99.9-percentile PAPR 2.38 dB and 3 dB more than SC-FDMA for one RB and three RBs, respectively. Consequently, MC-LDSMA requires 2.38 dB and 3 dB more back-off to avoid

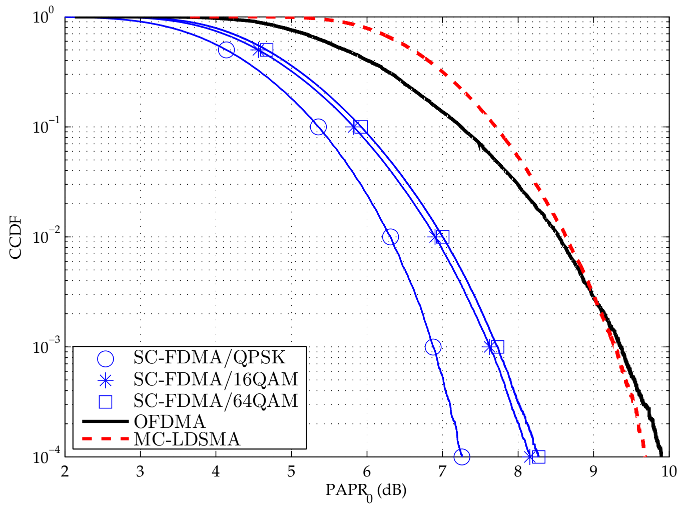


Fig. 3. PAPR comparison for SC-FDMA, OFDMA and MC-LDSMA with one resource block per user.

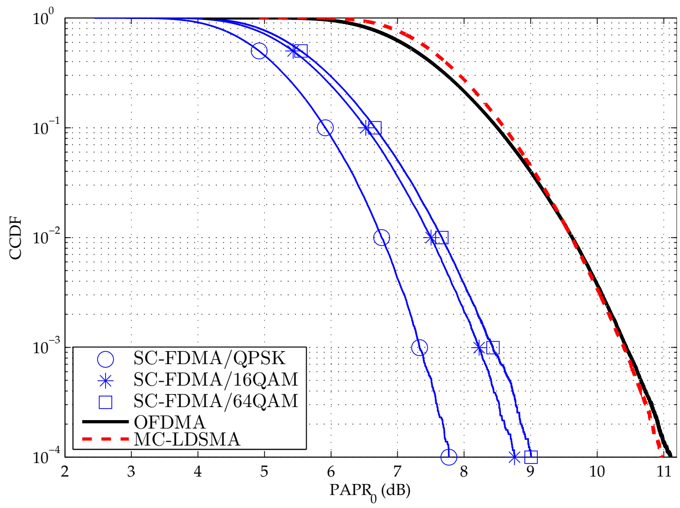


Fig. 4. PAPR comparison for SC-FDMA, OFDMA and MC-LDSMA with three resource blocks per user.

TABLE II
COMPARISON OF 99.9-PERCENTILE PAPR.

| | SC-FDMA | | MC-LDSMA | | Difference | |
|---|---|---|---|---|---|---|
| | 1RB | 3RB | 1RB | 3RB | 1RB | 3RB |
| QPSK | 6.9 | 7.4 | 9.28 | 10.4 | 2.38 | 3 |
| 16QAM | 7.62 | 8.23 | | | 1.66 | 2.17 |
| 64QAM | 7.73 | 8.43 | | | 1.56 | 1.97 |

the non-linear region of high-power amplifiers. This shows SC-FDMA outperforms MC-LDSMA in the PAPR. However, as we will show in the next section, this loss will be compensated by better link-level performance.

### B. Link-Level Comparison

Here, we present the link-level performance comparison between the three multiple access techniques in terms of BER. As shown earlier in the PAPR evaluation, the highest gain of SC-FDMA over MC-LDSMA is with QPSK modulation. Therefore, here we will focus on the BER performance for QPSK modulation. Figures 5 and 6 show the BER versus $Eb/N_0$ (energy per bit to noise power spectral density ratio) for OFDMA, SC-FDMA and MC-LDSMA with one RB and three RBs per user, respectively. It can be observed from Fig. 5 that SC-FDMA and OFDMA have the same BER performance. As only one RB is allocated to each user, the same frequency diversity is achieved by both systems. However, MC-LDSMA achieves better performance, by about 5.5 dB at $10^{-3}$ BER, due to the frequency diversity gained by spreading on more than one RB. In Fig. 6, we can see that OFDMA outperform SC-FDMA. This is easily justified by taking into account that while in SC-FDMA the RBs allocated to each user has to be adjacent [12], in OFDMA the allocated RBs can be distributed. Consequently, the achieved frequency diversity is higher in OFDMA, which resulting in better link-level performance. MC-LDSMA maintains its superiority to OFDMA and SC-FDMA by approximately 1.6 dB and 4.3 dB, respectively, at $10^{-3}$ BER.

### C. Summary of PAPR and Link-Level Comparison

As it has been shown from the simulation results in the previous sections, MC-LDSMA has the same PAPR as OFDMA and achieves better link-level performance. This shows that MC-CDMA outperforms the performance of OFDMA thanks for the frequency diversity achieved by spreading the data on more resource blocks. On the other hand, comparing to SC-FDMA, MC-LDSMA has higher PAPR values, especially for QPSK modulation. Nevertheless, the loss due to the required back-off is compensated by better link-level performance. For example, with QPSK modulation, the loss due to high PAPR comparing to SC-FDMA are 2.38 dB and 3 dB for 1RB and 3RB allocation, respectively, but the link-level gains are 5.5 dB and 4.3 dB. Hence, MC-LDSMA outperforms SC-FDMA with net gain 3.12 dB and 1.3 dB, for 1RB and 3RB allocation, respectively. So, by using MC-LDSMA technique the overall required transmit power can be reduced by at least 1.3 dB and 1.6 dB comparing to SC-FDMA and OFDMA. This power saving will be reflected on the battery life of the user equipment and cell coverage can be increased.

## IV. System-Level Performance

In multiuser systems, the signal of each user experiences independent channel realizations. Therefore, radio resource allocation plays a key role in optimizing the performance of multiuser systems by exploiting the frequency and multiuser

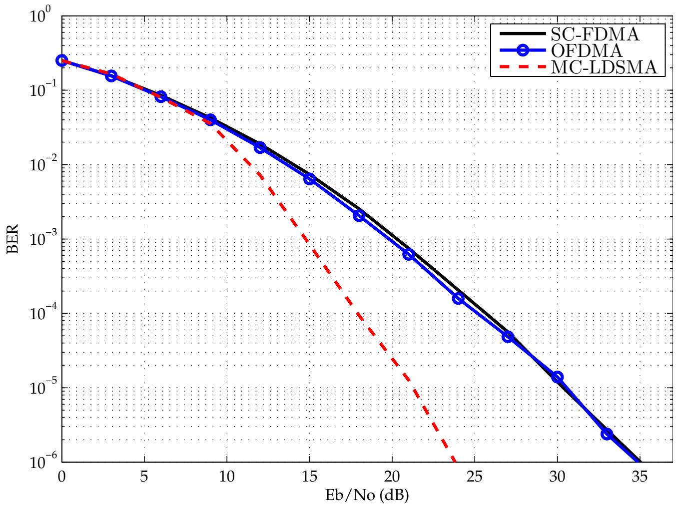


Fig. 5. BER comparison for SC-FDMA, OFDMA and MC-LDSMA with one resource block per user.

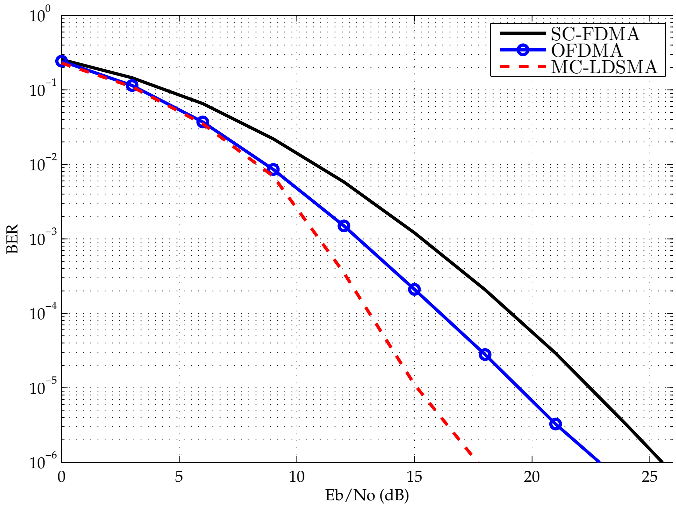


Fig. 6. BER comparison for SC-FDMA, OFDMA and MC-LDSMA with three resource blocks per user.

diversity gains [13]. Important performance measures in multiple access systems are the spectral efficiency and fairness among users. The radio resource should be allocated such that the spectral efficiency is maximized with maintaining fairness among the users. Using only the spectral efficiency as the optimization criterion is unfair to the cell-edge users. One of the most-used fairness criterion is to assign weights to prioritize the users, where users with bad channel conditions will be assigned higher weights to give them more priority in the allocation algorithm. So, the optimization problem for radio resource allocation can be formulated as a weighted sum-rate maximization as follows;

$$\max_{p_{k,n},u_{k,n}} \quad \sum_{k=1}^{K} w_k \sum_{n=1}^{N} u_{k,n} r_{k,n}(p_{k,n}), \tag{7}$$

subject to:

$$\sum_{n=1}^{N} p_{k,n} \leq P_k, \tag{8}$$

where $w_k$ is the weight associated with user $k$ and $r_{k,n}(p_{k,n})$ is the rate of user $k$ on subcarrier $n$. $u_{k,n}$ is the subcarrier allocation index, where $u_{k,n}$ equal to 1 if subcarrier $n$ allocated to user $k$ and 0 otherwise. In addition to the power constraint (8), each multiple access technique has a specific constraint on the optimization problem in (7). For OFDMA and SC-FDMA, there is an exclusivity constraint where the subcarrier cannot be allocated for more than one user. The exclusivity constraint can be formulated as follows;

$$\sum_{k=1}^{K} u_{k,n} = 1. \tag{9}$$

Another constraint for SC-FDMA is that users can only be allocated subcarriers that are adjacent [12]. For MC-LDSMA, there is no exclusivity in the subcarrier allocation and up to $d_c$ users can share the same subcarrier. So, the exclusivity constraint in (9) can be replaced by more relaxed one for MC-LDSMA as follows;

$$\sum_{k=1}^{K} u_{k,n} \leq d_c. \tag{10}$$

This subcarrier allocation flexibility in MC-LDSMA can significantly improve the system spectral efficiency by allowing the subcarrier to be reused by other users. Selecting a high value of $d_c$ (i.e. allowing more users to share the same subcarrier) increase the spectral efficiency. However, the receiver complexity will be increased by increasing the value $d_c$. So, it is essential to trade-off between the system spectral efficiency and receiver complexity. In our system-level performance evaluation of MC-LDSMA, we will choose different values of $d_c$ to see the effect on the spectral efficiency.

In this section, the system-level performance of MC-LDSMA is compared with OFDMA and SC-FDMA. Spectral efficiency and cell-edge users' average rate are used as the performance evaluation metrics. We will compare the system-level performance for these three multiple access techniques under dynamic resource block and power allocation. For MC-LDSMA, we will use the radio resource allocation algorithm we proposed in [14]. For OFDMA and SC-FDMA, the algorithms proposed in [15] and [12] will be used, respectively. We consider a single base station with 1 km radius and assume that users' locations are randomly generated and uniformly distributed over the cell. The maximum transmit power of each user is 1 Watt, and the system bandwidth is 5 MHz consisting of 32 resource blocks. The link gain between the base station and a user is given as the product of path loss, shadowing and fast fading effects. ITU pedestrian B channel model [16] is adopted for generating fast fading. The path loss is obtained by the modified Hata urban propagation model, which is given

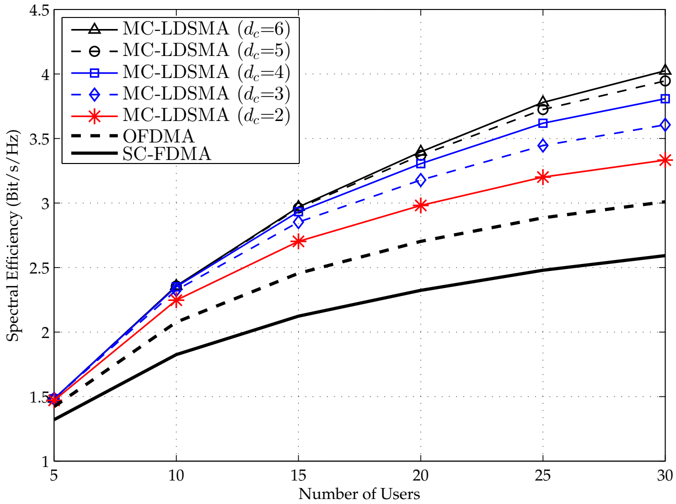


Fig. 7. Spectral Efficiency comparison for SC-FDMA, OFDMA and MC-LDSMA.

by (in dB) [17]:

$$\begin{cases} 122 + 38\log(d), & \text{if} \;\; d \geq 0.05 \; km, \\ 122 + 38\log(0.05), & \text{if} \;\; d < 0.05 \; km, \end{cases} \tag{11}$$

where $d$ (in kilometers) is the distance between the base station and the user. Lognormal shadowing is considered with mean value 0 and standard deviation of 8 dB. The noise power spectral density is assumed to be $-120$ dB/Hz. The users' weights are calculated as the inverse of the users' path losses to ensure fairness among the users by giving high priority to users far from the base station (cell-edge users). For MC-LDSMA system, the number of users per subcarrier is chosen to be between 2 and 6.

Fig. 7 shows the spectral efficiency versus the total number of users for MC-LDSMA, OFDMA and SC-FDMA. As it is evident from the figure, MC-LDSMA achieves spectral efficiency significantly higher than OFDMA and SC-FDMA. In OFDMA and SC-FDMA, resource blocks used by cell-edge users (users with bad channels and high weights) are not available for users with high channel gain due to the exclusivity constraint (9). On the other hand, in MC-LDSMA as there is no exclusivity constraint, the resource blocks allocated to cell-edge users can be used by the users with good channel conditions, which results in the high spectral efficiency.

Moreover, as it can be seen from the figure as the total number of users increased, OFDMA and SC-FDMA techniques become less and less competitive comparing to MC-LDSMA. This because MC-LDSMA can support more users, which increase the sum of users' transmitted power. OFDMA and SC-FDMA cannot support more than $N_{RB}$ users in the same time (where $N_{RB}$ is the number of recourse blocks), while in MC-LDSMA more than $N_{RB}$ users can be supported at the same time. Furthermore, it can be observed that for MC-LDSMA, more spectral efficiency is achieved as the selected

TABLE III
CELL-EDGE USERS' AVERAGE RATE (MBPS).

| | Total Number of Users | | | | |
|---|---|---|---|---|---|
| | **10** | **15** | **20** | **25** | **30** |
| MC-LDSMA | 1.15 | 0.99 | 0.86 | 0.78 | 0.69 |
| OFDMA | 0.55 | 0.48 | 0.42 | 0.4 | 0.35 |
| SC-FDMA | 0.56 | 0.48 | 0.4 | 0.36 | 0.32 |
| Gain over OFDMA | 109% | 106% | 105% | 95% | 97% |
| Gain over SC-FDMA | 105% | 106% | 115% | 116% | 115% |

value of $d_c$ is increased. However, high values of $d_c$ are not required as only a marginal increase in the spectral efficiency can be achieved. For example, as it is clear from the figure, $d_c = 5$ achieves almost the same spectral efficiency as $d_c = 6$. Consequently, high spectral efficiency can be achieved with small values of $d_c$ to keep the receiver complexity affordable. The number of users per subcarrier can be adjusted flexibly to offer a trade-off between system-level performance and receiver complexity. In order to evaluate the fairness among the users we show the cell-edge users' average rate in Table III. The results of MC-LDSMA is for $d_c = 4$. The results show that with MC-LDSMA, cell-edge users can achieve average data rates more than $100\%$ higher than OFDMA and SC-FDMA. This shows that in addition to high spectral efficiency as we see from Fig. 7, MC-LDSMA is fairer comparing to the other techniques. These improvements can be translated to an increase in the number of supported users in the cell and to an increase in the area of coverage.

Considering the results altogether, it can be concluded that the MC-LDSMA achieves superior performance comparing to OFDMA and SC-FDMA in link-level and system-level performance. MC-LDSMA requires less transmission power to achieve the targeted BER comparing to SC-FDMA and OFDMA, thereby conserving battery life and extending the cell range. Also, MC-LDSMA achieves higher spectral efficiency and fairness. Due to its superior performance, MC-LDSMA is an attractive candidate for next generation of mobile communications systems.

## V. CONCLUSION

In this paper, we have investigated the suitability of using MC-LDSMA as a multiple access technique for next generation mobile communications. Our approach was by analysing the performance of MC-LDSMA and comparing it with current multiple access techniques, namely SC-FDMA and OFDMA. The PAPR, link-level and system-level performance are evaluated through extensive Monte Carlo simulations.

The simulation results show that MC-LDSMA has significant performance improvements over SC-FDMA and OFDMA. Using MC-LDSMA can significantly improve the system performance in terms of required transmission power, spectral efficiency and fairness. These improvements can be translated into longer battery life of the user equipment, increase in the cell coverage, increase in the number of supported user in the cell and high data rates. Consequently, MC-LDSMA can be considered as a promising candidate for next generation of mobile communications systems.

## ACKNOWLEDGMENT

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