Order-Statistic based Spectrum Sensing for Cognitive Radio

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***Abstract*—Spectrum sensing for cognitive radio is challenging. In this letter, a spectrum sensing method based on quintiles of Order-Statistics is proposed. We derive the test statistic and evaluate the performance of the proposed method by Monte Carlo simulations. Simulation results show that order statistics based sensing considerably outperforms both energy detection and anderson darling based sensing in an Additive White Gaussian Noise (AWGN) channel; especially in a lower signal to noise ratio region.**

***Index Terms*—Cognitive Radio, Spectrum Sensing, Statistical Tests.**

I. Introduction

Cognitive Radio (CR) is an enabling technology that will allow cognitive users to operate in licensed spectrum and share spectrum with Primary Users (PUs) in an opportunistic manner. This paradigm enhances spectrum utilization and helps overcome the problem of spectrum scarcity in wireless communications. However, the introduction of opportunistic users employing cognitive radio technology inevitably increases the interference and thus degrades the Quality of Service (QoS) of the PUs [1]. To keep the interference to PU at an acceptable level, CRs should be able to perform continuous spectrum sensing, quickly seize opportunities to transmit their data and promptly vacate the spectrum when a PU starts transmitting data.

In the 2010 Federal Communication Commission (FCC) ruling, a geo-location database query mechanism for TV Band Devices (TVBD) was proposed for the protection of PUs [2]. Although the FCC proposed geo-location database oriented approaches, at the same time the report states that spectrum sensing is expected to play a vital role in the efficient management of spectrum in the future [2]. Similarly, other regulatory bodies including Ofcom in UK and European regulatory agencies represented by the CEPT continue to encourage researchers to investigate on spectrum sensing techniques.

A number of spectrum sensing techniques are available in literature, for a good review refer to [1]. Among various spectrum sensing techniques, *blind spectrum sensing* techniques have attracted much attention, as most of the time the signaling scheme of the PU is unknown to the CR's; this may also correspond to the case where an agile PU has considerable flexibility and agility in choosing its modulation and pulse shaping [3].

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One of the most widely known blind spectrum sensing technique is energy detection. Energy Detector (ED) is an optimal detector for independent and identically distributed (i.i.d.) signal samples and gives satisfactory detection performance with low computational complexity if the received Signal-to-Noise Ratio (SNR) is high. However ED based sensing (ED-sensing) performance deteriorates significantly if the received SNR is low [1]. Anderson-Darling (AD) sensing has been recently proposed in [4] and is based on the Anderson-Darling Goodness of Fit (GoF) test. For AD-sensing, the noise distribution does not necessarily have to be Gaussian but it must be known a-priori. Authors in [4], show by simulations that under the same sensing conditions, AD-sensing provides much higher sensitivity to detect an existing signal than ED-sensing.

In this letter, we propose another blind spectrum sensing method using GoF testing. We applied modified GoF testing based on Order Statistics (OS) to the problem of spectrum sensing and propose a novel sensing method. Like AD and ED-sensing, our method can also be classified as a non-parametric method as it does not require any prior information about the PU signal and/or channel conditions. Similarly, our method is applicable to any noise distribution but known otherwise. In this letter, we test proposed algorithm in an AWGN channel, and further investigations in the presence of fading channels is subject to our future work. Our results show that OS based sensing (OS-sensing) outperforms both AD and ED-sensing in an AWGN channel particularly when received SNR is low. The computational complexity of OS-sensing is higher than ED-sensing and is less than AD-sensing.

The rest of this letter is organized as follows. In section II, a brief review of the spectrum sensing as GoF testing is provided. Section III, presents AD-sensing briefly. In section IV, OS-sensing is introduced and an approximate expression for decision threshold and test statistic is derived. Section V evaluates the performance of proposed scheme by Monte Carlo simulations. We compare the performance of OS-sensing with those achieved by the ED and the AD test. Finally, conclusions are drawn in the section VI. We use following notations in this letter: all column vectors are represented by small boldface letters, $|\mathcal{S}|$ denotes cardinality of set $\mathcal{S}$ and $[.]^{\mathrm{T}}$ represents the transpose operation.

II. Spectrum sensing as GoF testing

Let $\mathbf{x} = [x_1, x_2 \dots x_N]^{\mathrm{T}}$ is the received signal vector where N is the total number of received signal samples after down-conversion, band pass filtering and sampling at a CR.

We assume received samples are i.i.d. i.e. x_i and x_j are independent of each other $\forall i, j \in \mathcal{S}$ where $\mathcal{S} = \{1, 2, \ldots, N\}$. In addition, without loss of generality and for the sake of simplicity, we assume that x_i is real-valued. Let $\mathrm{G}_X(x)$ denote the empirical Cumulative Distribution Function (CDF) of the received signal $\mathbf{x}$, defined as,

$$\mathrm{G_X}(x) = \frac{|\{i : x_i \le x, 1 \le i \le N\}|}{N} \tag{1}$$

If there is no signal transmission by the PU, $\mathrm{G_X}(x)$ defined in equation (1) is approaching the known noise distribution, $\mathrm{F_0}(\mathrm{x})$, asymptotically by the Glivenko-Cantelli theorem. Otherwise, in the presence of PU signal, $\mathrm{G_X}(x)$ is different from the known noise distribution.

Therefore, spectrum sensing can be formulated as a GoF testing problem which is a one sided hypothesis test for $\mathcal{H}_0$ i.e. received signal was drawn from a known noise distribution, $\mathrm{F_0}(x)$ [4]. Alternatively, $\mathcal{H}_1$ is the hypothesis that received signal characterized by $\mathrm{G_X}(x)$ is not drawn from the known $\mathrm{F_0}(x)$. Hence, the spectrum sensing problem can be defined as,

$$\begin{aligned} &\mathcal{H}_0: \ \mathrm{G_X}(x) = \mathrm{F_0}(x) && \text{Channel is idle} \\ &\mathcal{H}_1: \ \mathrm{G_X}(x) \neq \mathrm{F_0}(x) && \text{Channel is busy} \end{aligned} \tag{2}$$

Without loss of generality, we assume that noise samples are taken from a Gaussian distribution with zero mean and unit variance. Hence,

$$\mathrm{F_0}(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-y^2/2} dy \tag{3}$$

Depending on the metric used to measure discrepancy between $\mathrm{G_X}(x)$ and $\mathrm{F_0}(x)$, many GoF tests are available in Statistical theory including the Kolmogorov-Smirnov, Anderson Darling and Order Statistics test. In this letter, GoF test based on order statistic is used due to its high sensitivity for low SNR values [6].

III. Anderson-Darling based sensing

Wang et. al. in [4] proposed a spectrum sensing technique, AD-sensing, which used Anderson-Darling GoF test. In section V, we compare the performance of our proposed method with those achieved by AD-sensing and ED based sensing. Therefore, in this section, a brief overview of the steps involved in AD-sensing is given:

Step 1: Find out the threshold, λ_{AD}, for a desired probability of false alarm, P_{FA}, by solving following equation:

$$\begin{aligned} P_{\mathrm{FA}} = 1 - \frac{\sqrt{2\pi}}{\lambda_{\mathrm{AD}}} \sum_{i=0}^{+\infty} a_i (4i+1) \exp\left(-\frac{(4i+1)^2 \pi^2}{8\ \lambda_{\mathrm{AD}}}\right) \\ \times \int_0^{+\infty} \exp\left(\frac{\lambda_{\mathrm{AD}}}{8(w^2+1)} - \frac{(4i+1)^2 \pi^2 w^2}{8\ \lambda_{\mathrm{AD}}}\right) dw \end{aligned} \tag{4}$$

where $a_i = (-1)^i \Gamma(i+0.5)/(\Gamma(0.5)\, i!)$ and Γ is the Gamma function [4].

Step 2: Arrange received signal samples, $\mathbf{x} = [x_1, x_2, \ldots, x_N]^{\mathrm{T}}$, in an ascending order magnitude as,

$$x_{(1)} \le x_{(2)} \le \ldots \le x_{(N)} \tag{5}$$

Let,

$$\widetilde{\mathbf{x}} = \left[x_{(1)}, x_{(2)}, \ldots, x_{(N)}\right]^{\mathrm{T}} \tag{6}$$

Step 3: Elements of $\widetilde{\mathbf{x}}$ are transformed by known noise CDF, $\mathrm{F_0}(x)$ as,

$$z_i = \mathrm{F_0}\left(x_{(i)}\right), \quad i \in \mathcal{S} \tag{7}$$

Step 4: Calculate the test statistic τ_{AD} ,

$$\tau_{\mathrm{AD}} = -\frac{\sum_{i=1}^{N} (2i-1)\left(\ln(z_i) + \ln(1 - z_{N+1-i})\right)}{N} - N \tag{8}$$

Step 5: Make a decision: if $\tau_{\mathrm{AD}} \le \lambda_{\mathrm{AD}}$ that channel is idle ($\mathcal{H}_0$ is true), else if $\tau_{\mathrm{AD}} > \lambda_{\mathrm{AD}}$ ($\mathcal{H}_1$ is true), we consider channel is busy.

IV. Order-Statistic based sensing

OS-sensing is based on the quantiles of each ordered observation in its respective distribution, here referred to as $\boldsymbol{\rho}$-vector. In OS based GoF testing, components of $\boldsymbol{\rho}$-vector are considered as a measure of the GoF test metric. Intuitively, the extreme component of the $\boldsymbol{\rho}$-vector indicates a poor fit with $\mathrm{F_0}(x)$. To perform GoF testing based on order statistics, several test statistics are proposed in literature, e.g. see [6]. To achieve higher probability of detection, we proposed a test statistic, τ_{OS}, based on quantiles of order statistic (elements of $\boldsymbol{\rho}$-vector).

A. Computation of the $\boldsymbol{\rho}$-vector

The calculation of $\boldsymbol{\rho}$-vector is summarized as follows:

Step 1: Transformation Elements of received signal vector, $\mathbf{x}$, are transformed by known noise CDF, $\mathrm{F_0}(x)$ as:

$$z_i = \mathrm{F_0}(x_i) \qquad i \in \mathcal{S} \tag{9}$$

Define,

$$\mathbf{z} = [z_1, z_2, \ldots, z_N]^{\mathrm{T}} \tag{10}$$

Step 2: Sorting Elements of $\mathbf{z}$ are arranged in an ascending order of magnitude as:

$$z_{(1)} \le z_{(2)} \le \ldots \le z_{(N)} \tag{11}$$

Let,

$$\widetilde{\mathbf{z}} = \left[z_{(1)}, z_{(2)} \ldots z_{(N)}\right]^{\mathrm{T}} \tag{12}$$

where $\widetilde{\mathbf{z}}$ is a sorted vector, obtained by sorting $\mathbf{z}$ in an ascending order.

Step 3: β Transformation $\boldsymbol{\rho}$-vector is obtained by transforming elements of $\widetilde{\mathbf{z}}$ defined in (12) using beta CDF [6],

$$\rho_i = \beta\left(z_{(i)}; i, N-i+1\right), \quad i \in \mathcal{S} \tag{13}$$

$$\boldsymbol{\rho} = [\rho_1, \rho_2, \ldots, \rho_N]^{\mathrm{T}} \tag{14}$$

where $\beta(y; \alpha, \beta)$ denotes beta CDF with α and β as shape parameters of the distribution. It is easy to show that ρ_i for $\alpha = i$ and $\beta = N - i + 1$ (by applying integration by part) can be simplified to the following expression:

$$\rho_i = \sum_{j=i}^{N} \frac{N!}{j!\,(N-j)!} {z_{(i)}}^j \left(1 - z_{(i)}\right)^{N-j}, \quad i \in \mathcal{S} \tag{15}$$

B. Test Statistic

In [6], different test statistics based on $\boldsymbol{\rho}$-vectors are introduced, we modified a test statistic detailed in [6] that gives maximum probability of detection (P_{DE}) based on simulations. For our proposed test statistic, elements of $\boldsymbol{\rho}$-vector should be arranged in an ascending order;

$$\rho_{(1)} \leq \rho_{(2)} \leq \ldots \leq \rho_{(N)} \tag{16}$$

$$\widetilde{\boldsymbol{\rho}} = \left[\rho_{(1)}, \rho_{(2)} \ldots \rho_{(N)}\right]^{\mathrm{T}} \tag{17}$$

where $\widetilde{\boldsymbol{\rho}}$ is a vector, obtained from $\boldsymbol{\rho}$-vector by sorting in an ascending order, and then elements of $\widetilde{\boldsymbol{\rho}}$ are applied to test statistic, τ_{OS} :

$$\tau_{\mathrm{OS}} = \sum_{i \in \mathcal{S}} \left| \rho_{(i)} - \frac{i}{(N+1)^2} \right| \tag{18}$$

Smaller values of τ_{OS} indicate a departure from the known noise distribution, $\mathrm{F}_0(x)$. The hypotheses to be tested are then formulated as follows:

$$\begin{array}{ll} \mathcal{H}_0 : \tau_{\mathrm{OS}} \leq \lambda_{\mathrm{OS}} & \text{Channel is idle} \\ \mathcal{H}_1 : \tau_{\mathrm{OS}} > \lambda_{\mathrm{OS}} & \text{Channel is busy} \end{array} \tag{19}$$

where λ_{OS} is the decision threshold that is dependent on the desired probability of false alarm.

C. Threshold for test statistic

Although the received signal samples are i.i.d., $\boldsymbol{\rho}$-vector elements are inevitably dependent [6]. Therefore, it is mathematically intractable to derive a close form expression for the decision threshold [6]. Thus, we performed extensive simulations to approximate the threshold (λ_{OS}) of the test statistic (τ_{OS}) and probability of detection. Fig. 1 demonstrates how λ_{OS} varies with the number of samples (N) and the desired probability of false alarm (P_{FA}). A wide range of values of N was covered (i.e. $20 \leq N \leq 100$) and range of values of false alarm probability $(0.05 \leq P_{\mathrm{FA}} \leq 0.95)$ was chosen to generate Fig. 1. We approximated the threshold as a function of N and P_{FA} for the mentioned domain as,

$$\lambda_{\mathrm{OS}} = 2.599 + 0.8228N - 30.79P_{\mathrm{FA}} + 73.79P_{\mathrm{FA}}^2 - 49.08P_{\mathrm{FA}}^3 - 0.6466P_{\mathrm{FA}}N \tag{20}$$

The Root Mean Square (RMS) error between simulation results for threshold (Fig. 1) and corresponding approximated threshold λ_{OS} is about 0.35. OS-sensing algorithm is summarized in Algorithm 1, where $\mathbb{R}^+$ and $\mathbb{N}$ represent positive real numbers and natural numbers respectively.

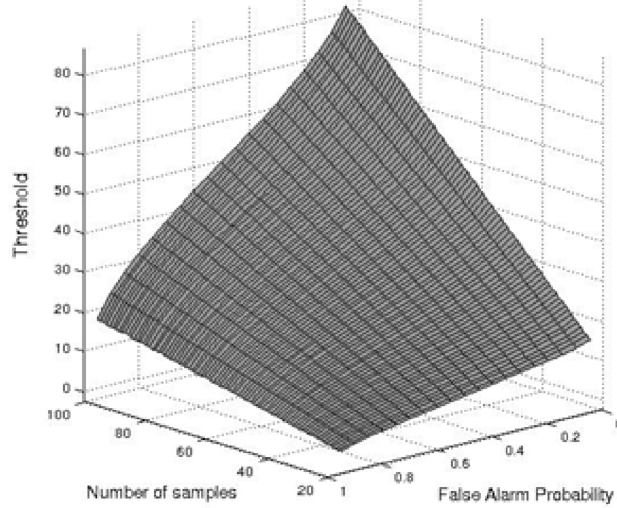

Fig. 1. Simulation results for threshold based on 10^4 iterations for each sample size, $20 \leq N \leq 100$ and $0.05 \leq P_{\mathrm{FA}} \leq 0.95$

Algorithm 1 OS-Sensing Algorithm

Input: $\lambda_{\mathrm{OS}} \in \mathbb{R}^+, \; N \in \mathbb{N}$
Output: $S \in \{\mathcal{H}_0 \; , \; \mathcal{H}_1\}$
  for each sensing event **do**
    Calculate $\mathbf{z}$ using (9);
    Ascend elements of $\mathbf{z}$ to obtain $\widetilde{\mathbf{z}}$ vector;
    Calculate $\boldsymbol{\rho}$-vector using (15);
    Ascend elements of $\boldsymbol{\rho}$-vector to obtain $\widetilde{\boldsymbol{\rho}}$-vector;
    Calculate τ_{OS} using (19);
    if $\tau_{\mathrm{OS}} \leq \lambda_{\mathrm{OS}}$ **then**
      $S \leftarrow \mathcal{H}_0$
    else
      $S \leftarrow \mathcal{H}_1$
    end if
  end for

V. SIMULATION RESULTS

The performance of our proposed sensing method in an AWGN channel is numerically evaluated in this section. Assume the received signal samples x_i at a CR, under $\mathcal{H}_0$ and $\mathcal{H}_1$ can be represented as follows [4]:

$$\begin{array}{l} \mathcal{H}_0 : \; x_i = w_i \\ \mathcal{H}_1 : \; x_i = \sqrt{h}\, m + w_i \end{array} \tag{21}$$

where $i \in \mathcal{S}$, m is the PU signal and w_i is the white gaussian noise sample with zero mean and unit variance. Without loss of generality, we assume $m = 1$ and hence channel gain h is the received SNR. Fig. 2 compares Receiver Operating Characteristic (ROC) curves achieved by OS, AD and ED based sensing for SNR $= -10$db and SNR $= -15$db cases with $N = 32$. The ROC curves were obtained from Monte Carlo simulations using the signal model defined in equation (21). From Fig. 2, it is clear that the detection probability approaches 1 much faster in the case of OS-sensing, and the performance of OS-sensing is indeed better than that of ED and AD-sensing. To examine further the impact of SNR, P_{DE} values were plotted in Fig. 3 for fixed $P_{\mathrm{FA}} = 0.1$ as the SNR was varied from -25db to 0db (keeping the samples number N constant at 32 samples). For example, with the number of samples $N = 32$, SNR=-15dB, the detection probability P_{DE} of OS-sensing is 0.39, while the one for ED and AD-sensing can achieve 0.13 and 0.26 respectively.

Both figures show that for a fixed number of samples, the sensing performance of the OS-sensing method is superior

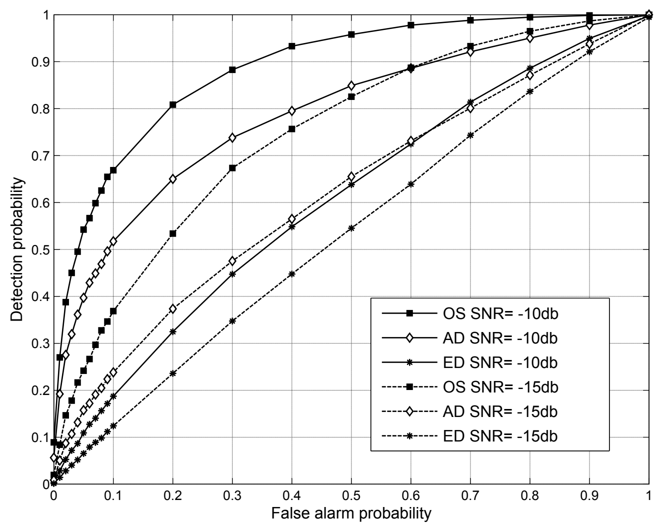

Fig. 2. ROC curves for OS, AD and ED-sensing with different SNR values, $N = 32$

to the sensing performance of the ED and AD-sensing. AD-sensing is more complex than OS-sensing due to its excessive calculations for threshold compared with OS-sensing. However, both of them are more complex than ED based sensing.

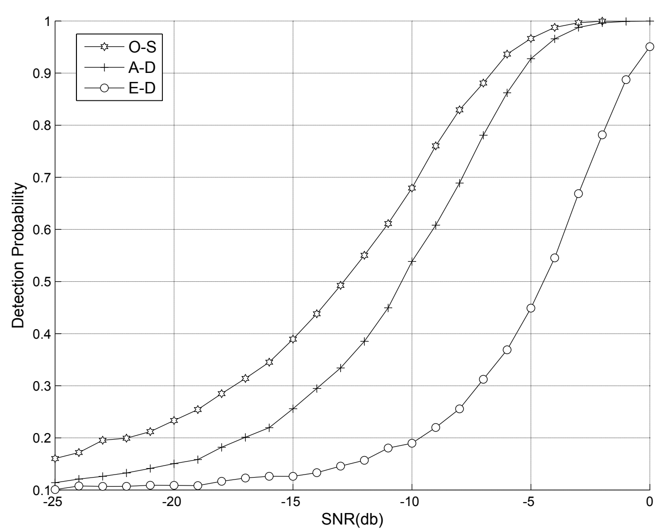

Fig. 3. Comparison of detection probability for OS, AD and ED-sensing with different SNR values, $N = 32$ and $P_{\mathrm{FA}} = 0.1$

VI. Conclusions

The performance of the OS, AD and ED based sensing method is investigated in an AWGN channel. In general, it can be concluded that among all the three spectrum sensing techniques considered, OS is the most powerful test particularly for low SNR values whereas ED is the least powerful method. However, the computational complexity of OS is higher than ED and lower than AD.

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