Modelling of Spacecraft Structures
Incorporating Damped Bolted Joints

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ABSTRACT

In a small satellite developed by Surrey Satellite Technology Ltd. (SSTL) many bolted joints have been used to connect the honeycomb panels in the construction of the main body of the spacecraft. Damping provided by these joints is a key property of the structure. The current understanding of it is limited and forms the focus of this work.

The Finite Element Method has been used as the primary investigative tool in this work. This is complemented and informed by selected experimental studies.

Initially the energy dissipated in the joints of the prototype satellite was assessed, which is of concern to the designers in SSTL. A detailed bolted joint model was created and non-linear static analyses were carried out to obtain the force-energy relationship for this joint. The forces in the satellite joints were found from frequency response analyses of a satellite model. These forces were used in conjunction with the joint force-energy relationships to obtain the total energy dissipated. It was found that when all the joints function in the micro-slip states, their energy dissipating capability was low. The energy dissipated from the joints was less than 5% of the energy input to the satellite when the satellite was base-excited at an acceleration of 2g and a structural damping coefficient of 0.04. Static tests were carried out on aluminium bolted lap joints to determine the effect of bolt pre-load and to quantify the micro-slip and macro-slip regions.

Following this, two ways of increasing damping capacity of bolted joints in a simple satellite model were investigated. The effect of the bolted joints functioning in macro-slip was studied first. A pair of elastoplastic solid elements was used to represent a bolted joint region in the satellite model. The energy dissipated by the joints was obtained by calculating the area under the force-displacement hysteresis loop in the elements through non-linear transient analyses. It was found that if some of the joints operate in the macro-slip region the energy dissipating capability of the joints could be
increased significantly. For example, if the joints on four chosen edges of the satellite operate in macro-slip, the energy dissipated by the joints can be increased to around 40% of the energy input to the satellite at the same excitation and global damping level used in the micro-slip analyses. Experiments carried out on real satellite joint coupons were used to define the response of the joints for use in the modelling work.

There may be some reluctance to use joints in macro-slip in a satellite. Thus an alternate way of increasing damping capacity of bolted joints, using viscoelastic layers in the joints, was introduced and investigated. Spring-dashpot systems were used to model the bolted joints with viscoelastic layers and then developed into the non-linear domain. These representations were incorporated into satellite models to estimate the energy dissipation capability. It was found that this is also an effective way to increase the damping. For example if viscoelastic layers with thickness of 0.6 mm were used in the joints on the same four edges of the satellite as those used in macro-slip, the energy dissipated from the joints can be increased to nearly 25% of the input energy at the same excitation and global damping level used in previous analyses. Dynamic tests were carried out on aluminium lap joints with viscoelastic (VersaSil) layers at different displacement amplitudes and frequencies to determine the rate and amplitude dependency of the material. The correlation between the numerical results and the test data were good.

By increasing damping capacity of bolted joints in these two ways the response of the satellite can be decreased. Thus a better protection of the satellite or a better design of the structure can be obtained.
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I declare that the work presented in this thesis is original and appropriate to the subject area. It is all my own work under supervisions of my supervisors except for some satellite information which was kindly provided by Dr. Guy Richardson.
Publications

- Modelling of spacecraft structures incorporating bolted joints operating in macro-slip (in progress)
- Modelling of small satellite structures incorporating bolted joints with non-linear viscoelastic layer, 19th Annual AIAA/USU Conference on Small Satellites, 08/2005, Logan, USA, SSC05-IX-7 (presented)
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SSTL is developing a small satellite, the Galileo System Test Bed V2A (GSTB-V2/A), for the European Space Agency as part of the Europe’s Galileo navigation program. The current design is a 400-kg, 3-axis stabilised satellite with a cubic body approximately 1.3 metres on each edge (with solar panels stowed). The CAD model of the satellite is shown in Figure 1.1.

Many bolted joints have been used in GSTB-V2/A to connect honeycomb panels in the main construction of the satellite to make assembly, design and manufacture easier. The damping or energy dissipation of the bolted joints is one area that SSTL would like to explore. This forms the focus of this work.

During a spacecraft's operation life, it will undergo different environments and have
different loads on it. One of them is the launch environment during which the mechanical loads are very harsh. There are not only relatively steady-state loads (such as thrust while a rocket engine burns) but also transient loads (such as thrust when a rocket ignites or shuts down), acoustic loads (which include waves with many different frequencies and cause structures to vibrate randomly) and pyro shock (which is high-intensive, high frequency vibration caused by the explosives commonly used to separate stages). Damping in a structure is very important under dynamic loads. With little or no damping, the structure can vibrate to damage itself at the natural frequencies.

The damping in the bolted joints of the spacecraft is especially interesting because their properties can be more easily changed than other parts of the structure. If the damping behaviour is known it can be used or changed to make a better design of spacecraft so that it can survive the launch environment more easily.

A great deal of research exists on bolted structures. Many analytical and numerical models for bolted joints have been created. This will be shown later in the literature review. However, for a large structure with many bolted joints like GSTB-V2/A, there are almost no existing theories or methods which can be used directly to produce useful conclusions. This is why this research was carried out.

Finite element analyses (FEA) and their correlation to test results form the basis of this research.

1.2 Different types of FE analyses

Numerous types of FE analyses need to be carried out in a structural design. Those used in this work include linear and non-linear static analysis, normal modes analysis, frequency response analysis, linear and non-linear transient analysis.
The static analyses are very basic analyses of a structure. When a structure is in a static state, the forces and moments on any part of it are in equilibrium, that is, the sum of them is equal to zero in any direction. Based on this and the material properties, the deformation under certain force or the strain under certain stress can be obtained. The stiffness and the strength of a structure or any component can be checked. Alternatively through the analyses, the size of the structure can be determined to provide stiffness and strength required. This analysis can also be used to find underlying errors in the Finite Element model of a structure. For example it can be used to check if each part of the structure is properly constrained.

Besides static loads, many structures also experience dynamic loads. When the applied load changes with time, the structure, normally as a flexible body, will vibrate. The natural frequency of a structure is the vibrating frequency of the system when it is unforced. It is very important for analysis because when the forcing frequency equals the natural frequency there will be resonance. The response of a system at natural frequency can reach very high levels. Normal modes analyses can indicate the natural frequencies of the system and how the system vibrates at a certain frequency, the mode shape of the system.

Frequency response analysis is a method used to compute the structural response to steady-state oscillatory excitation. There are two main methods to undertake frequency response analyses. One is the direct method in which the coupled partial differential equations of motion are solved directly using complex algebra. The other is a modal method in which the equations of motion are solved by using modal coordinates. When the satellite is launched it experiences forces of different frequencies. Frequency response analyses must be carried out to make sure the response is within the allowable range.

Sometime the response in time domain is also important for assessing a structure.
Transient response analyses are then necessary. It should be noted that frequency response analyses are linear. If non-linearity is involved in the structure, and dynamic analyses are necessary, non-linear transient analyses must be carried out.

There are three basic types of non-linearity in a structure: boundary conditions, as in the contact problem; material, as in plasticity; and geometric nonlinear, as in large displacement. If the structure has any of these properties, then nonlinear static or transient analyses have to be carried out.

1.3 Experimental tests

Besides analyses, physical tests are also essential parts of design. Tests can provide parameter information for analyses and verification of the designed structure. For a spacecraft, there are system level tests and component level tests. System level tests are often carried out to test the electrical and mechanical connections. It is also used to measure the spacecraft's response and verify that the environment used to qualify the spacecraft's components is appropriate. However, it is very expensive to do a system level test, so it is carried out very rarely. Component tests are normally not so expensive and are easy to deal with. Many component tests are carried out during the design of a spacecraft.

Several component experiments were undertaken in this work including bolt torque-tension tests, static and dynamic tests on aluminium lap joints and dynamic tests on the same joints with viscoelastic layers of various materials. Some test results of spacecraft joint coupons carried out in SSTL have also been used. System level tests of the satellite were carried out in SSTL and some of those results were used in this work, for example the structural damping coefficient was evaluated from the frequency response tests of the spacecraft. All the test results were used to validate the numerical analyses and are a necessary part of this research.
1.4 Structure of the thesis

This thesis has eight chapters. Besides this introductory chapter, a literature review is found in chapter 2. The main body of this review is composed of five parts and provides the necessary knowledge base and up-to-date research techniques in the field for the research.

In chapter 3 the study of a plain bolted joint is introduced including numerical modelling and experimental testing. Static tests were carried out on an aluminium lap bolted joint controlled by displacement. An FE model of this joint was created and displacement loading was applied. It was found that the predicted force-displacement curve was almost exactly the same as the one obtained from the experiment. Thus, the FE model of bolted joint was validated. This joint was also used to find the behaviour of the joint at different preloads, friction coefficients and displacement amplitudes. This forms the basis of the next chapter.

The energy dissipation capacity of a satellite with plain bolted joints in micro-slip, with plain bolted joints in macro-slip and with viscoelastic layered bolted joints are presented in chapters 4, 5, 6 and 7 separately (chapter 6 for linear model of viscoelastic layered joints and chapter 7 for non-linear one). All chapters include substantial FE modelling work. The tests on real satellite joint coupons and on aluminium bolted lap joints with viscoelastic materials are included in chapter 5 and 7. Finally conclusions and future tasks are presented in chapter 8.
Chapter 2

Literature Review and Principal Knowledge

2.1 Introduction

In order to get a broad and deep background knowledge and to see what others are doing in this field, various publication databases were searched. Many relevant books and papers were read. In section 2.2, the knowledge of spacecraft structure development is introduced. The role of FEA in the procedure is emphasised. The knowledge of structural dynamics and damping is addressed in section 2.3. These two sections provide very general knowledge for the research work. As mentioned in chapter 1, the research work in this thesis includes energy dissipation assessment of spacecraft with plain and viscoelastic layered bolted joints. In section 2.4 plain bolted joints are discussed, while viscoelastic materials are presented in section 2.5. When the macro-slip behaviour of the plain bolted joints was modelled, elastoplastic materials were used. So in the last section, section 2.6, the properties of the elastoplastic material are discussed briefly. A summary of literature and major points of research work in the following chapters are given in section 2.7.

2.2 Spacecraft structure development

Most of the basic knowledge about spacecraft structures can be found in the book by Sarafin [1]. This discusses the requirements, analysis, verification and quality assurance, design and final verification of a spacecraft.

Generally, a spacecraft is made up of payloads and six subsystems: the structure, the propulsion, the orbit and attitude control, the thermal control, the power, the
communication and data relay. The structure is a very important part of a spacecraft. It supports all the components, protects them from negative environmental influences and enables the spacecraft to work as a whole. Designing an optimal spacecraft structure is a difficult task, needing an iterative procedure of designing, evaluating and testing.

The first thing in developing spacecraft structures is to define the mission requirements. Typically, the following characteristics must be defined: strength, structural life, structural response, natural frequency, stiffness, damping, mass properties, dynamic envelope, positional stability and mechanical interface. In order to derive requirements, identify desirable design features and verify requirements, a large amount of analyses must be undertaken. These include the analyses of dynamics and loads, stress, stiffness, thermal, thermo-elastic, mass properties, mechanism life, probability and statistics and failure. Dynamics and loads include predicting natural frequencies, modes of vibration, damping and responses to time-varying forces and vibrations. It is a very important and a very complex part of the whole analysis task.

In most modern analyses on spacecraft structures, the most efficient and commonly used technique is the finite element method (FEM) [2-4], especially for complicated structures. FEM is involved in almost every stage of developing a spacecraft structure, as shown in Figure 2.1. It uses models and input properties from the design, and analyses the static and dynamic performances to verify that the spacecraft can survive and work efficiently in all life environments. The analysis results will be fed back to the design procedure to help optimise and modify the design. After several iterations, a structure will be determined and manufactured and then a range of tests will be undertaken. The test results will be correlated with the FEA model to make sure the model is appropriate. Then further information can be drawn from the FEA to verify that the whole spacecraft will perform successfully.

FEA plays an important rule in all these procedures. Sarafin [1] gives the steps to use FEA efficiently:
1. Understand the problem. Define the objectives of analysis and identify any constraints such as cost, schedule, and available hardware and software.

2. Define the desired output from FEA and plan what you will do with it.

3. Decide on the modelling strategy, including class of model, boundary conditions, manner of loading, and method of generating the model.

4. Identify the appropriate level of documentation.

5. Estimate cost and time for the analysis to make sure your plan is acceptable.

6. Generate the model.

7. Check the model.

8. Prepare the input file and submit the job for computer analysis.

9. Check the analysis result.

10. Check how sensitive the results are to modelling assumption.

11. Process the FEA results.

12. Document the analysis and control the model’s configuration.

---

**Figure 2.1 - Process of developing spacecraft structures [1]**

Besides this book, many papers [5-21] have provided useful information in this field. Research work includes structure design, software and hardware development, investigation of new methodologies, FE model creation and analysis, and test methods etc. They provide good examples for understanding the spacecraft structure development and are considered briefly below.

Masure et al [5] described the structural design and verification for spacecraft PROBA.
The structure consists of an aluminium machined interface plate, an H-structure composed of aluminium sandwich panels and structural solar panels. Aglietti et al [6] presented the design of MiniSIL. The primary structure of this is an aluminum cylinder. The second structure is composed of the accommodation module and the outer panels.

Obst et al [11] described the design requirements and key features of the ROSETTA Lander which posed the challenge of finding a minimum weight structure that can provide all the functions required by the mission (ROSETTA is an ESA mission designed to explore the comet P/Wirtanen). The structure’s design history and the FEA results for the current structure are also discussed.

Fullekrug and Sinapius [12] proposed a new test method known as the modal force combination method. This can be used to replace the base excitation method when the spacecraft is very large and difficult to test with traditional methods. The method was explained and basic equations were derived. The method was verified to be feasible by an analytical case study based on experimental modal data performed on the PPF satellite.

Nair et al [13] described some new design and development efforts carried out for the INSAT-2 series of spacecraft including the realisation of an efficient structural subsystem, the definition of the mechanical design specifications and the qualification philosophy involving the whole design procedure. The overall INSAT-2 structural/mechanical design, analysis, test activities and results were given.

Stahle et al [14] discussed two things: one was the need for, and benefit from, applying damping treatments to satellites to reduce component failure due to launch and ground testing vibration; the other was the benefits of using viscoelastic damping materials. A computer program known as OCTAVE (Optimised Cost of Testing for Acoustic and Vibration Environments) was used.
Kolsch et al [15] discussed the damping design in a more general sense. They divided the effort into five main subjects: the reduction of responses in the primary structure; the protection of sensitive equipment from launch loads; the provision of micro-gravity environment; the reduction and control of displacements and deformations of large flexible structures; the motion control in mechanisms. Depending on different structural configurations and requirements, the identification of the type and location of different damping measures - either active actuators or passive devices, such as functional dampers, viscoelastic or material dampers, sectioned layer applications, degressive springs, progressive springs, magnetic bearings etc were discussed. The approaches were demonstrated on representative spacecraft designs. It was found that damping could be increased up to one order of magnitude at a fairly low mass penalty, by applying to a spacecraft structure additional measures after the final assembly. The incorporation of high damping in initial phases of hardware developments will be investigated in future.

Gil et al [16] investigated the damping property of ISO spacecraft. The strain energy method was used to obtain the local damping factors. A FE model was developed using Patran/Nastran software. Local damping was considered by using a condensed matrix incorporating material damping. Tests were carried out to correlate the results. Although local damping can improve the results of analysis, some differences between the FEM and the test results remain. More investigation will be done to improve the knowledge of damping.

The book and the papers provided useful background knowledge about spacecraft structure design and analyses for the research.

2.3 Structural dynamics and damping theories

Books and papers [22-39] were read to provide knowledge in this field that is essential
for the project. Salient points will be outlined in this section.

2.3.1 The dynamic analysis procedure and testing hardware

For a general structure, the dynamic analysis steps are as Figure 2.2 [22]. According to the existing set of requirements a preliminary definition of the structure is developed. An initial model is established and the identification of the relevant parts initiated. Then the component tests are launched. From these tests, the data can be derived to implement the mathematical model. Then a full scale (global) test must be performed to ensure that the structure dynamic behaviour corresponds to the analytical prediction. The analysis/test approach follows a process of trial and error. When differences between analysis and test behaviour are observed, a correlation must be done in order to obtain a reliable structure model. This model can be used to do all kinds of further analyses to investigate the behaviour of the structure. A more detailed description can be found in Lopez’s paper [22].

![Figure 2.2 - Dynamic analysis steps][22]

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[22]: Lopez's paper

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Analysis and test always have a close relationship. Because of the lack of the damping information for a structure many people use testing data to obtain or update the damping property [23,24,25]. Some people also use testing data to update the structure mass and stiffness property [25,26].

For a vibration test, the general hardware used is shown in Figure 2.3. The basic hardware elements required consist of a source of excitation, called an exciter, for providing a known or controlled input force to the structure, a transducer to convert the mechanical motion of the structure into an electrical signal, a signal conditioning amplifier to match the characteristics of the transducer to the input electronics of the digital data acquisition system and an analysis system (or analyzer) in which signal processing and modal analysis computer programs reside. Although the specific hardware has changed rapidly due to the advance of technology, they can still be included in these functional groups [27].

![Figure 2.3 - Schematic of hardware used in performing a vibration test](image)

**Figure 2.3 – Schematic of hardware used in performing a vibration test [27]**

### 2.3.2 Damping models

The damping mechanisms of structures are quite different. Some have well defined damping mechanisms which may be specific dampers or natural damping mechanisms.
These include fluid flow past the structure, pressure wave radiation into the medium surrounding the structure, the friction between two parts having relative movements and some special energy absorbing materials in the structures etc. Others have no specific damping mechanisms. The damping arises from a wide range of damping effects which are small and difficult to assess separately.

There are several damping models [27,29, 30] to represent damping mechanisms. The one most often used is the viscous damping model. Others include the hysteresis damping and the Coulomb damping models.

a) Viscous damping

This is the simplest and most often used where damping is proportional to velocity. The equation of motion of single-degree-of-freedom (SDOF) system is

$$m \ddot{x} + c \dot{x} + kx = F \quad (2.1)$$

where $x$ is the displacement of the system, $m$ is the mass, $c$ is the viscous damping coefficient, $k$ is the stiffness and $F$ is the excitation force.

b) Material (Hysteresis) damping

This is so called because when a material is cyclically loaded its stress strain curve describes a hysteresis loop. The energy dissipated in this procedure is proportional to the area of the hysteresis loop. It is assumed that the energy dissipated per cycle by internal energy dissipation is independent of the frequency of excitation. Thus material damping is idealised as viscous damping with the coefficient divided by the frequency of excitation. So the equation of motion is

$$m \ddot{x} + \frac{h}{\omega} \dot{x} + kx = F e^{i \omega t} \quad (2.2)$$

where $h$ is the material damping constant, $\omega$ is the radian frequency of the excitation
force, $F_0$ is the magnitude of the force and $t$ is the time. It can be seen that this form is only valid for harmonic excitation. Material damping can also be incorporated into the equation as a part of the complex stiffness:

$$m\ddot{x} + (k + ih)x = F_0 e^{iat}$$  \hspace{1cm} (2.3)

In this equation, there is a force $ihx$ which is ninety degrees out of phase with displacement $x$. Thus a force-displacement hysteresis loop forms and damping can be represented.

c) Coulomb damping

When two surfaces slide over each other, the friction between them dissipates energy and Coulomb damping appears. This is another common form of energy dissipation. The friction force is constant, but opposite to the direction of motion. This is a non-linear model. In practice it can be linearised by using a viscous form of damping assuming that the same amount of energy per cycle is dissipated. The equation of motion is:

$$m\ddot{x} + \frac{4\mu P}{\pi \omega x_0} \dot{x} + kx = F_0 e^{iat}$$  \hspace{1cm} (2.4)

where $\mu$ is the coefficient of friction, $x_0$ is the harmonic response amplitude, $P$ is the normal force.

When the damping sources are not clear or are complicated, modal damping or proportional damping will be used. For modal damping, a damping coefficient will be setup for each mode as shown in next section. For proportional damping, the damping is assumed to be proportional to a linear combination of mass and stiffness. Other forms of the damping model can be chosen for convenience of the numerical solution. Experiences or test results are normally necessary for these models to set parameters.
2.3.3 Frequency response analysis method

One of the vibration analyses that are most often used on a spacecraft structure is the frequency response analysis. It is a method used to compute structural response to steady-state oscillatory excitation. Basic knowledge of this can be found in reference books [27,31].

There are two methods to undertake frequency response analyses [31]. One is the direct method in which the coupled partial differential equations of motion are solved directly using complex algebra. Beginning with the damped forced vibration equation of motion with harmonic excitation

$$\mathbf{M} \ddot{x}(t) + \mathbf{B} \dot{x}(t) + \mathbf{K} x(t) = \mathbf{P}(\omega)e^{iat}$$  \hspace{1cm} (2.5)

where \( x \) is the displacement vector, \( \mathbf{M} \) is mass matrix, \( \mathbf{B} \) is damping matrix, \( \mathbf{K} \) is stiffness matrix and \( \mathbf{P}(\omega)e^{iat} \) is the excitation force vector. The load is introduced as a complex vector, which is convenient for the mathematical solution of the problem. For harmonic motion assume a harmonic solution of the form:

$$x = u(\omega)e^{iat}$$  \hspace{1cm} (2.6)

Then the motion equation is simplified to:

$$(-\omega^2\mathbf{M} + i\omega\mathbf{B} + \mathbf{K})u(\omega) = \mathbf{P}(\omega)$$  \hspace{1cm} (2.7)

and can be solved using complex algebra.

The other method is a modal method in which the decoupled equations of motion can be solved easily if no damping or only modal damping is considered. The first step in the formulation is to transform the variables from a physical coordinate \( u(\omega) \) to a
Chapter 2 – Literature Review and Principal Knowledge

modal coordinate $\xi(\omega)$ by assuming

$$x = \Phi \xi(\omega) e^{i\omega t}$$  \hspace{1cm} (2.8)

where $\Phi$ are mode shapes. When modal damping is used, each mode has damping $b_i$ where

$$b_i = 2m_i \omega_i \zeta_i$$  \hspace{1cm} (2.9)

The equations of motion can be uncoupled by using the mode shapes and have the form

$$-\omega^2 m_i \ddot{\xi}_i(\omega) + i\omega b_i \dot{\xi}_i(\omega) + k_i \xi_i(\omega) = p_i(\omega)$$  \hspace{1cm} (2.10)

where $m_i$ is i-th modal mass obtained from modal mass matrix $\Phi^T M \Phi$, $k_i$ is i-th modal stiffness obtained from modal stiffness matrix $\Phi^T K \Phi$, $p_i$ is i-th modal force obtained from the modal force vector $\Phi^T P$, $\zeta_i$ is i-th damping ratio.

The damping ratio is defined as:

$$\zeta = c / c_{cr}$$  \hspace{1cm} (2.11)

where $c_{cr}$ is critical damping which indicates the turning point of underdamped motion and overdamped motion.

The solution technique used in the direct method is also used on the smaller modal coordinate matrices if non-modal damping is present.

2.4 Bolted joint design, modelling and damping

A bolted joint should have sufficient strength and stiffness under both static and dynamic loads. The joint behaviour under static load is easier to obtain. There are even
some standards [40,41] which can be referred to. Under dynamic loads it is not as easy because not only the mass and stiffness but also damping information is necessary to determine the behaviour. Damping causes many problems because its behaviour is not clearly known. Much research has been undertaken to investigate the behaviour of bolted joints [42-92]. For almost every structure with bolted joints, a new model and analysis are required because the behaviour of each structure is different due to different configurations and properties of joints. It is almost impossible to develop a standard concerning the dynamic behaviour of bolted joints for use in joint design. Therefore, many models and techniques can be found in literature.

This research has generally been undertaken by computer modelling or laboratory testing. Quite often both are carried out using experimental results to validate the modelling. The modelling can be divided into analytical modelling and numerical modelling, for convenience. Modelling work with straightforward formulations for bolted joints as a whole is called analytical modeling, while modelling work using techniques such as the finite element method for detailed analysis of the bolted joints is called numerical modelling in this review.

2.4.1 Analytical models of bolted joints

Researchers have sought to analyse bolted joint behaviour analytically [44-55]. Various types of analytical models for bolted joints have been established. The influence of the parameters can be investigated using these models. The models can be validated or improved by experiments. Then the models can be used in complex structures. Here some of the different models are introduced:

a) Coulomb damping model
This model was introduced in section 2.2.2 and can be used directly as the bolted joint model.
b) functional model [45]

Some researchers just used the following simple model:

\[ X = C_1 \left( \frac{T}{P} \right)^n + C_2 \left( \frac{T}{P} \right) \]  

(2.12)

where \( X \) is tangential displacement, \( T \) is tangential load, \( P \) is normal load, \( C_1, C_2 \) and \( n \) are constants depending upon joint characteristics.

c) Mass-spring-dashpot model [46,47]

Esmailzadeh et al [47] used a spring-mass-damper model (as shown in Figure 2.4) to analyse the dynamic behaviour of a bolted closure system and assumed that the quantities identifying the system were known. They found that optimal pre-stress values that minimised the peak bolt deformation and stress existed and damping was quite important especially for larger values of natural frequencies, longer loading duration and lower levels of pre-stress.

![Figure 2.4 - Model for a closure bolted system [47]](image)

The motion equation of the system is

\[ M\ddot{x} + (C_m + C_b)\dot{x} + (K_b + K_m)x = f(t) + K_b x_b + K_m x_m \]  

(2.13)

where \( C_m \) is the damping coefficient of the bolted material, \( C_b \) is the damping coefficient of the bolt, \( K_m \) is the stiffness of the bolted material, \( K_b \) is the stiffness of the bolt, \( x \) is the instantaneous bolt length, \( x_b \) is the unstretched length of the
bolt, \( x_m \) is the bolt length after applying the pre-load.

Equation (2.13) can be solved and the conclusions given above were obtained from the results.

d) Elasto Slip Model (ESM) [44]

The Elasto Slip Model (ESM) is composed of a series of elastic spring and ideal Coulomb element units. The transverse force can be expressed as

\[
F(u, \dot{u}) = c_0 u + \sum_{i=1}^{m} r_i(t) \left| r_i(t) \right| < h_i \\
\text{sgn} \dot{u} \\
\]

\[
u = u_k - u_i, \left| r_i \right| = \left| c_i (u - u^*) - h_i \text{sgn} \dot{u} \right|
\]

where \( u^* \) denotes the displacement prior to velocity reversal. The symbols are shown in Figure 2.5a. Figure 2.5b is a 7 parameter system loop.

It can be seen clearly from Figure 2.5b that this model can provide varying stiffness and hysteresis.

![Figure 2.5 - General ESM and hysteresis loop from a 7-parameter model [44]](image-url)
e) Valanis Model [44,45,48]

The Valanis model is governed by the equation

\[
\dot{F} = \frac{E_o \dot{u} [1 + \frac{\lambda}{E_o} \text{sgn}(\dot{u})(E_o u - F)]]}{1 + \kappa \frac{\lambda}{E_o} \text{sgn}(\dot{u})(E_o u - F)} = \dot{F}(u, \dot{u}, F)
\]

where \( F \) is friction force, \( u \) is relative displacement, \( E_o, E_t \) and \( \lambda \) are material parameters and \( \kappa \) reflects the influence of micro-slip (where the contact surfaces first begin to slip). All parameters can be obtained from the hysteresis loop shown in Figure 2.6, where

\[
\lambda = \frac{E_o}{\sigma_0 (1 - \kappa \frac{E_t}{E_o})}
\]

Lenz et al [49] investigated the energy dissipated in a bolted joint using the Valanis model. Special resonators were designed in experiments to study the joint behaviour under both longitudinal and torsional forces. The variation of the hysteresis loop and the energy dissipation with relative displacements were given. They found that the Valanis model could be adapted to serve as a flexible joint hysteresis description for cyclic as well as transient loads.

f) LuGre Model [44,50]

In this model the friction interface was visualised as contact between bristles shown in Figure 2.7. For simplicity the lower part was assumed to be rigid. Because the contact condition is highly random the result is based on an average behaviour of the bristles. It is a dynamic friction model. The governing equations are:

\[
F = \sigma_o \phi + \sigma_1 \dot{\phi} + \sigma_2 v = \mu(\phi, \dot{\phi}, v)
\]

\[
\dot{\phi} = v - \sigma_o \frac{|v|}{g(v)} \phi, \phi(t = 0) = \phi_0
\]

\[
g(v) = F_c + (F_t - F_c) \exp(-v/v_s)^2
\]

20
where $F$ is the friction force, $v$ is the relative velocity. $\sigma_0$ describes the stiffness of the bristles. The dynamic friction velocity dependency is controlled by $\sigma_1$ and $\sigma_2$. The parameter $F_c$ is the Coulomb friction threshold whereas $F_s$ corresponds to the stiction force.

\begin{equation}
F = \frac{v^{1/4}}{\alpha N k |\Delta x|}
\end{equation}

where $F$ is the friction force, $v$ is the velocity, $\alpha$ is the average fraction of bristles in contact, $N$ is the number of bristles, $k$ is the bristle stiffness, $\Delta x$ is the change in displacement over the time interval of interest. By comparing with experimental results it was found that the bristle construct appeared to offer a plausible model for joint friction.

**g) Empirical model**

Ungkurapinan et al [52] investigated bar connections with bolted joints in transmission towers. The mathematical expressions to describe slip and load-deformation behaviour (shown in Figure 2.8) were developed. A total of 36 joint tests were carried out to collect the slip data. Multi bolt connections were studied. The expressions obtained were:
\[ P = \theta_0 \delta \quad \text{for } 0 \leq \delta \leq A/\theta_1 \text{ (Region 1)} \]
\[ P = A \quad \text{for } A/\theta_1 \leq \delta \leq (A/\theta_1 + 0.85) \text{ (Region 2)} \]
\[ P = ((B - A)/\theta_1) (\delta - A/\theta_1 - 0.85) + A \quad \text{for } (A/\theta_1 + 0.85) \leq \delta \leq (A/\theta_1 + 0.85 + Q) \text{ (Region 3)} \]
\[ P = B + ((C - B)/R) (\delta - A/\theta_1 - 0.85 - Q) \quad \text{for } (A/\theta_1 + 0.85 + Q) \leq \delta \leq (A/\theta_1 + 0.85 + Q + R) \text{ (Region 4)} \]

(2.19)

where \( P \) is load on the joint in kN; \( \delta \) is axial deformation of the joint in mm; \( n \) is number of bolts in the joint; \( \theta_1 \) is 36.3436 + 0.44; \( A \) is 12.212n - 4.115; \( B \) is 36.488n + 29.68; \( Q \) is -0.2n + 2.68; \( C \) is 34.6n + 82.86; and \( R \) is -1.501n + 6.735.

Figure 2.8 - The variation of idealized load with deformation diagram for joints [52]

2.4.2 Numerical models of bolted joints

With the development of computer techniques researchers began to model the bolted joints using detailed 2-D or 3-D models of joints which included solid and contact elements. These normally included the bolt, nut and bearing part.

This method can be traced back to 1976 when Krishnamurthy established a 3-D model of connections. He used 8-node subparametric bricks, whose geometry is more refined than the displacement expansion, in order to reproduce the behaviour of bolted end
plate connections. The contact was embodied artificially by attaching and releasing nodes at each loading step on the basis of stress distribution. It was linearly elastic but very time intensive [56].

Others have written code to solve the problem. Krzyzanowski et al [57] undertook research on static and vibration energy dissipation in flange bolted joints using a load-displacement hysteresis loop. Analysis was performed using the FE method with their own contact macro-element. The relationship between vibration damping of the system, the interface surface finish and the applied load was given. Also considered was the relationship between vibration damping of the system, flange thickness and applied pre-load. Kukreti et al [58] developed a three-dimensional finite element programme to analyse the moment-rotation hysteretic behaviour of end-plate connections subjected to seismic loading. The results were compared with those of experiments.

Then, with the development of commercial FE software, researchers used the contact elements within the software to solve the problem. Much modelling and analysis have been undertaken using such software. The most common used software include ANSYS, ABAQUS, MARC and NASTRAN (MARC was bought by NASTRAN's company and became a part of NASTRAN).

ANSYS was used by Sun [59], Sherbourne et al [60, 61] and Ramadan et al [62]. Sun [59] described a 3D finite element method. The superelements, gap elements, constraint equations and submodelling have been used to model a bolted flange interface in jet engines and undertake thermal/stress analysis. Substructuring techniques were a key feature of this modelling. It was used to reduce the model size and complexity. These techniques have been used very successfully in life management problems, field investigations and design changes in GE aircraft engines. Sherbourne [60] established a 3-D finite element model to study the stiffness and strength of the T-stub to unstiffened column flange bolted connection subjected to a moment. The prying action and gradual
plasticity of components was investigated. Ramadan et al [62] studied the static behaviour of a bolted steel single lap shear connection. The load displacement relationship was studied. The yield behaviour of the connection and stress of the hole was analysed. The analysis results were validated by experiments.

ABAQUS was used by Bursi [56] and De Matteis et al [63] etc. Bursi [56] scrutinised some basic issues about the numerical modelling of bolted connections: constitutive relationships, step size, number of integration points, kinetic description, element types and discretisations. Bursi used 3-D finite element models based on solid and contact elements of ABAQUS and LAGAMINE to investigate the behaviour of rotation, force and stress with the displacement in extended end plate connections. The experimental results validated the numerical model. De Matteis et al carried out FEA on aluminium alloy T-stub joints. The procedure was calibrated with existing experimental results and failure mechanisms were investigated. The bolt pre-loading, the effect of the heat affected zone due to welding and the effect of material strain hardening were taken into account in a sensitivity analysis.

MARC and NASTRAN have been used by Mistakidis et al [64], Sawa et al [65] and Gantes et al [66] etc. Mistakidis et al established a numerical 2-D finite element model to study the behaviour of a steel T-stub connection subjected to tensile loading in MARC. The development of zones of plasticity and unilateral contact effects on the interfaces between connection members and bolts were taken into account. The results were compared with those obtained from laboratory tests. Sawa et al studied the stress and sealing performance in pipe flange connections. The finite element analysis was undertaken in MARC where contact conditions including friction could be considered. The gasket was modelled using elastoplastic material. Gantes et al developed a finite element model for simple T-stub steel connections subjected to tensile loading in Nastran. Gap elements were used to model the contact behaviour. The bolt length was varied to take into account the actual flexibility provided by the bolt shank and the nut. The results were compared with experimental data found in the literature.
No comparison among these software packages have been made concerning the modelling of bolted joints, but good results seem to have been obtained from all of them. There are also other FE software packages that have not been so widely used. Pratt et al [67] investigated single and dual conical-head bolted lap joints. The relationship between the load and elongation was obtained by finite element model as well as experimentally. A non-linear finite element code named NIKE3D was used. Slidelines were used in the code to model the contact condition. The yield strength and the deformation energy were also found from testing.

2.4.3. Experimental methods to obtain the viscous damping of bolted joints

Viscous damping is most commonly used in structural analysis and other forms of damping are often converted into an equivalent viscous damping coefficient. So in this section the experimental methods to obtain the viscous damping of bolted joints are discussed.

There are several ways to obtain damping ratio or viscous damping coefficient from testing.

a) Half power bandwidth approach [51]

This is most commonly used way. Compliance is the transfer function between the excitation force and the response displacement \( H(\omega) = \frac{X(\omega)}{F(\omega)} \). It is derived from the single-degree-of-freedom (SDOF) system, but can be used approximately in multi-degree-of-freedom (MDOF) systems.

\[
\zeta = \frac{\omega_b - \omega_d}{2\omega_d} \quad (2.20)
\]

where \( \zeta \) is the damping ratio, \( \omega_d \) is the natural frequency of the system, \( \omega_a \) and
\(\omega_a\) and \(\omega_b\) are two frequencies shown in Figure 2.9.

![Compliance magnitude graph](image)

**Figure 2.9 – The variation of compliance magnitude with frequency [51]**

b) Log decrement approach [51]

This was implemented in a local linear construct to approximate energy dissipation in structures. The oscillatory decaying acceleration signal was subjected to a low pass filter between the first and second oscillatory modes to eliminate higher mode contributions. The analytic function of an approximate envelope of the signal was calculated. The decay curve was divided into 10 segments. The average amplitude of each segment was taken. They were fit into a straight line using a least square approach. The system damping factor of the first mode was inferred from this.

![Acceleration graph](image)

**Figure 2.10 – The variation of acceleration with time**
c) Hysteresis loop approach (damping from friction) [68]

Hysteresis loops are formed by the applied force and the slip of bolted joints. An approximate loop is shown in Figure 2.11. Equivalent viscous damping ratio can be obtained equating the energy dissipated by a viscous damper with the energy dissipated from non-linear behaviour shown in Figure 2.11. It was found the equivalent damping ratio is given by

$$\zeta_{eq} \approx \frac{1}{4\pi} \frac{\text{area of hysteresis loop}}{\text{area under skelton}} \quad (2.21)$$

The area $\text{OABD_3O}$ in Figure 2.11 represents the surface under skelton, while the parallelogram $\text{BCDE}$ is the hysteresis loop. That is

$$\zeta_{eq} \approx \frac{1}{4\pi} \frac{4F_r(\Delta_m - \Delta_r)}{F_r(\Delta_m - \frac{\Delta_r}{2})} = 2 \frac{(1 - \frac{\Delta_r}{\Delta_m})}{\pi (2 - \frac{\Delta_r}{\Delta_m})} \quad (2.22)$$

where $\Delta_m$ is the amplitude of macro-slip (the whole part of contact surfaces slip) and $\Delta_r$, the amplitude of micro-slip.

![Figure 2.11 - Hysteresis loop [68]](image-url)
d) Frequency response function approach [69,70]

The frequency response function \( H^{(3)} \) of the structure in Figure 2.12 can be written as

\[
H^{(3)} = \begin{pmatrix}
H_{aa}^{(1)} - H_{ab}^{(1)} H_{ba}^{-1} H_{ba}^{(1)} & H_{ab}^{(1)} H_{ba}^{-1} H_{bc}^{(2)} \\
H_{cb}^{(2)} H_{bc}^{-1} H_{ba}^{(1)} & H_{bc}^{(2)} - H_{cb}^{(2)} H_{bc}^{-1} H_{bc}^{(2)}
\end{pmatrix}
\]

(2.23)

where

\[
H_{bc} = H_{bb}^{(1)} + H_{bb}^{(2)} + H_{J}
\]

(2.24)

and if the mass effect of the bolt can be ignored, then

\[
H_{J} = (K + j\omega C)^{-1}
\]

(2.25)

The subscripts of \( H \) (except \( H_{J} \)) indicate where the input force was added (second subscript) and where the output response was measured (first subscript). The numbers in the equations refer to the substructures or the whole structure.

During experiments \( H \)'s (except \( H_{J} \)) were measured at different frequencies. More equations than variables were obtained and \( K \) and \( C \) were then solved using the least squares method.

![Figure 2.12 – Sketch of idealised structure [69]](image)

e) Laser vibrometry measurement approach [71]

The laser vibrometer enabled measurements to be made of mobility frequency response functions and velocity r.m.s. scans at a single frequency over a selected area of the
structure. Then the normal mode, the critical damping ratio, the undamped natural frequency and the damped natural frequency could be estimated by means of modal analysis carried out on the mobility frequency response function.

2.4.4 Torque-tension relationship

The torque-tension relationship is important in a bolted joint. So the literature in this area will be discussed briefly in this section.

It is important to know the pre-load in a bolt, but it is not easy to obtain. Normally, there are three ways to measure the pre-load of a bolt [65]. The most widely used method is by a torque wrench. Although it is not considered to be a reliable method of obtaining an accurate pre-load, it is easy to use. The relationship between the torque applied and the axial force has been studied experimentally [72] and the results are discussed below in a little more detail. A better way to acquire the pre-load is by measuring the bolt elongation using an internal bolt strain gauge [73]. Karamis et al [74] mentioned that the angle of twist of the nut can also be used to determine the axial force.

Jiang et al [72,75] undertook an experimental study on the torque-tension relationship for bolted joints. The relationship can be expressed as

\[ T = KDP \]  \hspace{1cm} (2.26)

where \( T \) is the torque, \( D \) is the nominal diameter of the bolt, \( P \) is the pre-load achieved and \( K \) is the torque coefficient. It can also be expressed as

\[ T = P \left( \frac{p}{2\pi} + \frac{\mu_r r_e}{\cos \beta} + \mu_n r_n \right) \]  \hspace{1cm} (2.27)

where \( p \) is the pitch of the thread, \( \mu_r \) is the coefficient of friction between the nut and bolt thread, \( r_e \) is the effective contact radius of the thread, \( \beta \) is the half-angle of the
Chapter 2 – Literature Review and Principal Knowledge

thread, $\mu_n$ is the coefficient of friction between the face of the nut and the bearing surface and $r_n$ is the effective radius of contact area between the nut and its bearing surface.

It can be seen from the equation (2.27) that the torque required to tighten a bolt consists of three parts: the torque to stretch the bolt and the torques to overcome friction on the two parts of the contact surfaces. In the paper some special fixtures were used to measure $\mu_r$, $\mu_n$ and $K$. The influence of the number of tightening/loosening cycles was also investigated. Figure 2.13 and 2.14 have been selected from their results. It can be seen that the torque-tension relationship is non-linear. This non-linearity is mainly from the second part of equation (2.27). It can also be seen that $K$ varies with repeated tightening-loosening and depends very much on the washer materials. For a steel washer, it is almost constant, for an aluminium washer, it increases significantly. The influence of the size and shape of the hole, the use of a slot in a bolted joint, contact area and position, turning speed, coating, and the use of wax on the bearing surface were also investigated. It was noted that the scatter in the friction may overshadow the influence of size, speed, and contact positions.

Hagiwara [76] undertook research on the axial force in bolts. It was directed towards determining the strength of the bolt. Experiments were carried out to verify the analytical results.

![Figure 2.13 - The variation of K with axial load [72]](image1)

![Figure 2.14 - The variation of K with number of tightening [72]](image2)
2.4.5 The variability of damping ratios in bolted structures

The damping ratios of structures are very important parameters for assessing the dynamic response. The variation of damping under different conditions in general bolted structures are very helpful for any study in this field. A selection of results are shown in Table 2.1. The damping ratios in different structures under different experimental conditions can vary significantly, say from 0.001 to 0.036.

From these papers as well as [80-83], it can be seen that factors that influence the damping ratios may include the geometry and material properties of the structures and the joints, the torque of the bolt, the magnitude of the frequency and the excitation, the contact area and even the history of load. Important conclusions include:

- The bolt tension generally decreased with time. The loss starts from the time when the joint is tightened and asymptotically reaches a drop of five to six percent. But most of the loss happened within a day of tightening the bolts.
- The coefficient of friction decreases with increasing clamping pressure.
- The modal damping ratios of the structure with bolted joints exceed by a large factor, say 10–40 under some conditions, those of the structure without bolted joints.
- The damping ratios significantly increase with increasing levels of excitation.
- The damping decreases as frequencies increase.
- The preload torque affects the damping ratio to some degree. Over a certain range, the resonant peak continued to shift to the lower frequencies as the torque decreased and the response amplitudes decreased significantly, especially in higher frequency range. In some structures it was found that the load history affects this trend significantly.
- The damping ratio generally increased when using joints with viscoelastic material. When rubber sheets of increasing thickness were used in the joint the
natural frequencies decreased slightly and the damping ratios increased.

- The machining of the contact surfaces will affect the energy dissipation, for example, joints with a ground surface demonstrated better dissipative properties than those with milled surfaces.

These all give some guidance to the future study and experimentation.

**Table 2.1 – Some experimental results**

<table>
<thead>
<tr>
<th>Ref. ID</th>
<th>Test structures</th>
<th>Test objectives</th>
<th>Variables</th>
<th>Modal damping ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>77</td>
<td>Bolted rod frame</td>
<td>Obtain the modal damping ratios of structures with and without bolted joints</td>
<td>Eigenfrequency of the structure without bolted joints: 228Hz</td>
<td>0.055%</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Eigenfrequency of the structure with bolted joints: 174–217Hz</td>
<td>0.43%</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Bare joints</td>
<td>2.01%</td>
</tr>
<tr>
<td>78</td>
<td>Bolted plate and shell type structure</td>
<td>Obtain the relationship between the damping ratio, the preload and the frequency range. Test the effects of viscoelastic material on damping</td>
<td>Bare joints: Torque range: 0-100% of the maximum torque; Frequency range: 15–300Hz</td>
<td>0.1%</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Structure with viscoelastic material</td>
<td>0.7%</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Bare joints</td>
<td>1.2%</td>
</tr>
<tr>
<td>79</td>
<td>Bolted beams</td>
<td>Obtain the relationship between the damping ratio and the preload. Test effects of rubber sheets on damping</td>
<td>Bare joints: Torque range: 50-300 kgf-cm; Frequency range: 17.2–730.0Hz</td>
<td>0.6%</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Structure with rubber sheets: Sheet thickness range: 2-6mm Frequency range: 15–727Hz</td>
<td>3.6%</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Structure with rubber sheets: Sheet thickness range: 2-6mm Frequency range: 15–727Hz</td>
<td>4.6%</td>
</tr>
<tr>
<td>51</td>
<td>Bolted lap joints</td>
<td>Obtain the relationship between the damping ratio and the velocity amplitude in the jointed beam and the contact areas</td>
<td>Velocity range: 0.5 in/s-3.5 in/s</td>
<td>0.3%</td>
</tr>
<tr>
<td>50</td>
<td></td>
<td></td>
<td>Velocity range: 0.5 in/s-7.0 in/s Contact areas: 0.17-1.36 in²</td>
<td>1.0%</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Contact areas: 0.17-1.36 in²</td>
<td>0.04%</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Contact areas: 0.17-1.36 in²</td>
<td>0.28%</td>
</tr>
<tr>
<td>24</td>
<td>Two overlaid beams connected with bolts</td>
<td>Obtain the relationship between the damping ratio, the preload and the frequency range.</td>
<td>Frequencies varied from 100 to 1500Hz</td>
<td>1.8%</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Torques varied from 16.9 to 98.3N-m</td>
<td>0.25%</td>
</tr>
</tbody>
</table>

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2.4.6 Literature in other fields of bolted joints

There is much literature which is useful for investigating bolted joints but cannot be put into any of the sections above. They are reviewed below.

Tsuji et al [84] investigated the method of tightening a flange joint to achieve uniform bolt tension and precise flange alignment. Changes in bolt tension and torque under repeated tightening were also given.

Padmanabhan and Murty [85] experimentally studied the damping in structural joints subjected to tangential loads. They calculated the area of the measured hysteresis loop to find the energy dissipated in the joints. The relationship between the energy lost and the tangential force, normal preload and joint materials were studied using a response surface methodology (RSM). RSM is a combination of mathematical and statistical techniques used in the empirical study of relationships and optimisation, where several independent variables or factors \((x_1, x_2, \ldots, x_k)\) influence a dependent variable or response \(Y\). In the paper, the logarithm of the energy dissipated in the bolted joint per cycle was chosen as \(Y\); \(x_1\) and \(x_2\) were functions of the peak cycle tangential force and the normal preload respectively. The empirical expression of \(Y\) was established and the parameters were calculated from the experimental data. It was found that:

\[
\begin{align*}
Y_s &= 3.288078 + 0.344478x_1 - 0.118485x_2 \\
Y_c &= 3.284069 + 0.293733x_1 - 0.179684x_2 \\
Y_p &= 3.012732 + 0.296685x_1 - 0.120111x_2 \\
Y_a &= 3.162771 + 0.485564x_1 - 0.230323x_2 \\
&\quad+ 0.092030x_1^2 + 0.186723x_2^2 - 0.138485x_1x_2 \\
\end{align*}
\]

(2.28)

where \(Y_s\) is the logarithm of the energy loss per cycle for mild steel joints, \(Y_c\) is for grey cast iron joints, \(Y_p\) is for phosphor bronze joints, \(Y_a\) is for the aluminium alloy
\[ x_1 = \frac{\ln T - \ln 141}{\ln 181 - \ln 141} \]
\[ x_2 = \frac{\ln P - \ln 3464}{\ln 5108 - \ln 3464} \]  

(2.29)

where \( T \) is the peak cycle tangential force; \( P \) is the normal preload.

Sidorov [83] did experiments on a cantilever tank-type structure consisting of 132 structural elements connected by bolted joints. A total of 840 M6 bolts, 3264 M5 bolts, 188 M8 bolts were used. It was found that, with an increasing number of load cycles at the stage of running-in, and with increasing tightening torque used on the bolted joints, the damping of the tank vibrations decreased. After passing the stage of running-in and alignment, the influence of the tightening torque applied to the bolted joints on the damping of the vibrations is slight (for a very large range of torque changes).

Yura et al [86] introduced a test method. It was found that the slip coefficient under short-term static loading was independent of clamping force, paint thickness and hole diameter. The slip coefficient is calculated as follows:

\[ k = \frac{\text{SlipLoad}}{2 \times \text{ClampingForce}} \]  

(2.30)

[Figure 2.15 - Definition of slip load [86]]
Three types of curves were usually observed and the slip loads were defined as:

Curve (a). Slip load is the maximum load, provided this maximum occurs before a slip of 0.02 in. (0.5mm) is recorded.
Curve (b). Slip load is the load at which the slip rate increases suddenly.
Curve (c). Slip load is the load corresponding to a deformation of 0.02 in. (0.5mm).

The last definition applies when the variation of load with slip curves show a gradual change in response.

In Groper's paper [68], the phenomenon of changing the coefficient of friction was mentioned. It seems hard to define the coefficient in bolted joints as supplementary factors must be considered. These include variation of the clamping pressure with the distance away from the bolt, the region of partial slip and nonslip and the variation of the friction with the pressure. The pattern of the variation of the coefficient of friction with the cumulative slip of ductile steel is shown in Figure 2.16a. The variation of the coefficient of friction with cycles for various average values of the clamping pressure in bolted joints is shown in Figure 2.16b.

Figure 2.16 – The variation of the coefficient of friction with a) the cumulative slip and b) the number of cycles [68]
2.5 Properties and application of viscoelastic materials

There are many materials whose behaviour lies between elastic solids and viscous liquids and these are called viscoelastic materials. They have attracted attention for a long time. Many of these materials have been used as dampers. Viscoelastic dampers have long been used in the control of vibration and noise in aerospace structures and industrial machines. They have also been used in civil engineering structures. It is well known that the behaviour of viscoelastic materials is both strain and strain rate dependent. It also depends on temperature. A widely used model for viscoelastic materials is represented by a complex modulus. Both the real and the imaginary part change with frequency and temperature. This model forms a good approximation in many situations. However, when the strain is large and more accuracy is required, more complicated models need to be created. As the first types of models do not consider strain amplitude dependency they have been called linear models in this thesis. The second types of models which include both strain and strain rate dependency have been called nonlinear models.

2.5.1 Linear models of viscoelastic materials

2.5.1.1 Typical properties of viscoelastic material

Researchers have used a variety of techniques to present the properties of viscoelastic materials. A normal and simple way to do this is to express the viscoelastic material as a complex modulus. Many viscoelastic materials are used to bear shear force, so the shear modulus can be express by

\[ G^* = (1 + i\eta)G \]  

(2.31)

where \( \eta \) is referred to as the material damping loss factor and \( G \) is the real part of the
complex shear modulus. Both $\eta$ and $G$ are temperature and frequency dependent. The real part of the modulus represents the stiffness of the material and is called storage modulus ($G$) and the imaginary part represents the damping of the material and is called damping modulus. The ratio of damping modulus to storage modulus is called loss factor ($\eta$). Figures such as those in Figure 2.17 have become an industry standard.

Figure 2.17 – Damping and stiffness properties for 3M ISD 112 [109]

To use this figure find the frequency of interest on the right vertical coordinate. A horizontal line through this frequency will cross the oblique line which represents the temperature of interest. A vertical line through this crossing point will intersect the $G$ and $\eta$ curves and give these two properties at the frequency and temperature. The values shown on the bottom coordinate are known as reduced frequencies and were used to create the figure. It can be seen that the modulus of viscoelastic material increases with frequency and decreases with temperature. The loss factor increases initially and then decreases.
2.5.1.2 Structural dynamics and viscoelastic material

From the literature [93], it can be seen there are three different ways of solving the equations of motion for structures with viscoelastic material in Nastran:

a) The complex eigenvalue method

Suppose the discretized equations of motion take one of the following two forms:

\[ M\ddot{x}(t) + C\dot{x}(t) + Kx(t) = l(t) \]  \hspace{1cm} (2.32)

or

\[ M\ddot{x}(t) + (K_1 + iK_2)x(t) = l(t) \]  \hspace{1cm} (2.33)

where

\[ M, C, K = \text{physical coordinate mass, damping and stiffness matrices (all real and constant)} \]

\[ K_1, K_2 = \text{real and imaginary parts of the stiffness matrix calculated with complex material constants} \]

\[ x, \dot{x}, \ddot{x} = \text{vector of nodal displacements, velocities and accelerations} \]

\[ l = \text{vector of applied node loads} \]

The solution for either form of the equations of motion can be carried out in terms of damped normal modes. This method is too expensive and limited with respect to problem size. Its materials must have dynamic stress-strain behaviour of a certain type. Equation (2.33) implies that both storage and loss moduli are constant. Equation (2.32) requires that storage moduli be constant and loss moduli increase linearly with frequency.

b) The modal strain energy method (SEM)

This is an approximation to the more expensive complex eigenvalue method. It avoids
the use of complex matrices entirely by assuming that the real mode shapes obtained by
suppressing the term $C\mathbf{x}$ or $K_2\mathbf{x}$ are approximations to the true complex mode
shapes. The damped structure is thus represented in terms of its undamped mode shapes
with appropriate damping terms inserted into the uncoupled modal equations of motion.
That is:

$$\ddot{\alpha}_r + \eta^{(r)}\omega_r\dot{\alpha}_r + \omega^2_r\alpha_r = l_r(t)$$
$$\ddot{\mathbf{x}} = \sum \tilde{\phi}^{(r)}\alpha_r(t)$$
$$r = 1, 2, 3, \ldots$$

where

- $\alpha_r = r$'th modal coordinate
- $\omega_r = $ natural radian frequency of the $r$'th mode
- $\mathbf{x} = $ approximation of physical coordinates
- $\tilde{\phi}^{(r)} = r$'th mode shape vector of the associated undamped system
- $\eta^{(r)} = $ loss factor of the $r$'th mode

The modal loss factors are calculated by using the undamped mode shape and the
material loss factor for each material. For example, the material loss factor of the metal
face sheets of a sandwich structure is very small compared to that of the viscoelastic
core. In this situation the modal loss factor is found from

$$\eta^{(r)} = \frac{\eta}{\eta} \frac{I_r^{(r)}}{I^{(r)}}$$

where

- $\eta = $ material loss factor of viscoelastic core evaluated at the $r$'th calculated resonant
  frequency
  
  $$\frac{I_r^{(r)}}{I^{(r)}} = $ fraction of elastic strain energy attributable to the sandwich core when the
  structure deforms in the $r$'th mode shape.
It is well suited to design work because it leads to optimum choices for both viscoelastic material and geometry of the damping treatment. It can account for variations in material properties with frequency in an approximate but simple way.

The modal damping ratios can be adjusted to obtain better results.

$$\eta^{(r)} = \eta^{(r)} \sqrt{\frac{G_2(f_r)}{G_{2,\text{ref}}}}$$  \hspace{1cm} (2.36)

where

- $\eta^{(r)}$ = adjusted modal damping ratio for the r'th mode
- $\eta^{(r)}$ = modal damping ratio for the r'th mode obtained by iteration
- $G_{2,\text{ref}}$ = core shear modulus used in final normal modes calculation to obtain modal frequencies, shapes, and masses
- $G_2(f_r)$ = core shear modulus at $f = f_r$, where $f_r$ is r'th mode frequency calculated with $G_2 = G_{2,\text{ref}}$

c) The direct frequency response method

The applied load varies sinusoidally in time. The steady state equations of motion have the form:

$$(-\ddot{M}\omega^2 + \ddot{K}_1(\omega) + i\ddot{K}_2(\omega))\ddot{X}(\omega) = \ddot{L}(\omega)$$  \hspace{1cm} (2.37)

where

- $\ddot{K}_1(\omega), \ddot{K}_2(\omega)$ = stiffness matrices calculated using the real and imaginary parts of the material properties, respectively
- $\ddot{L}(\omega), \ddot{X}(\omega)$ = complex amplitude vectors of applied nodal loads and responses, respectively.
This can account for the material property variations with frequency exactly but at a substantial cost penalty. The method does not give information of direct use in improving a candidate design.

These three methods are quite commonly used in Finite Element software. Mokeyev [94] introduced the fourth one:

d) Generalized complex eigenvector method

This method is based on approximation of viscoelastic properties by differential operators and the mode superposition technique. It assumed:

\[ \sigma = D^* \varepsilon \]  \hspace{1cm} (2.38)

where \( D^* \) is the matrix of differential operators. If the material is isotropic, the matrix is completely determined by two different operators

\[ D^* = H_1^* D_1 + H_2^* D_2 \]  \hspace{1cm} (2.39)

where \( D_1 \) and \( D_2 \) are matrices of constants, \( H_1^* \) and \( H_2^* \) are differential operators corresponding to the bulk and shear moduli. The differential operator \( 1/H_j^* \) is presented as a sum of elementary fractions

\[ 1/H_j^* = \sum_{i=1}^{m} \frac{1}{a_{ij} + b_{ij} \frac{d}{dt}} \]  \hspace{1cm} (2.40)

Coefficients \( a_{ij} \) and \( b_{ij} \) are defined from the condition of best coincidence of complex characteristic of viscoelastic material and complex characteristic of different operator in preset frequency range.

Based on these assumptions, the forced oscillation of heterogeneous viscoelastic viscoelastic structures can be described by the matrix equation
This equation can be solved by some complex matrix derivation. The advantage of this method is that it allows real changes of the viscoelastic property in frequency range. Further, the complex eigenvalue problem can be solved for small matrices.

2.5.1.3 Analytical models of viscoelastic material

The most common analytical models for a viscoelastic material are spring dashpot systems.

a) Maxwell model, Kelvin (Voigt) model and SMM

In the Maxwell model a spring and a dashpot are used in series. In the Kelvin and Voigt model they are used in parallel. These two models are the simplest analytical forms for a viscoelastic material. They are good for the preliminary study of bolted joints with a viscoelastic layer and will be discussed in detail in the next section. Many other analytical models are different combinations of these two models. In fact, this kind of models was called SMM in Park’s paper as mentioned in section 5.2.1.1 and is more efficient than those listed blow.

b) The fractional derivate and the Zener model [101]

The fractional derivative of order \( \alpha \) of a function \( f(t) \) is

\[
\frac{d^\alpha f(t)}{dt^\alpha} = \frac{1}{\Gamma(1-\alpha)} \frac{d}{dt} \int_0^t \frac{f(\tau)}{(t-\tau)^\alpha} d\tau
\]

(2.42)

where \( \Gamma \) is the Gamma function, \( \tau \) a dummy variable.

The following can be obtained by using fractional derivation in the Zener model (Figure 2.18):

\[
M_s \ddot{x}_s + C_s \dot{x}_s + K_s x_s = F_s
\]

(2.41)
where $M_d$ is the storage modulus, $M_l$ is the loss modulus, $M_0$ is the static modulus.

c = $M_\infty / M_0$, $M_\infty$ is the value taken by the elastic modulus in the limit of infinite frequency and $\omega_n = \omega \tau$, is the normalized frequency.

\[ \tau_r = \frac{\mu}{M_l} \]
\[ M_\infty = M_0 + M_l \]
\[ M(\omega) = \frac{M_0 + M_\infty j \omega \tau_r}{1 + j \omega \tau_r} \]

\[ \text{Figure 2.18 - The Zener model [101]} \]

c) Havriliak-Negami model

This model is a little similar to the Zener model. It is called MPL in Park's paper as mentioned in section 5.2.1.1. Kalgaonkar et al [96] said it is one of the most frequently used empirical equations for describing the dynamic mechanical and dielectric relaxation behaviour of polymers. The HN equation that relates the complex modulus ($E^*$) to the rubbery regime modulus at low frequencies ($E_0$) and the glassy regime modulus at high frequencies ($E_\infty$) can be written as

\[ E^*(\omega) = E_\infty + \frac{E_0 - E_\infty}{[1 + (i \omega \tau)^\alpha]^\beta} \]

(2.44)

where $\omega$ is the angular frequency, $\tau$ is the relaxation time, $i$ is the unit imaginary number, and $\alpha$ and $\beta$ are the two adjustable fitting parameters. There are five parameters in this model. Szabo and Keough proposed a simpler approach to perform
dynamic mechanical thermal analysis in the context of the five-parameter HN model. This parameter estimation procedure made some assumptions: the complex plane representation of the modulus is independent of the relaxation time $\tau$, and $\tau$ is the only temperature dependent parameter in the HN model.

The estimation procedure comprises of two steps. First plot the modulus in the complex plane and estimate $\alpha, \beta, E_0, E_\infty$. Then solve for $\tau$ at each temperature over which the $E'$ measurement is conducted. The paper successfully employed the HN relationship to predict the dynamic mechanical properties of polymer/clay nanocomposites over a wide range of temperature and frequencies.

2.5.1.4 Other relative research in this field

Kodiylam et al [97] used a viscoelastic layer to give passive vibration control of structures. A new approach using function sensitivity was introduced to characterise the viscoelastic material property variation with frequency. An iteration optimisation process was used to design the structure. Hybrid approximations to damping and frequency constraints were used to reduce the computational effort required for solving the non-linear programming problem.

Bergen [98] undertook research on the vibration attenuation of CASSINI by using Tuned Vibration Absorbers (TVAs). The TVAs consisted basically of a rigid mass bolted onto the structure through the Viscoelastic Material (VEM) pad. Furon CE-5530C was chosen as VEM because it possessed a high damping loss factor with low sensitivity to temperature and negligible outgassing characteristics. The analyses and tests showed that significant vibration attenuation was achieved.

Maly et al [99] described the design of a solar array damper that was built into each of two new solar arrays installed on the Hubble Space Telescope (HST) during Servicing
Mission 3. The objective of the damper was to reduce the dynamic interaction of these new wings with the telescope spacecraft. The damper which was integral to the mast of the solar array suppressed the fundamental bending modes of the deployed wings. The unique damper design, a combination of titanium spring and viscoelastic damper was developed. The viscoelastic material selected was 3M ISD 142R. Shear modulus and loss factor as functions of frequency and temperature were measured directly. The damper performance was verified using Direct Complex Stiffness component testing.

Park [100] reviewed and compared three analytical models of viscoelastic dampers: the standard mechanical model (SMM), the modified power law (MPL) and the fractional derivative model (FDM). The parameters of the models were all derived by fitting the experimental data. Two examples were given. It was found that the classical SMM is more efficient than FDM and MPL in the mechanical characterization of viscoelastic dampers.

Ouis [101] presented a new viscoelastic model which was based on the classical model as proposed by Zener. A slight modification was incorporated through introducing the concept of fractional derivative model. The author has named this model the fractional Zener model and it was shown to give excellent agreement with the experimental results.

Melo et al [102] experimentally investigated the frequency and temperature dependence of the viscoelastic properties of composite structures. Dynamic mechanical analysis (DMA) equipment was used. The lamina properties were measured and compared with theoretical results and good agreement was obtained. The laminate properties were obtained from the lamina properties using a theoretical laminate model. The test results also matched the theory well. This provided a way (by using DMA data) to study the viscoelastic properties of composite structures.

Horr et al [103] studied the damping behaviour of a viscoelastic beam. Linear
viscoelastic theory and energy based damping models were discussed. Based on this and a beam model considering the shear deformation, a formula was obtained for the damping ratio as a function of frequency.

2.5.2 Nonlinear models of viscoelastic materials

2.5.2.1 Constitutive models in literature

A very common idea used was introduced in Lee's book [104]. Namely, the uniaxial nonlinear viscoelastic constitutive equation of Schapery can be written for an isotropic material as

\[ r(t) = g_0 \sigma(t) D_0 + g_1 \int_0^t D_c(\psi' - \psi^1) \frac{d}{ds} [g_2 \sigma] ds \tag{2.45} \]

where \( r(t) \) represents uniaxial kinematic strain at current time \( t \); \( \sigma(t) \) is the stress at time \( t \); \( D_0 \) is the elastic compliance and \( D_c(\psi') \) is a transient creep compliance function. The factor \( g_0 \) defines the effect of stress and temperature on the elastic compliance. The transient (or creep) compliance factor \( g_1 \) has similar meaning, operating on the creep compliance component. The factor \( g_2 \) accounts for the influence of load rate on creep, and depends on stress and temperature. The function \( \psi' \) represents a reduced time-scale parameter defined by

\[ \psi' = \int_0^t (a_{\sigma t})^{-1} ds \tag{2.46} \]

where \( a_{\sigma t} \) is a time scale "shift factor". For thermorheologically simple materials, \( a = a(T) \) is a function of temperature \( T \) only. This function modifies, in general, the viscoelastic response as a function of temperature and stress. In this model, four material parameters (\( g_0, g_1, g_2 \) and \( a \)) are available to characterise nonlinear
Banks et al [105] developed general constitutive models for both quasi-static and dynamic deformations of a viscoelastic rod. A Boltzmann-type stress-strain law was assumed:

\[
\sigma(t) = g_\varepsilon(e(t)) + \int_0^t Y(t-s) \frac{d}{ds} g_\nu(e(s), \dot{e}(s)) ds
\] (2.47)

where \( Y \) is the memory kernel, and \( g_\varepsilon \) and \( g_\nu \) are non-linear functions accounting for the elastic and viscoelastic non-linear response respectively of the elastomer. An exponential form for \( Y \) was chosen based on additional studies:

\[
Y(t) = e^{-Ct}
\] (2.48)

A number of linear and non-linear functions for \( g_\varepsilon \) and \( g_\nu \) were assessed. By carrying out curve fitting studies, cubic polynomials were chosen:

\[
g_\varepsilon(e(t)) = \sum_{i=1}^{3} E_i e(t) \quad , \quad g_\nu(e(t)) = \sum_{i=1}^{3} a_i e(t) \] (2.49)

The parameters \( C \), \( g_\varepsilon \) and \( g_\nu \) were fitted from the test results. It was shown that the model was quite successful.

Kitagawa et al [106] gave a review of literature about defining a constitutive law in non-linear stress and strain space. Experiments were carried out to correlate and validate the use of a model based on overstress theory for strongly time dependent materials. A linear theory of viscoelasticity was expressed by

\[
k \dot{\gamma} + \tau = m \dot{\gamma} + g[\gamma]
\] (2.50)

where \( \tau \) and \( \gamma \) are shear stress and strain respectively; \( k \) and \( m \) are material constants; \( g[\gamma] \), which is a linear function of \( \gamma \), is an equilibrium stress-strain curve.
obtained at the strain rate of $\dot{\gamma} = 0$. Non-linearity is introduced in the equation by assuming $k$ and $m$ are not constants, but are given by

$$
k = k[\tau - g[\gamma], \tau, \gamma, \dot{\tau}, \dot{\gamma}] 
m = kG$$

where $G$ is the instantaneous elastic shear modulus. The quantity $\tau - g[\gamma]$ is called an overstress. The forms of $g[\gamma]$ and $k$ can be empirically determined and the other parameters can be found by comparison with the test results.

Bergstrom et al [107] established a new constitutive model for time dependent behaviour of elastomers. In this model the mechanical behaviour was decomposed into two parts: an equilibrium network corresponding to the state that is approached in long time stress relaxation tests; and a second network capturing the non-linear rate-dependent deviation from the equilibrium state. The word network was used because the material was thought of as a network of molecular chains. This chain model was also taken into account when establishing some parameters. Many experiments were carried out to validate the model.

It can be concluded that there is no perfect model for viscoelastic material. All of them can represent the material to some degree, under certain conditions and with the help of experiments.

### 2.5.2.2 Examples of nonlinear behaviour of viscoelastic materials in literature

From the experimental data shown in the literature, an obvious non-linearity in the stress strain response can be observed, as shown in the following examples. Kitagawa et al [106] did torsional tests on polypropylene hollow cylinders. Figure 2.19 shows
two experimental results at a strain rate of $1.4 \times 10^{-3} / s$. Banks et al [105] did experiments on filled rubber rods in simple uniaxial tension. Figure 2.20 compares the experimental and numerical results for the same rubber. Bergstrom et al [107] did experiments on carbon black filled and unfilled Chloroprene rubber. Figure 2.21 shows the uniaxial compression results. Figure 2.21a is for 15pph carbon black to different final strains at a strain rate of -0.01/s. Figure 2.21b is for Nitrile rubber to a final strain of -0.8 at different strain rates.

Figure 2.19 – The stress strain hysteresis loops in paper [106] (a - in a constant strain amplitude  b - at descending strain amplitude)

Figure 2.20 – The stress strain hysteresis loops in paper [105] - model prediction data and calculated
2.6 The behaviour of elastoplastic materials

Elastoplasticity has been used during this PhD research. However, it was not used to model a real elastoplastic solid, but to represent a bolted joint in macro-slip because they have similar behaviour under certain conditions. Various books [108-111] were read to have a clear understanding of the materials and make a proper use of them.
Elastoplasticity is so called because it has both elastic and plastic properties. The behaviour of the material is represented by a stress-strain curve. For some materials like aluminum the curve is quite continuous as shown in Figure 2.22a, so the curve itself needs to be defined. For other materials like mild steel there is a clear turning point in the stress-strain curve named the yield point as shown in Figure 2.22b. Before that point the material behaves elastically and after that, it behaves plastically. That is: before yielding when the load is taken away it is reversible. If the load is applied beyond the yielding part, there will be permanent deformation and the unloading path will be different from the loading path. If it is loaded sinusoidally, the stress and strain will make a hysteresis loop.

There are several different types of elastoplastic models. One is called elastic perfectly plastic and has no stress increase over the yielding region. Another one is called elastic-linear hardening. That is, after yielding, the stress increases linearly with strain, but at a reduced rate. If the elastic part is very small and can be ignored, it is called rigid-perfectly plastic or rigid-linear hardening. These models are shown in Figure 2.23.

Much research has been undertaken on elastoplastic materials and different rules for defining yielding and hardening have been established. Examples of yielding functions include von Mises, Tresca, Mohr-Coulomb and Drucker-Prager. All are functions of stress. When this function meets a certain condition, yield begins. At this point, the function makes a surface in the stress space. This surface is called the yield surface. For perfectly plastic material, the yielding surface remains the same for different loading
histories. For strain hardening material, it changes according to the loading history. Thus different hardening rules appear. The hardening rules include isotropic, kinematic and combined. In isotropic hardening, the yield surface maintains its shape, centre and orientation but expands uniformly about the origin as the material is loaded beyond yield. With kinematic hardening, the yielding surface retains its size, shape, and orientation, but is free to translate in stress space.

![Diagram of hardening rules](image)

(a) Linear Hardening  
(b) Perfectly plastic  
(c) Rigid-linear hardening  
(d) Rigid-perfectly plastic

Figure 2.23 – Stress-strain curves of different elastoplastic models [108]

### 2.7 Conclusions

A large amount of literature was investigated before and during the research work, as shown above. It can be seen that people have been doing significant work in all the fields involved in this research. For example, Sarafin [1] created a “bible” for the spacecraft engineer. Inman [27] introduced very important knowledge for engineering vibration. Bathe [1] wrote a classic book in FEA. Hitchings [29] discussed the very basic and widely used damping models. All of them provided the base knowledge for this research. Other researchers created very useful analytical models for bolted joints,
such as the Coulomb damping model, the functional model, the mass-spring-dashpot model, the Elasto Slip Model, the Valanis Model, the LuGre Model and some empirical models [44-55]. Although most are not used in satellite modelling, they informed the researcher and helped to investigate the behaviour of bolted joints in chapter 3. Some other important information and conclusions in literature about bolted joints played almost the same role. The methods outlined in literature [27,93] and the software help documents [31] (i.e. frequency response analysis, equations of viscoelastic material etc) were invaluable in the FEA of the spacecraft as described in chapter 4-7 of this thesis. Goodier and Hodge [108] gave basic information about elastoplastic materials so that they can be used properly for the bolted joint in macro-slip. When the viscoelastic layer was introduced in bolted joints, the basic idea of the linear model came from the literature [100,95,93,96,109,94] and a figure in paper [14] was chosen to provide data for the research in chapter 6. There are many good nonlinear constitutive models as those introduced by Banks[105] and Kitagawa [106]. Those models were not appropriate for use in the satellite model directly. However, they provided very good information about nonlinear behaviour of viscoelastic material especially those experimental results. They are very helpful in the research in nonlinear behaviour of viscoelastic layered bolted joints in chapter 7.

All this literature forms an indispensable part of the research carried out here. However, none of the information could be used directly to obtain the answers needed. The main reasons are: a) the models of the bolted joints depend heavily on the properties of a joint: the materials used, the geometry of the joint, the preload of the joint and the surface conditions etc. It is impossible to use other peoples’ results without much experimental and numerical exploration. b) With the development of FEA, some analytical methods were used less because FEA can provide accurate results in a simpler way with the help of commercial software. c) Constitutive models for bolted joints sometimes are very difficult (if not impossible) to use in a large satellite model due to the dimension problem. d) People undertaking research are concerned about different things.
New detailed bolted joint models are created in chapter’s 3 and 4 for the purpose of researching the behaviour and energy dissipation capacity of bolted joints. A systematic method is established in chapter 4 to estimate the energy dissipated in bolted joints in the satellite. A new model of using elastoplastic solid elements to represent the behaviour of bolted joints operating in macro-slip is introduced in chapter 5. And efficient ways of estimating energy dissipated in viscoelastic layered bolted joints in satellite structures are introduced in chapter’s 6 and 7.
Chapter 3

Bolted Joint Modelling and Testing

3.1 Introduction

In order to investigate the mechanical behaviour of a structure, a numerical model can be established. This is a common way of undertaking research on a real structure because, if only testing is used, development could be very time consuming and expensive. From the literature, most of the research on bolted joints has been undertaken in the following way: a) establishing an analytical model; b) undertaking analysis using this model; c) performing experiments; d) comparing the results to prove the analytical model. This is a good way to model the bolted joints. However, as computers become faster and numerical modelling methods become more mature, FEA is used more frequently. It can provide more accurate results particularly for complex structures and after many years of development it is also simple to use. All modelling work in this thesis was done in Nastran with Patran as the pre and post processor.

It is very difficult to incorporate detailed bolted joint models in a FE model of the whole satellite because of the high level of detail and the large number of joints. The behaviour of bolted joints should be known clearly before they can be incorporated into the whole satellite model. Moreover, the energy dissipated in a joint, determined from a detailed joint model, can be used in conjunction with a global satellite model to estimate the energy dissipated in all joints in the satellite.

In this chapter, a numerical model of a bolted joint structure is presented. Non-linear static analysis was undertaken on the model to observe the influence of different parameters. Some experiments were carried out to validate the model. The joint model
Chapter 3 - Bolted Joint Modelling and Testing

consisted of two aluminium plates connected by straps and two bolts. This was the experimental specimen. The specimen was easy to make and test and established some general conclusions about bolted joint structures, which formed the basis of the research on the bolted joints in the satellite.

3.2 Numerical modelling of bolted joints

3.2.1 Gap elements

Gap elements formed an important part of the bolted joint model. They are point-to-point contact elements. Two points lying on separate contact surfaces are connected by a gap element. Figure 3.1 shows the force-displacement curves which define the stiffness and force computations for the elements. When surfaces are in contact with each other, the gap elements are closed. There are normal contact forces along the gap axis and shear forces (friction forces) perpendicular to the gap axis. When it is closed, there can be three different conditions: sliding (no friction), sticking (static friction) and slipping (kinetic friction) under different applied forces. When contact surfaces are not in contact at the points, the gap elements are open, there is no axial or tangential load transfer.

![Figure 3.1 - Showing (a) axial force and (b) shear force of Gap element](image)

The most important parameters in gap elements include the axial stiffness for a closed...
gap (KA), the axial stiffness for an open gap (KB), the transverse stiffness when the gap is closed (KT), the coefficient of static friction (\( \mu_s \)) and the coefficient of kinetic friction (\( \mu_k \)). In Figure 3.1, MU1 stands for \( \mu_s \). MU2 stands for \( \mu_k \). The parameter FO is the preload in element while U0 is initial distance between the two points of the gap and \( U_A - U_B \) is relative movement of the two points along the gap axis. \( \Delta V \) and \( \Delta W \) are relative movements in two directions perpendicular to gap axis. Nastran has an adaptive gap element in which the stiffness can be adjusted to facilitate convergence. Initial values for these parameters must be assumed based on experience. According to the recommendation of Nastran, KA should be three orders of magnitude higher than the stiffness of the neighbouring grid points. The default value of KB is equal to \( 10^{-14} \) times KA. It is recommended that KT 0.1 x KA and the default value is equal to KA x \( \mu_k \). The parameters \( \mu_s \) and \( \mu_k \) are determined by the materials used. FO was set to 0 as the preload was modelled by applying pressure directly on the model to an area corresponding to the washer under the bolt. The default value of U0 is equal to 0. This is the unloaded status. The joint was loaded to the full pressure first to give the bolted condition and then shear forces were applied incrementally.

3.2.2 The finite element model of a bolted joint

In order to investigate the behaviour of a bolted joint, a simple specimen was designed, see Figure 3.2. Numerical modelling and experimental testing were carried out. The joint used was a double lap joint composed of two 80 x 25 x 6 mm aluminium main plates with 34 x 24 x 2 mm aluminium clamping plate on either side. Two M6 steel bolts were used to join the assembly together. There was a 2 mm space between the two main plates. The diameter of the holes in all plates was 7 mm to give enough clearance to accommodate bolt movement due to slipping.
Making use of symmetry, only one eighth of the joint was modelled. The friction between the main plates and the clamping plates was of primary concern, so the bolts and nuts were not included in the model explicitly. The preload was modelled as a uniform pressure applied over a ring of 3.5 mm inner radius and 6 mm outer radius around the hole. This ring of pressure represents the action of the washer. A total of 1172 four noded shell elements were used for both plates, as these reduced the complexity of the mesh compared with solid elements, and were appropriate for this structure. In addition, 344 gap elements were used to model the contact condition. The size of the element was between around 0.25 \( \text{mm}^2 \) and 1.4 \( \text{mm}^2 \).

The boundary conditions were established according to the rules of symmetry. If numbers 1-6 are used to represent the constraints in translation in the \( x \), \( y \) and \( z \) directions and the rotation about the \( x \), \( y \), and \( z \) axes respectively the boundary condition applied to the bottom surface was 345, to the front edges was 246 and to the right curve was 156. The FE model and the boundary surface and edges are shown in Figure 3.3.
The material properties for aluminum used in the model were an elastic modulus of 75000 \( N/mm^2 \), a Poisson's ratio of 0.3 and a density of \( 3.699 \times 10^{-9} t/mm^3 \). The closed stiffness for the gap elements was \( 7.5 \times 10^5 \ N/mm \).

### 3.2.3 Finite element analysis of the model

Non-linear static analyses were carried out on this model using displacement control. The relationship between the applied force and displacement at the free end for different coefficients of friction are shown in Figure 3.4. The same response but for different preloads is shown in Figure 3.5. The coefficient of friction and the preload directly affect the force at which the bolt starts to slip. It can be seen that a good approximation for the slipping force \( (F_s) \) is

\[
F = \mu P
\]

where \( \mu \) is the coefficient of friction and \( P \) is the preload.

![Figure 3.4](image-url)  
**Figure 3.4 – The variation of shear force with displacement for different coefficients of friction**

A simple check on the model stiffness was made. Using the “0.4” curve in Figure 3.4, the displacement at which macro-slip began was about 0.022 mm. Based on simple
tensile loading an estimate of the displacement from the aluminium main plate outside the contact region is about 0.0139 mm. Thus the remaining extension was taken up by the micro-slip of the gap elements and extension within the contact region and added up to less than 0.008 mm. By comparing with the test result carried out later this seems entirely reasonable.

![Image of force vs displacement graph]

**Figure 3.5 – The variation of shear force with displacement for different preloads**

From Figure 3.4 and 3.5 it can be seen that the curves have two obviously different parts. Before the force reached a certain value, it increased steadily with small but discernable non-linearity, representing extension of the constituent parts and micro-slip in the gap elements. After a certain value, the movement increased with no significant increase in force. In the former stage only part of the contact surface experienced slipping and this will be referred to as the micro-slip stage. In the latter stage the entire contact surface slipped and this will be referred to as the macro-slip stage.

In this kind of joint configuration, the gap elements outside the washer experienced only a small contact pressure. Thus, only a small applied displacement was required to cause some of these gap elements to slip. The contact states were checked from the results to the "0.4" curve in Figure 3.4. It was found that even at the beginning (point 1) some gap elements had begun to slip. At point 3, all gap elements had slipped. Point 2 lies between point 1 and point 3 and at this point there were a significant number of gap elements in the slip state.
The force and slipping behaviour of the gap elements at these three points are shown in Figures 3.6 and 3.7 in which (1), (2) and (3) correspond to points 1, 2 and 3 in Figure 3.4. It was found that only the gap elements under, and immediately adjacent to, the pressure area were closed. This is shown in Figure 3.6 where the zero shear force corresponds to gap elements that were not closed. It can be seen that the shear force increased with increasing applied load (points 1 to 3) as expected.

Figures 3.6 - Shear forces (N) in the gap elements (at points 1-3 in the loading curve in Figure 3.4)

Plots of gap element displacement are shown in Figure 3.7. It can be seen from Figure 3.7 that the displacements increased with increasing distance from the bolt centre.
Further, an element on the right side experienced a larger displacement than the corresponding element on the left side. It was found from the results file that the closed gap elements farthest from the bolt centre began to slip first. As the applied load increased the number of elements moving from the stick to the slip state increased as the band of slipped elements moved in towards the centre of the bolt. From the point 3 onwards, no gap elements were in the stick state, all had slipped.

Figure 3.7 – Relative displacements (at points 1-3 in the loading curve in Figure 3.4) in Gap elements (mm)

Figure 3.8 and Figures 3.10-3.11 show how the joint behaved under cyclic forces. All the modelling was carried out in displacement control. In Figure 3.8 the controlling
displacement was fixed at $\pm 0.03\ mm$ and the preload was fixed at $1.5\ kN$. The analysis was static and the loading procedure is shown in Figure 3.9. The force-displacement functions for different coefficients of friction (0.4, 0.5 and 0.6) are given. In Figure 3.10 the coefficient of friction was fixed at 0.4 and the preload was fixed at $1.5\ kN$. The force-displacement functions for different displacement amplitudes ($0.01\ mm$, $0.02\ mm$ and $0.03\ mm$) are given. In Figure 3.11 the coefficient of friction was fixed at 0.4 and the controlling displacement was fixed at $0.02\ mm$. The force-displacement functions at different preloads ($1\ kN$, $1.5\ kN$ and $2\ kN$) are given. In all of the modelling the static analyses were carried out and the loading procedure was similar to the one shown in Figure 3.9.

**Figure 3.8 – Hysteresis loops with different coefficients of friction**

**Figure 3.9 – Loading procedure for static analyses**
The forces and displacements formed hysteresis loops. The energy dissipated can be calculated from the area of the loop. If the curve is accurate, the energy will be estimated accurately. It can be seen from the figures that the amount of energy dissipated depended on the coefficient of friction, the preload and the excitation amplitude.

In Figure 3.8, with increasing coefficient of friction, the joint state changed from macro-slip to micro-slip. As the macro-slip force was the product of the coefficient of friction and the preload, with increasing coefficient of friction it was more difficult for macro-slip to happen. The maximum shear forces became larger, but the energy dissipated per cycle became less due to the faster decrease of the amount of macro-slip.

In Figure 3.10, with increasing displacement amplitude, the joint state changed from micro-slip to macro-slip. The maximum shear forces and the energy dissipated per cycle became larger. In Figure 3.11, with increasing preload, the joint changed from macro-slip to micro-slip. Again, because the macro-slip force was proportional to the preload, with increasing preload it was more difficult for macro-slip to occur. The maximum shear force increased, but the amount of macro-slip decreased more rapidly and the energy dissipated per cycle became less.

![Hysteresis loops with different displacement amplitudes](image)

Figure 3.10 – Hysteresis loops with different displacement amplitudes
3.3 Experimental testing of the bolted joint

3.3.1 Specimen and equipment

In order to obtain the mechanical parameters for the bolted joints, a series of experiments were undertaken including preload measurement, static testing and cyclic testing.

Preload is so important in the response of a bolted joint that it was necessary to measure it with considerable accuracy. A special strain gauge, Kyowa KFG-1.5-120-C20-11 internal bolt gauge, was used. A hole of 2mm diameter and 8mm depth was drilled in the head of a bolt. The gauge was put into the hole and bonded using adhesive EP-18. A bolt with this kind of gauge is shown in Figure 3.12. The strain, which was proportional to the preload, was measured using a digital gauge indicator (Figure 3.13) making use of Wheatstone bridge theory. Although the strain-preload relationship can be calculated theoretically it is better to measure both parameters and hence calibrate the bolt gauge experimentally.

Figure 3.11 – Hysteresis loops with different bolt preloads
In order to calibrate the bolt gauge a special tool was designed as shown in Figure 3.14. It consists of two bars, an image of which is shown in Figure 3.14 (b), a longer one with a threaded hole and a shorter one with a clearance hole and slot. The shorter one is a little complicated and a cross-section view is shown in Figure 3.14 (a). The section place and view direction are indicated in Figure 3.14 (b) with a line and arrows. The bolt was put through the clearance hole and fastened in the threaded hole. The two bars were pinned to the tensile machine. There is a slot in the shorter bar through which the strain gauge wires can pass. The strains and forces were recorded and plotted and the calibration factor calculated. In every subsequent joint test the preload was found from the factored reading of the stain gauge.
Figure 3.15 (a) shows the specimen in the machine wedge grips. The static and cyclic tests were then carried out. In order to prevent the grip displacement from affecting the results, an extensometer was used to measure the relative displacement. The dynamic tests were also controlled from this extensometer. A picture of extensometer is shown in Figure 3.15 (b). Abrasive paper of grade 400 was used to restore similar surface condition between different tests.

Figure 3.15 – (a) Joint specimen with extensometer set-up in the machine and (b) extensometer

Three Instron test machines, a 1341, a 6025 (with a 5500R control system) and a 8511, were used to do all experiments reported in this thesis. Instron 1341 and 8511 are servo-hydraulic machines. The maximum load for the former is 20kN and for the latter is 50kN. Both can be used for tensile, compression and fatigue tests. The former has powerful grips, while the latter has wedge grips which may loose when sustaining a large compression force. The Instron 6025 is a screw driven machine with maximum force capacity of 100kN. It cannot be used to do cyclic tests with high frequency.

Experiments in this chapter were carried out on the Instron 8511 dynamic testing machine and the Instron 6025 tensile machine. Additionally some limited dynamic experiments were also carried out on the Instron 1341 to assess the effect of the grips. In
the hydraulic grips the slipping was not a problem, so the tests were controlled from the actuator position.

3.3.2 Test results

3.3.2.1 Torque and preload

Using the calibrating fixtures in Figure 3.14 and the bolt in Figure 3.12 the calibration was undertaken in the Instron 6025.

The relationship between force and strain was obtained and is shown in Figure 3.16. Two experiments were done and the same results were obtained. The slope is 0.0052 kN/µε (For every strain gauge used, this calibration was repeated and the slope was only a little different). Using this calibration, the preloads caused by different torques (applied by a torque wrench, Torqueleader TSC 10) were obtained and are shown in Figure 3.17. It can be seen that the preload was almost proportional to the torque. The stain gauge will be used directly to obtain the preload in the experiments, so not many tests were undertaken to investigate the preload-torque relationship.

![Figure 3.16 - The variation of bolt forces with bolt strain in calibration tests](image-url)
3.3.2.2 Static tests

Many static tests were carried out to investigate the behaviour of the joint. Some will be discussed in this section.

Two static tests were undertaken on the Instron 8511 in manual displacement control. These are shown in Figure 3.18 (a) and (b). In Figure 3.18 (a), the difference between static and dynamic coefficient of friction is obvious. This phenomenon did not appear in Figure 3.18 (b) because fewer force values were obtained in the second test than in the first one. Figure 3.18 (a) was carried out after the clamping parts were abraded. The preload was 3.038 kN. It can be seen that the load plateau began at about 2.4 kN, so the coefficient of friction $\mu$ was 0.395, noting that there were two clamping plates.

Figure 3.18 (b) was plotted after 100 cycles of loading. The preload was 3.413 kN and shear force at the beginning of macro-slip was about 3.4 kN, so the coefficient of friction has increased to about 0.50.

Two static tests were also carried out on the Instron 6025 and the results are shown in
Figure 3.19. Figure 3.19 (a) was carried out just after the contact surfaces were abraded. The preload was 3.132 kN and the shear force at the beginning of the macro-slip was about 3.1 kN. Thus the coefficient of friction was about 0.49. Figure 3.19 (b) was undertaken after a 100 cycle test (on the Instron 8511). The preload was 3.038 kN and the shear force at the beginning of the macro-slip was about 3.7 kN. The coefficient of friction was thus about 0.61.

The different coefficients of friction obtained from different machines should not arise from the machine itself. The surfaces were abraded by hand, so the conditions were not identical. In addition, due to the manual set-up, the contact condition may change from test to test. However, in each case it was observed the coefficient of friction increased with cumulative displacements. After the tests, the joint was undone and it was observed
Chapter 3 - Bolted Joint Modelling and Testing

that the contact surface near the hole exhibited increased roughness. This increased the resistance to surface movement and can explain the increase of the coefficients of friction. In the macro-slip stage, there was generally a small but observable increase in shear force. Sometimes (Figure 3.18a and Figure 3.19b) the differences between kinetic and static coefficients of friction were seen.

The experimental results in Figure 3.19a are compared with corresponding numerical results in Figure 3.20. It can be seen that the loading stiffness of the numerical model is a little lower than the experimental results. This is possibly because the numerical model is not exactly the same as the real one (the washer was not explicitly included in the model). Another numerical result is given in Figure 3.20 where thickness of shells in the washer area was increased to incorporate the washer. It can be seen that the numerical result is almost the same as the test result. There is some difference in the initial slope. This may come from the flexibility of the test machine. It seems that the model with gap elements can be used to represent the joint with considerable accuracy.

![Experimental and numerical results](image)

**Figure 3.20 - Experimental and numerical results**

### 3.3.2.3 Dynamic tests

A low-frequency (0.1Hz) cyclic displacement was applied to the joint using the test
machine with hydraulic grips, Instron 1341. There was insignificant compliance in these grips, so the extensometer was not used for displacement control. The hysteresis loops are formed from displacement of the actuator and the force. Typical data is shown in Figure 3.21 where the preload was 3.18 kN. This machine provided a good displacement control as shown in Figure 3.22.

![Figure 3.21 - The variation of cyclic forces with displacements](image)

**Figure 3.21 – The variation of cyclic forces with displacements**

It was found that the force-displacement loop changed with increasing cycles. The plateau force increased, but at a reducing rate. This can be accounted for by an increase in the coefficient of friction, which resulted in it becoming increasingly more difficult to cause the joint to slip. There is some evidence that the surface condition approached a stable condition.

![Figure 3.22 - Controlling displacements (every 5 cycles)](image)

**Figure 3.22 – Controlling displacements (every 5 cycles)**
The variation of energy dissipated per cycle with the number of cycles is presented in Figure 3.23 and the maximum applied force in each cycle are shown in Figure 3.24. It can be seen that the energy increases initially and then decreases. The maximum force, however, increases monotonically.

The macro-slip force changing with number of cycles is also shown in Figure 3.25. It has the same trend as the maximum force, but becomes stable sooner. This also illustrates the variation of coefficient of friction, which equals the slip force divided by the preload, which is assumed to be constant.
3.4 Conclusions

Important conclusions that arise from the investigation observed above include:

a) There were obvious micro- and macro-slip stages in bolted joints as the applied force increased.

b) The present gap element model can provide a good simulation of this phenomenon which was validated by comparison with the experimental results.

c) The coefficient of contact friction in the bolted joints increased with the cumulative relative displacements and tended toward a stable value.

d) When the joints are used, special consideration should be given to prior cycling of the joint.

e) The bolted joints from the satellite should behave in a similar way to the specimen discussed in this chapter. The hysteresis loop obtained from the satellite bolted joint model can be used to estimate the energy dissipated in the satellite.
Chapter 4

Energy Dissipated in the Satellite Joints Operating in Micro-slip

4.1 Introduction

In the SSTL satellite GSTB/V2A at the time of this research all joints were designed to operate in the micro-slip range. From the research outlined in the previous chapter it can be seen that the FE model with gap elements can represent the behaviour of a bolted joint in micro-slip very well. In this chapter a detailed model of the satellite joint will be used to estimate the energy dissipated by the joints of the satellite. The dimension and the material properties of the joints are derived from the real satellite. The techniques developed in chapter 3 were then used for the joint analyses. These were combined with results from a global satellite model to find the energy dissipated in the joints.

The procedure used to estimate the energy dissipated by the bolted joints in a satellite can be summarised in three steps. First, a detailed joint model for stress analysis was created. The relationship between the energy dissipated by the joint and the excitation force was found. Then the forces in the actual joints were calculated from a dynamic frequency response analysis of the satellite model. This was repeated for excitations in all three directions and for a number of natural frequencies and assumed global damping levels. The energy dissipated by each joint was estimated from the relationship determined from the detailed joint analysis. Finally, the energy dissipated from all the joints was compared with the excitation energy input to the satellite model to determine the influence of the joints. This will be discussed in the following sections.
4.2 Finite element model of the satellite joint

Figure 4.1 shows the detailed model for the satellite bolted joint. It includes the aluminium skin, the honeycomb core, the aluminium connecting plate, the foam and the aluminium bobbin. The bolted part is magnified and shown with different materials in different colors so that the construction can be seen more clearly. The geometry and the material properties were derived from the actual satellite panel (see Appendix A). The energy dissipation of the joint is important and this dissipation was mainly from the friction between the connecting plate and the panel when they experience relative movement. Again, the bolt did not appear explicitly in the model, the preload was applied as a pressure over the region corresponding to the bolt washer. The skin and the plate were modelled using 3-node shell elements and other parts were modelled using 4-node tetrahedron elements. The shell elements and the solid elements share nodes on the surface since the skin is very thin (0.5 mm). The contact condition was again modelled using Nastran gap elements.

![Figure 4.1 - Detailed satellite bolted joint model](image-url)
Because of the symmetry of the joint connection it was only necessary to model a quarter of one bolted connection with appropriate symmetry boundary conditions applied. The force on a complete bolt structure is thus four times the force on the FE model while the displacement is the same. Figure 4.2 shows a sketch of a real joint structure. Figure 4.3 shows the boundary conditions that enforce the symmetry of the joints. Number 1-6 in the figure represent the six degrees of freedom as outlined in section 3.2.2.

![Sketch of the bolted honeycomb panels](image)

**Figure 4.2 – Sketch of the bolted honeycomb panels**

![Boundary conditions of the satellite joint model](image)

**Figure 4.3 – Boundary conditions of the satellite joint model (a) front (b) back (c) bottom and right**
4.3 Joint model analysis and results

Non-linear static analyses were carried out on the detailed joint model shown in Figure 4.1. A preload of 8 kN was applied and the coefficient of friction was set at 0.35, the former matching the intended assembly preload values and the latter being determined from the preliminary joint testing work. The resulting force-displacement response is shown in Figure 4.4.

![Figure 4.4 - The variation of the applied shear force with the displacement of the joint](image)

Figure 4.4 shows the two different stages of joint behaviour: micro-slip and macro-slip. The macro-slip force in this model is about 1380 N. In the current design of the satellite the bolted joints are required operate within the micro-slip level. So the model excitation force should not exceed 1380 N for this configuration of joint (a quarter joint).

Different magnitudes of excitation force were applied statically to the left end of the model in Figure 4.1. Each sub-case has 50 increments. A typical saw tooth loading profile is shown in Figure 4.5. It goes up in increments to the maximum (positive) force first (sub-case 1), then goes down to minimum (negative) force (sub-case 2) and then goes up to the maximum force again (sub-case 3). Due to the non-linear contact response the resulting force-displacement response of the joint formed a hysteresis loop as shown in Figure 4.6. The area of the loop is the energy dissipated per cycle under the
Chapter 4 – Energy Dissipated in the Satellite Joints Operating in Macro-slip

given excitation force by a quarter of a bolted joint. This analysis was repeated with different excitation forces and different energy values were obtained. The relationship between energy and excitation force is plotted in Figure 4.7.

![Figure 4.5 - Typical loading applied to the joint model](image1)

![Figure 4.6 - Typical hysteresis loop obtained from the joint model](image2)

![Figure 4.7 - The variation of energy dissipated with excitation force in (a) an in-plane joint and (b) a corner joint](image3)

In Figure 4.7a the energy and force have both been multiplied by a factor of four to provide the total force-energy relation for an in-plane one-bolt joint. At the corner of the satellite the panel connections have only one contact surface as shown in Figure 4.8. Thus the stiffness of the corner joint is half that of the plane joint and only half the in-plane joint load can be sustained. Therefore, the energy and force in Figure 4.7b are half the values in Figure 4.7a.
A fourth order polynomial was used to fit these data for subsequent processing of the satellite results. For the in-plane joint (Figure 4.7a), the polynomial is

$$E = 0.0005F^4 + 0.0009F^3 + 0.0013F^2 + 0.0031F + 0.0023$$

(4.1)

For the corner joint (Figure 4.7b), it is

$$E = 0.0002F^4 + 0.0004F^3 + 0.0007F^2 + 0.0016F + 0.0012$$

(4.2)

where $E$ is the energy dissipated in a joint, $F$ is an modified force that makes the curve fitting more accurate. The form of $F$ is:

$$F = (F_{\text{real}} - F_{\text{mean}}) / F_{\text{std}}$$

(4.3)

where $F_{\text{real}}$ is the real force in a joint, $F_{\text{mean}}$ is the mean value of the sample forces used to create the curves and $F_{\text{std}}$ is the standard deviation of the sample forces. The force samples used for in-plane joints are [200 400 800 1600 2400 3200 4000 4400 5200], so $F_{\text{mean}} = 2200$ and $F_{\text{std}} = 1900.8$. Similarly it can be shown that $F_{\text{mean}} = 1100$ and $F_{\text{std}} = 950.4$ for corner joints. All forces are in Newtons.
These two formula are used to calculate the energy dissipated in the bolted joints in the satellite model.

### 4.4 Satellite model analysis

#### 4.4.1 Model description

The satellite consists of honeycomb panels connected by bolted joints forming the bus to hold all the equipment and the two propulsion tanks. The model is shown in Figure 4.9.

The majority of the structure is modelled using 4-node shell elements. Most of the equipment was considered as non-structural mass. However, some tall subsystems, such as the power system and the laser reflectors, were modelled using 8-node brick elements skinned by 4-node shell elements. Shell elements were used to model the outside boxes and solid elements to represent all interior components. The propulsion tank shell was modelled using 4-node shell elements and the propellant was modelled using low modulus solid 6-node wedge elements. The joints lay on the lines indicated by the circles. The (joint) nodes on connected panels were linked together using multipoint...
Chapter 4 – Energy Dissipated in the Satellite Joints Operating in Macro-slip

constraints (MPC) causing the nodes to have the same translational displacements. The forces in the bolted joints were obtained by extracting the MPC forces.

For all analyses the four separation system attachment points were connected to a single external control point via translation MPC’s shown in Figure 4.10. Base loads and base constraints were applied to this point. These four points were connected to the nodes above them on the bottom of the satellite model by MPC’s enforcing the same x and y displacements. In the z direction four high stiffness springs were used to connect the nodes. This modelled the launch release springs, used in practice. The model coordinate system is also shown in the figure.

Figure 4.10 – View for separation system attachment points

Due to the symmetry of the satellite, only the energy dissipated in the joints in a quarter of satellite need to be determined. The total energy will be four times this value. The joints were divided into the 8 groups shown in Figure 4.9, according to locations and the connecting configurations. This grouping made the energy calculation easier.
4.4.2 General analysis of the model

In this model, there are no non-linear elements. The damping was modelled using a structural damping coefficient (G). When using structural damping, the damping force \( f_s \) of an element is:

\[
f_s = iGku
\]

where \( k \) is stiffness, \( u \) is displacement, \( i = \sqrt{-1} \). This damping parameter was used for calculation convenience as only one parameter was needed to model the damping of the whole structure. A value for G was found by comparing the measured displacements during vibration testing with the modelling results. An approximate value for this parameter for the satellite is 0.04, based on the experimental testing.

Normal modes analysis was undertaken on the model first, to determine the natural frequencies of the satellite. For this analysis, all six degrees of freedom of the external control point were constrained. The modal effective mass fractions (MEMF) of the model were also found. This parameter indicated how much of the whole satellite mass was involved in a given modal motion [111]. The natural frequencies and their corresponding MEMF were given in Table 4.1 and 4.2. Only those frequencies with large modal mass fraction values are assumed to be important. The six frequencies whose modal effective mass fractions were over 10% were chosen to do the analyses and these have been highlighted in the Table 4.1 and 4.2 and listed in Table 4.3. They ranged from 27 Hz to 109 Hz. In this work, \( f_1, f_2, ... f_6 \) are used to indicate these six frequencies.

Normal modes at these 6 frequencies are described in Table 4.3 and shown in Figure 4.11 - 4.16. They are helpful to understand the behaviour of the satellite and the analysis.
results.

Table 4.1 – Modal effective mass fraction - translational

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<th>Mode No.</th>
<th>Frequency (Hz)</th>
<th>T1 Fraction</th>
<th>T2 Fraction</th>
<th>T3 Fraction</th>
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### Table 4.2 – Modal effective mass fraction - rotational

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<td>$5.234 \times 10^{-5}$</td>
</tr>
<tr>
<td>19</td>
<td>124.47</td>
<td>$2.632 \times 10^{-4}$</td>
<td>$4.084 \times 10^{-4}$</td>
<td>$1.730 \times 10^{-4}$</td>
</tr>
<tr>
<td>20</td>
<td>125.23</td>
<td>$5.619 \times 10^{-5}$</td>
<td>$1.849 \times 10^{-4}$</td>
<td>$2.972 \times 10^{-4}$</td>
</tr>
<tr>
<td>21</td>
<td>128.65</td>
<td>$1.028 \times 10^{-7}$</td>
<td>$2.066 \times 10^{-4}$</td>
<td>$5.816 \times 10^{-4}$</td>
</tr>
<tr>
<td>22</td>
<td>130.22</td>
<td>$4.049 \times 10^{-8}$</td>
<td>$3.399 \times 10^{-5}$</td>
<td>$5.181 \times 10^{-5}$</td>
</tr>
<tr>
<td>23</td>
<td>132.87</td>
<td>$5.756 \times 10^{-6}$</td>
<td>$5.543 \times 10^{-6}$</td>
<td>$8.752 \times 10^{-5}$</td>
</tr>
<tr>
<td>24</td>
<td>134.02</td>
<td>$1.511 \times 10^{-5}$</td>
<td>$1.488 \times 10^{-6}$</td>
<td>$3.814 \times 10^{-6}$</td>
</tr>
<tr>
<td>25</td>
<td>134.18</td>
<td>$1.009 \times 10^{-4}$</td>
<td>$1.021 \times 10^{-5}$</td>
<td>$5.789 \times 10^{-6}$</td>
</tr>
<tr>
<td>26</td>
<td>135.79</td>
<td>$1.103 \times 10^{-4}$</td>
<td>$6.781 \times 10^{-7}$</td>
<td>$1.360 \times 10^{-4}$</td>
</tr>
<tr>
<td>27</td>
<td>137.28</td>
<td>$4.044 \times 10^{-5}$</td>
<td>$1.302 \times 10^{-5}$</td>
<td>$6.693 \times 10^{-4}$</td>
</tr>
<tr>
<td>28</td>
<td>137.66</td>
<td>$8.424 \times 10^{-7}$</td>
<td>$1.345 \times 10^{-6}$</td>
<td>$1.807 \times 10^{-3}$</td>
</tr>
<tr>
<td>29</td>
<td>137.79</td>
<td>$7.849 \times 10^{-5}$</td>
<td>$1.316 \times 10^{-5}$</td>
<td>$2.703 \times 10^{-3}$</td>
</tr>
<tr>
<td>30</td>
<td>138.12</td>
<td>$6.371 \times 10^{-5}$</td>
<td>$5.829 \times 10^{-8}$</td>
<td>$1.428 \times 10^{-5}$</td>
</tr>
</tbody>
</table>
Frequency response analyses were then undertaken on the satellite model. These analyses were of a base driven type, using a large mass approach. A large mass was attached to the control node and a force applied, resulting in a known acceleration. Additional translation constraints were applied to the control node to prevent unwanted rotation. The frequency recovery points were chosen as the six natural frequencies mentioned above. When the influence of the load excitation level was investigated, the response was, as anticipated, simply scaled by the appropriate load factor. The response analyses were repeated for different excitation directions and with different structural damping coefficients. The forces at each node of the “joints” were extracted from the result files.

<table>
<thead>
<tr>
<th>Mode No.</th>
<th>Frequencies (Hz)</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>27.123</td>
<td>Lateral mode +X +Y</td>
</tr>
<tr>
<td>2</td>
<td>27.801</td>
<td>Lateral mode +X -Y</td>
</tr>
<tr>
<td>3</td>
<td>73.634</td>
<td>Axial z mode</td>
</tr>
<tr>
<td>4</td>
<td>74.938</td>
<td>Torsional mode*</td>
</tr>
<tr>
<td>9</td>
<td>97.768</td>
<td>Torsional mode*</td>
</tr>
<tr>
<td>11</td>
<td>108.38</td>
<td>Lateral mode with propellant tank in phase</td>
</tr>
</tbody>
</table>

*The difference between these two modes can be seen in the relative movement of the main body and the solar panels.
Chapter 4 – Energy Dissipated in the Satellite Joints Operating in Macro-slip

Figure 4.12 – Satellite body in mode 2

Figure 4.13 – Satellite body in mode 3
Figure 4.14 – Satellite body with solar panels in mode 4

Figure 4.15 – Satellite body with solar panels in mode 9
Figure 4.16 – Satellite body in mode 11 (a) whole body and (b) propellant tanks
4.4.3 Results and data processing

Using a structural damping coefficient of 0.04 and excitation acceleration of 2g, the MPC forces at each node in all the groups were extracted and the resultant forces were calculated. It was found that the resultant forces at \( f_1 \) and \( f_2 \) under \( x \) and \( y \) excitation and at \( f_3 \) and \( f_4 \) under \( z \) excitation were much bigger than those in other conditions. They are shown in Figure 4.17. The axis labelled \( \text{distance} \) refers to the \( x \), \( y \) or \( z \) location along the group of joints. The origin of the coordinate system is at the bottom centre of the satellite model, as shown in Figure 4.10.

It was found that the forces could vary considerably within each group. Ideally the forces in the bolted joints should be almost the same to avoid excess weight and to maximise the damping. So the positions of the bolts have been optimized, increasing the pitch in areas of high load transfer and reducing it in areas of low load transfer. However, in reality, the joint distribution can be optimised only for one excitation and one frequency. Thus it was necessary to develop a rule to choose the conditions for which the joints were to be optimised. In this work the maximum summed force of all nodes in a group was used as a selection criterion. They appeared in different planes at different frequencies and different excitation directions as shown in Table 4.4.

<table>
<thead>
<tr>
<th>Excitation Direction</th>
<th>Group1</th>
<th>Group2</th>
<th>Group3</th>
<th>Group4</th>
<th>Group5</th>
<th>Group6</th>
<th>Group7</th>
<th>Group8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Excitation Direction</td>
<td>( x )</td>
<td>( y )</td>
<td>( y )</td>
<td>( y )</td>
<td>( z )</td>
<td>( x )</td>
<td>( x )</td>
<td>( x )</td>
</tr>
<tr>
<td>Frequent Number</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>3</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Resultant Force</td>
<td>( xz )</td>
<td>( xz )</td>
<td>( xz )</td>
<td>( yz )</td>
<td>( yz )</td>
<td>( yz )</td>
<td>( yz )</td>
<td>( yz )</td>
</tr>
</tbody>
</table>
Figure 4.17 – The variation of MPC force (N) with node location (m)
The von Mises stress distributions are given in Figure 4.18 - 4.21 for the four excitation directions and frequencies shown in Table 4.4. These correspond to the maximum force curves in Figure 4.17. By comparing these figures with the curves in Figure 4.17, it can be seen that the MPC forces are entirely reasonable and consistent. For example, the MPC forces of the nodes near the middle of the satellite in group 3 and near the bottom of the satellite in group 8 (the coordinate is close to 0) are quite high in Figure 4.17. The stresses at the corresponding points in Figure 4.18 and 4.19 are also quite high. This is because these two locations are near the excitation points. From Figure 4.17, it can be seen that there are some peaks in MPC force curve of group 5 and group 7 joints. This was because there were localised instrumentation located there. They changed the local stress, as can be seen from Figure 4.20 and 4.21.

Figure 4.18– The von Mises stress (Pa) distribution for x excitation at fl
Figure 4.19 – The von Mises stress (Pa) distribution for y excitation at f1

Figure 4.20 – The von Mises stress (Pa) distribution for y excitation at f2
After optimisation, the suggested joint locations for each group of joints are shown in Table 4.5. The locations refer to the global coordinate along the edge of the particular joints (ie x, y or z).

The forces in the joints after optimisation are shown in Figure 4.22. For those with two contact surfaces the joints are optimised so that, ideally, all the joints transmit a force of 5000 N. For those with one contact surface the joints are optimised so that all the joints transmit a force of 2500 N. Although it was impossible to control all the forces at 5000 N or 2500 N (some of them even exceed these limits in other modes of excitation), the distribution was much better after this single pass optimisation strategy and the energy dissipated by the joints could then be investigated.

The joint forces were put into equations (4.1) to (4.3) as appropriate, and then summed to obtain the energy dissipated by all the joints. The total energy input to the satellite was calculated from the force-displacement hysteresis loop at the excitation points.
### Table 4.5 – Joint locations (positions along the line of joints (m))

<table>
<thead>
<tr>
<th>Joint ID</th>
<th>Group 1</th>
<th>Group 2</th>
<th>Group 3</th>
<th>Group 4</th>
<th>Group 5</th>
<th>Group 6</th>
<th>Group 7</th>
<th>Group 8</th>
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<td>1</td>
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<td>-0.3660</td>
<td>0.1425</td>
<td>0.2055</td>
<td>0.1199</td>
<td>0.0412</td>
<td>0.2903</td>
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<tr>
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<td>-0.2750</td>
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<tr>
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</tr>
<tr>
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<td>0.3764</td>
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<td></td>
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<td>0.9762</td>
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</table>
Figure 4.22- The variation of MPC Forces (N) with node locations (m) after joint location optimisation
The effects of structural damping coefficient and excitation levels on the satellite energy were investigated. Figure 4.23 and 4.24 show the energy from the x excitation at the first natural frequency. The variation of the energy with structural damping coefficient is shown in Figure 4.23 and the energy variation with the excitation level is shown in Figure 4.24. It can be seen that the energy dissipated from the bolted joints and the energy input to the satellite both decrease with an increased of structural damping coefficient and increased with an increase in the excitation level.

The ratio of the energy dissipated from all the joints to the excitation input energy also varies with structural damping coefficient and excitation level. A smaller damping coefficient and a smaller excitation level cause the ratio to increase. The ratios at a damping value of 0.04 and an excitation level of 2g (typical launch conditions) are shown in Table 4.6.

<table>
<thead>
<tr>
<th></th>
<th>f1</th>
<th>f2</th>
<th>f3</th>
<th>f4</th>
<th>f5</th>
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<tbody>
<tr>
<td>x excitation</td>
<td>3.1</td>
<td>1.5</td>
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<td>0</td>
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</tr>
<tr>
<td>y excitation</td>
<td>2.9</td>
<td>1.6</td>
<td>0.37</td>
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<td>1.0</td>
<td>1.6</td>
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<tr>
<td>z excitation</td>
<td>2.0</td>
<td>2.1</td>
<td>1.6</td>
<td>1.6</td>
<td>0.83</td>
<td>0.39</td>
</tr>
</tbody>
</table>

It can be seen in Table 4.6 that some energy dissipated in the joints are very small. For
example, at frequency \( f_3 \) and \( f_4 \) and \( x \) excitation the energy dissipated in the joints can be ignored. The reason is that at this excitation, the corresponding modes are difficult to be activated. This makes the force in the joints small, so is the energy.

It should be noted that the results above were obtained with the following assumptions:

a) The forces derived from the edges of the satellite model were the forces at the joints

b) The summed force along an edge of the satellite model was independent of how many bolts are used

c) The amount of energy dissipated by all joints with the same number of contact surfaces was the same

d) The coefficient of friction in the joints was constant and the preload was distributed uniformly over the washer area

4.5 Conclusions and models for the next stage of research

4.5.1 Conclusions

In this chapter an effective three step approach was established to estimate energy dissipation in the satellite joints. The relationship between the energy dissipated and the applied harmonic joint force was successfully established using FEA. It was found that a fourth-order polynomial can represent this relationship very well. This was used in the satellite structure to find the energy dissipated in the bolted joints which is of concern to the designers in SSTL. The conclusion is, of the energy dissipated in the satellite GSTB-V2/A a relatively modest amount (not over 4%) is from the bolted joints, when the structural damping coefficient is 0.04 and excitation acceleration is \( 2g \).
4.5.2 Models for the next stage of research

Design engineers seek to decrease the vibration of a spacecraft during launch to protect it from damage. One way of achieving this is to introduce damping on the spacecraft to decrease vibration. And another way is to isolate the structure from the vibration sources. At the time of this research some damping mechanisms were used between the GSTB-B2/A and the launch vehicle. In the rest part of this thesis, ways of improving the damping in the joints have been investigated. These include investigating the use of joint macro-slip and using viscoelastic materials in the joints as mentioned before. These methods and techniques can be used in the future for designing and optimising the bolted structures on the satellite.

Numerous non-linear analyses have been necessary to validate the methods. If they were carried out on complicated models, like the real satellite model used in section 4.4 it would be very time consuming. In order to reduce the complexity, a simple idealised satellite model was created. This included only the main body of the satellite as shown in Figure 4.25. It was composed of seven honeycomb panels, six on the outside surfaces and one in the middle, parallel to the top and bottom panels. The honeycomb and the dimensions are the same as a real satellite model, but all the payloads and supporting systems like the power and propulsion have been removed. It is a good idealisation of the real satellite for investigative research. This model consisted of 2412 shell elements.

In Figure 4.25 the small circles (MPC) represent the bolted joints. The joints on the all edges lying in the y direction have formed the focuses of subsequent research. All research will be carried out on this model in the chapters that follow. This section presents the results for the model with joints operating in micro-slip in order to compare with the other joint models discussed in later chapters.

Normal modes analysis was carried out and it was found that the first five natural frequencies are: 35.033 Hz, 35.201 Hz, 68.3 Hz, 83.781 Hz and 161.02 Hz. In fact this
model behaves similarly in the x and y directions. The geometry of the vertical panels are the same but the mass is a little different. That is why the first and the second natural frequency are so close to each other.

In the first mode the satellite swayed in y direction and in the second mode the satellite swayed in x direction. The first modal shape can be seen in Figure 4.26. The second modal shape is almost the same but in the x direction. The modal effective mass fraction was also obtained. For the first mode 77% of the total mass was involved, for the second one 78% was involved. Here and in the later chapters the response at the first natural frequency has been investigated. The results should be almost the same for the second natural frequency, just in different direction.

The first modal shape (where the structure sways in the y direction) would be induced by an excitation in the y direction at the first natural frequency. In the following frequency response and transient analyses an excitation in the y direction has been used. At this excitation bolted joints lying on the four edges in the y direction will dissipate more energy because the predominant loading is shear, while bolted joints on other edges would experience some shear and some tensile loading. Therefore, research will
be focused on the joints lying in the y direction.

Frequency response analyses were carried out here. The energy input to the satellite was obtained from the four separation points on the satellite base. The joint forces were obtained from the MPC forces. Equation (4.2) was used again to obtain the energy dissipated in those joints. They are compared with the input energy and it was found that at the first natural frequency (35.033 Hz) the energy dissipated in the joints was 1.2% of the input energy when the structural damping coefficient is 0.04 and the excitation acceleration is 2g again.

The natural frequency of this simple satellite model is higher than the complicated satellite model. This result is reasonable because the stiffness of the simple satellite did not change much while the mass was much smaller (considering $\omega_n \propto \sqrt{k/m}$, where $\omega_n$ is the radian natural frequency, $k$ is the stiffness of the system and $m$ is the mass of the system). The energy dissipated in the joints of this satellite is also smaller, which is also quite reasonable because not all joints were taken into account.
5.1 Introduction

Bolted joints in spacecraft structures were investigated in the research outlined in the previous chapters. Under shear loading a normal bolted joint has two distinct responses; micro-slip and macro-slip. Experimental data is shown again in Figure 5.1 for convenience. During the micro-slip stage, the force increases with relatively small increments of displacement. The behaviour is slightly non-linear. During the macro-slip stage, the displacement increases while the force change is much less significant.

![Figure 5.1 - The variation of experiment shear force with displacement of a bolted joint](image)

The behaviour was investigated in Chapter 4 by creating a detailed bolted joint model. It was found that if all joints operated in the micro-slip stage the energy dissipation capacity of the joints is quite small. However, when macro-slip occurs, much more energy can be dissipated. This can be simply estimated as follows:
From the detailed satellite model it was found that at the first natural frequency with a 2g x-excitation and structural damping coefficient of 0.04 the energy input to the satellite was 122.8 J. If the optimised locations of joints in Chapter 4 were used, there would be 32 (8 x 4) joints with two contact surfaces and 232 (58 x 4) joints with one contact surface. From the research in Chapter 4, it was found that the sliding force \( F \) for two-contact-surface joint was 5520 N (1380N x 4) and for one-contact-surface joint was 2760 N (690N x 4). It would not be unreasonable to assume that 10% of the joints function in macro-slip and the amount of macro-slip is 0.15mm.

It would also be reasonable to assume that the force displacement hysteresis loop of a joint is a perfect parallelogram as illustrated in Figure 5.2. Based on these simple assumptions the energy dissipated in the joints is

\[
1.38 \times 4 \times 0.15 \times 2 \times 8 \times 4 \times 10\% + 0.69 \times 4 \times 0.15 \times 2 \times 58 \times 4 \times 10\% = 24.5 J
\]

This is around 20% of the input energy.

![Figure 5.2 – Assumed force displacement loop for macro-slip bolted joints](image)

It should be noted that this result is based on some very simple assumptions. For example, a constant macro-slip of 0.15mm was assumed. Much investigation has been carried out to achieve a more accurate estimation of the energy dissipated and this will be discussed in this chapter.
Modelling macro-slip is considerably more complex than modelling micro-slip. In micro-slip a three step procedure was used to assess the energy dissipation in the joints of the spacecraft structure. This included a) establishing the force-energy relationship from a detailed bolted joint model, b) finding the satellite joint forces (MPC forces) from a frequency response analysis of the satellite model and c) using the force-energy relationship of the joint forces to obtain the energy dissipated in the joints. In this approach the force in the joints was obtained from the multipoint constraint (MPC) force. The MPC ensured that the connected nodes had the same displacement. This is valid when the joints remain in the micro-slip stage. However, if they function in the macro-slip stage, significant errors may be introduced by this assumption and this needs to be investigated. For example, the natural frequency may change and at a certain frequency the forces may be redistributed in different joints due to the slip. The most difficult part may be that the amount of macro-slip is very difficult to predict even when the joint force is known.

Although a detailed FE joint model can provide a good approximation for both micro-slip and macro-slip, it cannot be used in the satellite model because the required level of detail is too high. It was necessary to find a way to incorporate the macro-slip behaviour into the satellite model.

The behaviour of an elastoplastic material is very close to the behaviour of the bolted joint shown in Figure 5.1. Thus solid elements, assigned an elastoplastic material response, may be used to represent the bolted joints if they are created properly. This is a novel approach and was successfully implemented which enabled some useful results to be obtained.

5.2 The elastoplastic material unit

In order to investigate how the solid element with an elastoplastic material works a
single element model shown in Figure 5.3 was created. An elastic modulus, a yield stress and a small hardening slope were given to the elastoplastic material. A von Mises yield function and isotropic hardening rules were used. The bottom of the element was fixed and the top was constrained by MPC so that all nodes have the same displacements. One node was assigned a cyclic displacement in the x direction. Non-linear static analysis was carried out on this configuration. The resulting force-displacement response is shown in Figure 5.4. Comparing this figure with the force-displacement response of a detailed bolted joint (Figure 5.1), it can be seen that they are very similar.

![Figure 5.3 - A single solid element model](image)

![Figure 5.4 - Force-displacement response at a top node of the single element model](image)

It is clear that if the right parameters were used in the solid element model, its behaviour will be equivalent to the bolted joint. The question is a) how to correlate them and b) how to then incorporate them into the satellite model. In the following sections the technique will be introduced. Some further experimental data were obtained (by SSTL) to test the strength of the satellite. These are discussed and the results are used to validate the method.

### 5.3 Experiments on satellite bolted joints

It was necessary to undertake strength and stiffness tests on satellite joints to validate
the design. Some panel coupons were manufactured for this purpose. Figure 5.5 and 5.6 show the coupons used. They are two views of the panel to panel (in-plane) joint and the corner joint respectively.

![Diagram of panel to panel joint](image1)

**Figure 5.5 – Panel to panel joint**

![Diagram of corner joint configuration](image2)

**Figure 5.6 – Corner joint configuration**

The materials used were the same as the satellite panels, aluminium (2014) skins and aluminium (5056) cores. The size of the coupons was $100 \times 60 \times 20$ mm. M5 titanium
fasteners and washers were used. The connecting strips were quite different. In the panel to panel joint there were aluminium (7075) connecting strips on both sides of joint in which there were five 6mm holes. The size of the connecting strips was $94 \times 21 \times 2$ mm. There were four pins on each strip to locate the bolts in the middle of the holes, this providing uniform clearance. A titanium nut plate on which there are five threaded holes was used instead of normal nuts. In the corner joint there were no connecting strips but a titanium "prism" with ten threaded holes. These configurations were designed to meet the joint functional and manufacturing requirements and were used in the real satellite.

Static tests were carried out on these joints in the Instron 6025 tensile testing machine. The test coupon was mounted in a steel fixture and shear load was exerted as shown in Figure 5.7. The tests were controlled by a LVDT (Linear Variable Differential Transformer) which is a sensitive displacement measuring instrument. The speed was set to 0.002mm/s.

![Figure 5.7 – Experimental setup for in-plane joint](image)
Some tests were carried out on a 5 fastener coupon, others were carried out on a 3 fastener coupon (the first and the fifth fastener were removed). Preload torques of 10Nm and 20 Nm were used. The shear load was obtained from the load cell and the relative displacement of the joint was obtained from the LVDT.

Figure 5.8 shows one of the test results. The micro-slip and macro-slip parts are obvious. It can be seen that for this kind of joint in macro-slip there is still some residual stiffness.

![Graph](image)

**Figure 5.8 – Test result of a 3 fastener corner joint torqued to 20 Nm**

### 5.4 Satellite model with bolted joints that can operate in macro-slip

#### 5.4.1 Description of the model

The simplified satellite model shown in Figure 5.9 was created. It was almost the same as the one discussed in Chapter 4 (Figure 4.16) except the joints have been modelled differently. In Chapter 4 the joints were represented by MPC constraints. In Figure 5.9 the joints are more complicated, as described below.

The behaviour of the corner joint shown in Figure 5.6 was investigated. It can be seen
that if macro-slip occurs it will take place on the two surfaces of the connecting prism. The macro-slip on the surface perpendicular to x axis will be in the y-z plane. The macro-slip on the surface perpendicular to z axis can be in the x-y plane. Since the response of an elastoplastic material is similar to joint macro-slip, two properly constrained elastoplastic elements can be introduced to model the macro-slip on these two planes. These two elements are called a bolted joint unit.

Figure 5.9 – Model of satellite with the bolted joint units in place

The following approach was used in this work. A joint unit was composed of two connected 8 node brick elements as shown in the inset in Figure 5.9. Clearly, this does not model the actual geometry of the joint, however, the overall behaviour of the unit is very close to the real joint. The four nodes on the top of element A were connected by MPCs so that they had the same displacement. The same was true for the four nodes on bottom of element A and the four nodes on the front and back of element B. The bottom of element A was connected to the top panel of the satellite and the back of element B was connected to the front panel of the satellite. These two elements themselves were connected by another MPC in all three directions. The top and bottom of element A
were connected by 4 rigid springs, as were the front and back of element B. In this way, element A modelled the relative movement of joints in the plane perpendicular to $z$ axis and element B modelled the relative movement of joints in the plane perpendicular to the $x$ axis.

Such a joint unit can model a single fastener joint or a joint with more than one fastener depending on the parameters used.

In the simple satellite model in Figure 5.9 nine bolted joint units have been included on two of the top edges while eight bolted joint units have been included on two of the lower edges giving 34 bolted joint units in total. Each unit represented 3 specific fasteners. Due to the symmetry of the structure the bolted joint units were divided into 9 representative groups as shown in Figure 5.9.

Four springs were used at the base of the satellite model. They connect the excitation points to the four satellite release points. These springs were used to model the real connection between the launch vehicle and the satellite. At the same time they can be used to calculate the energy input to the satellite.

5.4.2 Parameter correlation

In order to ensure that this model worked as expected, a simple model, as shown in Figure 5.10, was created. It was exactly the same as those used in Figure 5.9, but was independent of the satellite. There were two coincident nodes in the middle. One was constrained in all six degrees of freedom and connected to one element. The other was connected to another element and was excited at $z$ direction.

The experimental result shown in Figure 5.8 was used to determine whether the model in Figure 5.10 could be used to represent the behaviour of a real joint.
A bi-linear curve was drawn to represent the test results as shown in Figure 5.11. It was to be reproduced by the model in Figure 5.10. To do this the relationship between \( F_y \) (the force at which macro-slip occurs), \( K_1 \) (the slope of the micro-slip stage), \( K_2 \) (the slope of the macro-slip stage) and the material properties of solid element must be found. The important properties for the non-linear material model being used were the shear modulus \( G \), the Poisson’s ratio \( \nu \), the hardening slope \( K \) and the yield point \( \sigma_{yld} \).

\[
\sigma_{vm} = \frac{1}{\sqrt{2}} \left[ (\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 + 6(\tau_{yz})^2 + 6(\tau_{zx})^2 + 6(\tau_{xy})^2 \right]^{1/2}
\]

(5.1)
where \( \sigma_x, \sigma_y \) and \( \sigma_z \) are tensile stresses; \( \tau_{xy}, \tau_{xz} \) and \( \tau_{yz} \) are shear stresses.

Due to the manner of the constraint and the excitation of the joint unit in Figure 5.10, one shear stress \( \tau \) will dominate and all others can be neglected. The same has been found for the bolted joint unit in the satellite model excited in one direction at the base. Thus the yield point is

\[
\sigma_{yld} = \sqrt{3}\tau = \sqrt{3} \frac{F_y}{A}
\]  

(5.2)

where \( A \) is the area of the shear surface of the bolted joint unit.

The relationship between \( K_I \) and \( G \) can be obtained from the simple stress and strain relationship:

\[
K_I = G \frac{A}{L}
\]  

(5.3)

where \( L \) is the length of the edge of the brick element perpendicular to the shear surface.

That is

\[
G = K_I \frac{L}{A}
\]  

(5.4)

Poisson’s ratio \( \nu \) was also required to define the elastic properties of the material. Thus

\[
E = 2G(1 + \nu)
\]  

(5.5)

where \( E \) is the Young’s modulus.

The only remaining essential material property is the hardening slope \( K \) of the equivalent stress-plastic strain curve. According to [3], the hardening slope is defined as

\[
K = \frac{E^T}{1 - \frac{E^T}{E}}
\]  

(5.6)

where \( E \) is the elastic modulus and \( E^T \) is the slope of the uniaxial stress-strain curve in the plastic region.
From equation (5.4) it can be seen:

\[ G_2 = \frac{K_2 L}{A} \]  

(5.7)

where \( G_2 \) is the shear modulus of the plastic region. So

\[ E^T = 2G_2(1 + v) \]  

(5.8)

By using equations (5.4)-(5.8), it can be shown that:

\[ K = \frac{2K_1 K_2 L(1 + v)}{A(K_1 - K_2)} \]  

(5.9)

From Figure 5.11, it can be found:

\[ K_1 = 5.549286 \times 10^8 \text{ N/m} \]
\[ K_2 = 1.640441 \times 10^7 \text{ N/m} \]
\[ F_y = 7769 \text{ N} \]

In the model used, \( A = 0.0025 \text{ m}^2 \), \( L = 0.05 \text{ m} \), \( v = 0.4 \). Substituting these data into equations (5.2), (5.4) and (5.9), gives:

\[ \sigma_{yld} = 5382521 \text{ N/m}^2 \]
\[ G = 1.109857 \times 10^{10} \text{ N/m}^2 \]
\[ K = 946630.5 \text{ N/m}^2 \]

The force and displacement of the free node in Figure 5.10 is shown in Figure 5.12. The experimental results (the idealised black bi-linear curve in Figure 5.11) are also shown. It can be seen the elastoplastic model represented the joint behaviour very well.
Chapter 5 – Energy Dissipated in the Satellite Joints Operating in Macro-slip

5.4.3 Effects of preload on the stiffness of the joints

The macro-slip of a joint is controlled by the preload on the bolt. It can be imagined that when the preload of the joint changes the stiffness may also change. Two series of experiments were carried out on the corner joints to see the effects. The results are shown in Figure 5.13 where it can be seen that the stiffness does change. Two stiffnesses can be obtained from the figure. Stiffnesses for intermediate torques can be interpolated between these values at the corresponding torque.

Figure 5.12 – Comparison of experimental and numerical results of the single joint model

Figure 5.13 – Corner joint response under different preloads
It can be found from Figure 5.13 that:

\[ F_{y1} = 2841N \]
\[ F_{y2} = 7769N \]
\[ K_{11} = 1.56333 \times 10^8 \]
\[ K_{12} = 5.549286 \times 10^8 \]

where \( F_y \) and \( K \) were defined in the previous section. The second subscripts 1 and 2 represent the cases of torque of 10 \( Nm \) and torque of 20 \( Nm \) respectively. Assuming that the stiffness of the joint varied linearly with the macro-slip force it can be shown that:

\[ K_1 = 80883.8 F_y - 7.34579 \times 10^7 \]  \hspace{1cm} (5.10)

The assumption made here is only for primary analyses since there are only two groups of experimental results available. Further investigation needs to be carried out to validate it or improve it if higher accuracy is required.

Substituting into equation (5.4) and making use of equation (5.2), the following modulus-yield stress relation can be found:

\[ G = 46698.3 \sigma_{yld} L - \frac{L}{A} \times 7.34579 \times 10^7 \]  \hspace{1cm} (5.11)

5.4.4 Numerical results from the simple satellite model

5.4.4.1 Energy dissipation in the satellite with joints operating in macro-slip

As in the micro-slip modelling discussed in Chapter 4, a value of 0.04 was assumed for the structural damping coefficient of the satellite model. The excitation acceleration applied was 2g in the y direction. These parameters were used in both the frequency response analyses and the transient analyses discussed in the following sections. The energy dissipation capability at different macro-slip forces was investigated.

Firstly, it was assumed that the bolted joints were given a preloading torque of 20Nm. In
this case all the solid elements had \( G = 1.109857 \times 10^{10} \text{ N/m}^2 \). The hardening slope had a negligible effect on the energy dissipated and was ignored in the analyses. Normal modes analysis was carried out to find the first natural frequency. It was 34.97Hz. A frequency response analysis was carried out at this frequency and the von Mises stress in each joint unit was obtained. The corresponding stress in the satellite body is shown in Figure 5.14. It shows clearly that the joints in the middle of the edge transfer more force and the joints at the corner transfer less. It should be noted that in both normal modes analyses and frequency response analyses only the elastic properties of the joint units were used. This precludes any macro-slip.

![Figure 5.14](image)

Figure 5.14 – The von Mises stress (Pa) of the simple satellite model with 2g y-excitation at a frequency of 34.97 Hz

Following these analyses the yield stress of the elements were set to 80% and 50% of the real stress obtained from the analysis above (no macro-slip). The elastic stiffness of the elements were also changed according to equation 5.11. Due to the symmetry of model and load to the x and y axes only a quarter of the joints need to be investigated in detail. By further research into the two elements in each bolted joint unit it was found
that both elements experienced almost the same level of load and deformation. Thus only nine elements were considered. All the other elements had the same properties or behaviour as one of these nine elements. The nine elements are indicated in Figure 5.9. Data for these were obtained and are shown in Table 5.1. Due to the stiffness change the first natural frequency of the structure should also decreased slightly. Normal modes analyses were carried out again to get the frequency and it was found to be 34.92 Hz in the 80% case and 34.85 Hz in the 50% case.

Finally nonlinear transient analyses were carried out at the new natural frequencies to find the energy dissipation property.

Table 5.1 – The yield stress and shear modulus of solid elements in different elements

<table>
<thead>
<tr>
<th>Element ID</th>
<th>80% case</th>
<th>50% case</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(\sigma_{\text{yld}} (\times 10^6 \text{N/m/m}))</td>
<td>(G(\times 10^9 \text{N m/m}))</td>
</tr>
<tr>
<td>1</td>
<td>1.43</td>
<td>1.86</td>
</tr>
<tr>
<td>2</td>
<td>2.02</td>
<td>3.25</td>
</tr>
<tr>
<td>3</td>
<td>2.39</td>
<td>4.11</td>
</tr>
<tr>
<td>4</td>
<td>2.50</td>
<td>4.36</td>
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<td>1.80</td>
<td>2.73</td>
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<td>6</td>
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<td>2.78</td>
<td>5.02</td>
</tr>
<tr>
<td>8</td>
<td>2.92</td>
<td>5.36</td>
</tr>
<tr>
<td>9</td>
<td>2.96</td>
<td>5.45</td>
</tr>
</tbody>
</table>

The behaviour of the joints in the 80% case and the 50% case are shown in Figure 5.15 and 5.16 respectively. The stress and strain of one element in each joint unit is shown. The other element in the unit has almost the same behaviour. The stiffness of the elements are different as it was assumed the joint stiffness was proportional to the macro-slip force. The stress-strain data can be converted to force and displacement by use of the dimension of the joints (cube of 0.05m on each side). Then the energy dissipated in the joint which is proportional to the area of the loop can be calculated. The loop size of Element 1 is smallest. Those of Element 2-4 are intermediate and the
loop size of Element 5 is somewhat larger than those of Element 2-4. The loop sizes of Element 6-9 are largest. This indicates the different energy dissipating capabilities of joint groups. The energy dissipated in all these joints was summed and it was found to be 11.8 J in the 80% case and 28.2 J in the 50% case. The input energy was also calculated from the force-displacement at the excitation points on the base of satellite. In the 80% case the input energy was 67.1 J and in the 50% case it was 69.2 J. Thus, in the 80% case the energy dissipated in the bolted joints is 17% of the input energy and in the 50% case it is 41%.

Figure 5.15 – The stress-strain response of the joint elements when the slip stress is 80% of the rigidly bolted stress

Figure 5.16 – The variation of stress with strain of solid elements representing the joints when the slip stress is 50% of the rigidly bolted stress
The amount of macro-slip in each joint unit in the 50% case is shown in Figure 5.17. The yield stress is shown in Figure 5.18. It can be seen that the joint’s macro-slip increased with the yield stress in the joint, but was also constrained by the geometry. For example, joint unit 1 has almost no macro-slip because there is small force transferred from this position and it is close to a ‘rigid’ edge which constrains the movement. The macro-slip in joint unit 4 is smaller than joint unit 2 and 3 because this joint is close to the middle point where the excitation point was and the panels were connected rigidly to transfer the large excitation force. The top edges had more macro-slip than the bottom ones in general because they transfer more shear force and they are less constrained. The amount of macro-slip in the joints varied significantly, but the maximum macro-slip in all these joints was not over 0.15 \textit{mm} which is reasonable because the clearance between bolts and holes of the joints are normally 0.5 \textit{mm}.

![Figure 5.17 - The macro-slip of each joint unit](image1)

![Figure 5.18 - The yield stress of each joint unit](image2)
It can be seen that the joints on the top of the satellite play an important role in dissipating energy. The case where only these joints go into macro-slip was considered. It was found that the natural frequency of the satellite was 34.91 Hz and the energy dissipated in the joints is 18% of the total energy input to the satellite.

5.4.4.2 Comparison of responses of satellite systems with micro-slip and macro-slip joints

In order to see how the macro-slip force affects the response of the satellite, one node and one element on the spacecraft, (node M and element N in Figure 5.9) were chosen. The variation of the displacement of the node with the percentage yield stress is shown in Figure 5.19. The variation of the von Mises stress of element N is shown in Figure 5.20.

Figure 5.19 – The variation of the displacement of node M with different macro-slip forces

Figure 5.20 – The variation of the von Mises stress of element N with different macro-slip forces
It can be seen that when the macro-slip force decreased the response of the spacecraft decreased too. The rate of change of displacement increased with reducing macro-slip load. The same was true for the stress. From the micro-slip condition to the 50% case the displacement decreased by about 29% and the stress decreased by about 26%. This has considerable significance for a spacecraft. For example, a stress decrease means the weight of the spacecraft may be decreased because less material (or different materials) can be used to provide the necessary strength. This will decrease the whole cost of the spacecraft and the launch.

A more common way to represent the damping capability is through the damping ratio $\zeta$. Figure 5.21 shows the frequency responses of two systems near their first natural frequencies. The damping ratio $\zeta$ was obtained from the results through the half power bandwidth approach introduced in section 2.4.3.

The micro-slip response was obtained from the model in Figure 4.16. Thus the damping is only from the structural damping. Since the energy dissipated in the micro-slip joints is only 1.2% of the total energy it is reasonable to ignore its effect. Frequency response analysis is carried out on the model and value of damping ratio $\zeta$ was found to be 2%. This is entirely reasonable as the structural damping coefficient is double the damping ratio (see Appendix E).

![Figure 5.21 – Frequency response at node M in the satellite with different joints](image-url)
For the macro-slip data the effect of the joints should not be neglected because the energy dissipated in the joints is a significant part of the total energy. A series of non-linear transient analyses were carried out for the 50% case. These data are plotted in Figure 5.21. The damping ratio $\zeta$ was found to be 3.5%. Therefore, the damping ratio has increased significantly.

5.5 Conclusions

In this chapter a novel way of estimating the energy dissipated in bolted joints operating in macro-slip in a spacecraft structure was introduced. It was used in a simple satellite model to assess the effect of macro-slip. Two important conclusions were obtained:

a) By using proper parameters, the elastoplastic solid element pair was able to represent the macro-slip of bolted joint in a large structure accurately and efficiently.

b) By using macro-slip joints the modal damping ratio can increase significantly, thus the force and displacement response of a spacecraft can be decreased and it provides an efficient way to decrease vibration of a spacecraft.

In fact this novel unit of two elastoplastic solid elements can be used in structures of any complexity to represent the macro-slip behaviour of bolted joints where, in general, it is not possible to include the detailed FE joint models in the global model.

Although macro-slip in joints can dissipate a lot of energy designers may not like the joints in a structure going into macro-slip as it will cause some instability or become difficult to control. An alternative way of increasing damping will be investigated in the following chapters to provide designers with an alternate means of increasing the damping.
Chapter 6

Energy Dissipated in Linear Viscoelastic Layered Bolted Joints in Spacecraft Structures

6.1. Introduction

A viscoelastic material is a material that exhibits behaviour between that of a solid and a liquid. It reflects combined viscous and elastic response under mechanical stress. Using it to dissipate energy is quite common in many areas, including civil, marine and aerospace engineering. However, how to model the behaviour of such a material is a problem far from being solved fully, especially when used in a large structure.

It was well known that the behaviour of a viscoelastic material depends on many factors including temperature, stress and time. It is difficult to include all these factors in a unified model. It is also not necessary to consider everything because, depending on the situation in which it is used, the dependence on some factors is stronger than on others. For example, the material under consideration will be used primarily in the spacecraft for decreasing the vibration during launch. Therefore the temperature can be assumed as constant. In addition when the strain is not very large, it is normally assumed that the viscoelastic material is only frequency dependent. The modelling work in this chapter is based on these two assumptions.

When the viscoelastic material is used in the bolted joint, creating a detailed FE model is a good way to undertake the research on the behaviour of the joint. However, as mentioned in previous chapters, it is not appropriate to use a detailed joint model in the satellite model as the required level of detail would be too large. A simplified approximation must be used. The most commonly used model for viscoelastic materials is a spring-dashpot model.
Chapter 6 – Energy Dissipated in Linear Viscoelastic Layered Bolted Joints in Spacecraft Structures

From the literature review, it can also be seen that much research has been undertaken on viscoelastic materials and many results have been reported. Significant amongst these are frequency and temperature dependency curves.

The focus of this chapter is how to make use of viscoelastic material properties in the bolted joint model and how to incorporate the joint model into the satellite model. After completing this many useful results will be obtained and discussed.

6.2 Modelling of bolted joints with a viscoelastic layer

6.2.1 Viscoelastic material properties

A viscoelastic material has both elastic and viscous properties. So under dynamic loads there is a phase difference between the excitation load and the displacement response. Normally this is represented by a complex modulus \( G^* = (1 + i\eta)G \) as mentioned in section 2.5.2.1. It has been found that the shear modulus \( G \) and the loss factor \( \eta \) depend on frequency as well as temperature. Much research has been undertaken on different materials by other researchers. In this chapter Stahle’s [14] experiment results on SMRD 100F90 will be used in the modelling research on bolted joints with viscoelastic layers because this material has been used in spacecraft applications.

The values in table 6.1 were obtained from Figure 6.1 at a temperature 25°. This temperature is chosen because the focus of this research is on the launch situation. The values at frequencies of 10 Hz and 100 Hz have been shown by crosses on the figure. Other values are obtained in the same way. This approach has been discussed in some detail in section 2.5.1.1.
Figure 6.1 – Damping and stiffness properties for SMRD 100F90 [14]

Table 6.1 – Stiffness and damping properties of SMRD 100F90

<table>
<thead>
<tr>
<th>Frequency (Hz)</th>
<th>Storage modulus ($\times 10^6$ N/m²)</th>
<th>Damping modulus ($\times 10^6$ N/m²)</th>
<th>Loss factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.15</td>
<td>2.83</td>
<td>0.683</td>
</tr>
<tr>
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<td>4.17</td>
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<td>5.623</td>
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<td>7.57</td>
<td>0.924</td>
</tr>
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<td>10</td>
<td>10</td>
<td>10</td>
<td>1</td>
</tr>
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<td>17.78</td>
<td>13.7</td>
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<td>1.08</td>
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<td>31.62</td>
<td>18.9</td>
<td>21.3</td>
<td>1.13</td>
</tr>
<tr>
<td>56.23</td>
<td>25.9</td>
<td>29.2</td>
<td>1.13</td>
</tr>
<tr>
<td>100</td>
<td>33.4</td>
<td>36.2</td>
<td>1.08</td>
</tr>
</tbody>
</table>

6.2.2 Estimation of energy dissipated in a viscoelastic material

In order to estimate the energy dissipation capability of a viscoelastic material in shear, a simple calculation has been introduced. Let $F$ be transverse force acting on the element of material with shear area of $A$ and thickness of $h$ as shown in Figure 6.2.
According to the simple relationship between stress and strain, it can be shown that

$$\frac{F}{A} = (1 + i \eta)G \frac{x}{h}$$  \hspace{1cm} (6.1)

The relationship between the harmonic force $F$ and the displacement $x$ can be found and the area $E$ of the hysteresis loop they form can be calculated as

$$E = \pi F x \sin \theta = \frac{\pi F^2 h \eta}{AG(1 + \eta^2)}$$  \hspace{1cm} (6.2a,b)

where $\theta$ is the phase angle between the force and the displacement. The detailed derivation can be found in Appendix B.

6.2.3 Detailed model of a bolted joint with a viscoelastic layer

As with research into bolted joints operating in micro-slip (reported in Chapter 4), a detailed bolted joint model with a viscoelastic layer was created to investigate the behaviour of the joint.

6.2.3.1 Modelling of viscoelastic materials in Nastran

As discussed in section 2.5.1.2, Nastran can model viscoelastic material in three ways, the complex eigenvalue method, the modal strain energy method and the direct frequency response method. In this work the third method has been used because it is most accurate.

The following stiffness matrix of the viscoelastic material is used:

$$[K_{dd}]_v = [(1 + g_{REF} TR(f)) + i(g + g_{REF} TI(f))] [K_{dd}^1].$$  \hspace{1cm} (6.3)
where

\begin{align*}
TR(f) &= \frac{1}{g_{\text{REF}}} \left[ \frac{G'(f)}{G_{\text{REF}}} - 1 \right] \\
TI(f) &= \frac{1}{g_{\text{REF}}} \left[ \frac{G'(f)}{G_{\text{REF}}} - g \right]
\end{align*}

(6.4)

and \( g \) is the global structural damping coefficient. \( g_{\text{REF}} \) is the reference structural damping coefficient of the viscoelastic material. It can be seen from (6.3) and (6.4) that \( g_{\text{REF}} \) is finally cancelled. It is included for convenience and the value it takes is not important. The parameter \( G_{\text{REF}} \) is the reference modulus of viscoelastic material and is also just for initialisation of matrix. \( G'(f) \) is the frequency dependent storage modulus and \( G'(f) \) is the frequency dependent loss modulus of the viscoelastic material. These two parameters are really used in the calculation. \( [K_{\text{dd}}' \] is the stiffness matrix of the viscoelastic material computed on the basis of \( G_{\text{REF}} \). Details of explanation of equations (6.3) and (6.4) can be found in Appendix C.

This stiffness matrix was incorporated into the dynamic equations of motion for the elements and then the equations were solved directly by numerical calculation.

6.2.3.2 Detailed joint modelling

A detailed bolted joint with a viscoelastic layer is shown in Figure 6.3. The effective area of the viscoelastic layer in a whole joint is \( 25\,\text{mm} \times 16\,\text{mm} - \pi \times 3\,\text{mm} \times 3\,\text{mm} \). The thickness of the layer is 0.5 \( \text{mm} \). Ten node tetrahedron solid elements were used in the FE model in Figure 6.3b. A 1.5 \( \text{kN} \) preload was applied on the washer area. The left end of solid 3 was constrained by MPCs so that the whole surface will have the same displacement. A force was applied on the independent node of the MPCs. Only a quarter of one bolt joint is modelled according to symmetry conditions. Thus the boundary condition for the right surfaces of solid 1 and 2 is 156; the boundary condition for the bottom of solid 3 is 345 and the boundary condition for the front surfaces of solid 1, 2
and 3 is 246. Number 1 to 3 signify that the translational degree of freedom in x, y and z directions respectively are constrained. Number 4 to 6 signify that the rotational degree of freedom in x, y and z directions respectively are constrained.

Figure 6.3 – Detailed bolted joint with viscoelastic layer (a) dimension of the joint and (b) FE model

The material of solid 1 and 3 is aluminium with elastic modulus of 70000 $N/mm^2$, Poisson’s ratio of 0.33 and density of $2.8 \times 10^{-9} \ t/mm^3$. Assume the material properties of the viscoelastic material $G_{REF} = 10 N/mm^2$, $g_{REF} = 0.09$ and $g=0$. $G_{REF}$ and $g_{REF}$ are only used to initialise the stiffness matrix as stated in section 6.2.3.1. Their values don’t affect the calculated results. A value of $g=0$ means that there is no global structural damping in the model. The parameters used in the model are calculated according to equation (6.4) and shown in Table 6.2.

Table 6.2 – Viscoelastic material properties used in the detailed joint modelling

<table>
<thead>
<tr>
<th>f(Hz)</th>
<th>TR(f)</th>
<th>TI(f)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-6.50</td>
<td>3.15</td>
</tr>
<tr>
<td>1.778</td>
<td>-5.45</td>
<td>4.64</td>
</tr>
<tr>
<td>3.162</td>
<td>-3.64</td>
<td>6.63</td>
</tr>
<tr>
<td>5.623</td>
<td>-2.0</td>
<td>8.41</td>
</tr>
<tr>
<td>10</td>
<td>0</td>
<td>11.1</td>
</tr>
<tr>
<td>17.78</td>
<td>4.15</td>
<td>16.5</td>
</tr>
<tr>
<td>31.62</td>
<td>9.86</td>
<td>23.6</td>
</tr>
<tr>
<td>56.23</td>
<td>17.7</td>
<td>32.4</td>
</tr>
<tr>
<td>100</td>
<td>25.8</td>
<td>40.2</td>
</tr>
</tbody>
</table>
Many frequency response analyses have been undertaken. Here is just one example:

At 1 Hz and an excitation force of 1000N

\[ x_0 = 0.534304\text{mm} \]
\[ \phi = -33.7^\circ \]
\[ E = \pi x_0 \sin(33.7^\circ) = 1.863J \]

where \( x_0 \) is the amplitude of the displacement at the free (left) end of the joint and \( \phi \) is the phase angle between force and displacement.

From the estimation formula (equation 6.2b):

\[ F = 1000N \times 2, h = 0.5\text{mm}, G = 4148694 \text{N/m}^2, \eta = 0.683, \]
\[ A = 16 \times 25 - \pi \times 3^2 A = 371.7 \text{mm}^2 \]

\[ x = \frac{Fh}{A(1 + i\eta)G} \]
\[ x_0 = 0.00053550\text{m} \]
\[ \phi = -34.333^\circ \]
\[ E = \pi ^2 x \sin \theta = \frac{\pi ^2 h \eta}{lwG(1 + \eta^2)} = 1.898J \]

When the results from the FE model and the estimation formula are compared it is found that the displacement error of the estimation formula was 0.22%, the phase angle error was 1.9% and the error of dissipated energy was 1.9%. It can be seen that using the estimation formula in the preliminary analyses is quite reasonable.

6.2.4 Analytical models of bolted joints with viscoelastic layers

6.2.4.1 The Maxwell model

The Maxwell model consists of a spring and dashpot in series as shown in Figure 6.4.

The following equations can be obtained according to spring and dashpot behaviour.
Chapter 6 – Energy Dissipated in Linear Viscoelastic Layered Bolted Joints in Spacecraft Structures

\[ F_1 = k_m x_1 \]
\[ F_2 = c_m \dot{x}_2 \]
\[ F = F_1 = F_2 \]
\[ x = x_1 + x_2 \]

\[ F, x \]
\[ F_1, x_1, k_m \]
\[ F_2, x_2, c_m \]

Figure 6.4 – The Maxwell model

From these equations, the following equation can be obtained:

\[ \frac{dx}{dt} = \frac{1}{k_m} \frac{dF}{dt} + \frac{F}{c_m} \]  \hspace{1cm} (6.6)

Assuming

\[ F = F_0 \cos \omega t \]  \hspace{1cm} (6.7)

Equation (6.6) can be solved to give

\[ x = \frac{1}{k_m} F_0 \cos \omega t + \frac{1}{c_m \omega} F_0 \sin \omega t \]
\[ = F_0 \sqrt{\frac{1}{k_m} \frac{1}{c_m^2 \omega^2}} \left( \cos \omega t + \frac{1}{c_m \omega} \sin \omega t \right) \]
\[ = x_0 \cos (\omega t + \phi) \]  \hspace{1cm} (6.8)

where
\[ x_0 = \frac{F_0}{k_mC_m\omega} \sqrt{c_m^2\omega^2 + k_m^2} \]

\[ \cos \phi = \frac{c_m\omega}{\sqrt{c_m^2\omega^2 + k_m^2}} \]

\[ \sin \phi = -\frac{k_m}{\sqrt{c_m^2\omega^2 + k_m^2}} \]  \hspace{1cm} (6.9)

From above equations it can be shown that:

\[ k_m = \frac{F_0}{x_0 \cos \phi} \]

\[ c_m = -\frac{F_0}{\omega x_0 \sin \phi} \]  \hspace{1cm} (6.10)

From the detailed joint results using equation (6.10) it can be shown that

\[ k_m = 4.499 \times 10^6 \text{ N/m} \]

\[ c_m = 1.07 \times 10^6 \text{ N}
d\text{s/m} \]  \hspace{1cm} (6.11)

A spring dashpot FE model like the one shown in Figure 6.4 was created in Nastran. The parameters in equation (6.11) were used. The result is:

\[ x = 0.000534m \]

\[ \phi = -33.7^\circ \]

\[ E = 1.863J \text{(from equation 6.2a)} \]

It can be seen that the spring dashpot model gave the same displacement and the same phase angle under the same excitation force as those given by the detailed joint model. Since the error of the estimation model to the detailed joint model was found to be small in section 6.2.3, it can be used to obtain the relationship between \( A, G, \eta, h \) and \( k_m, c_m \). This approach makes the modelling procedure simpler.

Using the estimation formula to obtain \( k \) and \( c \) (through equation (6.1) and (6.10)): 

\[ \]
Chapter 6 – Energy Dissipated in Linear Viscoelastic Layered Bolted Joints in Spacecraft Structures

\[ k_m = \frac{A(1 + \eta^2)G}{h} \]
\[ c_m = \frac{A(1 + \eta^2)G}{\omega h \eta} \]  

(6.12)

6.2.4.2 The Kelvin (Voigt) Model

The Kelvin model consists of a spring and dashpot in parallel as shown in Figure 6.5. The motion equation of this model is:

\[ F = F_1 + F_2 \]
\[ x = x_1 = x_2 \]
\[ F_1 = k_k x_1 \]
\[ F_2 = c_k \dot{x}_2 \]  

(6.13)

![Figure 6.5 - The Kelvin or Voigt Model](image)

Assuming \( F = F_0 \cos \omega t \), equation \( F = k_k x + c_k \dot{x} \) can be solved to give

\[ x = \frac{F_0}{\sqrt{k_k^2 + c_k^2 \omega^2}} \cos(\omega t + \phi) \]  

(6.14)

where

\[ \tan \phi = \frac{-\omega c_k}{k_k} \]  

(6.15)

It can be seen that
Chapter 6 – Energy Dissipated in Linear Viscoelastic Layered Bolted Joints in Spacecraft Structures

\[ x_0 = \frac{F_0}{\sqrt{k_k^2 + c_k^2 \omega^2}} \]  

From equations (6.15) and (6.16) it can be shown that

\[ k_k = \frac{F_0}{x_0} \cos \phi \]  
\[ c_k = -\frac{F_0}{\omega x_0} \sin \phi \]  

Equations (6.12) and (6.18) give equations to evaluate the equivalent Maxwell and Kelvin spring dashpot parameters from a specification of the basic viscoelastic material parameters.

6.3 Modelling a simple satellite model with bolted joints with a viscoelastic layer

6.3.1 Satellite model

The simple satellite model used is shown in Figure 6.6. It is very similar to the model in Figure 4.16 except that spring-dashpot elements are used to connect the translation degrees of freedom of the two nodes at each joint. On each bottom edge there are 18 joints and on each top edge there are 19 joints. So there are 74 joints and they were divided into 19 groups according to the symmetry. It is the groups that are shown in Figure 6.6. The planes of symmetry are at the centre of the satellite and perpendicular to \( x \) and \( y \) axes. For example the four node pairs at the top vertices of the satellite belong to group 10. Other groups all have 4 node pairs except group 19 which is in the middle
Chapter 6 – Energy Dissipated in Linear Viscoelastic Layered Bolted Joints in Spacecraft Structures

of the top edges and has only 2 node pairs. The Kelvin spring dashpot system was used to incorporate the viscoelastic response in each joint. Each pair of coincident nodes along the four edges was connected by three springs and three dashpots. They transmitted loads in the x, y and z directions respectively.

Figure 6.6 – Simple satellite model

6.3.2 Modelling and results

The shear modulus $G$ and the loss factor $\eta$ of the viscoelastic material in Table 6.1 were used in equation (6.18) along with geometry of the viscoelastic layer (ie A and h in equation (6.18)) in the joint. Several thickness of the viscoelastic layer were used to investigate variations in the satellite response.

As the length of the satellite edge was 0.9 m and there were 19 bolts along each edge, the width of the joint was assumed to be 0.01 m and the hole in the joint for the bolts was 5 mm. Thus the shear area of each joint in equation (6.18) was calculated as
The parameters of the spring dashpot with a layer thickness of 0.6mm are shown in Table 6.3. Due to the configuration of the joints the shear areas of the viscoelastic material in the y direction is double the areas in the x and z directions. Therefore, the stiffness and the damping coefficient of spring dashpot in the y direction are half those in x and z directions.

**Table 6.3 – Frequency dependent properties of spring dashpot (for a joint with 0.6mm layer)**

<table>
<thead>
<tr>
<th>f(Hz)</th>
<th>$K_x$ $(x10^6N/m)$</th>
<th>$K_z$ $(x10^6N/m)$</th>
<th>$C_x$ $(x10^4N.s/m)$</th>
<th>$C_z$ $(x10^4N.s/m)$</th>
<th>$K_y$ $(x10^6N/m)$</th>
<th>$C_y$ $(x10^4N.s/m)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3.16</td>
<td>34.3</td>
<td>1.58</td>
<td>17.2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.778</td>
<td>3.87</td>
<td>28.4</td>
<td>1.94</td>
<td>14.2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3.162</td>
<td>5.11</td>
<td>22.9</td>
<td>2.56</td>
<td>11.4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5.623</td>
<td>6.24</td>
<td>16.3</td>
<td>3.12</td>
<td>8.15</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>7.61</td>
<td>12.1</td>
<td>3.80</td>
<td>6.05</td>
<td></td>
<td></td>
</tr>
<tr>
<td>17.78</td>
<td>10.4</td>
<td>10.1</td>
<td>5.23</td>
<td>5.06</td>
<td></td>
<td></td>
</tr>
<tr>
<td>31.62</td>
<td>14.4</td>
<td>8.14</td>
<td>7.18</td>
<td>4.07</td>
<td></td>
<td></td>
</tr>
<tr>
<td>56.55</td>
<td>19.7</td>
<td>6.27</td>
<td>9.86</td>
<td>3.13</td>
<td></td>
<td></td>
</tr>
<tr>
<td>100</td>
<td>25.4</td>
<td>4.38</td>
<td>12.7</td>
<td>2.19</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Frequency response analyses were carried out with an excitation acceleration of 2g and a structural damping of 0.04. It was found that the natural frequency was 34.76 Hz. At this frequency the force-displacement response at each joint was obtained and are shown in Figure 6.7. The energy dissipated in all these joints is about 17.3 J which is 25% of the input energy, 70.2 J.

The frequency response of node M, in Figure 6.6, with three different joint models micro-slip joint, macro-slip joint (50% case) and a viscoelastic layer of 0.6mm layer, are put together in Figure 6.8. The damping ratio $\zeta$ of for the viscoelastic case can be found by using half power bandwidth approach and is about 0.024.
Figure 6.7 – Force-displacement loop of joints (a) 1-9 and (b) 10-19

Figure 6.8 – Frequency response at one node on the satellite model
The damping ratio of satellite models with micro- and macro-slip bolted joints were also obtained in Chapter 5. They are shown together in Table 6.4. It can be seen that damping ratio increases with energy dissipated in the bolted joints.

Table 6.4 – Damping ratio and energy dissipated in the different bolted joints of satellite models

<table>
<thead>
<tr>
<th></th>
<th>Micro-slip</th>
<th>Macro-slip</th>
<th>Viscoelastic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Damping ratio $\zeta$</td>
<td>0.02</td>
<td>0.035</td>
<td>0.024</td>
</tr>
<tr>
<td>Energy dissipated in bolted joints</td>
<td>1.2%</td>
<td>41%</td>
<td>25%</td>
</tr>
</tbody>
</table>

Joints with 0.4mm viscoelastic layers were also considered. The displacement of node M in the satellite model with 0mm (none), 0.4mm and 0.6mm layers are shown in Figure 6.9. The von-Mises stress in element N is shown in Figure 6.10. The displacement decreased around 9% and the stress decreased around 10% from 0 mm to 0.6mm cases. Again this is significant for decreasing the satellite cost. The thickness of the viscoelastic layers can be used to control the response of the satellite.

Figure 6.9 – The variation of displacement of node M with thickness of viscoelastic layer
6.4 Conclusions

From the research reported in this chapter, several important conclusions can be obtained:

a) The simple energy estimation model of the bolted joint with a viscoelastic layer gave very close correlation to those from a detailed finite element model.

b) Both Maxwell and Kelvin spring-dashpot models can give the same results as the analytical estimation model or detailed finite element model if the displacement, the phase angle and energy dissipation in the frequency response analysis are the only parameters concerned.

c) Using a viscoelastic layer in bolted joints can dissipate a significant amount of energy. Such an approach can decrease the maximum response of a structure significantly.

d) As more energy is dissipated, larger modal damping ratios are obtained. However, a more specific relationship between these two values could not be found.
Chapter 7

Estimating Energy Dissipation of Non-linear Viscoelastic Layered Bolted Joints in Spacecraft Structures

7.1 Introduction

In Chapter 6, spring-dashpot systems were introduced in a simple satellite structure to model the bolted joints with a viscoelastic layer. The stiffness of the springs and the damping of the dashpot changed with frequency and the model represented the frequency dependence of the viscoelastic material well. A restriction of this approach is that at a given frequency the stiffness and the damping of the viscoelastic material are constants. However, it is well known that these two parameters depend on strain as well as strain rate. The frequency dependent model cannot represent this non-linear behaviour. Therefore a new model was developed. It was found that a displacement dependent non-linear spring - dashpot could model the strain – strain rate dependent behaviour of viscoelastic material well. Parameters for this model were obtained by correlation with experimental data. This will be discussed in detail in the following sections.

7.2 Experimental work

In order to obtain a better understanding of real viscoelastic behaviour and to generate data to define the material property for modelling, some experiments were carried out.

The joints were similar to those described in Chapter 3. They are a double lap joint composed of two 25x80x6mm aluminium bars with two 25x34x3mm aluminium
clamping plates. The clearance holes in the joints were a little larger than in the work reported in chapter 3 to let the viscoelastic layer develop significant strain. The hole has a diameter of 8mm and bolts a diameter of 6mm. Thin layers of various viscoelastic materials were used between the clamping plates and the clamped bars. A specimen is shown in Figure 7.1.

![Figure 7.1 - Bolted joint with viscoelastic layers](image)

The Instron 8511 test machine was used to undertake the experiments. Sinusoidal dynamic tests with various speed (0.1 Hz, 1 Hz and 10Hz) were carried out. The tests were controlled by the same extensometer used in Chapter 3. The displacement amplitude was varied from 0.1mm to 0.8mm. Bolt preload torques varying from 6 N.m to 14 Nm were applied using a torque wrench Torqueleader TCR 25.

There is a vast range of viscoelastic materials that can be chosen for the damping layer. For use in a real satellite the properties that should be considered include not only mechanical parameters such as strength, stiffness and damping but also those making it suitable for space use, like out-gassing and thermal properties. At this stage only the mechanical properties of the materials have been considered to obtain the data and to test the models.

A thermal interface material named Cho-Therm, which is a silicone binder with a boron nitride filler, was chosen for initial testing. This material has been used in a prototype satellite as gaskets to facilitate heat transfer across joints. It was interesting and appropriate to see how much energy that material absorbed. There are different versions
of Cho-Therm. Further detail is contained in the data sheet in Appendix D. Cho-Therm T500 was chosen for this work as it has a large elongation. Physically, it resembles a piece of thick paper (Figure 7.2) and can be cut into the shape required. Figure 7.3 shows two typical hysteresis loops from the tests. One test was carried out at 0.1Hz, another at 10Hz. Obviously the frequency affected the behaviour of the material and the energy it can absorb. However, Cho-Therm has a relatively small elongation, normally below 10%, and at this elongation it cannot transfer a significant force. In fact this material is not very strong. It produced cracks as shown in Figure 7.4 after the tests mentioned above. This limits the energy absorbing capacity of the material and reduces our interest in its use as damping material.

Figure 7.2 – A sheet of Cho-Therm

Figure 7.3 – Hysteresis loops of a bolted

Figure 7.4 – A failed layer of Cho-Therm T500
Another material considered was a kind of silicone rubber named Versasil manufactured by Nusil. The properties of the material supplied are shown in Table 7.1. It still cannot carry a significant load, but can be used to investigate the viscoelastic behaviour and validate the non-linear spring dashpot model being developed. Before curing, the rubber is very flexible. It was rolled into a thin sheet and then cured in the oven at 100°C for about 1.5 hours. Following this it is like a normal rubber (Figure 7.5) and can be cut into the shape needed for insertion into the joints.

Table 7.1 – Typical vulcanized properties of VersaSil (MED-4050)

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Specific gravity</td>
<td>1.15</td>
</tr>
<tr>
<td>Durometer hardness</td>
<td>50</td>
</tr>
<tr>
<td>Tensile strength, Die C, psi/MPa</td>
<td>1350/9.3</td>
</tr>
<tr>
<td>Elongation, %</td>
<td>810</td>
</tr>
<tr>
<td>Tear strength, Die B, ppi/kN/m</td>
<td>230/40.3</td>
</tr>
<tr>
<td>Stress @ 200%, psi/MPa</td>
<td>330/2.3</td>
</tr>
</tbody>
</table>

A VersaSil 4050 gasket as shown in Figure 7.5 having a thickness of about 0.8mm was used in cyclic tests. At this thickness the maximum shear strain could be 125% due to the limitation of the joint clearance of 1.0 mm. This did not make use of the full deformation ability of the material. Different frequencies and different amplitudes of
displacement were applied. Figure 7.6 shows two hysteresis loops. One is at 1 Hz, another one is at 10 Hz. The amplitude of the displacement is 0.1 mm. The stiffness of the rubber does increase with frequency as discussed in the literature.

Figure 7.6 – Hysteresis loops of a bolted joint with VersaSil 4050 at 1 Hz and 10 Hz

Figure 7.7 shows the results of tests when the displacement amplitude sustained by the joints are 0.1 mm and 0.3 mm separately. The frequency is 10 Hz. Obviously the hysteresis loops are quite different.

Figure 7.7 – Hysteresis loops of a bolted joint with VersaSil 4050 at different amplitudes of displacement (10 Hz)

More experiments were carried out at 1 Hz with different displacement amplitudes in extensometer control. The results are shown in Figure 7.8. From Figure 7.7 and 7.8, the
dependence of properties on both strain and strain rate can be observed, but it is dominated by the strain amplitude effects.

![Hysteresis loops of a bolted joint with VersaSil 4050 (1Hz)](image)

**Figure 7.8 – Hysteresis loops of a bolted joint with VersaSil 4050 (1Hz)**

In practice the joints would be required to sustain a higher shear force in order to be used in load carrying structures. However, the deformation of the joint is limited for two reasons, a) the clearance in the joint is limited and b) the deformation has to meet design requirements. For a fixed deformation when the rubber is thinner, the strain is higher and hence sustains a higher level of stress. As the elongation of the VersaSil material is quite large, the thickness of the rubber can be reduced to enable it to bear more force for a given displacement. It was also assumed that if a primer is used, the rubber should bond to the substrates and thus bear more shear loading. Thus a thin layer of VersaSil 4050 was used with primer to see if the shear force could be increased. For the results in Figure 7.9 the thickness of the rubber used was about 0.15 mm. The test was controlled by extensometer at amplitude 0.4mm and frequency 1Hz. The Mullins effect is clearly apparent in the figure. The Mullins effect is a stress softening effect in rubberlike materials, i.e. the stress at a given deformation on unloading a specimen is less than the stress at the same deformation when the specimen was originally being loaded.

It can be seen from Figure 7.9 that the shear force does increase compared with the data in Figure 7.8, changing from about 200 N to 500 N in the stabilised state as the strain
amplitude is increased. The stiffness of the rubber is still too small to be used for significant structural loading. However, the results can be used in correlating the numerical model.

Figure 7.9 – Hysteresis loops of a bolted joint with thin VersaSil 4050

Some experiments were undertaken to investigate the effect of preload of the bolted joints on the resulting hysteresis loop. The results are shown in Figure 7.10. Primer was used and the thickness of the rubber was about 0.5mm. A variety of preloading was given through the use of 8Nm, 10Nm, 12Nm and 14Nm torques. It seems that the effect of the preload is not very significant. The stiffness of the rubber increased slightly with the increase of the preload.

Figure 7.10 – The effect of pre-load on the hysteresis loops
7.3 Modelling of bolted joints with viscoelastic layers

It was necessary to incorporate the non-linear viscoelastic bolted joints into a large structure to investigate the global response of the structure. The question is how to achieve this. It can be seen from the literature review (section 2.5.2) that much research has been carried out on non-linear behaviour of viscoelastic materials. Some good constitutive models have been developed. However, these models need to be included in a detailed joint model and it is not possible to include this level of detail in the global satellite model. Thus a simpler representation of the non-linear bolted joint response has been developed in this chapter.

For a linear spring dashpot model like the Kelvin model, which is composed of a spring and a dashpot in parallel, as shown in Figure 6.5, the force-displacement hysteresis loop is shown in Figure 7.11 for various damping values. The excitation force is 2000 N at 20 Hz; the stiffness of the spring is 1000000 N/m; the damping coefficient used for the dashpot are 1000 N.s/m, 5000 N.s/m and 10000 N.s/m for the curves 1, 2 and 3 respectively. It can be seen that with an increase of the damping coefficient the area of the hysteresis loop becomes bigger (there is more energy dissipated).

![Figure 7.11 - The hysteresis loops from a linear Kelvin model using different damping coefficients](image)
In Figure 7.11, there is a slight change in the slope of the loops. A simple explanation is: with the increase of the damping coefficient, more force can be sustained by the dashpot. Therefore, at the same total force, smaller force is sustained by the spring, which causes smaller displacement. Thus, the whole system seems stiffer.

The curves in Figure 7.11 are ellipses. Based on this, one may think that if the spring becomes nonlinear, the hysteresis loop may change from the ellipsoid shapes in Figure 7.11 and become more like the experimental data shown in the earlier experimental figures. Further, if one can define the nonlinear values of stiffness and damping at any point, it may be possible to derive an exact match. The spring dashpot model used in this chapter is shown in Figure 7.12. It is a non-linear spring (whose stiffness depends on the relative movement of the two nodes) and a linear dashpot (whose damping coefficient is a constant). It may be easier to control the shape using stiffness instead of damping (or both) because the stiffness is more directly related to the stiffness of the system. The value of the stiffness $k(\Delta)$ and the constant damping coefficient $c$ can be determined from the experimental results so that the system can replicate the observed behaviour of the joints.

![Figure 7.12 - Non-linear spring dashpot model](image)

Different parameters for the spring and dashpot were used to assess the effect of the parameters on the resulting loop shape. The force deformation relationship is assumed to be
\[ F = a \Delta^b = k(\Delta)\Delta \]

Different parameters (as shown in Table 7.2) were used for the systems. The results are shown in Figure 7.13. The variation of the spring stiffness with deformation is shown in Figure 7.14. By comparing these curves it can be seen that the stiffness of the spring and spring-dashpot system change in a similar way. That is, when the stiffness of the spring is small, the stiffness of the system is also small and vice versa.

**Table 7.2 – Parameters for different spring-dashpot system**

<table>
<thead>
<tr>
<th>Parameters of curves in Figure 7.13 a</th>
<th>Parameters of curves in Figure 7.13 b</th>
</tr>
</thead>
<tbody>
<tr>
<td>curve 1</td>
<td>curve 2</td>
</tr>
<tr>
<td>curve 1</td>
<td>curve 2</td>
</tr>
<tr>
<td>excitation force (N)</td>
<td>100</td>
</tr>
<tr>
<td>a (N/m^b)</td>
<td>10000</td>
</tr>
<tr>
<td>b</td>
<td>3</td>
</tr>
<tr>
<td>c (Ns/m)</td>
<td>10</td>
</tr>
</tbody>
</table>

It can be see from Figure 7.13 that if a proper form of non-linear spring is chosen, we may be able to replicate the data seen in the experiments. By observing the loops in the figures showing the experimental data it can be seen that at the both ends of the loops the stiffness is bigger and in the middle the stiffness is smaller. A stiffness like that shown in equation (7.1) can be used to fit the curve.

\[ k = a_1 \times \Delta^2 + a_2 \times \Delta + a_3 \]  

(7.1)

One set of the experimental data (from Figure 7.9) was selected and used to find the corresponding stiffness of the spring so that the correlation between the spring-dashpot system is as close as possible to the experimental data. An optimisation function in Matlab was used to find the best coefficients \( a_1, a_2, a_3 \) in equation (7.1) and the best damping coefficient constant \( c \). The objective function used was
\[ \sum (f_i - F_i)^2 \]

where \( f_i \) is the experimental force, \( F_i \) is the numerical force at the same displacement. The results are shown in Figure 7.15.

![Figure 7.13 - Force displacement loops for spring-dashpot system (a)](image1)

\[ F = 10000\Delta^3 \text{ and (b) } F = 10000\Delta^{0.6} \]

![Figure 7.14 - Force-displacement of non-linear spring](image2)

(a) \( F = 10000\Delta^3 \) and (b) \( F = 10000\Delta^{0.6} \)

The same optimisation procedure was used to find the best stiffness and the best damping coefficient of a linear spring-dashpot system. This is also shown in Figure 7.15. It can be seen that the curve from the non-linear spring dashpot system is much closer to the experimental data. Interestingly it was found the energy error (4.5%) of the linear
system is smaller than the nonlinear system (5.7%). The energy error is calculated from:

\[
\frac{e - E}{e}
\]

where \(e\) and \(E\) are experimental energy and numerical energy separately.

![Graph showing experimental data and optimised spring dashpot system](image)

**Figure 7.15 – The variation of experimental data with data from optimised spring dashpot system for VersaSil**

This problem was resolved by defining another objective function including energy dissipated as well as force:

\[
\sum (f_i - F_i)^2 + w(e - E)
\]

where \(w\) is simply a weighting parameter used to balance force and energy errors. When \(w\) is large enough, the energy error can be quite small (1%) for both linear and non-linear system. Some results are shown in Table 7.3. To give a comprehensive evaluation of the systems the force error is also calculated, which is given as

\[
\frac{\sum (f_i - F_i)}{\sum f_i}
\]

<table>
<thead>
<tr>
<th>(w)</th>
<th>linear</th>
<th>non-linear</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>energy error</td>
<td>force error</td>
</tr>
<tr>
<td>0</td>
<td>2.6%</td>
<td>16.2%</td>
</tr>
<tr>
<td>10000</td>
<td>0.042%</td>
<td>16.3%</td>
</tr>
</tbody>
</table>
From above analyses it can be seen that if only the energy is of concern, both linear and non-linear systems can provide a good representation of the material. If the force and the energy are both of concern, the non-linear system is a much better representation.

In order to see if the same material property can be used in the joints at the same frequency but under different amplitude control, the experimental data in Figure 7.7 were analysed. The results are shown in Table 7.4.

<table>
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<tr>
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<th>non-linear system</th>
</tr>
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<td></td>
<td>energy error</td>
<td>force error</td>
</tr>
<tr>
<td>0.1mm</td>
<td>0.3mm</td>
<td>0.1mm</td>
</tr>
<tr>
<td>optimised according to 0.1mm curve</td>
<td>0.47%</td>
<td>24%</td>
</tr>
<tr>
<td>optimised according to 0.3mm curve</td>
<td>20%</td>
<td>1.1%</td>
</tr>
</tbody>
</table>

It can be seen from Table 7.4 that the non-linear spring-dashpot is a good representation of the viscoelastic material at certain frequency and certain displacement. However, if the same property is used for material with different amplitudes large errors may appear. The dependence of the stiffness of the viscoelastic material on displacement amplitude is obvious. This needs to be investigated further in the future.

7.4 Incorporating non-linear joints into the satellite model

Such non-linear spring-dashpot joints need to be included in the global satellite model. In order to see if they can be used in an FE model of the satellite where there are many bolted joints and to investigate their effect on the behaviour of the whole structure the
Chapter 7 – Energy Dissipated in Non-linear Viscoelastic Layered Bolted Joints in Spacecraft Structures

same simple satellite model shown in Figure 6.6 was used. This satellite is composed of 7 honeycomb panels which are modelled by 2412 shell elements. The non-linear joints were on the four edges of the top and bottom of the satellite, in the same positions as the linear ones in Figure 6.6. They are also divided into the same 19 groups as the linear ones.

A non-linear material was defined whose behaviour is shown in Figure 7.16. The parameters were chosen by comparing with a linear model, to ensure that their stiffness and the energy dissipated will be similar. The stiffness $k$ of $7 \times 10^6 \text{N/m}$ and the damping coefficient $c$ of $4 \times 10^4 \text{N.s/m}$ were chosen for the linear spring-dashpot model, which are close to the values used in simple satellite model in Chapter 6 when the thickness of the viscoelastic layer was 0.6mm and the frequency was around 30Hz. After several numerical trials the following parameters were used:

$$k = 5 \times 10^{14} \Delta^2 + 2 \times 10^6 \Delta (\text{N/m})$$
$$c = 10000 (\text{N.s/m})$$

(7.1)

The stiffness ($k$) of the spring changes with the deformation ($\Delta$). Their relationship is shown in Figure 7.17.

![Figure 7.16 - Comparison of a non-linear spring dashpot model and a linear one](image)

These parameters are used in the y direction at each joint. In the x and z directions linear
spring-dashpots with stiffness $k$ of $1.4 \times 10^7 \text{ N/m}$ and damping coefficient $c$ of $8 \times 10^4$ were used. This is because in these two directions they have twice the stiffnesses (see Table 6.3) and the energy dissipated in them is quite small compared with the value in the y direction.

It is difficult to find the natural frequency for this model because the techniques used (for example frequency response analysis) do not consider the deformation dependency of the spring stiffness. From equation (7.1) and the deformation in Chapter 6 it can be seen that the stiffness of the springs is between $2 \times 10^6 \text{ N/m}$ and $7 \times 10^6 \text{ N/m}$. Frequency response analyses with a structural damping coefficient of 0.04 were carried out with spring stiffness of $2 \times 10^6 \text{ N/m}$, $4 \times 10^6 \text{ N/m}$ and $6 \times 10^6 \text{ N/m}$. It was found the natural frequency is 34.18 Hz, 34.38 Hz and 34.52 Hz separately. The aim of this section is to see the effect of the non-linearity on the structural response, so small differences in the natural frequency are not significant.

In fact there should be ways to find the actual natural frequency. For example a fairly good result can be obtained by some time consuming iterative calculations. However, this is not the primary area of concern here. In the following analyses a frequency of 34.35 Hz was chosen to see the effect of the non-linear spring dashpot model.

Figure 7.17 – The variation of stiffness with deformation of the non-linear spring
Non-linear transient analyses were carried out at 34.35 Hz on the simple satellite model. The structural damping coefficient was 0.04 and the excitation acceleration was 2g. The hysteresis loop of each joint is shown in Figure 7.18. It can be seen that the non-linear spring-dashpot models work very well in the satellite structure.

The displacement of node N (Figure 6.6) was obtained. It was 0.0009104 m when the excitation acceleration is 2 \( \text{m/s}^2 \) and is 0.009751 m when the excitation acceleration is 2g. If the model is linear the displacement of node N should be 0.008922 m at excitation acceleration of 2g. The relative difference is 8.5%. That means if the non-linear model is an accurate model, then linear model may have an error of 8.5%.

![Figure 7.18 - Hysteresis loops of non-linear joints (a) 1-9 and (b) 10-19](image-url)
7.5 Conclusions

From the research in this chapter, the following conclusions can be obtained:

a) A technique has been developed to incorporate the non-linear response of bolted joints with viscoelastic layers.

b) The viscoelastic material VersaSil has a strong strain and strain rate dependency.

c) Non-linear spring dashpot models are more representative than linear ones for modelling viscoelastic joints in a complicated structure.

d) Depending on the level of the strain dependency of the material used, different material properties may need to be used for the joints in different positions to achieve higher accuracy.

e) The conclusions in chapter 6 are more accurate for viscoelastic materials that exhibit lower level of strain dependency.
Development work on a satellite in SSTL has raised a number of technical questions. The structural design is one of the challenges faced. The main body of the current satellite is composed of honeycomb panels connected by bolted joints. How these joints function and how they meet the requirements, especially in the severe launch environment, are of concern to the designers of the satellite.

The mass, stiffness and damping properties of a structure determine its response to prescribed loads. The former two are relatively easy to quantify. The latter is very difficult to determine and model.

Researchers have investigated the response of bolted structures for some time. Many analytical and numerical models have been created. However, modelling alone is not generally sufficient. Experimental methods play a very important role in the research because joint properties depend on many factors which sometime are not readily definable. For example, one cannot know the coefficient of friction of joints unless some experiments are carried out. Further, these parameters often change with the working conditions of the joint. An example of this is the abrasion of the metal-metal contact surfaces. Joint preload is often difficult to predict accurately because it can change with loading speed and may decrease with time. All these add to the difficulties in modelling bolted joints.

From the literature review it can was seen that researchers have already made considerable progress in modelling the behaviour of bolted joints. However, knowledge is still very limited concerning how to incorporate bolted joints into a large structure efficiently. Almost all engineering work has been facilitated by experiments. A focus of
this thesis has been the incorporation of bolted joints into the spacecraft model. The methods created here can be used in other bolted structures and the methods and results are very helpful both for the satellite under development and for future developments.

In general, the following main conclusions can be obtained from the thesis:

1) The damping capability of bolted joints operating in micro-slip is generally low. With a typical structure damping coefficient of 0.04 and base excitation acceleration of 2g the energy dissipated in the joints are less than 4% of the energy dissipated in the whole satellite.

2) The three step method developed in the thesis is a good way to evaluate the energy dissipated in the joints of a complicated structure. That is, i) a detailed joint model is created and the force energy relationship obtained; ii) the bolted joints in the structure are modelled using MPCs and the force in the joint (MPCs) can be obtained from the structure model, iii) this joint force can be used as the force in the detailed joint and applied to the force-energy relationship. In this way the total energy can be obtained by summing the energies dissipated by all joints. It was found during this procedure that a fourth order polynomial can fit the joint force-energy curve very well.

3) Macro-slip can be an efficient way to improve the damping of a bolted joint. If a subset of the joints are set to slip at 50% of the corresponding rigid joint load, the energy dissipated in the joints can exceed 40% of the energy dissipated in all the structure, assuming a typical structural damping of 0.04 and an excitation acceleration of 2g.

4) A novel way of modelling the macro-slip behaviour was introduced and it was shown to work well. The method is established by setting the behaviour of an elastoplastic element pair set to match experimental results of bolted joints. Two elastoplastic solid elements forming a unit appropriately constrained by MPCs were used to model the corner joint.
5) Introducing a viscoelastic layer is another good way to improve the damping dissipated in a bolted joint. If the layer is a 0.6 mm thick film of a commercial elastomer (SMRD 100F90) the energy dissipated by a subset of the joints can exceed 25% of the total energy assuming a typical structural damping of 0.04 and an excitation of 2g (while the same subset of the joints operating in micro-slip can only dissipate 1.2% of the total energy at the same structural damping and excitation).

6) Spring-dashpot can be used to model the joints with viscoelastic layers. A simple relationship between the viscoelastic properties and the spring-dashpot properties was found. By using this model, the behaviour of complicated structures with bolted joints with viscoelastic damping layers can be readily simulated. This is a linear model so it can be expected to be accurate for relatively small material strains.

7) A non-linear spring-dashpot model was found to be a better model for the amplitude dependent behaviour of the viscoelastic material. A second order polynomial was found to be good for the VersaSil used in the experiments. Technique for incorporating the model into a satellite model was developed and validated.

From the above it can be seen that this thesis provides some novel and practical ways to incorporate bolted joint response into large scale structures. It also provides useful information for improving damping in a satellite for future design. By improving damping, the whole cost of a satellite can be decreased.

However, there is still much work that can be carried out in this field in the future. The following should provide some interesting lines of enquiry.

1) This thesis presented a method to model the macro-slip in the bolted joints. Information was obtained based on certain assumptions. These include allowing
the joint to slip at specific levels of load. It is still open as to how to implement this in real structures.

2) Based on the experiments carried out on the bolted aluminium joint it can be seen that the dynamic behaviour of the bolted joints can be very different from their static response. Therefore when the joints become stable and factors that affect this stable condition is a subject worthy of further research.

3) Another task which can be addressed is how to develop the constrained elastoplastic element unit to handle more complex joint loadings than were encountered in this work?

4) The non-linear spring dashpot model used in the thesis still has limitations if used with large dynamic strains. The improvement of this model is another task to be considered. Although different spring dashpot properties can be used at different positions with different strains, this is not an ideal solution. A more complicated material model needs to be investigated. Such constitutive models do exist but how to incorporate such model in a complicated structure needs to be resolved.

5) The optimum viscoelastic materials for use in the bolted joints in spacecraft structures form another topic for further investigation.

6) In this work it was found the damping coefficient of the structure increased with the energy dissipated in the joints. It would be interesting to determine a relationship between these two parameters.
Appendix A – Information about the satellite GSTB-V2/a

Figure A.1 – GSTB-V2/a Structure - components
Figure A.2 – GSTB-V2/a Structure – assembly
## Table A.1 – Equipment masses

<table>
<thead>
<tr>
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<th>Unit Mass (kg)</th>
<th>System Mass (kg)</th>
<th>Location</th>
</tr>
</thead>
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<td>Payload</td>
<td></td>
<td></td>
<td></td>
</tr>
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<td></td>
<td>Antenna</td>
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<td>Thruster Panels</td>
</tr>
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</tr>
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<tr>
<td></td>
<td>5</td>
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<td>Payload Bay EFF</td>
</tr>
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</table>
### Appendices

<table>
<thead>
<tr>
<th>Panel</th>
<th>Equip Mass (kg)</th>
<th>Inserts Mass (kg)</th>
<th>Skin Mass (kg)</th>
<th>Core (1)/Adh Mass (kg)</th>
<th>Total Panel Mass (kg)</th>
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</table>

**Notes**

1. Mass per unit area per mm thickness for cores is as defined below. This value multiplied by the panel thickness (mm) and included in the non-structural mass per unit area.

   a. \( 5.2 \text{ lb/ft}^3 \) core = \( 0.084 \text{ kg/m}^3 \)
   
   b. \( 3.4 \text{ lb/ft}^3 \) core = \( 0.055 \text{ kg/m}^3 \)

2. The total panel masses include relevant equipment and insert masses as well as the materials used in the sandwich panel’s construction.
### Table A.3 – Panel material and core specifications

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<th>Panel</th>
<th>Skin material</th>
<th>Core weight (lb/ft$^3$)</th>
<th>Panel thickness (mm)</th>
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</tbody>
</table>

### Table A.4 – Panel FE properties

<table>
<thead>
<tr>
<th>Panel</th>
<th>Thickness</th>
<th>Bending stiffness</th>
<th>Thickness ratio</th>
<th>Non-structural Mass (kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prop bay SFF</td>
<td>0.0008</td>
<td>999.19</td>
<td>17.75</td>
<td>5.88</td>
</tr>
<tr>
<td>Prop Bay Radiators</td>
<td>0.0010</td>
<td>1140.75</td>
<td>19.00</td>
<td>15.77</td>
</tr>
<tr>
<td>Prop Tank Support Panels</td>
<td>0.0010</td>
<td>1140.75</td>
<td>19.00</td>
<td>45.17</td>
</tr>
<tr>
<td>Prop Bay Avionics Support</td>
<td>0.0080</td>
<td>12.00</td>
<td>1.50</td>
<td>17.90</td>
</tr>
<tr>
<td>Thruster Panels</td>
<td>0.0010</td>
<td>1140.75</td>
<td>19.00</td>
<td>33.28</td>
</tr>
<tr>
<td>Access panels</td>
<td>0.0010</td>
<td>1140.75</td>
<td>19.00</td>
<td>5.75</td>
</tr>
<tr>
<td>Avionics Plate</td>
<td>0.0010</td>
<td>2610.75</td>
<td>29.00</td>
<td>33.41</td>
</tr>
<tr>
<td>Payload Bay EFF</td>
<td>0.0010</td>
<td>2610.75</td>
<td>29.00</td>
<td>82.62</td>
</tr>
<tr>
<td>Payload Bay Radiators</td>
<td>0.0010</td>
<td>1140.75</td>
<td>19.00</td>
<td>41.92</td>
</tr>
<tr>
<td>Antenna</td>
<td>0.0020</td>
<td>3000</td>
<td>48.00</td>
<td>6</td>
</tr>
<tr>
<td>Antenna Platform</td>
<td>0.0020</td>
<td>2610.75</td>
<td>29.00</td>
<td>11.55</td>
</tr>
<tr>
<td>Antenna Platform Flexure Support</td>
<td>0.0040</td>
<td>36.75</td>
<td>3.00</td>
<td>11.31</td>
</tr>
<tr>
<td>Solar panels</td>
<td>0.0020</td>
<td>630.75</td>
<td>14.00</td>
<td>6.06</td>
</tr>
</tbody>
</table>
### Table A.5 – Shell and solid element properties

<table>
<thead>
<tr>
<th>Equipment</th>
<th>Material</th>
<th>Shell thickness (mm)</th>
<th>Solid element density (note 1)</th>
<th>Mass (kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Propellant tank</td>
<td>See below</td>
<td>See below</td>
<td>See below</td>
<td>5.5 (each)</td>
</tr>
<tr>
<td>Tank wall</td>
<td>Ti</td>
<td>0.66</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>Tank eq wall</td>
<td>Ti</td>
<td>4</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>Tank eq ring</td>
<td>Ti</td>
<td>6</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>Tank flange</td>
<td>Ti</td>
<td>8</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>Tank bracket</td>
<td>Al</td>
<td>10</td>
<td></td>
<td>0.15 (each)</td>
</tr>
<tr>
<td>Payload strut mount</td>
<td>Al</td>
<td>3</td>
<td></td>
<td>0.18</td>
</tr>
</tbody>
</table>

### Notes

1. For units modelled as a low modulus solid skinned with shell elements the solid material name is not listed but its density is given.

### Table A.6 – Beam element properties

<table>
<thead>
<tr>
<th>Beams</th>
<th>Material</th>
<th>Diameter (mm)</th>
<th>Wall thickness (mm)</th>
<th>Mass (kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Payload support strut</td>
<td>Al</td>
<td>40</td>
<td>1</td>
<td>0.35</td>
</tr>
</tbody>
</table>

### Table A.7 – Material properties use for analysis

<table>
<thead>
<tr>
<th>Material</th>
<th>E (Pa)</th>
<th>G (Pa)</th>
<th>v</th>
<th>CTE (x10^6)</th>
<th>p (kg/m^3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aluminium (AL)</td>
<td>$70 \times 10^9$</td>
<td>-</td>
<td>0.33</td>
<td>23</td>
<td>2800</td>
</tr>
<tr>
<td>Titanium (Ti)</td>
<td>$114 \times 10^9$</td>
<td>-</td>
<td>0.34</td>
<td>8.6</td>
<td>4400</td>
</tr>
<tr>
<td>Aluminium core (5.2)</td>
<td>-</td>
<td>$320 \times 10^6$</td>
<td>0.03</td>
<td>23</td>
<td>84</td>
</tr>
<tr>
<td>Aluminium core (3.4/3.1)</td>
<td>-</td>
<td>$170 \times 10^6$</td>
<td>0.03</td>
<td>23</td>
<td>50</td>
</tr>
</tbody>
</table>
### Table A.8 – Mass properties

<table>
<thead>
<tr>
<th>Mass property</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass</td>
<td>458.3</td>
<td>Kg</td>
</tr>
<tr>
<td>CoG X</td>
<td>0.0027</td>
<td>m</td>
</tr>
<tr>
<td>CoG Y</td>
<td>-0.0009</td>
<td>m</td>
</tr>
<tr>
<td>CoG Z</td>
<td>0.5982</td>
<td>m</td>
</tr>
<tr>
<td>Ixx</td>
<td>107.8</td>
<td>Kgm²</td>
</tr>
<tr>
<td>Iyy</td>
<td>100.0</td>
<td>Kgm²</td>
</tr>
<tr>
<td>Izz</td>
<td>101.4</td>
<td>Kgm²</td>
</tr>
<tr>
<td>Ixy</td>
<td>0.653</td>
<td>Kgm²</td>
</tr>
<tr>
<td>Ixz</td>
<td>-0.708</td>
<td>Kgm²</td>
</tr>
<tr>
<td>Iyz</td>
<td>-0.144</td>
<td>Kgm²</td>
</tr>
</tbody>
</table>
Appendix B – Energy dissipated in a viscoelastic material

When the phase angle between the force and response displacement is \( \theta \), it is assumed that

\[
F = F_0 \cos \omega t \\
x = x_0 \cos(\omega t - \theta)
\]  
\( \text{(A.1)} \)

The energy dissipated \( \text{W} \) can be calculated as follows:

\[
W = \int_0^{2\pi} F x dt = \int_0^{2\pi} F_0 \cos \omega t (-\omega x_0 \sin(\omega t - \theta)) dt
\]
\[
= -\omega F_0 x_0 \int_0^{2\pi} \cos \omega t (\sin \omega t \cos \theta - \cos \omega t \sin \theta) dt
\]
\[
= -\omega F_0 x_0 \left( \frac{1}{2\omega} \cos \theta \sin^2 \omega t - \frac{\sin \theta \sin 2\omega t}{4\omega} - \frac{\sin \theta}{2} \right)_{\omega t}^{2\pi}
\]
\[
= \pi F_0 x_0 \sin \theta
\]  
\( \text{(A.2)} \)

From the definition of complex modulus of viscoelastic material the following equation can be obtained:

\[
\frac{F}{A} = (1 + i\eta)G \frac{x}{h}
\]  
\( \text{(A.3)} \)

that is

\[
x = \frac{F_0 h}{AG(1 + \eta^2)} (1 - i\eta)
\]  
\( \text{(A.4)} \)

From (A.4) it can be seen that

\[
x_0 = \frac{F_0 h}{AG \sqrt{1 + \eta^2}}
\]  
\( \text{(A.5)} \)

\[
\sin \theta = \frac{\eta}{\sqrt{1 + \eta^2}}
\]

Substitute (A.5) into (A.2) the energy dissipated in the viscoelastic material can be obtained:

\[
W = \pi F_0 x_0 \sin \theta = \pi \frac{F_0^2 h}{AG \sqrt{1 + \eta^2 \sqrt{1 + \eta^2}}} \frac{\eta}{\sqrt{1 + \eta^2}} \frac{\eta}{AG(1 + \eta^2)} = \frac{\pi F_0^2 h \eta}{AG(1 + \eta^2)}
\]  
\( \text{(A.6)} \)
Appendix C – Damping in direct frequency response analysis in Nastran

The stiffness and damping component of the dynamic matrices for direct frequency response analysis are:

$$K_{dd} = (1 + ig)K^{1}_{dd} + K^{2}_{dd} + iK^{4}_{dd}$$  \hspace{1cm} (A.7)

$$B_{dd} = B^{1}_{dd} + B^{2}_{dd}$$  \hspace{1cm} (A.8)

where \( g \) = Overall structural damping

\( K^{1}_{dd} \) = Stiffness matrix for structure elements

\( K^{2}_{dd} \) = Stiffness terms generated through direct matrix input

\( K^{4}_{dd} \) = Element damping matrix generated by the multiplication of individual element stiffness matrices by an element damping GE

\( B^{1}_{dd} \) = Damping matrix of some damping elements

\( B^{2}_{dd} \) = Damping terms generated through direct matrix input

The viscoelastic material properties are reflected in terms of equation (A.7). If the stiffness properties for the viscoelastic elements are initially computed on the basis of a representative reference modulus, \( G_{REF} \), the stiffness matrix for the viscoelastic elements (denoted by the subscript \( V \)) may be written in the form

$$K^{V}_{dd} = \frac{G'(f) + iG^*(f)}{G_{REF}} K^{-1}_{dd V}$$  \hspace{1cm} (A.9)

where \( G' \) = shear storage modulus

\( G^* \) = shear loss modulus

To cast equation (A.7) into a form that may be compared to equation (A.9), the
following conditions are utilized:

1. The use of the \([K_{dd}^4]\) matrix will be restricted to the viscoelastic elements. All viscoelastic materials must have representative reference values of \(G_E, g_{REF}\) and \(G_{REF}\) entered on their material data entries. Then by definition,

\[
K_{dd}^4 = g_{REF} K_{dd}^1
\]  
(A.10)

2. The tabular function \(TR(f)\) and \(TI(f)\) are defined to represent the complex moduli of the viscoelastic materials.

Those two conditions may be combined in equation (A.7) to provide the following expression:

\[
K_{dd} = (1 + ig)K_{dd} + \{TR(f) + iTI(f)\}K_{dd}^4
\]

\[
= \{[1 + g_{REF} TR(f)] + i(g + g_{REF} TI(f))\}K_{dd}^1
\]  
(A.11)

A comparison of equation (A.10) and equation (A.11) yields the form of the tabular functions \(TR(f)\) and \(TI(f)\).

\[
TR(f) = \frac{1}{g_{REF}} \left[ \frac{G'(f)}{G_{REF}} - 1 \right]
\]  
(A.12)

\[
TI(f) = \frac{1}{g_{REF}} \left[ \frac{G'(f)}{G_{REF}} - g \right]
\]
## Appendix D – Properties of CHO-THERM

### Table A.9 – Properties of Cho-Therm T500

<table>
<thead>
<tr>
<th>Construction</th>
<th>Binder</th>
<th>Silicone</th>
</tr>
</thead>
<tbody>
<tr>
<td>Filler</td>
<td>Boron Nitride</td>
<td></td>
</tr>
<tr>
<td>Colour</td>
<td>Green</td>
<td></td>
</tr>
<tr>
<td>Thickness, inch(mm)</td>
<td>0.010(0.25)</td>
<td></td>
</tr>
<tr>
<td>Tensile strength, psi (MPa)</td>
<td>1000(6.89)</td>
<td></td>
</tr>
<tr>
<td>Tear strength, lb/in (kN/m)</td>
<td>100(17.5)</td>
<td></td>
</tr>
<tr>
<td>Elongation, %</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>Hardness (Shore A)</td>
<td>82</td>
<td></td>
</tr>
<tr>
<td>Specific Gravity</td>
<td>1.55</td>
<td></td>
</tr>
<tr>
<td>UL flammability rating</td>
<td>V-0</td>
<td></td>
</tr>
<tr>
<td>Outgassing: % TML/% CVCM</td>
<td>0.40/0.10</td>
<td></td>
</tr>
</tbody>
</table>

### Table A.10 – Properties of Cho-Therm 1671

<table>
<thead>
<tr>
<th>Construction</th>
<th>Binder</th>
<th>Silicone</th>
</tr>
</thead>
<tbody>
<tr>
<td>Filler</td>
<td>Boron Nitride</td>
<td></td>
</tr>
<tr>
<td>Carrier</td>
<td>Fiberglass</td>
<td></td>
</tr>
<tr>
<td>Colour</td>
<td>White</td>
<td></td>
</tr>
<tr>
<td>Thickness, inch(mm)</td>
<td>0.015(0.38)</td>
<td></td>
</tr>
<tr>
<td>Tensile strength, psi (MPa)</td>
<td>1000(6.89)</td>
<td></td>
</tr>
<tr>
<td>Tear strength, lb/in (kN/m)</td>
<td>100(17.5)</td>
<td></td>
</tr>
<tr>
<td>Elongation, %</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>Hardness (Shore A)</td>
<td>90</td>
<td></td>
</tr>
<tr>
<td>Specific Gravity</td>
<td>1.55</td>
<td></td>
</tr>
<tr>
<td>UL recognized</td>
<td>File No. E57104</td>
<td></td>
</tr>
<tr>
<td>Outgassing: % TML/% CVCM</td>
<td>0.40/0.10</td>
<td></td>
</tr>
</tbody>
</table>

### Table A.11 – Properties of Cho-Therm 1674

<table>
<thead>
<tr>
<th>Construction</th>
<th>Binder</th>
<th>Silicone</th>
</tr>
</thead>
<tbody>
<tr>
<td>Filler</td>
<td>Aluminium oxide</td>
<td></td>
</tr>
<tr>
<td>Colour</td>
<td>Blue</td>
<td></td>
</tr>
<tr>
<td>Thickness, inch(mm)</td>
<td>0.010(0.25)</td>
<td></td>
</tr>
<tr>
<td>Tensile strength, psi (MPa)</td>
<td>1500(10.34)</td>
<td></td>
</tr>
<tr>
<td>Tear strength, lb/in (kN/m)</td>
<td>100(17.5)</td>
<td></td>
</tr>
<tr>
<td>Elongation, %</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>Hardness (Shore A)</td>
<td>90</td>
<td></td>
</tr>
<tr>
<td>Specific Gravity</td>
<td>2.20</td>
<td></td>
</tr>
<tr>
<td>UL flammability rating</td>
<td>V-0</td>
<td></td>
</tr>
<tr>
<td>Outgassing: % TML/% CVCM</td>
<td>0.45/0.20</td>
<td></td>
</tr>
</tbody>
</table>
Appendix E – The relation between structural damping coefficient and equivalent viscous damping in Nastran

Structural damping is introduced to motion equations by means of complex stiffness. Structural damping force $f_s$ is

$$f_s = iGku$$

(A.13)

where $G$ is structural damping coefficient, $k$ is stiffness and $u$ is displacement.

However, transient response analysis does not permit the use of complex coefficients. Therefore, structural damping is included by means of equivalent viscous damping. A relation between these two parameters must be defined.

Viscous damping force $f_v$ is

$$f_v = cu$$

(A.14)

where $c$ is damping coefficient.

Assuming constant amplitude oscillatory response for a SDOF system, the two damping forces are identical if

$$Gk = c\omega$$

(A.15)

where $\omega$ is radian frequency

The relation between the natural frequency, the damping coefficient and the damping ratio $\zeta$ of a SDOF system is

$$\omega = \sqrt{\frac{k}{m}}$$

$$\zeta = \frac{c}{2\sqrt{k}m}$$

(A.16)
where \( m \) is the mass of the system.

Substitute equation (A.16) into (A.15) it can be obtained:

\[
G = 2\zeta
\]

(A.17)
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