Friction and camber influences on the static stiffness properties of a racing tyre

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Abstract

Investigation of the stationary load-deflection behaviour of tyres reveals many details of the structure and the rubber to road friction properties. These characteristics are fundamental to the understanding of the behaviour of both the stationary and the rolling tyre. In connection with racing, tyre static stiffness characteristics are of interest as they reflect on the controllability of the vehicle to which they are fitted.

In this paper, the construction of a finite element model capable of predicting the static tyre behaviour in great detail is presented. The model is validated against extensive experimental data, including contact pressure distributions and load-deflection characteristics. Static loading tests which involve variations in wheel camber angle and in the friction rules coupling tread rubber with the ground surface, are simulated with the tyre model. The results of the simulations reveal that a friction coefficient of 0.5 is sufficient to prevent sliding in static loading tests for this particular tyre. Lower levels of friction lead to tread sliding and reduced vertical tyre stiffness. Sliding is primarily lateral, leading to narrowing of the contact patch of the tyre. Narrowing also increases the local sidewall curvature which is part of the softening mechanism for both upright and cambered tyres.

1. Introduction

Tyre behaviour on a running car depends on carcass stiffness and rubber to road friction properties with complex interactions between them. In general terms, the small-effort behaviour primarily involves adhesion between tread rubber and ground surface and is dominated by carcass stiffness attributes, while the large-effort or near-limit behaviour has sliding of rubber relative to ground at its core. In the latter case, the behaviour is ruled by rubber to road friction forces. In racing, the main concern is with limit-behaviour but nevertheless, the small-effort demeanour is of interest, since it reflects on the controllability of the concerned vehicle.

Many structural details of a tyre can be revealed by static load-deflection testing, in which the non-spinning tyre is loaded against a flat surface and detailed observations of deformations and stresses are made. If this kind of test is done in reality, many of the measurements needed are intricate and difficult but if the testing is virtual, no such difficulties arise. They are replaced by the need to develop a virtual tyre, the results from which can be taken to represent the real item.
This article is an account of the construction of such a virtual racing tyre and results which derive from the model relating to static loading tests. The numerical tests involve variations in the friction rules coupling tread rubber with the ground surface and in wheel camber angle. Corresponding laboratory tests are conducted on an actual racing tyre. Results obtained are compared and fully interpreted.

Within the following discussion, the ISO axis system is employed. Note that the directions of the shear stresses at the tyre road interface are defined such that positive lateral shear stresses push the tyre in the direction of the negative y-axis (i.e. to the right side) and positive longitudinal shear stresses push the tyre towards the negative x-axis (i.e. rearwards).

2. Tyre model description

For this static deflection study a radial tyre with plain tread containing four grooves is considered. The model is built by using the finite element (FE) method and all analyses are performed with the implicit commercial code Abaqus/Standard V6.7.

Due to the nature of the investigated loading cases a full three dimensional (3D) tyre model is required. The 3D model is generated by revolving a two dimensional (2D) cross-section (see Figure 1) about the spindle axis, employing the symmetric model generation procedure [1].

Structure and materials of FE model

The 3D model is constructed from rubber and reinforcement materials which represent a contemporary racing tyre. The rubber materials are used for the sidewalls and the tread compound, and are simulated with a hyperelastic Mooney-Rivlin model [1]. To avoid convergence difficulties due to the near incompressibility of rubber and the contact simulation
itself, reduced linear brick elements with hybrid formulation (C3D8RH) are selected. The reinforcement materials – the belts, the carcass and the two beads – are modelled with fully integrated elements. For the belts and carcass, linear brick elements (C3D8) are employed and the asymmetric lay-up is recreated with an anisotropic material model. The beads are modelled with linear prismatic elements (C3D6) and the bead material is simulated with an orthotropic material model.

**Mesh details**

To minimize computational cost while maintaining modelling accuracy, the mesh is locally refined near the contact region. A suitable mesh density that predicts the static tyre behaviour accurately was determined with an initial mesh refinement study. The final tyre model is constructed from 80 non-uniform tyre segments, from which 34 segments are used in the contact patch region to span a 50° sector angle. The remaining segments are distributed as outlined in Table 1. Accordingly, the mesh of the full 3D tyre model comprises 24,890 elements and the total number of degrees of freedom (including contact variables) is approximately 105,000.

<table>
<thead>
<tr>
<th>sector</th>
<th>sector angle</th>
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<tr>
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<td>13</td>
<td>10°</td>
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<tr>
<td>transition</td>
<td>25°</td>
<td>10</td>
<td>2.5°</td>
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<tr>
<td>contact patch</td>
<td>50°</td>
<td>34</td>
<td>~1.5°</td>
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<td>transition</td>
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<td>outside</td>
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**Model boundaries**

The boundaries of the tyre model are two contact conditions which involve (1) contact between the rim and the tyre and (2) contact between the tyre and the ground surface. Both contact conditions are set up in the same way. The road and the rim are modelled with smooth analytical surfaces. In each case, contact with the tyre is established with the node-to-surface discretisation formulation, whereby the tyre forms the slave surface and the analytical rigid components are defined as the master surfaces. The tangential contact behaviour is simulated with a Coulomb friction model using a constant friction coefficient, denoted by $\mu$. For the rim-tyre boundary the friction coefficient is set to 0.5 which corresponds to values found in the literature [2]. The friction coefficient between the road and the tyre is varied for the different simulations as outlined below.
To replicate the interaction between the tyre and the rim more realistically, the FE model of the wheel is made in two halves [2, 3]. By this device, the mounting of the tyre on the rim can be simulated by moving each rim half from the outside towards the centre-plane of the tyre.

3. Virtual testing

The static load-deflection behaviour of the tyre is investigated for two types of variations in operating conditions: (1) friction level between the road and the tyre and (2) camber angle. Static loading simulations are performed at four different friction coefficients (0, 0.25, 0.5 and 1.0) and with two different camber angles (0 and -3°).

Procedure

The virtual testing of the tyre itself is broken down into the following four stages:

1. Generation of the three dimensional model by revolving the 2D cross-section about the spindle axis.
2. Mounting of the tyre on the rim by moving the two rim halves from the outside towards the centre-plane of the tyre.
3. Inflation of the tyre by a uniform internal pressure. In accordance with widely accepted modelling methodology [2, 3], the inflation pressure is assumed to stay constant during the vertical loading simulation and hence is treated as providing a distributed load acting on the inside surface of the tyre, see Figure 2.
4. Loading of the tyre in the direction normal to the ground surface. The three dimensional model is held stationary by constraining the rim in all rotational and translational degrees of freedom. Then, depending on the analysis objective, the loading of the tyre is controlled via the ground plane by either specifying a displacement or a certain vertical load at the reference node of the ground surface.

By restarting the simulation after the tyre inflation (step 3), the virtual testing at the varying operating conditions outlined above requires only the last analysis stage to be repeated with the changed parameters. As the camber angle is introduced at the road surface, the changes for the camber investigation involve tilting of the ground plane about the longitudinal axis before it is brought into contact with the tyre, see Figure 2.
Figure 2: Loading conditions of the static deflection test

Analysis details
All analyses are performed with double precision to guard against numerical imperfections. Typically, the total CPU time for the multi-step deflection analysis varies between ~20 minutes (frictionless simulation) and ~500 minutes ($\mu = 1.0$) on a dual 3.6 GHz Xeon processor workstation.

4. Validation
In order to ensure the accuracy of the proposed FE methodology, the tyre model is correlated with experimental data. In particular, the model is validated in two ways: (1) the stationary load against deflection behaviour and (2) the static contact pressure distribution. Therefore, experimental investigations are conducted with the actual racing tyre mounted in a load-deflection machine. It should be noted that the investigated tyre had been employed cambered in previous rolling tests, which caused minor wear of the tread surface. As a result of the wear, the contact pressure distribution will have been affected slightly as described below.

Procedure
For the experimental load-deflection investigation, a known displacement is applied to the wheel axle and the vertical load is recorded with load cells placed under the ‘contact ground surface’. A linear variable differential transformer (LVDT) is employed to measure the vertical displacement of the rig. To obtain the true tyre deflection during the investigation the measured displacement is corrected by the deflection of the rig itself. Due to frictional influences this
procedure allows reliable measurements only for vertical loads greater than 1 kN and the load-deflection curves shown in Figure 3 and Figure 4 are presented accordingly. For the contact pressure investigation, a pressure mapping system provided by XSENSOR Technology Corporation is placed on top of the contact ground surface. The pressure sensing system consists of a matrix of capacitive pressure sensors separated by 1.15 mm. Measurements are taken at four different normal loads ranging from 1.5 kN to 6.0 kN.

Vertical stiffness correlation

![Vertical stiffness correlation](image)

**Figure 3:** Vertical load against relative vertical displacement of the tyre at zero camber – simulation ($\mu = 0.5$) and experiment

![Vertical stiffness correlation](image)

**Figure 4:** Vertical load against relative vertical displacement of the tyre at 3° camber – simulation ($\mu = 0.5$) and experiment
As indicated by Figure 3 and Figure 4, the predicted load against deflection curves for the upright and the cambered tyre match the corresponding experimental investigations very well. In addition, by exhibiting a near linear relationship for the specified vertical load range, the experimental and predicted load-deflection characteristics are in accordance with published data [4, 5].

**Contact pressure distribution correlation**

![Contact Pressure Distribution](image)

*Figure 5: Normalised measured contact pressure distribution of the upright tyre subjected to a vertical load of 4507 N*
Predicted and measured contact pressure distributions for the same vertical load of 4507 N are shown in Figure 5 and Figure 6 as exemplary results. In order to facilitate comparison, the FE results and the measurements are presented with the same normalised pressure scale.

Investigation of the contact pressure distributions gives rise to three observations. Firstly, the lower half of the contact patch of the real tyre (positive side of the y-axis) shows to be slightly bigger than the upper half (negative side of the y-axis). This difference in size results from a slight uneven wear of the tread surface caused by previous rolling tests as described above.

Secondly, the overall contact patch shapes and dimensions of the real tyre and the FE tyre model match very well. Both show concave leading and trailing contact patch edges and exhibit a small asymmetry about the longitudinal and lateral centre-lines. This asymmetric contact patch shape is caused by a distortion/twisting of the loaded tyre which, in turn, results from the asymmetric carcass lay-up. Thirdly, the predicted and measured pressure distributions also show good agreement and only local discrepancies exist. Especially, the predicted pressure distribution of the centre rib and the lower middle rib (on the positive y-axis) differ to a small extent from the experimental results. These differences can be attributed to tread surface wear of the tested tyre, as mentioned above.

Figure 6: Normalised predicted contact pressure distribution of the upright tyre pressed against a friction surface ($\mu = 0.5$) with a vertical load of 4507 N
5. Results obtained

With the validated FE model, the virtual static loading tests were performed. Results on the static load-deflection behaviour of the tyre, the shear stress and contact pressure distributions in the contact region, the contact area against load characteristics and the sidewall deflection are presented and discussed below.

The load against deflection curves at the different friction levels, for both upright and cambered tyres (see Figure 7 and Figure 8) exhibit the expected shape, i.e. after an initial non-linear region, the relationship between the load and the displacement becomes linear. Some of the initial nonlinearity can be ascribed to the curvature of the crown profile [4]. For the case of the cambered tyre, the initial nonlinearity of the load-displacement graph (see Figure 8) also results from the fact that the formation of the ‘full contact patch’ requires more tyre deflection. As a consequence, compared to the upright tyre, the initial nonlinearity extends over a greater displacement range.

![Figure 7: Vertical tyre load against displacement as a function of friction coefficient (zero camber)](image-url)
The differences in the vertical tyre load against displacement graphs with changing friction levels imply that the contact conditions have an effect on the static vertical tyre stiffness. In particular, the influence of friction level is more pronounced at higher tyre loads. At normal loads greater than \( \sim 1.5 \text{ kN} \) the carcass stiffens as the coefficient of friction is increased. This rise in vertical tyre stiffness can be attributed to the occurrence of frictional stresses at the tyre-road interface which oppose the shrinkage of the contact patch of the statically loaded tyre [5, 6]. Shrinkage of the contact patch, in turn, results from the relaxation of the inflation-induced pre-strains of the tyre cords in the contact patch [5] which occur due to the deformation of the tyre (compression, stretching and bending of the tyre structure) as it is pressed against the ground surface [7]. Shrinkage involves the tread moving towards the centre of the contact patch.

On a ‘friction surface’, the horizontal movement is opposed and restricted by frictional shear forces. As indicated by the shear stress distributions along the longitudinal centre-line at a friction coefficient of 0.25, see Figure 9, the stresses in the lateral and longitudinal directions stay well below the friction limit (i.e., contact pressure multiplied by the friction coefficient). The lateral shear stresses along the wheel centre plane exist only because of the twisting of the tyre due to the asymmetric carcass lay-up as described above. In contrast, at the same friction level of 0.25, the lateral shear stresses along the centre cross-section, see Figure 10, match the stress limit in the region of the outer and middle tread ribs. As a result, these tread ribs slide laterally. The longitudinal shear stresses, which again arise solely due to the twisting of the tyre, stay below the friction limit. Therefore, at low friction levels, the tread is restrained from sliding in...
the longitudinal direction, whereas it will slide laterally. When the friction level is increased, the lateral shear stresses reach the stress limit only in the region of the outer ribs of the contact patch, see Figure 11. Consequently, tread sliding only occurs in this region and, hence, the movement of the tread as a whole is restricted. These observations also indicate that the contact patch predominately tends to shrink in the lateral direction, i.e. the contact patch narrows on loading.

Figure 9: Shear stress distributions along the longitudinal centre-line at 6000 N vertical load and $\mu = 0.25$. Positive and negative stress limits are shown, marked by open circles

Figure 10: Shear stress distribution along the lateral centre-line at 6000 N vertical load and $\mu = 0.25$. Positive and negative stress limits are shown, marked by open plot symbols
Figure 11: Shear stress distribution along the lateral centre-line at 6000 N vertical load and $\mu = 1.0$. Positive and negative stress limits are shown, marked by open plot symbols.

Figure 12: Lateral displacement of the left sidewall (on the side of the positive y-axis) along the cross-section relative to the inflated, unloaded condition. The results are presented for a vertical load of 6000 N and for different friction coefficients.

The friction dependent narrowing of the contact patch affects the sidewalls as well. As shown by Figure 12, an increase in friction reduces the lateral displacement and thus the curvature of the
sidewalls. Reducing the curvature, in turn, effectively stiffens the sidewalls [8] and consequently causes the overall vertical tyre stiffness to increase.

The frictional stiffening effect also influences the contact area because the flattening of the tyre in the contact region is dependent on the structural stiffness of the tyre. The contact area increases as the structure becomes softer [9] and thus, as shown by Figure 13, the size of the contact patch increases with lower friction levels. Consequently, a tyre on a frictionless ground is effectively more compliant than the same one on a high friction surface.

Furthermore, the growth in contact area with reducing friction levels is accompanied by increasing lateral shrinkage of the contact patch (due to the relaxation of the cord tension) as described above. As a result of these two opposing effects, the frictionless simulation exhibits the greatest contact length of all investigated configurations as indicated by Figure 14. This behaviour is also reported in the literature [5].

Figure 13: Contact area of the upright tyre against vertical load as a function of friction level
Figure 14 also shows that the contact pressure along the longitudinal centre-line reduces as the coefficient of friction increases. In particular, results show that the normal pressure around the centre of the contact patch decreases noticeably. As explained above, the sidewalls become effectively stiffer with increasing friction coefficient. Then, as stiffer sidewalls raise the normal load near the shoulder region, the contact pressure in the interior of the contact patch is reduced. This effect can be seen in Figure 15 and Figure 16. Similar experimental observations are reported by Pottinger [5].

All the results discussed above show that the tyre behaviour changes only marginally once the coefficient of friction exceeds 0.5. Hence, a contact condition exists at which the available friction forces at the tyre road interface exceed the shear forces so that the entire tread surface will stick to the ground (i.e. no sliding will occur on increasing the loading up to a reasonable maximum). Also, this observation implies that the stiffening effect of the tyre due to friction level is limited.
Figure 15: Normalised contact pressure distribution of the upright tyre model at 4500 N vertical load and $\mu = 0$

Figure 16: Normalised contact pressure distribution of the upright tyre model at 4500 N vertical load and $\mu = 1.0$
A similar ‘friction level related behaviour’ can be studied for the cambered tyre, i.e. the load-displacement curves at $3^\circ$ camber (see Figure 8) become steeper with rising coefficient of friction. As with the upright tyre, this behaviour results from narrowing of the contact patch of the cambered tyre, see Figure 17, due to the relaxation of the cord tension and, as a result, the vertical tyre stiffness is dependent on the prevailing contact conditions. The introduction of the camber angle slightly lowers the overall tyre stiffness. These observations agree with the studies by Kim, Kondo and Akasaka [8].

![Figure 17: Cross-section views of the inflated, unloaded tyre and the cambered tyre at 6000 N vertical load on a frictionless surface and on a friction surface with $\mu = 1.0$. Wheel is shown cambered and the road surface level.](image)

The vertical stiffness of a cambered tyre is affected in many ways, with two particular mechanisms occurring. Firstly, by tilting the tyre with respect to the road, the direction of loading of the tyre and thus of the sidewalls is inclined. Also, the vertical load is unevenly distributed across the width of the tyre and hence on the sidewalls. Therefore, the deflection on the laden side increases compared to the zero camber configuration, see Figure 18, and, as a result, the curvature of the sidewall rises. As already explained above, the stiffness of the sidewalls reduces with increasing curvature [8] and consequently the vertical stiffness of the tyre is changed. Secondly, as the tyre is pressed against the ground at an angle, the belt layers are twisted. This twisting provides a resistance to the deflection and thus affects the global tyre stiffness. These two mechanisms may oppose each other so that the resulting tyre stiffness is only affected slightly.
6. **Conclusions**

The construction of a virtual tyre model to simulate the real static tyre behaviour in great detail was reported. Also, experimental investigations conducted with a load-deflection machine and with the XSENSOR pressure sensing system were presented and were shown to work well for the validation of the tyre model.

The results of the simulations and the measurements show substantial agreement in respect of (1) static vertical load-deflection properties, (2) pressure distributions over the contact region and (3) the influence of a 3° camber angle.

Over the load range studied, a friction coefficient of 0.5 is sufficient to prevent sliding in static loading tests for this particular tyre. Lower levels of friction lead to sliding and lower vertical tyre stiffnesses. Sliding is primarily lateral, leading to “waisting” of the tyre in the contact region. Waisting increases the local sidewall curvature which is part of the softening mechanism for the cambered and upright tyre.

Current work is concerned with simulating the rolling tyre with the same basic model and new solution methods.
References

1 Abaqus 6.7 HTML Documentation, ABAQUS, Inc. 2007


8 Kim, S., Kondo, K., Akasaka, T., “Contact Pressure Distribution of Radial Tire in Motion With Camber Angle”, Tire Science and Technology, TSTCA, Vol. 28, No. 1, January – March 2000, pp. 2-32