Codebook Based Single-User MIMO System Design with Widely Linear Processing

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***Abstract*— This work addresses joint transceiver optimization for multiple-input, multiple-output (MIMO) systems. In practical systems the complete knowledge of channel state information (CSI) is hardly available at transmitter. To tackle this problem, we resort to the codebook approach to precoding design, where the receiver selects a precoding matrix from a finite set of predefined precoding matrices based on the instantaneous channel condition and delivers the index of the chosen precoding matrix to the transmitter via a bandwidth-constraint feedback channel. We show that, when the symbol constellation is improper, the joint codebook based precoding and equalization can be designed accordingly to achieve improved performance compared to the conventional system.**

I. Introduction

MIMO systems have attracted significant interest due to their ability to satisfy the increasing demand of high bit-rate services. In MIMO systems, performance improvements can be achieved by exploiting channel state information (CSI) at the transmitter. In this case, the quality of the communication link can be improved by jointly designing the precoder and equalizer. CSI can be estimated in the receiver in time division duplex (TDD) system and fedback to the transmitter. Joint design of precoding at the transmitter and equalization at the receiver for multicarrier MIMO channels under a variety of design criteria was addressed in [1], where the authors formulated the design problem within the framework of convex optimization theory, in which a number of design criteria can be easily accommodated and efficiently solved. Joint design of optimum linear precoder and equalizer for a MIMO channel using a weighted minimum mean square error (MMSE) criterion subject to a transmit power constraint was treated in [2]. Closed form solutions are derived for the optimum precoder and equalizer as functions of error weights, transmit power, receiver noise variance, and eigenvalues of the MIMO channel. A unified framework for joint optimization of nonlinear Tomlinson-Harshima precoding at the transmitter and linear equalization at the receiver was proposed in [3]. It was shown that nonlinear precoding provides better performance than linear precoding when the cost function is multiplicatively Schur-convex. In contrast, when the cost function is multiplicatively Schur-concave the nonlinear scheme converges to the linear scheme.

The joint transceiver design mentioned above assumes perfect CSI at transmitter, which is not a realistic assumption. CSI is usually imperfect due to channel estimation errors, time-variation of channel gains, bandwidth constraint of the feedback channel, etc. To tackle this problem, the codebook approach was introduced, e.g., in [4], to design precoders with limited channel feedback. In such a quantized precoding system, the optimal precoder is chosen from a finite codebook known to both receiver and transmitter. The index of the optimal precoder, which is chosen according to the instantaneous channel information, is conveyed from the receiver to the transmitter using a limited number of bits over a low-delay feedback link.

In radio communications contexts, there has been an increasing interest in optimal widely linear (WL) filters [5] for improper signal modulations such as binary phase shift keying (BPSK), amplitude shift keying (ASK) modulations, or those corresponding to a complex filtering of real-valued modulations (after a derotation operation), such as minium shift keying (MSK), Gaussian MSK (GMSK) or offset quadrature amplitude modulations (OQAM) [6]. In principle, real-valued ASK modulation is less power efficient than the corresponding complex QAM modulation as the constellation points cannot be packed as densely in the complex plane. However, since only one signal dimension is used for data transmission, additional degrees of freedom are available and can be exploited. It was demonstrated in [7] that transmission with real-valued data symbols can lead to a higher spectral efficiency for DS-CDMA systems than using a complex symbol alphabet. In [8], a WL strategy was proposed for a coded downlink OFDM system which combines real-valued ASK modulation and single antenna interference cancellation. The proposed scheme has been shown to yield superior performance compared to the QAM system with the same spectral efficiency. It was also shown in [9] that WL receivers achieve better error performance, lower sensitivity to channel estimation errors and are more robust to the fading unbalance problem than conventional receivers. Moreover, they enable MIMO systems to operate with a number of transmit antennas larger (up to a factor of two) than the number of receive antennas. Consequently, the use of an M-ary ASK constellation coupled with WL receivers is not a limitation and may even bring advantages in terms of power and spectral efficiencies compared to an M-ary QAM constellation with linear receivers [9].

Here we study a closed-loop single-user MIMO system with improper modulations, such as 4ASK. By utilizing the improperness property of the transmitted signal, we propose a new joint precoder and equalizer design which is shown to outperform the corresponding QPSK system with the same spectral efficiency.

The remainder of the paper is organized as follows. In Section II, we present the system model and briefly describe the conventional approach to transceiver design. Two novel joint precoder-equalizer algorithms are proposed in Section III. The proposed schemes are evaluated and compared to the conventional scheme in Section IV. Finally, in Section V, conclusions are drawn based on the simulation results.

Notations: $(\cdot)^{\mathcal{T}}$ denotes matrix transpose, $(\cdot)^{\mathcal{H}}$ matrix conjugate transpose, $(\cdot)^*$ matrix conjugate, $\mathrm{E}[\cdot]$ expectation, $\|\cdot\|$ Euclidian norm, $\mathrm{trace}(\cdot)$ trace operation, and $\mathbf{I}_N$ an $N \times N$ identity matrix.

II. Conventional Approach to Joint Precoder-Equalizer Design

A generic MIMO communication system model is shown in Fig. 1. Input symbol streams are passed through a linear precoder optimized for a known channel. The precoder is a matrix with complex elements and can add redundancy to the input symbol streams to improve system performance. The precoder output is transmitted over the MIMO channel through N_t transmit antennas. The signal is received by N_r receive antennas and processed by a linear equalizer, which is optimized for the fixed and known channel. The linear equalizer also operates in the complex field and removes any redundancy that has been introduced by the precoder. The received signal can be expressed as

$$\tilde{\mathbf{s}} = \mathbf{GHFs} + \mathbf{Gn}, \tag{1}$$

where $\mathbf{H} \in \mathbb{C}^{N_r \times N_t}$ is the channel matrix, $\tilde{\mathbf{s}} \in \mathbb{C}^{M_t \times 1}$ is the received signal vector, $\mathbf{s} \in \mathbb{C}^{M_t \times 1}$ is the transmitted symbol vector, $\mathbf{n} \in \mathbb{C}^{N_r \times 1}$ is the noise vector, each element of which has zero mean and variance σ_n^2, $\mathbf{F} \in \mathbb{C}^{N_t \times M_t}$ is the precoding matrix and $\mathbf{G} \in \mathbb{C}^{M_t \times N_r}$ is the equalization matrix. The precoder adds a redundancy of $N_t - M_t$ across space since it has M_t input symbols and N_t precoded symbols that are transmitted simultaneously through N_t transmit antennas. The estimate of the transmitted symbol vector $\hat{\mathbf{s}}$ is obtained by making hard decision on the received signal vector $\tilde{\mathbf{s}}$.

In this section, we present the conventional joint precoder-equalizer solution. The proposed design utilizing the improperness property of the signal constellation will be introduced in the following section. In both cases, the precoders are designed using codebook and limited channel feedback.

With the conventional approach, the precoding and equalization matrices $\mathbf{F}, \mathbf{G}$ are optimized according to the MMSE criterion, i.e.,

$$\begin{aligned}\mathbf{G}, \mathbf{F} &= \arg\min_{\mathbf{G},\mathbf{F}} \mathrm{E}[\|\tilde{\mathbf{s}} - \mathbf{s}\|^2] = \arg\min_{\mathbf{G},\mathbf{F}} \mathrm{E}[\|\mathbf{Gr} - \mathbf{s}\|^2] \\ &= \arg\min_{\mathbf{G},\mathbf{F}} \mathrm{E}[\|\mathbf{G}(\mathbf{HFs} + \mathbf{n}) - \mathbf{s}\|^2],\end{aligned} \tag{2}$$

where

$$\mathbf{r} = \mathbf{HFs} + \mathbf{n}. \tag{3}$$

The optimal precoding matrix can be derived by minimizing the mean square error (MSE) [4]

$$MSE(\mathbf{F}) = \mathrm{trace}\{(\mathbf{F}^{\mathcal{H}}\mathbf{H}^{\mathcal{H}}\mathbf{HF}/\sigma_n^2 + \mathbf{I})^{-1}\}. \tag{4}$$

Given the knowledge of the precoding matrix $\mathbf{F}$, the equalization matrix $\mathbf{G}$ can be obtained by

$$\mathbf{G} = (\mathbf{F}^{\mathcal{H}}\mathbf{H}^{\mathcal{H}}\mathbf{HF} + \sigma_n^2\mathbf{I})^{-1}\mathbf{F}^{\mathcal{H}}\mathbf{H}^{\mathcal{H}}. \tag{5}$$

As indicated by Eq. (4), the dependency of $\mathbf{F}$ on $\mathbf{G}$ has been removed. Now the question is how to derive the optimal precoder $\mathbf{F}^{\mathrm{opt}}$ based on Eq. (4). Apparently, the derivation of $\mathbf{F}^{\mathrm{opt}}$ requires the knowledge of the channel matrix $\mathbf{H}$ which can be fedback from receiver to transmitter. Much of prior work in this area was conducted based on the assumption of perfect knowledge of CSI at the transmitters. Due to the bandwidth constraint in practical wireless systems, the feedback channel is only able to communicate a finite number of bits per block. The receiver can either perform quantization on the channel matrix and feedback the quantized channel information to the transmitter; or predefine a finite set of precoding matrices called codebook and instruct the transmitter to select the best precoder from the codebook based on the channel condition. It was discovered in [10] that the latter approach is much preferred, directly quantizing the channel with 16 bits of feedback performs much worse than a 6-bit feedback codebook based precoder. It was also observed in [10] that the number of feedback bits q in practical systems needs not be large. Assuming perfect CSI (which is equivalent to $q = \infty$) does not lead to substantial gain compared to $q = 6$, the limited feedback precoder obtains performance close to that of the unquantized precoder.

For a q-bits feedback channel, the system needs to prepare a total of $L = 2^q$ precoding matrices, denoted as $\mathbf{F}_1, \mathbf{F}_2, \ldots, \mathbf{F}_L$ and collected into a codebook $\mathcal{F}$ as $\mathcal{F} := \{\mathbf{F}_1, \mathbf{F}_2, \ldots, \mathbf{F}_L\}$. Based on the current channel realization, the receiver will decide which codeword (precoder) from the codebook $\mathcal{F}$ is the most favorable and inform the transmitter to switch to that precoder by feeding back its q-bit codeword index. Based on the block fading channel model, channel feedback and transmitter adaptation are done on a per block basis. The codebook $\mathcal{F}$ which consists of a finite number of matrices represent a set of subspaces in the Grassmann manifold. Designing sets of L matrices that maximize the minimum subspace distance (where distance can be chosen in a number of different ways, such as the chordal distance, the projection two-norm distance and the Fubini-Study distance between two subspaces [4]) is known as Grassmannian subspace packing [11]. It was observed in [10] that the performance of different codebooks are not clearly distinguishable. Hence, sticking to the codebook with any distance optimized will be comparably good. One simple method for designing good packings with arbitrary distance functions is to use the non-coherent constellation designs demonstrated in [12], which has been shown to yield codebooks with large minimum distances and can be easily modified to work with any distance function on the Grassmann manifold. This codebook design also requires the least amount of storage at both transmitter and receiver. For those reasons, we will use the structured block-circulant codebook proposed in [12] in this work. In this case, a codebook is fully specified once the first codeword $\mathbf{F}_1$ and a diagonal rotation matrix $\mathbf{Q}$ are

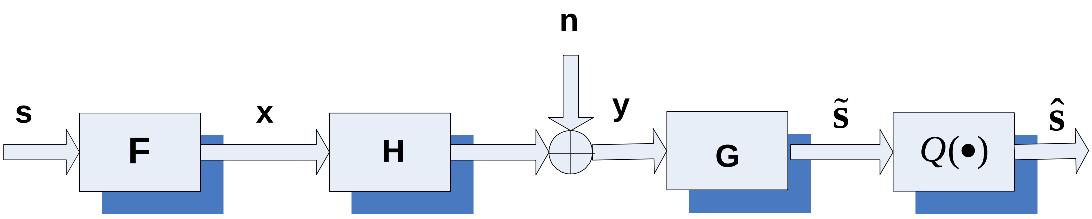

Fig. 1. Joint transmitter and receiver design.

provided. The other codewords in the codebook are given by

$$\mathbf{F}_l = \mathbf{Q}^l \mathbf{F}_1, \quad (6)$$

for $l = 2, \ldots, L$. The matrix $\mathbf{Q}$ is a diagonal matrix fully parameterized by an integer vector $\mathbf{u} = \begin{bmatrix} u_1 & \ldots & u_{N_t} \end{bmatrix}$, i.e.,

$$\mathbf{Q} = \begin{bmatrix} \exp\left(j\frac{2\pi}{L}u_1\right) & & 0 \\ & \ldots & \\ 0 & & \exp\left(j\frac{2\pi}{L}u_{N_t}\right) \end{bmatrix}. \quad (7)$$

The first codeword $\mathbf{F}_1$ is chosen to be a $N_t \times M_t$ submatrix of the $N_t \times N_t$ DFT matrix $\mathbf{D}_{N_t}$ whose (m,n)th element is $(\mathbf{D}_{N_t})_{m,n} = \exp\left(j\frac{2\pi}{N_t}(m-1)(n-1)\right)$, where $1 \leq m, n \leq N_t$. Denoting $\mathbf{d}_c$ as the c-th column of the matrix $\mathbf{D}_{N_t}$, the first codeword is a collection of M_t columns parameterized by the set of columns indices $\mathbf{c} = \begin{bmatrix} c_1 & \ldots & c_{M_t} \end{bmatrix}$, i.e., $\mathbf{F}_1 = \begin{bmatrix} \mathbf{d}_{c_1} & \ldots & \mathbf{d}_{c_{M_t}} \end{bmatrix}$.

In Table I, we tabulate the choices of $\mathbf{u}$ and $\mathbf{c}$ for different transmit antenna numbers N_t and spatially multiplexed data stream numbers M_t. Note that the choice of $L = 2^q$ is a result of trading off performance with the number of feedback bits.

Once the codebook is specified, the receiver observes a channel realization $\mathbf{H}$, selects the best precoding matrix (codeword) to be used, and feeds back the index of the codeword to the transmitter. We know that the optimal precoder in a conventional system is chosen by minimizing MSE defined by (4). Substituting $\mathbf{F}$ with $\mathbf{F}_l$ in this equation (where $\mathbf{F}_l$ is the lth codeword in the codebook), the index of the precoding matrix to be conveyed from the receiver to the transmitter is selected as

$$\begin{aligned} l^{\mathrm{opt}} &= \arg \min_{l \in \{1,2,\ldots,L\}} MSE(\mathbf{F}_l) \\ &= \arg \min_{l \in \{1,2,\ldots,L\}} \mathrm{trace}\{(\mathbf{F}_l^{\mathcal{H}}\mathbf{H}^{\mathcal{H}}\mathbf{H}\mathbf{F}_l/\sigma_n^2 + \mathbf{I})^{-1}\}. \end{aligned} \quad (8)$$

Once the precoding matrix $\mathbf{F}$ is determined, the receiver uses Eq. (5) to calculate the equalization matrix $\mathbf{G}$.

III. Proposed Joint Precoder-Equalizer Design

The conventional approach introduced in Section II is optimum for MIMO systems with proper signal modualtion, such as M-QAM an M-PSK. However, for improper modulations, such as M-ary ASK, the conventional joint precoder-equalizer design expressed by Eqs. (5) and (8) is no longer optimum. In what follows, we show how the joint design can be improved by utilizing the improperness property. Denote $\mathrm{Re}(\mathbf{x})$, $\mathrm{Im}(\mathbf{x})$ as the real and imaginary part of $\mathbf{x}$, respectively. The improved receive filter $\mathbf{G}$ and the precoder $\mathbf{F}$ are derived as

$$\begin{aligned} \mathbf{G}, \mathbf{F} &= \arg\min_{\mathbf{G},\mathbf{F}} \mathrm{E}[\|\mathbf{e}\|^2] = \arg\min_{\mathbf{G},\mathbf{F}} \mathrm{E}[\|\,\mathrm{Re}\{\tilde{\mathbf{s}}\} - \mathbf{s}\|^2] \\ &= \arg\min_{\mathbf{G},\mathbf{F}} \mathrm{E}[\|\,\mathrm{Re}\{\mathbf{G}\mathbf{r}\} - \mathbf{s}\|^2]. \end{aligned} \quad (9)$$

It was shown in [13], [14] that for real-valued $\mathbf{s}$, minimization of (9) will result in a better estimator than minimization of (2). Next we derive two transceiver algorithms based on the modified cost function (9).

A. Algorithm I

$\mathrm{Re}\{\tilde{\mathbf{s}}\}$ in Eq. (9) can be reformed as

$$\mathrm{Re}\{\tilde{\mathbf{s}}\} = \mathrm{Re}\{\mathbf{G}\mathbf{r}\} = \underbrace{\begin{bmatrix} \mathrm{Re}(\mathbf{G}) & -\mathrm{Im}(\mathbf{G}) \end{bmatrix}}_{\mathbf{G}_a} \underbrace{\begin{bmatrix} \mathrm{Re}(\mathbf{r}) \\ \mathrm{Im}(\mathbf{r}) \end{bmatrix}}_{\mathbf{r}_a}. \quad (10)$$

For real-valued $\mathbf{s}$, $\mathbf{r}_a$ can be expressed according to (3) as

$$\underbrace{\begin{bmatrix} \mathrm{Re}(\mathbf{r}) \\ \mathrm{Im}(\mathbf{r}) \end{bmatrix}}_{\mathbf{r}_a} = \underbrace{\begin{bmatrix} \mathrm{Re}(\mathbf{H}) & -\mathrm{Im}(\mathbf{H}) \\ \mathrm{Im}(\mathbf{H}) & \mathrm{Re}(\mathbf{H}) \end{bmatrix}}_{\mathbf{H}_a} \underbrace{\begin{bmatrix} \mathrm{Re}(\mathbf{F}) \\ \mathrm{Im}(\mathbf{F}) \end{bmatrix}}_{\mathbf{F}_a} \mathbf{s} + \underbrace{\begin{bmatrix} \mathrm{Re}(\mathbf{n}) \\ \mathrm{Im}(\mathbf{n}) \end{bmatrix}}_{\mathbf{n}_a}. \quad (11)$$

Substituting (10) and (11) into (9), yields

$$\begin{aligned} \mathbf{G}_a, \mathbf{F}_a &= \arg\min_{\mathbf{G}_a,\mathbf{F}_a} \mathrm{E}[\|\mathbf{G}_a\mathbf{r}_a - \mathbf{s}\|^2] \\ &= \arg\min_{\mathbf{G}_a,\mathbf{F}_a} \mathrm{E}[\|\mathbf{G}_a(\mathbf{H}_a\mathbf{F}_a + \mathbf{n}_a) - \mathbf{s}\|^2] \end{aligned} \quad (12)$$

We have now converted (9) into the standard optimization problem shown in (2). Therefore, the solutions expressed by (5) and (8) can be applied in a straightforward manner, but with $\mathbf{H}$ and $\mathbf{F}_l$ replaced by their augmented version, $\mathbf{H}_a$ and $\mathbf{F}_{la}$, respectively, i.e.,

$$l_a^{\mathrm{opt}} = \arg \min_{l \in \{1,2,\ldots,L\}} \mathrm{trace}\{(\mathbf{F}_{la}^{\mathcal{H}}\mathbf{H}_a^{\mathcal{H}}\mathbf{H}_a\mathbf{F}_{la}/\sigma_n^2 + \mathbf{I})^{-1}\}. \quad (13)$$

Given the knowledge of the precoding matrix $\mathbf{F}_a$, the equalization matrix $\mathbf{G}_a$ can be obtained by

$$\mathbf{G}_a = (\mathbf{F}_a^{\mathcal{H}}\mathbf{H}_a^{\mathcal{H}}\mathbf{H}_a\mathbf{F}_a + \sigma_n^2\mathbf{I})^{-1}\mathbf{F}_a^{\mathcal{H}}\mathbf{H}_a^{\mathcal{H}}. \quad (14)$$

TABLE I

PARAMETERS FOR $\mathbf{F}_1$ AND $\mathbf{Q}$. N_t IS THE NUMBER OF TRANSMIT ANTENNA, M_t IS THE NUMBER OF SPATIALLY MULTIPLEXED DATA STREAMS, L IS THE CODEBOOK SIZE, AND $q = \log_2 L$ IS THE NUMBER OF FEEDBACK BITS.

N_t	M_t	$q = \log_2 L$	$\mathbf{c} = [c_1 \quad \ldots \quad c_{M_t}]$	$\mathbf{u} = [u_1 \quad \ldots \quad u_{N_t}]$
2	1	3	$[1]$	$[1 \quad 0]$
4	2	3	$[0 \quad 1]$	$[1 \quad 7 \quad 52 \quad 56]$
3	1	5	$[1]$	$[1 \quad 26 \quad 28]$
3	2	5	$[1 \quad 2]$	$[1 \quad 26 \quad 28]$
4	1	6	$[1]$	$[1 \quad 8 \quad 61 \quad 45]$
4	2	6	$[0 \quad 1]$	$[1 \quad 7 \quad 52 \quad 56]$
4	3	6	$[0 \quad 2 \quad 3]$	$[1 \quad 8 \quad 61 \quad 45]$

B. Algorithm II

$\mathrm{Re}\{\tilde{\mathbf{s}}\}$ in Eq. (9) can be expanded as

$$\begin{aligned}\mathrm{Re}\{\tilde{\mathbf{s}}\} &= \mathrm{Re}\{\mathbf{G}(\mathbf{HFs}+\mathbf{n})\} \\ &= 0.5\mathbf{G}(\mathbf{HFs}+\mathbf{n}) + 0.5\mathbf{G}^*(\mathbf{HFs}+\mathbf{n})^* \\ &= 0.5\mathbf{GHFs} + 0.5(\mathbf{GHF})^*\mathbf{s} + 0.5[\mathbf{Gn} + (\mathbf{Gn})^*].\end{aligned} \tag{15}$$

The MSE function can be reformed as

$$\begin{aligned}\mathrm{E}[\|\mathbf{e}\|^2] &= \mathrm{E}[\|\,\mathrm{Re}\{\tilde{\mathbf{s}}\} - \mathbf{s}\|^2] \\ &= \mathrm{trace}\{\mathrm{E}([0.5\mathbf{GHFs} + 0.5(\mathbf{GHF})^*\mathbf{s} + 0.5(\mathbf{Gn} \\ &\quad + \mathbf{G}^*\mathbf{n}^*) - \mathbf{s}][0.5\mathbf{s}^{\mathcal{H}}(\mathbf{GHF})^{\mathcal{H}} + 0.5\mathbf{s}(\mathbf{GHF})^{\mathcal{T}} \\ &\quad + 0.5(\mathbf{Gn})^{\mathcal{H}} + 0.5(\mathbf{Gn})^{\mathcal{T}} - \mathbf{s}])\} \\ &= \mathrm{trace}(0.25\mathbf{J}),\end{aligned} \tag{16}$$

where

$$\begin{aligned}\mathbf{J} &= \mathbf{GHFF}^{\mathcal{H}}\mathbf{H}^{\mathcal{H}}\mathbf{G}^{\mathcal{H}} + \mathbf{GHFF}^{\mathcal{T}}\mathbf{H}^{\mathcal{T}}\mathbf{G}^{\mathcal{T}} \\ &\quad + \mathbf{G}^*\mathbf{H}^*\mathbf{F}^*\mathbf{F}^{\mathcal{H}}\mathbf{H}^{\mathcal{H}}\mathbf{G}^{\mathcal{H}} + \mathbf{G}^*\mathbf{H}^*\mathbf{F}^*\mathbf{F}^{\mathcal{T}}\mathbf{H}^{\mathcal{T}}\mathbf{G}^{\mathcal{T}} \\ &\quad - 2[\mathbf{GHF} + \mathbf{G}^*\mathbf{H}^*\mathbf{F}^* + \mathbf{F}^{\mathcal{H}}\mathbf{H}^{\mathcal{H}}\mathbf{G}^{\mathcal{H}} + \mathbf{F}^{\mathcal{T}}\mathbf{H}^{\mathcal{T}}\mathbf{G}^{\mathcal{T}}] \\ &\quad + \sigma_n^2(\mathbf{GG}^{\mathcal{H}} + \mathbf{G}^*\mathbf{G}^{\mathcal{T}}) + 4\mathbf{I} \\ &= 2\sigma_n^2(\mathbf{HF})^{-1}\mathbf{G}^{\mathcal{H}}.\end{aligned} \tag{17}$$

The proof for Eq. (17) is given in Appendix. To design the system, we first derive the optimum equalization matrix $\mathbf{G}^{\mathrm{opt}}$ assuming the precoding matrix is fixed. Subsequently, the optimal precoding matrix $\mathbf{F}^{\mathrm{opt}}$ is determined based on $\mathbf{G}^{\mathrm{opt}}$. Refer to [1], [15] for detailed descriptions of this two-step optimization approach. Note that there are some other alternatives to solve the transceiver optimization problem under question, e.g., an iterative semidefinite programming (SDP) based framework was proposed in [16]. The reader is referred to [17] for a full account of the bi-convex optimization.

Next we show how $\mathbf{G}^{\mathcal{H}}$ can be derived from Eq. (32) in Appendix. We substitute $\mathbf{F}$ with $\mathbf{F}_l$ in this equation, where $\mathbf{F}_l$ is the lth codeword in the codebook, and denote

$$\begin{aligned}\mathbf{G}^{\mathcal{H}} &= \mathrm{Re}(\mathbb{G}) + j\,\mathrm{Im}(\mathbb{G}) \\ \mathbf{HF}_l\mathbf{F}_l^{\mathcal{H}}\mathbf{H}^{\mathcal{H}} &= \mathbf{X} = \mathrm{Re}(\mathbf{X}) + j\,\mathrm{Im}(\mathbf{X}) \\ \mathbf{HF}_l\mathbf{F}_l^{\mathcal{T}}\mathbf{H}^{\mathcal{T}} &= \mathbf{Y} = \mathrm{Re}(\mathbf{Y}) + j\,\mathrm{Im}(\mathbf{Y}) \\ \mathbf{HF}_l &= \mathbb{H} = \mathrm{Re}(\mathbb{H}) + j\,\mathrm{Im}(\mathbb{H}).\end{aligned} \tag{18}$$

Eq. (32) can be reformed as

$$\begin{aligned}2\,\mathrm{Re}(\mathbb{H}) + 2j\,\mathrm{Im}(\mathbb{H}) &= [\mathrm{Re}(\mathbf{X})\,\mathrm{Re}(\mathbb{G}) - \mathrm{Im}(\mathbf{X})\,\mathrm{Im}(\mathbb{G}) \\ &\quad + \mathrm{Re}(\mathbf{Y})\,\mathrm{Re}(\mathbb{G}) + \mathrm{Im}(\mathbf{Y})\,\mathrm{Im}(\mathbb{G}) + \sigma_n^2\,\mathrm{Re}(\mathbb{G})] \\ &\quad + j[\mathrm{Re}(\mathbf{X})\,\mathrm{Im}(\mathbb{G}) + \mathrm{Im}(\mathbf{X})\,\mathrm{Re}(\mathbb{G}) + \mathrm{Im}(\mathbf{Y})\,\mathrm{Re}(\mathbb{G}) \\ &\quad - \mathrm{Re}(\mathbf{Y})\,\mathrm{Im}(\mathbb{G}) + \sigma_n^2\,\mathrm{Im}(\mathbb{G})].\end{aligned} \tag{19}$$

Splitting the real and imaginary parts yields

$$\begin{aligned}2\,\mathrm{Re}(\mathbb{H}) &= \mathrm{Re}(\mathbf{X})\,\mathrm{Re}(\mathbb{G}) - \mathrm{Im}(\mathbf{X})\,\mathrm{Im}(\mathbb{G}) + \mathrm{Re}(\mathbf{Y})\,\mathrm{Re}(\mathbb{G}) \\ &\quad + \mathrm{Im}(\mathbf{Y})\,\mathrm{Im}(\mathbb{G}) + \sigma_n^2\,\mathrm{Re}(\mathbb{G}) \\ &= [\mathrm{Re}(\mathbf{X}) + \mathrm{Re}(\mathbf{Y}) + \sigma_n^2]\,\mathrm{Re}(\mathbb{G}) + [\mathrm{Im}(\mathbf{Y}) - \mathrm{Im}(\mathbf{X})]\,\mathrm{Im}(\mathbb{G}) \\ 2\,\mathrm{Im}(\mathbb{H}) &= \mathrm{Re}(\mathbf{X})\,\mathrm{Im}(\mathbb{G}) + \mathrm{Im}(\mathbf{X})\,\mathrm{Re}(\mathbb{G}) + \mathrm{Im}(\mathbf{Y})\,\mathrm{Re}(\mathbb{G}) \\ &\quad - \mathrm{Re}(\mathbf{Y})\,\mathrm{Im}(\mathbb{G}) + \sigma_n^2\,\mathrm{Im}(\mathbb{G}) \\ &= [\mathrm{Im}(\mathbf{X}) + \mathrm{Im}(\mathbf{Y})]\,\mathrm{Re}(\mathbb{G}) + [\mathrm{Re}(\mathbf{X}) - \mathrm{Re}(\mathbf{Y}) + \sigma_n^2]\,\mathrm{Im}(\mathbb{G}),\end{aligned} \tag{20}$$

or in matrix form

$$\begin{bmatrix}2\,\mathrm{Re}(\mathbb{H}) \\ 2\,\mathrm{Im}(\mathbb{H})\end{bmatrix} = \begin{bmatrix}\mathrm{Re}(\mathbf{X}) + \mathrm{Re}(\mathbf{Y}) + \sigma_n^2\mathbf{I} & \mathrm{Im}(\mathbf{Y}) - \mathrm{Im}(\mathbf{X}) \\ \mathrm{Im}(\mathbf{X}) + \mathrm{Im}(\mathbf{Y}) & \mathrm{Re}(\mathbf{X}) - \mathrm{Re}(\mathbf{Y}) + \sigma_n^2\mathbf{I}\end{bmatrix} \cdot \begin{bmatrix}\mathrm{Re}(\mathbb{G}) \\ \mathrm{Im}(\mathbb{G})\end{bmatrix}. \tag{21}$$

Therefore

$$\begin{bmatrix}\mathrm{Re}(\mathbb{G}) \\ \mathrm{Im}(\mathbb{G})\end{bmatrix} = \begin{bmatrix}\mathrm{Re}(\mathbf{X}) + \mathrm{Re}(\mathbf{Y}) + \sigma_n^2\mathbf{I} & \mathrm{Im}(\mathbf{Y}) - \mathrm{Im}(\mathbf{X}) \\ \mathrm{Im}(\mathbf{X}) + \mathrm{Im}(\mathbf{Y}) & \mathrm{Re}(\mathbf{X}) - \mathrm{Re}(\mathbf{Y}) + \sigma_n^2\mathbf{I}\end{bmatrix}^{-1} \cdot \begin{bmatrix}2\,\mathrm{Re}(\mathbb{H}) \\ 2\,\mathrm{Im}(\mathbb{H})\end{bmatrix}. \tag{22}$$

Denoting $\mathbf{A} = \mathrm{Re}(\mathbf{X}) + \mathrm{Re}(\mathbf{Y}) + \sigma_n^2\mathbf{I}$, $\mathbf{B} = \mathrm{Im}(\mathbf{Y}) - \mathrm{Im}(\mathbf{X})$, $\mathbf{C} = \mathrm{Im}(\mathbf{X}) + \mathrm{Im}(\mathbf{Y})$, $\mathbf{D} = \mathrm{Re}(\mathbf{X}) - \mathrm{Re}(\mathbf{Y}) + \sigma_n^2\mathbf{I}$, Eq. (22) can be reformed as

$$\begin{aligned}\begin{bmatrix}\mathrm{Re}(\mathbb{G}) \\ \mathrm{Im}(\mathbb{G})\end{bmatrix} &= \begin{bmatrix}\mathbf{C}_{00} & \mathbf{C}_{01} \\ \mathbf{C}_{10} & \mathbf{C}_{11}\end{bmatrix}\begin{bmatrix}2\,\mathrm{Re}(\mathbb{H}) \\ 2\,\mathrm{Im}(\mathbb{H})\end{bmatrix} \\ &= \begin{bmatrix}2(\mathbf{C}_{00}\,\mathrm{Re}(\mathbb{H}) + \mathbf{C}_{01}\,\mathrm{Im}(\mathbb{H})) \\ 2(\mathbf{C}_{10}\,\mathrm{Re}(\mathbb{H}) + \mathbf{C}_{11}\,\mathrm{Im}(\mathbb{H}))\end{bmatrix},\end{aligned} \tag{23}$$

where $\mathbf{C}_{00} = (\mathbf{A} - \mathbf{BD}^{-1}\mathbf{C})^{-1}$, $\mathbf{C}_{11} = (\mathbf{D} - \mathbf{CA}^{-1}\mathbf{B})^{-1}$, $\mathbf{C}_{01} = -\mathbf{C}_{00}\mathbf{BD}^{-1}$, and $\mathbf{C}_{10} = -\mathbf{C}_{11}\mathbf{CA}^{-1}$. Since $\mathbf{G}^{\mathcal{H}} =$

$\text{Re}(\mathbb{G}) + j\,\text{Im}(\mathbb{G})$, according to (17), we have

$$\begin{aligned}\mathbf{J}_l &= 2\sigma_n^2(\mathbf{H}\mathbf{F}_l)^{-1}\mathbf{G}^{\mathcal{H}} \\ &= 4\sigma_n^2(\mathbf{H}\mathbf{F}_l)^{-1}[(\mathbf{C}_{00}\,\text{Re}(\mathbb{H}) + \mathbf{C}_{01}\,\text{Im}(\mathbb{H})) \\ &\quad + j(\mathbf{C}_{10}\,\text{Re}(\mathbb{H}) + \mathbf{C}_{11}\,\text{Im}(\mathbb{H}))]. \end{aligned} \tag{24}$$

The optimal codeword index can now be chosen according to

$$\begin{aligned} l^{opt} &= \arg\min_{l\in\{1,2,\ldots,L\}} MSE(\mathbf{F}_l) = \arg\min_{l\in\{1,2,\ldots,L\}} \text{trace}(\mathbf{J}_l) \\ &= \arg\min_{l\in\{1,2,\ldots,L\}} \text{trace}\{(\mathbf{H}\mathbf{F}_l)^{-1}[(\mathbf{C}_{00}\,\text{Re}(\mathbb{H}) + \mathbf{C}_{01}\,\text{Im}(\mathbb{H})) \\ &\quad + j(\mathbf{C}_{10}\,\text{Re}(\mathbb{H}) + \mathbf{C}_{11}\,\text{Im}(\mathbb{H}))\}. \end{aligned} \tag{25}$$

After the codeword has been selected, the receiver filter $\mathbf{G}^{\mathcal{H}} = \text{Re}(\mathbb{G}) + j\,\text{Im}(\mathbb{G})$ can be determined according to Eq. (22).

IV. Simulation Results

We compare different closed-loop algorithms by applying them to a 4×2 MIMO system ($N_t = 4, N_r = 2$). The number of transmitted symbol streams is set to $M_t = 2$, and the transmit power is $M_t\sigma_s^2 = 2$, i.e., unit average transmit power. We assume uncorrelated Rayleigh fading channel and the channel coefficients are normalized such that the average channel gain for each transmitted symbol is equal to unity. The simulation results are averaged over at least 10,000 channel realizations.

We use the structured block-circulant codebook designed in [12]. According to Table I, for systems with $N_t = 4, M_t = 2$ configuration, the first codeword $\mathbf{F}_1$ is formed by the first two columns of the 4×4 DFT matrix, i.e.,

$$\mathbf{F}_1 = \frac{1}{\sqrt{4}}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & \omega & \omega^2 & \omega^3 \end{bmatrix}^{\mathcal{T}}, \tag{26}$$

where $\omega = \exp(-\pi i/2)$. The diagonal matrix $\mathbf{Q}$ is formed by

$$\mathbf{Q} = \begin{bmatrix} \exp\left(j\frac{2\pi\cdot 1}{L}\right) & 0 & 0 & 0 \\ 0 & \exp\left(j\frac{2\pi\cdot 7}{L}\right) & 0 & 0 \\ 0 & 0 & \exp\left(j\frac{2\pi\cdot 52}{L}\right) & 0 \\ 0 & 0 & 0 & \exp\left(j\frac{2\pi\cdot 56}{L}\right) \end{bmatrix}. \tag{27}$$

The remaining codewords ($\mathbf{F}_l \in \mathbb{C}^{4\times 2}$ for $l = 2, \ldots, L$) can be determined by $\mathbf{F}_l = \mathbf{Q}^l\mathbf{F}_1$.

Figs. $2 - 4$ show the BER and MSE performance comparison of different schemes with 3-bit and 6-bit feedback, respectively. The employed modulation schemes are 4ASK, QPSK, which are chosen such that both systems have the same data transmission rate or spectrum efficiency. The conventional system with 4ASK and QPSK modulations is implemented according to Eqs. (5) and (8). The proposed scheme I is implemented according to Eqs. (13) and (14); the proposed scheme II is implemented according to Eqs. (22) and (25). The 4ASK system with WL receiver is implemented with the conventional design shown in (8) at transmitter, but with widely linear processing (WLP) shown in (22) at receiver to utilize the signal's improperness property.

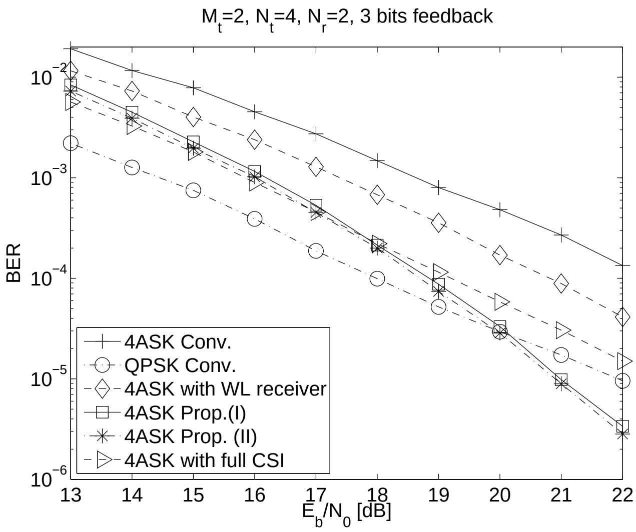

Fig. 2. BER Performance comparison of the closed-loop MIMO with 3-bit feedback.

The BER performance comparison of different schemes is presented in Figs. 2 and 3. As expected, the QPSK system performs much better than the conventional 4ASK system, however, it is outperformed by the improved 4ASK system with the proposed joint precoder-equalizer design when E_b/N_0 goes beyond 20 dB in the case of 3-bit feedback and 18 dB in the case of 6-bit feedback. The performance advantage of the proposed schemes becomes more obvious when SNR further increases. This is due to the fact that the system performance at low SNRs is dominated by the proper channel noise, whereas the performance gain by the proposed schemes become larger as SNR goes higher since it benefits more from exploiting the improper nature of the transmitted signal. Comparing Fig. 2 with Fig. 3, one can also see that the performance gain achieved by the proposed system in comparison with the conventional 4ASK and QPSK systems becomes more obvious when the number of feedback bits increases.

Among two proposed alogrithms for the 4ASK system, Algorithm II has a marginal performance advantage compared to Algorithm I. It is worth noting that the two step optimization procedure adopted in this work (and others, e.g, in [1], [15]) is a tractable approach to joint transceiver design, but not necessarily optimal. However, Algorithm II appears to be slightly closer to optimality than Algorithm I.

Another important observation from Figs. 2 and 3 is that joint WLP at both transmitter and receiver (as in proposed algorithms) achieve better performance than WLP only at receiver. For example, with 6 bits feedback and a target BER of 10^{-4}, the proposed schemes outperform their conventional counterpart by 4.2 dB, among which only 1.6 dB is achieved by WLP at receiver. This indicates that the maximum gain can only be obtained when the signal's improperness property is utilized both at transmitter and receiver.

The performance of the conventional 4ASK system with full CSI is also shown in the figures for comparison purposes. Comparing Fig. 2 to Fig. 3, it is evident that the

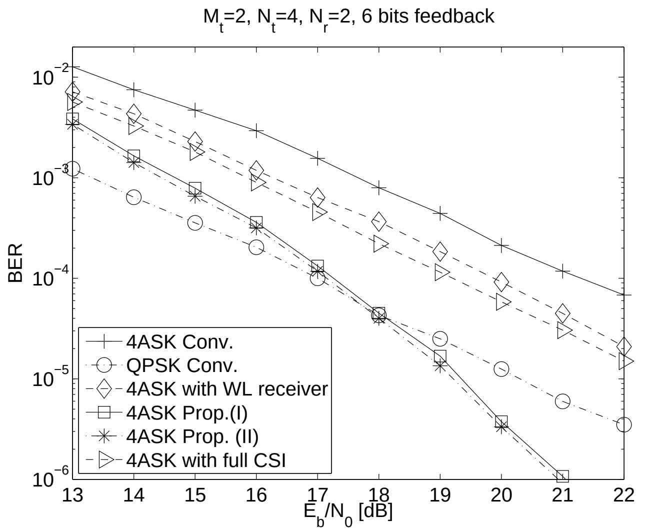

Fig. 3. BER Performance comparison of the closed-loop MIMO with 6-bit feedback.

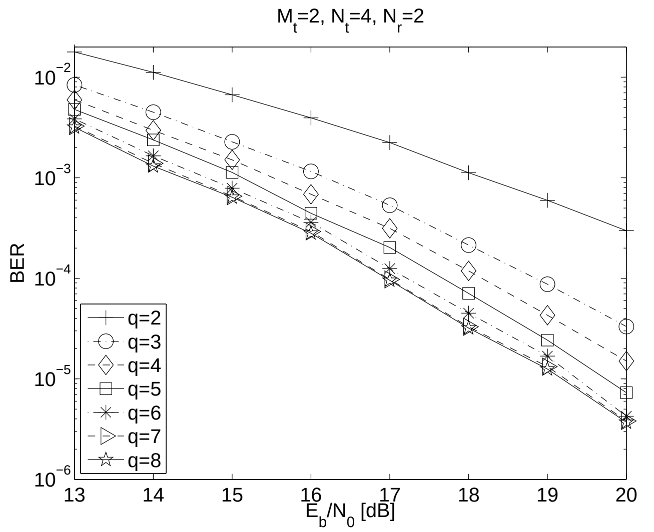

Fig. 5. Performance of the proposed scheme with different number of feedback bits q.

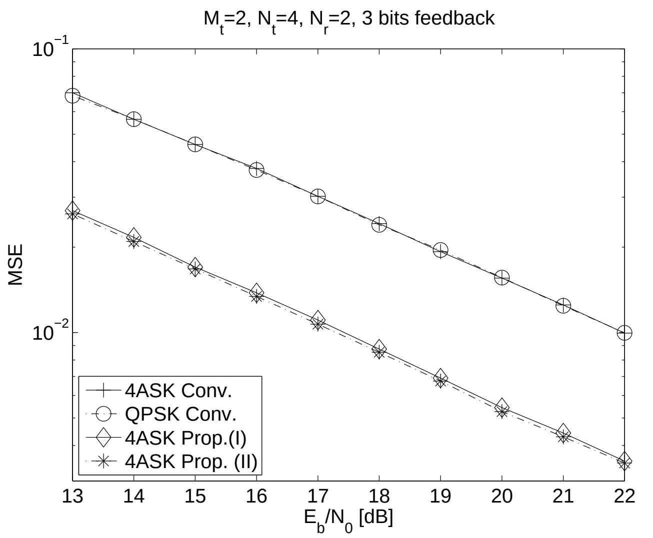

Fig. 4. MSE Performance comparison of the closed-loop MIMO with 3-bit feedback.

system performance improves as the number of feedback bits increases. For example, the conventional 4ASK system with 6-bit feedback has closer performance to the system with full CSI compared to that with 3-bit feedback, and the proposed 4ASK system with 6-bit feedback yields better performance than the proposed 4ASK system with 3-bit feedback.

Fig. 4 shows the MSE performance comparison of different systems. The conventional 4ASK system and QPSK system have the same MSE performance since they use the same optimization approach expressed by Eqs. (2) $-$ (5). The 4ASK system with the proposed schemes achieves a significantly lower MSE compared to the conventional one, leading to better BER performance at full range of SNR compared to the conventional system and in the high SNR region compared to the conventional QPSK system.

We study the performance of the 4ASK system with the proposed Algorithm I with different number of feedback bits q in Fig. 5. One can see that the system performance improves as the number of feedback bits increases. The most obvious gain is observed when the value of q increases from 2 to 3. However, the performance gain gradually diminishes as the number of feedback bits further increases. No noticeable gain can be observed when the value of q goes beyond 7, meaning that the system with full CSI ($q = \infty$) does not lead to substantial gain compared to $q = 7$.

V. Conclusions

We proposed a novel approach to joint precoder-equalizer design for closed-loop single-user MIMO systems where the precoder is designed using the codebook approach with limited channel feedback. The proposed schemes involve WLP and utilize the signal's improperness property. The simulation results show that the proposed MIMO transceivers outperform the conventional solution at high SNRs, which suggests that conventional systems can be used when the noise level is high, whereas the proposed system can be applied to achieve improved performance in less noisy channels. It has also been demonstrated that the best performance can only be achieved by applying WLP both at transmitter and receiver, and that the performance of full CSI can be approached when the number of feedback bits equal to 7.

Appendix

Proof of Equation (17)

Recall that

$$\begin{aligned}\mathbf{J} = &\mathbf{GHFF}^{\mathcal{H}}\mathbf{H}^{\mathcal{H}}\mathbf{G}^{\mathcal{H}} + \mathbf{GHFF}^{\mathcal{T}}\mathbf{H}^{\mathcal{T}}\mathbf{G}^{\mathcal{T}} \\ &+ \mathbf{G}^*\mathbf{H}^*\mathbf{F}^*\mathbf{F}^{\mathcal{H}}\mathbf{H}^{\mathcal{H}}\mathbf{G}^{\mathcal{H}} + \mathbf{G}^*\mathbf{H}^*\mathbf{F}^*\mathbf{F}^{\mathcal{T}}\mathbf{H}^{\mathcal{T}}\mathbf{G}^{\mathcal{T}} \\ &- 2[\mathbf{GHF} + \mathbf{G}^*\mathbf{H}^*\mathbf{F}^* + \mathbf{F}^{\mathcal{H}}\mathbf{H}^{\mathcal{H}}\mathbf{G}^{\mathcal{H}} \\ &+ \mathbf{F}^{\mathcal{T}}\mathbf{H}^{\mathcal{T}}\mathbf{G}^{\mathcal{T}}] + \sigma_n^2(\mathbf{GG}^{\mathcal{H}} + \mathbf{G}^*\mathbf{G}^{\mathcal{T}}) + 4\mathbf{I}. \qquad (28)\end{aligned}$$

It can be shown that [18], [19]

$$\frac{\partial\,\mathrm{trace}(\mathbf{GHFF}^{\mathcal{H}}\mathbf{H}^{\mathcal{H}}\mathbf{G}^{\mathcal{H}})}{\partial\mathbf{G}} = \mathbf{G}^*(\mathbf{HFF}^{\mathcal{H}}\mathbf{H}^{\mathcal{H}})^{\mathcal{T}};$$
$$\frac{\partial\,\mathrm{trace}(\mathbf{GHFF}^{\mathcal{T}}\mathbf{H}^{\mathcal{T}}\mathbf{G}^{\mathcal{T}})}{\partial\mathbf{G}} = 2\mathbf{GHFF}^{\mathcal{T}}\mathbf{H}^{\mathcal{T}};$$
$$\frac{\partial\,\mathrm{trace}(\mathbf{G}^*\mathbf{H}^*\mathbf{F}^*\mathbf{F}^{\mathcal{H}}\mathbf{H}^{\mathcal{H}}\mathbf{G}^{\mathcal{H}})}{\partial\mathbf{G}} = \mathbf{0};$$
$$\frac{\partial\,\mathrm{trace}(\mathbf{G}^*\mathbf{H}^*\mathbf{F}^*\mathbf{F}^{\mathcal{T}}\mathbf{H}^{\mathcal{T}}\mathbf{G}^{\mathcal{T}})}{\partial\mathbf{G}} = \mathbf{G}^*\mathbf{H}^*\mathbf{F}^*\mathbf{F}^{\mathcal{T}}\mathbf{H}^{\mathcal{T}};$$
$$\frac{\partial\,\mathrm{trace}(\mathbf{GHF})}{\partial\mathbf{G}} = (\mathbf{HF})^{\mathcal{T}};$$
$$\frac{\partial\,\mathrm{trace}(\mathbf{F}^{\mathcal{T}}\mathbf{H}^{\mathcal{T}}\mathbf{G}^{\mathcal{T}})}{\partial\mathbf{G}} = (\mathbf{HF})^{\mathcal{T}};$$
$$\frac{\partial\,\mathrm{trace}(\mathbf{F}^{\mathcal{H}}\mathbf{H}^{\mathcal{H}}\mathbf{G}^{\mathcal{H}})}{\partial\mathbf{G}} = \frac{\partial\,\mathrm{trace}(\mathbf{G}^*\mathbf{H}^*\mathbf{F}^*)}{\partial\mathbf{G}} = \mathbf{0};$$
$$\frac{\partial\,\mathrm{trace}(\sigma_n^2\mathbf{GG}^{\mathcal{H}})}{\partial\mathbf{G}} = \frac{\partial\,\mathrm{trace}(\sigma_n^2\mathbf{G}^*\mathbf{G}^{\mathcal{T}})}{\partial\mathbf{G}} = \sigma_n^2\mathbf{G}^*. \quad (29)$$

Setting the partial derivative of $\mathrm{trace}(\mathbf{J})$ with respect to $\mathbf{G}$ to zero (i.e., $\frac{\partial\,\mathrm{trace}(\mathbf{J})}{\partial\mathbf{G}} = \mathbf{0}$), and utilizing (29), result in the matrix equation

$$\mathbf{G}^*(\mathbf{HFF}^{\mathcal{H}}\mathbf{H}^{\mathcal{H}})^{\mathcal{T}} + 2\mathbf{GHFF}^{\mathcal{T}}\mathbf{H}^{\mathcal{T}} + \mathbf{G}^*\mathbf{H}^*\mathbf{F}^*\mathbf{F}^{\mathcal{T}}\mathbf{H}^{\mathcal{T}} + 2\sigma_n^2\mathbf{G}^* = 4(\mathbf{HF})^{\mathcal{T}}. \quad (30)$$

Taking transpose operation on both sides of Eq. (30) yields

$$\mathbf{HFF}^{\mathcal{H}}\mathbf{H}^{\mathcal{H}}\mathbf{G}^{\mathcal{H}} + 2\mathbf{HFF}^{\mathcal{T}}\mathbf{H}^{\mathcal{T}}\mathbf{G}^{\mathcal{T}} + \mathbf{HFF}^{\mathcal{H}}\mathbf{H}^{\mathcal{H}}\mathbf{G}^{\mathcal{H}} + 2\sigma_n^2\mathbf{G}^{\mathcal{H}} = 4\mathbf{HF}, \quad (31)$$

which can be reformed as

$$\mathbf{HFF}^{\mathcal{H}}\mathbf{H}^{\mathcal{H}}\mathbf{G}^{\mathcal{H}} + \mathbf{HFF}^{\mathcal{T}}\mathbf{H}^{\mathcal{T}}\mathbf{G}^{\mathcal{T}} + \sigma_n^2\mathbf{G}^{\mathcal{H}} = 2\mathbf{HF}; \quad (32)$$
$$\mathbf{H}^*\mathbf{F}^*\mathbf{F}^{\mathcal{H}}\mathbf{H}^{\mathcal{H}}\mathbf{G}^{\mathcal{H}} + \mathbf{H}^*\mathbf{F}^*\mathbf{F}^{\mathcal{T}}\mathbf{H}^{\mathcal{T}}\mathbf{G}^{\mathcal{T}} + \sigma_n^2\mathbf{G}^{\mathcal{T}} = 2\mathbf{H}^*\mathbf{F}^*. \quad (33)$$

Utilizing (32) and (33), $\mathbf{J}$ in Eq. (17) can be simplified as

$$\begin{aligned}\mathbf{J} &= \mathbf{G}(\mathbf{HFF}^{\mathcal{H}}\mathbf{H}^{\mathcal{H}}\mathbf{G}^{\mathcal{H}} + \mathbf{HFF}^{\mathcal{T}}\mathbf{H}^{\mathcal{T}}\mathbf{G}^{\mathcal{T}} + \sigma_n^2\mathbf{G}^{\mathcal{H}}) \\ &\quad - 2\mathbf{GHF} + \mathbf{G}^*(\mathbf{H}^*\mathbf{F}^*\mathbf{F}^{\mathcal{H}}\mathbf{H}^{\mathcal{H}}\mathbf{G}^{\mathcal{H}} + \mathbf{H}^*\mathbf{F}^*\mathbf{F}^{\mathcal{T}}\mathbf{H}^{\mathcal{T}}\mathbf{G}^{\mathcal{T}} \\ &\quad + \sigma_n^2\mathbf{G}^{\mathcal{T}}) - 2\mathbf{G}^*\mathbf{H}^*\mathbf{F}^* + 4\mathbf{I} \\ &\quad - 2(\mathbf{F}^{\mathcal{H}}\mathbf{H}^{\mathcal{H}}\mathbf{G}^{\mathcal{H}} + \mathbf{F}^{\mathcal{T}}\mathbf{H}^{\mathcal{T}}\mathbf{G}^{\mathcal{T}}) \\ &= 4\mathbf{I} - 2(\mathbf{F}^{\mathcal{H}}\mathbf{H}^{\mathcal{H}}\mathbf{G}^{\mathcal{H}} + \mathbf{F}^{\mathcal{T}}\mathbf{H}^{\mathcal{T}}\mathbf{G}^{\mathcal{T}}). \end{aligned} \quad (34)$$

From Eq. (32), we have $\mathbf{F}^{\mathcal{H}}\mathbf{H}^{\mathcal{H}}\mathbf{G}^{\mathcal{H}} + \mathbf{F}^{\mathcal{T}}\mathbf{H}^{\mathcal{T}}\mathbf{G}^{\mathcal{T}} = 2\mathbf{I} - \sigma_n^2(\mathbf{HF})^{-1}\mathbf{G}^{\mathcal{H}}$, therefore

$$\mathbf{J} = 4\mathbf{I} - 2(\mathbf{F}^{\mathcal{H}}\mathbf{H}^{\mathcal{H}}\mathbf{G}^{\mathcal{H}} + \mathbf{F}^{\mathcal{T}}\mathbf{H}^{\mathcal{T}}\mathbf{G}^{\mathcal{T}}) = 2\sigma_n^2(\mathbf{HF})^{-1}\mathbf{G}^{\mathcal{H}}. \quad (35)$$

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