Magnetic Induction Tomography

and Techniques for Eddy-Current Imaging

by

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Abstract.

One method used to detect and characterise defects in conducting materials is eddy-current testing. The technique requires the measurement of the scattered magnetic field produced by a defect in a conductor which is in the vicinity of an oscillating magnetic source.

A novel method, based on Tomographic principles, and referred to as "Magnetic Induction Tomography", has been developed in order to measure the 2-dimensional distribution of a magnetic field. The algorithm for reconstructing the vector field is developed from first principles, and is based on the "Fourier Central Projection" theorem. It is then verified using simulated data for the cases of magnetic monopole and dipole sources. A practical demonstration of magnetic induction tomography is presented using a series of experimental examples.

A second approach based on the mutual induction between two orthogonal coils has also been investigated. In this case the transducer is based on two rectangular polarised coils. Two different transducers are presented. A pair of polarised coils when scanned over the region of interest can reveal the presence of cracks. By using an array of coils it is possible to make measurements in two dimensions without moving the transducer.

Finally, a microcomputer-controlled scanning rig is described together with the development of a data acquisition system suitable for evaluating eddy-current transducers.
To:

Mum, Dad, Nan, Mike, Andy

& last but not least

Margaret.
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Chapter 1

Introduction.

An aircraft in flight is subject to considerable stress which arises due to the continual flexing of the structure. As a result of this, cyclical stress fatigue cracks can initiate and propagate from localised areas of high stress, for example fastener holes. The detection of these cracks at an early stage is vital not only from a safety point of view, but also for reasons of economy. One established crack detection method uses eddy currents. It is only during recent years though, that full exploitation using imaging techniques has been considered. To a large extent, this has been influenced by the rapid advance in semiconductor technology. Certain data-gathering techniques are only practical using sophisticated electronic control systems. This thesis describes the development of new methods and transducers for use in eddy-current inspection instruments. First though, a brief description of the varied techniques currently used by the aviation industry is presented, which is a resume of Masons [1980] review paper.

Ultrasonic instruments are capable of measuring sub-millimeter cracks. One technique, referred to as a transmission technique, is based on measuring the attenuation of the sound waves as they travel through the specimen. An alternative technique measures the time of flight and the amplitude of sound waves which are reflected by the defects. This is referred to as a pulse echo technique. There are two main drawbacks
associated with ultrasonic evaluation. The first is concerned with acoustically coupling the source and the specimen. The second limitation is caused by the poor transmission of the sound through the first and second layer interface. Hence ultrasonic techniques can only reliably detect top layer cracks. The latter restriction is important since an aircraft is inherently a multi-layer structure.

Another well established technique is X-ray radiography. Its main advantage is the ability to detect cracks that are located deep within the conductor. It is also possible to identify defects in second and subsequent layers, although the method requires access to both sides of the specimen. The radiation source is placed on one side, whilst the detector is placed on the other. The detector in its simplest form consists of photographic film, which is sensitive to X-rays. The technique is expensive and slow since the film must be developed, although the film does provide a permanent record of the inspection. Alternative sensors include X-ray sensitive television, which reduces the cost of consumable items, but increases the initial capital outlay of the system. Other drawbacks include the problems associated with strong ionizing radiation, and the potential hazards of handling the sources.

One process that can reveal the presence of surface-breaking defects, which are smaller than the size which can be detected by the eye alone, involves the use of dye penetrants. Initially the surface of the specimen must be cleaned. The penetrant is applied and allowed to seep into any cracks. The surface coating is then removed. The remaining penetrant is developed and can be observed using ultraviolet light. Apart from the
inconvenience of removing and replacing the protective coatings, this method is limited to testing in the top layer only, and is restricted to surface-breaking cracks.

Eddy currents have been used in the aircraft industry for many years. The method enables portable instruments to be constructed, which are capable of detecting small surface, sub-surface, and 2nd layer cracks. In comparison with other methods, it is quicker and cheaper than radiographic film techniques, and does not have the coupling problems related with ultrasonic instruments. Furthermore it is not necessary to remove the protective paint layers before inspection. On the other hand, it is not possible to detect cracks, such as delamination, that do not interrupt the flow of the eddy currents. This method can fail to find deeply buried cracks and the signals can be misinterpreted as a result of complicated sub-surface structures.

The Materials and Structures Department of the Royal Aircraft Establishment (RAE), Farnborough, have a research programme that includes the design and development of eddy-current inspection instruments. A collaborative research project was instigated between the Physics Department, University of Surrey, and the RAE to expand their eddy-current research programme. This thesis presents some of the findings of that project.

One particular area within the RAE research programme is the detection of small radial cracks that propagate from rivet holes. An instrument, the Eddiscan, was designed to measure the impedance change in

--- 1.3 ---
A constant current coil, which is scanned around the circumference of the rivet [Harrison 1985]. A one dimensional distribution of the measured impedance change with angular position can reveal the presence of a defect. This is possible since a radial crack breaks the cylindrical symmetry of the impedance. The existing Eddiscan system was constructed for use with non-ferrous plates, rivets, and sub-structures. Research is currently in progress to develop a 2nd generation instrument, which can also be used to inspect ferrous materials.

At the start of this project, the Mark I Eddiscan had been successfully completed. The main strength of the instrument is the adoption of a scanning measurement technique. The detection coil is scanned along a path which follows the circumference of the rivet. The result is a highly optimised instrument to investigate defects under rivets. An initial objective for this project, was to design and implement a piece of apparatus which could be used for general scanning experiments. As scanning techniques invariably generate large quantities of data, an automated control and processing system also had to be developed. A description of the complete data acquisition system is presented in chapter 6.

A typical simple eddy-current transducer is based on a constant-current coil. In the presence of a conductor the impedance of the coil will change. The potential drop across the coil is a measure of the impedance of the coil. To maximise the information from a single coil, it is necessary to measure a distribution of the potential drop as the coil is moved in a plane close to the surface of the specimen. From this
distribution the presence and characterisation of cracks must be inferred. The impedance of the coil in the presence of a crack-free conductor is much larger than the change of impedance resulting from moving the detector to a region which contains a crack. Consequently as the transducer is moved over the cracked region of the conducting specimen, a small change in the source coil voltage will occur. It is the change in voltage that provides the information regarding the cracks. Before the change in the voltage can be amplified, it is necessary to remove the component of the voltage due to the coil in the presence of the uncracked conductor. Typically this can be achieved by incorporating the coil as one arm of a balanced bridge circuit.

An alternative transducer, considered in chapter 5, is based on two rectangular coils that are positioned symmetrically at right angles to one another. This type of coil is referred to as a polarised coil. One coil is used as the magnetic field source, the other operates as a detector. The induced emf in the detector is theoretically zero except in the presence of an asymmetry. Under certain circumstances a crack can be such an asymmetry. Consequently the necessity to remove large voltages by balancing has been avoided. A 2-D scanning procedure was adopted and the induced emf was plotted as a function of position. The basic idea was extended as a 2-D array of perpendicular coils. The underlying objective of using the array transducer is to simulate a mechanical scanning technique by electronic means.

Although 2-D mechanical scanning techniques are acceptable in a laboratory, for example to verify theoretical results, it is impracticable...
in an operational eddy-current instrument. In chapter 3 a technique is described that enables the magnetic field to be measured over a 2-D plane, yet minimises the translational movement required to achieve this objective. The theory behind this technique is based on tomographic reconstruction from projections. The algorithm which has been fully developed, is then verified in chapter 4 using both simulated and measured data. Chapter 4 also contains the derivations of the expressions used to generate the simulated data.

The fundamental electromagnetic equations and the simplifying assumptions are presented in chapter 2. Also included is a discussion of imaging and the basic eddy current models.
Chapter 2

Basic Principles of Electromagnetic Theory.

2.1 Introduction.

The use of eddy currents in the detection and characterisation of cracks in conductors is well known. A typical instrument consists of a source of magnetic field to establish the eddy currents, and a detector to measure the magnetic field. It also contains an electronic system to process the amplified signals, and a method of quantifying the field. The latter can vary from an analogue meter to sophisticated surface and contour plots on a computer graphics terminal. The excitation transducer is usually a current source of some form, and normally the detector is a coil. There are various methods of driving the current source, eg constant single and multiple frequencies Libby [1971], or pulsed excitation Morris [1975], Libby [1971]. The shape, size and orientation of the coil can also vary a great deal. These will normally be chosen to suit each specific application. If the source and detector transducers are the same coil, then the coil's impedance is measured. If the transducers are two separate coils, then the induced emf in the detector is measured. The presence of defects is inferred from the variation of the measured values with position.

To a large extent the characterisation of cracks to date is based on empirical techniques. Work is currently in progress ECG [1985] to
provide a better theoretical understanding of the relationship between the measured magnetic fields and the size and position of cracks within a conductor. In principle the solutions of all electromagnetic problems are derived from Maxwell's equations. The major difficulties arise when the relevant boundary conditions are applied. The point at which the description changes from being quantitative to being qualitative is governed by the complexity of the problem. Simple configurations with a high degree of symmetry have been solved analytically. In general most physically-realisable problems are too complicated to solve in this way, and so numerical methods such as finite element and difference techniques must be used, Stoll [1974].

All electromagnetic problems can be formulated in the first instance, using Maxwell's equations. In section 2.2, the general electromagnetic theory is simplified by applying physically realistic constraints. From these expressions the 1-D wave equation in a conducting medium and the expression for the skin depth are derived. This leads conveniently to a description of eddy currents. Finally the chapter will conclude with a discussion of eddy-current imaging.

2.2 Basic Theory.

The solution of any electromagnetic problem ultimately derives from Maxwell's equations.

\[ \nabla \times E(t) = -\frac{\partial B(t)}{\partial t} \]  

(2.1)
\[ \nabla \times \mathbf{H}(t) = \mathbf{J}(t) + \frac{\partial \mathbf{D}(t)}{\partial t} \quad (2.2) \]

\[ \nabla \cdot \mathbf{E}(t) = \frac{\rho}{\varepsilon} \quad (2.3) \]

\[ \nabla \cdot \mathbf{B}(t) = 0 \quad (2.4) \]

where \( \mathbf{E} \) and \( \mathbf{H} \) are the electric and magnetic fields, \( \mathbf{B} \) is the magnetic induction, \( \mathbf{J} \) is the current density, \( \mathbf{D} \) is the displacement current, \( \varepsilon \) and \( \mu \) are the permittivity and permeability of the medium. \( \mathbf{B} \) and \( \mathbf{D} \) can be expressed as:

\[ \mathbf{B}(t) = \mu \mathbf{H}(t) \quad (2.5) \]

\[ \mathbf{D}(t) = \varepsilon \mathbf{E}(t) \quad (2.6) \]

and Ohm's law gives

\[ \mathbf{J}(t) = \sigma \mathbf{E}(t) \quad (2.7) \]

where \( \sigma \) is the conductivity of the material.

The analysis of general problems can be greatly simplified by restricting the current sources to be time harmonic. The above
expressions can be described by a complex phasor vector and a time dependent term, e.g. the field intensity can be written as

\[ H(t) = \text{Re} H e^{-i\omega t} \quad (2.8) \]

where \( H \) is the phasor, \( \omega \) is the angular frequency, \( t \) is the time and \( \text{Re} \) denotes the real part of the complex vector. From (2.1) and expressing the magnetic induction in terms of \( H \)

\[ \nabla \times E(t) = -\mu \frac{\partial H}{\partial t} e^{-i\omega t} = i\omega \mu H(t) \quad (2.9) \]

As the time dependent terms cancel, this expression and the other expressions consists only of phasor vectors and constants. Substituting expressions (2.6) and (2.7) into (2.2) and changing to phasors

\[ \nabla \times H = \sigma E - i\omega E \quad (2.10) \]

At low frequencies, the conductivity is much greater than the product of the permittivity and the frequency. Consequently the second term on the right of expression (2.10) can be neglected. This implies that in an isotropic homogeneous medium, the divergence of the electric field is zero, and there can be no build up of electric charge. This does not necessarily apply at boundaries.

A wave equation can be derived from Maxwell's equations by taking the curl of (2.2) and substituting for \( E \) using (2.1) and (2.5).
\[ \nabla \times \nabla \times \mathbf{H} = i \omega \sigma \mu \mathbf{H} \]  
\hspace{1cm} (2.11)

Using the identity
\[ \nabla \times \nabla \times \mathbf{H} = \nabla \nabla \mathbf{H} - \nabla^2 \mathbf{H} \]  
\hspace{1cm} (2.12)

Only non-ferrous materials are considered during this project. Hence the relative permeability may be assigned to 1 everywhere. Therefore

\[ \nabla \cdot \mathbf{B} = \nabla \cdot (\mu \mathbf{H}) = \nabla \mu \cdot \mathbf{H} + \mu \nabla \cdot \mathbf{H} = 0 \]  
\hspace{1cm} (2.13)

(2.11) can be expressed as

\[ \nabla^2 \mathbf{H} = -i \omega \sigma \mu \mathbf{H} \]  
\hspace{1cm} (2.14)

This is the vector Helmholtz equation. In air the expression reduces to the Laplace equation (2.15) since the conductivity of air is zero.

\[ \nabla^2 \mathbf{H} = 0 \]  
\hspace{1cm} (2.15)

Consider a conducting half space, with the boundary at \( z=0 \), and an incident plane wave, which has a single magnetic component in the \( x \) direction. If the excitation is confined to being a function of \( z \) only, then expression (2.14) reduces to

\[ \frac{\partial^2 \mathbf{H}}{\partial z^2} = -i \omega \sigma \mu \mathbf{H} = k^2 \mathbf{H} \]  
\hspace{1cm} (2.16)
inside the conductor. This has the solution

\[ H = A e^{-kz} + B e^{kz} \]  (2.17)

where \( k = (1-i) \sqrt{\frac{\omega}{\mu \sigma}} \)

The coefficient A, must be zero since the field must fall to zero as \( z \) tends to negative infinity, and B is obtained from the boundary conditions at the surface. The well known skin depth term (2.18), can now be defined as

\[ \delta = \sqrt{\frac{2}{\omega \mu \sigma}} \] (2.18)

Hence

\[ H = H_0 e^{-z/\delta} e^{iz/\delta} \] (2.19)

This expression represents a wave travelling into the conductor. These fundamental results will now be used in defining the basic eddy-current model.

2.3 Applications to Eddy Currents.

The expressions defined in the previous section are perfectly general for harmonic oscillating current sources. The equations are constrained by considering only non-ferrous materials in the low frequency limit.
Analytic solutions of eddy-current problems have been obtained for some cases with simple boundary conditions. A particularly useful model consists of a semi-infinite isotropic homogeneous conducting half space (see fig 2.1.),

where $J_s$ and $J_e$ are the oscillating current source and the induced eddy current distributions. Stoll [1974] considered the problem for an infinitely long straight conducting wire as the source, whilst Hammond [1962] considered the source as a horizontal circular current loop. The solution for a vertical current loop was derived by Kriezis and Xypteras [1979]. The above solutions are specific to the particular source geometries. Harrison proposed that a general dyadic Greens function could be derived, which would enable the solutions of the electric and magnetic fields to be calculated for arbitrary source distributions. Bowler [1985a] has formulated the solution to this problem rigorously, and has subsequently further simplified it in terms of scalar potentials [Bowler 1985b].
During the formulation of the scalar Greens function solution, certain physical properties became apparent. The net normal component of the electric field at a conducting/non-conducting boundary inside the conductor is zero. This can be justified on physical grounds since the current flow across the boundary is zero. The implication of this to the semi-infinite conducting half space fig(2.1), is that no normal components of the electric field can exist within the conductor. This can be deduced from a zero normal component of $\mathbf{E}$ at the boundary, and no internal sources within the conductor. This is also discussed by Hammond [1982].

A crack within a conductor can be thought of as a 2-Dimensional discontinuity in the conductivity. The normal component of the electric field at such a boundary is zero. The assumption is made, that the electric field at the same position in an equivalent non-cracked conductor, is not zero. In the case of the cracked conductor charge must build up at the surface until the net normal component of $\mathbf{E}$ is zero. The divergence of the current from a volume which totally encloses the crack is zero. Hence the net charge leaving the volume is zero. This implies that a positive charge build up on one side of the crack has an equivalent charge depletion on the other. Therefore a crack can be described as a distribution of electric dipoles.

Consider a crack, of infinitesimally small width which is parallel to the surface of a semi-infinite conductor as shown in fig (2.2). Since the normal component of $\mathbf{E}$ on the crack surface is zero, there will be no charge build up. It is the oscillating dipole distribution that provides the information regarding the defect. Hence it is not possible to detect
this kind of crack using eddy currents. A vertical 2-D crack will in general generate a dipole distribution. For this to happen the equivalent electric field in the "defect free conductor", and the direction of the defect must not be parallel, see fig(2.3).

Fig 2.2: 2D crack (Parallel to the surface.) in conducting half space.

Fig 2.3: Plan view of cracks (perpendicular to the surface.) in a conductor.

The dipole distribution is an additional source inside the conductor. In air, The magnetic field will consist of 3 components:

1 : The free space magnetic field due to the primary current source.

2 : The magnetic field due to eddy currents within the uncracked conducting half space.

3 : The magnetic field due to the current flow in the conductor

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caused by the oscillating dipole distribution.

The description of the dipole distribution is complicated because the boundary conditions at each point on the surface of the crack must be met using the total electric field. This depends on the contributions from the other dipoles. The variation of the distribution and phase of the dipoles over the surface will vary with the position, shape, and size of the crack, and the uncracked electric field solution.

The solution of the forward problem should provide valuable information to be used in the solution of the inversion problem, which is defined as: "Given a measured magnetic field at the surface, the primary current source and the solution to the defect free equivalent problem, can the dipole distribution be obtained?". With the knowledge of the dipole distribution, better characterisation of defects should be possible.

2.4 Eddy-Current Imaging.

"When a function of two independent variables (such as light intensity as a function of direction) is approximately reproduced elsewhere as another function of two variables, we speak of image formation". This is the definition of a 2-D image as proposed by Bracewell [1965] and is the definition adopted throughout this thesis. It is common practice for imaging techniques to be titled with an identifying process preceding it, such as "Nuclear Magnetic Resonance Imaging". However the name does not imply that an image of the identifying process is produced, ie X-ray imaging does not produce an image of X-rays. Similarly
the term "Eddy-Current Imaging" does not necessarily imply the end result is an image of the distribution of eddy currents. Rather it is an image obtained by the use of eddy currents.

Images of various 2-Dimensional distributions are considered during this project. In the magnetic induction tomography chapters, the aim is to reconstruct an image of the magnetic field in a plane immediately above the conductor. This distribution varies from conventional images because it is a vector quantity. This presents immense problems of image display. These can be simplified by transforming the fields into scalar potentials, which are then plotted.

The objective of using eddy-current imaging is to map the distribution of defects and characterise them within a general conducting specimen.
Chapter 3.

Magnetic Induction Tomography.

3.1 Introduction

Consider an arbitrary oscillating magnetic source in air, that has finite support, i.e., outside a given region the field has decayed to a level which can be considered to be zero. A problem that frequently arises is to determine the components of the magnetic field that lie in a plane at a fixed distance from the source.

The conventional solution to this problem is to place a suitable transducer, e.g., search coil, at the desired position so that its axis is parallel to the direction of the component of the field to be measured. If the transducer is rotated by 90 degrees, the other field component can be measured. To construct a 2-D distribution of magnetic field samples, the transducer must be physically translated to each measurement position in the plane. This technique will be referred to as localised field sampling. The resolution with which the field can be measured is governed by the size of the transducer.

The localised measuring system has 3 degrees of freedom, the X and Y lateral positions, and the rotational angle, $R$ of the transducer. The fourth degree of freedom, the height $(H)$ of the plane above the source, is constrained in this instance to be constant. Such a system was constructed

--- 3.1 ---
during this project and is described in chapter 6. This piece of apparatus is acceptable in a laboratory environment, but is very impracticable as a portable instrument for operational use (for example on the underside of an aircraft wing).

An alternative technique is proposed which enables the localised magnetic field to be determined without requiring the lateral translation of the transducer. This is achieved by selecting a data-gathering method which makes two of the three degrees of freedom redundant. The localised distribution of the field can then be mathematically reconstructed from these measurements. The transducer consists of a uniformly-spaced array of parallel rectangular coils, see fig 3.1. The size of the induced emf in each coil is determined by Faraday's law of induction (3.1)

\[ v = -\frac{d}{dt} \int_{S} B \cdot da \]  (3.1)

where the integration is taken over the surface (s), \( v \) is the induced voltage, \( t \) is time, \( B \) is the magnetic induction, and \( da \) is the elemental area. The induced emf depends on the total net flux that passes through the surface of the coil. As the induced emf has both a magnitude and a phase relative to the field source, and as the array consists of \( n \) coils, then there are \( 2n \) pieces of information available at each frequency. A projection is defined as the set of ordered values which represent the integral of the magnetic field over the coils surface at each sample point. The set of induced emfs is equal to the time derivative of the projections. A sequence of projections can be obtained by measuring a projection at a set of regular angular intervals. The array of coils is

--- 3.2 ---
rotated about its midpoint, see fig 3.2.

![Diagram of rectangular coil array]

Fig 3.1: Isometric view of rectangular coil array.

The initial formulation of the problem is: "Given a set of projections, measured in a plane at equi-angular intervals, can an image of the magnetic field be reconstructed?".

Problems associated with the reconstruction of distributions in a plane from projections can be solved using tomographic reconstruction techniques. The word tomography is derived from the Greek word 'Tomi'.
which means slice. Examples of the disciplines that extensively use
tomographic reconstruction are nuclear magnetic resonance, x-ray,
gamma-ray, and ultrasonic imaging. If tomographic techniques are to be
considered for the reconstructions, the underlying differences between
reconstructing a magnetic field and for example the reconstruction of the
density distribution of a body using gamma rays, must be identified. The
primary consideration is the final image. The magnetic field distribution
is a vector quantity, whilst the density distribution is scalar. The task
is to formulate a reconstruction algorithm that is capable of coping with
general vector distributions, or to define a suitable transformation of
the electromagnetic field such that existing scalar reconstruction
algorithms can be used.

3.2 Conventional Tomographic Reconstruction from Projections.

Existing scalar reconstruction algorithms have been reviewed and a
summary is presented along with an example of their application to gamma-
ray tomography.

3.2.1 Gamma-Ray Tomography.

One technique used to gather data in gamma-ray imaging is to generate
a fixed collimated beam of photons, which is incident on a collimated
detector (see fig 3.3). The specimen under test is placed on a table so
that it intercepts the beam. The table has two degrees of freedom, lateral
movement and rotation. The lateral movement is in the direction which is
perpendicular to the beam. The attenuation of the beam is measured as a
function of both the angle of the table, and its position. At any constant angle, the set of attenuation values obtained at different lateral positions constitute a projection. As the height of the specimen is unaltered during the experiment, the data only contains information about a single slice of the object. An attenuation coefficient is obtained by measuring the ratio of the attenuation of the beam through air, and the same beam through the object. A relationship can be derived between the attenuation coefficients and the density distribution in the slice. The reconstruction process is often referred to as 'Computed Tomography' (CT) or 'Computer Assisted Tomography' (CAT).

![Diagram of Single Beam Gamma-Ray Tomographic Imaging System](image)

**Fig 3.3:** Single Beam Gamma-Ray Tomographic Imaging System.

Digitally-computed image reconstructions are inherently discrete. If the size of the image is $n \times m$ pixels, then it can be described using an abstract vector space of dimension $n \times m$. Each basis vector represents a small area in real space. The coefficient assigned to that basis vector will represent the average value of the image within that area. Each basis vector will be referred to as a pixel, loosely derived from "picture.
The ray, which follows the same path as the beam of photons, is defined as the straight line which is perpendicular to the line of the projection, that also passes through the point \((a, t)\) where \(a\) defines the angle of the projection, and \(t\) defines a displacement within the projection. The raysum is defined as the weighted sum of all the basis vector coefficients that lie along the path of the ray. The weighting in the raysum is a function of the path length of the ray through the pixel. A projection is the ordered set of all raysums for a fixed projection angle. Alternatively, the image can be defined as a 2-dimensional function, \(f(x,y)\) in Cartesian space, see fig(3.4). The number of raysums in the projection corresponds to the number of translational sample points. The process of measuring the raysums is described by Herman [1980].

![Diagram](image.png)

Fig 3.4: Discrete model of an image and its projections.
3.2.2 The Radon Transform.

Radon [1917] first formulated an expression to evaluate the raysum which is referred to as the "Radon Transform". It is also the definition of a projection. If the image is expressed in polar coordinates \( f(r, \varphi) \) and \( W \) is the ray path which is a normal that passes through the point \( p(t, \alpha) \), then the Radon transform (and the projection) is

\[
p(t, \alpha) = \int_{-\infty}^{\infty} f(\sqrt{t^2 + W^2}, \alpha + \tan^{-1}(W/t))dW \quad t \neq 0 \quad (3.2)
\]

\[
p(t, \alpha) = \int_{-\infty}^{\infty} f(W, \alpha + \pi/2)dW \quad t = 0
\]

The simplest reconstruction technique is to select each pixel in turn, calculate which ray in each projection passes through the selected pixel, sum their corresponding raysums, and assign the total value to the pixel. This technique is referred to as 'Back-Projection Reconstruction'. It can be expressed as

\[
f(r, \varphi) = \int_{0}^{\pi} P(rcos(\alpha - \varphi), \alpha)d\alpha \quad (3.3)
\]

This reconstruction algorithm accentuates any high frequency components, i.e., sharp contrasts in densities. The effect is to produce the classic star artifact in the reconstructed image. These may be eliminated if the projections are filtered prior to back projecting. There are three types of filtering, Fourier, Radon, and Convolution [Brooks and Di Chiro 1976], which are collectively referred to as 'Filtered Back Projection'.

--- 3.7 ---
Radon proposed the first reconstruction algorithm (referred to as the inverse 'Radon Transform'). which can be described as:

(1) partially differentiate the projection with respect to the length variable (t).

(2) Hilbert transform the result of (1) with respect to the length variable.

(3) Backproject the result of (2) to form an image.

(4) Normalise the image by multiplying throughout by $-1/2\pi$

The expression for the inverse radon transform is therefore

$$f(r, \varphi) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{0}^{\pi} \frac{\partial P(t, \alpha)}{\partial t} \frac{1}{(r \cos(\alpha - \varphi) - t)} \ dt \ d\alpha$$  \hspace{1cm} (3.4)$$

This is an exact method which requires $P(t, \alpha)$ to be known for all $t$ and $\alpha$.

[Herman 1980, Radon 1917]

3.2.3 The Algebraic Techniques.

Iterative techniques have been referred to as 'brute force' methods. The image is represented in an abstract vector space, with each pixel mapping into a single basis vector. If there are $n \times m$ pixels, then the image is uniquely defined by the image vector $f$, which has dimension of $nm$. The projections are also defined to be a linear vector $P$, of dimension $N$, where $N$ is the number of sample points per projection. The relationship between the image and the projections is defined as

$$P_j = \sum_{i=1}^{nm} W_{ij} f_i$$  \hspace{1cm} (3.5)$$

--- 3.8 ---
The weighting matrix $W_{ij}$ represents the contribution of the $i$'th pixel to the $j$'th raysum. In principle, the image may be reconstructed by inverting the matrix, $W$. Problems can occur if the number of unknowns exceed the number of equations, in which case there is no unique solution. If there is excessive noise in the data, then the solutions can be meaningless. A final drawback is the size of the inverted matrix, which rapidly becomes too large to handle in practice.

The algorithm employed for iterative reconstruction assumes a defined initial image, (this may take advantage of a priori knowledge, with a consequential reduction in convergence time.) and to correct repeatedly the image to match the projection data. Two techniques are commonly used. The first establishes the corrections required so that a single projection fits the image. A single iteration is complete when each of the projections has been considered in turn. This is referred to as the 'Algebraic Reconstruction Technique' (ART). The second method evaluates the corrections so that a single pixel fits all of the projections simultaneously - 'Simultaneous Iterative Reconstruction Technique' (SIRT). A single SIRT iteration is complete when each pixel has been corrected. [Brooks & Di Chiro 1976, Budinger & Gullberg 1974, Herman 1976, Gilbert 1976, Oppenheim 1974].

The iterations are repeated until the image converges. It is possible (especially ART) for the image to eventually diverge. In such cases it is necessary to apply optimisation criteria to select the most appropriate image. One such method is to minimise the least squares solution.
Optimisation criteria are discussed in Herman & Lent [1976]. A comparison of the merits and rates of convergence between ART and SIRT is presented in Gilbert [1971], and it is clearly shown that ART converges more rapidly than SIRT, but is susceptible to noise, with the possibility of eventual divergence of the image. The final image available using SIRT is more accurate than with ART.

3.2.4 Derivation of the Central Projection Theorem.

The final and most elegant methods are the 'Analytic Reconstruction Techniques'. They are based on the direct solution of the projection operator. The derivation of the 'Fourier Reconstruction' algorithm is presented below. It is included because a modified version will be used in the magnetic induction reconstruction algorithm. This method is preferred to the others as it is an exact method. Analytic test cases can be used to generate exact data. This has obvious advantages during the verification of the implementation.

--- 3.10 ---
The "Central Projection Theorem" relates the Fourier transform of the measured projections \( P(k, \alpha) \) to the Fourier Transform of the image \( F(u, v) \). The projection expression (3.2) may be re-defined from a line integral along the ray path, to a surface integral over the image. If a delta function is chosen which maps directly onto the path of the ray, then its sifting properties will ensure that only the contributions of the image which are coincident with the ray path will be included in the raysum. The path of the ray (see fig 3.5) is expressed as:

\[
x \cos \alpha + y \sin \alpha - t = 0
\]  

The projection expression is re-defined as

\[
p(t, \alpha) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y) \delta(x \cos \alpha + y \sin \alpha - t) \, dx \, dy \tag{3.7}
\]

The expression for the Fourier Transform of the projection is

\[
F(k', \alpha) = \int_{-\infty}^{\infty} p(t, \alpha) e^{-ik't} \, dt \tag{3.8}
\]

Substitute for (3.7) into (3.8) and integrate with respect to \( t \) first:

\[
F(k', \alpha) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y) e^{-ik'(x \cos \alpha + y \sin \alpha)} \, dx \, dy \tag{3.9}
\]

The 2-dimensional Fourier Transform of the image, \( F(u, v) \) is

\[
F(u, v) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y) e^{-i2\pi(ux + vy)} \, dx \, dy \tag{3.10}
\]

--- 3.11 ---
If \( u = k' \cos \alpha \) and \( v = k' \sin \alpha \) then \( \mathcal{F}(u,v) = \mathcal{F}(k',\alpha) \).

Thus the Fourier Transform of the projection is equal to the Fourier transform of the image along the line \( \cos \alpha = \frac{v}{u} \cdot \sin \alpha = 0 \). This is the Central Projection Theorem. It will be used as a comparison for the resulting expression of the 'Modified Central Projection Theorem' as applied to magnetic induction tomography which is derived below.

![Fig 3.6: The central projection line in the Fourier space of the image.](image)

3.3 The Modified Central Projection Theorem.

3.3.1 Introduction.

It was proposed by Jones [1984] that since the magnetic field is conservative in air, then it can be described in terms of a scalar potential. Since under these circumstances the image reconstruction is transformed from a vector to a scalar distribution, existing scalar algorithms can be considered. To achieve a conservative field, the region

--- 3.12 ---
of the reconstruction must not contain any current sources, ie in the low frequency limit

\[ \nabla \times \mathbf{H} = 0 \]  

The magnetic induction expressed as the negative gradient of a scalar potential is given by (3.12). The components of the gradient in polar coordinates is expressed as (3.13)

\[ \mathbf{B} = -\nabla \psi(r) \]  

where \( \psi(r) \) is a scalar potential.

\[ \begin{align*}
B_r &= -\frac{\partial \psi}{\partial r} \\
B_\phi &= -\frac{1}{r} \frac{\partial \psi}{\partial \phi}
\end{align*} \]  

Based on the transformation of the magnetic field to a scalar distribution, the derivation of a reconstruction algorithm based on the central projection theorem is presented in the next section.

3.3.2 Derivation of the Magnetic Induction Projection Equation.

Consider a rectangular coil of unit height and length \( l \), sampling the component of the magnetic field in the plane of a region, see fig (3.7). The rectangular coil samples the field at a distance 'a' above the source. The position of the coil is expressed using a polar coordinate system. The distance in the plane of reconstruction from the origin, to the point on the coil which coincides with the surface normal that passes through the origin, is 't'. The angle subtended by the coil surface normal and the \( x \) axis is \( \alpha \), see fig(3.7). Any point \( L \), can be described by the coordinates \( L(r, \phi) \).
Fig 3.7: Plan view of coordinate system used in Induction Tomography.

Fig 3.8: Oblique view of Induction Tomography coordinate system.
A delta function can be used to describe the locus of points of the line of the coil on the plane, which is given by

\[ r \cos(\varphi - \alpha) = t \]  
(3.14)

Assuming that the excitation field is sinusoidal, then the time dependence of the signal reduces to a constant. The voltage induced in the coil is defined by Faraday's law of induction, (3.1). This is a measure of the total flux linking the coil.

\[ v = i \omega \int_B \cdot da \]  
(3.15)

In polar coordinates, the magnetic induction is given by

\[ B = B_r \hat{r} + B_\varphi \hat{\varphi} \]  
(3.16)

The component of the magnetic field which is normal to the surface of the coil at the point \( L(r, \varphi) \) is

\[ B \cdot n = B_r \cos(\varphi - \alpha) - B_\varphi \sin(\varphi - \alpha) \]  
(3.17)

The emf induced in the coil is proportional to the integral of the normal component of \( B \), over the surface of the coil. If the height of the coil is small, we may assume that the field is constant in the Z direction. The surface integral may be replaced by a line integral, multiplied by a constant which depends on the height of the coil. The integral limits
Fig 3.9: The plan view of the normal to the coil at point \( L(r, \varphi) \) in polar coordinates.

are defined over the entire plane of reconstruction, but contributions are restricted to that of the line of the coil by the use of a delta function (3.14).

\[
\begin{align*}
\mathbf{v} &= i\omega \int \left[ B_r \cos(\varphi - \alpha) - B_\varphi \sin(\varphi - \alpha) \right] \delta(r \cos(\varphi - \alpha) - t) \, r \, dr \, d\varphi \\
&= i\omega f_\alpha(t) \tag{3.19}
\end{align*}
\]

If the angle between the normal of the coil and the x axis is maintained, but the coil is shifted laterally, then the sequence of magnetic field measurements, which are related to the induced emfs by the expression

\[
\mathbf{v} = i\omega f_\alpha(t) \tag{3.19}
\]

constitute a projection. Hence a magnetic field projection is defined as
The Fourier transform of the projection is

$$\tilde{F}_\alpha (k) = \int_{-\infty}^{\infty} f_\alpha (t) e^{-ikt} \, dt$$  \hspace{1cm} (3.21)

substituting for the projection into (3.21)

$$\tilde{F}(k) = \int_{-\infty}^{\infty} \int_{0}^{2\pi} \int_{0}^{\infty} \left[ B_r \cos(\varphi-\alpha) - B_\varphi \sin(\varphi-\alpha) \right] e^{-ikt} \rho \, dr \, d\varphi \, dt$$  \hspace{1cm} (3.22)

Integrating first with respect to \( t \)

$$\tilde{F}(k) = \int_{0}^{\infty} \int_{0}^{\infty} \left[ B_r \cos(\varphi-\alpha) - B_\varphi \sin(\varphi-\alpha) \right] e^{-ikt} \rho \, dr \, d\varphi$$  \hspace{1cm} (3.23)

substituting for \( B_r \) and \( B_\varphi \) from (3.13)

$$\tilde{F}(k) = \int_{0}^{\infty} \int_{0}^{\infty} \left[ -\frac{\partial \psi \cos(\varphi-\alpha)}{\partial r} \rho \right] e^{-ikt} \, dr \, d\varphi$$  \hspace{1cm} (3.24a)

$$+ \int_{0}^{\infty} \int_{0}^{\infty} \left[ \frac{1}{r} \frac{\partial \psi \sin(\varphi-\alpha)}{\partial \varphi} \rho \right] e^{-ikt} \, dr \, d\varphi$$  \hspace{1cm} (3.24b)

Evaluating the first half of \( F(k) \) (3.24a) using integration by parts

$$3.24a = - \int_{0}^{2\pi} \left[ r \psi \cos(\varphi-\alpha) \right]_{\varphi=0}^{\varphi=2\pi} d\varphi$$  \hspace{1cm} (3.25a)

$$+ \int_{0}^{\infty} \int_{0}^{\infty} \frac{1}{r} \psi \left[ r \cos(\varphi-\alpha) \right] dr \, d\varphi$$  \hspace{1cm} (3.25b)
The scalar potential at infinity is defined to be zero, consequently term (3.25a) is zero. Differentiating with respect to $r$:

$$3.24a = \int_0^\infty \int_0^{2\pi} \psi [-\text{i}kr \cos(\varphi - \alpha) + \cos(\varphi - \alpha)] e^{-i\text{i}kr \cos(\varphi - \alpha)} \, dr \, d\varphi$$

(3.26)

Evaluating the term (3.24b), again by parts

$$3.24b = \int_0^\infty \int_0^{2\pi} \psi \sin(\varphi - \alpha) e^{-i\text{i}kr \cos(\varphi - \alpha)} \, dr \, d\varphi$$

(3.27a)

$$3.24b = \int_0^\infty \int_0^{2\pi} \frac{\partial}{\partial \varphi} \left[ \int_0^\infty \psi \sin(\varphi - \alpha) e^{-i\text{i}kr \cos(\varphi - \alpha)} \right] \, dr \, d\varphi$$

(3.27b)

As the magnetic field is conservative, we have

$$\psi(r, \varphi) = \psi(r, \varphi + 2\pi)$$

(3.28)

Consequently 3.27a is zero. Taking the differential of 3.27b:

$$3.24b = \int_0^\infty \int_0^{2\pi} \psi [-\cos(\varphi - \alpha) - \text{i}kr \sin^2(\varphi - \alpha)] e^{-i\text{i}kr \cos(\varphi - \alpha)} \, dr \, d\varphi$$

(3.29)

The Fourier Transform of the projection is given by (3.26) + (3.29)

$$\mathcal{F}_\alpha (k) = \int_0^\infty \int_0^{2\pi} \psi [-\text{i}k \cos^2(\varphi - \alpha) - \text{i}k \sin^2(\varphi - \alpha)] e^{-i\text{i}k \cos(\varphi - \alpha)} \, r \, dr \, d\varphi$$

(3.30)
Therefore

\[ F_{\alpha}(k) = -ikr \int_0^{2\pi} \int_0^\infty \psi \cos(\phi-\alpha) e^{-ikr \cos(\phi-\alpha)} r dr d\phi \tag{3.31} \]

but the definition of the Fourier Transform of \( \psi \) in polar coordinates is given by

\[ \tilde{\psi}_\alpha(k, \beta) = \int_0^{2\pi} \int_0^\infty \psi e^{-i|k| r \cos(\phi-\beta)} r dr d\phi \tag{3.32} \]

If \( |k| \cos \beta = k \cos \alpha \) where \( \beta = \alpha + \tau \) for all \( k < 0 \) then

\[ F_{\alpha}(k) = -i k \tilde{\psi}_\alpha(|k|, \beta) \tag{3.33} \]

This is the 'Modified Central Projection Theorem' as applied to magnetic induction tomography. It is similar to the gamma ray central projection theorem in that it relates the Fourier transform of the projection at angle \( \alpha \), to the Fourier Transform of the 2-dimensional image along the line that passes through the origin at an angle \( \alpha \). The MIT projection must first be divided by the wavenumber, whereas the gamma projection does not. Another point to note is, if the source data is real, then the reconstructed image will be imaginary. This again does not apply to the gamma ray algorithm.
3.4 Application of the Modified Central Projection Theorem.

A theoretical expression relating the Fourier transform of the image of the in-plane components of the magnetic field to the Fourier transform of the magnetic field projections obtained by a long narrow rectangular coil was derived in the previous section. The practical application of this theory will now be considered. The general algorithm is first presented, along with the phase corrections and data ordering required for the fast Fourier transform. A flow diagram of the algorithm to reconstruct the scalar magnetic potential from projection data is shown in fig 3.11. The source data consists of \( m \) projections, sampled at equi-angular increments of \( \pi / m \), see fig 3.10. Each projection consists of \( N \) samples, with equal intervals of \( 2r/N \) along the radius, ranging from \( -r \) to \( r \).

As a consequence of the potentially large number of FFTs to be computed, it was considered advantageous to use an array processor (Floating Point Systems). This type of system contains an FFT subroutine as a library function.

![Figure 3.10: Description of the projection sample positions.](image)
Measure Projection data

Fourier Transform
the projections.

Apply Phase Correction

Re-order data to natural
ordered form using N/2 rotation.

Divide transformed projection
by wavenumber K.

Interpolate projection data to fill the
distribution in cartesian Fourier space.

Shuffle data into inverse FFT
form by two rotation operators.

Inverse 2-D FFT.

Re-shuffle data to
restore natural ordering.

Normalise data.

Fig 3.11: Implementation flow chart of the algorithm to reconstruct the Magnetic Scalar Potential from projections.
The FFT algorithm operates on N data points, where N is a radix 2 integer. This is typical of the standard routine that may be available on other systems. The discrete Fourier transform pair is given in (3.34) and (3.35).

\[
F(k) = \frac{1}{N} \sum_{r=0}^{N-1} f(r) e^{-i2\pi kr/N} \quad (3.34)
\]

\[
f(r) = \sum_{k=0}^{N-1} F(k) e^{i2\pi rk/N} \quad (3.35)
\]

The source data array consists of a sequence of values in natural ordered form, with element zero representing \(f(-N/2)\) through to element \(N-1\) representing \(f(N/2-1)\). The ordering of the data, which is a requirement of the FFT algorithm is shown in fig (3.12). To restore the data to its natural ordered form, a rotation (defined by 3.36) must be applied to the data.

\[
R(k) = F(k+N/2) \quad \text{where } k = -N/2 \text{ to } N/2 - 1 \quad (3.36)
\]

Fig 3.12: Data ordering before (top) and after (bottom) applying the 1D Fourier Transform.

The range over which the summation is taken is significant, as a shift in the range will result in a change in the phase characteristics of
the transformed signal. This problem was encountered and has been catered for in the reconstruction. The problem originates as a result of the difference in the ranges of the analytic transform and its discrete computed counterpart. The range of the discrete data is from \(-N/2\) to \(N/2\) -1 and for the computed transform from 0 to \(N-1\). The derivation of the phase factor is given below. The computed transform is denoted by \(F_c(k)\) and the analytic transform is given by \(F_a(k)\).

\[
\sum_{r=-N/2}^{N/2-1} f(r) e^{-i2\pi kr/N} = F_a(k) \tag{3.37a}
\]

\[
\sum_{r'=0}^{N-1} f(r') e^{-i2\pi kr'/N} = F_c(k) \tag{3.37b}
\]

\[
\sum_{r=-N/2}^{N/2-1} f(r) e^{-i2\pi k(r+N/2)/N} = F_c(k) \tag{3.37c}
\]

\[
F_c(k) = e^{-i\pi k} \sum_{r=-N/2}^{N/2-1} f(r) e^{-i2\pi kr/N} = e^{-i\pi k} F_a(k) \tag{3.37d}
\]

Then the phase factor = \((-1)^{\lvert k \rvert}\).

The relationship between the transformed projections and the transformed magnetic scalar potential is:

\[
\tilde{F}_\alpha(k) = -i k \tilde{\psi}(k, \alpha) \tag{3.38}
\]

The next operation is to construct the distribution in the Fourier space from the Fourier transformed projections. The distribution is sampled at regular points in polar coordinates, see fig(3.13). If a matrix consists of the values of the distribution at regular points in cartesian
space obtained from the samples taken at regular polar points, then the matrix will in general be sparse.

Fig 3.13: The polar sample points in Fourier space obtained by transforming the measured projections.

The magnetic scalar potential may be evaluated directly from the polar samples by inverting the Fourier space distribution using a polar coordinate version of the inverse Fourier transform. Alternatively the polar sampled distribution in Fourier space can be interpolated prior to inverse transforming to fill a matrix which represents the distribution at regular points in cartesian space. The distribution in real space can then be obtained by using a cartesian inverse transform. This has the drawbacks that any interpolation errors which exist will be accentuated by the inverse transform. The advantage of using the cartesian transformation is the FFT algorithm [Cochran, et al 1967, Cooley 1965] which is not available when using the polar form. Having opted for the cartesian inverse transform, the choice of dividing the distribution in the Fourier space by the wavenumber before or after interpolation must be

--- 3.24 ---
considered. Before interpolation, the projections and the wavenumber are both sampled at regular intervals in polar coordinates. The purpose of interpolating is to provide samples at regular cartesian intervals. To divide the interpolated distribution by \( K \) would require the wavenumber to be transformed. Hence it is computationally frugal to divide the distribution by the wavenumber before interpolating.

The interpolation to obtain the function in cartesian rather than polar coordinates is implemented using a linear nearest neighbour algorithm. Each point on the cartesian grid is transformed to its polar form. The nearest projection either side of the selected point is then calculated, along with the corresponding nearest radial sample point, see fig (3.14).

![Diagram of polar and cartesian points](image)

Fig 3.14: Example of the polar and cartesian points used in the linear nearest neighbour interpolation algorithm.

The expressions used in the interpolation are:

\[
\begin{align*}
f(r', \theta_1) &= \frac{[f(r_2, \theta_1) - f(r_1, \theta_1)](r' - r_1) + f(r_1, \theta_1)}{r_2 - r_1} \quad (3.39a) \\
f(r', \theta_2) &= \frac{[f(r_2, \theta_2) - f(r_1, \theta_2)](r' - r_1) + f(r_1, \theta_2)}{r_2 - r_1} \quad (3.39b) \\
f(r', \theta') &= \frac{[f(r', \theta_2) - f(r', \theta_1)](\theta' - \theta_1) + f(r', \theta_2)}{\theta_2 - \theta_1} \quad (3.39c)
\end{align*}
\]
Alternative higher order interpolation algorithms are discussed by Stoer [1980], Stark [1982] and Sezan [1984]. The expression used to compute the inverse 2D FFT is:

$$\psi(r) = \sum_{v=-N/2}^{N/2-1} \sum_{u=-N/2}^{N/2-1} F(u,v) e^{i2\pi(ux+vy)/N}$$

At this stage the data is in natural ordered Fourier form. The order required by an inverse 2-dimensional Fast Fourier transform, can be determined by examining the order of the data due to a forward 2-D FFT operating on naturally ordered source data, see fig (3.15). To re-order $F(K, \alpha)$, it is necessary to apply two sets of rotations, the first rotates all of the rows by $N/2$. The second operation acts on the results of the first operation by rotating the columns by $N/2$.

$$R'(x,y) = f(x, y+N/2) \quad -N/2 \leq y \leq N/2 -1$$

$$R(x,y) = R'(x+N/2, y) \quad -N/2 \leq x \leq N/2 -1$$

Fig 3.15: The ordering of a 2-D array before (left) and after (right) applying a 2D FFT.
The resulting data is ordered as \( R = 0 \) to \( N-1 \). The discrete transform is periodic every \( N \) data points. The desired range of \(-N/2\) to \( N/2-1\) may be recreated by applying two rotations. These rotations are the same as in expressions 3.41 and 3.42.

The result of applying this algorithm to the projection data is the naturally ordered magnetic scalar potential. The magnetic induction, \( B \), may be generated by taking the gradient of the reconstructed scalar potential. It is more convenient to display the reconstruction in terms of the scalar potential as it consists of a single 2-dimensional function, whereas the field requires the display of a 2-dimensional distribution for each of the 3 vector components.

3.5 Summary.

In this chapter the problem of measuring the magnetic field in a plane above an arbitrary oscillating magnetic source, by using a non-localised measurement system has been considered. This is made possible by expressing the magnetic field in terms of a scalar potential. Having reviewed existing scalar reconstruction techniques, the "Modified Central Projection Theorem" was then formulated. The chapter concluded by formulating a practical implementation of the algorithm. In the next chapter, the algorithm will be verified.
Chapter 4

Verification of Magnetic Induction Tomography.

4.1 Introduction

In the last chapter an expression was derived relating the Fourier transform of the magnetic scalar potential in a plane above an arbitrary oscillating magnetic source, to the Fourier transform of the measured projections of the magnetic field. An implementation of the algorithm was presented, which overcame the problems associated with the phase shifts due to the sampling range, and the ordering of the data for the 2-D FFT. In this chapter the algorithm is verified for the cases of the magnetic monopole and dipole using analytically generated data. This is followed by a presentation of reconstructed potentials using experimentally measured projections. The chapter concludes with a summary of magnetic induction tomography.

Michel in 1750 was the first to consider magnetic poles. He found that an inverse square law of force between the poles existed [Hammond 1978], ie the magneto-motive force is given by

\[ F = \frac{cQ_a Q_b}{R^2} \]  \hspace{1cm} (4.1)

where \( Q_a \) and \( Q_b \) are the magnetic pole strengths, \( c \) is a constant which depends on the units of the system and \( R \) is the distance between the
poles. In electrostatics it is convenient to use fields and potentials. The analogy can be applied to the magnetic case, ie

\[ H = \frac{cQa}{R} \]  

(4.2)

where \( H \) is the magnetic field strength due to a magnetic monopole. Since electrostatic fields are conservative, they can be described as a negative gradient of a scalar potential. This also applies to magnetic fields in current-free regions, ie

\[ \psi = -\frac{cQa}{R} \]  

(4.3)

where \( \psi \) is the magnetic scalar potential, which has units of amps.

The following analysis starts from the definition of the unit monopole scalar potential as given above. This is used to derive the magnetic dipole potential, and the analytic monopole and dipole projection expressions. The reconstruction algorithm is tested using noise free data generated from these expressions. Reconstruction data that is not measured, but is generated, is referred to as simulated data. The selected examples highlight the effects of the sampling size and range. These are followed by examples of reconstructions using measured projections. The source field is generated by small current loops that approximate to oscillating magnetic dipoles.

--- 4.2 ---
4.2.1 Derivation of the Projection Expressions for the Magnetic Monopole.

Consider a magnetic monopole situated at the origin at a distance 'a' below a plane, see fig (4.1). The plane will be the surface at which the magnetic scalar potential is to be reconstructed. The scalar potential for the unit monopole is given by

\[ \psi_m = \frac{cQa}{(r^2 + a^2)^{1/2}} \quad (4.4) \]

where \( r \) is the radial distance of any point on the surface from the point on the plane immediately above the source.

The Magnetic Induction \( B \), is defined as the negative gradient of the magnetic scalar potential, which in polar coordinates is given by

\[ B_r = -\frac{\mu \partial \psi_m}{\partial r} \quad B_\phi = -\frac{\mu \partial \psi_m}{r \partial \phi} \quad (4.5) \]

For convenience a system of units is chosen such that
\[ c = \frac{1}{Qa\mu} \] (4.6)

Hence the induction becomes

\[ B_r = \frac{r}{(r^2 + a^2)^{3/2}} \quad B_\varphi = 0 \] (4.7)

Using the definition of the projection from (3.20)

\[ f_\alpha(t) = \int_0^{2\pi} \int_0^\infty \left[ B_r \cos(\varphi - \alpha) - B_\varphi \sin(\varphi - \alpha) \right] \delta(r \cos(\varphi - \alpha) - t) r \, dr \, d\varphi \] (4.8)

and substituting for \( B_r \) and \( B_\varphi \) from (4.7)

\[ f_\alpha(t) = \int_0^{2\pi} \int_0^\infty \frac{r \cos(\varphi - \alpha) \delta(r \cos(\varphi - \alpha) - t) r \, dr \, d\varphi}{(r^2 + a^2)^{3/2}} \] (4.9)

Substituting (See fig 4.2) \( x = r \cos(\varphi - \alpha), y = r \sin(\varphi - \alpha) \)

and \( r = (x^2 + y^2)^{1/2} \)

then changing the variables of integration gives

\[ f_\alpha(t) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{x \delta(x - t)}{(x^2 + a^2 + y^2)^{3/2}} \, dx \, dy \] (4.10)

Integrating with respect to \( x \) gives

\[ f_\alpha(t) = \int_{-\infty}^{\infty} \frac{t}{(t^2 + a^2 + y^2)^{3/2}} \, dy \] (4.11)
This may be evaluated by substituting

\[ b^2 = a^2 + t^2, \quad y = b \tan \phi \]

\[
\begin{align*}
    f(\alpha, t) &= \frac{\pi/2}{b^2(1+\tan^2 \phi)^{3/2}} \int_{-\pi/2}^{\pi/2} \frac{t}{b^2} \frac{1}{\sec^2 \phi} d\phi \\
    &= \frac{\pi/2}{b^2} \int_{-\pi/2}^{\pi/2} \frac{1}{\sec^2 \phi} d\phi 
\end{align*}
\]  \hspace{1cm} (4.12)

The projection for a single unit monopole positioned at a distance 'a' above the origin of the reconstruction plane is therefore

\[
f(\alpha, t) = \frac{2t}{(a^2 + t^2)} \]  \hspace{1cm} (4.13)

4.2.2 Derivation of the Projection Expression for the Magnetic Dipole.

The magnetic dipole projection 4.14, can be derived from the sum of the monopole projections due to 2 magnetic monopoles of opposite polarity separated by a distance 2Xo in the limit as Xo tends to zero, fig 4.3.

\[
f_d(t, \alpha) = \lim_{Xo \to 0} \left( f_m^+ - f_m^- \right) \frac{1}{2Xo} 
\]  \hspace{1cm} (4.14)

where \( f_m^+ \) and \( f_m^- \) are the positive and negative monopole projections.
respectively.

Using the result for the single monopole projection 4.13, then

\[
\begin{align*}
\lim_{x \to a} f(x) &= \lim_{x \to a} f'(x) \\
&= \lim_{x \to a} g(x) = \lim_{x \to a} g'(x) \\
\end{align*}
\]

Hence by differentiating

\[
\begin{align*}
\frac{df^+}{dx} &= \frac{2(t^2 - a^2) \cos \alpha}{(a^2 + t^2)^2} \\
\frac{df^-}{dx} &= \frac{2(a^2 - t^2) \cos \alpha}{(a^2 + t^2)^2}
\end{align*}
\]
Thus the projection for a dipole situated at a distance \(a\) below the surface is

\[
f(t) = 2\cos\alpha \frac{(t^2-a^2)}{(a^2+t^2)^2}
\]  

(4.20)

4.2.3 Derivation of the Fourier Transform of the Magnetic Monopole and Dipole.

To account for the effects that occur in the reconstructed potentials, it is useful to compare the computed and analytic Fourier transforms of the monopole and dipole projections. The analytic Fourier transforms are derived in this section, but the comparisons are left until the next.

The Fourier Transform of the Monopole projection (4.13) is expressed as

\[
F_{\alpha}(k) = \int_{-\infty}^{\infty} f_{\alpha}(t)e^{-ikt} dt
\]

(4.21)

substituting for the projection

\[
F_{\alpha}(k) = \int_{-\infty}^{\infty} \frac{2t}{(a^2+t^2)} e^{-ikt} dt = \int_{-\infty}^{\infty} \frac{2t}{(t+ia)(t-ia)} e^{-ikt} dt
\]

(4.22)

Evaluating by the Cauchy Residue Theorem (Jordans Lemma)

\[
F_{\alpha}(k) = -2\pi i \sum_{i \in \mathbb{P}} R_i = -2\pi i e^{-ka} \quad k > 0
\]

(4.23)

--- 4.7 ---
\[ F(k) = 2\pi i \sum_{i \in \text{up}} R_i \quad k < 0 \tag{4.24} \]

where 'lp' represents the lower half plane and 'up', the upper half. When \( k=0 \), the function is odd, hence the integral is zero. Consequently if we define the function \( \text{sgn}(k) = \) the sign of \( k \) when \( k \) is not zero, and zero when \( k=0 \), then the monopole Fourier transform is

\[ F_{\alpha}(k) = -\text{sgn}(k)2\pi i e^{-|k|a} \quad -\infty < k < \infty \tag{4.25} \]

To obtain the Fourier transform of the dipole projection, expression (4.20) must be substituted into expression (4.21), ie

\[ \tilde{F}(k) = \int_{-\infty}^{\infty} \frac{2\cos \omega e^{-ikt}}{(a^2 + t^2)} dt \tag{4.26a} \]

\[ -\frac{4a^2 \cos \omega e^{-ikt}}{(a^2 + t^2)^2} dt \tag{4.26b} \]

Both terms may be evaluated by Cauchy's Residue Theorem (Jordans Lemma). The contours are taken in the upper half plane (up) when \( k < 0 \) and the lower half plane (lp) when \( k > 0 \).

\[ \text{when } k > 0 \quad (4.26a) = -2\pi \int_{\text{lp}}^{\infty} R_n = 2\pi \cos \omega \frac{k}{a} \tag{4.27} \]

\[ \text{when } k < 0 \quad (4.26a) = 2\pi \int_{\text{up}}^{\infty} R_n = 2\pi \cos \omega \frac{k}{a} \tag{4.28} \]

hence (4.26a) \[ = 2\pi \cos \omega \frac{|k|}{a} \tag{4.28} \]

Evaluating term 4.26b

--- 4.8 ---
Expanding the integral as a Laurent series about $t=-ia$ to find the residues.

Let $\eta = (t+ai)\eta$

Cauchy's residue theorem in this case is

$$\tag{4.26b} \left( -2\pi i \sum_{n \in \text{lp}} R_n \right) (k>0)$$

substituting for $\eta$

$$\tag{4.26b} = -4a^2 \cos \alpha \int_{-\infty}^{\infty} \frac{e^{-ikt}}{(t+ai)^2(t-ai)^2} dt$$

The exponential may be expanded as a Taylor series ie

$$-ik(\eta-ia) = e^{-ika} (1-ik\eta)$$

to 2 terms

And the binomial expansion of

$$\frac{1}{(\eta-2ia)^2}$$

is $-1 \left(1 + \frac{\eta}{4a^2} \right)$

to 2 terms

The residues of the integral are given by the coefficients of $\eta$ which are raised to the power of 1, ie

$$R = e^{-ka(-ik+1)2\pi \cos \alpha}$$

$k>0$

$$\tag{4.26b} = 2\pi i \sum_{n \in \text{lp}} R_n$$

--- 4.9 ---
The above analysis is repeated to obtain the residues for the integral when \( K < 0 \). This time the Laurent expansion is about the point \( t = ia \).

Hence (4.26b) \[ (2\pi k - 2\pi) e^{\frac{ka}{a}} \] (4.36)

since \( |k| = k, k > 0 \) and \( |k| = -k, k < 0 \)

(4.26b) \[ = (-2\pi |k| - 2\pi) \cos \alpha e^{-|k|a} \] for all \( k \). (4.37)

The Fourier transform of the dipole projection is the sum of the two terms 4.26a and 4.26b (or 4.28 + 4.37), which is

\[ F_\phi(k) = -2\pi |k| \cos \alpha e^{-|k|a} \] (4.38)

These results will be used in the discussion of the simulated reconstructions in the next section.

4.2.5 Discussion of Simulated Projection Reconstruction Results.

Projection data was generated using the monopole (4.13) and dipole (4.20) expressions derived in the previous sections. This data was used to reconstruct an image of the magnetic scalar potential. The theoretical monopole potential is given by expression (4.4). The theoretical dipole potential can be obtained as the superposition of the two potentials due to two monopoles separated by a distance \( 2X_0 \) taken in the limit as \( X_0 \) tends to zero, ie
Direct comparisons of the theoretical and reconstructed potentials are presented and discussed in this section.

32 projections were used in all of the reconstructions using simulated data. The number is fairly arbitrary for an axially symmetric potential such as the monopole, but can be important for more complex distributions. This will be discussed again in the section on the reconstructions of real data, where there is a trade-off between data gathering time, and image quality. Both the monopole and dipole projection expressions are functions of the distance between the point on the plane of reconstruction and the source. The number of samples in the projection is 64. To fill a 32 by 32 point distribution in cartesian Fourier space using polar sampled data, it is necessary to measure \( \sqrt{2} \) times the number of samples in the X or Y direction. This is done to obtain values for all points along the 45 degree line, see fig 4.4. To ensure the number of samples is still compatible with the radix 2 requirements of the FFT, 64 points were generated in each projection.

![Fig 4.4: Polar sampling range for nxn cartesian array.](image)
The library routines used to plot the potential, auto-scale in the Z direction. Consequently the user has no control over the Z range. Careful note should therefore be made of the Z axis scaling factor as there is invariably a different power of 10 in the scales between the reconstructed and theoretical potentials. The X and Y axis represent the lateral position in the plane of reconstruction. The position of the source in the plots are shifted from the origin, to half the full scale values in both the X and Y directions.

The simulated unit height monopole projection is shown in fig 4.5a. All the projections in this case are the same, since the monopole projection is independent of angle. Fig 4.6 and 4.7 are the theoretical and reconstructed potentials in a plane at unit height above the monopole source.

The theoretical and reconstructed monopole potentials for the 10 units height are shown in figs 4.8 and 4.9. The corresponding projection is shown in fig 4.5b. Clearly these are not the same. A high frequency ripple can be seen on the Fourier transform of the projections, see fig 4.10 and 4.11. The discrete transform is considered to be periodic. Hence as the signal does not decay to zero at the edges, and has a discontinuity at the periodic boundary, the transformed signal oscillates. This may be reduced by forcing the signal to decay at the boundaries, eg by using an alternative window in the sampling space such as a Hanning window [Gold, 1969].

--- 4.12 ---
Fig 4.5: Examples of magnetic monopole and dipole projections.
Fig 4.6: Contour and surface plots of the theoretical magnetic scalar potential in a plane at unit height above a unit magnetic monopole.
Fig 4.7: Contour and surface plots of the reconstructed magnetic scalar potential in a plane at unit height above a unit magnetic monopole using simulated projection data.
Fig 4.8: Contour and surface plots of the theoretical magnetic scalar potential in a plane at 10 units height above a unit magnetic monopole.
Fig 4.9: Contour and surface plots of the reconstructed magnetic scalar potential in a plane at 10 units height above a unit magnetic monopole using simulated projection data.
Fig 4.10: The Fourier transform of the unit height monopole projection. The dashed line is the theoretical transform, the solid line is the computed version.
Fig 4.11: The Fourier transform of the 10 unit height monopole projection. The dashed line is the theoretical transform, the solid line is the computed version.
Fig 4.12: Contour and Surface plots of the theoretical magnetic scalar potential in a plane at unit height above a unit magnetic dipole source.
Fig 4.13: Contour and surface plots of the reconstructed magnetic scalar potential in a plane at unit height above a unit magnetic dipole source using simulated data.
Fig 4.14: Contour and Surface plots of the theoretical magnetic scalar potential in a plane at 4 units height above a unit magnetic dipole source.
Fig 4.15: Contour and surface plots of the reconstructed magnetic scalar potential in a plane at 4 units height above a unit magnetic dipole source using simulated data.
Fig 4.16: Contour and Surface plots of the theoretical magnetic scalar potential in a plane at 10 units height above a unit magnetic dipole source.
Fig 4.17: Contour and surface plots of the reconstructed magnetic scalar potential in a plane at 10 units height above a unit magnetic dipole source using simulated data.
Fig 4.18: Fourier transform of the unit height magnetic dipole projection. The dashed line is the theoretical transform, the solid line is the computed version.
Fig 4.19: Fourier transform of the 10 units height magnetic dipole projection. The dashed line is the theoretical transform, the solid line is the computed version.
Three theoretical and reconstructed magnetic potentials are shown in figs 4.12 to 4.17, for a dipole at 1, 4, and 10 units below the surface of the reconstruction plane. Fig 4.13 shows a ripple in the reconstruction. This is caused by under sampling in the projection. The effects are caused by aliasing of the high frequency components of the signal, see fig 4.18.

The reconstruction algorithm cannot cope with structure at the edge of reconstruction. In the formulation of the problem, a constraint was imposed that the potential must decay to zero at the boundary of reconstruction. In the 10 Units height example (fig 4.16), the scalar potential is not zero at the boundary. The algorithm does not reconstruct the true potential at these points (fig 4.17). The transformed projection of this example is shown in fig 4.19. The signal has a non-zero DC frequency term, which is not shown in the reconstruction. This highlights the fact that there is a pole in the reconstruction algorithm at \( K=0 \). Since this term represents the DC potential, it is assigned to be zero always. The potentials for the 4 units height is included as an intermediary example between the two extremes of the 1 and 10 units height.

In this section, the reconstruction algorithm has been verified by way of analytic test cases. These have highlighted the importance of the sampling step size and range. In the next section, reconstructions of measured data are presented and discussed with the experimental techniques that were adopted.
4.3 Reconstruction Using Measured Projections.

4.3.1 Introduction.

In the last section, expressions for the monopole and dipole projections were derived. These were used to verify the magnetic induction tomography reconstruction programs. An advantage of using simulated data is that the projections are free from noise. As previously discussed, noisy projections can prevent convergence of the image. This problem is associated more with algebraic than analytic reconstruction techniques. Another advantage of simulated data is that the scalar potential is completely defined. Hence direct comparisons between the theoretical and the reconstructed potentials are possible. The previous section highlighted the importance of the step size and range over which the samples should be taken. Bearing these in mind, experimental projections have been measured and their corresponding scalar potentials reconstructed. A presentation of the experimental techniques adopted in measuring the projections is described in the next section. This is followed by a discussion of the reconstructed potentials.

4.3.2 Experimental Techniques.

The purpose of the following experiments was to investigate the feasibility of implementing a practical tomographic magnetic field measuring system. The transducer was conceived as a parallel array of rectangular coils. To automatically measure the induced emf in each coil,
a digitally-controlled analogue multiplexer is required, which connects each coil in turn to the analogue sub-system. The transducer was simulated using a single rectangular coil which was scanned and measurements taken at appropriate points. These points correspond to the equivalent positions of the coils on the rectangular array. The array transducer was simulated to save the design and constructional time required to implement the full version. It also allowed greater positional flexibility, for example the step size could be varied, which would otherwise require the construction of a new transducer.

The rectangular detector former was made of perspex. Its shape and dimensions are shown in fig 4.20. The coil was wound using 100 turns of 45 SWG enamled wire.

![Fig 4.20: Rectangular Detector Coil.](image)

The scanning rig (chapter 6) is incapable of sampling at regular polar coordinates since it was designed to move along cartesian axes only. Polar sampling can be simulated if the source field is mounted on a turntable. If the detector traverses the source in a single direction, with the coil perpendicular to the direction of travel, and the table is rotated after each projection, then sampling of the field at regular polar coordinates is accomplished. This method or rotating the object is
commonly used in gamma ray imaging.

A source field is required that can be mounted on a turntable and rotated. It should also have a simple distribution for comparison purposes. A small current carrying coil produces a magnetic field which is similar to a dipole. This is an obvious phantom, since the measured reconstructions can be compared with the distributions in the previous section. A phantom is a term used in medical imaging for a known test specimen. The source coil consisted of 100 turns of 45 SWG enamelled copper wire wound on a 2mm diameter perspex rod. The current through the coil was 2.7 mA (error = 0.01 mA). The signal generator was buffered from the coil by a type 4741 op-amp. A resistor was placed in series with the coil to act as a current limiter. The frequency of the signal was nominally set at 10KHz.

The turntable consists of a free-standing perspex base with a rotatable perspex table on top, see fig 4.21. Since the experiment was a simulation of the array transducer, the table was rotated by hand. The angle of the table angle was measured using a protractor, which was attached to the underside of the table top. Two marks were made on the top and bottom surface of the base, which were used to reduce the parallax errors during alignment. If further work is to continue using the single coil to simulate the array of coils, it is suggested that the turntable rotation is automated, since this is the main source of measurement error. The projection angle can only be specified to about half a degree. The operator must remain close to the system, as the table must be rotated after each projection at approximately 2-3 minute intervals. A typical
experiment requires 36 projections. Care should be taken to prevent movement of the free standing turntable. Another improvement to the system therefore is to fix the turntable base plate to one of the side walls of the scanning rig.

![Diagram of Perspex Table Top, Front Elevation, Protractor, Perspex Base, Plan View.](image)

Fig 4.21: Perspex Turntable Used in Tomographic Measurement.

Using the small coil source, as described above, the range and step size used during the measurements of the projections were determined empirically. 64 samples at 2 mm steps were measured, with the source positioned in the middle of the projection. The width of the detector coil was chosen such that a wider coil would produce a negligible increase in the induced emf. To verify that the width of the coil was satisfactory, the coil was scanned either side of its normal path, and the induced emf observed. Fig 4.22 shows the normal path (A) of the detector. Paths B and C are adjacent to A. If the induced emf in the coil is negligible, then the width of the coil is adequate.
Fig 4.22: Scanning Paths of the Tomographic Detector Coil.

4.3.3 Discussion of Results.

The first experimental investigation was the effect on the quality of the reconstructed potential when the number of projections is varied. This information is important as the data-gathering time can be substantially reduced if the minimum number of projections is measured. The source coil, as described in the previous section, is positioned relative to the detector coil as shown in fig 4.23.

Fig 4.23: Source coil position for tomography measurements.

Figs 4.27 to 4.29 are the reconstructed potentials using 9, 18 and 36 equally spaced projections. Note that the angle of the source in the reconstruction is not squarely positioned in the reconstructed region, and that there is an artifact at the boundary. The data for 9 simulated projections was generated and the potential was reconstructed to try to recreate the artifact, but this was unsuccessful. A comparison of the
measured and simulated projections revealed close similarity except at the null projection, see fig 4.25a.

![Diagram](image)

**Direction of Scan.**

**Detector Coil.**

**Source Coil.**

**Direction of Rotation.**

Fig 4.24: Source and Tomographic Detector Position for Null Projection.

The net flux through the detector for the null projection should be zero at all scan positions. This was so for the simulated projection, but not for the measured data. One reason for this is possible mis-alignment of the source coil with respect to the detector. This can be simulated by offsetting the projections by half the angular increment so that the null projection is never generated. The reconstructed potential is shown in fig 4.30. A slight artifact is observed in the contour plot. Considering that the potential is reconstructed from 9 projections to accentuate any problems, the effects due to small angular rotational misalignments is negligible. A second reason for the non-zero null projection is the finite extent of the coil. Further examination of the measured null projection shows variations in the expected signal. It is these perturbations that cause the reconstruction artifacts, as the null projection should be either constant or zero, since it is virtually impossible to wind a coil such that all the wires are parallel. This coupled with the finite size of the coil results in the theoretically null projection being non-null in the measured examples. The effect of reducing the number of projections is to extend the region over which the artifact
To verify that the reconstruction is independent of large rotations of the source, two separate experiments were performed. First the source was rotated by 90 degrees, see fig 4.25(a), with the corresponding potential shown in fig 4.31. The second case is with the source angled at 45 degrees, as in fig 4.25(b), with the reconstruction shown in fig 4.32.

![Fig 4.25: Rotated Source Positions for Tomographic Reconstructions.](image)

One advantage of formulating the reconstruction in terms of scalar potentials is the use of the principle of superposition. Consider the reconstruction of the potential caused by two sources. If the potential from source 'A' above, (see fig 4.26) is subtracted from the dual potential, then the resulting distribution is the potential caused by source 'B' alone. This was investigated using two similar small coils as sources. The two coils were driven from the same current source so that the signals were of the same phase. The scalar potentials for the dual coil source were reconstructed, followed by the potential with source 'B' removed. The potential for the coil configuration in fig 4.26(a) is shown in fig 4.33. Fig 4.34 is the reconstructed potential corresponding to the source coil configuration shown in fig 4.26(b). The reconstructed
potential which has coil 'A' as it's source is shown in fig 4.35. This is subtracted from the potentials in fig 4.33 and 4.34. Hence the potential from source 'B' alone at the position shown in fig 4.26(a) is shown in fig 4.36, and the corresponding potential for source 'B' positioned as in fig 4.26(b) is shown in fig 4.37.

Direction of scan.
Detector
Coil.
Source 'B'.
(a).

Direction of Scan.
Detector
Coil.
Source 'B'.
(b).
Source 'A'.

--- 4.36 ---

Fig 4.26: Tomographic Reconstruction Using Two Sources.

Since the reconstructions are of scalar potentials, it is possible to use the principle of superposition. This was demonstrated by reconstructing a dual coil source, then reconstructing with one of the sources removed. If the two potentials are subtracted, then the remaining distribution in theory should be solely caused by source that was removed. This is useful in calibrating a transducer, ie if two distributions are reconstructed of identical source distributions over a conductor, except one contains a defect, then the difference between the two distributions is due to the defect. The variation of the trough artifact in figs 3.33 and 3.35 makes the figs 3.36 and 3.37 more complicated. It should be possible in this simple cases to apply a priori knowledge to improve the image, eg the potential at the boundary should be zero.
Fig 4.27: Contour and surface plots of the reconstructed magnetic scalar potential using 9 measured projections above a magnetic dipole source.
Fig 4.28: Contour and surface plots of the reconstructed magnetic scalar potential using 18 measured projections above a magnetic dipole source.
Fig 4.29: Contour and surface plots of the reconstructed magnetic scalar potential using 36 measured projections above a magnetic dipole source.
Fig 4.30: Contour and surface plots of the reconstructed potential using simulated projections of a dipole source with the projection angles offset by half an angular increment.
Fig 4.31: The reconstructed scalar potential from measured projections of a dipole source that has been rotated by 90 degrees relative to the scanning transducer direction.
Fig 4.32: The reconstructed magnetic scalar potential from measured projections that has been rotated by 45 degrees relative to the direction of the scanning transducer.
Fig 4.33: The reconstructed scalar potential of two dipole sources positioned as in fig 4.26a.
Fig 4.34: The reconstructed scalar potential of two dipole sources positioned as in fig 4.26b.
Fig 4.35: The reconstructed scalar potential of dipole source A, as shown in fig 4.26a and 4.26b.
Fig 4.36: The reconstructed scalar potential of the dipole source B, as shown in fig 4.26a.
Fig 4.37: The reconstructed scalar potential of the dipole source B, as shown in fig 4.26b.
4.4 Conclusions to Magnetic Induction Tomography.

Magnetic induction tomography is a mathematical technique which is used to reconstruct an image of the magnetic scalar potential from projections. The problem of reconstructing the potential arose as a direct consequence of the primary objective which was to minimise the mechanical movement of the transducer required to completely measure the magnetic field over a plane. The reconstruction is only valid if the field decays to zero at the border of the reconstructed plane.

The reconstructions that are presented are of potentials due to small sources. Even with these simple cases care must be taken to select the correct sampling range and step size. The transducer used to gather the experimental data is a single rectangular coil. The original concept was a transducer consisting of an array of rectangular coils. A projection is measured by electrically selecting each coil, then sampling the induced voltage. The sequence of projections is obtained by repeating the measurements at regular angles.

In conclusion, a scanning method has been developed which is capable of measuring the distribution of a conservative magnetic field in a plane. The need to scan the transducer in the lateral directions has been made redundant and measurement is reduced to a single rotational movement. The theory of magnetic induction tomography is full developed, and it has been demonstrated experimentally.

--- 4.48 ---
Chapter 5

Polarised Coils.

5.1 Introduction.

Consider an eddy-current system which measures the impedance change of a single current-driven coil. The voltage drop across that coil is proportional to its impedance. The drive signal is likely to be several orders of magnitude larger than the voltage change when the coil is moved from a cracked to an uncracked conducting specimen. An example of a current driven system is the Eddiscan [Harrison, 1985]. The size of the defect signal can be as small as microvolts superimposed on a drive signal of volts. High amplification of the defect signal is necessary before it can be digitised with reasonable resolution. This is only possible if the drive voltage is first removed, since otherwise the source signal is likely to drive the amplifier into saturation as the voltage rails are reached. Methods of removing these signals include using the detector coil as one element of a bridge circuit [Libby, 1971]. An alternative method was sought that was inherently null, in that the voltage in the transducer should be zero except in the presence of a crack.

There is no induced voltage in a flat coil placed perpendicularly to an infinitely long, straight, current source as shown in fig 5.1. If the conductor lies along the X axis, then A coil must be placed in the plane, X=constant. This would still apply if a conducting half-space is
introduced underneath the source. Suppose that a crack is present in the conductor, then the magnetic field above the plate will be distorted as a result of the eddy-current perturbation due to the crack. Under the right circumstances, a net flux will pass through the detector coil, which will result in an induced voltage.

Fig 5.1: Infinite Straight Conducting Source and a Perpendicular Detector.

Fig 5.2: Perspective View of a Single Polarised Coil Pair.

An application of this idea is based on two rectangular coils placed symmetrically at right angles to one another. This transducer is referred to as a single polarised coil pair, see fig 5.2. An alternating current is driven through one of the coils, which is the primary source, the other operates as a detector. Simple analysis based on symmetry shows that the electric field due to the source coil is parallel to the detecting coil in the region of the detector. Kriezis and Xypteras [1979] have shown that the current sources which are normal to an adjacent conducting half-space,
induce no eddy currents in the conductor. Hence in fig 5.2, the current elements ab and cd do not induce eddy currents. The parallel element bd, which is nearest to the conducting surface is the main source of eddy currents. The contributions from the second parallel element depends on the height of the source coil. The direction of the eddy currents is parallel to the source in the immediate vicinity of the source.

3-dimensional analysis of the eddy current paths in the vicinity of cracks is extremely difficult, consequently a qualitative model is presented. In section 2.3 it was noted that the normal component of the electric field in the conductor at a conducting/non-conducting interface is zero. Consider therefore a crack in a conductor in the presence of an electric field. Charge must build up on the surface until the net normal component of $E$ is zero. In section 2.2 it was shown that the net build up of charge is zero except at boundaries, and that the defect can be considered as a distribution of electric dipoles. The magnetic field above the surface can be thought of as three elements, the field due to the source current, the field due to the eddy currents in an uncracked conductor, and the scattered field due to the electric dipole distribution on the crack surface. The corresponding electric fields are denoted by their sources, ie $E_0$ denotes the primary free space field, $E_c$ denotes the field due to the eddy currents in the crack-free conductor, and $E_d$ is the field due to the dipole distribution. The polarised coil is only sensitive to the component of the fields due to the dipole distribution. This is considered in isolation, although it cannot physically exist without the other two.

--- 5.3 ---
The polarised coil pair is not capable of detecting all types of defects inside a conductor. The limitations of the transducer are presented by way of examples. Consider a straight crack situated normally to $\mathbf{E}_c$, see fig 5.3a. The net flux cutting the detector coil is proportional to the current which flows perpendicularly to $\mathbf{E}_c$. The line $ab$ in fig 5.3a represents a particular position for the detecting coil above the defect. As can be seen, the component of the current due to $\mathbf{E}_d$ which is parallel to $AB$, is symmetric about $C$. The net induced voltage in the parallel elements of the detector is zero, and the voltages induced in the normal elements are equal and opposite. Hence, the net induced voltage is zero.

Consider a similar crack, which is parallel to $\mathbf{E}_c$, see fig 5.3b. In the case of a narrow crack, the size of dipole distribution is much less than the previous case, since the surface area of the crack presented to $\mathbf{E}_c$ is much smaller. The symmetry arguments that were used in the last example still apply. Hence the net voltage induced in the detector is again zero. The 3rd example is a crack that is orientated at approximately 45 degrees to $\mathbf{E}_c$, see fig 5.4. Clearly the component of the current

--- 5.4 ---
distribution which is parallel to AB is not symmetric about the point C. Consequently a net flux will exist which results in an induced voltage in the detector. The polarised coils can detect the presence of line defects if the crack is neither parallel or perpendicular to the detecting coil.

Fig 5.4: Plan view of crack at 45 degrees to Ec.

The analysis of polarised coil transducers in the presence of defects is too complex for existing theory to cope with. Consequently it is presented as an empirical technique. Even so, it has to date produced some promising results, eg the detection of flaws and edges in second layers. In the following sections, the experimentation and results are presented. The technique will be extended by considering an array of rectangular coils. The objective of using the array transducer is to infer 2-D sub-surface structure in a conductor without requiring translational or rotational movement.
5.2 Experimental Techniques.

The details of the transducers and experimental techniques for the single polarised coils are presented in this section. The methods include both 1-D and 2-D raster scan techniques.

Three polarized coils of different sizes were wound on perspex formers. Details of the size of the formers are presented in figs 5.5 to 5.7. The magnetic field produced by the source coil is proportional to the product of the number of turns in the coil and its current. It was decided to use a low current and large number of turns for two main reasons. Firstly it is easier to wind thin wire around the corners of the former. Secondly if the current is small the coil can be driven directly by the signal generator. Consequently the detectors were wound using the finest wire that could be handled without undue difficulty. Each coil consists of 100 turns of 45 SWG enamelled copper wire. This has a working current of approximately 5 mA. This current can easily be supplied directly from an operational amplifier, or the signal generator. A resistor is connected in series with the source coil to act as a current limiter. The size of the resistor was chosen such that any change in the coils impedance was negligible by comparison. The 3 coils shown in figs 5.5 to 5.7 are referred to as PC1, PC2 and PC3 (Polarised Coils) respectively.

The detector coil also consisted of 100 turns of 45 SWG enamelled wire. The detector should be wound with as many turns (N) as possible, since the induced emf is proportional to N. The current to be carried by
Fig 5.5: Polarised Coil (PC1) dimensions.

Fig 5.6: Polarised Coil (PC2) dimensions.

Fig 5.7: Polarised Coil (PC3) dimensions.
this detector is small, so that the only constraint on the wire thickness is its physical strength and the practicalities of winding the coil without damage.

The transducer was connected to the scanning gantry by a perspex rod, see fig 5.8. Details of the scanning rig are presented in chapter 6. A perspex collar enabled the height of the transducer above the conducting specimen to be adjusted manually. When making 2 dimensional scans, the measurements were taken in one direction of travel only. This reduces the positioning error due to backlash between the drive shaft thread and the bearings, though the method extends considerably the length of the scan time. If the spatial variation of the induced emfs is small in relation to the step size between samples, then measurements can be taken in both scan directions.

The induced voltage in the detector is amplified using a low-noise pre-amplifier, which is located as close to the transducer as possible. The output of the pre-amp is then fed into a lock-in amplifier before it is digitised. Details regarding the amplitude and phase characteristics of the pre-amp, the lock-in amplifier and the ADCs are presented in

--- 5.8 ---
The total amplification of the system required to detect small, sub-surface, and second-layer cracks is of the order of 4000. At these levels of gain, it is apparent that the free space detector signal is non-zero. This induced voltage is referred to as the residual signal. It arises as a consequence of the relative positional and winding asymmetries between the source and detector coils. In air, the residual signal is 90 degrees out of phase with respect to the source. The amplitude is also a linear function of frequency. The dependence of the residual signal on the height of the transducer above the specimen (lift off) is negligible. The lock-in amplifier (Ortholoc) has an offset voltage facility which allows the residual signal to be nulled. The maximum offset voltage is 10 times the full scale deflection voltage on the meter. As the sensitivity is increased, the maximum voltage that can be removed decreases. This can limit the detection of very small cracks. In practice the transducer is positioned in a region which is free from cracks and away from edges. The Ortholoc is adjusted until an approximate null reading occurs on the meters. The signal is sampled by the ADCs and the remaining residual signal is removed by software. The transducer is then positioned so that the crack is at the centre of a 2-D scan region.

5.3 Discussion of Single Polarised Coil Results.

The specimens used during the experiments are all made from aluminium plates, which are at least 400 mm square. The polarised coil is very insensitive to small symmetric defects such as circular holes. Some
specimens had slots which passed right through the plate. Surface-breaking cracks though were fairly easy to detect, consequently the results that are presented in this section are directed towards the harder task of detecting sub-surface, and second-layer defects.

The first experiment PCEXI uses a 5mm plate which has a 2.5 mm cut milled halfway through it, see fig 5.9(a). The orientation of the source and detector coils relative to the defect is shown in fig 5.9(b).

![Specimen Arrangement for Polarised Coil experiment PCEXI & 2.](image)

The PCl transducer (fig 5.5) was used during experiments PCEXI to PCEX5, with the source current set at 3mA. The source frequency in this case was 1 KHz. The scanned region was 30 mm square and samples were taken every mm. At each point, the in-phase and quadrature signals were measured. These are shown in figs 5.13 and 5.14 respectively. The amplitude and phase of the distribution are shown in figs 5.15 and 5.16 respectively. They can be evaluated using

\[
\text{amp} = \left( r^2 + q^2 \right)^{1/2} \tag{5.1}
\]

\[
\text{ph} = \tan^{-1} \left( q/r \right) \tag{5.2}
\]
where \( r \) is the in-phase component, \( q \) is the quadrature component, 'amp' is the amplitude and 'ph' is the phase. It can be seen in the phase image (fig 5.16.) where the amplitude of the signal is low, that the effects of noise makes the distribution meaningless. Hence without any processing, the phase image is extremely cluttered. One simple technique that improves the clarity of the image, is to apply a window which suppresses the low amplitude signals. It is defined such that if the amplitude of the signal is less than a threshold level, then the phase is set to \(-\pi/2\). Figs 5.17 and 5.18 are the phase images after processing with the corresponding thresholds set to 150 and 300 units respectively (1 unit = 0.6 nanovolts). The signal to noise ratio (S/N) was 151 where S/N is defined as

\[
S/N = \frac{PS}{RMSN}
\]  

and PS is the peak amplitude of the signal, and RMSN is the root-mean-square noise. The noise value is obtained by taking a suitable area of the image, for instance 10 points by 10 points, where the signal contains only noise, then calculating the mean amplitude in that area. The absolute variation of the signal about the mean is summed and the total is divided by the number of samples in the area. Hence RMSN is

\[
RMSN = \frac{1}{N} \sum_{i=1}^{N} |x_i - \bar{x}|
\]  

Experiment PCEX2 was identical to PCEXI except that the frequency was set to 4 KHz. The amplitude and phase plots are shown in figs 5.19 and 5.20. The peak signal is less than in PCEXI. This is not too surprising...
since the skin depth is less in this case. A frequency will eventually be reached where the skin depth is too small for any detectable dipole distribution to arise on a sub-surface crack. By comparing figs 5.18 and 5.20 it is clear that the phase change with frequency is measurable. The phase image in fig 5.20 has its threshold level set at 60 nano-volts and the S/N was 120.

The ability to detect second-layer defects is very important in multilayer structures. Fig 5.10a shows the slot in the top side of the 2nd layer plate (Experiment PCEX3). Fig 10b is the orientation of the slot with respect to the source and detector coils.

Fig 5.10: Second Layer slot in Aluminium Conducting Plate (PCEX3).

Fig 5.21 is the amplitude image of the scan. The frequency of the drive signal was 453Hz. Apart from the frequency, which was selected to penetrate the top plate, the experimental arrangements for PCEX3 was the same for PCEX1. In this case the S/N drops substantially to 20. An amplitude window, like the phase window, can aid the clarity of the image. This window is defined such that any signal of amplitude less than the selected threshold, is assigned to zero. The threshold in fig 5.22 is set to 42 nanovolts.
One application of the polarised coils is to detect the presence of second layer structures and edges. Fig 5.11 shows the arrangement of two aluminium plates. The purpose of the experiment is to try to locate the lower plate through the top one. The frequency of the source was again set to 453 Hz. 40 by 40 samples were measured at 2 mm step sizes. The amplitude plot of scan PCEX4 is shown in fig 5.23. The corresponding phase plot can be seen in fig 5.24. The phase image has been windowed with a threshold of 90 nanovolts and the S/N was 116.

![Diagram of experimental arrangement](image)

**Fig 5.11:** 2nd Layer Edge Detection for PCEX4.

![Diagram of experimental arrangement](image)

**Fig 5.12:** 2nd Layer Corner Detection for PCEX5.

Fig 5.12 shows the experimental arrangement of the specimens for locating a corner of a second layer plate (PCEX5). The amplitude plot is shown in fig 5.25. The setup for PCEX5 is the same as PCEX4. The S/N was
Fig 5.13 (PCEX1) Plot of the in-phase component of the induced voltage over a region containing a sub-surface slot.
Fig 5.14 (PCEX1) Plot of the quadrature component of the induced voltage over a region containing a sub-surface slot.
Fig 5.15 (PCEX1) Plot of the amplitude of the induced voltage over a region containing a sub-surface slot.
Fig 5.16 (PCEX1) Phase plot of the induced voltage over a region containing a sub-surface slot.
FIG 5.17 (PCEX1) Plot of the windowed phase distribution. Threshold level is 90 nano Volts.
FIG 5.18 (PCEX1) Plot of the windowed phase distribution. Threshold level is 180 nano Volts.
Fig 5.19 (PCEX2) Amplitude as for fig 5.15 except frequency is set at 4 KHz as opposed to 1 Khz during PCEX1.
Fig 5.20 (PCEX2) Plot of windowed phase distribution. Threshold level is 180 nano Volts and the frequency is 4 KHz.
Fig 5.21 (PCEX3) Amplitude plot of 2nd layer defect. The frequency is set at 453 Hz.
Fig 5.22 (PCEX3) Windowed amplitude plot of 2nd layer defect. The threshold is set at 42 nano Volts.
Fig 5.23 (PCEX4) Amplitude plot of 2nd layer plate edge.
Fig 5.24 (PCEX4) Windowed phase plot of 2nd layer plate edge. The threshold is set at 90 nano Volts.

--- 5.25 ---
Fig 5.25 (PCEX5) Amplitude plot of 2nd layer plate corner.

--- 5.26 ---
Fig 5.26 (PCEX6) Amplitude plot of sub-surface slot using small transducer, PC2. The top distribution is unwindowed, the lower one has a threshold voltage of 30 nano Volts.
Fig 5.27 (PGEC7) Amplitude plot of sub-surface slot using the large transducer, PC3.
calculated to be 76.

A small polarised-coil transducer was used in experiment PCEX6 to investigate the effects of reducing the size of the coils. The sub-surface slot experiment described in PCEX1 was repeated using transducer PC2 (fig 5.6.), at a source frequency of 453 Hz. The amplitude plot has a low S/N of only 20, see fig 5.26. The lower image is the amplitude plot which has been windowed at a threshold level of 30 nanovolts.

Transducer PC3 (see fig 5.7.) was much larger than the previous two. The reason for constructing it was to examine the possibilities of extending a single polarised coil transducer to an array of them. Hence a large coil was wound and the experimental arrangement for PCEX6 repeated. Fig 5.27 is the amplitude image of the scan. The step size in this case though was extended to 2mm between samples. Thus the effect of using a larger coil is to spread the region over which the slot is sensed by the detector. The S/N for PCEX7 was 61.

In this section the results using a single polarised coil transducer have been presented. They are based on the 2-D distribution of the induced voltage in the detector coil. The system has proven to be extremely sensitive to line defects. High signal-to-noise ratios were possible which enabled sub-surface and 2nd layer slots to be inspected. The next section describes the aims, problems and results of implementing an array of polarised coils.
5.4 Polarised Coil Array.

A logical extension to a single polarised coil pair, is an array of source and detector coils. One set operates as the field source, the other perpendicular set operates as the detector, see fig 5.28. A set of measurements of the voltages induced in each detector coil can be made as each source coil is selected. Only one source and detector coil pair is enabled at any instant. The objective of using the array transducer is to scan various positions by electrically selecting the appropriate coils rather than by mechanically moving a single pair. The prototype transducer consists of a 7 by 7 array, in which the coils are selected by manually operated switches.

--- 5.30 ---
The voltage induced in the coil array was measured in free space and found to be non-zero. The actual voltage distribution can be seen in fig 5.29. The distribution will now be shown to be caused by the asymmetries in the windings of the z elements of the 'source coil relative to the detector, as shown in fig 5.30. All coils except the central ones are necessarily asymmetric.

Fig 5.29: Free Space Induced emf in Polarised Coil Array.

Fig 5.30: Asymmetric source and detector coil pair.
The magnitude of the induced voltage is proportional to the flux linkage which cuts the detector coil. It is convenient to consider the source coil as 4 separate finite length, current-carrying wire elements. The two elements which lie in the \(xy\) plane have currents in opposite directions. The position and length of these elements are such that the integral of the flux that cuts the detector due to both is zero. The flux through the detector due to the elements in the \(z\) direction can be calculated by integrating the magnetic induction over the area of the detector coil, i.e., by integrating \(B(x,y,z)\) with respect to \(z\) and \(y\). Using Biot-Savart's law

\[
\mathbf{B} = \frac{\mu_0 I}{4\pi} \int \frac{\mathbf{dl} \times \mathbf{r}}{r^2} \tag{5.5}
\]

an expression for the magnetic induction at a point for a finite current carrying conducting element can be derived

\[
\mathbf{B} = \frac{\mu_0 I}{4\pi \rho} (\sin \alpha_2 + \sin \alpha_1) \mathbf{\hat{\phi}} \tag{5.6}
\]

where \(I\) is the current and \(\mathbf{\hat{\phi}}, \mathbf{\hat{z}}, \) and \(\mathbf{\hat{\rho}}\) are basis vectors in cylindrical polar coordinates. Fig 5.31 is a description of the coordinates used in the above expression.

Fig 5.31: Coordinate system at \(P(x,y,z)\) Due to a Small Current Element.
Expressing the two angles $\alpha_1$ and $\alpha_2$ in terms of $z$ and $\rho$, the value of $B$ at a point is

$$B = \frac{\mu_0 I}{4\pi \rho} \left( \frac{z^2}{(z^2 + \rho^2)^{3/2}} \right) \nu^2 + \frac{(L - z)}{[(L - z)^2 + \rho^2]^{1/2}} \right)^\nu$$

To calculate the flux $\Phi$, through an area in the plane $X$-constant, it is necessary to integrate the component of (5.7) which is perpendicular to the area, over the surface of that area. First integrating with respect to $z$ over the limits $z=0$ to $z=L$ gives

$$\Phi(x) = \frac{\mu_0 I}{2\pi \rho} \left( [L^2 + \rho^2]^{1/2} - \rho \right)$$

then integrating (5.8) the result, with respect to $y$, by substituting

$$\rho = (x^2 + y^2)^{1/2}$$

gives

$$\Phi(x) = \frac{\mu_0 I}{2\pi} \int \frac{Y^2}{(x^2 + y^2)^{1/2}} \left[ \frac{L^2 + (x^2 + y^2)}{(x^2 + y^2)^{1/2}} - 1 \right] dy$$

Expression (5.9) is the total flux that cuts the detector coil due to a single current carrying conductor in the $z$ direction. Expression (5.9) is an elliptic integral which in this case is evaluated using numerical techniques. The net flux through the detector is a function of the non-symmetric area of the detector about the source coil (see fig 5.30b) which in this case is area EC.

The area EC depends on the currently selected coil pair. Although there are 49 combinations, 13 positions are symmetric. There are only 6 unique positions which are shown in fig 5.32. At each source coil position, the
flux due to both z direction elements must be evaluated and summed.

Fig 5.32: Plan View of Unique Coil Positions of the Free Space emf's.

A plot of the measured and theoretical free space distributions, both of which have been normalised to the maximum sample point is shown in fig 5.33. To use the array transducer, the free space distribution must first be sampled. If the transducer is subsequently placed in a defect region, and the signals are again sampled, then as long as the frequency is kept constant, the free space contribution to further measurements can be removed. Two examples are presented in figs 5.34 and 5.35. The former is the amplitude distribution for a surface breaking slot which has the same length as the coil spacing. The second case has a surface beaking slot that extends beyond the length of the transducer. The free space distribution has been removed from both images.

--- 5.34 ---
The last experiment conducted was to measure the relative peak sensitivity of the various source detector combinations using the same specimen. The array transducer was positioned so that for each selected coil pair, a maximum voltage was measured. The distribution of voltages was then normalised relative to the central coil voltage. Fig 5.36 shows the percentage sensitivity of the array transducer.

This section has demonstrated a transducer that is capable of measuring information regarding the presence of cracks in conductors over two dimensions without the need to move the sensor. The resolution of this particular array was extremely coarse. Furthermore the digitally-controlled analogue multiplexer and the feedback electronics have not been
Fig 5.34: Short Surface Breaking slot Amplitude Distribution.

Fig 5.35: Long Surface Breaking slot Amplitude Distribution.
implemented. It was considered that repeating the experiments with a higher resolution array and constructing the associated control electronics must be classified as development. As such it was not pursued further during this project.

5.5 Summary.

The single polarised coil was conceived as an eddy current transducer which was insensitive to the magnetic field caused by the source current. Any non-zero voltage induced in the detection coil is caused by the presence of defects in the conductor or edge effects. The single polarised coil was used in a 2-D scanning process. The results demonstrate a high sensitivity to line defects and non-symmetries. Defects that are symmetric, such as small drill holes are easily missed. The induced emf in the detector in the absence of edge effects, is solely caused by the
perturbations of the eddy currents due to the asymmetry. Consequently the amplifying electronics was extremely straightforward, as it was unnecessary to suppress the induced voltage due to the source current. This is the main advantage of the polarised coil transducer.

The idea behind the rectangular coil pair was extended by considering an array of polarised coils. By electrically selecting each source in turn, and measuring the induced emf in each detector, a 2-D distribution of the coil field couplings can be obtained. When this transducer was implemented, it was found that as the symmetry of some of the coils had been broken, then the main property of the polarised coil, which was the null measurement in the absence of defects, had been lost. The cause of the free space emf distribution has been accounted for, and as such can be calibrated out. Future work for the polarised coil array consists primarily of implementing the analogue switching and the free space coupling balancing electronics.

The use of the single polarised coil pair has produced extremely good results. With further development, it should be possible to achieve similar results with the array transducer.
Chapter 6

Data Acquisition System

6.1 Introduction

During this project, a multipurpose data acquisition system was designed that fulfilled all of the experimental requirements. The complete system was built from scratch, based on the following specification. The transducer is systematically translated to the measurement positions, where the analogue signal in the transducer is sampled. This movement and measurement process should be fully automated. The signal in the transducer is an induced emf caused by a time harmonic magnetic field. The measurement system must be capable of converting the induced voltage, which can be as small as one microvolt, to a digital form with reasonable resolution so that any spatial variations of the signal can be observed. The system must also be capable of saving the digitised data for further processing and analysis.

This chapter presents the details of the approach adopted in amplifying, sampling and recording the signal in the transducer. Fig 6.1 is a global diagram of the complete system. It demonstrates the division of the system into distinct sub-systems. Section 6.2 describes the mechanical aspects of the scanner. This is followed by a presentation of the analogue and digital electronic systems. Finally a discussion of the control software and data processing is presented.
Fig 6.1: Block diagram of the data acquisition system.
6.2. Scanning Rig.

A procedure which is common to most of the experiments undertaken during this project, is to move the transducer and make spatially distributed measurements. These are normally made at regular intervals in one or two dimensions. The transducer is scanned over the surface of the specimen, and the signal is sampled. In designing an eddy-current scanner, certain essential precautions must be observed. The primary concern is the effect on the scattered field that may result from induced eddy currents in the conducting members of the scanner. An ideal system would contain no such conducting elements. This is not physically realistic so the conducting members should be kept to a minimum, and as far away from the region of interest as possible.

The transducers are very susceptible to edge effects. These will greatly increase the complexity of any analysis. Some problems can be solved theoretically when considered as either conducting half spaces, or very thin, conducting sheets of infinite size. In the laboratory, this is achieved by ensuring that the size of specimens are as large as is convenient. A realistic size is one where the effects due to the edges, are negligible in comparison with the effects in the region of interest.

There are four degrees of freedom with which the transducer can be positioned. They are the lateral positions, the height above the surface of the conductor and the rotational angle that the coil makes with the specimen.
Given the above criteria, a metallic scanning rig supported on a perspex frame was designed. The departmental mechanical workshop had constructed similar rigs for use in ultrasonic experiments. A photo of the complete rig is shown on page 6.5.

The pitch size of the gantry drive rods is 1mm per revolution. The stepping motor is capable of 200 steps per revolution. In theory this enables an incremental step size of 0.005 mm which in real terms is approximately 10 wavelengths of visible light. When distributed measurements are taken, the direction of movement is always the same. This ensures that the bearings are always driven on the same surface of the drive shaft to minimise the errors due to backlash. The move command issued by the computer is in units of 0.5 mm. A resolution of 1 mm was adequate for all the experiments undertaken during this project.

6.3 Analogue Sub-System

The movement of electric charge is the sole cause of all magnetic fields. If current is allowed to flow in the detecting coil it will act as a magnetic field source, perturbing the specimen eddy-current distribution. The primary task of the pre-amplifier is to provide a high impedance load for the detector, so that the detector current is minimised. The pre-amplifier is constructed from 2 low-noise bi-fet operational amplifiers, fig(6.3). The first stage is configured as a band-pass filter. Initially the detector coil was attached directly to the non-inverting input. On further investigation, a first order low-pass filter using the resistor and capacitor R1 and C1 could be included

--- 6.4 ---
without noticeably affecting the eddy current distribution. This is justified since the induced detector signal is of the order of microvolts. The size of the primary current is of the order of a few milli-amps. Hence the adverse contribution to the eddy currents resulting from the secondary current is approximately 7 orders of magnitude smaller. The gain of each of the op-amps is set to 20. The resistor and capacitor R5, and C2 act as a high-pass filter, to reduce the mains pickup. Resistors R6 and R9 are included to reduce ringing in the cable between the pre-amp and the ortholoc amplifier. The amplitude and phase response of the preamplifier is shown in fig 6.4 and 6.5.

![Fig 6.3: Low noise Pre-amplifier and filter.](image)

The signal is passed from the pre-amp to a lock-In amplifier (Ortec Brookdeal Ortholoc Model 9502). It contains both high and low pass filters, with manually adjustable signal amplification. Two analogue
Fig 6.4: Phase response of preamplifier.

Fig 6.5: Amplitude response of preamplifier.
outputs are available, for data-logging purposes. These outputs track the meter readings, giving a linear output voltage in the range of -10 to +10 volts. This piece of equipment is acceptable in a laboratory environment, but is unsuitable for operational use. A customised lock-in amplifier is essential for a field instrument.

The Ortholoc requires a reference signal which is taken from the drive signal of the source coil. All phase measurements are made relative to this reference. The Ortholoc can be used in two-phase or vector mode. The former provides two measurements, one represents the component of the signal which is in-phase with the reference, the other is the quadrature component (i.e., 90 degrees out of phase with the reference.). The output signals can be positive or negative. A negative signal represents a 180 degree phase shift in the respective component. In vector mode, the two readings represent the amplitude and phase of the signal. Initially this seemed to be more appropriate than the two phase mode. All of the manual measurements were taken in this manner. Unfortunately problems were encountered when the system was automated. The phase range in vector mode is -90 to +90 degrees. Since the signal often inverts, this range is inadequate, resulting in overloading of the amplifier. This can be compensated for by manually switching incremental phase shifts of 90 degrees into the signal. There is no such facility to do this electronically on this model. Consequently when the system is used as part of an automatic data-logging system the Ortholoc is restricted to its two phase mode. The high frequency noise can be reduced by adjusting the time constants in the filters. The rise time is approximately 2.2 times the selected time constant [Ortholoc 9502 users manual.]. This is set to 1

--- 6.8 ---
second for the cross coils, and 300 milliseconds for the tomography experiments. An appropriate time delay is required after the transducer is moved before the signal settles.

Any imperfections or asymmetry in the polarised coil windings will result in a coupling of the source and the detector coils. In the absence of defects and boundaries a perfect transducer should have a null measurement. In practice though, a residual signal will in general always exist. There is a facility on the Ortholoc to null such offsets. Unfortunately the maximum offset voltage is 10 times the 'Full Scale Deflection' voltage. This implies that as the signal sensitivity is increased, the maximum offset voltage decreases. A point will be reached where the residual signal cannot be nulled sufficiently to prevent it swamping the defect signal. An improvement to the system consists of a hardware null circuit as shown in block form in fig(6.6). This circuit is a proposed improvement, and has not yet been implemented.

Fig 6.6: Hardware null block diagram.
The analogue to digital converters (ADCs) are both 12-bit successive approximation devices, which are equivalent to the AD574J. They are configured in their 12-bit bipolar mode, with the input range set to +/- 10 volts [Radio Spares data sheet, 4383, Nov 81]. The conversion time is approximately 25 micro seconds. Each time a measurement is requested, several conversions take place and the average is returned. The number of conversions used in the averaging process is determined by the ADC control software. The usual precautions of separating the analogue and digital supplies, etc, have been observed.

6.4 Digital Sub-Systems.

VLSI technology has greatly simplified the implementation of scanning measurement techniques. For example, stepping motor controllers have been reduced from a design chore, to a simple constructional task by the use of dedicated integrated circuits. The aims of this project specifically included the application of such devices, which include 16-bit microprocessors, and their associated families of peripheral chips. Consequently the design and implementation of the computer system was of high priority.

The digital processing requirements can be split into three sections: control, measurement by reading the ADCs, and data processing. The main task of the control section is to position the transducer. The scanning system has 4 mechanical degrees of freedom, which govern the transducers spatial coordinates and its angle relative to the specimen. Measurements of the detector signal are made as a function of the transducers
position. The resolution of the system is governed by the step size between sampling points. A 2-D scan consists of samples measured on an \( nxn \) cartesian grid, where \( n \) is typically not less than 30. Two measurements are made at each position. These are the in-phase and quadrature components of the signal. Hence it is readily apparent that an automated system is required. The path of the transducer is chosen to suit each experiment. Therefore the scanner must be easy to re-sequence [Kyte 1984d]. The two most common sequences are 1-D, and 2-D raster scans.

The digital measurement section consists of reading the ADCs. The value of the digitised signal is stored on a floppy disk. The final computational task is the processing of the data. This will depend on the experiment, but the minimum requirement is some form of graphical output. The measured distributions are displayed either on a graphics terminal or plotter.

At the start of this project, it was decided to base the system on the Zilog Z8000 which represented a typical "state of the art" 16-bit microprocessor. Hence the system was originally designed around a single board computer (Z8000 Development Module, (DM) Copyright Zilog Inc, USA.). In these circumstances the hardware development consisted of designing and building the additional interfaces which the Z8000 lacked, such as the ADCs. The board was to have its own dedicated video subsystems, but due to time constraints, this was abandoned. Alternative graphics facilities were provided by a 6502 contained in an Acorn model 'B' microcomputer. Its main role was to act as an intelligent programmable terminal. The 6502 also provides a means by which additional peripherals

--- 6.11 ---
can be controlled such as printers and floppy disks, which are not necessarily required on a final instrument. Communications between the Z8000 and the 6502 is through a RS423/RS232C serial line, operating at 9600 baud. The 6502 issues a sequence of commands which are essentially "move the probe" and "sample the signal". When a command is transmitted, the 6502 must wait for a reply from the Z8000. This handshake is essential as it is the only method by which the two independent systems maintain their synchronisation. The handshake is either the value of the sampled signal, or in the case of a move instruction, a "command completed" message. Absolute position cannot be measured in the system. Hence the 6502 control program must maintain the relative spatial coordinates of the transducer by recording the path the transducer has previously taken.

The 6502 also provides the user with a real time display of the sampled signals by plotting graphs of the measured values using its high resolution graphics mode (640 by 256 pixels in monochrome.). Plots of the phase, amplitude, in-phase and quadrature components of the transducer signal can be displayed. The data is normally transferred directly to floppy disk at run time. Various options are available at the end of the scan such as screen dumps to the printer.

The 6502 is used as a preprocessor, filing the data in an ASCII text format. This is different to the coded binary form which is the default for Acorns "Basic" commands (eg inputf, outputf, etc). Most of the data analysis such as the tomographic reconstruction, is carried out on the central computing systems. The existing communications software does not to date support binary file transfers, only ASCII text. A further
advantage of storing the data in text format is the ease with which the
data can be locally inspected, i.e., using a standard text or word processor.
In text form the data is also compatible with Primes high-level language
filing system used by Fortran, C, and Pascal. The central system consists
of 9 Prime minicomputers linked together in a local area network. Apart
from the inherent power of the Primes, specialized hardcopy peripheral
facilities are available, such as the Calcomp plotter. There is an
additional software benefit when using high-level languages which is their
associated libraries (e.g., graphics-GINO, maths-NAG). All the surface plots
in this thesis were drawn using the Calcomp plotter which was controlled
by subroutines in the GINO graphics library (which can be called from any
of Primes high-level languages.). The tomographic reconstruction algorithm
was written in 'C' and used the FPS array processor (AP). There are
specific library routines for the AP which were called from 'C'. It was
considered necessary to use the AP considering the large number of FFTs
required for 2-D tomographic inversion.

The devices provided on the Z8000 development module consist of a
Z8002 microprocessor, two serial RS232 lines to support the transfer of
programs from the software development system, an interrupt timer and
four 8-bit parallel ports. The address space of the Z8002 is 64k bytes, of
which 16k is dedicated to ROM/EPROM. The only software resident on the
Z8000 is a monitor program, and the utilities to transfer programs and
data. These are stored as firmware in two 2k EPROMS. The available RAM is
sufficient to hold a small control program and its data segments. Fig[6.7]
shows the block diagram of the software development system, and the
development module.
Fig 6.7: Block diagram of the Z8000 and the PDS8000

The control programs were written using a PDS8000 Software Development System. The Z8000 is connected between the terminal and the PDS8000. Code is developed with the Z8000 in transparent mode, and then downloaded. Initially the development system consisted of a Z80 based disk system, with an accessible RAM of approximately 50k bytes. Program assembly was consequently disk based, from source file to relocatable object file, then eventually to a load file. The process of assembling code involves the use of a separate assembler, linker and loader program. These are very cumbersome and time consuming to use.
An upgrade board was installed, which contained a Z8001 processor with 256k of RAM. This enhancement was accompanied by new software which included 'Y', an advanced Zilog assembler. All of the control programs on the Z8000 were written in 'Y', [Kyte 1984b] which is very structured. It uses a combination of Pascal-like blocks such as: 'begin'; 'end'; 'case', 'if then else' and 'C' I/O instructions such as: 'printf'; 'getc', and 'putc'. The latter commands should be avoided if the software is to run on the DM because these routine are specific to the PDS8000.

Although the Z8000 provides several 8-bit parallel ports, it does not contain a 16-bit equivalent. It also lacks both ADCs and digital to analogue converters (DACs). To accommodate such extensions a Eurocard rack was connected to the main bus. Future expansion of the system has been made easier as a result. The rack consists of a back plane, which carries the full Z-Bus (Copyright Zilog Inc. USA) plus the power supplies. The two ADCs were to be mounted directly in the rack, but they were later separated due to the level of electromagnetic pickup caused by radiation from the digital switching. The Z-Bus has a multiplexed data and address bus. It also supports separate memory and I/O address segments. Hence one card in the rack is dedicated to demultiplexing the data/address bus, decoding the status and address lines and providing the chip selects for the expansion I/O devices [Kyte 1982]. Another board contains the 16 bit parallel input ports, which are polled, and the output ports, which are latched.

The primary task of the Z8000 is to control the position of the
transducer which has four degrees of freedom. They are the lateral positions \( X \) and \( Y \), the angle of rotation of the probe \( R \), and the height \( H \), of the probe above the surface of the specimen. The settings of \( X, Y \), and \( R \) are under microprocessor control. The height is at present only adjustable manually. The \( X \) and \( Y \) movement is controlled by driving two stepping motors, which are both attached to threaded shafts. The rotation movement is controlled by a stepping motor which is connected directly to the former of the transducer.

The pulse sequence to step the motors is generated by software. A 40 microsecond high going pulse is required, followed by a 1.6 ms delay. The pulses clock the stepping motor driver IC, (SA1027), which in turn enables the power transistors. There are 4 coils per motor, each is switched by a separate transistor. The stepping motor controller contains the high current power supplies, the digital and manual control interfaces, the power transistors, and the over-limit protection circuitry.

Details of the exact design of the interfaces, and their associated software drivers are very specific to this particular hardware configuration. Although constructing the data acquisition system consumed a considerable amount of time during this project, the innovative content (one must reluctantly admit) is not substantial enough to warrant inclusion within this thesis.
Chapter 7

7 Concluding Remarks.

The objective of this project was to develop methods using eddy-current imaging techniques that could be used to infer the presence of cracks within a conductor. To obtain the maximum information using any particular transducer, it is necessary to make spatially extensive measurements at the surface of the conducting specimen. Up until now this has implied mechanically scanning the transducer in 2-dimensions. To minimise the movement required to completely scan a region, a transducer was considered that consisted of a regular array of parallel rectangular coils. By electronically selecting the coils and rotating the transducer, samples of the magnetic field can be taken. To reconstruct the distribution from these samples, it is necessary to represent the field by a scalar potential. A reconstruction algorithm is developed based on tomographic reconstruction from projections, and is referred to as magnetic induction tomography (MIT). The theory is derived from first principles, and an implementation using simulated and measured data is presented. The problems associated with encompassing MIT into a complete eddy-current instrument have not yet been investigated. The formulation of the magnetic field in terms of scalar potentials has secondary advantages, which are primarily associated with displaying the results. The presentation of scalar 2-D distributions in the form of surface, contour, and grey scale plots is easy compared with the display of equivalent vector distributions.
An alternative inspection technique is based on the polarised coil pair. This transducer is scanned and the 2-D distribution of the induced voltage in the detector coil is measured. This method ideally produces a null signal in the absence of asymmetries such as defects, and has proven extremely successful at locating 2nd layer cracks and edges. An extension to this idea is an array of polarised coils. This transducer enables a 2-D distribution of measurements to be made without any mechanical movements. This is achieved by selecting the detector coils electronically.

This project has primarily been concerned with 2-D distributions of magnetic fields based on the requirements of eddy-current inspection instruments. It has only been possible as a direct result of the rapid advance of integrated circuit technology. Research is currently in progress to incorporate arrays of magnetic sensitive semiconductor devices onto a single substrate, which can then be used as an eddy-current transducer. This coupled with an increased theoretical commitment to understanding the physics of the scattering process associated with electromagnetic waves incident on a crack in a conducting medium promises a healthy future for eddy-current inspection.
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Further Discussion.

One problem that is often encountered in conventional eddy-current transducers is lift-off. The effect manifests itself as a change in voltage of the detected signal with the distance between the transducer and the specimen. This especially applies to direct impedance-measurement systems. The apparatus designed during this project and described in chapter 6, was optimised to investigate fields in a 2-dimensional plane above a magnetic field source. Consequently lift-off effects were not studied during this project and a fixed lift-off was assumed. The height of the coil above the specimen was set by adjusting a sliding collar on the transducer connecting rod. This was secured during the experiment by a locking bolt. If detailed investigations on the effects of lift-off are to be studied, then the apparatus will require some modifications.

An ideal polarised coil pair (chapter 5) has no induced voltage in the detector coil when placed in the presence of a defect free conductor away from any edges and structure. Also since there is no induced voltage in the detector in free space, then the effect of lift-off from the defect-free specimen is zero. When a residual signal or a signal due to asymmetries exist, a lift-off effect may occur. Hence future work should include the investigation of the effect of variation in lift-off.

Further research should also consider the problems of integrating Magnetic Induction Tomography (chapter 3 and 4) into a crack-detection system. The main problem is removing from the detecting coils the induced voltage, which is caused by the direct coupling with the excitation
source. Additional experiments that can be conducted using the existing system include: (i) positioning the source in the region of the boundaries and observing the resultant change in the reconstructions; (ii) quantifying the errors that occur as a result of selecting a nearest neighbour linear interpolation algorithm; (iii) quantifying the improvements in the interpolation when a higher order interpolation algorithm is used; (iv) introducing noise in the simulated data projections and observing the effects that this has on the reconstructions. Measurements of the signal-to-noise ratio for the tomographic experiments were not considered appropriate, since for experimental measurements the noise level was extremely low (-54dB), and as such it was masked by the digitising errors when the analogue signals were converted by the ADCs.

One problem which was encountered with the single polarised coil (chapter 5) is a blind spot see fig 5.26. This occurs as a result of the symmetry which exists as the transducer is positioned directly over the 90 degree corner of a plate. This problem may be surmounted by scanning the area twice. The second time though with the transducer rotated by 45 degrees. If the specimen has circular symmetry, as in the case of a hole, then rotating the transducer will be to no avail. Since this transducer does not detect small circular holes, it may be of use in detecting holes with radial cracks. As such this is an application that should be pursued further, including the determination of the minimum defect size for detection at a particular frequency.

The fundamental ideas concerned with the array of polarised coils
have been demonstrated in chapter 5, together with some of the problems. Further work should consider the possibilities of using magneto-sensitive semiconductor substrates instead of detector coils. This would result in higher spatial resolution of the measured field distributions. A major problem expected with array transducers is the direct coupling of the excitation source and the detector. This also applies to the tomography detectors.

It should be noted that all the work presented in this thesis is based on theoretical and simulated data. The specimens used for the experimental work contained simulated defects, i.e., slots milled in alloy plates. The low frequency limit approximation (less than 10 MHz.) was assumed to be valid, which is defined to be the frequency below which any displacement current terms in the fundamental equations (chapter 2) can be neglected. Since the performance of the lock-in amplifier is restricted to frequencies between a few Hz to 150 kHz, any frequency selected in this range would not violate this limit. The skin depth at any particular frequency can be calculated using the expressions derived in chapter 2. The skin depth is highly dependent on the composition of the specimen. The frequencies chosen during the experiments were obtained empirically to maximise the transducer signals.

All of the ideas suggested in this thesis should be consolidated with further development so that they are optimised for the purpose with which they were conceived, namely the detection of cracks in conducting plates with emphasis on the practicalities of creating inspection instruments that are suitable for use in the aircraft industry.