UNIVERSITY OF SURREY

THE STRUCTURAL BEHAVIOUR OF BRACED BARREL VAULTS
WITH PARTICULAR REFERENCE TO WIND EFFECTS

by

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in the Department of Civil Engineering

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SUMMARY

Barrel vaults are a popular structural form of space coverage. Development of new materials and improvements in engineering constructional technology have led to a significant reduction in the self-weight of barrel vault. Wind load is therefore, one of the main parameters which govern the design of structural elements. Codes of practice, published so far, throw a limited light on the matter of wind pressure distributions. Existing data are rather approximate for design purposes.

The present study of wind effects on barrel vaults was carried out experimentally using the wind tunnel as a tool. Two different cross-sections of barrel vault were studied, each having a circular curvature and each being produced in different length to width ratios including one that spanned the complete tunnel to represent a two-dimensional case. They were tested in a simulated boundary layer flow with different oncoming flow directions. Models were rigid, springing directly from the base board. The roughness approach was introduced to achieve the simulation of high Reynolds number flow, preliminary work having been carried to investigate this approach. The static mean pressures obtained were expected to be similar to the prototype in the natural wind.

The evolution of effective techniques of analysis was one of the most significant factors contributing to the rapid development and widespread use of barrel vault. A discussion on several available analytical methods was carried out. Adopting the finite element method of skeletal structure and applying the wind loads obtained from the wind tunnel tests, the structural behaviour and the efficiency of stress distribution of structures in a family of two-way double-layer braced barrel vault under the wind loads were investigated.

The present study indicates that further detailed studies of the subject may, however, lead to a more precise understanding of the behaviour of barrel vault structures subjected to wind and this may bring about increased structural safety and serviceability and also economy in cost.
TO MY PARENTS
ACKNOWLEDGEMENTS

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INTRODUCTION

Any structure which is built on the earth's surface should be capable of withstand ing the loads imposed on it by the environment. The wind, in particular, contributes one of the major forms of structural loading and even moderate winds are capable of imposing high forces on structures. As a result, most building codes of practice incorporate fairly lengthy sections denoted specifically to those aspects of the design and construction of structure which are concerned with the resistance of wind load.

Prior to the nineteenth century, most barrel vaults were constructed by traditional methods in traditional building materials, such as timber, stone, brick or later, concrete. As a result of the high self-weight of such structures, the detailed prediction of wind forces was not the prime concern to architects and builders, for the simple reason that this was not necessary. However, as the century progressed, more and more use was made of the new lighter structural materials, for example, steel or aluminium. The ease with which these materials would be used came from the fact that they could be worked into bars and plates. It led to the widespread adoption of lattice or grid as forms of construction suitable for roofing large areas. The material was expensive compared to the traditional materials, but simultaneous developments in the techniques of structural analysis provided the means of calculating stresses in skeletal structure and this made the possibility of designing the light, economic braced barrel vaults with a high degree of accuracy.

The aim of the design is to ensure that the performance of structures subjected to the action of any environmental loading such as wind loads, snow loads etc, will be adequate during their anticipated life from the standpoint of both structural safety and serviceability. Therefore, an understanding of the flow of wind around any structure, leading to the accurate prediction of wind pressures and forces, is an essential require-
ment for modern economic structural design.

Although many barrel vaults have been built during the past decade, the influence of the wind still represents the greatest unknowns in the design. It becomes necessary to develop tools enabling the designer to estimate wind effects with a higher degree of refinement than was previously required. Generally, the designer needs information regarding the wind environments, the relation between that environment and the wind forces it induces on the structure and the behaviour of the structure under the action of these forces. The aim of the present study is to provide some information on wind forces on the barrel vaults and the structural behaviour of the vaults under the wind forces.

In the present state of knowledge, neither the prediction of the wind environment nor the estimation of the properties of flow around curved shape structures is without its difficulties. Although considerable advances have been made in recent years, the process of estimating the wind load on a curved structure still involves a large number of unknowns and a considerable amount of inspired guesswork.

In the first part of the present study, experimental analysis on barrel vaults is carried out using the wind tunnel as a tool. Models are tested in a simulated boundary layer flow. The roughness approach is employed to simulate a high Reynolds number flow condition. Mean pressure distributions on models are obtained in the wind tunnel tests and are expected to be generally similar to the prototype. Due to the limitation of time and knowledge, only two different geometrical forms of barrel vault have been taken into consideration in this study. They both are in circular curved form and are taken as springing from the ground surface.

The great development of computing science led to the progression of structural analysis techniques, especially on large space skeletal structures. A review and discussion on several analytical techniques on braced barrel vaults, still in favour for designers, is presented in the second part of the thesis. Finally, choosing a suitable analytical method and using the wind pressures obtained in the first part, work on
understanding the structural behaviour of barrel vaults under the action of wind is carried out. Since the double-layer braced barrel vault will be built widely due to its own advantages, three different configurations of double-layer braced barrel vault are chosen to study.
PART I

AERODYNAMIC ASPECT
## NOTATION

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<th>SUFFIX</th>
<th>DESCRIPTION</th>
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<tr>
<td>C</td>
<td>D</td>
<td>drag coefficient, dimensionless</td>
</tr>
<tr>
<td>C</td>
<td>p</td>
<td>pressure coefficient, dimensionless</td>
</tr>
<tr>
<td>C</td>
<td>pb</td>
<td>base pressure coefficient, dimensionless</td>
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<tr>
<td>C</td>
<td>pm</td>
<td>maximum -ve pressure coefficient, dimensionless</td>
</tr>
<tr>
<td>D</td>
<td></td>
<td>typical length parameter of structure or diameter of circular curved barrel vault</td>
</tr>
<tr>
<td>H</td>
<td></td>
<td>height of structure or rise of barrel vault</td>
</tr>
<tr>
<td>h</td>
<td></td>
<td>height about ground</td>
</tr>
<tr>
<td>p</td>
<td></td>
<td>local mean pressure</td>
</tr>
<tr>
<td>p</td>
<td>o</td>
<td>reference static pressure</td>
</tr>
<tr>
<td>R</td>
<td></td>
<td>radius of curvature of circular arc</td>
</tr>
<tr>
<td>Re</td>
<td></td>
<td>Reynolds number, dimensionless</td>
</tr>
<tr>
<td>S</td>
<td>X</td>
<td>longitudinal span or length of barrel vault</td>
</tr>
<tr>
<td>S</td>
<td>Y</td>
<td>transverse span or width or chord of barrel vault</td>
</tr>
<tr>
<td>V</td>
<td></td>
<td>mean wind velocity</td>
</tr>
<tr>
<td>V</td>
<td>o</td>
<td>mean wind velocity outside the boundary layer</td>
</tr>
<tr>
<td>ε</td>
<td></td>
<td>effective height of rough material</td>
</tr>
<tr>
<td>μ</td>
<td></td>
<td>viscosity of air</td>
</tr>
<tr>
<td>ρ</td>
<td></td>
<td>density of air</td>
</tr>
<tr>
<td>θ</td>
<td></td>
<td>angular position of curved surface measured from stagnation point</td>
</tr>
<tr>
<td>θ</td>
<td>w</td>
<td>angular position of the beginning of wake region</td>
</tr>
<tr>
<td>ψ</td>
<td></td>
<td>angle of which the chord of the circular curved barrel vault subtended</td>
</tr>
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</table>
CHAPTER 1

SCOPE OF PRESENT STUDY ON WIND EFFECTS

1.1 INTRODUCTION

The estimation of wind effects on barrel vaults involves the problem of wind flow around a curved surface. Since a number of uncertain factors need to be taken into consideration and there are numerous unknowns involved in the analysis, determination of the wind effects on curved surfaces is a challenging field for study. Looking back into the history of wind engineering, the study of wind flow around a circular cylinder has been carried out for several decades. Relf (1914) made an early investigation into the effect of Reynolds number on the drag of a circular cylinder and then an investigation on the vortex shedding, the influence of upstream turbulence and the surface roughnesses has followed and still continues to this day (more detailed reference in Chapter 2, Section 2.2.1). Information on the wind loading on curved shape structures and on the flow pattern on which it is dependent is limited. Variables such as the geometry of the barrel vaults and the characteristics of the atmospheric boundary layer need to be taken into consideration in determining the wind effects; the present study is limited to certain cases. In this chapter, the scope of the present study is outlined. An understanding of flows over a curved surface is necessary to an understanding of wind loading; a basic treatment of such flow is therefore included.

1.2 SURVEY OF RELEVANT INFORMATION OF WIND LOADS ON BARREL VAULTS

Most structural engineers, when predicting wind flow around a proposed structure, will turn first to the relevant code of practice for the particular country. In the case of barrel vaults, it is possible to find
some information about wind pressures on a curved roof, either with an enclosure wall or springing from the ground surface, in several codes of practice. In Figure 1.1, the wind pressures given by American code (ANSI A58.1-1972), Russian code (BC & R II-A.11-62), Australian code (AS 1170, Part 2-1975), Indian code (IS: 875-1964) and U.K. Wind Loading Handbook (Newberry and Eaton, 1974) are quoted and shown. These codes of practice basically provide mean pressures for a variety of shapes under different wind regimes (e.g. open country, towns). In order to predict the actual load or pressure, the coefficient must be multiplied by the dynamic head of the wind ($\frac{1}{2} \rho V^2$), where $V$ is the design wind speed. Codes of practice usually provide criteria for determining the design wind speed, which is dependent on ground roughness, building size, height above ground, required life time of the structure and the topography of the site. These are intended to cover the contingencies affecting the incident wind and the way in which gustiness influences loading. However, the magnitude of the design wind speed may well be a somewhat uncertain element of a wind load calculation. This uncertainty shows clearly in British code CP3 Chapter V: Part 2-1972. The design wind speed should be the local velocity at the height which should be taken to the top of the structure or the height of the structure may be divided into convenient part. The question of which reference wind speed to use in the given pressure coefficients must be resolved.

In fact, many of the data given by codes of practice were obtained from wind tunnel tests. However, the characteristics of the incident flows in the wind tunnel test were often far from those of atmospheric winds (Stansly and Wootton, 1975). Again, for the wind pressure coefficient of a curved roof springing from the ground surface, all the codes of practice give a high positive pressure at the windward portion of the roof surface including the leading edge at ground surface level. Since the velocity in the boundary layer near the ground is small compared with the freestream velocity and theoretically approaches zero on the flat surface itself, the information given in codes of practice, must be, to that extent, an oversimplification.
1.3 Methods of Assessing Wind Effects on Barrel Vaults

In general, there are many different methods available to assess the effects of wind on a proposed structure. One quick way, already mentioned, is to refer to a code of practice. However, it has been mentioned before that the information on the wind loads on barrel vaults given by codes of practice may be misleading. On the other hand, the theoretical and experimental knowledge, for a given shape of structure, may have been evolved that will enable the designer to estimate with confidence certain types of wind loads and the associated structural response, for example, the estimation of the mean load acting on a circular cylinder in terrain with reasonably uniform roughness and the prediction of flutter instability of a certain type of suspension bridge in horizontally homogeneous wind flow (Simiu, 1978). Unfortunately, in many situations encountered in wind engineering practice, no analytical tool capable of describing the wind loading phenomenon presently exists. As in the case of barrel vaults having a curved surface, the difficult situations involve one or both of the following factors: 1) unknown aerodynamic properties of the structures involved, for example, position of boundary layer separation, effects of upstream turbulence, velocity of flow and surface condition. 2) significant local disturbances of the oncoming wind flow such as may be created in cities by the presence of large buildings near the structure under consideration. Therefore, in the absence of workable analytical models or of previous experience with similar structures and environments, the effects of winds on barrel vaults must be determined experimentally. The experimental methods in predicting the wind effects on structures mainly depend upon wind tunnel tests on scaled model or full-scale measurements.

Currently, the use of a wind tunnel for the prediction of both structural and environmental wind effects associated with structures may offer the most promising solution from an engineering point of view. In fact, it must simulate in a reasonably adequate fashion the conditions that will prevail in the full-scale situation. For the present case of estimation wind effects on barrel vaults by wind tunnel test, several problems arise. The main problems are representing the atmospheric boundary layer, achieving the same flow patterns as for high Reynolds
number flow and measuring the quantities that are relevant to the process of design. The sensitivity of various quantities to be used in the design to the properties of the atmospheric turbulence has been the object of research by many authors; references could be found in many theses and literature. To establish ways of simulating high Reynolds number flow was one of the main tasks in the present study.

In addition, since no suitable theoretical analysis could be used to compare with the experimental results, ideally the full-scale studies would be used to verify wind tunnel observations. However, to measure the wind loading on full-scale structure exposed to the natural wind is expensive. For structures in the design stage or for research in the short term, full-scale measurements are obviously out of the question. Therefore, to obtain experimental information on wind effects in the present situation, it is necessary to rely heavily upon wind tunnel tests, because they can be carried out under carefully controlled conditions at considerably lower cost.

1.4 MAIN CONSIDERATION IN THE STUDY OF WIND EFFECTS ON BARREL VAULTS

Wind engineering may be thought of as the discipline of developing tools enabling the designer to estimate wind effects on structures. The aim of providing adequate information of wind effects on structures is mainly concerned with the induced wind forces, localised surface pressures and dynamic response of structures due to wind loads. During the last few years, wind engineering has covered not only structural considerations but also environmental considerations such as pedestrian safety, the problem of noise caused by wind flowing and air pollutant movement. In the limited time of the present study, work has been concentrated on structural considerations in simple fundamental cases.

1.4.1 WIND ENVIRONMENT FOR STRUCTURES SITUATED

In determining wind effects on structures by wind tunnel tests, a
satisfactory model of the natural wind is required first of all and then a scaled structural model may be tested in this wind environment. Generally, information on the wind environment includes

1) a description and explanation of the basic features of atmospheric flows such as tornado or typhoon zones.

2) a description of the detailed structure of atmospheric flows near the ground including the variation of mean velocity with height above ground, atmospheric turbulence and the dependence of the mean velocity and of turbulence upon roughness of terrain.

3) the prediction of wind conditions at given geographical location such as probability of future wind speeds.

This information is usually provided by meteorology, micrometeorology and climatology. For wind engineering, the requirement is to produce a suitable simulated wind environment in the wind tunnel. If the natural wind, the boundary layer of the earth, is to be modelled accurately, the ideal requirements are similarity of velocity profile, turbulence intensity and power spectral density of the longitudinal component. The last two factors are probably less important than the first when static pressure distribution is the main concern. There is a variety of techniques which have been used to achieve these requirements. Generally, the basic categories of wind tunnels used for simulating atmospheric flow are:

1) long tunnels having a length of the order of 30m so that the boundary layer develops naturally over a rough floor. 2) tunnels with passive devices placed at the entrance of the test section. Currently, a thick boundary layer may be generated by grids, fences or vortex generators and the flow is then allowed to pass over a fetch of roughness elements.

The wind tunnel in the Hydraulics Laboratory of the Civil Engineering Department of University of Surrey, which was to be used for the present study, belongs to the second category as mentioned above. A barrier and vortex generators could be used on conjunction either with the original smooth tunnel wall or with a surface covered by a regular pattern of roughness elements. This system is able adequately to simulate the mean velocity, turbulent intensities, turbulent shear stress and scale of turbulence of a naturally grown boundary layer of thickness approximately equal to the height of the vortex generators (Dianat, 1980). This simulated boundary layer, with a rough surfaced wall in use, has a
velocity distribution \( V = h^{1/3} \), similar to the distribution in suburban areas or small towns, while with a smooth surface wall, it has a velocity distribution \( V = h^{1/7} \), similar to the distribution over open sea or bare, flat land.

1.4.2 WIND FLOW PATTERN AND PRESSURE DISTRIBUTION

The overall induced wind forces (e.g. drag forces and lift forces) and the localised pressures on the surface of a structure are a particular concern of structural engineering. This is because the stability of the structure mainly depends upon the overall wind forces, while for designing the roofing or cladding, the localised pressures may have an extremely significant effect. The determination of wind loads (including wind forces and localised pressures) on a structure can be materially improved and assisted by an understanding of the nature of the wind flow pattern around the structure since the variations of pressures produced on the surface are indeed influenced by the flow pattern. As the structural design of barrel vault roofing is the particular concern of this thesis, detailed measurements were taken to determine the localised pressures on the surface of the structure.

Barrel vaults may be in circular arc form or in elliptical curved form, either with supporting columns or simply springing from the ground surface. In all cases obviously a curved surface is involved. From the present knowledge, it is known that the effect of the Reynolds number of the flow may be pronounced in the case of flow over curved surfaces because the flow can separate from the surface at different position at different Reynolds number. Further, the surface condition and turbulence of approaching flow may affect the separation position. Therefore, the flow pattern and hence, the pressure distribution are different at different Reynolds number, different atmospheric flow and different surface finished. Present knowledge from theoretical and experimental works on the circular cylinder provides certain fundamental information for understanding the wind effects on curved surface, in particular for structures with circular curvature. Thus, a barrel vault in circular curved shape, springing from the ground surface may be
studied on the light of this knowledge.

Actually, barrel vaults may have openings such as entrance door, window, etc. so that internal pressures may be induced. The internal pressure in a structure usually depends on the relative ease of inflow and outflow of the air and on the relative size and position of openings and leakage points in the roof relative to the external pressure distribution. The total wind pressures on the roof then depend on the difference of pressure between the outer and the inner faces. Besides, it is also necessary to consider the disturbance of flow by any particular neighbouring building. The effects of neighbouring buildings could be taken into account in assessing the consequential pressure distribution. For example, surrounding buildings may give shelter or they may so channel the air-flow as to give rise to wind pressures that are more severe than would have occurred on the structure in isolation. However, it is impossible to consider so many influential factors in the present study, which will therefore, be limited to the fundamental situation, which is a rigid enclosed structure situated in fairly open terrain.

1.4.3 Static Response to Wind Loads

The determination of wind loads on a structure is basically a dynamic problem because the turbulence in the wind flow produces fluctuating loads which are dependent on time. However, it has been conventional practice to treat such loads as static loads. This approach is only satisfactory provided that the relationship between the time variation of the wind loads and the natural frequencies of the structure is such as to cause an essentially static response. Eventually, a wind which is not exceptionally strong can cause vibration, structural deterioration, fatigue problems or human discomfort. Currently, observation of dynamic response to wind load is mostly carried out for flexible and low-damping structures. In the present study, surface pressures on the structure which are determined by the wind tunnel tests are the mean static pressures. Therefore, the test models are rigid so that they do not oscillate unduly.
1.5 Boundary Layer Development and Separation of Flow

When investigating flow past a curved surface both experimentally and theoretically, the difficulties encountered are the boundary layer development on the curved surface and the position of separation. Boundary layer development and separation of flow are important in fluid mechanics because they are closely connected to the flow pattern and the pressure distribution. However, within the field of fluid mechanics, the problem of boundary layer development, together with separation of flow, is undoubtedly one of the most complex subjects, so that a brief discussion of this phenomenon is felt necessary here.

When air flows over any surface, a frictional force is exerted between the two. Due to viscous and eddy stresses in the air this frictional force is spread through a thin layer of air adjacent to the surface, producing a velocity gradient in that layer. This region of flow is called the boundary layer. As the frictional force exerted by the surface continually opposes the motion of the air, the layers of air adjacent to the surface lose momentum. If the pressure gradient is falling with distance, sufficient momentum is transferred through the boundary layer for the layers at the surface to maintain some momentum. However, in the situation of adverse pressure gradient, the transference of momentum is less. Eventually, the layers on the surface have lost all their momentum and come to rest. This phenomenon is called separation and at this point, \( \frac{\partial v}{\partial y} y = 0 \), and the surface shear stress may be zero. In addition, flow separation may also be caused by a severe discontinuity of the tangent to the surface. For example, separation occurs at the sharp edge or corner of rectangular buildings.

Since the boundary layer is dominated by the effects of viscosity, there are two distinct types of boundary layer—laminar or turbulent. The laminar boundary layer is usually characterised by stratified layers of fluid sliding over each other with very little mixing across the layer. On the other hand, the turbulent boundary layer is characterised by turbulent eddies causing considerable mixing through the layer. Based on their characteristics, the losses of momentum in the turbulent boundary layer are mainly associated with dissipation by the eddies rather than
with the direct shearing action as in the laminar boundary layer. For this reason, a turbulent boundary layer tends to be thicker than a laminar one, giving rise to a later separation.

From observation of flow past a circular cylinder, an idea of the flow past a highly curved surface may be deduced. The flow might be characterised by a very thin laminar boundary layer in the forward portion of the curved surface followed by transition and eventual turbulent separation which results in a wake. In fact, the transition from laminar to turbulent boundary layer on a curved surface may be governed by 1) Reynolds number 2) the turbulence of the approaching flow and 3) the roughness of the surface. To understand fully the flow features around a curved surface, a technique of predicting shear stress on the surface of curved shape model and a concept for the treatment of boundary layer flow near the separation would be required. It is, nevertheless, intended in the present study that at least a first approximation to the flow features of a circular curved surface at a high Reynolds number flow may be produced.
American code (ANSI A58.1-1972)

**Table 8**

<table>
<thead>
<tr>
<th>Rise-to-Span Ratio, r</th>
<th>Windward Quarter</th>
<th>Center Half</th>
<th>Leeward Quarter</th>
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<td>Roof on elevated structure</td>
<td>0&lt;r&lt;0.2</td>
<td>-0.9</td>
<td>(-0.7-r)</td>
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<tr>
<td></td>
<td>0.2&lt;rs0.3</td>
<td>(1.5r-0.3)*</td>
<td>(2.75r-0.68)</td>
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<td></td>
<td>0.3&lt;rs0.6</td>
<td>1.42r</td>
<td>(-0.7-r)</td>
</tr>
<tr>
<td>Roof springing from ground level</td>
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<td>1.42r</td>
<td>(-0.7-r)</td>
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</tbody>
</table>

*When the rise-to-span ratio is (0<2rs0.3), alternate coefficients given by (6r-2.1) shall also be used for the windward quarter.*

Australian code (AS 1170, Part 2-1975)

![Diagram](image)

<table>
<thead>
<tr>
<th>Rise-to-span ratio, G</th>
<th>Windward quarter</th>
<th>Center half</th>
<th>Leeward quarter</th>
</tr>
</thead>
<tbody>
<tr>
<td>Roof on elevated structure</td>
<td>0&lt;G&lt;0.2</td>
<td>-0.9</td>
<td>(-0.7-G)</td>
</tr>
<tr>
<td></td>
<td>0.2&lt;Gs0.3</td>
<td>1.5G-0.3*</td>
<td>2.75G-0.675</td>
</tr>
<tr>
<td></td>
<td>0.3&lt;Gs0.6</td>
<td>1.4G</td>
<td>(-0.7-G)</td>
</tr>
<tr>
<td>Roof springing from ground level</td>
<td>0&lt;G&lt;0.6</td>
<td>1.4G</td>
<td>-0.7-G</td>
</tr>
</tbody>
</table>

*The alternative coefficient 6G-2.0 shall also be used.*

**Fig. C8 CURVED ROOFS**

Indian code (IS: 875-1964)

![Diagram](image)

**Fig. 7**

**Fig. 9**

Figure 1.1 Wind pressure distributions on curved structures —— quoted from codes of practice
Values $c$, $c_1$, and $c_2$

<table>
<thead>
<tr>
<th>$f$</th>
<th>$c$</th>
<th>$c_1$</th>
<th>$c_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>0.8</td>
<td>-0.8</td>
<td>0.8</td>
</tr>
<tr>
<td>0.2</td>
<td>0.9</td>
<td>-0.9</td>
<td>0.7</td>
</tr>
<tr>
<td>0.3</td>
<td>1.0</td>
<td>-1.0</td>
<td>0.4</td>
</tr>
<tr>
<td>0.4</td>
<td>1.1</td>
<td>-1.1</td>
<td>0.6</td>
</tr>
<tr>
<td>0.5</td>
<td>1.2</td>
<td>-1.2</td>
<td>0.7</td>
</tr>
</tbody>
</table>

Remarks:
Aerodynamic coefficients for intermediate values $f/1$ shall be determined by linear interpolation.

**Table 11 Aerodynamic Coefficients**

---


**Table 11.5 $C_{pe}$ for arched roof as in Figure 11:17 for wind angle $\alpha = 90^\circ$**

<table>
<thead>
<tr>
<th>Ratio $y/d$</th>
<th>Ratio $h/d$</th>
<th>$C_{pe}$ for segments $\alpha = 90^\circ$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>$+0.3$</td>
</tr>
<tr>
<td>1/8</td>
<td>-0.5</td>
<td>$+0.5$</td>
</tr>
<tr>
<td>1/4</td>
<td>-0.9</td>
<td>$+0.6$</td>
</tr>
<tr>
<td>1/2</td>
<td>-1.2</td>
<td>$+0.7$</td>
</tr>
<tr>
<td>1</td>
<td>-1.4</td>
<td>$+0.8$</td>
</tr>
<tr>
<td>5</td>
<td>-1.8</td>
<td>$+1.0$</td>
</tr>
<tr>
<td>1/8</td>
<td>0</td>
<td>$-0.4$</td>
</tr>
<tr>
<td>1/4</td>
<td>-1.2</td>
<td>$-0.5$</td>
</tr>
<tr>
<td>1/2</td>
<td>-1.5</td>
<td>$-0.7$</td>
</tr>
<tr>
<td>1</td>
<td>-1.6</td>
<td>$-1.0$</td>
</tr>
</tbody>
</table>

**Table 11.6 $C_{pe}$ for arched roof as in Figure 11:17 for wind angle $\alpha = 90^\circ$**

<table>
<thead>
<tr>
<th>Section</th>
<th>$C_{pe}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>-0.8</td>
</tr>
<tr>
<td>B</td>
<td>-0.6</td>
</tr>
<tr>
<td>C</td>
<td>-0.3</td>
</tr>
<tr>
<td>D</td>
<td>-0.2</td>
</tr>
</tbody>
</table>

NB All forces calculated from pressure coefficients should be assumed to act normal to the face.

Wind pressure distributions on curved structures — quoted from codes of practice
CHAPTER 2
PRELIMINARY STUDY

2.1 PURPOSE

The pressure distribution on barrel vaults may depend upon a number of factors such as the Reynolds number, the surface roughness, the velocity distribution and the turbulence characteristics of the approaching stream and the presence of other large structures in the vicinity. The manner in which these factors affect the pressure distribution is not completely known. In order that results of wind tunnel tests on models of reduced scale be applicable to the prototype, it is necessary, in principle, to simulate these factors in a wind tunnel.

To study flows around structures immersed in the natural wind, adequate simulation of atmospheric boundary layer flow in a wind tunnel is one of the main similarity requirements. The nature of the wind in the earth's boundary layer is complex. In this investigation, attention was mainly directed to the effects of appropriate Reynolds number and velocity distribution; the effect of variation in intensity and scale of turbulence, probably minor, is worthy of further study.

There is in no doubt that models should be geometrically similar to the prototype and also, if possible, be tested at the same Reynolds number ($\frac{\rho_{VD}}{\mu}$). The Reynolds numbers for full scale barrel vaults in the real environment are large, typically of the order $10^7$ to $10^8$ (for a barrel vault with a 20m width in span and wind velocity of say 30m/sec., Reynolds number is approximately $4\times10^7$). For direct simulation of these high Reynolds number on models, it requires either a high speed tunnel of compressed air (having higher density) or a large size of model to ensure that the product $\rho_{VD}$ has the same value. Unfortunately, tunnels speeds are limited, compressed air tunnels are costly and hence, very rare, and
the size of model is limited by tunnel size but, more, by constraint due to blockage in the tunnel which may distort the pressure distribution around the body. A typical value of Reynolds number for a wind tunnel is, therefore, of the order of $10^5$ or $10^6$. The simulation of the atmosphere boundary layer in the wind tunnel may be achieved with some success but the modelling of a barrel vault with a reasonable value to the Reynolds number is extremely difficult. The purpose of the preliminary study was to establish a possible approach to the simulation of a representative pressure distribution of the high Reynolds number flow past the prototype.

2.2 SIMULATION OF HIGH REYNOLDS NUMBER FLOW IN A WIND TUNNEL

It should be noted that flow past a bluff body at relatively high Reynolds number may affect the nature of the boundary layer that forms on the surface of the body in the region of flow separation. The boundary layer flow usually changes from laminar to turbulent. From the present knowledge of two-dimensional flow, the wind flow past a sharp-edged structure separates at the sharp corner so that the development of flow field, pressure distribution, the drag and lift forces depend very little upon Reynolds number. For a curved structure such as the fundamental case of uniform flow past a circular cylinder, the separation point is not so clearly defined and varies with Reynolds number. It is on this account, therefore, that the development of flow pattern, pressure distribution and aerodynamic forces are Reynolds number-dependent.

For a bluff body with a highly curved surface such as a barrel vault, the wind flow past the body is no doubt dependent on Reynolds number. The subject of the present study is barrel vaults without a supporting column or wall, i.e. a part of cylinder rested upon a base board. It may be expected that these types of structures will show some similarity with the case of flow over a circular cylinder with, however, considerable differences in detail.
2.2.1 Two-Dimensional Uniform Flow over Circular Cylinder

For a smooth two-dimensional cylinder in uniform low turbulence flow, a number of flow situations can be created by increasing the flow velocity, each situation being identified by a specific Reynolds number range. The effects of Reynolds numbers on the boundary layer separation point, the resultant width of the wake and the drag coefficient for the case of smooth circular cylinder are shown in Figures 2.1 and 2.2.

At low Reynolds number (less that 40), approximate solutions of Navier-Stokes equations appear to represent the observed flow phenomena well (Dennis, 1964, 1970). At extremely low values of Reynolds number (less than 5), the flow is entirely laminar and remains attached. At Reynolds number near or equal to 5 and 6, the flow form remains symmetrical but flow separation occurs at the rear of the cylinder and a pair of attached eddies are formed. As Reynolds number increases the eddies become elongated and the laminar wake becomes more unstable. At Reynolds number of about 30, the instability of the wake leads to the development of a vortex street with eddies being shed alternately from either side of the cylinder at a periodic rate which increases with increasing Reynolds number.

When the Reynolds number is greater than 40, the establishment of the flow pattern is almost entirely a matter of experimental observations. The flow pattern is a function of the boundary layer flow in the vicinity of the point of separation on the cylinder. At Reynolds number between 150 and \(10^5\), denoted the subcritical flow regime, the vortex shedding becomes less regular and turbulence in the wake spreads progressively upstream toward the cylinder (Tritton, 1959 and Gerrard, 1961). The wake is wide and, hence, the drag is high and nearly constant. The separation boundary layer flow in the cylinder which is laminar moves progressively forward with increasing Reynolds number. When Reynolds number reaches a value of the order \(10^5\), separation occurs on the forward part of the cylinder at a value of \(\theta\) between 70° and 80°, where \(\theta\) is angular position of the cylinder measured from the frontal stagnation point (Achenbach, 1968).
As Reynolds number increases above a value of about $10^5$, denoted the critical flow regime, there is a transition from laminar flow to turbulent flow at the rear of the cylinder (Bearman, 1968 and Batham, 1973). The Reynolds number at which this actually occurs depends strongly on the freestream turbulence level and surface roughness. The laminar boundary layer separates initially at about $\theta$ between $90^0$ to $100^0$ just ahead of the transition point to turbulent flow. Since the turbulent boundary layer can withstand a greater adverse pressure gradient, the boundary layer reattaches just downstream of where transition occurs to form a localised separation bubble (Roshko, 1961 and Bearman, 1968). Turbulent reseparation occurs further back on the cylinder at about $125^0$ and $140^0$. This rearward movement of the final separation point is accompanied by a rapid narrowing of the wake in which there is no dominant periodicity of vortex shedding, giving rise to a rapid drop in drag coefficient (E.S.D.U., 1970).

As Reynolds number further increases and exceeds the Reynolds number of the minimum drag, the supercritical flow regime is reached. The transition to turbulent flow occurs further forward on the cylinder accompanied by a rewidening of the wake, giving rise an increasing drag. The separation point moves forward and occurs in the range of $\theta$ between $115^0$ and $120^0$ (Achenbach, 1968).

At Reynolds number above $10^7$, denoted the transcritical flow regime, the flow around the cylinder becomes almost entirely turbulent and the drag approaches a constant value. Thus the pressure distribution becomes largely independent of Reynolds number (Roshko, 1961).

Increasing surface roughness has the effect of the disturbing the laminar flow over the cylinder and thus of promoting earlier transition to turbulent flow on the cylinder so that any given flow pattern occurs at progressively lower Reynolds number (Armitt, 1968, Achenbach, 1971, Batham, 1973 and Szechenyi, 1975). For Reynolds number above the critical and at even higher values, the turbulent flow becomes less significantly influenced (Batham, 1973), but the adverse effect of roughness on the boundary layer growth leads to earlier flow separation with a wider wake and a higher drag coefficient. The drag increases again from the lower limit and nearly reaches a constant value in the transcritical range.
The transcritical drag coefficient increases with increasing roughness parameter (Achenbach, 1971). The variation of drag coefficient with Reynolds number for rough and smooth cylinder is simply shown in Figure 2.3.

2.2.2 SURFACE ROUGHNESS APPROACH

In the previous section it is suggested that the reproduction of full scale flows around a smooth cylinder may be attempted by artificially disturbing the laminar boundary layer that would form on the model surface near the region of separation. This can be done by covering the model surface with roughness elements, for example emery paper, sand or glass beads. Szechenyi (1975) attempted to simulate transcritical Reynolds number flow over a two-dimensional circular cylinder by this surface roughness technique. The results obtained in his study show that the separating boundary layer becomes turbulent at a Reynolds number based upon roughness height of about 200 in a manner independent of the diametral Reynolds number, provided that the latter is greater than $10^5$. The turbulent separation is probably fully developed when the roughness Reynolds number reaches a value of about 1000, at which stage the various parameters such as drag coefficient become constant.

Cooling towers with their circular cross-section pose a very similar problem with Reynolds numbers in the region from $10^7$ to $10^8$. The surface roughness approach has been used for cooling towers in recent years (Armitt, 1968 and Farell, Güven, Maisch and Patel, 1976). It is suggested by Armitt that if a body of cylindrical cross-section is coated with sand where roughness Reynolds number is at least 600 (or better 1000) at diametral Reynolds number between $10^5$ and $10^6$, the mean pressure distribution obtained in this way would approximate to full scale conditions and would be adequate for stress analysis purpose.

This high Reynolds number simulation depends entirely on the transformation of the boundary layer from laminar to turbulent. If the barrel vaults in the present investigation show that their flow characteristics are entirely governed by this modified boundary layer in much the same way as the two-dimensional flow over a circular cylinder,
this surface roughness approach, therefore, can be applied on the models to simulate the high Reynolds number flow in the wind tunnel tests.

2.3 WIND TUNNEL TESTS

It was decided therefore that it would be useful to explore the surface roughness approach on barrel vaults in the following steps:

(1) beginning with the simple case of smooth as well as rough circular cylinders in two-dimensional uniform flow to check that the experimental results accorded reasonably with previous work and to establish that flow in the supercritical region can be achieved by the use of roughness elements.

(2) subsequently to place a splitter plate along the centre-line of the wake of a cylinder in two-dimensional cross flow and to investigate the influence of pressure distribution and wake formation.

(3) moving further towards the case of the barrel vaults, to set smooth and rough semi-circular vaults on a base board in a turbulent shear flow and to investigate the influence of surface roughness.

2.3.1 GENERAL REMARKS

The experimental instruments used in all wind tunnel tests of the preliminary study and the presentation of results are described in the following sections.

2.3.1.1 WIND TUNNELS

A. In Civil Engineering Department (as shown in Plate 2.1)
Type: low speed blow-down open circuit wind tunnel
Dimensions of working section: 1.372m height x 1.067m width x 9.0m length
Maximum tunnel speed: 16m/sec.

B. In Mechanical Engineering Department
   Type: low speed blow-down open circuit wind tunnel
   Dimensions of working section: 0.76m height x 0.61m width x 2.20m length
   Maximum tunnel speed: 8m/sec.

2.3.1.2 MEASUREMENT INSTRUMENTATION

A. The reference velocity of the freestream uniform flow in all the tests was obtained from the dynamic pressure \(\frac{1}{2} \rho V^2\) which was measured by a pitot static tube. Since the air in a tunnel was heated considerably during operation and then changed density, the dynamic pressure of the approaching freestream flow was continuously monitored during the experiments to ensure a constant value.

B. The mean pressures on the models were taken from stainless tubes set in holes drilled in the surface of the model. The outside diameter and inside diameter of the stainless tubes were 1.6mm and 1.2mm respectively. The stainless tubes were set flush with the model's surface and were cemented into the holes. Under this situation, the stainless tubes should not interfere with the flow and the termination of the tube would not significantly modify the model contour. The pressure can then be determined at each tapping by connecting each with a plastic tube to a Furness micromometer. The most sensitive range of the micromometer could give the pressure to 1 millimetre water gauge full scale. In the present work, by transferring the reading from the micromometer to a digital voltmeter, a mean pressure was calculated immediately by a Tektronix 31 programmable calculator.

C. It is realised that the mean velocity at the point of reattachment in the downstream flow after separation from the surface of a bluff body tends to zero so that the mean velocities before and after the reattachment must be of opposite sign. This principle was used in
the design of a twin tube detecting the point of reattachment in the downstream flow. The twin tube was made from two stainless tubes which were 1.7mm in diameter and 150mm in length, fastened together in parallel along their length. A pressure hole of 0.6mm diameter was drilled in each tube at 50mm measured from the closed ends. These two holes faced towards the left and right hand sides of the twin tube. Details of the twin tube are shown in Figure 2.4. The twin tube, set at right angles to the flow and taped flat to a surface at a chosen position, was connected to the pressure measuring system (previously described) and the mean pressure difference for the position was read out. By placing the twin tube at a series of positions in the downstream flow, an approximate position of reattachment could be obtained at the null position where the mean pressure difference started to change sign.

2.3.1.3 Surface Roughness

The uniformly distributed roughnesses used in this study were provided by sand which was sieved from ordinary builders sand using sieves giving an effective diameter of sand between the upper and lower sieve size of 0.6mm and 1.1mm. The choice of sand diameter, which accorded with suggestion of Armitt (1968) and Szechenyi (1975), would give a roughness Reynolds number in the present wind tunnel test of between 200 to 1000, in order to produce a simulated high Reynolds number flow. The sand was coated on to the model's surface by painting on a thin film of glue and then sprinkling on the sand whilst slowly rotating the model. A uniform layer of sand over the whole surface was produced.

2.3.1.4 Pressure Coefficients and Drag Coefficients

The mean pressures obtained in the tests were referred to the uniform freestream static pressure and the pressure coefficients were then referred to the uniform freestream dynamic pressure. The drag coefficients were calculated by the equation $C_p \cos \theta (d\theta)$ using Simpson's rule.
2.3.1.5 Blockage Constraint

The flow past a bluff body immersed in an air stream bounded by rigid tunnel walls is subject to what is called blockage constraint. The rigid boundaries prevent a free lateral displacement of the air flow by the body, in the neighbourhood of which velocities are higher than they would be in an unlimited stream. In a case of circular cylinders, with their large associated wake, the effect should be considerable in preventing the lateral expansion of the streamlines around the wake, thus, increasing the local velocities in this region. It is probable, therefore, that the blockage effect may be mostly associated with the downstream face exposed to the wake pressure while on the upstream face there may be little apparent effect in the pressure distribution. However, Maskell (1963) pointed out that the drag of non-lifting bluff bodies is invariant or less critical under blockage constraint. Later, McKeon and Melbourne (1971) investigated a circular plate and a rectangular bluff body in a uniform stream and found out that the form of the pressure distribution on the downstream face does not vary significantly with constraint. According to the observation mentioned above, if the size of model is made to be as small as possible to obtain a blockage ratio (the ratio of model reference area and cross-sectional area of the wind tunnel) not more than 10%, the pressure distribution should not be greatly affected by the blockage constraint. No blockage correction on pressure and drag coefficients is quoted in the present results.

2.3.2 Circular Cylinder in Two-Dimensional Uniform Cross Flow

The experiment was carried out in the wind tunnel of the Civil Engineering Department. A perspex circular cylinder, of diameter either 11cm or 16cm, spanned the tunnel horizontally on the centre-line of the tunnel. The cylinder was connected to a gear box so that the cylinder could be rotated around its longitudinal axis from 0° to 360°. The mean static pressures around the cylinder could thus be measured by using a single pressure tapping at the mid-span. Due to the fact that the cylinder was axisymmetrical, only half of the circumference would be measured. Both sizes of cylinder were investigated in their original smooth condition
and also when coated with sand. For a rough surface cylinder, the effect of different roughness ratio (the ratio of effective height of sand to diameter of cylinder) was investigated by using the same size of sand, between 0.6mm and 1.1mm, coated on the surface of the two different sizes of cylinders. The details of the experimental arrangement are shown in Figure 2.5. The experimental data are summarized in the following table.

<table>
<thead>
<tr>
<th>Model diameter</th>
<th>Roughness ratio</th>
<th>Blockage ratio</th>
<th>Diametral Reynolds number</th>
<th>Roughness Reynolds number</th>
</tr>
</thead>
<tbody>
<tr>
<td>11cm</td>
<td>$5.54 \times 10^{-3}$, $1.00 \times 10^{-2}$</td>
<td>0.08</td>
<td>$3.0 \times 10^4$, $9.0 \times 10^4$</td>
<td>200, 700</td>
</tr>
<tr>
<td>16cm</td>
<td>$3.75 \times 10^{-3}$, $6.88 \times 10^{-3}$</td>
<td>0.117</td>
<td>$4.0 \times 10^4$, $2.0 \times 10^5$</td>
<td>200, 1000</td>
</tr>
</tbody>
</table>

The results of pressure distribution are shown in Figures 2.6 and 2.7. The drag coefficients are shown in Figure 2.8. It may be seen that for both sizes of rough cylinder, the coefficient has fallen to a minimum and then risen again. The flow then is in the supercritical region although the final constant value of the coefficient may not have been fully attained.

For two different diameter cylinders with different roughness ratios, the results of drag coefficient versus Reynolds number as shown in Figure 2.9 are in good agreement with the results of Fage and Warsap (1930) but not in good agreement with Achenbach (1971) especially in the supercritical range. The blockage ratios were 0.125 for Fage and Warsap and 0.1666 for that of Achenbach. The blockage ratios are so close in those experiments that this factor could not be expected to produce the large differences in drag. Furthermore, the span to diameter ratio was 20.2 or 7.88 for Fage and Warsap, 3.33 in the experiments of Achenbach and 6.67 in the present tests. It is possible that three-dimensional effects are present only at the ends for longer models, but are present throughout the length for shorter model such as Achenbach's so that the flow does not approximate to two-dimensional case even at mid-span.
2.3.3 Circular Cylinder with Splitter Plate in Two-Dimensional Uniform Cross Flow

The experiment was carried out in the wind tunnel of the Mechanical Engineering Department. The smooth cylinders, of diameter 5.08cm or 2.54cm, spanned the tunnel vertically on the centre-line of the working section. The splitter plate which was 0.12cm thick and 30.5cm in length (six times of the diameter of the larger cylinder), made of mild steel, was installed on the centre-line. The method of mean pressure measurement was the same as in the previous test of circular cylinders. Details of the experimental arrangement are shown in Figure 2.10.

The results of pressure distribution are shown in Figures 2.11 and 2.12.

When a splitter plate is placed downstream of a circular cylinder in a cross flow, the wake will be altered from that found in the case of a plain cylinder. From the present results, the suction decreases and hence, the drag decreases compared with the plain cylinder. The base pressures for both sizes of cylinder with splitter plate are approximately 0.6 and the drag coefficients are 0.85. It is in good agreement with the similar work of Apelt and West (1973, 1975) who found that the base pressure was 0.6 and the drag coefficient was 0.8 in the same range of Reynolds number.

The reattachment of the flow after separation was investigated by placing the twin tube at a number of positions along the centre-line of the splitter plate in the windward direction. For the 2.54cm and 5.08cm diameter cylinders, the flow reattached to the plate at 4.5 and 4.75 times the corresponding diameter respectively. Vortex shedding from the plain cylinder is eliminated by the fitting of a splitter plate at subcritical flow regime.

Unfortunately, it was not possible to go to Reynolds numbers as high as for the cylinder without the splitter plate because at the higher tunnel speed, severe flutter developed at the trailing edge of the splitter plate. However, Roshko (1961) has investigated the influence upon pressure distribution and drag of a splitter plate in the transcritical flow region. The results indicated that the variation in the drag
coefficient is small, from 0.7 to 0.63 and that the effect on the pressure distribution is not significant. Similar results might then have been ensured with the present geometry, had circumstances permitted.

2.3.4 SEMI-CIRCULAR VAULT IN THIN BOUNDARY LAYER CROSS FLOW

For a semi-circular vault or a portion of a circular cylinder rested upon a base board, it was impossible to test in an arrangement similar to that used in the test of circular cylinder with a uniform upstream flow condition since the velocity must approach zero at the surface of the board. The possibility was investigated of installing a wooden base board parallel with the floor in the wind tunnel. The vault, therefore, could be firmly situated on the base board and a very thin boundary layer flow with a flow very nearly uniform outside the layer could be obtained by controlling the leading edge with a flap. The experiment was carried out in the wind tunnel of the Civil Engineering Department. Details of the base board are shown in Figure 2.13. Two perspex semi-circular vaults of diameter 16cm and 32cm with smooth and rough surfaces were tested. Seventeen pressure tappings were drilled in the surface of vault at an angular position of every 10°. Sand of the size, 0.6mm to 1.1mm, was coated on to the surface of the two different sizes of vaults to obtain different roughness ratios. The experimental arrangement is given in Figure 2.14. A semi-circular vault of length equal to the full tunnel width rested upon the base board. The mean velocity profiles of the approaching flow were determined by a pitot traverse in each testing case and in three different positions along the width of the base board. The typical mean velocity profile is shown in Figure 2.15 and represented an approximately two-dimensional flow. Further, the tests data are summarized and are given in the following table.

<table>
<thead>
<tr>
<th>Model diameter</th>
<th>Roughness ratio</th>
<th>Blockage ratio</th>
<th>Diametral Reynolds number</th>
<th>Roughness Reynolds number</th>
</tr>
</thead>
<tbody>
<tr>
<td>16cm</td>
<td>3.75x10⁻³</td>
<td>0.07</td>
<td>3.0x10⁴</td>
<td>150</td>
</tr>
<tr>
<td></td>
<td>6.88x10⁻³</td>
<td></td>
<td>1.2x10⁵</td>
<td>600</td>
</tr>
<tr>
<td>32cm</td>
<td>1.88x10⁻³</td>
<td>0.14</td>
<td>4.0x10⁴</td>
<td>100</td>
</tr>
<tr>
<td></td>
<td>3.44x10⁻³</td>
<td></td>
<td>2.5x10⁵</td>
<td>650</td>
</tr>
</tbody>
</table>
The results of drag coefficients are shown in Figure 2.16 and pressure distributions are shown in Figures 2.17 and 2.18.

Comparing the pressure distribution around the semi-circular vault under a thin boundary layer flow and the circular cylinder under a uniform flow in a similar range of Reynolds number, the distribution pattern shows some obvious similarities and the following detailed observation may be made:

(1) A portion of the front part of the semi-circular vault is affected by the boundary layer so that the positive pressure is markedly reduced in comparison with the circular cylinder either with or without splitter plate.

(2) In particular, the pressure at the leading edge of the semi-circular vault is not +1.0 as it is for a circular cylinder but tends to zero. This is because the velocity in the boundary layer near the base board is small compared with the freestream velocity and is zero on the surface of base board itself.

(3) For the semi-circular vault rested on the base board, the base pressures between approximately 120° and 180° remain fairly constant. The base pressures decrease to nearly half of the value in the case of plain circular cylinder but are close to the circular cylinder with splitter plate.

From the results of drag variation with Reynolds number, Figure 2.16, the drag decreases steadily with increasing Reynolds number for the smooth vault. For the rough vault, the drag decreases rapidly and increases again in the range of Reynolds number near 10^5. The drag becomes nearly constant as the Reynolds number increases above 10^5. In addition, the vault with larger diameter and hence, larger blockage ratio, has a higher value of drag. This fact shows that the blockage effect influences the pressure distribution around the body. For the smooth vault in the subcritical region of flow, the blockage effect may be high because the pressure distribution depends upon Reynolds number. In the situation of the rough vault at Reynolds number above 10^5 with approximately constant
drag, the blockage effect should be small because the pressure distribution is independent of Reynolds number.

From the pressure distribution pattern and the variation of drag coefficient with Reynolds number, it is possible to verify that a semi-circular vault within a thin boundary layer flow shows some similar features to a circular cylinder within a uniform flow. It also shows that while the roughness Reynolds number between 200 and 1000 and the diametral Reynolds number is above $10^5$, an approximately constant drag is obtained so that a transcritical range of flow is simulated. If then the approaching boundary layer flow could be changed to give a reasonable simulation of the earth's boundary layer flow, the characteristics of flow around the semi-circular body rested on the base board should provide a reasonable simulation of the flow of the wind around a full-scale barrel vault.
Flow development around two-dimensional smooth circular cylinder with increasing Reynolds number (from E.S.D.I. 70013, 1977)

Variation of drag coefficient with Reynolds number for two-dimensional smooth circular cylinder (from Tritton, 1977)
Figure 2.3 Variation of drag coefficient with Reynolds number for two-dimensional smooth and rough circular cylinder (from E.S.D.I. 70013, 1977)

Figure 2.4 Details of a twin tube
Figure 2.5  Experimental arrangement of circular cylinder in two-dimensional uniform cross flow
Figure 2.6 Pressure distributions around circular cylinder of 11cm diameter in two-dimensional uniform cross flow.
Figure 2.7 Pressure distributions around circular cylinder of 16cm diameter in two-dimensional uniform cross flow
Figure 2.8 Variation of drag coefficient with Reynolds number—present results of circular cylinders of 11cm and 16cm diameter in two-dimensional uniform cross flow

Figure 2.9 Comparison of present result with previous work on circular cylinder in two-dimensional uniform cross flow
Figure 2.10 Experimental arrangement of circular cylinder with splitter plate in two-dimensional uniform cross flow
Figure 2.11 Pressure distributions around circular cylinder of 2.54cm diameter with splitter plate in two-dimensional uniform cross flow

Figure 2.12 Pressure distributions around circular cylinder of 5.08cm diameter with splitter plate in two-dimensional uniform cross flow
Figure 2.13 Details of a base board

Figure 2.14 Experimental arrangement of semi-circular vault in thin boundary layer cross flow
Figure 2.15  Velocity profile of approaching thin boundary layer flow at section A-A (referred to Figure 2.14)

Figure 2.16  Variation of drag coefficient with Reynolds number of semi-circular vaults of 16cm and 32cm diameter in thin boundary layer cross flow
Figure 2.17 Pressure distributions around semi-circular vault of 16cm diameter in thin boundary layer cross flow
Figure 2.18 Pressure distributions around semi-circular vault of 32cm diameter in thin boundary layer cross flow

(i) Smooth surface

(ii) Rough surface $\epsilon/D = 1.88 \times 10^{-3} \sim 3.44 \times 10^{-3}$
Plate 2.1  Wind tunnel in the Hydraulics Laboratory of the Civil Engineering Department
3.1 INTRODUCTION

Barrel vaults with two different geometries were the object of this main experimental study. The models were tested in a simulated boundary layer flow. Three different directions of oncoming wind and three different ratios of length to chord for each type of barrel vault were investigated. Surface roughness on the model was used to simulate a high Reynolds number flow (expected to be in the supercritical flow region) in the wind tunnel. The measurement of mean pressure distribution on the models was the principal aim of the experimental investigation.

3.2 EXPERIMENTAL ARRANGEMENT

Experiments were carried out in the wind tunnel in the Hydraulics Laboratory of the Civil Engineering Department. Details of the wind tunnel have been mentioned in Chapter 2, Section 2.3.1.1.

3.2.1 MODELS

Barrel vaults of two different forms were studied. The cross-section of the barrel vaults was a circular arc subtending an angle $\psi$ at the centre. Two different value of $\psi$, $180^\circ$ and $120^\circ$, were considered.

The two types of models, with subtended angles of $180^\circ$ and $120^\circ$, were made from plastic circular pipe of 0.15m and 0.20m diameter respectively. This choice of diameter gave an approximately equal chord in these two different type of models. Blockage ratio was of the same order, 10% or
less in both cases.

Three different lengths of each type of barrel vault were used in the investigation of three-dimensional flow effects. First, one having the longitudinal length exactly equal to the span of the tunnel was made to obtain a two-dimensional flow condition. The other two were made so as to have a value of length to chord ratio of 1 and 2 respectively.

In considering the surface roughness of models, the plastic material could be taken as representing a smooth surface condition. To obtain a rough surface condition, sand applied as described in Chapter 2, Section 2.3.1.3, was again used to produce uniformly distributed roughness on the models. In this case, the sand was sieved from ordinary builders sand using sieves giving an effective diameter of sand between the upper and lower sieve size of 1.0mm and 2.0mm.

For the two-dimensional models, with longitudinal length the same as the tunnel height, nineteen pressure tappings were drilled at equal intervals around the circular arc at the mid-span of the model. For the three-dimensional models, with length to chord ratio of 1 and 2, pressure holes were drilled over the whole surface (included two sides as well). All pressure tappings which were made by inserting stainless steel tubes flush with the model's surface were in 1.6mm outside and 1.2mm inside diameter. Plastic tubing was used to transmit the pressure to the mean pressure measurement system mentioned below. Details of the models and the position of pressure holes are shown in Figures 3.1 and 3.2 and Plate 3.1. The relevant data of the testing models are summarized in Table 3.1.

3.2.2 Wind Tunnel Simulation of Atmospheric Boundary Layer

In order to study the flow around barrel vaults immersed in the atmosphere, there must be a means of simulating a boundary layer in a wind tunnel. The simulation system fitted in the tunnel and used for the present experiments was based on that developed originally by Counihan (1969). This system consisted of a barrier wall, vorticity generators and a surface covered by a regular pattern of roughness elements and is shown
clearly in Figure 3.3 and Plate 3.2. For the 'rough' boundary layer which was the principal object of investigation, a castellated barrier was used and the surface roughness was represented by 'lego' blocks. On the other hand, for purposes of comparison, it was possible to produce a rather different boundary layer of the same thickness using a simple flat-topped barrier in conjunction with the plywood tunnel wall to give a 'smooth' boundary layer. The whole simulation system was mounted on the tunnel wall. The barrier was installed just downstream of the inlet end of the working section together with vorticity generators behind. This system is able adequately to simulate a naturally grown boundary layer approximately equal in height to the vorticity generators. However, the 'smooth' boundary layer in the wind tunnel was not as fully developed as the 'rough' boundary layer. Most of the experimental work therefore involved the investigation of flow around barrel vaults mounted on the rough surface. Details of the arrangement, results and discussion of this system have been reported by Dianat(1980). Some relevant parameters of the 'rough' and 'smooth' boundary layers are given in Table 3.2 and the velocity profiles and the longitudinal turbulence distributions of each of the boundary layers as they approach the models were shown in Figure 3.4.

Thus the system produced a boundary layer flow of depth equal to several times the intended height of the models. For many full-scale barrel vaults in the natural wind, the thickness of the earth's boundary layer is likely to be considerably greater relative to building height. In studies such as the present however, which are concerned with mean pressure distributions on curved bodies in the Reynolds number-independent region, it is probably the velocity distribution, conveniently expressed here in the form of power-laws, $V \propto h^{1/3}$, that is of more importance. For the simulation system used for the present experiments, the 'rough' boundary layer has a velocity distribution $V \propto h^{1/7}$, similar to the distribution in suburban areas or small towns and the 'smooth' boundary layer has a velocity distribution $V \propto h^{1/4}$, similar to the distribution over open sea or bare, flat land.

With the appropriate velocity distribution and a flow regime well into the supercritical range, the pressure distribution should give a reasonable indication of that to be expected full-scale in the natural wind. The
### Table 3.1 Relevant data of the models

<table>
<thead>
<tr>
<th>Model type</th>
<th>Subtended angle (ψ)</th>
<th>Radius of curvature (R)</th>
<th>Chord (S_y)</th>
<th>Rise/Chord (H/S_y)</th>
<th>Blockage ratio</th>
<th>Roughness ratio</th>
<th>Length (S_x)</th>
<th>Length/Chord (S_x/S_y)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>180°</td>
<td>0.075m</td>
<td>0.15m</td>
<td>0.5</td>
<td>7.0%</td>
<td>7.3x10^{-3}</td>
<td>1.372m</td>
<td>9.147</td>
</tr>
<tr>
<td></td>
<td>120°</td>
<td>0.10m</td>
<td>0.17m</td>
<td>0.289</td>
<td>4.7%</td>
<td>5.5x10^{-3}</td>
<td>0.30m</td>
<td>2.0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Length (S_x)</th>
<th>0.15m</th>
<th>1.372m</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length/Chord (S_x/S_y)</td>
<td>1.0</td>
<td>8.07</td>
</tr>
</tbody>
</table>

### Table 3.2 Parameters of the 'smooth' and 'rough' boundary layers

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Thickness (mm)</th>
<th>Displacement thickness (mm)</th>
<th>Momentum thickness (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>'smooth' boundary layer</td>
<td>256</td>
<td>28.6</td>
<td>21.4</td>
</tr>
<tr>
<td>'rough' boundary layer</td>
<td>298</td>
<td>59.5</td>
<td>38.3</td>
</tr>
</tbody>
</table>

Table 3.1 Relevant data of the models

Table 3.2 Parameters of the 'smooth' and 'rough' boundary layers
whole question of the exact effect of the intensity and scale of turbulence in the upstream flow does certainly, however, merit detailed systematic study. The best practical solution, for wind tunnel testing, will probably be that of simulating only the lower fraction of the earth's boundary layer correctly to the same scale as the model over the entire width of the tunnel. The parameters of turbulence could then be more nearly correct throughout.

3.2.3 MEASUREMENT INTRUMENTATION

3.2.3.1 FREESTREAM VELOCITY

The reference freestream velocity could be obtained from the dynamic pressure which was measured by a pitot static tube and its position in the wind tunnel is shown in Figure 3.5. The dynamic pressure of the reference freestream flow was continuously monitored during the experiments to ensure steadiness.

3.2.3.2 MEAN PRESSURE

The mean pressure data were obtained automatically by means of the Furness micromometer and a Solartron time domain analyser JM 1860. The pressure tubing from the models as well as the tubes from a pitot static probe used to measure the reference dynamic pressure could be connected to the micromometer. The reference pressure was taken from the static tapping of the pitot static tube above, set in the freestream remote from the body. Before each series of experiments, the pressure measuring system was calibrated statically to ensure proper operation and linearity of its response.

3.2.3.3 REATTACHMENT OF SEPARATED FLOW

The reattachment of flow after separation from the models' surface
was investigated by two methods. These two methods were simple and easy to handle in the test and served to establish a close approximation to the position of reattachment. For the models tested under a 'smooth' boundary layer, the reattachment was found by estimating the position on the base board at which the twin tube (previously described in Chapter 2, Section 2.3.1.2) showed a zero pressure difference. For the models tested under a 'rough' boundary layer, the twin tube was difficult to place on the rough lego board so that a probe with silk thread at one end was used instead. By moving the probe along the lego board with the silk thread touching the surface of the board and observing when the silk thread changed from one direction to the other side, the approximate position of reattachment was obtained.

3.2.4 Setting Up and Testing

The two simulated boundary layers were generated on the wind tunnel wall. Models were firmly mounted on this wall with their centre kept to the same position in each test. A support was provided outside the tunnel wall to fasten the model securely. Details of the setting up in the wind tunnel are shown in Figure 3.5 and Plate 3.3.

The two-dimensional models, with length the same as the tunnel height, were tested at several different dynamic pressures (hence, at different velocities and Reynolds numbers). Only one direction of oncoming flow, normal to the axis of the model, was investigated. Models with length to chord ratio of 1 and 2 were tested at approximately the same dynamic pressure (i.e., the maximum value that could be obtained in the wind tunnel) with three different oncoming flow directions; they were 0°, 45° and 90° to the axis of the model and were achieved by rotating the axis to set it at the appropriate angle to the centre-line of the tunnel. The position of the model under the different oncoming wind flow directions is shown in Figure 3.6.

For each type of model, a smooth and a uniformly distributed rough surface were studied in the experimental analysis. The smooth model experiments were expected to give a useful reference for the rest of the
experiments and also to assess the experimental procedures before the rough model experiments. The rough models were expected to obtain mean pressure distributions reasonably similar to the results of a prototype under a high Reynolds number flow.

3.3 EXPERIMENTAL RESULTS

3.3.1 PRESENTATION OF RESULTS

All pressure measurements from the surface of the models were reduced to pressure coefficient form using the definition $C_p = (p - p_o)/(\frac{1}{2} \rho V^2)$ where $p$ is the local pressure on the model surface, $p_o$ is the reference static pressure and $\frac{1}{2} \rho V^2$ is the dynamic pressure. In deciding which dynamic pressure to use, problems occur for barrel vaults immersed in a thick boundary layer. For the case of a circular cylinder in uniform flow, there is only one velocity affecting the surface pressures, that is, the freestream velocity. It is, therefore, sensible to use the dynamic pressure based on the freestream velocity. For the case of barrel vaults, the surface pressures are affected by the velocity profile of the approaching flow. According to the Clause 5.5.2 from the British Standard Code of Practice CP3: Chapter V: Part 2-1972, it is suggested that the local velocity to be used should be that at a height equal to that of the top of the structure or, alternatively, that the height of the structure may be divided into convenient parts. The best choice of dynamic pressure in the pressure coefficient is not obvious; the pressure distribution pattern, however does not change no matter which dynamic pressure is used. It has been decided to use the approaching freestream velocity in the present study because the pressure coefficients which are referred to some other local velocity within the boundary layer may be simply deduced as follows:
\[ C_{P_{\text{freestream}}} = \frac{p - p_0}{\frac{1}{2} \rho V_{\text{freestream}}^2} \]

\[ C_{p_{\text{local}}} = \frac{p - p_0}{\frac{1}{2} \rho V_{\text{local}}^2} \]

\[ C_{P_{\text{freestream}}} \cdot \frac{1}{2} \rho V_{\text{freestream}}^2 = C_{p_{\text{local}}} \cdot \frac{1}{2} \rho V_{\text{local}}^2 \]

\[ C_{p_{\text{local}}} = \left( \frac{V_{\text{local}}}{V_{\text{freestream}}} \right)^2 \]

where \( \left( \frac{V_{\text{local}}}{V_{\text{freestream}}} \right) \) of the 'rough' and 'smooth' boundary layers are given in Figure 3.4.

The mean pressure data would be analysed to evaluate the drag coefficient \( C_D \) (by integrating the pressure distribution using Simpson's rule), the base pressure coefficient \( C_{pb} \) (defined as the average of the pressure coefficients in the wake region), the minimum pressure coefficient \( C_{pm} \) and the approximate location \( \theta_W \) of the beginning of wake region, defined as suggested by Niemann(1971) (shown in Figure 3.7). The wake angle \( \theta_W \) was found to be useful both as a parameter characterising the pressure distribution and as a rough indicator of the mean location of separation, since separation usually occurs within a short distance. Moreover, an examination of these parameters is important not only since they are useful in summarizing the main characteristics of the pressure distribution but also shed some light on the overall effects of surface roughness.

Figure 3.7 Definition of parameters characterising pressure distribution
Finally, in all the data presented, no correction has been made for tunnel blockage. In the case of the models which were expected to be tested in a simulated high Reynolds number flow condition in which the pressure distribution is independent of Reynolds number, it is perhaps hardly necessary to correct for tunnel blockage while the blockage ratio is less than 10% (suggested by McKeon and Melbourne, 1971, as has been discussed in Chapter 2, Section 2.3.1.5). However, some experiments might be at Reynolds numbers in the region in which the pressure distribution is dependent on Reynolds number. Since no suitable established method could be applied in the blockage correction for a Reynolds number-dependent flow, it is better to present all the results as actually measured.

Further, for the purposes of comparing the present results of barrel vaults to circular cylinder, Reynolds numbers which are presented in this chapter are based on the approaching freestream velocity and the diameter of the full circular section (i.e. twice the radius of curvature of the barrel vaults).

3.3.2 MODELS WITH SUBTENDED ANGLE OF 180°

Experiments of models with subtended angle of 180° (that is, in a semi-circular curved form) were tested separately in the 'rough' and 'smooth' boundary layers. A smooth surface and a uniformly distributed roughness surface of roughness ratio between 7.3x10^{-3} to 1.3x10^{-2} were studied in the experimental analysis. Results of the experiments are given in the following sections.

3.3.2.1 RESULTS OF TWO-DIMENSIONAL MODELS

A. Models Immersed in 'Smooth' Boundary Layer

Mean pressure distributions on the surface of two-dimensional models with smooth and rough surfaces are shown in Figures 3.8 and 3.9. Drag coefficients were evaluated from the mean pressure results and the
parameters characterising the mean pressure distributions of smooth and rough surface models were plotted against Reynolds number are shown in Figure 3.10.

B. Models Immersed in 'Rough' Boundary Layer

Mean pressure distributions on the surface of two-dimensional models with smooth and rough surfaces are shown in Figures 3.11 and 3.12. The mean pressure distributions had been analysed; the plot of drag coefficients and parameters versus Reynolds number are shown in Figure 3.13.

3.3.2.2 RESULTS OF THREE-DIMENSIONAL MODELS

Wind tunnel tests of the three-dimensional models (i.e. models of length to chord ratio of 1 and 2) were carried out with Reynolds number of the order of $10^5$ and roughness ratio between $7.3 \times 10^{-3}$ to $1.3 \times 10^{-2}$. It is expected that a pressure distribution pattern reasonably close to that of the prototype could be obtained in this situation, particularly with the rough boundary layer.

A. Models Immersed in 'Smooth' Boundary Layer

Mean pressure distributions on the surface of model for different oncoming wind directions were obtained. Results of smooth surface models are shown in Figures 3.14 to 3.17 for oncoming flows at $0^\circ$ and $90^\circ$ to the axis of the model. Results of rough surface models are shown in Figures 3.18 to 3.23 for oncoming flows at $0^\circ$, $45^\circ$ and $90^\circ$ to the axis of the model. For the rough models, positions of the lines of reattachment on the plane downstream are shown in Figure 3.24 for oncoming flows at $45^\circ$ and $90^\circ$ to the axis of the model.

B. Models Immersed in 'Rough' Boundary Layer

Only the rough surface models were tested in the 'rough' boundary
layer in three different oncoming flow directions. Mean pressure distributions on the surface of models and position of downstream reattachment were investigated. Those results are shown in Figures 3.25 to 3.31.

3.3.3 MODELS WITH SUBTENDED ANGLE OF 120°

Experiments of models with subtended angle of 120° were carried out only in the 'rough' boundary layer. A smooth surface and a uniformly distributed roughness surface of roughness ratio between 5.5x10⁻³ to 1.0x10⁻² were tested. Results of the experiments are given in the following sections.

3.3.3.1 RESULTS OF TWO-DIMENSIONAL MODELS

Mean pressure distributions on the surface of two-dimensional models with smooth and rough surfaces are shown in Figures 3.32 and 3.33. Drag coefficients were evaluated from the mean pressure results and the parameters characterising the mean pressure distributions of smooth and rough surface models were plotted against Reynolds numbers and are shown in Figure 3.34. Since for the purposes of comparing these models with those subtending an angle of 180°, the angle used as the base for plotting mean pressure distribution was measured from the frontal stagnation point plus 30°, i.e. for both types of model the angle for any point on the surface represents the angle between the vertical and the tangent to the surface at that point.

3.3.3.2 RESULTS OF THREE-DIMENSIONAL MODELS

Again, the three-dimensional models (i.e. models of length to chord ratio of 1 and 2) were tested in the 'rough' boundary layer with Reynolds number of the order of 10⁵. Models with a smooth surface and with a rough surface of the same roughness ratio as for the two-dimensional experiments were employed in the investigation. Again, it is
expected that a mean pressure distribution similar to that of a prototype could be obtained in this circumstance. Mean pressure distributions on the smooth and rough surfaces of the models for three different oncoming wind directions are shown in Figures 3.35 to 3.46. Positions of downstream reattachment are shown in Figure 3.47 for oncoming flows at 45° and 90° to the axis of the model.

3.4 DISCUSSION OF EXPERIMENTAL RESULTS

Before the discussion of the above experimental results, it is necessary to mention that most flows have a three-dimensional character principally as a result of their contact with boundaries. For example, if a laminar flow consisting of an air mass displaced uniformly as a single unit encounters an object, it will be diverted in several directions and the passage of that flow along a surface sets up boundary layer velocity gradients. In addition, three-dimensionality is clearly inherent in turbulent flows. However, some actual flows retain certain two-dimensional features, at least to a first approximation. For example, considering the present case of a long barrel vault in two-dimensional structural form with flow normal to one face, the mean flow may be considered for practical purposes as two-dimensional except near the ends of the vault. In the following sections, the two-dimensional and three-dimensional structural form of models are considered to have two-dimensional and three-dimensional flow patterns respectively.

3.4.1 MODELS IN TWO-DIMENSIONAL FLOWS

3.4.1.1 PRESSURE DISTRIBUTIONS AND DRAG COEFFICIENTS

In the previous sections, the mean pressure distributions of the two different types of models with smooth and rough surface conditions at Reynolds numbers between 3x10⁴ to 1.5x10⁵ were presented. Since the speed of wind tunnel is limited, the Reynolds numbers obtained could not have been increased in the present experimental arrangement. However,
the tests still show that the present results are consistent among themselves and the distribution patterns of mean pressures are similar in a general way to the case of circular cylinder immersed in a two-dimensional uniform cross flow except the pressure at the leading edge of the barrel vault is not +1.0 due to the effect of the boundary layer flow (has been discussed in Chapter 2, Section 2.3.4).

As seen from Figures 3.10, 3.13 and 3.34, $C_{pb}$, $C_{pm}$ and $\theta_w$ show a generally similar systematic variation with Reynolds number for all the smooth models and for all the rough models. For a circular cylinder in uniform cross flow, drag coefficient is closely related to $C_{pb}$. This fact also happens on the present case for the barrel vaults in the 'smooth' and 'rough' boundary layers.

Examination of the results for models with smooth surface in the 'smooth' or 'rough' boundary layer shows that $|C_{pm}|$ increase and $|C_{pb}|$ decreases and $\theta_w$ increases slowly with increasing Reynolds number. As $\theta_w$ may be regarded as an approximate indicator of location of separation, the plot of $\theta_w$ versus Reynolds number indicates that the separation point moves downstream yielding a narrower wake so that $|C_{pb}|$ decreases and hence, the drag coefficient decreases. It is not surprising that these features are similar to the case of uniform flow past a circular cylinder at subcritical flow region since these barrel vaults were in circular curved form and the Reynolds number of flow lay in the subcritical range as for the smooth circular cylinder. The flow past the smooth models, then, is strongly dependent upon Reynolds number in this range.

Investigating the results of rough surface model with subtended angle of 180° in the 'smooth' and 'rough' boundary layers (shown in Figures 3.10 and 3.13) and with subtended angle of 120° in the 'rough' boundary layers (shown in Figure 3.34), it clearly indicates that surface roughness has a significant influence on pressure distribution whether in the 'smooth' or 'rough' boundary layer. Speaking overall, the influence of surface roughness on positive pressures at the front part of barrel vaults is not significant. For $|C_{pm}|$ and $|C_{pb}|$, however, the manner of variation with Reynolds number is clearly different from the smooth surface models.
Observing the results of the rough surface models with subtended angle of 180° and 120° immersed in the 'rough' boundary layer, it shows that $|C_{pb}|$ becomes a minimum at the Reynolds number for which the corresponding drag coefficient curve in Figures 3.13 and 3.34 indicates a minimum value of $C_D$. As the Reynolds number increases beyond this value, $|C_{pm}|$ continuously decreases while $|C_{pb}|$ and $C_D$ increase, until all attain nearly constant values at Reynolds number near the order of $10^5$. This similarity and connection between the behaviours of $C_{pm}$, $C_{pb}$ and $C_D$ have been observed in the rough circular cylinder in uniform cross flow by Güven(1975).

Furthermore, the observed variation of the angle $\theta_w$ with Reynolds number shows that $\theta_w$ changes with Reynolds number in a manner corresponding to the changes in $C_{pb}$, $C_{pm}$ and $C_D$, i.e. $\theta_w$ starts to decrease from the peak value at a Reynolds number just beyond that for the minimum value of $C_D$. A similar variation of $\theta_w$ with Reynolds number for rough circular cylinder has been observed by Achenbach(1971). In contrast with the smooth models, then, the rough models show little dependence upon Reynolds number at values above $10^5$.

The detailed pressure distributions indicate further that, for model with subtended angle of 180° and of large roughness ratio between $7.3 \times 10^{-3}$ to $1.3 \times 10^{-2}$ at Reynolds number near the order of $10^5$, the position of separation moves upstream of $\theta_w$ around 90° for 'smooth' boundary layer and 100° for the 'rough' boundary layer while the location of minimum pressure $C_{pm}$ remains substantially unaffected at 75° to 80°. It is in close agreement with the results of Roshko(1961) and Niemann(1972) for the smooth circular cylinder in flow of high Reynolds number of the order of $10^7$, i.e. $\theta_w$ of 100° or 101° and the location of minimum pressure $C_{pm}$ at 75° or 72°. For the model with subtended angle of 120°, no similar previous work exists for comparison. However, $\theta_w$ and location of minimum pressure $C_{pm}$ show a value close to that of the model with subtended angle of 180°. Since the boundary layer separation depends on the presence of an adverse pressure gradient in a streamwise direction along the boundary, it appears that as the barrel vaults were both of circular curved form but with different rise to chord ratio, the separation occurs in a similar region in both cases, i.e. one where the surface makes a similar angle to the main flow.
The experimental analysis of barrel vaults in the two-dimensional boundary layer flows confirms that the flow features of the model studies at present follow similar trends to the case of uniform flow over a circular cylinder so that in a similar manner prototype mean pressure distributions could be reproduced on scaled models by roughening the model surface. The roughness ratios and the highest Reynolds number used in the two-dimensional flows tests were, therefore, employed in the tests of three-dimensional models to obtain a high Reynolds number flow results.

3.4.1.2 Different Boundary Layers

Using the local velocity within the boundary layer at a height corresponding to the height of the barrel vaults and the mid-height of the barrel vaults instead of the freestream velocity to compute the dynamic pressure, the pressure coefficients of the models with subtended angle of 180° (in a semi-circular curved form) and with smooth and rough surface conditions in the 'smooth' and 'rough' boundary layer flows of Reynolds number near the order of 10^5 are obtained and are shown in Figure 3.48. In this case the following observation concerning the effect of the two different boundary layers on the pressure distribution may be made:

(1) For both smooth and rough models, the positive pressure region in the front of the barrel vault appears to have a slightly larger value for the barrel vault in the 'smooth' boundary layer.

(2) For both smooth and rough models in the 'rough' boundary layer, there is a higher value of the minimum pressure and the base pressure.

It must be remembered that the different boundary layers have different velocity distributions. Even if the reference velocity and dynamic head are taken at the same point in each case, other velocities will be dependent and hence the pressure coefficients are not closely comparable directly.
3.4.1.3 COMPARISON WITH CODES OF PRACTICE

As mentioned in previous chapter, some of the codes provide the pressure coefficients of curved structures. Most of the pressure coefficients are applied on the two-dimensional flow with oncoming flow normal to the axis of the vault. For arbitrarily chosen barrel vaults having the same geometry as the testing models and with assigned dimensions, a comparison of the present results obtained in the 'rough' boundary layer to the American code (ANSI A58.1-1972) and the Australian code (AS 1170, Part 2-1975) was made. The barrel vaults, having subtended angles of 180° and 120°, were set to be of 10m height. The basic wind speed was chosen to be 40m/sec. at the top height of the vaults. The 'rough' boundary layer in the present experimental work was probably equivalent to the case of Exposure B (suburban areas, towns, city outskirts, wooded areas and rolling terrain) given in American code and lay between the Terrain category 2 (open terrain with well scattered obstructions having heights generally 1.5 to 10m) and Terrain category 3 (terrain with numerous closely spaced obstructions having the size of domestic houses) given in Australian code. With this assumption, the design wind speeds at the top height of the vaults were then obtained. The local pressures around the examplified barrel vaults with subtended angles of 180° and 120°, obtained from the pressure coefficients given by the present wind tunnel tests and the codes are shown in Figure 3.49. The comparison shows that the pressure coefficients provided by the American code and the Australian code are of an approximate character, not as precise as might be desired for the understanding of the wind pressure distribution on a barrel vault.

Further, the Indian code (IS:875-1964) implies the pressure coefficients for the central half of the vault (springing from the ground level) are independent of varying rise to chord ratio; this is incorrect.

3.4.2 MODELS IN THREE-DIMENSIONAL FLOWS

In the experimental analysis of three-dimensional models, tests on models with smooth and rough surfaces were carried out separately. Since
it has been proved that the roughness approach could be employed in the present geometrical form of models, the models with smooth surface were used for reference. Because of the considerable complexities involved in the three-dimensionality of flow, discussion will concentrate mainly on the models with rough surface.

3.4.2.1 PRESSURE DISTRIBUTIONS

In the previous sections, the mean pressure distributions, based on freestream dynamic pressure, over the complete barrel vaults are shown as lines of constant pressure. Under different oncoming wind directions, the mean pressure distributions on the barrel vaults of two different geometries, boundary layers and length to chord ratios show a similar pattern but with some differences in details.

With the oncoming wind blowing normal to the axis of the barrel vault, theoretically, the pressure distribution should be symmetrical about the centre-line of the barrel vault in the windward direction. However, the present results show a pressure distribution that is not exactly symmetrical but only approximately so. This might be caused by the misalignment during the setting up of the models or the fact that the models might not be perfectly symmetrical. Observing the cross-sectional pressure distributions of each geometrical type of models, it is clearly seen that the distribution had pronounced similarities to the corresponding two-dimensional model. Near the centre part of models, the cross-sectional pressures were higher than the portion near two ends and tend to become uniform. Since this portion was far from the two sides, the end effects are probably small. Detailed investigation of the pressure distribution on the two-dimensional models (i.e. of high length to chord ratios) and on the models of length to chord ratios of 1 and 2 shows that for the two different geometrical forms of barrel vaults, the peak suction and the base pressure at the mid-span of the models were higher for model of a higher length to chord ratio. This indicates that the effect of three-dimensionality is more significant for a short vault.

With the oncoming wind blowing parallel to the axis of the barrel
vault, the windward end wall experienced positive pressures and the leeward end wall experienced suctions. Pressures on the vault surface were all suctions with higher values at the leading part. This indicates that the flow first strikes the windward side wall and then separates at the edge of the wall. In addition, observing the pressure distributions of the smooth and rough surface models for this wind direction, it is found out that they showed a similar pattern and a similar magnitude because the edge separation is in this case independent of Reynolds number or surface roughness.

With the oncoming wind at 45° to the axis of the barrel vault, the distribution patterns were absolutely different to the other two oncoming wind directions. If a cross-section is taken from the upper windward corner along the wind flow direction, it could be observed that there were positive pressures at the front part with a peak suction in the middle and the base pressure at the rear; it may be considered that in the portion of the body where the cross-section, in the direction of the wind, forms a complete curved surface elliptical rather than circular, the flow past that region must be in a generally similar pattern to the case of circular curved body. Considering the region above that section, the flow first strikes the windward end wall then separates at the edge of the wall. Therefore, positive pressures occurred at the windward end wall and suctions occurred on the vault surface. For the region further to the rear, the cross-section is still a curve but not so complete as in the middle portion of the body so that positive pressures occurred in a small area at the front part and a fairly uniform suction occurred on most of the surface. Further, comparing the positive pressure, the peak suction and the base pressure to those occurring with the wind direction normal to the axis of the barrel vault, it is found out that the present wind direction gives a lower positive pressure but a higher peak suction and base pressure on the vault surface.

3.4.2.2 Different Boundary Layers

Rough surface barrel vaults with subtended angle of 180°(a semi-circular curved form) were tested in both the 'smooth' and 'rough' boundary layers.
For each of the three different oncoming wind directions and for both the different length to chord ratios, the mean pressure distributions display a generally very similar pattern in these two boundary layer flows.

Using the local velocity within the boundary layer at a height corresponding to the height of the barrel vaults and to the mid-height of the barrel vaults instead of the freestream velocity to compute the dynamic pressure, some important pressure coefficients are listed in Table 3.3 for comparison.

The observation of Figures 3.18 to 3.31 and Table 3.3 shows that the pressure distributions of barrel vaults (i.e. magnitude of pressure, position of separation ... and so on) in the two different boundary layer flows are different in detail. Further, if a reference dynamic pressure at the same height is chosen to compute the pressures on the vault surface, then roughly speaking, the 'rough' boundary layer flow gave slightly higher pressures in all the studied cases.

3.4.2.3 LENGTH TO CHORD RATIOS

As seen from the mean pressure distributions given in the previous section, different lengths of barrel vaults would not alter the general character of pressure distribution but the pressures would be slightly changed.

With the wind normal or at 45° to the axis of the barrel vaults, for the models of higher length to chord ratio, 2 in this case, a higher positive pressure, peak suction and base pressure on the vault surface were obtained. The pressures on the end walls, on the other hand, were very much the same for both the length to chord ratios, 1 and 2. Therefore, there is no doubt that part of the three-dimensionality is caused by the boundary side flows.

With the wind parallel to the axis of barrel vault, pressures on the windward end wall and the curved surface were nearly the same for models with different length to chord ratios. With the model with the shorter length to chord ratio of 1, the suctions on the leeward end wall were higher
Table 3.3 Comparison of the pressure coefficients of rough surface barrel vaults with subtended angle of 180° and with roughness ratio $e/D=7.3\times10^{-3}$ and $1.3\times10^{-2}$ obtained in the 'smooth' and 'rough' boundary layers.
than for the model with length to chord ratio 2. This could be explained
by the possibility that the flow separated at the end wall boundary and
was slowed down while passing downstream. Therefore, for longer models,
the velocity of the separated flow at the leeward end could be less than
for the short models.

3.5 CONCLUDING REMARKS

Since little relevant work on wind pressure distributions on barrel
vaults has been carried out, the present work is a first step in this
research study. Perhaps the most useful contribution of the present
results is the understanding of surface roughness effects not only on the
mean flow around circular cylinders, but also on the vaults of circular
curved form. For large barrel vaults of circular curved form in a
'rough' or 'smooth' boundary layer flow, in the supercritical regime, the
surface roughness effects arise from the greater retardation of the
boundary layer flow on the surface by larger surface roughness, which
results in this pattern of flow being established at a lower Reynolds
number. Prototype mean pressure distributions can therefore be
reproduced on scaled models by roughening the model surface with a suitable
size of roughness material. Although in the present experimental work,
no attempt has been made to predict the separation point, the formation of
the wake nor to estimate what effect the displacement or momentum
thickness or the turbulence intensity or different roughness ratios may
have on the pressure distributions, the results of the three-dimensional
models may not have represented the mean pressures on prototype very
precisely but gave an idea of the distribution pattern.
Figure 3.1 Details of models with subtended angle of 180°
Figure 3.2  
Details of models with subtended angle of 120°
Figure 3.3 Dimensions of simulated boundary layer system

<table>
<thead>
<tr>
<th>Boundary layer</th>
<th>H(mm)</th>
<th>h/H</th>
<th>$X_{1}/H$</th>
<th>b/H</th>
</tr>
</thead>
<tbody>
<tr>
<td>ROUGH</td>
<td>300</td>
<td>0.140</td>
<td>1.36</td>
<td>0.043</td>
</tr>
<tr>
<td>SMOOTH</td>
<td>300</td>
<td>0.067</td>
<td>1.36</td>
<td>0</td>
</tr>
</tbody>
</table>

Figure 3.4 Velocity profiles and longitudinal turbulent intensities of the 'smooth' and 'rough' boundary layers as they approach the barrel vaults
Figure 3.5  Experimental arrangement of barrel vault mounted on the simulated boundary layer system

Figure 3.6  Position of barrel vault under three different oncoming wind directions
Figure 3.8 Pressure distributions around the two-dimensional smooth surface barrel vault with subtended angle of 180° in the 'smooth' boundary layer

Figure 3.9 Pressure distributions around the two-dimensional rough surface barrel vault with subtended angle of 180° in the 'smooth' boundary layer —
\[ \varepsilon/D = 7.3\times10^{-3} \approx 1.3\times10^{-2} \]
Figure 3.10 Variation of drag coefficient and parameters characterising pressure distribution with Reynolds number — two-dimensional barrel vaults with subtended angle 180° in the 'smooth' boundary layer.
Figure 3.11 Pressure distributions around the two-dimensional smooth surface barrel vault with subtended angle of 180° in the 'rough' boundary layer
Figure 3.12 Pressure distributions around the two-dimensional rough surface barrel vault with subtended angle of $180^\circ$ in the 'rough' boundary layer — $\epsilon/D=7.3\times10^{-3}$ to $1.3\times10^{-2}$
Figure 3.13  Variation of drag coefficient and parameters characterising pressure distribution with Reynolds number — two-dimensional barrel vaults with subtended angle of 180° in the 'rough' boundary layer
Figure 3.14  Pressure distribution on barrel vault in 'smooth' boundary layer
Re $= 10^5$
$\varepsilon/D$: Smooth

Geometrical form and oncoming wind direction:
DEVELOPED SURFACE

Figure 3.15 Pressure distribution on barrel vault in 'smooth' boundary layer
Re = 105
\(\varepsilon/D\): Smooth
Geometrical form and oncoming wind direction:
Figure 3.16  Pressure distribution on barrel vault in 'smooth' boundary layer
Re = 10^5
ε/D: Smooth

Geometrical form and oncoming wind direction:
Figure 3.17  Pressure distribution on barrel vault in 'smooth' boundary layer
Re = 10^5
ε/D: Smooth

Geometrical form and oncoming wind direction:
Pressure distribution on barrel vault in 'smooth' boundary layer geometrical form and oncoming wind direction.

Figure 3.19

$Re = 10^5$

$\epsilon/D: 7 \times 10^{-3} \sim 1.3 \times 10^{-2}$
DEVELOPED SURFACE

Figure 3.20 Pressure distribution on barrel vault in smooth boundary layer. Geometrical form and oncoming wind direction:

Re = 10^5

e/D: 7.3x10^-3 ~ 1.3x10^-2
Figure 3.21  Pressure distribution on barrel vault in 'smooth' boundary layer
Re = 10^5
c/D: 7.3x10^{-3} \sim 1.3x10^{-2}

Geometrical form and oncoming wind direction:
DEVELOPED SURFACE

Figure 3.22  Pressure distribution on barrel vault in 'smooth' boundary layer
Re = 10^5
\( e/D: \ 7.3 \times 10^{-3} \sim 1.3 \times 10^{-2} \)

Geometrical form and oncoming wind direction:
Figure 3.23 Pressure distribution on barrel vault in smooth boundary layer. Geometrical form and oncoming wind direction:

- \( \text{Re} = 105 \)
- \( \epsilon/\text{D} = 7.3 \times 10^{-3} \sim 1.3 \times 10^{-2} \)

Developed surface
Figure 3.24  Reattachment of downstream flow — rough surface barrel vaults with subtended angle of 180° and roughness ratio \( \varepsilon/D = 7.3 \times 10^{-3} \approx 1.3 \times 10^{-2} \) in 'smooth' boundary layer at Re = 10^5
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Re = $10^5$
$\epsilon/D$: $7.3 \times 10^{-3} \sim 1.3 \times 10^{-2}$
Geometrical form and oncoming wind direction:
Figure 3.26  Pressure distribution on barrel vault in 'rough' boundary layer
Re = 10^5
ε/D: 7.3x10^{-3} \sim 1.3x10^{-2}

Geometrical form and
oncoming wind direction:
Figure 3.27  Pressure distribution on barrel vault in 'rough' boundary layer
Re = 10^5
ε/D: 7.3x10^-3 ~ 1.3x10^-2
Geometrical form and oncoming wind direction:
Figure 3.28  Pressure distribution on barrel vault in 'rough' boundary layer
Re = 10^5
e/D: 7.3x10^-3 ~ 1.3x10^-2

Geometrical form and oncoming wind direction:
Figure 3.29  Pressure distribution on barrel vault in 'rough' boundary layer
Re ≈ 10^5
ε/D: 7.3x10^{-3} ~ 1.3x10^{-2}

Geometrical form and oncoming wind direction:
Figure 3.30  Pressure distribution on barrel vault in 'rough' boundary layer
\( Re = 10^5 \)
\( \varepsilon/D: 7.3 \times 10^{-3} \sim 1.3 \times 10^{-2} \)

Geometrical form and oncoming wind direction:
Figure 3.31  Reattachment of downstream flow — rough surface barrel vaults with subtended angle of 180° and roughness ratio ε/D = 7.3x10^{-3} ~ 1.3x10^{-2} in 'rough' boundary layer at Re = 10^5
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Figure 3.33a  Pressure distributions around the two-dimensional rough surface barrel vaults with subtended angle of 120° in the 'rough' boundary layer when $\varepsilon/D = 5.5 \times 10^{-3} \sim 1.0 \times 10^{-2}$.
Figure 3.33b  Pressure distributions around the two-dimensional rough surface barrel vaults with subtended angle of 120° in the 'rough' boundary layer ——

\[ \epsilon/D = 5.5 \times 10^{-3} \sim 1.0 \times 10^{-2} \]
Figure 3.34 Variation of drag coefficient and parameters characterising pressure distribution with Reynolds number — two-dimensional barrel vaults with subtended angle 120° in the 'rough' boundary layer
Figure 3.35  Pressure distribution on barrel vault in 'rough' boundary layer
Re = $10^5$  Geometrical form
$\varepsilon/D$: Smooth and oncoming wind direction:

Figure 3.36  Pressure distribution on barrel vault in 'rough' boundary layer
Re = $10^5$  Geometrical form
$\varepsilon/D$: $5.5 \times 10^{-3}$ and oncoming wind direction:
$1.0 \times 10^{-2}$
Figure 3.37 Pressure distribution on barrel vault in 'rough' boundary layer
\( \text{Re} = 10^5 \) Geometrical form
\( \varepsilon/D: \) Smooth and oncoming wind direction:

Figure 3.38 Pressure distribution on barrel vault in 'rough' boundary layer
\( \text{Re} = 10^5 \) Geometrical form
\( \varepsilon/D: 5.5 \times 10^{-3} \) and oncoming wind direction:
\( 1.0 \times 10^{-2} \)
Figure 3.39 Pressure distribution on barrel vault in 'rough' boundary layer
Re = 10^5  Geometrical form
ε/D: Smooth and oncoming wind direction:

Figure 3.40 Pressure distribution on barrel vault in 'rough' boundary layer
Re = 10^5  Geometrical form
ε/D: 5.5x10^-3 and oncoming wind direction:
1.0x10^-2
**Figure 3.41** Pressure distribution on barrel vault in 'rough' boundary layer
Re = 10^5  
ε/D: Smooth and oncoming wind direction:

**Figure 3.42** Pressure distribution on barrel vault in 'rough' boundary layer
Re = 10^5  
ε/D: 5.5x10^-3 and oncoming wind direction:
Figure 3.43  Pressure distribution on barrel vault in 'rough' boundary layer
Re = $10^5$  Geometrical form 
$e/D$: Smooth and oncoming wind direction:

Figure 3.44  Pressure distribution on barrel vault in 'rough' boundary layer
Re = $10^5$  Geometrical form 
$e/D$: $5.5 \times 10^{-3}$ and oncoming wind direction:
Figure 3.45  Pressure distribution on barrel vault in 'rough' boundary layer
Re = 10^5  Geometrical form
ε/D: Smooth and oncoming wind direction:

Figure 3.46  Pressure distribution on barrel vault in 'rough' boundary layer
Re = 10^5  Geometrical form
ε/D: 5.5x10^{-3} and oncoming wind direction:
1.0x10^{-2}
Figure 3.47 Reattachment of downstream flow — rough surface barrel vaults with subtended angle of 120° and roughness ratio $\epsilon/D = 5.5 \times 10^{-3}$ $\sim 1.0 \times 10^{-2}$ in the 'rough' boundary layer at $Re \approx 10^5$
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Figure 3.48b Pressure distributions around the two-dimensional rough surface barrel vault with subtended angle of 180° and roughness ratio $\epsilon/D = 7.3 \times 10^{-3} \sim 1.3 \times 10^{-2}$ in the 'smooth' and 'rough' boundary layers at $Re = 10^5$
Figure 3.49 Comparison of the present wind tunnel test results to the codes of practice
(i) **Two-dimensional models**

Plate 3.1 Details of the models

(ii) **Three-dimensional models**

\[
\frac{\text{Length}}{\text{Chord}} = 2
\]

\[
\frac{\text{Length}}{\text{Chord}} = 1
\]

Subtended angle:

- 180°
- 120°
Plate 3.2  The 'rough' boundary layer system
(i) Two-dimensional model mounted on the 'lego' board of the 'rough' boundary layer system

(ii) Three-dimensional model mounted on the plywood tunnel wall of the 'smooth' boundary layer system

Plate 3.3 Models mounted on the base board
PART II

STRUCTURAL ASPECT
CHAPTER 4

ANALYSIS ON BRACED BARREL VAULTS

4.1 INTRODUCTION

Analysis in one form or another forms an integral part of the design process applied to most forms of structure. In the case of braced barrel vaults, there is no doubt that the evolution of effective techniques of analysis was one of the most significant factors contributing to their rapid development and widespread use. In this chapter, a brief review of the various techniques of analysis that are currently applied to braced barrel vaults is given.

4.2 TECHNIQUES OF ANALYSIS

The purpose of analysis is to investigate the behaviour of a structure, either existing or proposed. The feature common to all techniques of analysis is the use of a model which may be studied and used to predict the behaviour of an actual structure. An analytical model may be described as a device that is closely related to an actual structure such that observations on the model may be used to predict accurately the performance of the actual structure in the desired respect. The model may be a physical system giving rise to an experimental analysis or it may be a mathematical model giving rise to a purely mathematical analysis.

The magnitudes of internal stresses and deformations resulting from the application of external forces or deformation are the aspects of structural behaviour that are most commonly studied. However, there are a large range of other behaviour variables that can be studied including dynamic response, the effects of temperature distribution, elastic buckling, plasticity, fatigue, creep, crack propagation, natural
frequencies of vibration and the determination of collapse loads.

The main characteristics of the various analytical methods applied in braced barrel vaults are next briefly examined.

4.2.1 EXPERIMENTAL ANALYSIS

Experimental analysis is the direct method in which the strain and deflection of a structural model is directly measured, when the structural model itself is under the action of a system of external forces or deformations. The experimental models need not necessarily physically resemble their prototype but the majority of structural models do appear physically similar to their prototypes (Redshaw, 1965). To ensure an accuracy of prediction on the experimental model that is at least as good as that to be obtained from a mathematical model, requires a high standard of laboratory facilities, a skilled technician and a good deal of patience.

Some years ago, various experimental and analytical studies on the stress distribution in single-layer braced barrel vaults with six different types of member configuration were carried out by Makowski (1964). The use of physical models is mainly to confirm a new mathematical idealisation or method of analysis. Since the mathematical methods of analysis for a linear elastic structure are well proven and available, nowadays experimental studies are, mostly, interested in the investigation of non-linear behaviour, collapse load estimation, instability phenomena etc.

4.2.2 EARLY 'SIMPLIFIED' METHODS

4.2.2.1 TENSION COEFFICIENT APPROACH

This method is based upon the assumption that the joints on the two ends of a braced barrel vault are constrained in all directions and that the structure is pin-connected. This method employs the classical
analytical technique. The structure is first set to be statically determinate by the insertion of redundant forces, then the primary structure is analysed under the external applied load as well as those unknown redundant forces. The member forces in the primary structure are obtained by considering the equilibrium of each joint and the redundant forces are obtained by the application of Castigliano's theorem. This method gives a correct solution of a braced barrel vault if the boundary condition and the jointing system are the same as the assumption (King, 1974).

4.2.2.2 LATTICE TRUSS METHOD

This method is partly based upon Föppl's method (1892). It is assumed that a braced barrel vault is composed of lattice trusses with common top and bottom longitudinal members. The uniformly distributed load on the structure is assumed to be acting along the ridges of this latticed structure and is termed the 'ridge load'. The 'skin forces' which are evenly distributed along the respective lattices are obtained by solving the 'ridge loads' into the planes of the lattices. The 'skin forces' on each lattice are then reduced to a system of concentrated nodal forces, hence each lattice is analysed individually by assuming it to be a pin-connected truss simply supported at the end diaphragms. The forces in the common longitudinal members are then summed up to give the actual forces in the longitudinal members. This method gives reasonable results in the central regions of long braced barrel vaults but overestimates the values of forces in the longitudinal members near the edges (Velankar, 1967).

4.2.2.3 HOLLOW BEAM THEORY

The beam theory is normally used for predicting the forces in the crown and the springing of a cylindrical shell in a continuous medium. It only gives reasonable results for a long shell. In applying this theory to analyse a braced barrel vault, the calculation is extremely simple. The braced barrel vault system is first transformed into an equivalent hollow beam. The equivalent hollow beam consists of equivalent
longitudinal boom members which are laid on the same position of the longitudinal members of the braced barrel vault. The effective areas of the equivalent longitudinal boom members are related to the cross-sectional areas of the longitudinal and inclined members and the direction of the inclined members. The equivalent hollow beam is assumed to be simply supported at the two end diaphragms so that the longitudinal moment is determined as for a simply supported beam. Further to assume that the stress distribution along the depth of the equivalent hollow beam section is linear, the forces in each of the equivalent longitudinal boom members are obtained. This method only gives an approximation of forces in the longitudinal members but fails to give the forces in the inclined members and also cannot be used for the boundary condition of the longitudinal edges other than the one with free edges or the one with constraint in the horizontal directions only (Velankar, 1967 and King, 1974).

4.2.3 Finite Element Method for a Linear Elastic Skeletal Structure

Structures composed only of beams, struts, ties etc. are called skeletal structure. Single-layer or double-layer braced barrel vaults are a typical type of skeletal structure. In the finite element analysis, the initial idealisation into elements of the skeletal structure may be made to coincide with 'natural' elements or members of the structure. The only approximation that needs to be employed is that the structural elements are one-dimensional so that their relevant properties can be expressed as a function of the distance along the axis of the member. If it is further assumed that the elements are linearly elastic as well as their force-displacement behaviour is linear, then the overall structural behaviour is linear. The solution may be obtained by the analysis without further approximation and is essentially exact subjected to arithmetical accuracy.

Probably the most important step in the analysis of a structure by the finite element method is the idealisation process. It is generally a compromise between accuracy and a faithful representation on the one hand,
and the dictates of practicality and limited computer resource on the other. Consequently, an important factor in the process is the judgement and experience of the engineer.

For braced barrel vaults, there is no doubt that a reasonable approach to the idealisation is to consider them as a skeletal structure with a finite number of degrees of freedom. Obviously, the number of degrees of freedom is related to the actual joint conditions. However, the actual joint conditions that exist in all practical cases, where a wide variety of jointing techniques are used, range from nearly pinned through to nearly fully rigid. A choice of a pin-jointed structure gives rise to a particularly simple formulation and a minimal consumption of computer resources but the assumption of pinned joints conflicts with the actual joint conditions. To resolve this conflict it is necessary to observe that the joint fixity conditions have very little effect on the magnitudes of mean axial stresses in the members and their major influence is on the magnitudes and nature of the distribution of bending, shear and torsional stresses induced locally by joint eccentricities and lateral loading applied along the members (Butterworth, 1978). For a single-layer braced barrel vault, previous studies (Bligh, 1971 and King, 1974) show that the joint fixity conditions have an effect on the deflections and the stress distributions. The deflections are always reduced and the axial stresses are smaller and more evenly distributed for a rigid-jointed barrel vault. However, the degree of the effect varies in different types of member configuration, geometry or external load etc. Since there is no fundamental obstacle to formulating an idealisation which models the actual joint conditions as accurately as desired, it is better to assume that a single-layer braced barrel vault is a rigid-jointed space frame. For a double-layer braced barrel vault, experience and knowledge are still not enough as a basis for choice and therefore, the assumption of pin-jointed with all external loadings applied at the joints is normally used in the first stage of analysis. Local bending and shearing stresses would then be determined by local analyses in the vicinity of each joint and member, taking account of actual joint fixity, member eccentricities at the joints and member loading. This approach, as stated above, will give rise to a simple formulation of the method of analysis and reduce the computational efforts tremendously.
The flexibility and the stiffness approaches are two distinct ways of formulation in the finite element method. The flexibility approach was favoured in the early days possibly due to its being an extension of traditional hand computation techniques. Now, however, the vast majority of practical structural analysis programs are based on the stiffness method due to its systematic procedure.

In this part of study, it is decided to use the standard stiffness method for skeletal structure as a means of analysis of the braced barrel vaults under consideration. This analytical method is the most commonly used in the analysis of a linear elastic skeletal structure and gives an exact solution under the idealisation in such an extent. In addition, the standard stiffness method could be extended to non-linear analysis.

4.2.4 METHODS EMPLOYING ANALOGOUS CONTINUUM

Analogous continuum methods generally have the objective of providing estimation of the behaviour of skeletal structures with a minimum of calculation or may be calculated by hand techniques. In the case of braced barrel vaults, the structure is replaced by an equivalent continuum, generally a shell, with appropriately chosen properties such that its macroscopic behaviour bears some relation to the original structure. The 'equivalent shell' is then analysed by the shell theory which is already available and the results interpreted in terms of behaviour of the original structure. These methods will be investigated in detail in Chapter 5.

4.2.5 ANALYTICAL FINITE DIFFERENCE METHODS

Analytical finite difference methods (Tarzi, 1971 and Dean, 1975) as applied to skeletal structures depend on the existance of skeletal structures which are so regular that each internal joint (or group of joints) is connected to the adjacent ones by repeating patterns of members. If the stiffness equations of the typical group of joints are written in finite difference operator form, an elastic analysis may be obtained by means of a finite Fourier series. In some cases of braced barrel vaults,
there would be symmetry of translation in at least one direction and the boundaries parallel to this direction would also be simply supported; the analysis is only for a rectangular portion of the regular structure (Awni, 1972).

These methods have been shown to yield solutions in a good agreement with those obtained by 'exact' discrete element analysis (the stiffness method) also with less computational effort for single-layer braced barrel vault (Awni, 1972). For pin-connected double-layer braced barrel vaults, these methods give a reliable force in the members except the member in the boundary regions (Yazdani, 1973).

4.3 AREA OF SPECIAL INTEREST

In the design of a braced barrel vault, in practice, the initial analysis always treats the braced barrel vault as a skeletal structure which behaves in a linearly elastic manner. Referring to the previous sections, where the braced barrel vault is assumed to be a linear elastic skeletal structure, those analytical methods may be classified as an approximation method except the stiffness method. In the past, a great deal of work has been done by Makowski (1957), Velankar (1967), Awni (1972), Yazdani (1973) and King (1974) on investigating the approximation methods such as the tension coefficient approach, the lattice truss method, the hollow beam theory and the finite difference methods. Their works are mainly concerned with the difference of the analytical results obtained by those approximation methods and the stiffness method. Velankar (1967) has attempted to investigate the shell analogy approach but only gives in conclusion a sentence to the effect that shell analogy does not give tenable results for forces in members along the longitudinal edges when compared with the 'exact' method (the stiffness method).

The analogous continuum methods are used only very occasionally in the analysis of braced barrel vaults in the recent years due to the convenience of use of the computer. Nevertheless, it is interesting to investigate the methods which have been used in designing many braced barrel vaults in the past and are still favoured by some designers in this
day. It is better to present a detailed discussion of the methods rather than simply to dismiss them, saying that the method is too approximate or the results are not reliable.
CHAPTER 5

A STUDY OF THE ANALYTICAL METHODS
EMPLOYING ANALOGOUS CONTINUUM

5.1 INTRODUCTION

This chapter represents an attempt to investigate the two analogous continuum methods - the shell analogous method for braced barrel vaults and the equivalent shell replacement method for 'diamond' barrel vaults. The present study firstly investigates the analytical results obtained by the analogous continuum methods on several examples and secondly by the use of the stiffness method. Since these analytical methods are numerical methods, an experimental study is also carried out to give a counter check later. Finally, detailed discussion will be made regarding the analogous continuum methods.

5.2 GENERAL REMARKS

The linear elastic analysis of a braced barrel vault of any practical size by any method may involves a considerable volume of arithmetical operations. It becomes essential to carry out these operations on a computer. The structural analytical programs developed as a tool for the present investigation are mentioned herein.

Further, the definitions and notation involved in the analytical methods are also given in this section.

5.2.1 COMPUTING PROGRAMS
5.2.1.1 Analogous Continuum Methods

According to the analogous continuum methods, a computing program was developed in three main parts. They are:

1) Calculate the properties of an analogous shell (in the shell analogous method) or an equivalent shell (in the equivalent shell replacement method) by the given data of a braced barrel vault.

2) Calculate the membrane forces and moments in the analogous shell or the equivalent shell. Since the braced barrel vaults chosen for investigation were in cylindrical form, a general theory of cylindrical shell of homogeneous material developed by Gibson (1968, 1980) was applied in the analysis of the analogous or the equivalent shell.

3) Interpret the results of the analogous shell or the equivalent shell in terms of behaviour of the original skeletal structure.

The program was written in Fortran language; part 2 of the program had been checked by an example which was given in the test used by Gibson (1968).

5.2.1.2 The Stiffness Method for Linear Elastic Rigid-Jointed Space Frame

The program developed was based on the matrix formulation of the stiffness method as applied to linearly elastic rigid-jointed space frame. The stiffness matrix was a one-dimensional array for decomposition using Gaussian elimination, taking the advantage of the properties of the matrix which is symmetric, positive definite and banded. The program was written in Fortran language. In order to make sure that the program was logically correct and arithmetically accurate, it had been checked by the standard 'STRESS' program which was developed by the Computing Unit of University of Surrey.
5.2.2 DEFINITIONS AND NOTATION

5.2.2.1 DEFINITIONS

A. Frame Coordinate System

B. Member Coordinate System

C. Sign Convention of Forces and Moments

5.2.2.2 NOTATION

A  Cross-sectional area
E  Modulus of elasticity
F  Force
G  Shear modulus
I  Second moment of area
J  Polar moment of area
L  Member length
M  Moment
N  Membrane force
where suffix $X$, $Y$, $Z$ are related to frame coordinate system;
suffix $x$, $y$, $z$ are related to member coordinate system;
suffix 1 is related to horizontal member and 2, 3 are related to
inclined members in the three-way braced barrel vault structure;
suffix $s$ applies to the skeletal structure;
and primed quantities apply to the continuum shell structure.

5.3 GENERAL ASSUMPTIONS IN THE ANALOGOUS CONTINUUM METHODS

The two analogous continuum methods which are studied in this
chapter are suitable for application to different kinds of three-way
braced barrel vaults. The basic assumptions made in the analysis of
both methods, more or less, are the same. They are as follows:
(1) The effects of shell curvature on elastic constants are neglected.

(2) The length of each structural member is small in comparison with
the shell geometry.

(3) The thickness of the analogous shell or the equivalent shell is
small in comparison with the shell geometry so that a thin shell
theory could be applied.

(4) The analogous shell or the equivalent shell is homogeneous and
linear elastic given that the structural members must be linear
elastic.

(5) Members of the structure running in different directions (that is,
horizontal and inclined directions) may have different cross-
sectional areas but with same material properties. It means that
the analogous shell or the equivalent shell be isotropic or
anisotropic.
5.4 THE SHELL ANALOGOUS METHOD

5.4.1 INTRODUCTION

The treatment of the braced barrel vault as a continuum was first done by Del Pozo (1956). His work deals with cylindrical vault with equal size members and an equilateral triangular configuration of members. Later, Wright (1965) introduces a shell analogy which appears to be the most feasible and complete for braced barrel vaults with the same form of member configuration as Del Pozo's work. Wright also proves that the Poisson's ratio given by Del Pozo is in error. Further, Lang (1965) extends the previous theory of Wright to include the effect of shell bending and torsional moments in addition to the membrane forces. Besides, some work has also been done in Soviet Union by Vlasov (1958) and Pshenichnov (1958, 1960) on braced barrel vaults in a diamond-shaped grid form. Matsuoka and others (1972) and Sollazzo (1976) have worked on the continuum method in single-layer vaults with any form of member configuration. The shell analogous method which is discussed herein is based on the most familiar works of Wright (1965) and Lang (1965).

5.4.2 ELASTIC PROPERTIES OF THE ANALOGOUS SHELL

The shell analogous method developed by Wright (1965) is based on an equilateral triangular form of member configuration. The elastic properties of an analogous shell are obtained by considering that the strains of a basic structural unit must be equal to the strains of an analogous shell element when under the same load. The basic structural unit and the analogous shell element are shown in Figure 5.1.

If members of the structure are of identical cross-sectional area, Wright shows that the elastic properties of the analogous shell are as follows:

\[
E' = \frac{2A_s E_s}{\sqrt{3} I_s t'}
\]
If the horizontal and the inclined members of the structure are of different cross-sectional area, $A_{1s}$ and $A_{2s}$ respectively, Wright shows that the elastic properties of the analogous shell are as follows:

$$E'_{Y} = \frac{2A_{s}E_{s}}{\sqrt{3} l_{s}t'}$$
$$G' = \frac{\sqrt{3} A_{s}E_{s}}{4l_{s}t'}$$
$$\nu_{X}' = \frac{1}{3}$$
$$\nu_{Y}' = \frac{1}{3}$$

Considering the effective thickness of the analogous shell, the basic structural unit is assumed to be subjected to pure bending and then the effective thickness is obtained by equating the radius of curvature between the basic structural unit and the analogous shell element.

If members of the structure are of identical cross-sectional area, Wright shows that the effective thickness is as follows:

$$t' = 2\sqrt{3} \frac{I_{s}}{A_{s}}$$
Where the horizontal and the inclined members of the structure are of different cross-sectional area, $A_{1s}$ and $A_{2s}$ respectively, Wright shows that the effective thicknesses of the analogous shell in the $X$ and $Y$ directions are as follows:

$$t_{X}'^2 = \frac{3}{2} \frac{I_{1s}}{A_{1s}} \left( \frac{I_{2s}}{I_{1s}} \right) (1-\nu_X'\nu_Y')$$

$$t_{Y}'^2 = \frac{3}{2} \frac{I_{2s}}{A_{2s}} \left( \frac{A_{1s}}{2A_{2s}} \right) (1-\nu_X'\nu_Y')$$

5.4.3 SHELL FORCES AND MOMENTS RELATED TO STRUCTURAL MEMBER FORCES AND MOMENTS

By considering statical equilibrium on the structural unit on which the analogous shell forces and moments act, the axial forces, torsional moments and bending moments of each individual member are obtained. The forces and moments on the basic structural unit as well as the analogous shell element are shown in Figures 5.2 and 5.3.

Interpreting the membrane forces of an analogous shell element in terms of the axial force in the members of a basic structural unit, Wright(1965) shows that the member forces are as follows:

$$P_1 = \frac{\ell_s}{2\sqrt{3}} (3N_x'-N_y')$$

$$P_2 = \frac{\ell_s}{\sqrt{3}} (N_y'+\sqrt{3}N_{xy}')$$

$$P_3 = \frac{\ell_s}{\sqrt{3}} (N_y'-\sqrt{3}N_{xy}')$$

Interpreting the torsional and bending moments of an analogous shell element in terms of the torsional and bending moment in the members of a basic structural unit, Lang(1965) shows that the bending and torsional moments in the members are as follows:
\[ M_1 = \frac{\rho_s}{2} (\sqrt{3} M_x' \pm M_{xy}') \]
\[ T_1 = -\frac{\rho_s}{2} (\sqrt{3} M_{xy}' \pm M_y') \]
\[ M_2 = \frac{\rho_s}{4} (\sqrt{3} M_y' + M_{xy}') \]
\[ T_2 = \frac{\rho_s}{4} (\sqrt{3} M_{xy}' - M_y') \]
\[ M_3 = \frac{\rho_s}{4} (\sqrt{3} M_y' - M_{xy}') \]
\[ T_3 = \frac{\rho_s}{4} (\sqrt{3} M_{xy}' + M_y') \]

For \( M_1 \) and \( T_1 \), the positive sign applies to the right hand end while the negative sign applies to the left hand end.

5.4.4 COMPARISON OF ANALYTICAL RESULTS OF STRESS ANALYSIS OF BRACED BARREL VAULTS OBTAINED BY THE SHELL ANALOGOUS METHOD AND THE STIFFNESS METHOD

5.4.4.1 GEOMETRY AND CHARACTERISTIC OF BRACED BARREL VAULTS A AND B

Braced barrel vaults A and B were chosen for study. They were of different geometry but had the same member configuration, member properties, jointing system, boundary conditions and external applied load. Details are given in the following sections.

A. Configurations and Dimensions

Braced barrel vaults A and B were of circular arc form, having different subtended angles of 80° and 180° respectively. Each of them had two different lengths, 24m and 15m long and were named as A24m long,
A 15m long, B 24m long and B 15m long. Member configuration was in the form of an equilateral triangle. Details of configurations and dimensions of those four structures are shown in Figures 5.4 and 5.5.

B. Member Properties

(1) Material properties of members:
   (a) Modulus of elasticity of steel $E_s = 2.0 \times 10^8$ kN/m$^2$
   (b) Shear modulus of steel $G_s = 0.75 \times 10^8$ kN/m$^2$

(2) Member section:
   (a) Outside diameter = 0.143 m
   (b) Thickness = 0.005 m
   (c) Cross-sectional area $A_s = 1.72 \times 10^{-3}$ m$^2$
   (d) Second moment of area $I_{ys} = I_{zs} = 2.57 \times 10^{-6}$ m$^4$
   (e) Polar moment of area $J_s = 5.14 \times 10^{-6}$ m$^4$

C. Joints

All members in the four braced barrel vaults were rigid jointed, giving rise to translational displacements in the X, Y and Z directions and rotational displacements about axes in those directions at each joint.

D. Boundary Condition

(1) Transverse edges

Braced barrel vaults were assumed to be supported at the joints along the transverse edges, each support being constrained against translational displacements in the Y and Z directions and rotational displacement about an axis in the X direction.
(2) Longitudinal edges

Braced barrel vaults were assumed to be supported at the joints along the longitudinal edges. Three different support conditions were taken into investigation and are given as follows:

(a) Boundary Condition 1 (denoted as B.C.1) - free edges
Joints along the two longitudinal edges were assumed being free to move in the X, Y and Z directions and to rotate about axes in those directions.

(b) Boundary Condition 2 (denoted as B.C.2) - roller supports
Each support along the two longitudinal edges was assumed being constrained against translational displacement in the Z direction.

(c) Boundary Condition 3 (denoted as B.C.3) - fixed supports
Each support along the two longitudinal edges was assumed being constrained against translational displacements in the X, Y and Z directions.

E. Loading

An external load of 0.2kN/m² was assumed to be uniformly distributed on the whole structure. Self-weight of structure was included.

5.4.4.2 ANALYTICAL RESULTS

Two different methods were used to analyse the four braced barrel vaults. They were the shell analogous method and the stiffness method. For the shell analogous method, axial forces, bending moments and torsional moments of members in the structures were obtained. For the stiffness method, at two ends of each member, forces and moments in x, y and z directions with respect to the corresponding member coordinate were obtained. The member axial forces given by those two different analytical methods are shown in Figures 5.6 to 5.11.
Since the interpretation of shell moments in terms of torsional and bending moments in members by the shell analogous method was not a reasonable approach (as will be discussed in Section 5.7.2), it is not necessary to show all the results and to compare with the results of the stiffness method. Only two examples were chosen for investigation; they were Vault A_24m long and Vault A_15m long with free edges boundary condition (B.C.1). Their analytical results of torsional and bending moments obtained by the shell analogous method and the stiffness method are shown in Figures 5.12 and 5.13.

5.4.4.3 COMPARISON

As mentioned above, the interpretation of shell moments in terms of torsional and bending moments in members by the shell analogous method was not a reasonable approach, so that the comparison of the analytical results of stress analysis of the braced barrel vaults obtained by the stiffness method and the shell analogous method was concentrated on the member axial forces. From all the investigated examples, the distribution pattern of axial forces given by the shell analogous method showed a fair degree of agreement with the stiffness method but the magnitude of forces were not in good agreement. The differences of the maximum axial forces in all the examples obtained by the two different analytical methods are shown in Table 5.1. Observing the maximum axial forces of the barrel vaults (shown in italic letters in Table 5.1), most of the forces obtained by the shell analogous method differed by -15% to +15% from those obtained by the stiffness method except in few cases the forces differed nearly up to +50%.
<table>
<thead>
<tr>
<th>Vault</th>
<th>A&lt;sub&gt;24m long&lt;/sub&gt;</th>
<th>A&lt;sub&gt;15m long&lt;/sub&gt;</th>
<th>B&lt;sub&gt;24m long&lt;/sub&gt;</th>
<th>B&lt;sub&gt;15m long&lt;/sub&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Stiffness method (kN)</td>
<td>Shell analogous method (kN)</td>
<td>Difference (%)</td>
<td>Stiffness method (kN)</td>
</tr>
<tr>
<td>B.C.1</td>
<td>Tensile</td>
<td>52.7</td>
<td>61.6</td>
<td>16.8</td>
</tr>
<tr>
<td></td>
<td>Compressive</td>
<td>25.8</td>
<td>29.0</td>
<td>12.4</td>
</tr>
<tr>
<td>B.C.2</td>
<td>Tensile</td>
<td>32.2</td>
<td>43.6</td>
<td>35.4</td>
</tr>
<tr>
<td></td>
<td>Compressive</td>
<td>24.0</td>
<td>48.3</td>
<td>101.2</td>
</tr>
<tr>
<td>B.C.3</td>
<td>Tensile</td>
<td>1.4</td>
<td>2.1</td>
<td>50.0</td>
</tr>
<tr>
<td></td>
<td>Compressive</td>
<td>5.6</td>
<td>4.9</td>
<td>-12.5</td>
</tr>
</tbody>
</table>

Notes: \[ \text{Difference (\%)} = \frac{\text{Shell analogous method (kN)} - \text{Stiffness method (kN)}}{\text{Stiffness method (kN)}} \times 100\% \]

Table 5.1 Comparison of maximum axial forces obtained by the stiffness method and the shell analogous method
5.5 THE EQUIVALENT SHELL REPLACEMENT METHOD FOR 'DIAMOND' BARREL VAULTS

5.5.1 INTRODUCTION

'Diamond' structures have been developed by Dr. Fujio Matsushita of Tomoegumi Iron Work Ltd. (Japan). The basic unit of the 'diamond' structures is a diamond shape and has relatively high stiffness. For the structural members, equal angular or T-shaped members (cut from H-shaped) are mainly used as chord members and flat bars, equal angular or T-shaped members as inclined bracing members. The joints are designed to transfer all forces and moments. A typical structural member and joint are shown in Figure 5.14. 'Diamond' barrel vaults are space structures in which the three-way frameworks have curved surfaces.

One of the stress analysis methods of 'diamond' barrel vaults which applies the theory of shell to skeletal space framework has been known as the 'equivalent shell replacement method' developed by Matsushita. This continuum method will be investigated in the following sections.

5.5.2 ELASTIC PROPERTIES OF THE EQUIVALENT SHELL

Suppose a basic structural unit is as shown in Figure 5.15. In order to find the equivalent continuum to the basic unit, it is required to obtain the elastic properties such that the equivalent continuum and the basic unit yield the same deformation against the same forces.

Matsushita shows that the equivalent elastic properties of the equivalent shell are as follows:

\[
E'_X = \frac{E_A S}{h \xi S \cos \alpha}
\]

\[
E'_Y = \frac{E_A S \cos^3 \alpha}{h \xi S \sin \alpha (1+\sin^3 \alpha)}
\]
\[ G' = \frac{E_s A_s \sin \alpha \cos \alpha}{h L_s} \]

\[ v'_x = \tan^2 \alpha \]

\[ v'_y = \frac{\sin \alpha \cos^2 \alpha}{1 + \sin^3 \alpha} \]

where \( A_s \) is cross-sectional area of a structural member and the cross-sectional area of the horizontal member is regarded as \( 2A_s \) and \( h \) is depth of a structural member or thickness of an equivalent shell.

Since the equivalent monolithic shell section has the same cross-sectional area as the structural member, the second moment of area of the actual structural member is greater than the second moment of area of the equivalent monolithic shell section. A modified factor, \( \gamma \), is necessary for calculating the flexural and torsional rigidities. The modified factor which has been derived by Matsushita is the ratio of the second moments of area of the section of the shell element and the structural member. With this modified factor, the flexural rigidities \( D'_x \) and \( D'_y \) and torsional rigidity \( D'_{xy} \) are as follows:

\[ D'_x = \frac{E'_x (\gamma^3 h^3)}{12(1-v'_x v'_y)} \]

\[ D'_y = \frac{E'_y (\gamma^3 h^3)}{12(1-v'_x v'_y)} \]

\[ D'_{xy} = \frac{G'(\gamma^3 h^3)}{12} \]

therefore, on the other hand, \( (\gamma^3 h) \) could be treated as an effective thickness of the equivalent shell.
5.5.3 SHELL FORCES AND MOMENTS RELATED TO STRUCTURAL MEMBER FORCES

Having the elastic properties of the equivalent shell, the membrane forces and moments of the equivalent shell may be obtained by applying the shell theory. Again, as the shell analogous method operates by considering statical equilibrium on the basic unit on which the forces and moments of the equivalent shell act, the axial forces in the chords of the structural members are obtained. The forces and moments of the equivalent shell acted on the basic unit are shown in Figure 5.16.

The member forces in the basic structural unit are given in the following table:

<table>
<thead>
<tr>
<th>Member</th>
<th>Member axial forces</th>
</tr>
</thead>
<tbody>
<tr>
<td>2/3 Top chord</td>
<td>$P = \frac{N'' M''}{2 j_s} \cos \theta \pm \frac{N_{yx} M_{xy}}{2 j_s} \sin \theta$</td>
</tr>
<tr>
<td>Bottom chord</td>
<td>$P = \frac{N'' M''}{2 j_s} \cos \theta \pm \frac{N_{yx} M_{xy}}{2 j_s} \sin \theta$</td>
</tr>
</tbody>
</table>

($'+'$: member 2,5 ; '−': member 3,4 )

| Top chord | $P = \frac{N' M'}{2 j_s} \cos \theta \pm \frac{N_{yx} M_{xy}}{2 j_s} \sin \theta$ |
| Bottom chord | $P = \frac{N' M'}{2 j_s} \cos \theta \pm \frac{N_{yx} M_{xy}}{2 j_s} \sin \theta$ |

5.5.4 COMPARISON OF ANALYTICAL RESULTS OF STRESS ANALYSIS OF 'DIAMOND' VAULTS OBTAINED BY THE EQUIVALENT SHELL REPLACEMENT METHOD AND THE STIFFNESS METHOD
5.5.4.1 GEOMETRY AND CHARACTERISTIC OF 'DIAMOND' VAULTS A AND B

The 'diamond' vaults A24m long, A15m long, B24m long and B15m long that were chosen for the investigation were similar to the braced barrel vaults A24m long, A15m long, B24m long and B15m long mentioned in Section 5.4.4. The member layout, dimensions, joints, boundary conditions and loading were the same, only the structural members were different.

A. Structural Members

For the structural members, equal angular members were used as top and bottom chord and flat bars as inclined bracing members. A typical structural member is shown as follows:

![Structural Member Diagram]

(1) Material properties of members:
(a) Modulus of elasticity of steel $E_s = 2.0 \times 10^8 \text{kN/m}^2$
(b) Shear modulus of steel $G_s = 0.75 \times 10^8 \text{kN/m}^2$

(2) Structural member section:
- For angle $\angle 51 \times 51 \times 4.6 \text{mm}$
  - Cross-sectional area = $4.56 \times 10^{-4} \text{m}^2$
  - Second moments of area about $y$ and $z$ axis = $11 \times 10^{-8} \text{m}^4$
(a) Cross-sectional area of structural member $A_s = 9.12 \times 10^{-4} \text{m}^2$

(b) Second moment of area $I_{ys} = 1.7039 \times 10^{-5} \text{m}^4$

(c) Second moment of area $I_{zs} = 2.2 \times 10^{-7} \text{m}^4$

(d) Polar moment of area $J_s = 1.7259 \times 10^{-5} \text{m}^4$

(e) Sectional distance between the centre of gravity of top and bottom chords $j_s = 0.2716 \text{m}$

5.5.4.2 ANALYTICAL RESULTS

Two different methods were used to analyse the four 'diamond' vaults. They were the equivalent shell replacement method and the stiffness method.

From the equivalent shell replacement method, the axial forces at top and bottom chords of each structural member were found so that the normal stress distributions of the top and bottom chords were obtained.

Since the joints were designed to transfer all forces and moments, a three-dimensional rigid-jointed linear elastic stiffness method was employed to analyse the structures. The forces and moments in $x$, $y$ and $z$ directions with respect to the member coordinate at two ends of each structural member were obtained.

At stated above, the equivalent shell replacement method only gives the normal stresses in the top and bottom chords. The comparison of the analytical results obtained by these two different methods and the stiffness method relates, therefore, to the normal stress distributions only.

The results of normal stress distribution of top and bottom chords of all examples are shown in Figures 5.17 to 5.22.
5.5.4.3 COMPARISON

Since the stiffness method approach employed in the analysis of 'diamond' vaults A_24m long, A_15m long, B_24m long and B_15m long took account of the shearing forces, the normal stresses were distributed linearly along the top and bottom chords. In the equivalent shell replacement method, the interpretation of shell shearing forces in terms of shearing forces of structural members was neglected so that only a uniformly distributed normal stresses on top and bottom chords were given. From the results obtained by the two different analytical methods, it is observed that the stress distribution patterns were similar but the detailed stresses were not in close agreement. The comparison of the maximum normal stresses obtained by those two different analytical methods are given in Table 5.2. Observing the maximum normal stresses in the chords of the barrel vaults (shown in italic letters in Table 5.2), the stresses obtained by the equivalent shell replacement method differed by -27% to +23% from those obtained by the stiffness method.
<table>
<thead>
<tr>
<th>'Diamond' vault</th>
<th>A&lt;sub&gt;24m long&lt;/sub&gt;</th>
<th>A&lt;sub&gt;15m long&lt;/sub&gt;</th>
<th>B&lt;sub&gt;24m long&lt;/sub&gt;</th>
<th>B&lt;sub&gt;15m long&lt;/sub&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Maximum normal stresses (Top and bottom chords)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Stiffness method</td>
<td>Equivalent shell replacement method</td>
<td>Difference</td>
<td>Stiffness method</td>
</tr>
<tr>
<td>B.C.1 Tensile</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Top</td>
<td>19.31</td>
<td>18.91</td>
<td>-2.07</td>
<td>10.16</td>
</tr>
<tr>
<td>Bottom</td>
<td>26.51</td>
<td>26.55</td>
<td>0.15</td>
<td>14.51</td>
</tr>
<tr>
<td>B.C.1 Compressive</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Top</td>
<td>17.18</td>
<td>19.32</td>
<td>12.45</td>
<td>11.45</td>
</tr>
<tr>
<td>Bottom</td>
<td>11.65</td>
<td>14.24</td>
<td>22.23</td>
<td>6.84</td>
</tr>
<tr>
<td>B.C.2 Tensile</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Top</td>
<td>13.55</td>
<td>14.54</td>
<td>7.31</td>
<td>6.07</td>
</tr>
<tr>
<td>Bottom</td>
<td>14.15</td>
<td>13.20</td>
<td>-6.71</td>
<td>6.98</td>
</tr>
<tr>
<td>B.C.2 Compressive</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Top</td>
<td>12.01</td>
<td>11.61</td>
<td>-3.33</td>
<td>5.06</td>
</tr>
<tr>
<td>Bottom</td>
<td>11.94</td>
<td>11.31</td>
<td>-5.27</td>
<td>6.51</td>
</tr>
<tr>
<td>B.C.3 Tensile</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Top</td>
<td>0.72</td>
<td>0.73</td>
<td>1.38</td>
<td>0.79</td>
</tr>
<tr>
<td>Bottom</td>
<td>1.56</td>
<td>0.93</td>
<td>-40.38</td>
<td>1.62</td>
</tr>
<tr>
<td>B.C.3 Compressive</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Top</td>
<td>2.48</td>
<td>3.07</td>
<td>23.79</td>
<td>2.46</td>
</tr>
<tr>
<td>Bottom</td>
<td>2.46</td>
<td>3.12</td>
<td>26.82</td>
<td>2.34</td>
</tr>
</tbody>
</table>

Notes: $\text{Difference (\%)} = \frac{\text{Equivalent shell replacement method (N/mm}^2\text{)} - \text{Stiffness method (N/mm}^2\text{)}}{\text{Stiffness method (N/mm}^2\text{)}} \times 100\%$

Table 5.2 Comparison of maximum normal stresses obtained by the stiffness method and the equivalent shell replacement method.

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5.6 COMPARISON OF EXPERIMENTAL AND NUMERICAL ANALYTICAL RESULTS

After the investigation of the continuum methods and the stiffness method in analysing several braced barrel vaults, an experimental study was carried out for a counter check of those numerical methods.

5.6.1 GEOMETRY AND CHARACTERISTIC OF MODELS

5.6.1.1 CONFIGURATION AND DIMENSIONS

Two models, Vault C_{1.35m long} and C_{0.75m long}, in different length of 1.35m and 0.75m but with the same circular arc form with a subtended angle of 80° were chosen for analysis. The models were a single-layer braced barrel vault and the member configuration was in an equilateral triangular form. Details of the configuration and dimensions of the models are shown in Figure 5.23 and Plates 5.1 and 5.2.

5.6.1.2 MEMBER PROPERTIES

A. Main Member (circular hollow section)
(1) Outside diameter=6.5mm
(2) Inside diameter=4.2mm
(3) Cross-sectional area=19.33mm²
(4) Second moment of area about y axis=72.35mm⁴
(5) Second moment of area about z axis=72.35mm⁴
(6) Polar moment of area=144.7mm⁴

B. Members Along Transverse Edges (rectangular section)
(1) Width=16mm
(2) Thickness=5mm
(3) Cross-sectional area=80mm²
(4) Second moment of area about y axis=1706.7mm⁴
(5) Second moment of area about z axis=166.67mm$^4$
(6) Polar moment of area=1873.3mm$^4$

5.6.1.3 MATERIAL PROPERTIES

Perspex was used as the model material. For the purpose of model-making, perspex is tough and easy to cut and it also allows the cementation of several adjacent members within a structural configuration together. Perspex is homogeneous and has sufficient tensile and compressive strength. Perspex behaves in a linearly elastic manner up to about 400 microstrain and its creep may be disregarded for short term loading if its elasticity is within this range. When higher values of strain is required, creep of the material must be considered.

The elastic properties of the model material in the analysis were obtained separately as follows:

(1) Modulus of elasticity $E=3747.3$N/mm$^2$
   The young's modulus was obtained from the stress versus strain curve by the tensile test of a perspex tube with same cross-sectional area as the main member. The stress versus strain curve obtained by the tensile test is shown in Figure 5.24.

(2) Poisson's ratio $\nu=0.35$
   The poisson's ratio was obtained by the ratio of the strain in longitudinal direction to the strain in transverse direction of a perspex strip under direct tensile force.

(3) Shear modulus $G=1387.889$N/mm$^2$
   The shear modulus was obtained by the relationship of $G=\frac{E}{2(1+\nu)}$, assuming that the material is homogeneous and Hookean.

5.6.1.4 JOINTING SYSTEM

Members in the models were rigidly connected. For a rigid-jointed
structure, the stresses at the joints would be a function of the area stiffness, the flexural stiffness of the members at the joint and the eccentricity of the member loads. However, in the analysis of a skeletal structure, the joints of the structure are usually assumed to be perfectly rigid and of zero dimensions. Under such circumstance, the jointing system in the model should fulfill the following requirements:

1. The size of joints should be as small as possible.
2. Centre lines of component members must meet at the centre of the joint.
3. Joints should be able to transmit forces and moments from any direction.

Two different types of jointing system were used in the models of a three-way braced barrel vault. There were six members connecting at a joint except the joints at the transverse edges, where only three to five members were connected.

The joints in the main members were made of perspex discs. The minimum disc of a joint, 20mm in diameter by 9.5mm in thickness, was connected by six tubular members of 6.5mm outside diameter. The members along the transverse edges were made up of perspex strips so that the adjacent tubular members were glued directly to the perspex strips. Details of the jointing system are shown in Plate 5.3.

5.6.1.5 BOUNDARY CONDITION

All the joints along the longitudinal edges of the models were supported by columns which restrained the translational displacements in the Y and Z directions of the supporting joints. The supporting joints rested on ball bearings by which the Z direction movements were constrained. The Y direction movements were constrained by screws which were just in contact with the joints. Details of supports are shown clearly in Plate 5.4.
5.6.1.6 APPLIED LOADING

Two different models were analysed with the same applied load. Since the perspex behaves in a linearly elastic manner up to about 400 microstrain, the stresses and strains in the members of the models induced by the applied loading were kept within its elastic range.

A uniformly distributed load of 0.2kN/m² covering the whole structure was chosen in the present experimental study. This distributed load had to be converted to an equivalent concentrated load applied on each joint in the experimental test. It was assumed that the distributed load was shared by the joint which took the load on the surface area formed by connecting the mid-point of the connecting members to the centroid of the surface areas of the adjacent unit of openings. Figure 5.25 represents the plan of the braced barrel vault under uniformly distributed load and the assumed shares of load for joints are indicated by the shaded areas.

5.6.2 EXPERIMENTAL ANALYSIS

5.6.2.1 STRAIN MEASUREMENT

Strains in component members of the models were measured with the help of electrical strain gauges which connected directly to a data logger. Two gauges were stuck diametrically opposite each other onto the midspan of the members. Members used to measure axial and bending strains in the models are shown in Figure 5.26. The technique for undertaking this investigation was to use a quarter-bridge system in which each active gauge (gauges at top and bottom fibres of members) reading was recorded separately and the axial and bending strains could be obtained independently by the following formulae:

\[
\text{Axial strain} = \frac{\text{strain top} + \text{strain bottom}}{2} \\
\text{Bending strain} = \frac{\text{strain top} - \text{strain bottom}}{2}
\]
A half-bridge system was also used to give the bending strains for checking the bending strains obtained by the quarter-bridge readings.

The strain gauges were manufactured by Showa (Japan). Their resistance was 1.2052 and their length was 8.0mm.

5.6.2.2 Calibration Factors

The purpose of the calibration of strain gauges was to convert axial strains and bending strains into axial forces and bending moments. The calibration factors of axial and bending strains were found by using two specimens of the same cross-sectional area as the main members, one in pure tension and the other in pure bending.

A. Axial Force Calibration Factor

A specimen of same cross-sectional area as the main member was loaded axially in increments of 0.5kg from zero to 4.0kg. The readings were taken 1 minute after loaded and 1.5 minutes unloaded. The average of the axial force calibration factor obtained by the five different tests was 0.07243N/μstrain.

B. Bending Moment Calibration Factor

A specimen of the same cross-sectional area as the main member was subjected to pure bending. The pure bending system is shown in Figure 5.27. The loading increments were of 1/20 of a pound and it was loaded from zero to 4/20 of a pound. The time between readings and the number of calibrations were the same as for the axial force calibration. The average of the bending moment calibration factor obtained by the tests was 0.097N-mm/μstrain.
5.6.2.3 Model Testing and Results

The geometry of the model is one of the most important factors in the experimental analysis. Tests were carried out on the models to check for its symmetry when the model was under symmetrical loading. Strain gauges stuck on either side of the plane of symmetry were used to check for its symmetry. In addition, the boundary supports were checked carefully to ensure that the joints were just touching the ball bearings as well as the transverse screws before each test was carried out.

The models were loaded by weights hanging on each joints. The weight was made up of a container which was filled with sand or lead shot of the desired amount. All the weights were hung onto the centre hole of the disc joint by means of a wire.

The models were loaded 2 to 2.5 minutes before any reading on the strain gauge was taken. This time lag which has been recommended by the previous research studies in the Experimental Laboratory of the Civil Engineering Department of University of Surrey, was long enough either to provide correct reading in each strain gauge or to eliminate the preliminary creep set up in the models.

Initial reading was regained after the model was unloaded and left for 5 minutes. This shows that a period of 5 minutes was enough for the model for complete recovery.

A short program was written in Fortran language to calculate the axial forces and bending moments of members from the readings given by the data logger.

The axial forces and bending moments of selected members in the two models are shown in Figure 5.28.
5.6.3 Comparison of Experimental Results to Numerical Analytical Results

Two different numerical methods— the shell analogous method and the stiffness method were employed to analyse the Vault C₁.₃₅m long and Vault C₀.₇₅m long. The analytical results of member axial forces and bending moments are shown together with the experimental results in Figure 5.28.

Observing the results shown in Figure 5.28, it was found out that the results given by the stiffness method were in a fair agreement with the experimental results. The member axial forces obtained by the stiffness method and the experiments were close to each other except for the members at or near the boundary supports. As a perfect roller support was impossible to produce, frictional forces at the contact surface of the boundary joints would exist. This imperfection of boundary joints might have caused an effect upon the estimation of the stresses in the region at or near the boundary. The bending moments obtained by the stiffness method were not very close in magnitude with the experimental results. It might be considered that several factors would affect the experimental results such as the area stiffness of joints, the eccentricity of joint loads and existing of built in bending stresses. However, those factors were difficult to eliminate and estimate in the experimental work.

Investigating the results given by the shell analogous method, nearly all the member axial forces and bending moments were of the same sign as the experimental results but showed a great difference in the magnitude.

There is no doubt that many factors may affect the precision of experimental analysis, nevertheless, the present experimental work verified that the shell analogous method was inferior to the stiffness method in accuracy.
5.7 Discussion

In the previous sections, the continuum methods have been investigated by application to several three-way braced barrel vaults with different geometry, length and boundary condition. The analytical results obtained by the continuum methods were not in good agreement, either with the stiffness method or the experimental analytical results. The continuum methods sometimes gave an overestimation of stresses but sometimes gave an underestimation. This depended upon different geometry, boundary condition etc. Generally speaking, the continuum methods gave a similar distribution pattern of normal stresses (or member axial forces) to the stiffness method; the magnitude, however, was not close to the stiffness method. In this section, it is decided to give a detailed discussion on the approximate characteristics of the continuum methods.

5.7.1 Effective Rigidities

When a linearly elastic skeletal structure is substituted by an equivalent continuum, it is essential to evaluate correctly the effective rigidities of the original structure. For rigid-jointed space structures such as braced barrel vaults, they have in general six kinematic degrees of freedom (three translational and three rotational displacements in strain state) at each nodal joint. Therefore, the derivation of elastic constants* of the equivalent continuum and the theory of shell which is employed to analyse the equivalent continuum should have included the effects of the local deformation of the nodal joints.

In the shell analogous method and the equivalent shell replacement method, they both consider a braced barrel vault with equilateral or isosceles triangular mesh with members of the same cross-sectional area or differing only for inclined and horizontal directions members so that the equivalent continuum is an isotropic or an anisotropic shell.

* Based on Hooke's Law, the stresses are linearly related to the strains for a linearly elastic material. The elastic constants are the constants related to the stresses and strains.
Further it is assumed that the equivalent shell is very thin compared to the overall dimensions; the shell element is, therefore treated as a thin shell having four independent elastic constants. The elastic constants are then solved in those methods by considering that the in-plane deformations and shear deformation of the shell element and of the basic structural unit must be equal to each other under the same in-plane forces. The changes in curvatures and twists of the equivalent shell and the rotational displacements of the original structure are not included in the derivation of elastic constants since they are assumed so small that could be neglected. This contradicts the assumption of bending and twisting of the basic structural unit and the shell element which has been used simultaneously in deriving the effective thickness of the equivalent shell. Therefore, in these continuum methods, the macro-displacements govern the overall behaviours of the structure. Since the structural joint is one of the main factors governing the overall behaviours of the structure, the analytical results of stress analysis obtained by these continuum methods would be approximate.

5.7.2 INTERPRETED SHELL FORCES AND MOMENTS IN TERMS OF MEMBER FORCES AND MOMENTS

The two investigated continuum methods both employ a similar approach to convert the shell forces and moments into member forces and moments of the original structure (the approach has been described in previous sections).

For the shell analogous method, the conversions of the shell membrane forces into axial force of the three members of a basic structural unit is successfully solved by the three statical equilibrium equations in terms of $N_x'$, $N_y'$ and $N_{xy}'$. For a rigid-jointed skeletal structure, bending moments at the two ends of the members may be different because shearing forces along the member exist and also torsional moments at the two ends are equal in magnitude but in opposite directions according to the condition of equilibrium. The shearing forces are assumed to be very small and are neglected in the shell analogous method so that a basic structural unit has three unknown torsional moments and three unknown
bending moments. Unfortunately, only four equilibrium equations could be obtained in terms of $M_x'$, $M_y'$, $M_{xy}'$ and $M_{yx}'$. This is insufficient to solve the six unknowns. Lang (1965) has solved this problem by adding four equations. First, it is considered that the equilibrium of the basic unit both at left and right vertical sections of the basic unit are different. Two additional equations are thus obtained. Secondly, two more equations are set up in the consideration of the equilibrium of $M_y'$ and $M_{yx}'$ in horizontal section of the basic unit by assuming that the bending moments and torsional moments in the members are separately related to $M_y'$ and $M_{yx}'$. A set of eight linearly dependent equations is obtained leading to solutions that are not unique. Lang then assumes that the bending moment and torsional moment of the horizontal member are different at the left and right hand ends so that the eight equations become linearly independent. However, a question of the equilibrium of the horizontal member arises and this approach to obtain those eight equilibrium equations is not a reasonable way. It was shown clearly in Figures 5.12, 5.13 and 5.28 that the torsional and bending moments obtained by the shell analogous method differed considerably from those obtained by the stiffness method analysis or by the experimental analysis.

For the equivalent shell replacement method, member axial forces at top and bottom chords are obtained by considering the static equilibrium of the basic structural unit on which the shell forces and moments acted. The shell moments are divided by the distance between the centre of gravity of top and bottom chords so that they would be treated in the same manner as membrane forces. Therefore, the axial forces at the top and bottom chords in terms of the shell forces and moments are evaluated successfully and reasonably in this type of structural member.

Further, the axial forces or moments of the members are calculated by taking the average of the shell forces and moments distributed along the length of the basic unit. However, the shell forces or moments are varied along any section and sometimes change sign and have a large variation over a short distance. Averaging the shell forces and moments to give the member axial forces or moments is an approximation. It was clearly observed in the investigated examples that the axial forces obtained by the continuum methods which had a bigger difference in
magnitude or a different sign from the results obtained by the stiffness method were mostly at the region when the shell forces tended to change sign.

5.7.3 BOUNDARY CONDITIONS

The boundary restraints of the braced barrel vaults are usually at the boundary joints but the boundary restraints applied on the equivalent shell are continuous in the continuum methods. This different treatment of the boundary would affect the structural behaviour. It was clearly observed in all investigated examples that the analytical results of the members near the boundary were always very different from those yielded by the stiffness method.

5.8 CONCLUDING REMARKS

After the investigation on several examples of braced barrel vaults by employing the continuum methods and the stiffness method, conclusions are drawn as follows:

(1) Accuracy

Compared with the stiffness method, the continuum methods give a similar normal stress (or axial force) distribution pattern but the magnitude of stresses (or axial forces) is not in good agreement. Since the continuum shell assumed in the analysis only compares in its macroscopic behaviour with the original skeletal structure, the solution given by the continuum methods must be approximate. Generally, these method would give a rough idea about stress distributions but are inferior to stiffness method in accuracy for linearly elastic braced barrel vaults.

(2) Efficiency

The continuum methods could be calculated by hand technique or by computer. When the stress analysis is carried out on a computer, the continuum methods take less computing time and storage in comparison
with the stiffness method. This is the reason why some designers still like to employ this method in the preliminary design.

(3) Limitations

The continuum methods are suitable to apply to a braced barrel vault composed of fairly fine grid configuration with a basic structural unit. The whole shape of the vault must be continuous. Analysis is based on the linearly elastic design and can hardly be extended to elastic-plastic or limit-state design. These methods are also suitable to analyse a structure under a simple form of loading such as uniformly distributed load, symmetrical or asymmetrical radial load which would be expressed properly by a Fourier series when applied to the shell theory. For the wind loads obtained by the wind tunnel tests shown in the previous chapter, a mathematical problem may be encountered in finding a Fourier series to express the wind pressure distributions.
Figure 5.1 Basic unit of the shell analogous method

Figure 5.2 Axial forces on the basic unit and membrane forces on the shell element

Figure 5.3 Bending and torsional moments on the basic unit and the shell element
Figure 5.4 Configuration and dimensions of Vault A_{24m} long and Vault A_{15m} long
Figure 5.5  Configuration and dimensions of Vault $B_{24\,m\text{ long}}$ and Vault $B_{15\,m\text{ long}}$
Figure 5.6 Analytical results of member axial forces — B.C.1
Figure 5.7  Analytical results of member axial forces — B.C.2
Figure 5.9 Analytical results of member axial forces – B.C.1
Vault B_{24m} long

Vault B_{15m} long

Figure 5.10 Analytical results of member axial forces — B.C.2
Figure 5.11 Analytical results of member axial forces — B.C.3
Figure 5.12 Analytical results of torsional moments — B.C.1
Figure 5.13 Analytical results of bending moments — B.C.1
Type 1: suitable for structure with relatively small scale

Type 2: suitable for structure with relatively large scale

Figure 5.14 Typical structural members and joints used in 'Diamond' structures (from Matsushita, 1980)
Figure 5.15  Basic structural unit of the equivalent shell replacement method.

Figure 5.16  Shell forces and moments acted on the basic structural unit
Figure 5.17 Analytical results of normal stresses — B.C.1
Figure 5.17 (continued)  Analytical results of normal stresses — B.C.1
Figure 5.18 Analytical results of normal stresses — B.C. 2
'Diamond' vault A15m long

Figure 5.18(continued)   Analytical results of normal stresses — B.C.2
Figure 5.19  Analytical results of normal stresses — B.C.3
Figure 5.19(continued) Analytical results of normal stresses — B.C.3
'Diamond' vault B_{24m} long

Figure 5.20  Analytical results of normal stresses — B.C.1
'Diamond' vault B_{24m} long

Figure 5.21  Analytical results of normal stresses — B.C.2
Figure 5.21(continued) Analytical results of normal stresses — B.C.2
Figure 5.22  Analytical results of normal stresses — B.C.3

'Diamond' vault B24m long

+ : TENSILE
- : COMPRRESSIVE

10.0 N/mm²
Figure 5.22 (continued) Analytical results of normal stresses — B.C.3
Figure 5.23 Configuration and dimensions of Vault C1.35m long and Vault C0.75m long
Figure 5.24 Stress vs strain curve of perspex tube

Notes: Each joint takes the distributed load on the shaded area

Figure 5.25 Equivalent concentrated loads on joints
VAULT C0.75m long

VAULT C1.35m long

Figure 5.26 Positions of strain gauge on models

Figure 5.27 Arrangement of pure bending system for obtaining bending moment calibration factor
Figure 5.28  Experimental and analytical results of member axial forces and bending moments
Figure 5.28(continued)  Experimental and analytical results of member axial forces and bending moments
Figure 5.28(continued) Experimental and analytical results of member axial forces and bending moments
Plate 5.1  Vault C1.35m long under concentrated point loads at joints

Plate 5.2  Vault C0.75m long under concentrated point loads at joints
Plate 5.3  Details of the joints

Plate 5.4  Details of the supports
CHAPTER 6

STRUCTURAL BEHAVIOUR OF TWO-WAY DOUBLE-LAYER BRACED BARREL VAULTS UNDER WIND LOADS

6.1 INTRODUCTION

Braced barrel vaults have been a popular form of three-dimensional skeletal structure during the last decade. Over the years, many braced barrel vaults have been designed and constructed using a variety of configurations, jointing methods and materials.

The world-wide interest in braced barrel vaults is due not only to their aesthetics, but also due to the fact that they provide an economic answer to many design requirements, such as large spans and a capacity to carry extensive overhead services. In construction, components permit fast production methods in the workshop, easy transportation and speedy erection.

In the past braced barrel vaults have been usually constructed as single-layer structures. Nowadays, with a demand for structures with a large uninterrupted span and area, a survey to establish a new stage of barrel vault has been undertaken. Since double-layer grids are an extension and progression of the single-layer grid, from this survey, double-layer braced barrel vault emerged as a practical proposal. To achieve the advantages as the single-layer barrel vaults, two conditions need to be satisfied. They are: 1) quick and easy means of calculating the forces in structural members and 2) an available method of jointing members together. In recent years several double-layer barrel vaults have been built in Japan and Europe. Two examples are shown in Plate 6.1.

In the past, research work was concentrated on single-layer braced barrel vaults. A large range of variables on a barrel vault such as
configuration, support condition, rise to width ratio, loading etc, were
taken into account on the study of their effects on the structural
behaviour. The work on double-layer braced barrel vaults is limited.
In this chapter, work is attempted to investigate the structural behaviour
of three types of configuration of two-way double-layer braced barrel
vaults under dead load and wind loads. The efficiency in stress
distribution of these three configurations is one of the main scopes of
the study.

Double-layer braced barrel vaults are usually on a substantial scale
and involve a large number of members and joints. A considerable volume
of arithmetic operations may be required during the initial linear elastic
analysis. Since the double-layer barrel vaults have a fine arrangement
of members so that a uniform and continuous pattern of member configuration
is displayed, the finite difference methods and the shell analogous
methods could be employed in the analysis. The lesser computational
effort in those analytical methods, no doubt, attracts the designer.
However, the finite difference methods (as described in Chapter 4) and the
shell analogous methods (as investigated in Chapter 5) only give an
approximate solution of stresses and are also limited the analysis of the
structure to under certain types of loading. For the present purpose of
understanding the structural behaviour of double-layer braced barrel
vaults under different loading conditions, the stiffness method is the
most suitable one since it could apply in any type of linear elastic
skeletal structure and give an exact solution.

6.2 Double-Layer Braced Barrel Vaults

A double-layer braced barrel vault can be described as two parallel
curved grids (referred to as the 'top-layer' and the 'bottom-layer')
which are connected together by 'bracing members'. The components
forming the top-layer and bottom-layer are usually referred to as the
'chord members'. In general, the layout of the two layers may not be
identical and the interconnection of the top-layer and the bottom-layer
by the bracing members may result in various forms of pyramidal units
with triangle, square, rectangular or hexagonal bases.
A number of terms and notation which are used in this chapter to describe a double-layer braced barrel vault are defined herein and are also shown in Figure 6.1.

- $S_X$: longitudinal span of barrel vault
- $S_Y$: transverse span of barrel vault (or width of barrel vault)
- $H$: rise of barrel vault which is measured from the base line to the crown of the top-layer
- $h$: height of pyramidal unit
- $\phi$: angle that the bracing member makes with the plane of the top-layer
- $n_X$: number of pyramidal units along the longitudinal length of barrel vault
- $n_Y$: number of pyramidal units along the transverse curve of barrel vault
- $n_X \times n_Y$: total number of pyramidal unit of barrel vault
- $l$: member length
- $\alpha = H/S_Y$: rise to transverse span (or width) ratio
- $\beta = S_X/S_Y$: longitudinal span to transverse span ratio

where lower case suffix X, Y and Z (longitudinal, transverse and vertical) are referred to frame coordinate system (as described in Chapter 5, Section 5.2.2.1);
- T is referred to top-layer;
- B is referred to bottom-layer and
- BR is referred to bracing members.

6.3 CHARACTERISTIC OF ANALYTICAL MODELS

6.3.1 CONFIGURATIONS

It is considered that the number of cases which could be studied in this chapter is limited; it is better to concentrate on a relatively small number of basic cases which are the most practical ones. Based on this objective, it has been decided to consider three different configurations
in a family of two-way double-layer barrel vaults. For the two-way double-layer barrel vaults, each of the top-layer and bottom-layer consists mainly of two sets of parallel lines (actually, one is parallel to the longitudinal axis and the other is parallel to the transverse curve) and intersecting at right angles or setting diagonally. Therefore, a structure is formed with pyramidal units of square or rectangular bases. The arrangements of the three configurations are as follows:

TYPE 1 'SQUARE-ON-SQUARE'

In this arrangement, the layout of the top-layer and bottom-layer are the same with one layer being offset from the other in a curve surface. The members of both layers are parallel to the boundary lines of the barrel vault. An example of its plan and perspective view is shown in Figure 6.2.

TYPE 2 'SQUARE-ON-DIAGONAL'

In this arrangement, the top-layer members are set parallel to the outer edges but the bottom-layer members are set diagonally. An example of its plan and perspective view is shown in Figure 6.3.

TYPE 3 'DIAGONAL-ON-SQUARE'

In this arrangement, the top-layer members are set diagonally whilst the bottom-layer members are set parallel to the edges of the barrel vault. An example of its plan and perspective view is shown in Figure 6.4.

6.3.2 GEOMETRICAL AND LAYOUT DIMENSIONS

When wind loads in the analysis are taken from the results of the wind tunnel tests which have been given in Chapter 3, it is necessary to consider a structure having the same geometrical and aspect ratio as the model used in the wind tunnel tests. In the present study, barrel vaults were chosen to have a circular curved arc with two different subtended angles, $180^\circ$ and $120^\circ$ (as shown in Figures 6.5 to 6.7) entailing the ratios of rise to transverse span $\alpha$ of 0.5 and 0.289. Besides, two different ratios of the longitudinal to the transverse span $\beta$ of 1 and 2 which represented a square and rectangular covering area were also taken into investigation.
In all cases barrel vaults were assumed to have the same transverse span thus longitudinal spans were simply determined from the $\beta$.

Since part of the scope of the present study is to compare the structural behaviour of different configurations, it is better to keep the same values of $n_X$ and $n_Y$ in the three different configurations for the same $\beta$. In the present work, $n_Y$ was set to be 13 and $n_X$ was set to be 9 and 18, while $\beta$ was 1 and 2 respectively. However, the 'SQUARE-ON-DIAGONAL' type can be arranged only with an odd number of pyramidal units along the edges. If an odd number was chosen for the $\beta$ of 1, and even number was obtained for the $\beta$ of 2. In such situation, 19 pyramidal units were chosen instead of 18 for $\beta$ of 2 and the longitudinal span was same as the 'SQUARE-ON-SQUARE' and 'DIAGONAL-ON-SQUARE' types. Indeed, it is a pity that the $n_X$ could not be kept the same for all configurations. However, a barrel vault with $\beta$ of 2 was considered as a comparatively long structure so that even increasing one pyramidal unit along the longitudinal span would not have significant effects on the structural behaviour. This choice of pyramidal units $n_X$ and $n_Y$ was applied in the barrel vaults with subtended angle of $180^\circ$ ($\alpha=0.5$) and $120^\circ$ ($\alpha=0.289$).

In all barrel vaults under analysis, the length of member of the top-layer in the longitudinal direction, $L_{T,X}$, was set as a parameter so that the height of pyramidal unit, the length of other members and the longitudinal and transverse spans were related to this parameter. Further, single height of pyramidal unit was taken into consideration. It was taken to be 0.5 of $L_{T,X}$ or 1/18 of the transverse span. This height gave an angle $\phi$ between the bracing and the plane of the pyramidal unit of $34^\circ$ and $37^\circ$ for $\alpha$ of 0.5 and 0.289 respectively.

Details of geometry and dimensions of the investigated barrel vaults are shown in Figures 6.5 to 6.7.

6.3.3 SUPPORTS

Single support condition was considered in all the studied barrel
vaults. To provide a clear span equal to the entire width of the structure, barrel vaults were assumed to be supported at the joints of the top-layer along the longitudinal and the transverse edges. Each support along the longitudinal edges was assumed being constrained against translational displacement in the Z direction. Each support along the transverse edges was assumed being constrained against translational displacements in the Y and Z directions.

It is noted that with the above assumption, the rigid body movement of a vault will not be restrained since it can move in the longitudinal direction. Hence some constraints would be imposed to prevent the structure from becoming a mechanism. Keeping symmetry for the structural layout, horizontal constraints in the longitudinal direction were imposed to the supports at the four corners.

6.3.4 PROPERTIES OF MEMBERS

In practice, the modulus of elasticity is constant for all members in a structure and the cross-sectional area of members in a given layer again is constant or nearly the same. In the present study, therefore, it was considered that the modulus of elasticity was constant and the cross-sectional area of the top-layer members, bottom-layer members and bracing members was the same.

6.3.5 JOINTING SYSTEM

In the analysis, all barrel vaults were assumed to be pin-jointed structures so that loads applied at joints were transmitted through the structure to the supports mainly by the axial forces in the members.

6.4 LOADING CASES

All external loads were applied as point loads at the joints of the top-layer. Dead load and wind loads were taken into consideration but
were analysed separately. Under linear elastic analysis, the results of combining dead load and wind load are simply obtained by adding up the results of the separate loadings.

6.4.1 Dead Load

Dead load which included self-weight and cladding was considered to be a uniformly distributed load covering the whole top-layer. To convert this distributed load to equivalent concentrated loads applied at joints, it was assumed that the distributed load was shared by the joint which carried the load on the surface area formed by connecting the mid-point of the connecting members to the centroid of the surface areas of the adjacent unit of openings. Figure 6.8 represents the plan of the top-layer of the double-layer barrel vaults under distributed load and the assumed shares of loads for joints are indicated by the shaded areas.

6.4.2 Wind Load

In the previous wind tunnel tests on barrel vaults, three different wind directions 0°, 45° and 90° which were measured from the longitudinal axis were chosen to investigate the wind pressure distribution. From the results of the wind tunnel tests, it was shown that wind blowing parallel to the longitudinal axis of the barrel vault caused a high pressure on the windward end wall and caused a comparatively low suction on the roof surface. The barrel vault under this direction of wind is not a critical condition when the roof is considered but caution is needed when designing the end truss. Usually, the end truss and the roof of the barrel vaults are considered separately. In the present work, analysis is mainly concerned with the roof structure, so that only wind directions of 45° and 90° were studied.

With the wind blowing normal to the longitudinal axis of the barrel vault (i.e. the case of 90°), a two-dimensional case (denoted as 2-D) and a three-dimensional case (denoted as 3-D) were considered. In the 2-D case, wind pressures were obtained under the assumption that the model
was infinitely long. The wind pressures were uniformly distributed along the longitudinal span so that the wind load on barrel vaults of different length but with same geometry was the same. In the 3-D case, the length and the end effect on the pressure distributions were taken into consideration so that barrel vaults with the same geometry in cross-section but of different length had different distribution pattern of wind loading.

The distributed wind pressures on the barrel vaults were determined by multiplying a dynamic pressure of the oncoming wind at a reference height by the wind pressure coefficients referred to that reference height. In accordance with the British Standard Code of Practice CP 3: Chapter V: Part 2, the dynamic pressure of the oncoming wind in the present study was taken at the height equal to the level of the crown of the barrel vault. The wind pressure coefficients of different oncoming wind directions obtained by the wind tunnel tests have already been given in Chapter 3. The distributed wind pressures had to be converted to equivalent concentrated wind forces applied at joints. Each joint was assumed to carry the distributed wind pressures on a surface area which was same as that used for uniformly distributed dead load.

6.5 Method of Analysis

The standard stiffness method of structural analysis was used as a means of analysis of the double-layer braced barrel vaults under consideration. This analytical method could give an exact solution of a linearly elastic skeletal structure. A brief description of the method for the linear elastic analysis of skeletal structure has been mentioned in Chapter 4.

6.6 Computer Implementation of Analysis

A program was developed to analyse a double-layer braced barrel vault. It was successfully applied to all the studied cases. The main steps in the computer program are explained herein.
6.6.1 DATA GENERATION

In a double-layer braced barrel vault, input data generally consist of information about topological properties of the structure, geometrical and elastic properties of the members and kinematical characteristics of the joints. In order to minimize the amount of effort and reduce the possibility of making errors in the preparation of the data for the program, a method was developed to generate the data automatically.

The main steps in the program for data generation are as follows:
(1) Setting an integer coordinate system on which the topological model of the structure was represented.

(2) The integer coordinates of the joint were generated by defining a function. The joints were numbered in the same order as they were generated.

(3) The planar configuration of the structure was generated and was presented as a matrix which contained the joint numbers. The position of an element of this matrix corresponded to the integer coordinates of that joint.

(4) Defining different functions and using the matrix generated in step 3, the member list, coordinates of the joints, member type list, constraint list, load vector or any other required data could be generated.

6.6.2 CONVERSION OF EXTERNAL LOADINGS

6.6.2.1 DEAD LOAD

A simple program was developed to convert the uniformly distributed dead load to equivalent concentrated joint loads. The joint loads only had vertical components (i.e. in the Z-direction) and were simply calculated by defining a function which related to the curvature of barrel
vault and the position of joints.

6.6.2.2 WIND LOADS

Since the wind pressures are distributed normal to the surface of a body, the wind forces on barrel vaults have two components, one in the Y direction and the other in the Z direction. These two components in the Y and Z directions are separately determined by integrating the wind pressures over the projection areas in the X-Z plane and the X-Y plane. Their mathematical forms are expressed as follows:

\[ F_Y = \int_X \int_Z p(X) \ p(Z) \ dZ \ dX \]
\[ F_Z = \int_X \int_Y p(X) \ p(Y) \ dY \ dX \]

where \( F_Y \) and \( F_Z \) are the Y and Z components of the wind forces; \( p(X) \), \( p(Y) \) and \( p(Z) \) are the expressions of wind pressure distribution with respect to the X, Y and Z coordinates.

From the results of the wind tunnel tests, the wind pressure distributions on barrel vaults showed an irregular pattern. It was difficult to express such distributions properly by a simple function. It was, therefore, decided to divide the surface into several small regions (in the present study, each region was taken as the surface area assigned to each joint for the purposes of converting the distributed wind pressures or dead load to equivalent concentrated point loads (see Section 6.4.2)) so that the pressure distributions over each of these regions could be fitted by an appropriate function by the method of least-squares.

The wind pressure distributions along the longitudinal length of the vault (i.e. in the X direction) were taken as varying in an approximate linearly way. The wind pressure distribution with respect to the X coordinate was fitted by a first degree polynomial and the trapezoidal rule was chosen to integrate the wind pressures. On the other hand, the wind pressures around the cross-section of a barrel vault (i.e. in the Y-Z
plane) had a curvilinear distribution. The wind pressure distributions with respect to the Y and to the Z coordinates were then fitted by a second degree polynomial and Simpson's rule was chosen to integrate the wind pressures.

Since the conversion of wind load to equivalent joint loads from the wind pressure distributions on barrel vaults involved the integration of an irregular curve over a curved surface, a computer program was developed to overcome this complicated and tedious calculation in the present study.

6.6.3 ANALYSIS

The analysis is based on the matrix formulation of the stiffness method as applied to linearly elastic pin-jointed space frames. The stiffness matrix is a one-dimensional array for decomposition using Gaussian elimination, taking advantage of the properties of the matrix which is symmetric, positive definite and banded.

In the cases of barrel vaults under the uniformly distributed dead load and under the two-dimensional wind load with oncoming wind direction at 90° to the longitudinal axis of the vault, both the structure and the external loading have a symmetrical layout. The program is capable of analysing a portion containing \( \frac{1}{2m} \)th of the structure, where \( m \) is the number of planes of symmetry.

6.6.4 RESULTS FROM THE ANALYSIS

A displacement vector contains joint displacements in the form of three translations per joint for each joint in the structure. These are in terms of the frame coordinate system.

A force vector contains member axial force of each member in the structure. These are in terms of the member coordinate system for each members.
A reaction vector contains reactive forces in the form of three forces for each joint at which a constraint is applied. These forces are in terms of the frame coordinate system.

6.7 ANALYTICAL RESULTS

This section presents the results of the theoretical stress analysis of the barrel vaults which have been mentioned in previous Section 6.3. Those results are presented in such a way to show the behaviour of a double-layer braced barrel vault under different external load.

Analytical results of member forces, support reactions and displacements are presented in a dimensionless form as coefficients of member forces, support reactions and displacements. The coefficients of member forces, support reactions and displacements were obtained by analysing a barrel vault with unit length of member of top-layer in the longitudinal direction $a_{T,X}$ (it has been mentioned in Section 6.3.2 that $a_{T,X}$ was set to be a parameter so that the length of the other members and the dimensions of the barrel vaults were in terms of it), unit intensity of dead load or unit dynamic wind pressure, unit of modulus of elasticity and unit of cross-sectional area of members. As a linear analysis was presumed, the member forces, support reactions and displacements of a barrel vault of assigned length of $a_{T,X}$, intensity of dead load or dynamic wind pressure, modulus of elasticity and cross-sectional area of members could be simply determined by multiplying the assigned values to the corresponding coefficients according to the following relationships:

\[
\text{Member forces} = C_{mf} \times q \times 0.1 \\
\text{Support reactions} = C_{sr} \times q \times 0.1 \\
\text{Displacement} = C_d \times q \times \frac{S_X}{n_X} \times \frac{1}{E/A}
\]

where $C_{mf}$ coefficient of member forces, $C_{sr}$ coefficient of support reactions, $C_d$ coefficient of displacements.
6,8 INVESTIGATION AND DISCUSSION

6.8.1 STRUCTURAL BEHAVIOUR UNDER THE EXTERNAL LOADS

In this section, a brief investigation and discussion of structural behaviour of all the studied barrel vaults under the dead load and wind loads are given. The maximum member forces of top-layer members, bottom-layer members and bracing members and the maximum deflection in all the structures are summarized in Tables 6.1a and 6.1b. In addition, it is noted that the deflection of a double-layer barrel vault mentioned in the following sections is referred to the vertical displacement (i.e. in the Z direction) of the joint of the bottom-layer.

6.8.1.1 DEAD LOAD

For the double-layer barrel vaults under uniformly distributed dead load, the maximum tensile and the maximum compressive forces in the barrel vaults of 'SQUARE-ON-SQUARE' and 'SQUARE-ON-DIAGONAL' types were very close to each other for $\beta$ of 1 and the maximum compressive forces were
### Table 6.1a: Maximum member forces and deflections of the double-layer braced barrel vaults with subtended angle of 180°

- **Notes:** All member forces and deflections are presented in dimensionless form.
- The actual values are obtained by the expressions which have been given in Section 6.7.
<table>
<thead>
<tr>
<th>$H/S_Y = \alpha = 0.289$</th>
<th>'SQUARE-ON-SQUARE'</th>
<th>'SQUARE-ON-DIAGONAL'</th>
<th>'DIAGONAL-ON-SQUARE'</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_X/S_Y = \beta$</td>
<td>$\beta = 1$</td>
<td>$\beta = 1$</td>
<td>$\beta = 1$</td>
</tr>
<tr>
<td>$\beta = 2$</td>
<td>$\beta = 2$</td>
<td>$\beta = 2$</td>
<td>$\beta = 2$</td>
</tr>
<tr>
<td>$n_X \times n_Y$</td>
<td>9x13</td>
<td>18x13</td>
<td>9x13</td>
</tr>
<tr>
<td>Total no. of joints</td>
<td>257</td>
<td>500</td>
<td>218</td>
</tr>
<tr>
<td>No. of members</td>
<td>256</td>
<td>212</td>
<td>468</td>
</tr>
<tr>
<td>DEAD LOAD</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Max. tensile</td>
<td>35</td>
<td>8</td>
<td>45</td>
</tr>
<tr>
<td>Max. deflect.</td>
<td>-94</td>
<td>-383</td>
<td>-62</td>
</tr>
<tr>
<td>WIND 90° (2-D)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Max. tensile</td>
<td>43</td>
<td>12</td>
<td>41</td>
</tr>
<tr>
<td>Max. deflect.</td>
<td>74</td>
<td>285</td>
<td></td>
</tr>
<tr>
<td>WIND 90° (3-D)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Max. tensile</td>
<td>23</td>
<td>13</td>
<td>21</td>
</tr>
<tr>
<td>Max. deflect.</td>
<td>40</td>
<td>188</td>
<td>31</td>
</tr>
<tr>
<td>WIND 45° (3-D)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Max. tensile</td>
<td>32</td>
<td>12</td>
<td>24</td>
</tr>
<tr>
<td>Max. deflect.</td>
<td>51</td>
<td>196</td>
<td>38</td>
</tr>
</tbody>
</table>

Notes: All member forces and deflections are presented in dimensionless form. The actual values are obtained by the expressions which have been given in Section 6.7.

Table 6.1b Maximum member forces and deflections of the double-layer braced barrel vaults with subtended angle of 120°
approximately twice the maximum tensile forces for $\beta$ of 2. For the barrel vaults of 'DIAGONAL-ON-SQUARE' type with $\beta$ of 1 and 2, the maximum compressive forces were approximately twice the maximum tensile forces. Besides, the maximum deflections occurred in the central region of the barrel vault in all the studied cases.

6.8.1.2 Wind Loads

Double-layer braced barrel vaults under the wind loads for the oncoming wind blowing at $45^\circ$ and at $90^\circ$ to the longitudinal axis of the vault show a reversal stress distribution pattern, compared the case of the same structures analysed under the uniformly distributed dead load.

The detailed investigation of the barrel vaults under the wind loads is summarized as follows:

(1) The 2-D case and 3-D case of wind loads for the oncoming wind blowing at $90^\circ$ to the longitudinal axis of the barrel vault:

(a) Stress distributions of generally similar patterns were shown for the barrel vaults using the 2-D case and the 3-D case of wind loads to analyse.

(b) The stress analysis of barrel vaults using the 2-D case of wind loads produced the results of higher magnitude than those using the 3-D case of wind loads. The 3-D case of wind load represented more closely the proper wind load on an actual structure. By taking the results analysed by using the 3-D case of wind loads as a basis of comparison, the overestimations of the analytical results of the barrel vaults using the 2-D case of wind loads are shown in Table 6.2. Certain observations are drawn as follows:

(i) For barrel vaults with subtended angle of $180^\circ(\alpha=0.5)$, the overestimations of maximum deflection were between 50% to 60% for $\beta$ of 1 and were between 40% to 50% for $\beta$
<table>
<thead>
<tr>
<th>WIND 90°</th>
<th>'SQUARE-ON-SQUARE'</th>
<th>'SQUARE-ON-DIAGONAL'</th>
<th>'DIAGONAL-ON-SQUARE'</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$S_X/S_Y=\beta=1$</td>
<td>$\beta=2$</td>
<td>$\beta=1$</td>
</tr>
<tr>
<td></td>
<td>2-D 3-D Difference(%)</td>
<td>2-D 3-D Difference(%)</td>
<td>2-D 3-D Difference(%)</td>
</tr>
<tr>
<td>H/S_Y=\alpha=0.5</td>
<td>32 21 52 129 87 48</td>
<td>37 25 48 131 90 45</td>
<td>54 31 74 223 136 63</td>
</tr>
<tr>
<td>Max. tensile</td>
<td>32 21 52 129 87 48</td>
<td>37 25 48 131 90 45</td>
<td>54 31 74 223 136 63</td>
</tr>
<tr>
<td>Max. deflection</td>
<td>46 30 53 176 118 49</td>
<td>43 27 59 152 107 42</td>
<td>54 33 63 211 139 51</td>
</tr>
<tr>
<td>H/S_Y=\alpha=0.289</td>
<td>43 23 86 150 102 47</td>
<td>39 24 62 169 117 44</td>
<td>82 42 95 278 175 58</td>
</tr>
<tr>
<td>Max. tensile</td>
<td>43 23 86 150 102 47</td>
<td>39 24 62 169 117 44</td>
<td>82 42 95 278 175 58</td>
</tr>
<tr>
<td>Max. deflection</td>
<td>74 40 85 285 188 51</td>
<td>54 31 74 270 185 45</td>
<td>82 45 82 340 222 53</td>
</tr>
</tbody>
</table>

Notes: All member forces and deflections are presented in dimensionless form.

Difference (%) = \( \frac{(2-D) - (3-D)}{3-D} \) x 100%

Table 6.2 Comparison of the maximum member forces and deflections of the double-layer braced barrel vaults under the 2-D case and the 3-D case of wind loads

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of 2. When \( \beta \) was 1 and 2, the overestimations of maximum member forces were between 30% to 50% for the 'SQUARE-ON-SQUARE' and 'SQUARE-ON-DIAGONAL' types but were nearly between 60% to 70% for the 'DIAGONAL-ON-SQUARE' type.

(ii) For barrel vaults with subtended angle of 120° (\( \alpha = 0.289 \)), the overestimations of maximum member forces were between 50% to 100% for \( \beta \) of 1 and were between 40% to 60% for \( \beta \) of 2. The overestimations of maximum deflection were approximately 80% for \( \beta \) of 1 and were approximately 50% for \( \beta \) of 2.

This fact showed clearly that the 2-D case of wind load could only be used for a very long barrel vault; otherwise, it would give an overestimation of stresses. In addition, the influence of different length of barrel vault on the overestimation of maximum member forces and deflection was not obvious for the barrel vaults with subtended angle of 180° but was obvious for the barrel vaults with subtended angle of 120°.

(2) The 3-D case of wind loads for the oncoming wind blowing at 90° and at 45° to the longitudinal axis of the barrel vault:

(a) Stress distributions of generally similar patterns but with differences in detail were shown for the barrel vaults under these two different oncoming wind directions.

(b) By taking the maximum member forces and deflections of the barrel vaults caused by the wind direction of 90° as a basis of comparison, Table 6.3 shows the differences of maximum member forces and deflections of the barrel vaults given by the wind direction of 45°. Certain observations from Table 6.3 are drawn as follows:

(i) For barrel vaults with subtended angle of 180° (\( \alpha = 0.5 \)) and with the wind direction of 45°, the 'SQUARE-ON-SQUARE'
### Table 6.3

Comparison of the maximum member forces and deflections of the double-layer braced barrel vaults under the wind loads for the wind blowing at 90° and at 45° to the longitudinal axis.

<table>
<thead>
<tr>
<th></th>
<th>'SQUARE-ON-SQUARE'</th>
<th>'SQUARE-ON-DIAGONAL'</th>
<th>'DIAGONAL-ON-SQUARE'</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( S_x/S_y = \beta = 1 )</td>
<td>( \beta = 2 )</td>
<td>( \beta = 1 )</td>
</tr>
<tr>
<td></td>
<td>( 90° )</td>
<td>( 45° )</td>
<td>Difference(%)</td>
</tr>
<tr>
<td>Max. tensile</td>
<td>21</td>
<td>21</td>
<td>0</td>
</tr>
<tr>
<td>Max. compressive</td>
<td>-21</td>
<td>-20</td>
<td>-4</td>
</tr>
<tr>
<td>Max. deflection</td>
<td>30</td>
<td>28</td>
<td>-6</td>
</tr>
</tbody>
</table>

**Notes:**

All member forces and deflections are presented in dimensionless form.

\[
\text{Difference (\%)} = \frac{(\text{WIND } 45°) - (\text{WIND } 90°)}{(\text{WIND } 90°)} \times 100\%
\]
and 'SQUARE-ON-DIAGONAL' types showed a slightly lower magnitude of maximum member forces and deflections. The maximum member forces and deflections were not more than 10% lower for $\beta$ of 1 and were less than 5% lower for $\beta$ of 2. On the contrary, the barrel vaults of 'DIAGONAL-ON-SQUARE' type showed a slightly higher magnitude of maximum member forces and deflections.

(ii) For barrel vaults with subtended angle of $120^\circ (a=0.289)$ and with the wind direction of $45^\circ$, all three different types of configuration showed a higher magnitude of maximum member forces and deflections. The maximum member forces and deflections were approximately 20% to 40% higher for $\beta$ of 1 and were slightly higher for $\beta$ of 2.

6.8.1.3 COMBINATION OF SELF-WEIGHT AND WIND LOADS

According to Makowski (1980), the development of new structural materials and improvements in engineering constructional technology have led to a significant reduction in the self-weight of space structures and an increase in their clear spans. For double-layer braced barrel vaults constructed in aluminium or steel tubes, the lower bound and the upper bound of self-weight are usually 0.1kN/m$^2$ and 0.3kN/m$^2$ (reference to the double-layer braced domes, Makowski, 1980). If a double-layer braced barrel vault is considered as being built in the situation of suburban areas or small towns (similar to the 'rough' boundary layer in the present wind tunnel test) with the oncoming wind velocity at the height equal to the level of the crown of the barrel vault between 35m/sec. to 55m/sec., the dynamic pressure is then 0.75kN/m$^2$ to 1.85kN/m$^2$. If these intensities of dead load and dynamic pressures of wind are used with the coefficients of member forces, deflections and support reactions given in the previous section and the results then added up, the member forces, deflections and support reactions of the barrel vaults under the combination of self-weight and wind load could be obtained. Since the dynamic wind pressure in some situations may be several times higher than
the intensity of self-weight of the structure, the stress distribution of
the combination of self-weight and wind load would then be mainly governed
by the wind load. This fact is shown clearly by an example given in
Figure 6.45.

6.8.2 STRUCTURAL BEHAVIOUR OF THE THREE DIFFERENT CONFIGURATIONS

For the double-layer braced barrel vaults of 'SQUARE-ON-SQUARE' type,
the maximum member forces existed in the top-layer and bracing members.
The overall member forces in the bottom-layer were relatively small.

For the double-layer braced barrel vaults of 'SQUARE-ON-DIAGONAL'
type, stresses were evenly distributed in the top-layer, bottom-layer and
bracing members. Compared with the other two configurations, this type
of configuration gave a small deflection.

For the double-layer braced barrel vaults of 'DIAGONAL-ON-SQUARE'
type, the maximum member forces existed on the top-layer. The overall
member forces in the bottom-layer and bracing members were relatively
small. Among those three configurations, this configuration gave a
large deflection in the structure.

Comparing the three different configurations, the barrel vaults of
'DIAGONAL-ON-SQUARE' type had the greatest number of joints and number
of members. It also gave large deflections and high member forces. In
such situation, it could be stated that the 'DIAGONAL-ON-SQUARE' type of
configuration was not efficient in stress distribution and was not
economic.

In Tables 6.4a and 6.4b, a comparison of total number of joints,
total number of members, total length of members, maximum member forces
and maximum deflections of the barrel vaults of 'SQUARE-ON-SQUARE' and
'SQUARE-ON-DIAGONAL' types are shown. It is observed that the maximum
member forces in the barrel vaults given by the 'SQUARE-ON-DIAGONAL' type
were mostly higher than those given by the 'SQUARE-ON-SQUARE' type. In
some situations, the maximum member forces were nearly 30% higher so
<table>
<thead>
<tr>
<th>$H/S_Y = \alpha = 0.5$</th>
<th>$S_X/S_Y = \beta = 1$</th>
<th>$S_X/S_Y = \beta = 2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n_X \times n_Y$</td>
<td>9x13</td>
<td>9x13</td>
</tr>
<tr>
<td>Total no. of joints</td>
<td>257</td>
<td>218</td>
</tr>
<tr>
<td>Total no. of members</td>
<td>936</td>
<td>704</td>
</tr>
<tr>
<td>Total length of members ($L_{T,X}$)</td>
<td>890.6</td>
<td>723.68</td>
</tr>
<tr>
<td>DEAD LOAD</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Max. tensile</td>
<td>31</td>
<td>30</td>
</tr>
<tr>
<td>Max. compressive</td>
<td>-37</td>
<td>-37</td>
</tr>
<tr>
<td>Max. deflection</td>
<td>-67</td>
<td>-52</td>
</tr>
<tr>
<td>WIND 90° (2-D)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Max. tensile</td>
<td>32</td>
<td>37</td>
</tr>
<tr>
<td>Max. compressive</td>
<td>-30</td>
<td>-36</td>
</tr>
<tr>
<td>Max. deflection</td>
<td>46</td>
<td>43</td>
</tr>
<tr>
<td>WIND 90° (3-D)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Max. tensile</td>
<td>21</td>
<td>25</td>
</tr>
<tr>
<td>Max. compressive</td>
<td>-21</td>
<td>-27</td>
</tr>
<tr>
<td>Max. deflection</td>
<td>30</td>
<td>27</td>
</tr>
<tr>
<td>WIND 45° (3-D)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Max. tensile</td>
<td>21</td>
<td>24</td>
</tr>
<tr>
<td>Max. compressive</td>
<td>-20</td>
<td>-24</td>
</tr>
<tr>
<td>Max. deflection</td>
<td>28</td>
<td>25</td>
</tr>
</tbody>
</table>

Notes: All member forces and deflections are presented in dimensionless form.
Difference (\%) = \frac{'SQUARE-ON-DIAGONAL' - 'SQUARE-ON-SQUARE'}{'SQUARE-ON-SQUARE'} \times 100%

Table 6.4a Comparison of the maximum member forces and deflections of the double-layer braced barrel vaults of 'SQUARE-ON-SQUARE' and 'SQUARE-ON-DIAGONAL' types with subtended angle of 180°
<table>
<thead>
<tr>
<th>$H/Y = \alpha = 0.289$</th>
<th>$S_X/S_Y = \beta = 1$</th>
<th>$S_X/S_Y = \beta = 2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n_X \times n_Y$</td>
<td>9x13</td>
<td>18x13</td>
</tr>
<tr>
<td>Total no. of joints</td>
<td>257</td>
<td>500</td>
</tr>
<tr>
<td>Total no. of members</td>
<td>936</td>
<td>1872</td>
</tr>
<tr>
<td>Total length of members ($x_Tx$)</td>
<td>803.96</td>
<td>1610.0</td>
</tr>
<tr>
<td>DEAD LOAD</td>
<td>'SQUARE-ON-SQUARE'</td>
<td>'SQUARE-ON-DIAGONAL'</td>
</tr>
<tr>
<td>Max. tensile</td>
<td>45</td>
<td>45</td>
</tr>
<tr>
<td>Max. compressive</td>
<td>-55</td>
<td>-49</td>
</tr>
<tr>
<td>Max. deflection</td>
<td>-94</td>
<td>-62</td>
</tr>
<tr>
<td>WIND 90(^\circ) (2-D)</td>
<td>Max. tensile</td>
<td>43</td>
</tr>
<tr>
<td></td>
<td>Max. compressive</td>
<td>-33</td>
</tr>
<tr>
<td></td>
<td>Max. deflection</td>
<td>74</td>
</tr>
<tr>
<td>WIND 90(^\circ) (3-D)</td>
<td>Max. tensile</td>
<td>23</td>
</tr>
<tr>
<td></td>
<td>Max. compressive</td>
<td>-16</td>
</tr>
<tr>
<td></td>
<td>Max. deflection</td>
<td>40</td>
</tr>
<tr>
<td>WIND 45(^\circ) (3-D)</td>
<td>Max. tensile</td>
<td>32</td>
</tr>
<tr>
<td></td>
<td>Max. compressive</td>
<td>-21</td>
</tr>
<tr>
<td></td>
<td>Max. deflection</td>
<td>51</td>
</tr>
</tbody>
</table>

Notes: All member forces and deflections are presented in dimensionless form.

Difference (%) = \(\frac{'SQUARE-ON-DIAGONAL' - 'SQUARE-ON-SQUARE'}{'SQUARE-ON-SQUARE'} \times 100\%

Table 6.4b Comparison of the maximum member forces and deflections of the double-layer braced barrel vaults of 'SQUARE-ON-SQUARE' and 'SQUARE-ON-DIAGONAL' types with subtended angle of 120\(^\circ\)
that the 'SQUARE-ON-DIAGONAL' type should be built with members of larger cross-sectional area. However, the total number of joints and the total length of members of the 'SQUARE-ON-DIAGONAL' type were nearly 13% and 19% less than the 'SQUARE-ON-SQUARE' type. Usually, the structural cost includes the cost of fabricated tube, delivery, erection, joints, painting specification etc. The increase of cross-sectional area of member may increase the weight of the tube and then the cost of fabricated tube may be increased. On the other hand, the decrease of number of joints and members may reduce the cost. Since so many factors may affect the overall structural cost, it is difficult to give any general rule as to which type of configuration is an economic one in the present study.

Investigating the stress distributions, the barrel vaults of 'SQUARE-ON-DIAGONAL' type showed that the member forces in the top-layer, bottom-layer and bracing members were of the same order of magnitude and the deflections were small, compared with the 'SQUARE-ON-SQUARE' type. In some situations such as the barrel vaults with subtended angle of 120° and with $\beta$ of 1, the maximum deflections given by the 'SQUARE-ON-DIAGONAL' type were nearly up to 20% smaller than those given by the 'SQUARE-ON-SQUARE' type. Therefore, for double-layer braced barrel vaults of 'SQUARE-ON-DIAGONAL' type of configuration under the present studied geometries, member properties, boundary condition and loading cases showed a good efficiency in stress distribution compared with the 'SQUARE-ON-SQUARE' and 'DIAGONAL-ON-SQUARE' types of configuration.

6.9 Concluding Remarks

Although the configuration, size of structure, member section, boundary condition and loading have been taken into investigation, the range is limited in the present study. Nevertheless, several points observed from the study would shed some light in the design of double-layer barrel vaults. They are drawn as follows:

(1) For a light-weight double-layer barrel vault, the wind load may be one of the main parameters which govern the design of the components of the barrel vault.
(2) For barrel vaults with finite length, the wind flow around the vault is entirely a three-dimensional problem. Using the two-dimensional wind load to analyse a barrel vault could reduce the computational effort but at the same time it overestimates the stresses and deformations. The possibility of designing an economic barrel vault and of understanding precisely its structural behaviour requires a correct estimation of wind loads.

(3) Difference in the oncoming wind direction may give difference in the stress distribution and deformation patterns of a double-layer barrel vault. The oncoming wind direction which causes critical member force and deflection of the barrel vault is different with different geometry, dimension and configuration of vault. Several different oncoming wind directions need to be taken into consideration in the design.

(4) With the aim of obtaining a lower cost design for a double-layer barrel vault, the 'DIAGONAL-ON-SQUARE' and 'SQUARE-ON-SQUARE' types of configuration are preferable.
Figure 6.1 Notation used for describing a double-layer braced barrel vault

Square or rectangular base pyramidal unit

Triangular base pyramidal unit
Figure 6.2 A double-layer braced barrel vault of 'SQUARE-ON-SQUARE' type of configuration.
Figure 6.3  A double-layer braced barrel vault of 'SQUARE-ON-DIAGONAL' type of configuration
Figure 6.4 A double-layer braced barrel vault of 'DIAGONAL-ON-SQUARE' type of configuration
Figure 6.5  Details of geometrical and layout dimensions of barrel vaults of 'SQUARE-ON-SQUARE' configuration
Figure 6.6 Details of geometrical and layout dimensions of barrel vaults of 'SQUARE-ON-DIAGONAL' configuration

Relevant data (in terms of $\ell_{T,X}$)

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Figure 6.7 Details of geometrical and layout dimensions of barrel vaults of 'DIAGONAL-ON-SQUARE' configuration
'DIAGONAL-ON-SQUARE' configuration

Each joint takes the distributed load on the shaded area

Notes:

Equivalent concentrated loads on joints of double-layer barrel vaults

Figure 6.8
Notes: (for Figures 6.9 to 6.14)

The structure is under a uniformly distributed load $q$

or

The structure is under a wind load of dynamic pressure at the height equal to the top level of the structure $q$

1- Forces in TOP-LAYER

2- Forces in BOTTOM-LAYER

3- Forces in BRACING

4- Reactions

vertical displacement of the joints of BOTTOM-LAYER

\[ x \times q \times \frac{x}{n_X} \times 1 \times 1 \times \frac{1}{A} \]

--- (+) Tensile force

----- (-) Compressive force

\( X \)

\( Y \)

\( Z \)

Reactions

• (+) Upward displacement

• (-) Downward displacement

Figure 6.9  Stress distribution in a double-layer barrel vault of 'SQUARE-ON-SQUARE' configuration under a uniformly distributed load $\alpha = 0.5, \beta = 1$
Figure 6.10 Stress distribution in a double-layer barrel vault of 'SQUARE-ON-SQUARE' configuration under a wind load of wind blowing at 90° to the longitudinal axis of the vault — \( \alpha = 0.5, \ \beta = 1 \)
Figure 6.11 Stress distribution in a double-layer barrel vault of 'SQUARE-ON-SQUARE' configuration under a wind load of wind blowing at 45° to the longitudinal axis of the vault —— α = 0.5, β = 1
### Figure 6.11 (continued)

**BRACING**

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**Figure 6.11 (continued)**
Figure 6.12  Stress distribution in a double-layer barrel vault of 'SQUARE-ON-SQUARE' configuration under a uniformly distributed load —— $\alpha = 0.5$, $\beta = 2$
**Figure 6.13** Stress distribution in a double-layer barrel vault of 'SQUARE-ON-SQUARE' configuration under a wind load of wind blowing at 90° to the longitudinal axis of the vault —— $\alpha = 0.5$, $\beta = 2$
|   | 1  | 2  | 3  | 4  | 5  | 6  | 7  | 8  | 9  | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 |
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| 3 | 1  | 2  | 3  | 4  | 5  | 6  | 7  | 8  | 9  | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 |
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**WIND(2-D)**

**WIND(3-D)**

Figure 6.13 (continued)
Figure 6.14  Stress distribution in a double-layer barrel vault of 'SQUARE-ON-SQUARE' configuration under a wind load of wind blowing at 45° to the longitudinal axis of the vault —- α = 0.5,  β = 2
Figure 6.14(continued)
Figure 6.15  Stress distribution in a double-layer barrel vault of 'SQUARE-ON-SQUARE' configuration under
a uniformly distributed load  ——  \( \alpha = 0.289, \beta = 1 \)
Figure 6.16  Stress distribution in a double-layer barrel vault of 'SQUARE-ON-SQUARE' configuration under a wind load of wind blowing at 90° to the longitudinal axis of the vault —— α = 0.289, β = 1
Figure 6.17  Stress distribution in a double-layer vault of 'SQUARE-ON-SQUARE' configuration under a wind load of wind blowing at 45° to the longitudinal axis of the vault  —  $\alpha = 0.289$,  $\beta = 1$
Figure 6.18  Stress distribution in a double-layer barrel vault of 'SQUARE-ON-SQUARE' configuration under a uniformly distributed load — $\alpha = 0.289$, $\beta = 2$
**Figure 6.19** Stress distribution in a double-layer barrel vault of 'SQUARE-ON-SQUARE' configuration under a wind load of wind blowing at 90° to the longitudinal axis of the vault —— $\alpha = 0.289$, $\beta = 2$
Figure 6.19 (continued)
Figure 6.19 (continued)
Figure 6.20 Stress distribution in a double-layer barrel vault of 'SQUARE-ON-SQUARE' configuration under a wind load of wind blowing at 45° to the longitudinal axis of the vault — $\alpha = 0.289, \beta = 2$
Figure 6.20(continued)
Notes: (for Figures 6.21 to 6.26)

The structure is under a uniformly distributed load \( q \)

or

The structure is under a wind load of dynamic pressure at the height equal to the top level of the structure \( q \)

1- Forces in TOP-LAYER
2- Forces in BOTTOM-LAYER
3- Forces in BRACING
4- Reactions

Vertical displacement of the joints of BOTTOM-LAYER

\[ \text{Vertical displacement of joints of BOTTOM-LAYER} \]

\[ x \times q \times 0.1 \]

\[ x \times q \times \frac{S_X}{n_X} 	imes \frac{1}{E} \times \frac{1}{A} \]

\(-- (+) \) Tensile force

\(-- (-) \) Compressive force

\[ X \] Reactions

\[ Y \]

\[ Z \]

\[ \bullet (+) \] Upward displacement

\[ \bullet (-) \] Downward displacement

Figure 6.21 Stress distribution in a double-layer barrel vault of 'SQUARE-ON-DIAGONAL' configuration under a uniformly distributed load —— \( \alpha = 0.5, \ \beta = 1 \)
Figure 6.22 Stress distribution in a double-layer barrel vault of 'SQUARE-ON-DIAGONAL' configuration under a wind load of wind blowing at 90° to the longitudinal axis of the vault —— α = 0.5, β = 1
Figure 6.23 Stress distribution in a double-layer barrel vault of 'SQUARE-ON-DIAGONAL' configuration under a wind load of wind blowing at 45° to the longitudinal axis of the vault —— \( \alpha = 0.5, \ \beta = 1 \)
Figure 6.24  Stress distribution in a double-layer barrel vault of 'SQUARE-ON-DIAGONAL' configuration under a uniformly distributed load —— $\alpha = 0.5$, $\beta = 2$
Figure 6.25  Stress distribution in a double-layer barrel vault of 'SQUARE-ON-DIAGONAL' configuration under a wind load of wind blowing at 90° to the longitudinal axis of the vault —— α = 0.5,  β = 2
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Figure 6.26 Stress distribution in a double-layer barrel vault of 'SQUARE-ON-DIAGONAL' configuration under a wind load of wind blowing at 45° to the longitudinal axis of the vault —— α = 0.5, β = 2
Figure 6.26 (continued)
Notes: (for Figures 6.27 to 6.32)

The structure is under a uniformly distributed load \( q \) or

The structure is under a wind load of dynamic pressure at the height equal to the top level of the structure \( q \)

1- Forces in TOP-LAYER
2- Forces in BOTTOM-LAYER
3- Forces in BRACING
4- Reactions

Vertical displacement of the joints of BOTTOM-LAYER

\[ x \times q \times 0.1 \]

\[ x \times q \times \frac{S_X}{n_X} \times \frac{1}{E} \times \frac{1}{A} \]

--- (+) Tensile force
----- (-) Compressive force

\( X \) \( Y \) \( Z \) Reactions

• (+) Upward displacement
• (-) Downward displacement

Figure 6.27 Stress distribution in a double-layer barrel vault of 'SQUARE-ON-DIAGONAL' configuration under a uniformly distributed load --- \( \alpha = 0.289, \ \beta = 1 \)
Figure 6.28  Stress distribution in a double-layer barrel vault of 'SQUARE-ON-DIAGONAL' configuration under a wind load of wind blowing at 90° to the longitudinal axis of the vault —— α = 0.289,  β = 1
Figure 6.29 Stress distribution in a double-layer barrel vault of 'SQUARE-ON-DIAGONAL' configuration under a wind load of wind blowing at 45° to the longitudinal axis of the vault — $\alpha = 0.289, \ \beta = 1$
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**Figure 6.29 (continued)**
Figure 6.30 Stress distribution in a double-layer barrel vault with 'SQUARE-ON-DIAGONAL' configuration under a uniformly distributed load $\alpha = 0.289$, $\beta = 2$. 

Figure 6.31  Stress distribution in a double-layer barrel vault of 'SQUARE-ON-DIAGONAL' configuration under a wind load of wind blowing at 90° to the longitudinal axis of the vault —— $\alpha = 0.289$, $\beta = 2$.
Figure 6.31 (continued)
Figure 6.32  Stress distribution in a double-layer barrel vault of 'SQUARE-ON-DIAGONAL' configuration under a wind load of wind blowing at 45° to the longitudinal axis of the vault —— α = 0.289,  β = 2
Notes: (for Figures 6.33 to 6.38)

The structure is under a uniformly distributed load $q$

The structure is under a wind load of dynamic pressure

at the height equal to the top level of the structure $q$

1- Forces in TOP-LAYER

2- Forces in BOTTOM-LAYER

3- Forces in BRACING

4- Reactions

Vertical displacement

of the joints of

BOTTOM-LAYER

$\times q \times 0.1$

$\times q \times \frac{S_x}{n_x} \times \frac{1}{E} \times \frac{1}{A}$

--- (+) Tensile force

----- (-) Compressive force

Reactions

$X$

$Y$

$Z$

• (+) Upward displacement

• (-) Downward displacement

Figure 6.33 Stress distribution in a double-layer vault of 'DIAGONAL-ON-SQUARE' configuration under a uniformly distributed load $\alpha = 0.5, \beta = 1$
Figure 6.34 Stress distribution in a double-layer barrel vault of 'DIAGONAL-ON-SQUARE' configuration under a wind load of wind blowing at 90° to the longitudinal axis of the vault —— α = 0.5, β = 1
Figure 6.35  Stress distribution in a double-layer barrel vault of 'DIAGONAL-ON-SQUARE' configuration under a wind load of wind blowing at 45° to the longitudinal axis of the vault —— $\alpha = 0.5$, $\beta = 1$
Figure 6.35 (continued)
Figure 6.36 Stress distribution in a double-layer barrel vault of 'DIAGONAL-ON-SQUARE' configuration under a uniformly distributed load —— $\alpha = 0.5$, $\beta = 2$
Figure 6.37 Stress distribution in a double-layer barrel vault of 'DIAGONAL-ON-SQUARE' configuration under a wind load of wind blowing at 90° to the longitudinal axis of the vault —— $\alpha = 0.5, \ \beta = 2$
Figure 6.37 (continued)
Figure 6.37 (continued)
Figure 6.38 Stress distribution in a double-layer barrel vault of 'DIAGONAL-ON-SQUARE' configuration under a wind load of wind blowing at 45° to the longitudinal axis of the vault —— $\alpha = 0.5, \quad \beta = 2$
Figure 6.38 (continued)
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Figure 6.38 (continued)
Notes: (for Figures 6.39 to 6.44)

The structure is under a uniformly distributed load $q$

or

The structure is under a wind load of dynamic pressure at the height equal to the top level of the structure $q$

1- Forces in TOP-LAYER

2- Forces in BOTTOM-LAYER

3- Forces in BRACING

4- Reactions

Vertical displacement of the joints of BOTTOM-LAYER

$$ x q \times \frac{S_X}{n_X} \times \frac{1}{E} \times \frac{1}{A} $$

--- (+) Tensile force

----- (-) Compressive force

☐ $X$

☐ $Y$

☐ $Z$

● (+) Upward displacement

● (-) Downward displacement

Figure 6.39 Stress distribution in a double-layer barrel vault of 'DIAGONAL-ON-SQUARE' configuration under a uniformly distributed load $\alpha = 0.289, \beta = 1$
Figure 6.40  Stress distribution in a double-layer barrel vault of 'DIAGONAL-ON-SQUARE' configuration under a wind load of wind blowing at 90° to the longitudinal axis of the vault —— $\alpha = 0.289$, $\beta = 1$
Figure 6.41  Stress distribution in a double-layer barrel vault of 'DIAGONAL-ON-SQUARE' configuration under a wind load of wind blowing at 45° to the longitudinal axis of the vault —— $\alpha = 0.289, \quad \beta = 1$
Figure 6.42 Stress distribution in a double-layer barrel vault of 'DIAGONAL-ON-SQUARE' configuration under a uniformly distributed load $\alpha = 0.289, \beta = 2$
Stress distribution in a double-layer barrel vault of 'DIAGONAL-ON-SQUARE' configuration under a wind load of wind blowing at 90° to the longitudinal axis of the vault — $\alpha = 0.289, \beta = 2$
Figure 6.43 (continued)
Figure 6.44  Stress distribution in a double-layer barrel vault of 'DIAGONAL-ON-SQUARE' configuration under a wind load of wind blowing at 45° to the longitudinal axis of the vault —— α = 0.289,  β = 2
Figure 6.44 (continued).
### Stress Distribution

<table>
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<th>Forces in TOP-LAYER</th>
<th>Vertical displacement of the joints of BOTTOM-LAYER</th>
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<td>( q = 0.3 \text{kN/m}^2 )</td>
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#### Table 6.45

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<td>( q = 0.3 \text{kN/m}^2 )</td>
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**Figure 6.45** Stress distribution in a double-layer barrel vault of 'SQUARE-ON-SQUARE' configuration under a combination of self-weight and wind load. 

- \( \alpha = 0.5 \), \( \beta = 1 \)
DOUBLE-LAYER BARREL VAULT OF SQUARE-ON-SQUARE TYPE
SPORTS STADIUM BUILT BY MERÖ-Raumstruktur GmbH & Co. Würzburg

Plate 6.1 Examples of double-layer braced barrel vaults
CONCLUSIONS AND SUGGESTIONS OF FURTHER WORK

The aerodynamic and the structural aspects of barrel vault structures have been studied. The conclusions of each part of the study have been given in the previous corresponding chapters. Observations made in the present study draw attention to several aspects of the design of barrel vault structures. Since this research field is extensive and only a small part of the possible work has been carried out in the present study, there are still some areas which, it is suggested, could be investigated in future.

Today, nearly all the countries have their building codes of practice which supply the design loads for the most common types of structure so that engineers are quite used to returning to the code of practice during the design process. For the wind loading on barrel vault structures, it has been found that there is very little available reliable information and such existing data show significant differences from different countries and also differ in comparison with the wind tunnel test results. All the codes of practice give the wind pressures under a two-dimensional flow with a direction of flow normal to the axis of the vault. From the present study of the structural behaviour of double-layer braced barrel vaults and the previous study of single-layer braced barrel vaults (Wong, 1979) under wind loads, it has been observed that the two-dimensional wind loading produces an overestimation of stresses and deformations of the structure, compared to the actual three-dimensional wind load. In some situation, the higher stresses and deformations are given by the oncoming wind direction, which is not generally normal to the axis of the vault. It appears that the information provided by the codes of practice in the present stage is limited and of an approximate character, and is not sufficient for the design of barrel vaults.

Having regard to the reliability of the available information of wind loading on barrel vaults, it is therefore, necessary to undertake wind
tunnel tests in an attempt to establish more accurately the static pressure distribution and further, the dynamic response due to wind load. For similarity of flow around geometrically similar structures which differ in size, the Reynolds number should be the same on each structure. Simulation of the high Reynolds number of barrel vault in the wind tunnel test is one of the main difficulties. Fortunately, this difficulty may be largely overcome by employing the roughness approach. The introduction of the roughness approach on barrel vaults is based on the investigated facts that barrel vaults having a circular curvature have similar flow features to a circular cylinder. The wind pressures obtained from the present wind tunnel tests are carried out in a simulated boundary layer where the boundary layer thickness is four times of the model height. For many full-scale barrel vaults in the natural wind, the thickness of the earth's boundary layer is likely to be more than four times relative to the height of the vault. It is, therefore, suggested for further work to carry out the model test in a simulated boundary layer with the velocity profile and turbulent intensity profile having a scale of the same order as the structure in the natural wind condition if it is possible. However, such tunnel test may require not only a large wind tunnel and high precision intruments in the laboratory but, moreover, they will entail a very extended and systematic programme of work to isolate the effect of the different upstream conditions, such as velocity distribution and length and scale of turbulence.

During the design process, while the external loads, materials and structural system are known, structural analysis is carried out to obtain optimum dimensions and forms for the structural elements so that the structural system is capable to withstand the external loads. There are several different structural analytical methods for analysing braced barrel vaults. Usually, the choice of analytical methods is based on the accuracy and the mathematical calculation resources. In connection with the purpose of design, analytical methods such as the shell analogous method and the equivalent shell replacement method, having the idealised mathematical model related to the macroscopic behaviour of the actual structure and giving an approximate solution, cannot be effectively used in designing a braced barrel vault structure.
In the study of the structural behaviour of three different types of configuration of double-layer braced barrel vaults under wind loads, the finite element method for skeletal structure is used to carry out the analysis. Since the self-weight of the steel or aluminium braced barrel vault is small and may be several times less than the wind loads in some situations, wind loads may take an important position in estimating the stresses and deformations of structure under external loads. The present study concentrated only on the investigation of 'SQUARE-ON-SQUARE', 'SQUARE-ON-DIAGONAL' and 'DIAGONAL-ON-SQUARE' types of configuration with two different geometries, two different lengths, single boundary support condition and single cross-sectional area of members. Comparison of these three different types of configuration has shown that the 'SQUARE-ON-DIAGONAL' type is efficient in stress distribution and the 'SQUARE-ON-SQUARE' and the 'SQUARE-ON-DIAGONAL' types may give an economic design. Much more work on the effect on the structural behaviour of varying the size of structure and the boundary support condition, of different member cross-sectional area of each layer and bracing and of other types of configuration may be of interest to the designer.

In addition, the present study of the structural behaviour of a barrel vault is limited to the linear elastic analysis. The non-linear analysis of single-layer or double-layer braced barrel vault, however, also requires further studies about structural behaviour.
REFERENCES
PART I - AERODYNAMIC ASPECT

Achenbach, E.
"Distribution of local pressure and skin friction around a circular cylinder in cross-flow up to \( Re=5\times10^6 \)." J. Fluid Mech., Vol. 34, pp. 625-639, 1968.

Achenbach, E.

ANSI A58.1-1972

Apelt, C.J., West, G.S. & Szewczyk, A.A.

Apelt, C.J. and West, G.S.

Armitt, J.

AS 1170, Part 2-1975
Batham, J.P.

BC & R II-A.11-62
"Building code and regulations, Part II, Section A, Chapter 11: Loads and effects". State Committee on Construction, Moscow, 1965.

Bearman, P.W.

Bearman, P.W.

BSI CP3: Chapter V: Part 2-1972

Counihan, J.

Counihan, J.

Counihan, J.

Dennis, S.C.R.
"The steady flow of a viscous fluid past a circular cylinder". ARC 26, 104, August 1964.
Dennis, S.C.R. and Chang, G.S.

Dianat, M.
"Development and calibration of boundary layers in 1370mm×1060mm wind tunnel". Internal Report, Department of Civil Engineering, University of Surrey, 1980.

E.S.D.U.
"Fluid forces acting on circular cylinders for application in general engineering, Item 70013 and 70014". Engineering Sciences Data Unit, 1970.

Fage, A. and Warsap, J.H.

Farell, C., Guven, O. & Maisch, F.

Gerrard, J.H.

Guven, O.
"An experimental and analytical study of surface-roughness effects on the mean flow past circular cylinders". PhD Dissertation, University of Iowa, December 1975.

IS: 875-1964
"India standard code of practice for structural safety of building: loading standard (revised)". Indian Standards Institution, Delhi, 1964.
Maskell, E.C.
"A theory of blockage effects on bluff bodies and stalled wings in a closed wind tunnel". ARC R&M 3400, November 1963.

McKeon, R.J. and Melbourne, W.H.

Newberry, C.W. and Eaton, K.J.

Niemann, H.J.

Relf, E.F.
"Discussion of the results of measurements of the resistance of wires, with some additional tests on the resistance of wires of small diameter". ARC R&M 102, March 1914.

Roshko, A.

Simiu, E. and Scanlan, R.H.

Stansby, P.K. and Wootton, L.R.
Szechenyi, E.

Tritton, D.J.

Tritton, D.J.
PART II - STRUCTURAL ASPECT

Awni, A.A.
"Analysis of braced barrel vaults by the finite difference equations". MSc Thesis, University of Surrey, 1972.

Bligh, J.

Butterworth, J.

Dean, D.L. and Avent, R.R.

Del Pozo, F.
"Metallic cylindrical shells made of a triangular network". Proc. of the Fifth Congress of the International Association for Bridge and Structural Engineering, Lisbon, pp. 405-421, 1956.

Föppl, A.
"Das Fachwerk in Raume". Teubner Verlag, Leipzig, 1892.

Gibson, J.E.

Gibson, J.E.
Howley, M. and Makowski, Z.S.

King, P.G.

Lang, E.G.

Makowski, Z.S.

Makowski, Z.S.

Makowski, Z.S.
"Braced domes- history of development of various types of domes and review of recent achievements all over the world". Proc. of Course on the Analysis, Design and Construction of Braced Domes, Vol. I, University of Surrey, September 1980.

Matsushita, F.

Matsushita, F.
鉄骨立体屋根構造の応力解析に関する研究
Pshenichnov, G.I.

Pshenichnov, G.I.

Redshaw, S.C.

Sollazzo, A.
"The formulation of an analogy between lattice shells and equivalent anisotropic continua". Lectures of Plate and Shell Analysis Applied to Double-Layer Grids, University of Surrey, May 1976.

Tarzi, A.I.

Velankar, S.V.

Vlasov, V.Z.
"Cylindrical shells and new ways of developing thin-walled spatial systems on structural mechanics". Proc. of Second Symp. of Concrete Shell Roof Construction, Oslo- Teknisk Ukeblad, pp. 121-146, 1958.

Wong, C.W.

Wright, D.T.
Wright, D.T.

Yazdani, N.H.