HYBRID CONTINUOUS FIBRE CEMENT COMPOSITES

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degree of Doctor of Philosophy

by

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SUMMARY

This thesis describes the interaction between continuous fibres of widely different strength and elastic moduli in cement based composites. The fibres covered are combinations of polypropylene networks, glass fibres and carbon fibres.

Experimental tensile stress-strain behaviour for composites containing continuous polypropylene networks and glass fibres has indicated that there is a synergistic interaction between the fibres at strains up to about 1.5 %, which is thought to be the most important region for practical applications. The properties of the components of the hybrid composites have also been determined experimentally and microstructural observation has been made of the fibre-matrix interfaces.

Existing theories for predicting the tensile behaviour of fibre reinforced cements have been reviewed. When two or more types of fibres are used as reinforcement in a composite, existing theories cannot satisfactorily describe the behaviour because of the synergistic effect which is produced and therefore a new theoretical model for hybrid composites has been developed which should be applicable to other hybrid composites containing combinations of a variety of fibre types.

In each region of the tensile stress-strain curve, equations have been created to satisfy the force balance and strain balance in the composite. The reliability of the equations has been assessed by substituting the experimentally determined values for material properties of combinations of glass and polypropylene fibres. A satisfactory agreement was found between the theoretical curves and the experimental curves.

The fracture behaviour of glass fibre roving is important in the interpretation of the discrepancies between experiment and theory and therefore a new technique of light transmission down the glass fibres has been introduced to obtain independent information about the failure of individual glass fibres within the bundle of fibres.

For carbon/polypropylene fibre reinforced composites, the carbon fibre distribution was found to be an essential additional parameter to obtain better predictions of composite performance and attempt was therefore made to quantify the carbon fibre distribution.
TO MY FAMILY
ACKNOWLEDGEMENTS

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CHAPTER I

1. INTRODUCTION

1.1. Development of fibre reinforced cement composites

The technique of fibre reinforcement in articles such as clay bricks containing straw, has been developed in order to strengthen brittle materials since ancient times. The modern technique of reinforcing brittle cement and concrete with fibres began with asbestos cement manufactured with the Hatschek process in about 1900.

In the Hatschek process, the asbestos fibres are dispersed in a cement slurry, which is filtered out on a fine screen cylinder and dewatered to produce thin sheet materials. Asbestos cements can be built up to the required thickness by adding the appropriate number of layers. As the green sheets have a considerable tear strength, they can be easily handled, shaped on a former and cut to length using a knife. The flat and corrugated sheets have been widely used as roofing and cladding materials, fire-resistant insulating boards, decorative finishes and so on. Asbestos cement products had remained as the most successful fibre reinforced cement composites until the 1960s.

In 1963, pioneering research on steel fibre reinforced concretes (SFRC) was reported by Romualdi and Baston. In the early work on SFRC, only straight steel fibres were used. Since then, more complicated geometries such as indented, crimped, hooked ends and enlarged ends have been developed in order to improve the mechanical bonding with the cementitious matrix. SFRC has been applied in the civil engineering and construction industry as pavement overlays, industrial flooring, slabs and non-pressure pipes since the early 1970s.

In 1964, the first systematic study on glass fibre reinforced cement composites (GRC) was introduced by Biryukovich and his colleagues. E-glass was used in these investigations in conjunction with high alumina cement but the glass was not
durable enough in a highly alkaline Portland cement matrix. Therefore, an alkali-resistant glass fibre was required for the reinforcement of Portland cements. Zirconia containing glass fibres were found to be more suitable for reinforcing these cements than E-glass fibres by Majumdar\textsuperscript{(1-4)} at the Building Research Establishment in the U.K. Their subsequent efforts and the collaboration with Pilkington Brothers plc led to the commercial production of an alkali-resistant (AR) glass fibre, trademarked Cem-FIL, in 1971. GRC has been commonly used in the form of thin and light interior and exterior panels. However conventional GRC deteriorates and becomes more brittle as it ages so that low-alkaline cement for GRC was developed in Japan\textsuperscript{(1-5)} in 1985. This new GRC has been improved further by being made with cement which is less alkaline and less contractile, and with AR glass fibres containing higher proportions of zirconia\textsuperscript{(1-6)}. Recently, this combination has enabled the production of larger interior and exterior panels and led to the increased use of GRC in curtain walls and elsewhere\textsuperscript{(1-7)}.

In the early 1960s, polymeric fibres such as nylon, polypropylene and polyethylene began to be studied for the reinforcement of concretes. One of the first commercial products of the concrete reinforced with fibrillated polypropylene fibres was developed by Shell Company, under the tradename Caricrete\textsuperscript{(1-8)} in 1966. After Caricrete was confirmed to have excellent impact strength\textsuperscript{(1-9)}, concrete shell piles made of Caricrete went on the market. This type of fibre has subsequently been improved further either as short, discontinuous fibrillated material to produce fibre reinforced concrete\textsuperscript{(1-10)}, or as a continuous network-mesh to produce thin sheet composites made by impregnating fibrillated polypropylene networks with cement paste or mortar\textsuperscript{(1-11)}. Small volumes (~0.1\%) of polymeric fibres, mainly polypropylene fibres, are also used at present to control cracking in the early stages of setting, typically less than three hours after casting\textsuperscript{(1-12)}.

In 1972, Ali, Majumdar and Rayment\textsuperscript{(1-13)} reported excellent properties of carbon fibre reinforced cement. Although carbon fibres were technically attractive for use in concrete, their high cost restricted their use in Portland cement composites. Discontinuous pitch-based carbon fibres have been developed at a lower cost by Kureha
Chemical Company in Japan\textsuperscript{(1-14)}. These carbon fibres have higher strength and a higher modulus compared to other polymeric fibres. They have been successfully used to fabricate exterior wall panels. Current uses of carbon fibre cement composites include cladding, free-access floor panels, lightweight decorating frames, shell structures and protective coatings for structural elements exposed to harsh environments\textsuperscript{(1-7)/(1-15)}.

Since it has known that exposure to asbestos fibres can be harmful to health, a number of other types of fibres have been used in fibre reinforced concrete as the alternatives, including steel, glass, polypropylene, carbon, polyvinyl alcohol (PVA), aramid, cellulose, wood fibres and so on. These fibres have a wide variety of the properties, in particular with respect to the modulus of elasticity which is an important characteristic when fibres are used for producing composites. In order to increase the cracking strength of their composites, large volumes of fibres must be included with a modulus of elasticity greater than that of the matrix. For cement based materials, the modulus of elasticity ranges from about 10 to 45GPa. Some fibres such as steel, glass, aramid and carbon meet this condition, while most of other fibres do not. However, both experimental and theoretical research have shown that, even with low modulus fibres, considerable improvements can be obtained with respect to the strain capacity, toughness, impact resistance and crack control of the fibre reinforced cement composites\textsuperscript{(1-16)}. In many applications, the enhancement of these properties is much more significant than a modest increase in tensile and flexural strength\textsuperscript{(1-17)}. The role of the randomly distributed discontinuous fibres is to reduce the crack widths by bridging the cracks, which provides some ductility in the post-cracking region.

The combination of two or more fibre types has been studied to provide a synergistic effect in which the stronger and stiffer fibre improves the first crack stress and the ultimate strength, while the more flexible and ductile fibre leads to improved toughness and strain capacity in the post-cracking region. In hybrid fibre composites, different fibres appear to act in such a way that they maintain their individual reinforcing capabilities. This is significant from the point of view of designing fibre reinforced cement composites taking the performance and the cost into consideration. The hybrid
fibre reinforcing technology and newly developed synthetic fibres have led to new applications for fibre reinforced composites.

1.2. Purpose of this study

Two types of continuous fibre reinforced cement corrugated sheeting have been produced commercially. The one in Germany consists of continuous glass fibres and chopped random glass fibres. The other in Italy consists of continuous networks of polypropylene, continuous glass fibre rovings and chopped glass strands. These composites make much more efficient use of a given volume of fibre than the short random fibre materials because the continuous fibres can be placed in the direction and position of the main imposed stresses and fibre pull-out is restricted before fracture. The composites therefore have improved strength and toughness compared with short fibre materials. For these practical composites, it is important to improve the post-cracking performance of the material particularly in the immediate post-cracking strain range of up to about 1-2% tensile strain. In this post-cracking region, localised areas of continuous glass fibres and carbon fibres are the most likely candidates for improving the properties of composites. These concentrated stiff regions can be combined with continuous networks of fibrillated polypropylene fibres which give an uniform fibre distribution with very closely spaced cracking. However, these different types of fibres have widely different elastic performance and their interactions are not well understood.

In the field of theoretical modelling for bulk composite performance, much progress is still required so that most designs of composites are based on experimental results and empirical relations. Before any worthwhile application can be considered, it would be beneficial to understand the way in which the fibres act in a hybrid composite. This will allow the sensible design of the hybrid composite, and the prediction of the tensile and flexural behaviour. Even although much effort has been applied to develop theories to describe the properties of composites and their relationships to the properties of the components, the current theoretical models are available only for single fibre
reinforced composites. For hybrid composites, the synergistic effect of different fibre
types has to be taken into consideration.

The main objectives of this study are to obtain a better understanding of the
interaction between continuous fibres of widely different strength and elastic moduli in
cement based composites and to provide a mathematical model which enables the
prediction of bulk composite performance. In this study, continuous alkali-resistant
glass fibres and continuous PAN (polyacrylonitrile) type carbon fibres have been used as
high stiffness fibres together with continuous networks of polypropylene fibres which
give a uniform and closely spaced fibre distribution. The research programme is
summarised below.

(1) Properties of the hybrid composites were obtained in comparison to those of the
single fibre reinforced composites. The assessment was carried out with uniaxial
tensile tests because tensile behaviour is more easily understood from the theoretical
point of view than flexural behaviour.

(2) The microstructure of hybrid composites was observed to obtain detailed
information about the interface between fibres and matrix. Also, the crude relationship
between fibre failure and tensile behaviour of the composites was obtained.

(3) In order to know the condition of particular continuous glass fibre failure in the
hybrid composite, a new technique was developed. A quantitative assessment of the
failure of the glass fibres was attempted using the variation of the transmitted light
through the glass filaments.

(4) Based on the results of all the experiments, a mathematical model for the tensile
behaviour of continuous glass/polypropylene fibre cement composites was proposed.

(5) The proposed model was applied to the combination of polypropylene networks
and continuous carbon fibres. Regarding the carbon fibre, the fibre distribution was
taken into consideration as a new efficiency factor for fibre reinforcement.

It is hoped that the work reported in this thesis has contributed to the theoretical and experimental understanding of the tensile behaviour of hybrid continuous fibre cement composites and has led hybrid composites to new applications in the future.
CHAPTER 2

2. PROPERTIES OF MATERIALS FOR THE GLASS/POLYPROPYLENE/CARBON FIBRE REINFORCED CEMENT COMPOSITES

2.1. Introduction

In this thesis, experimental and theoretical work has been carried out, mainly on a cement based matrix reinforced with continuous glass fibres and fibrillated polypropylene networks (GF/PP composites). This composite has been developed as a non-asbestos cement composite sheet under the tradename Retiver in Italy for roofing materials. Additionally, the study has been extended to composites reinforced with continuous carbon fibres and polypropylene networks (CF/PP composites).

The composite specimens were all manufactured using, nominally, the same matrix mix proportions. This chapter describes the properties of the matrix, the glass fibre and the polypropylene as components which can be used in the mathematical model developed in Chapter 7.
2.2. The matrix

2.2.1. Matrix composition

The materials and the mix proportions of the matrix are shown in Table 2.1.

Table 2.1 Mix proportions of matrix.

<table>
<thead>
<tr>
<th>Material</th>
<th>Mix by weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ordinary Portland cement</td>
<td>1.00</td>
</tr>
<tr>
<td>Water</td>
<td>0.34</td>
</tr>
<tr>
<td>Silica sand</td>
<td>0.19</td>
</tr>
<tr>
<td>Pulverized fuel ash</td>
<td>0.25</td>
</tr>
<tr>
<td>Superplasticizer</td>
<td>0.016</td>
</tr>
<tr>
<td>(Sulfonated melamine formaldehyde resin : Melment L10 by Hoechst Chemicals)</td>
<td></td>
</tr>
</tbody>
</table>

This matrix is the basic composition which has been used for fibre reinforced cement composites at the University of Surrey. This was designed to obtain desirable properties for composite preparation and performance. The low water-cement ratio (W/C=0.34) is attained by means of the superplasticizer, which results in high strength and density. Silica sand (particle size : 150-300μm) acts as aggregate to make shrinkage lower than pure cement paste. Pulverized fuel ash (PFA) is added as filler to increase fluidity of the matrix due to its spherical particles. High fluidity is desirable to impregnate the cement matrix into fibrillated polypropylene networks. It was necessary to retain fluidity during composite manufacturing in the laboratory so that slight adjustments were made in the amount of superplasticizer added to obtain suitable workability.
2.2.2. Matrix properties in tension

Matrix properties in tension were obtained from uniaxial tensile tests on the composite. Experimental results will be discussed in Chapter 4. The range and the average of the measured matrix modulus, cracking stress and strain are given as follows.

Elastic modulus of matrix in tension: 30-33GPa  Average 31.6GPa
Matrix cracking stress: 4-11MPa  Average 8.5MPa
Matrix cracking strain: 140-370×10^{-6}  Average 273×10^{-6}

The results are in reasonable agreement with other published values for the same proportions\(^{2-1}\). The average values of the elastic modulus and matrix cracking strain will be used to develop a mathematical model of the composite tensile behaviour in Chapter 7.

2.3. Glass fibre

2.3.1. Alkali-resistant glass fibre

The glass fibre should be resistant to alkalies when it is used with Portland cement. Inorganic silicate glasses are inherently reactive to alkalies; the silicon-oxygen-silicon network which forms the main structural skeleton of silicate glasses is attacked by the hydroxyl ions. The addition of ZrO\(_2\) in glass compositions substantially reduces the effect of alkaline corrosion, which is shown in all commercially available alkali-resistant (AR) glass fibre compositions. Typical AR glass fibre composition is presented in Table 2.2 compared with the E-glass fibre's.
Continuous glass fibres are usually supplied in the form of roving. A roving consists of several collected strands in which are multiple glass filaments bound by the size. The size is an organic polymer applied to glass fibres during manufacturing to protect the fibres from abrasion by handling equipment.

In this study, the AR glass fibre rovings were supplied by Cem-FIL International Ltd. The glass type is Cem-FIL 2, Type 285/5. A chemical inhibitor is incorporated in the size used for Cem-FIL 2 AR glass fibres. This is slowly released in the cement matrix around the glass fibres, which results in a significant reduction in fibre strength loss due to the interaction between glass and cement\(^\text{(2-3)}\).

General properties of the AR glass fibre are shown in Table 2.3.
Table 2.3 General properties of the AR glass fibre \(^{(2-4)}\)

<table>
<thead>
<tr>
<th>Properties</th>
<th>Type 285/5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tensile strength (MPa)</td>
<td>1200-1700</td>
</tr>
<tr>
<td>Elastic modulus (GPa)</td>
<td>72</td>
</tr>
<tr>
<td>Ultimate elongation (%)</td>
<td>2.4</td>
</tr>
<tr>
<td>Density (g/cm(^3))</td>
<td>2.68</td>
</tr>
<tr>
<td>Filament diameter (µm)</td>
<td>14</td>
</tr>
<tr>
<td>Number of filaments per strand</td>
<td>100</td>
</tr>
<tr>
<td>Nominal number of strands per roving</td>
<td>64</td>
</tr>
</tbody>
</table>

2.3.2. Tensile properties of the glass strand

2.3.2.1. Test procedure

The continuous glass strands were cut to 600mm length (+40mm for gripping). Both ends of the strand were fixed on a graph paper by epoxy resin in order to make the strand aligned accurately to the direction of tensile loading. This treatment prevented the strand breaking at the grip ends. Then the strands were set on the clamp type grips. The tensile test was carried out at a rate of 10mm/min which was about 1.7% strain/min. The load signal and time which corresponded to the cross head displacement were collected by the data logging system and recorded on the X-Y recorder. The strain was calculated by the displacement divided by the strand length of 600mm. The cross section area was determined as the total area of mono-filaments, assuming the diameter of 14µm and 100 filaments per strand. Five strands were tested.
2.3.2.2. Experimental results and discussion

Tensile stress-strain curves for five glass fibre strands are shown in Figure 2.1.

![Tensile stress-strain curves of glass fibre strands.](image)

Figure 2.1 Tensile stress-strain curves of glass fibre strands.

Each line showed good linearity except for initial strain up to 0.1% and around failure strain. The elastic modulus of the glass fibre was calculated from each slope. The modulus, the ultimate strain and stress are presented in Table 2.4.
Table 2.4 Properties of glass fibre strands.

<table>
<thead>
<tr>
<th>Specimen No.</th>
<th>Elastic modulus $E_{fg}$ (GPa)</th>
<th>Ultimate strain $\varepsilon_{fg}$ (%)</th>
<th>Ultimate stress $\sigma_{fg}$ (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>72.0</td>
<td>1.19</td>
<td>822</td>
</tr>
<tr>
<td>2</td>
<td>68.7</td>
<td>1.25</td>
<td>886</td>
</tr>
<tr>
<td>3</td>
<td>69.9</td>
<td>1.33</td>
<td>991</td>
</tr>
<tr>
<td>4</td>
<td>84.0</td>
<td>1.47</td>
<td>1279</td>
</tr>
<tr>
<td>5</td>
<td>72.8</td>
<td>1.19</td>
<td>928</td>
</tr>
<tr>
<td>average</td>
<td>73.5</td>
<td>1.29</td>
<td>981</td>
</tr>
</tbody>
</table>

The average modulus was almost same as the trade literature (72GPa shown in Table 2.3). On the other hand, the ultimate strain and stress was lower than typical values ($\varepsilon_{fgu}=2.4\%, \sigma_{fgu}=1200-1700$MPa). Each strand consists of 100 filaments and the tensile strength depends on the uniformity of filaments. If a filament has a flaw such as micro crack, impurity, warp and so on, the fracture is induced at that point. Therefore, the fracture strain and stress become lower than intrinsic properties. With respect to the strength of the glass fibre in the theoretical approach for composites, the strength as a strand has to be taken into account.

2.4. Polypropylene

2.4.1. Fibrillated polypropylene fibre networks

Polypropylene fibres are produced by low-pressure technology using Ziegler-Natta catalysts. They possess a variety of shapes such as monofilaments, film and extruded tape, and different properties due to their structure. The main advantages of polypropylene fibres are their alkali resistance, high ductility and the low price of the raw material. Their disadvantages are poor fire resistance, sensitivity to sunlight and oxygen,
a low modulus of elasticity and a poor bond with the cement matrix. These disadvantages are not necessarily critical especially for durability. Embedding in the matrix avoids the direct exposure to fire and other environmental effects. Polypropylene used in this study contains agents to stabilize ultra-violet rays. Hannant\textsuperscript{(2-5)} completed a 10 years durability test on cement composites reinforced with fibrillated polypropylene networks exposed to both natural weathering and indoor storage in air. It was concluded that little detrimental effects were observed concerning the notional strength.

In order to improve the bonding with the cement matrix, fibrillated polypropylene networks were used in the present work. Fibrillated networks were produced from the film abraded on the surface to be fine and hairy. The film was slit by pins and fibrillated by a roller. The opened networks provide an interlocking effect to bond with the matrix\textsuperscript{(2-6)}.

The polypropylene network packs were supplied by Retiflex s.p.a. in Italy. Each pack consists of 12 layers, the longitudinal and lateral direction being in the ratio of 2:1. In this study, lateral nets were removed from each pack to make fibres align with the direction of applied load and make the theoretical model simple. A single longitudinal layer of the polypropylene network is shown in Figure 2.2. Typical properties of the polypropylene film are presented in Table 2.5.

\begin{table}[h]
\centering
\begin{tabular}{|c|c|}
\hline
Properties & Polypropylene film \\
\hline
Tensile strength (MPa) & 400 \\
\hline
Elastic modulus (GPa) & 4-12 \\
\hline
Ultimate elongation (%) & 8.0 \\
\hline
Density (g/cm\textsuperscript{3}) & 0.93 \\
\hline
Nominal thickness (\textmu m) & 60-80 \\
\hline
\end{tabular}
\caption{Typical properties of the polypropylene film\textsuperscript{(2-7)}}
\end{table}
2.4.2. Tensile properties of the polypropylene film

2.4.2.1. Test procedure

Polypropylene films which had not been abraded were used for tensile tests. The film was cut into strips 2mm wide and 600mm long (+40mm for gripping). Then the same procedure as the glass fibre strands described in 2.3.2.1 was carried out for the polypropylene film strips. The cross sectional area was determined by the weight of the strip with accuracy of 0.0001g divided by the length, assuming the density of 0.93 which was greater than that of pure polypropylene due to the inclusion of a filler. Five film strips were tested in this method.
2.4.2.2. Experimental results and discussion

Tensile stress-strain curves for five film strips of polypropylene are shown in Figure 2.3. The films were tested in the drawn direction.

Since each curve is not linear, a single value for the modulus of the polypropylene film cannot be determined. The curve was divided every 0.5% strain up to the strain where the first sudden load reduction was observed. In each strain region, the elastic modulus was determined by a regression analysis. These elastic moduli were plotted against the strain in Figure 2.4.

![Tensile stress-strain curves of polypropylene film strips.](image)

**Figure 2.3** Tensile stress-strain curves of polypropylene film strips.
Figure 2.4 Elastic modulus of polypropylene film with strain.

The elastic modulus of the polypropylene is variable against its strain. It decreases from 13-14GPa at initial strain to 4-5GPa at 5% strain. The same trend was observed in the previous research\(^{(2-1)}\). In order to estimate the change of the elastic modulus with strain, a logarithmic regression curve was applied. The elastic modulus of the film is expressed as follows.

\[
E_{fp} = X + Y \log(\varepsilon_{fp})
\]

where \(E_{fp}\) is the elastic modulus of the polypropylene film (unit : GPa), \(\varepsilon_{fp}\) is film strain (unit : %), \(X\) is a constant and \(Y\) is a regression coefficient. The result of the analysis was presented in Table 2.6. The obtained regression equation was:

\[
E_{fp} = 9.49 - 2.82 \log(\varepsilon_{fp})
\]  

(2-1)
Ohno\textsuperscript{(2,1)} obtained a similar equation for polypropylene films.

Table 2.6  \textbf{Logarithmic regression analysis for the elastic modulus of polypropylene films with strain.}

\begin{equation*}
E_{fp} = X + Y \log(E_{fp}) \quad \text{$R^2$: correlation coefficient}
\end{equation*}

<table>
<thead>
<tr>
<th>Specimen No.</th>
<th>$X$</th>
<th>$Y$</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>9.85</td>
<td>-2.87</td>
<td>0.976</td>
</tr>
<tr>
<td>2</td>
<td>8.90</td>
<td>-2.96</td>
<td>0.998</td>
</tr>
<tr>
<td>3</td>
<td>9.54</td>
<td>-2.68</td>
<td>0.982</td>
</tr>
<tr>
<td>4</td>
<td>9.44</td>
<td>-2.79</td>
<td>0.984</td>
</tr>
<tr>
<td>5</td>
<td>9.72</td>
<td>-2.79</td>
<td>0.982</td>
</tr>
<tr>
<td>average</td>
<td>9.49</td>
<td>-2.82</td>
<td></td>
</tr>
</tbody>
</table>

As described above, the elastic modulus of polypropylene is a variable with strain in tension rather than a constant. In the theoretical approach for the tensile behaviour of composites containing polypropylene, its elastic modulus as shown in Equation (2.1) has to be considered to obtain better agreement with experimental results.
2.5. Carbon fibre

The history of carbon fibre began with the industrialization of the general purpose grade made from the rayon fibres in 1959 in USA. Then the two main processes for making carbon fibres were industrialized with different starting materials, either polyacrylonitrile (PAN carbon fibres) or petroleum and coal tar pitch (pitch carbon fibres). Both processes involve heat treatments, and various grades of carbon fibres can be obtained with each, depending on the combination of heat treatment, stretching and oxidation. The PAN carbon fibres are of higher quality, and are sometimes classified into Type I and Type II. Type I fibres have been graphitised to give maximum stiffness but have a relatively low strength, whereas Type II fibres have been carbonised to produce maximum strength. The stress-strain curves of Type I and Type II fibres are compared with Kevlar and E-glass fibre in Figure 2.5. The high modulus fibres have a much lower strain to failure (0.50%) compared with the high strength fibres (1.0%).

![Stress-strain curves of the carbon fibre compared with Kevlar and E-glass.](image)

The pitch carbon fibres have much lower modulus of elasticity and strength, but their price is much lower than that of the PAN carbon fibres. The pitch fibres still have
superior properties to many other synthetic fibres, and their modulus of elasticity is equal
to or greater than that of the cement matrix. These have made them more attractive for
cement reinforcement. With regard to the pitch fibres, a number of studies have been
carried out in Japan.\(^{(2-10),(2-11)}\)

In this study, continuous PAN based carbon fibres were supplied by Courtaulds
Grafil Inc\(^*\). The fibre type is XA-S which is sized with glycerol. The glycerol sized
carbon fibre is designed to be dispersed easily in the slurry due to the dissolution of the
size in water. General properties of the carbon fibre are shown in Table 2.7. They are
quoted from the trade literature with respect to Grafil 33-500 which is identical with XA-
S\(^{(2-12)}\). The fibre bundle consists of 12000 filaments.

| Table 2.7 General properties of the carbon fibre\(^{(2-13)}\). |
|-----------------------------|-----------------------------|
| **Properties**              | **Grafil XA-S**             |
| Tensile strength (MPa)      | 3790                        |
| Tensile Modulus (GPa)       | 228                         |
| Elongation (%)              | 1.7                         |
| Weight per roving (g/m)     | 0.40                        |
| Density (g/cm\(^3\))        | 1.80                        |
| Filament diameter (μm)      | 7                           |
| Filament count              | 12000                       |
| Size level (%)              | 5                           |
| Typical carbon content (%)  | 94                          |

\(^*\) Courtaulds Grafil Inc. was acquired by Mitsubishi Rayon Co., Ltd (MRC). Although
MRC established a European marketing network for carbon fibre products, Grafil Inc still
markets both GRAFIL carbon fibres from its Sacramento USA production unit and high
performance PYROFIL carbon fibres from the MRC Toyohashi plant in Japan.
CHAPTER 3

3. COMPOSITE MANUFACTURE AND TEST PROCEDURES

3.1. Introduction

The effect of fibre reinforcement in a composite depends on the fibre type, volume, length, orientation and distribution. Also, the manufacturing process of the composite is one of the important factors. Various kinds of manufacturing processes have been developed, according to fibre types and shapes, to meet the required applications. The manufacturing processes which are currently available may be classified as follows.

(1) Spray process

This technique is used primarily with glass fibre reinforced cement (GRC). Cement paste or cement mortar which is pumped under pressure, and glass fibre which is cut to lengths by a chopped gun, are sprayed simultaneously onto the mould surface. This method is capable of moulding intricate shapes and is suitable for the production of custom-made products. With this technique, substantially higher fibre volumes, up to about 6%, can be incorporated into the composite.

(2) Premix process

Cement mortar and the fibres previously cut to appropriate lengths are combined in a mixer, and the materials are poured into the mould. The vibration is applied to the mould, which allows the 2-dimensional control of fibre distribution in the mortar. However, because the fibres reduce the workability, only about 2-3% by volume of glass fibres can be mixed by this method.
(3) Extrusion process

This process is the most suitable for the manufacture of long fibre reinforced cement composites with the same cross section over the entire length, such as bars and columns. The premixed fibre-containing mortar is continuously supplied to an extrusion machine and the materials are extruded as the product through the die at the other end of the machine. With this technique, continuous fibres can be used where extra strength is required.

(4) Hatschek process

Based on the principle of paper making, this process can produce thin sheets, several of which are then overlaid to form a sheet of the desired thickness. This is called the Hatschek process which conventionally uses asbestos fibres as the essential ingredient. Recently, cellulose fibres or other fibres are used as alternatives for the asbestos fibres. Fibres are partly aligned to the direction of manufacturing, which results in effective reinforcement for that direction. This process allows fibre contents typically from 5% to over 20% by volume.

(5) Hand lay up process

In this method, various forms of fibre reinforcements such as roving, fibre mats or fabrics can be used and products of various sizes and shapes can be made by using suitable mould or forming surfaces. Several layers of continuous fibre reinforcements are penetrated into the cement or mortar paste. In order to obtain efficient penetration of the reinforcement into the cement matrix, compaction by vibration is often applied. As a result, dense materials with very high fibre contents can be produced. In the industrial process, these basic processes may be modified to suit mass-production. For instance, the cement based composite reinforced with fibrillated polypropylene networks and glass fibres, tradenamed Retiver, is produced by a specially designed machine based on the principle of the lay-up method. The cement slurry is sprayed continuously onto the continuous felt. The polypropylene networks are
inserted on the cement slurry by the special feeding machine. The cement slurry is well 
impregnated into the networks by the compaction and dewatering. Then another 
cement slurry is sprayed on it. These procedures are repeated on the continuous felt 
until the required thickness is attained.

According to fibre types and shapes, the most suitable composite 
manufacturing process should be chosen to make the most effective use of the fibre 
reinforcement. In this study, the hand lay up method is suitable for continuous glass 
and carbon fibres and the fibrillated polypropylene networks. The hybrid composite 
manufacturing procedure will be described in the next section.

3.2. Composite manufacture

3.2.1. Glass/polypropylene fibre reinforced cement composites

The properties of the matrix, the glass fibre and the polypropylene as 
components have already been described in Chapter 2. Using these materials, all 
composites were produced in the form of flat sheets approximately 600mm long 
(parallel to the fibre alignment), 400mm wide and 5-6mm thick, by a hand lay-up 
method. The detailed process of manufacture is as follows.

1. 600mm lengths of alkali-resistant glass fibres were cut from the glass 
roving supplied by Cem-FIL International Ltd. The roving consists of 64 glass strands 
which can be divided to the appropriate number according to the required fibre volume 
fraction.

2. Both ends of the correct amount of glass fibres were fixed straight on graph papers 
by adhesive tapes. The width of graph paper was 25mm corresponding to that of
tensile test specimens. The fixed position of glass fibres was determined in advance according to the design of the fibre reinforcement.

(3) Fibrillated polypropylene networks were cut to 600mm length and 400mm width from the reel of the fibre network packs supplied by Retiflex s.p.a. Each pack consisted of 12 layers in which 8 layers are in longitudinal and 4 layers are in lateral direction. Lateral nets were removed from each pack to form uni-directional reinforcement.

(4) Polythene sheet was placed on a table as a release layer. The first layer of the polypropylene network was laid flat on a polythene sheet.

(5) Ordinary Portland cement, silica sand and pulverized fuel ash were dry-mixed in a polythene bag. These mixed powder materials and distilled water were mixed by a mortar mixer for about 2 minutes. Then materials stuck to the container were scraped off and the mortar was mixed for about another 3 minutes. During mixing, the superplasticizer was added to obtain suitable fluidity of the slurry. In the strict sense, the fluidity of the slurry was influenced by the water temperature so that the mixing time and the amount of superplasticizer were slightly adjusted.

(6) The mixed cement slurry was put onto the polypropylene networks laid on a polythene sheet and worked by hand until the networks were fully impregnated by the matrix.

(7) The second layer of polypropylene networks or continuous glass fibres fixed on graph papers were laid on top. It was carefully pressed onto the lower layer to be penetrated by excessive mortar from below. Then, more cement slurry was worked into the fibre layer again.

(8) The procedure was repeated until a sufficient number of layers and the required
composite thickness had been attained. The top surface was carefully finished with a
trowel to obtain as uniform thickness over the sheet as possible.

(9) The composite sheet was protected from rapid drying by covering with a polythene
sheet overnight.

(10) The composite sheet was cured under water at 20°C for 28 days.

The sheet was cut into strips about 300mm long by 25mm wide for tensile testing,
during the curing period. Prior to testing, the width and thickness of each specimen
were measured at three positions to accuracy of 0.1 and 0.01mm with a vernier calliper
and a micrometer, respectively.

3.2.2. Carbon/polypropylene fibre reinforced cement composites

The principles of composite manufacture were the same as for glass/
polypropylene fibre reinforced cement composites. However, the carbon fibre
preparation was different from glass fibres. In the preliminary tensile tests, the carbon
fibre indicated a poor bonding with the cement matrix. In order to fully utilise the high
performance of the carbon fibre, it is essential to obtain enough fibre surface area for
bonding and therefore the carbon fibre must be dispersed as well as possible.

The thick carbon fibre bundle consisting of 12000 filaments was opened on a
wire gauze in water. Immediately after the carbon bundle was immersed in water, the
glycerol size dissolved and the fibre bundle was easily opened. The carbon fibres were
formed into a thin layer of about 25mm wide. The opened fibre layer was scooped up
carefully by the wire gauze and transferred on to a paper towel. After drying properly,
it was fixed straight on a graph paper in the same way as the glass fibres. In order to
compare the effects of fibre dispersions, four different fibre conditions were fabricated
in approximately the same fibre volume fraction. Typical conditions for the carbon fibre are shown in Figure 3.1. They are placed on a graph paper and the unit scale is 1mm. It was difficult to open a bundle uniformly, however, the difference among these conditions can be seen easily. The opened spaces between filaments allow cement particles to be penetrated to obtain a good fibre-matrix bond.

![Figure 3.1 Carbon fibre conditions before matrix is penetrated.](image)

(a) Fibre bundle (PCN)  (b) Opened bundle (C2 and PC2)
(c) Opened 1/2 of a bundle (C4 and PC4)  (d) Opened 1/3 of a bundle (C6 and PC6)
3.3. Reinforcement of composites

3.3.1. Glass/polypropylene (GF/PP) cement composites

The target fibre volume fraction and the details of reinforcement of the GF/PP composites are shown in Table 3.1. The range of glass and polypropylene fibre volume fractions are 0 to 4% and 0 to 8%, respectively. According to the glass fibre volume fraction, the number of glass strands per bundle and the number of glass layers were changed. The number of polypropylene layers was constant at five layers and the number of nets per layer was adjusted.
Table 3.1 The schematic description of GF/PP composite reinforcement.

<table>
<thead>
<tr>
<th>PP vol%</th>
<th>0 (0)</th>
<th>2 (4)</th>
<th>4 (8)</th>
<th>6 (12)</th>
<th>8 (16)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 (0)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.2 (3)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.4 (5)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.6 (10)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.8 (10)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.0 (10)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.2 (10)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.6 (64)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.4 (64)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3.2 (64)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4.0 (64)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

GF(strands/bundle) — Continuous glass fibre bundle
Polypropylene networks
3.3.2. Carbon/polypropylene (CF/PP) cement composites

The average fibre volume fraction and the schematic description of CF/PP composite reinforcement are shown in Table 3.2. One of the purposes of CF/PP composites is to examine the effects of carbon fibre distributions. Thus, the carbon fibres were arranged as two bundles, two layers, four layers and six layers in the composites of about the same fibre volume fraction. Unopened carbon fibre bundles were used for series PCN. One opened bundle was used as a single layer for series C2 and PC2. A half of a bundle was opened and used as a single layer for series C4 and PC4. The best dispersion was achieved in series C6 and PC6 in which one third of a bundle was used for each of six layers.
Table 3.2 Description of the CF/PP composite reinforcement.

<table>
<thead>
<tr>
<th>Series No.</th>
<th>Average fibre volume fraction</th>
<th>Fibre distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Polypropylene (%)</td>
<td>Carbon fibre (%)</td>
</tr>
<tr>
<td>C2</td>
<td>0</td>
<td>0.55</td>
</tr>
<tr>
<td>C4</td>
<td>0</td>
<td>0.66</td>
</tr>
<tr>
<td>C6</td>
<td>0</td>
<td>0.61</td>
</tr>
<tr>
<td>PCN</td>
<td>4.5</td>
<td>0.65</td>
</tr>
<tr>
<td>PC2</td>
<td>4.1 and 5.7</td>
<td>0.58</td>
</tr>
<tr>
<td>PC4</td>
<td>3.9 and 5.7</td>
<td>0.56</td>
</tr>
<tr>
<td>PC6</td>
<td>3.9 and 5.8</td>
<td>0.56</td>
</tr>
</tbody>
</table>

- Polypropylene networks
- Carbon fibre bundle
- Carbon fibre layer
3.4 Determination of fibre volume fractions

The volume content of the polypropylene networks in the composite was determined using cut samples of the test specimens after testing, approximately 50mm long by 25mm wide. The pot containing water was placed on an electrical balance and then zero balance was set. After the sample was saturated with water, it was immersed in water as shown in Figure 3.2. Assuming the specific gravity of water is 1.0, the weight indicating on a balance is equal to the sample volume ($V_c$), which is known as the Archimedean method.

![Diagram](image-url)

**Figure 3.2** The Archimedean method to determine the volume of the sample.

The sample was placed in a container with a mixture of concentrated hydrochloric acid diluted with an equal volume of water. It took about 7 days to dissolve the matrix. The softened matrix was carefully washed away with flowing water and the polypropylene networks were taken out. The nets were dried in an oven at 90°C and weighed ($W_{fp}$) on an electrical balance to an accuracy of 0.001g. The experimentally obtained polypropylene fibre volume fraction $V_{fp(\text{exp})}$ can be calculated as follows.
\[ V_{fp(\text{exp})} = \frac{W_{fp}}{\rho_{fp}} \frac{1}{V_c} \]

where \( \rho_{fp} \), the specific gravity of the polypropylene, is 0.93 from Table 2.5. Also, the polypropylene fibre volume fraction, \( V_{fp(\text{cal})} \), can be calculated from the average fibre volume of a manufacturing sheet corrected by the actual width and thickness of each test specimen. The correlation between \( V_{fp(\text{exp})} \) and \( V_{fp(\text{cal})} \) is shown in Figure 3.3.

![Figure 3.3](image)

**Figure 3.3** The correlation between experimental fibre volume fraction of polypropylene \( V_{fp(\text{exp})} \) and calculated fibre volume fraction \( V_{fp(\text{cal})} \).

The calculated fibre volume fraction \( V_{fp(\text{cal})} \) indicates good agreement with the experimentally obtained fibre volume fraction \( V_{fp(\text{exp})} \). In practice, it was difficult to separate polypropylene networks from glass and carbon fibres in the hybrid composite. Therefore, the calculated fibre volume fractions \( V_{fp(\text{cal})} \) corrected by the actual width and
thickness of each test specimen were used throughout the experimental assessment for GF/PP and CF/PP hybrid composites.

3.5. Uniaxial tensile test

3.5.1. Reasons for the choice of uniaxial tension tests rather than flexural tension tests to measure and analyse mathematically the behaviour of hybrid composites.

Cementitious materials are strong for compression, but weak for tension so that the failure generally occurs in tension. Thus, the main objective of the fibre reinforcement in the cement matrix is to enhance the tensile strength of the composite. The behaviour in direct tension is different from that in bending, because the tensile stress and strain at any cross section are constant whereas for flexural loading, the stress distribution varies from compression to tension in the cross section. Simple bending theory is not really applicable to the fibre reinforced cement composites (FRC), because of the effect of post cracking ductility. The stress distributions in flexure are shown for ideally elastic and elastic-plastic materials in Figure 3.4, assuming that both of them have the same tensile strength.
<table>
<thead>
<tr>
<th>Elastic-Brittle</th>
<th>Elastic-Plastic</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Tensile stress-strain curve</strong></td>
<td></td>
</tr>
<tr>
<td><img src="image1" alt="Graph" /></td>
<td><img src="image2" alt="Graph" /></td>
</tr>
<tr>
<td><strong>Stress distribution for bending in elastic region A</strong></td>
<td></td>
</tr>
<tr>
<td><img src="image3" alt="Graph" /></td>
<td><img src="image4" alt="Graph" /></td>
</tr>
<tr>
<td><strong>Stress distribution for bending in plastic region B</strong></td>
<td></td>
</tr>
<tr>
<td><img src="image5" alt="Graph" /></td>
<td><img src="image6" alt="Graph" /></td>
</tr>
<tr>
<td><strong>Stress distribution for bending at ultimate plastic region C</strong></td>
<td></td>
</tr>
<tr>
<td><img src="image7" alt="Graph" /></td>
<td><img src="image8" alt="Graph" /></td>
</tr>
<tr>
<td><strong>Bending load-deflection curve</strong></td>
<td></td>
</tr>
<tr>
<td><img src="image9" alt="Graph" /></td>
<td><img src="image10" alt="Graph" /></td>
</tr>
</tbody>
</table>

Figure 3.4 Behaviour of ideally elastic and elastic-plastic materials in flexure$^{(3-2)}$. 

34
Bending theory is applicable up to the first crack stress in the elastic region. Figure 3.4 (A) is for the elastic region with neutral axis at mid-depth and the tensile strength is equal to the bending strength. Figure 3.4(B) shows that when the elastic limit in bending of an ideally elastic material is reached, it will fail. However, the elastic-plastic material can continue to support additional loads. In the plastic region, the fibres are extending or are pulling out at constant load across a crack throughout the tensile section. The stress distribution is modified with the neutral axis moving upward and the tensile stress distribution becomes rectangular in shape as shown in Figures 3.4(B) and (C). As a result, the load in flexure for the elastic-plastic material will continue to ascend with increase in deflection beyond the elastic limit. Thus the notional flexural strength, $\sigma_b$, will be greater than the ultimate tensile strength of the material, $\sigma_t$, and has no particular physical significance. It is important to understand the ratios between $\sigma_b$ and $\sigma_t$. Hannam\(^{(3-3)}\) has shown that the neutral axis in FRC at the ultimate stage of flexural loading may be 1/4 of the depth from the compression surface as shown in Figure 3.5.

![Figure 3.5](image)

**Figure 3.5** The stress distribution in flexure of an ideally elastic-plastic material at the instant of failure\(^{(3-3)}\).

Based on this assumption, $\sigma_b$ is calculated as:

$$\sigma_b = 2.44\sigma_t$$
Thus, the fibre reinforcement can lead to increase in the load bearing capacity of the composite in bending even if there is no increase in tensile strength accompanied by the enhancement of the post-cracking ductility.

As described above, the flexural behaviour is complicated and different from the simple tensile behaviour. These differences suggest that the flexural test usually used to characterise the properties of FRC may be difficult to interpret in resolving parameters of basic physical significance, such as first crack stress and multiple cracking. On the other hand, although the tensile test takes more time than the flexural test to be carried out, the interpretation of the tensile behaviour is simpler than that of the flexural behaviour. In this study, the uniaxial tensile test has been carried out in order to interpret stress-strain curves and develop the mathematical model for the dynamic behaviour of the hybrid fibre cement composite.

3.5.2. Test procedures

An Instron 1122 (maximum load capacity : 0.5 tons, 5kN) was mainly used for the uniaxial tensile tests. Regarding the glass fibre reinforced composites containing more than 1.6% glass fibre by volume, the tensile tests were carried out with Instron 1185 (maximum load capacity : 10 tons, 98kN), because the ultimate strengths of the composites were theoretically predicted to be beyond the maximum load capacity of an Instron 1122. Strains at each side of the specimen were measured, using linear variable differential transformers (LVDT) attached to a clip-on extensometer with 100mm gauge length, as shown in Figure 3.6.
The output was electrically averaged and a single load-strain plot obtained on an X-Y recorder (Task 1). In order to determine accurately the uncracked composite elastic modulus and cracking strain, an X-Y-Y recorder was used simultaneously, where the strain output from the LVDT on each side of the specimen was recorded up to 0.5% strain at a larger magnification (Task 2). The recorder strain scale was calibrated against known movements of the extensometer fixed to a micrometer bench before each day’s testing (Figure 3.7).
The load signal was taken from the load cell and the load scale was calibrated using the Instron's own internal reference calibration facility. Also these signals were input to an Autonomous Data Acquisition Unit (ADU) system to enable automated calculation, analysis and graph plotting to different scales. The ADU system was controlled by Dialog software. The scan rates of Task 1 and Task 2 were 1 second and 100 milliseconds, respectively.

Figure 3.8 shows the apparatus for the uniaxial tensile tests. On the test specimens of 25mm wide and 300mm long, their middle third positions were marked to attach the extensometer. In setting the test specimen, it was first gripped in the top jaw where a lead strip about 2mm thick and 30mm wide was placed between the specimen and the jaw to prevent grip failures. Then the extensometer was attached on the specimen and the pins fixing the knife edges were removed. The two X-Y chart
recorders and ADU system set to zero before starting tests. The bottom jaw was then tightened on to the specimen with a lead strip. The loading in tension and the collecting data by the ADU system were started at the same time. All specimens were tested at a constant cross-head speed of 10mm/min, equivalent to an overall composite strain rate of about 3.5%/min.

Figure 3.8 Apparatus for the uniaxial tensile test.
3.6. Crack spacing

After tensile testing, the number of cracks in 50mm length within the gauge span was counted under the microscope. Identification of cracks was assisted by highlighting the cracks by wetting the specimen surface. Typical cracks formed on the specimen are shown in Figure 3.9. The crack spacing is simply given by:

\[
\frac{50\text{mm}}{\text{the number of cracks}}
\]

Figure 3.9 Typical cracks formed on the composite reinforced with 4vol% of polypropylene networks and 0.8vol% of continuous glass fibres.
CHAPTER 4

4. TENSILE BEHAVIOUR OF GLASS/POLYPROPYLENE FIBRE HYBRID CEMENT COMPOSITES

4.1. Introduction

Tensile testing is usually carried out to characterize the effect of the fibre reinforcement on the toughness of the composite and the crack control performance of the fibre. The major role of the fibres in FRC is performed in the post-cracking region, where the fibres bridge across the cracked matrix. The fibres can increase the toughness of the composite by providing energy absorption mechanisms related to their debonding and pull-out processes. When the composite cracks in tension, the load is transferred to the fibres. In order to prevent the failure at that time, the load bearing capacity of the fibres should be greater than the load on the composite at first crack. These relations can be expressed as the following equation (4-1).

$$\sigma_{fu} V_f > E_m \varepsilon_{mu} V_m + E_f \varepsilon_{mu} V_f$$

where

- $\sigma_{fu}$: ultimate fibre strength
- $V_f$: fibre volume fraction
- $V_m$: matrix volume fraction
- $\varepsilon_{mu}$: ultimate matrix strain
- $E_f$: elastic modulus of fibre
- $E_m$: elastic modulus of matrix

The right hand term in equation (4-1) represents the first crack stress of the composite. When equation (4-1) is satisfied, the catastrophic failure is prevented and the fibres can perform their roles effectively. Therefore, the range of the fibre volume was mainly chosen to satisfy equation (4-1). Tensile stress-strain curves of various combinations of glass/polypropylene fibre cement composites have been obtained to assess the hybrid effect in this study. With regard to the salient points on the stress-strain curves such
as the first cracking point, the first yield point and the ultimate point, the correlations for the fibre volume fractions were examined.

4.2. Literature survey on hybrid fibre reinforcement

Hybrid fibre cement composites contain two or more fibre types to utilize the best properties of individual fibres. The combination of different fibre types such as high and low modulus combination of fibres, has been studied by various investigators. Walton and Majumdar\(^{(4-2)}\) found that satisfactory composites can be produced by using a mixture of organic and inorganic fibres as reinforcements. Organic polymer fibres such as nylon and polypropylene substantially improved the impact resistance of the matrix, when added to cement or concrete. However, they had very little effect on the tensile or bending strength. To enhance the tensile and bending strength, the addition of a second fibre such as glass, asbestos or carbon was effective.

The effect of using steel and polyethylene fibres together in concrete has been investigated, in particular with regard to the flexural properties of the composite by Kobayashi and Cho\(^{(4-3)}\). It was possible to obtain fibre reinforced concrete of superior toughness by dispersing short, discontinuous steel and polyethylene fibres, in randomly oriented states, in the concrete. The flexural load-deflection curve of hybrid reinforced concrete, using polyethylene and steel fibres, was presented in Figure 4.1. The load-deflection curve is broadly divided into Region (I) up to the first crack load and the subsequent Region (II) to failure. Region (I) is governed by the behaviour of steel fibre reinforced concrete containing the same amount of steel fibres as the hybrid reinforced concrete. In this region, polyethylene fibres make practically no contribution because of their low modulus of elasticity. In Region (II), a maximum load is reached mainly due to the reinforcing effect of steel fibres and after the maximum point, there is small reduction in load despite increases in deformation, which is derived from the ductility of polyethylene fibres. Thus, superior toughness is produced in this region.
In hybrid fibre reinforced concrete comprised of steel fibres and polypropylene fibres, Hasaba et al.\cite{4-4} indicated that the resistance against impact loads was greater than that for steel fibre reinforced concrete and Larsen and Krenchel\cite{4-5} observed that the increase in fracture energy was about 40% of its initial value after 10 years of outdoor exposure. Also, Zheng and Feldman\cite{4-6,4-7} have described that the stronger and stiffer steel fibre improves the ultimate strength, while the more flexible and ductile polypropylene fibre leads to improved toughness and strain capacity in the post-cracking region.

Nicholls\cite{4-8} evaluated the synergism of combining mineral wool to improve stiffness and polyethylene pulp to improve toughness in mortar sheet products. In a hybrid micro-reinforced system with carbon, steel and polypropylene fibres, the materials properties were found to lie somewhere between those for the equivalent mono-fibre systems. More steel fibres lead to an improvement in strength while more carbon fibres lead to a more pronounced improvement in toughness and polypropylene fibres provided better toughening at large crack opening.\cite{4-9,4-10,4-11}
Flexural tests were carried out on cement-based composites containing, individually or in combination, continuous polypropylene networks, chopped glass strands and continuous glass rovings by Xu and Hannant. The combined use of these fibres can yield higher limit of proportionality and improved flexural performance in the post-cracking zone compared with their individual composites. A minor increase in the volume fraction of aligned glass rovings (say, 0.2%) can significantly improve the load-carrying capacity of the hybrid fibre reinforced composites after cracking. They concluded that the combination of polypropylene networks and glass fibres in a composite can result in a synergistic interaction between the two in a flexural system. In another paper reported by Xu and Hannant, tensile tests have been carried out to assess the properties of the same hybrid fibre reinforced composite in various combinations of polypropylene networks, glass fibre strands and glass rovings. Figure 4.2 demonstrates schematically the significant differences between the behaviour of various fibre combinations.

![Figure 4.2 Schematic description of the effect for combined reinforcement of polypropylene networks and glass strands on the tensile stress-strain curve. (after Xu and Hannant)](after Xu and Hannant)
(1) Stage 1: Elastic stage, up to the bend over point a". The external load is shared in proportion to the stiffness of the three components, matrix, glass and polypropylene.

(2) Stage 2: Multiple cracking stage, point a" to b". The multiple cracking length in the combined fibre composite is relatively short so that the glass strands appear to mainly control stage 2 due to their high elastic modulus and adequate bond stress.

(3) Stage 3: Point b" to c" in which the load increases relatively rapidly for a small increase in strain. In this stage, the debonding of the glass fibre appears to be stabilised or partly prevented by the presence of the polypropylene. It is likely that both glass and polypropylene fibres carry a greater stress than in either of the single fibre composites. The stress transfer mechanism is both elastic and frictional and the dominant role is still glass fibres.

(4) Stage 4: Point c" to d" in which there is a large increase in strain for a small change in load. In this stage, the load is thrown gradually on to the polypropylene network from the glass fibre and then the polypropylene fibres extend across the cracks until failure.

A typical tensile stress-strain curve of the composite reinforced with polypropylene networks, glass rovings and glass strands is presented in Figure 4.3. The glass rovings had failed by 0.5% strain without polypropylene and it is surprising that they sustain nearly 2% strain with combined fibres. As shown in Figures 4.2 and 4.3, these kinds of combined reinforcement can yield a composite with a significant increase in load-carrying capacity compared with a polypropylene reinforced cement, and considerable increase in toughness over a glass reinforced cement.
As described above, the effects of combining different fibre types have been confirmed in various kinds of hybrid fibre reinforced composites. However, the mechanics of the hybrid effect have not been understood.
4.3. Tensile stress-strain curves

Typical stress-strain curves obtained in this investigation for the unreinforced matrix and composites reinforced with continuous glass fibres are shown in Figure 4.4. In the range of fibre volume fraction from 0% to 4%, seven different fibre contents are represented. To improve clarity, the zero points of the stress-strain curves of the matrix and the composite containing 0.44vol% glass fibres are offset.

Figure 4.4 Tensile stress-strain curves of matrix and composites reinforced with continuous glass fibres.
These curves were influenced by the condition of the specimens (warping, unevenness, porosity, micro-crack) and the gripping arrangement, especially in the case of the composites containing less than 1% by volume of glass fibres. This problem was not critical for the composites reinforced with higher contents of glass fibres due to their flexibility. In the case of the fibre volume of 0.44%, composites failed catastrophically. The theoretical critical fibre volume \(^{(4-14)}\) is calculated by:

\[
V_{fg(crit)} = \frac{\sigma_{cc}}{\sigma_{fgu}}
\]

where \(V_{fg(crit)}\) : critical glass fibre volume  
\(\sigma_{cc}\) : first crack stress of the composite  
\(\sigma_{fgu}\) : ultimate glass fibre stress  

Assuming \(\sigma_{cc}\) of 6.29MPa and \(\sigma_{fgu}\) of 1500MPa, the critical fibre volume is calculated at 0.42%. As shown in Figure 4.4, the composite containing 0.68 vol% of glass fibres keeps its initial cracking stress up to about 0.8% strain and then failed at lower strain compared with others. Therefore, the actual critical fibre volume is thought to be around 0.68vol%. The initial cracking stresses and the stress-strain curves were shifted upward with increasing glass fibre volume fraction.

The results for the composite reinforced with polypropylene networks alone are given as typical stress-strain curves in Figure 4.5. Also, representative stress-strain curves for the combination with about 0.4vol% and 0.8vol% glass fibres are shown in Figures 4.6 and 4.7, respectively. In each Figure 4.5 - 4.7, the curves for four different volume fractions of polypropylene, 2vol% to 9vol%, are indicated.
Figure 4.5 Tensile stress-strain curves of composites reinforced with polypropylene networks.

Figure 4.6 Tensile stress-strain curves of composites reinforced with polypropylene networks and continuous glass fibres of 0.43vol%.
The stress-strain curves for composites with combined reinforcement of polypropylene networks and continuous glass fibres rise more steeply than for polypropylene alone up to about 1.5% strain, and then drop down as shown in Figures 4.6 and 4.7 which is thought to be caused mainly by fracture of the glass fibre. Hereafter, this first maximum point of the curve is called first yield point (FYP). After this point, a horizontal or slightly ascending region can be seen, which requires further explanation. This composite still indicates higher stresses up to about 4% strain than those of composites reinforced with polypropylene alone. After about 4% strain, the stress and strain keep increasing until the specimens fracture.

As shown in Figure 4.5 a composite containing about 2vol% polypropylene failed at less strain than others with more than 4vol%. Also, hybrid glass fibre composites combined with 2-3vol% polypropylene failed immediately after FYP by 2% strain (Figure 4.6 and 4.7). The volume of 2-3% of polypropylene is not enough to carry the
stress in composites containing glass fibre after the glass fractures. Therefore, it is thought that although the critical volume of polypropylene is about 2.4% for only polypropylene composites (Figure 4.5), when a hybrid composite is considered the increased stress at FYP will require an increased polypropylene volume to sustain the stress after glass fracture leading to an apparently increased critical volume. This follows from the equation \(^{(4-14)}\)

\[
V_{fp(crit)}^{'} = \frac{\sigma_{cc}}{\sigma_{fpu}}
\]

where \(V_{fp(crit)}^{'}\) : critical fibre volume of polypropylene

\(\sigma_{fpu}\) : ultimate fibre stress of polypropylene

If the stress at FYP is taken instead of the first crack stress of the composite, \(\sigma_{cc}\), the equation is

\[
(FYP)V_{fp(crit)}^{'} = \frac{\sigma_{ul}}{\sigma_{fpu}}
\]

where \((FYP)V_{fp(crit)}^{'}\) : critical fibre volume of polypropylene at FYP

\(\sigma_{ul}\) : stress of the composite at FYP

As \(\sigma_{ul}\) increases with glass fibre volume fraction, \((FYP)V_{fp(crit)}^{'}\) will increase. Thus there will be a variable \((FYP)V_{fp(crit)}^{'}\) depending on the glass fibre volume fraction.

The stress-strain curves for composites containing various volume fractions of glass fibres, 0-1.2%, with the constant 4.6vol% polypropylene are shown in Figure 4.8. In combination with polypropylene networks, the effect of glass fibres can be seen even though their volume fraction is only 0.21%. The stress increased in proportion to the volume fraction of the glass fibre up to FYP. For composites with 0.21-0.87vol% of glass fibre, the stress was carried by the polypropylene after FYP, and the strain still increased up to 7-8%. The composites with more than 1.09vol% of glass fibres indicated higher stress at FYP. However, they failed soon after this point. The tensile behaviour after FYP therefore depends on the balance between the stress and the fibre volume fraction of the polypropylene.

i.e. at FYP of 18.4MPa, \(\sigma_{fpu}=373\)MPa (p.70), \((FYP)V_{fp(crit)}^{'}=18.4/373=4.9\%

51
Figure 4.8 Tensile stress-strain curves of composites reinforced with polypropylene networks of 4.6vol% and continuous glass fibres.
4.4. Method of determination for characteristic values from tensile stress-strain curves

Figures 4.4-4.8 show the differences between the behaviour of various fibre combinations. Figure 4.9 demonstrates schematically the tensile stress-strain curve for a hybrid composite with polypropylene networks and continuous glass fibres. Notations used in the Figure are explained below.

\( \varepsilon_{ma} \) : strain at BOP (bend over point = point a)
\( \sigma_{cc} \) : stress at BOP
\( E_c \) : elastic modulus up to BOP, point o to a
\( \varepsilon_{ul} \) : strain at FYP
\( \sigma_{ul} \) : stress at FYP
\( E_2 \) : modulus between point b and c
\( T_{ul} \) : toughness up to FYP (It is assessed by calculating the area under the stress-strain curves expressed as energy absorbed per unit volume of the composite)
\( E_3 \) : modulus at 5-6% strain, between point e and f
\( \varepsilon_u \) : ultimate strain
\( \sigma_u \) : ultimate stress
\( T_u \) : toughness up to the ultimate point f
Figure 4.9 Schematic tensile stress-strain curve of composite reinforced with polypropylene networks and continuous glass fibres.

After the elastic region (o-a), first cracking occurs at the point a. The composite cracks successively to generate multiple cracking at approximately constant stress which is equal to the first crack stress, $\sigma_{cc}$ (a-b). After the multiple cracking region, the stress and the strain increase again until one of the fibres fails at the point c named first yield point. In the glass/polypropylene hybrid cement composite, the glass fibre fails first because of the lower ultimate elongation than that of the polypropylene. The stress-strain curve for the composite reinforced with the single fibre ends at the point c. Beyond the first yield point, the sudden drop of the stress can be seen in most of the hybrid composites. However, the points d and e are variable in the dotted region depending upon the loading speed and the balance of each fibre volume fraction.
At the ultimate point $f$, the hybrid composite fails with the failure of the second fibre, polypropylene.

Measuring stress-strain points collected by the ADU system between 1MPa and 6MPa by stress were regressed to the straight line, because the stress-strain curves were sufficiently stable in this region. The elastic modulus $E_e$ was given as the slope (X coefficient) of the line. In this stress range, the good linearity was obtained. Also a simple linear regression line was obtained after the bend over point $a$ from the strain range, 1000 to 2500 micro-strains. The strain and the stress at BOP ($\varepsilon_{\text{mu}}, \sigma_c$) were determined by the cross point of these two straight lines. Representative examples of regression lines and their regression outputs are presented in Figures 4.10 and 4.11. The explanation of regression outputs is described in APPENDIX 1 including comments on regression lines $a$ to $b$.

As shown in these Figures, immediately after the elastic region, the stress-strain curves are occasionally unstable up to about 1000 micro-strains. In most of the composites containing more than 0.8 vol% glass fibres, their stresses began to rise again at a little over 3000 micro-strains. Hence, the regression lines obtained from the strain range, 1000 to 2500 micro-strains, are regarded as proper to determine the bend over point.
Figure 4.10  Regression lines for the bend over point and the modulus $E_c$.

Figure 4.11  Regression lines for the bend over point and the modulus $E_c$.  

Regression output:
Constant 0
Std Err of Y Est 0.2394000
R Squared 0.9531215
No. of Observations 13
Degrees of Freedom 12
X Coefficient(s) 0.0264555
Std Err of Coef 0.0004630

Regression output:
Constant 8.4608625
Std Err of Y Est 0.1586439
R Squared 0.1857083
No. of Observations 14
Degrees of Freedom 12
X Coefficient(s) 0.001678
Std Err of Coef 0.0001014

Regression output:
Constant 9.8421771
Std Err of Y Est 0.2394000
R Squared 0.3537605
No. of Observations 23
Degrees of Freedom 21
X Coefficient(s) 0.0001839
Std Err of Coef 0.0000542

Regression output:
Constant 0
Std Err of Y Est 0.1240386
R Squared 0.3537605
No. of Observations 23
Degrees of Freedom 21
X Coefficient(s) 0.0001839
Std Err of Coef 0.0000542

Regression output:
Constant 8.4608625
Std Err of Y Est 0.1586439
R Squared 0.1857083
No. of Observations 14
Degrees of Freedom 12
X Coefficient(s) 0.001678
Std Err of Coef 0.0001014

Regression output:
Constant 9.8421771
Std Err of Y Est 0.2394000
R Squared 0.3537605
No. of Observations 23
Degrees of Freedom 21
X Coefficient(s) 0.0001839
Std Err of Coef 0.0000542
The second elastic modulus $E_2$ was given as the slope of the simple linear regression line obtained from measuring points between $b$ and $c$. The representative regression line is shown in Figure 4.12. It was difficult to specify uniformly the range between $b$ and $c$. Therefore, the proper stress range for regression was determined individually according to the load-strain curve on a X-Y recorder.

Figure 4.13 demonstrates the regression line for the third elastic modulus $E_3$. The $E_3$ was given as the slope of the line regressed from measuring points between 5% and 6% strain. The first yield points and the ultimate points were determined directly from the stress-strain curves.
Figure 4.12 Representative regression line for the modulus $E_2$. 

Figure 4.13 Representative regression line for the modulus $E_3$. 
4.5. Correlation for the fibre volume fractions

In the hybrid cement composite reinforced with continuous glass fibres and polypropylene networks, the fibre orientation can be regarded constant so that the fibre volume fractions are the dominant variables for the tensile behaviour. Thus, the correlations between characteristic values in tensile stress-strain curves and the fibre volume fractions of glass fibres and polypropylene networks were examined.

4.5.1. Bend over point (BOP)

The strains at BOP ($\varepsilon_{\text{mu}}$) are plotted against the volume fraction of polypropylene ($V_{fp}$) and glass fibre ($V_{fg}$) in Figures 4.14 and 4.15, also the stresses at BOP ($\sigma_{ce}$) are plotted in Figures 4.16 and 4.17, respectively. Figures 4.14 and 4.16 show three lines corresponding to 0%, 0.43% and 0.82% by volume of glass fibres. In Figures 4.15 and 4.17, the volume fraction of polypropylene is constant, 4.6%. All the lines are drawn, based on the simple linear regression model where the value of $R^2$ is the coefficient of determination. The meaning of $R^2$ is mentioned precisely in APPENDIX I (p.280). It provides rough index of the degree to which a set of plotted points cluster about the regression line.
Figure 4.14  Strain at BOP ($\varepsilon_{mn}$) vs. volume fraction of polypropylene ($V_{fp}$).

Figure 4.15  Strain at BOP ($\varepsilon_{mn}$) vs. volume fraction of glass fibre ($V_{fg}$).
Figure 4.16  Stress at BOP ($\sigma_{ce}$) vs. volume fraction of polypropylene ($V_{fp}$).

Figure 4.17  Stress at BOP ($\sigma_{ce}$) vs. volume fraction of glass fibre ($V_{fg}$).
In Figure 4.14 and 4.16, the plotted points show considerable scatter about the regression lines at around 2-3% of $V_{fp}$. However, these lines can be regarded to have a positive trend in general. According to the values of $R^2$, the lines for composites without glass fibres show higher linearity than those with glass fibres.

The plots are scattered in Figure 4.15 and the regression line seems to be almost horizontal in Figure 4.17. Therefore, the correlation of the glass fibre volume fraction in this range, 0 to 1.2%, with the strain and the stress at BOP is low in combination with 4.6vol% of polypropylene. In general, the polypropylene fibre of which elastic modulus is lower than that of the matrix, is thought to have little effect for BOP. However, in the glass/polypropylene hybrid cement composite, the polypropylene was actually more effective than the glass fibre ($V_{fg}$<1.2%). At BOP, uniformly distributed polypropylene networks could suppress the crack initiation. The glass fibre volume fraction less than 1.2% is insufficient for the crack suppression.

4.5.2. First yield point (FYP)

The strains at FYP are plotted against $V_{fp}$ and $V_{fg}$ in Figures 4.18 and 4.19, the stresses at FYP are plotted in Figures 4.20 and 4.21, and the plots for toughness up to FYP are presented in Figures 4.22 and 4.23, respectively. As FYP can be seen only in hybrid composites, two lines corresponding to 0.43% and 0.82% of $V_{fg}$ are indicated in Figures 4.18, 4.20 and 4.22.
Figure 4.18  Strain at FYP ($\varepsilon_{ul}$) vs. volume fraction of polypropylene ($V_{fp}$).

Figure 4.19  Strain at FYP ($\varepsilon_{ul}$) vs. volume fraction of glass fibre ($V_{fg}$).
Figure 4.20  Stress at FYP ($\sigma_{ul}$) vs. volume fraction of polypropylene ($V_{fp}$).

Figure 4.21  Stress at FYP ($\sigma_{ul}$) vs. volume fraction of glass fibre ($V_{fg}$).
Figure 4.22 Toughness up to FYP ($T_{ul}$) vs. volume fraction of polypropylene ($V_{fp}$).

Figure 4.23 Toughness up to FYP ($T_{ul}$) vs. volume fraction of glass fibre ($V_{fg}$).
Regarding the strain at FYP, the correlation for $V_{fp}$ and $V_{fg}$ is not good. The strain for 4-8% of $V_{fp}$ seems to be almost constant in Figure 4.18. A slight negative trend can be seen as the correlation for $V_{fg}$ in Figure 4.19. On the other hand, the stress at FYP and the fibre volume fraction are correlated well as a positive linear relationship, shown in Figures 4.20 and 4.21. The stress at FYP can be regarded to be dominated by both $V_{fp}$ and $V_{fg}$. Although the toughness for $V_{fp}$ indicates the same trend as the stress in Figure 4.22, the positive correlation cannot be confirmed for $V_{fg}$. As presented in Figure 4.23, the toughness seems to be almost constant regardless of the glass fibre volume fraction. It is related to the negative trend between the strain at FYP and $V_{fg}$ (Figure 4.19). Since the plots in Figures 4.19, 4.21 and 4.23 are represented only for 4.6% of $V_{fp}$, they are not enough to enable a discussion about the mechanism of reinforcement for toughness at FYP. This aspect requires further research.

4.5.3. Ultimate point

The ultimate strains are plotted against $V_{fp}$ and $V_{fg}$ in Figures 4.24 and 4.25, the ultimate stresses are plotted in Figures 4.26 and 4.27 and the ultimate toughnesses are shown in Figures 4.28 and 4.29, respectively.

The ultimate strains are greatly influenced by whether the fibre volume fraction of the polypropylene is more than the critical value at FYP or not. As mentioned before, the critical fibre volume of polypropylene determined by FYP will vary depending on the glass fibre content. In Figure 4.24, the strains up to 3% of $V_{fp}$ are much smaller than that of over 4% of $V_{fp}$. This is caused by the lower volume of fibres being less than the FYP critical fibre volume. In the range of more than 4% of $V_{fp}$, the strains seem to be constant. On the other hand, hybrid composites with more than 1% of $V_{fg}$ failed immediately after FYP, and hence, the ultimate strain was equal to the strain at FYP. Therefore, the ultimate strains for over 1% of $V_{fg}$ are much smaller than those for up to 1%. In the range of less than 1% of $V_{fg}$, the correlation between the ultimate strain and $V_{fg}$ cannot be confirmed.
Figure 4.24  Ultimate strain ($\varepsilon_u$) vs. volume fraction of polypropylene ($V_{fp}$).

Figure 4.25  Ultimate strain ($\varepsilon_u$) vs. volume fraction of glass fibre ($V_{fg}$).
Figure 4.26  Ultimate stress ($\sigma_u$) vs. volume fraction of polypropylene ($V_{fp}$).

Figure 4.27  Ultimate stress ($\sigma_u$) vs. volume fraction of glass fibre ($V_{fg}$).
Figure 4.28  Ultimate toughness ($T_u$) vs. volume fraction of polypropylene ($V_{fp}$).

Figure 4.29  Ultimate toughness ($T_u$) vs. volume fraction of glass fibre ($V_{fg}$).
In Figure 4.26, a high positive linearity was obtained from the correlation between the ultimate stress and $V_{fp}$ which indicates a constant polypropylene strength of about 373MPa, assuming $\sigma_u=\sigma_{fp}V_{fp}$, where $\sigma_{fp}$ is the ultimate stress of the polypropylene. On the contrary, the ultimate stress seems to be constant regardless of $V_{fg}$ at 4.6% of $V_{fp}$ in Figure 4.27. In the case of $V_{fp}$ exceeding the critical fibre volume, the glass fibre may not affect the ultimate stress. When $V_{fp}$ is less than the critical fibre volume, the glass fibre may have an effect on the ultimate stress.

The ultimate toughness for $V_{fp}$ indicates the same trend as the stress, as shown in Figure 4.28. In Figure 4.29, the toughnesses for $V_{fg}$ are decreased remarkably in the range of more than 1% of $V_{fg}$ for the same reason as the ultimate strain (refer to Figure 4.25). i.e. the polypropylene volume has reduced below $(FYP)_{V_{fp}(crit)}$.

4.5.4. Modulus $E_1$, $E_2$ and $E_3$

The modulus $E_1$, $E_2$ and $E_3$ are plotted against $V_{fp}$ and $V_{fg}$ in Figures 4.30-4.35, respectively. As shown in Figures 4.30 and 4.31, there seems to be no correlation between $E_1$ and the fibre volume fraction. The modulus $E_1$ is thought to be mainly dominated by the matrix. According to Figure 4.33, the modulus $E_2$ is dominated obviously by glass fibres. The contribution of polypropylene networks is also confirmed in Figure 4.32. The main dominant role is thought to be glass fibres because there is no FYP without glass fibres. Therefore, both contribution of glass fibres and polypropylene networks should be taken into consideration regarding the modulus $E_2$. In Figure 4.34, there is a good correlation between $E_3$ and $V_{fp}$. This would be expected as the slope=$E_1V_f$ which for 0% glass fibre would give a modulus for the polypropylene at high strains of about 3.2GPa. Figure 4.35 shows no correlation between $E_3$ and $V_{fg}$ probably because the glass is broken into short lengths at strain over 4%.
Figure 4.30 Modulus $E_c$ vs. volume fraction of polypropylene ($V_{fp}$).

Figure 4.31 Modulus $E_c$ vs. volume fraction of glass fibre ($V_{fg}$).
Figure 4.32  Modulus $E_2$ vs. volume fraction of polypropylene ($V_{fp}$).

Figure 4.33  Modulus $E_2$ vs. volume fraction of glass fibre ($V_{fg}$).
Figure 4.34 Modulus $E_3$ vs. volume fraction of polypropylene ($V_{fp}$).

Figure 4.35 Modulus $E_3$ vs. volume fraction of glass fibre ($V_{fg}$).
4.5.5 Crack spacing

The crack spacings are plotted against $V_{fp}$ and $V_{fg}$ in Figures 4.36 and 4.37, respectively. As shown in Figure 4.36, the relationship between the crack spacing and $V_{fp}$ is not linear, but it is likely to be an inverse proportional correlation. The crack spacings are also reduced by the addition of glass fibres. In Figure 4.37, the effect of glass fibres is found when plots for 0.2% of $V_{fg}$ are compared with those for without glass fibres, but it cannot be seen for more than 0.2% of $V_{fg}$ at 4.6% of $V_{fp}$. However when $V_{fp}$ is less than the FYP critical fibre volume, the effect of glass fibres appears. This is obvious from Figure 4.36 at 2-3% of $V_{fp}$ where higher volumes of glass fibres give smaller crack spacings. Surprisingly the crack spacings for composites containing more than 0.8% of $V_{fg}$ at 4.6%$V_{fp}$ increase. This may indicate that the composites have failed before the completion of multiple cracking.
Figure 4.36 Crack spacing vs. volume fraction of polypropylene ($V_{fp}$).

Figure 4.37 Crack spacing vs. volume fraction of glass fibre ($V_{fg}$).
4.6. Multiple regression analysis for the second elastic modulus $E_2$

As described above, it is obvious that the salient points in the tensile stress-strain curve of the hybrid fibre cement composite are under the influence of the fibre volume fraction. Before discussing the mathematical model for the tensile behaviour, the contribution ratios of the glass fibre and the polypropylene networks will be discussed. In this section, the second elastic modulus $E_2$ in Figure 4.9, is focused on.

Regarding the first elastic modulus $E_c$ shown in Figure 4.9, it can be expressed using the simple mixture rule as follows.

$$E_c = E_{fg}V_{fg} + E_{fp}V_{fp} + E_mV_m$$  \hfill (4-5)

where $E_{fg}$, $E_{fp}$ and $E_m$ are the elastic moduli of the glass fibre, the polypropylene and matrix, respectively. $V_m$ is the volume fraction of matrix. In the second elastic region, there is assumed to be no influence from the cracked matrix. The contribution ratios of the glass fibre and the polypropylene networks for the second elastic modulus $E_2$ will be discussed from the statistical point of view. The modulus $E_2$ is assumed to be expressed as following equation.

$$E_2 = a_1E_{fp}V_{fp} + a_2E_{fg}V_{fg} + a_0$$  \hfill (4-6)

where $a_1$, $a_2$ : coefficient

$a_0$ : constant

This equation includes two independent variables, $V_{fp}$ and $V_{fg}$. In such a case, multiple regression analysis can be used to obtain values $a_1$ and $a_2$. In this section, the modulus $E_2$ is analysed for all data of $V_{fp}$ and $V_{fg}$ on trial.

The result of the multiple regression analysis is presented in Table 4.1. The form of the multiple regression equation with two independent variables is:
\[ Y = A_1 X_1 + A_2 X_2 + A_0 \]  

(4-7)

where \( Y \) is called response variable, \( X_1 \) and \( X_2 \) are called regressor variables, \( A_1 \) and \( A_2 \) are regression coefficients and \( A_0 \) is a constant. \( V_{fp} \) and \( V_{fg} \) were applied as \( X_1 \) and \( X_2 \), respectively. \( A_1 \) and \( A_2 \) correspond to \( \alpha_1 E_{fp} \) and \( \alpha_2 E_{fg} \) in the equation (4-6).

### Table 4.1 Multiple regression analyses for \( E_2 \).

\[
Y = A_1 X_1 + A_2 X_2 + A_0 
\]

Response variable \( Y: E_2 \)

<table>
<thead>
<tr>
<th>Regressor variable</th>
<th>Regression coefficients</th>
<th>F-value</th>
<th>confidence (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( X_1 : V_{fp} )</td>
<td>( A_1 : 6.587 )</td>
<td>204.8102</td>
<td>99</td>
</tr>
<tr>
<td>( X_2 : V_{fg} )</td>
<td>( A_2 : 83.091 )</td>
<td>1034.6493</td>
<td>99</td>
</tr>
<tr>
<td>( A_0 : -0.21949 )</td>
<td></td>
<td>7.18 at 1%</td>
<td></td>
</tr>
</tbody>
</table>

The coefficient of determination \( R^2 = 0.9551 \)

The multiple regression equation is expressed as below.

\[
E_2 = 6.587V_{fp} + 83.091V_{fg} - 0.21949 \]  

(4-8)

As a result of the F-test on individual regressor, each F-value of the regressor is sufficiently larger than the value at 1%, which indicates that these regressor variables, \( V_{fp} \) and \( V_{fg} \), are certainly related to the response variable, \( E_2 \), with 99% confidence. The analysis of variance for significance of regression is presented in Table 4.2. This F-value is also sufficiently larger than the value at 1%, which suggests that this multiple regression model is adequate with 99% confidence.
Table 4.2 Analysis of variance table for the multiple regression.

<table>
<thead>
<tr>
<th>Source of Variation</th>
<th>Sum of Squares</th>
<th>Degrees of Freedom</th>
<th>Mean Square</th>
<th>F-value</th>
<th>Confidence (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regression</td>
<td>2.92795</td>
<td>2</td>
<td>1.46398</td>
<td>521.6081</td>
<td>99</td>
</tr>
<tr>
<td>Residual</td>
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</table>

5.07 at 1%

In Figure 4.38, experimental values are plotted against calculated values obtained from the equation (4-8).

![Graph](image)

\[ y = 1.0002x \]
\[ R^2 = 0.9478 \]

**Figure 4.38** Comparison between calculated values and experimental values for the modulus \( E_2 \).

As shown in this Figure, this equation is in a good agreement with experimental values. Since \( E_{fp} \) and \( E_{fg} \) are constants, the equation (4-6) is expressed as follows.

Assuming \( E_{fp} = 10 \text{GPa}, \ E_{fg} = 72 \text{GPa} \).
\[ E_2 = 0.6587E_{fp}V_{fp} + 1.154E_{fg}V_{fg} - 0.21949 \] (4-9)

The coefficients, \( a_1 \) and \( a_2 \), are 0.6587 and 1.154 from the equation (4-9), therefore the degree of contribution of the polypropylene and the glass fibre for \( E_2 \) can be estimated as \( 0.6587/1.154 = 1/1.75 \). However, as described before, the \( E_{fp} \) for the polypropylene is not accurately known and varies with strain. If \( E_{fp} \) equals to 6GPa instead of 10GPa, then

\[ E_2 = 1.098E_{fp}V_{fp} + 1.154E_{fg}V_{fg} - 0.21949 \] (4-10)

and the degree of contribution of the polypropylene and the glass fibre is \( 1.098/1.154 = 1/1.05 \), which means the effect of each fibre is approximately equal. Therefore, it seems to be possible to use the mixture rule without matrix for the second elastic modulus \( E_2 \).

As mentioned above, multiple regression analysis is supposed to be useful to estimate the contribution ratio of several factors in a certain variable. Taking these statistical results into consideration, the mathematical model for tensile behaviour will be developed in Chapter 7.
4.7. Resume

Stress-strain curves for composites containing various combinations of polypropylene networks and continuous glass fibres, were obtained by tensile tests. The first yield point could be seen characteristically in the curve for hybrid composites. The critical fibre volume of the polypropylene at FYP will vary depending on the glass fibre volume, which controls FYP according to the equation (4-4). In this work, the benefits of combining polypropylene and glass fibres were apparent for glass fibre volumes greater than 0.2% at polypropylene fibre volumes of 4.6%.

The stress, strain and toughness at bend over points, first yield points and ultimate points, the modulus in several regions and the crack spacing were plotted against the fibre volume fraction. These properties were more or less dominated by both polypropylene networks and glass fibres.

Multiple regression analysis was used to obtain a model equation for $E_2$ which was expressed in terms of $V_{fp}$ and $V_{fg}$. The equation indicated a good agreement with experimental values. It can be assumed that both fibres affect the reinforcement of the hybrid composite synergistically up to FYP. In order to develop a mathematical model for the tensile behaviour, the synergistic effect of both fibres should be taken into account.
5. MICROSTRUCTURAL PARAMETERS IN COMPOSITES

5.1. Introduction

The fracture behaviour of glass reinforced cement is controlled by the bond strength at the interface between the fibre bundle and cement and also by the fracture of individual filaments within the bundle at stresses near to the ultimate stress of the composite. Both interfacial bond strength and filament fracture are important to the understanding, and prediction, of the behaviour of glass/polypropylene hybrid fibre composites. As part of a major programme of this study into hybrid composites, the perimeter of the glass fibre strand in contact with cement has been measured and the development of the glass filament fracture within the composite is examined in this Chapter.

5.2. Frictional shear bond strength

In FRC composites, if the following relation is satisfied, a process of multiple cracking is initiated.

\[ \sigma_{fu} V_f > \varepsilon_{mu} E_c \]  

where
- \( \sigma_{fu} \): ultimate strength of fibre
- \( V_f \): fibre volume fraction
- \( \varepsilon_{mu} \): composite strain at first cracking
- \( E_c \): elastic modulus of the composite

The additional load will be transferred back into the matrix over a transfer length \( x \) in Figure 5.1 (a) and the matrix will eventually fracture into segments of length between \( x \) and \( 2x \), where \( x \) is determined by the rate of stress transfer between the fibre and the matrix. The strain distributions in fibres and matrix between the matrix cracks are shown in Figure 5.1 (b).
Figure 5.1 Schematic description of multiple cracking\(^{(5-4)}\).

(a) crack spacing (x and 2x)
(b) strain distributions in fibres and matrix

The transfer length \(x\) is calculated from the simple balance of the load \(\sigma_{mu}V_m\) needed to break unit area of matrix and the load carried by fibres across the same area after cracking. This load is transferred over a distance \(x\) by the limiting maximum shear stress \(\tau_{fu}\). Assuming fibres of circular cross-section of radius \(r\), the value of \(x\) can be derived:

\[
\sigma_{mu}V_m = 2\pi r \frac{V_f}{\pi r^2} \cdot \tau_{fu} \cdot x \tag{5-2}
\]

therefore,

\[
x = \frac{V_m}{V_f} \cdot \frac{\sigma_{mu} \cdot r}{2 \tau_{fu}} \tag{5-3}
\]

where \(\sigma_{mu}\) : ultimate strength of matrix without fibres

\(V_m\) : matrix volume fraction
The same principles can be applied to fibres of irregular cross section as follows

\[ x = \frac{V_m}{V_f} \frac{\sigma_{mu} \cdot A_f}{\tau_f \cdot P_f} \]  

(5-4)

where \( A_f \): cross sectional area of fibre  
\( P_f \): perimeter of fibre

The actual average crack spacing was statistically obtained as \((1.364 \pm 0.002)x\) by Gale.\(^{(5-3)}\) Hence, the frictional bond stress is calculated based on the following equation.

\[ \tau_{fu} = 1.364 \frac{V_m}{V_f} \frac{\sigma_{mu} \cdot A_f}{C \cdot P_f} \]  

(5-5)

where \( C \) is average crack spacing.

Assessments of the fibre-matrix frictional shear bond strength \( \tau_{fu} \) from measured crack spacing in FRC relies on a knowledge of the perimeter of the fibre bundle in contact with cement and the fibre volume fraction. All of the parameters, except for \( P_f \), on the right hand side of equation (5-5) can be accurately calculated or measured and therefore the study was focussed on how to determine \( P_f \). The calculation of bond strength from Laws et al.\(^{(5-6)}\) also requires an accurate knowledge of \( P_f \) because these variables relate all their bond parameters to \((P_f \tau_{fu})\) rather than to \( \tau_{fu} \).

5.3. Scanning electron microscope (SEM) observation

The microstructure of the composites was observed using a Cambridge S250 scanning electron microscope (SEM). The cross-sections of the glass/polypropylene fibre hybrid cement composites were observed on the cut surface by detecting secondary electrons. Typical glass fibre strands on the cut cross-section of the composite are shown in Figure 5.2 (a) and enlarged in Figure 5.2 (b).
Figure 5.2 Glass fibre strands on the cut cross-section of the hybrid composite.
In these composites, the reinforcing unit for glass fibre is not a single filament but rather a bundle of filaments surrounded by the matrix. The spaces between filaments are too small for cement grains to penetrate and as a consequence, only the external filaments are in direct contact with the matrix. Similar conditions were observed by Stucke and Majumdar (5-7). The interface between the fibre and the matrix is magnified in Figure 5.3.

Figure 5.3 (a) shows the interface between glass fibre and matrix, and Figure 5.3 (b) shows that between polypropylene and matrix. The perimeter and cross sectional area of the glass strands and the polypropylene have to be obtained to calculate the bond strength. As shown in these Figures, the complex microstructural arrangement makes it difficult to determine an exact perimeter of the glass strand. Therefore, the image analysis was adopted for this purpose.
Figure 5.3 Interface between fibre and matrix.

(Spheres in (b) are P.F.A particles)
5.4. Image analysis

5.4.1. Test method

In order to obtain an actual value of the perimeter of the glass fibre strand in contact with the cement, an image analyser, Quantimet 920 (Cambridge Ltd.), was used. However, there was a practical problem in analysing the screen images directly, because they were not clear enough to be analysed with respect to the contrast between fibre strands and the matrix. Thus, an SEM observation was carried out to take pictures in advance. The composite specimens were cut in cross section and embedded in epoxy resin. They were polished to a #1000 grit level with a silicon carbide paper and finished with 6μm and 1μm diamond spray polishing. To improve the contrast in the photographic image, the specimens were observed by detecting backscattered electrons with an SEM. In SEM observations, abundant secondary electrons are emitted from a specimen and are most commonly used as the imaging signal. Backscattered electrons are not usually as numerous as secondary electrons but most of them carry high energies.[5-8] The number of backscattered electrons is strongly dependent on the atomic number of the specimen whereas those of secondary electrons are not. Therefore, a high resolution image with good contrast can be obtained because of the difference of the atomic number for the main components between glass and cement matrix.

The shapes of the glass fibre strands were traced on tracing papers and then analysed using the Quantimet 920. This equipment can control penetration of the reflected rays with particular wave lengths that make the different materials distinguishable by their colours. In this work, the individual perimeters (Pfg) and areas surrounded by perimeters (A'fg) of the glass fibre strand were measured (includes voids filled with size). It was also programmed to calculate A'fg/Pfg and Afg/Pfg for each strand. Afg is an area occupied by filaments in each strand.Thirty three strands each containing 100 filaments were analysed.
5.4.2. Results and discussion

A typical backscattered electron image of the glass fibre strand is presented in Figure 5.4. The strand in the hybrid composite is shown in Figure 5.4 (a). Figure 5.4 (b) is a single strand embedded in epoxy resin. In these photographs, the perimeter of the glass fibre strand can be determined clearly. Traced glass fibre strands on papers were analysed by the Quantimet 920. The result of the image analysis is presented in Table 5.1.

The perimeter in contact with cement of the glass fibre strand consisting of 204 filaments has been estimated to be 2640μm by Krenchel\textsuperscript{(5-9)} and 2830μm by Oakley and Proctor.\textsuperscript{(5-10)} Compared with these values, the obtained perimeter of 1097μm with a range between 802μm and 1363μm and a standard deviation of 143μm for our strand with 100 filaments seems to be a reasonable value. $A_f$ was calculated to be 15394μm$^2$, assuming 14μm of diameter and 100 filaments in each strand. Using $A'_f$ of 23339μm$^2$ from Table 5.1, this means that the glass occupies 66% of the strand volume. Since the applied load on the glass fibre strands is carried by the true glass filament area, $A_f/P_f$ should be used in calculations.

With respect to the polypropylene networks, since they are made from films, the shape of the cross section of the individual fibres after fibrillation is generally rectangular. Typical cross sections of the polypropylene networks in white cement paste are presented in Figure 5.5. White cement as the matrix was useful to obtain a good contrast between the polypropylene and the matrix. A number of hairs of the polypropylene causing physically good bond with matrix were observed. These hairs make it difficult to determine the perimeter of the polypropylene in direct contact with the matrix. Ohno\textsuperscript{(5-11)} measured the perimeter $P_{fp}$ and cross sectional area $A_{fp}$ of the polypropylene networks by the image analysis. $P_{fp}$ of 223μm, $A_{fp}$ of 3808 μm$^2$ and $A_{fp}/P_{fp}$ of 17.08μm were obtained as the average value of 682 fibres.
(a) In the hybrid composite

(b) Single strand

Figure 5.4 Backscattered electron image of the glass fibre strand.
Table 5.1 Results of the image analysis on the cross section of the hybrid composite.

\( A'_{fs} \) is area contained within strand perimeter, NOT area of glass fibre

<table>
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<th>sample No</th>
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<th>( P_{fs} (\mu m) )</th>
<th>( A'<em>{fs} / P</em>{fs} )</th>
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Figure 5.5 Polished cross-section of the polypropylene network in white cement.
5.5. Calculation of bond strength

Uniaxial tensile tests were carried out for composites reinforced with continuous glass strands aligned with the direction of applied load. The stress-strain curves are shown in Figure 4.4 in Chapter 4. Crack spacing was measured after failure and the glass fibre-matrix frictional shear bond stress was calculated from equation (5-5). Table 5.2 shows the calculated value of the bond stress $\tau_{fgu}$ at each fibre volume.

Table 5.2 The calculated fibre-matrix frictional shear bond strength of composites reinforced with continuous glass fibres.

<table>
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<tr>
<th>Sample No.</th>
<th>$V_{fg}$ (%)</th>
<th>$\sigma_{mu}$ (MPa)</th>
<th>C (mm)</th>
<th>$\tau_{fgu}$ (MPa)</th>
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$V_{fg}$ : glass fibre volume fraction

$\sigma_{mu}$ : matrix crack strength

C : crack spacing

$\tau_{fgu}$ : glass fibre-matrix frictional shear bond strength
It can be seen from Table 5.2 that the average bond strength apparently decreases from 2.19MPa at $V_{fg} = 1.57\%$ to 1.38MPa at $V_{fg} = 3.97\%$. These values compare with between 1MPa and 2MPa, depending on age and curing conditions, as determined by Laws et al.\(^{(5-6)}\) They investigated the bond strength between glass strands and cement with the value of $P_f\tau_{fgu}$ by fibre pull out test, where $P_f$ is the perimeter of the fibre strand. They used the Cem-FIL 2 fibre strands consisting of approximately 200 individual filaments. After 7 days initial cure period at 90% RH and 20°C, $P_f\tau_{fgu}$ was 4N/mm. They indicated that $P_f\tau_{fgu}$ was altered as a function of time and storage condition. For water storage at 20°C, $P_f\tau_{fgu}$ increased with storage time. $\tau_{fgu}$ is deduced to be 1.4MPa using a value of 2830μm for the perimeter of the strand obtained by Oakley and Proctor.\(^{(5-10)}\) This is of the same range as the value calculated here.

It can also be seen from Table 5.2 that the cracking stress of the matrix ($\sigma_{mm}$) apparently increases with glass fibre volume possibly due to crack suppression.\(^{(5-12)}\) This influences the calculated bond strength although probably not the real bond strength if such a parameter exists. However this would not explain the apparent reduction in bond strength with increase in fibre volume which may imply that equation (5-5) is an oversimplification of the real situation over a range of fibre volumes.
5.6. Glass fibre fracture in hybrid composites

A schematic tensile stress-strain curve of a hybrid composite reinforced with continuous glass fibres and polypropylene networks was presented in Figure 4.9 in Chapter 4. It is thought that the glass fibres fracture at just before point $c$ but it is not known whether all the filaments in a strand fracture simultaneously or whether progressive fracture occurs with the most highly stressed filaments failing first and transferring the load to less stressed filaments in the core of the strand. Therefore, the development of strand and filament fracture within the composite is examined in this section.

A hybrid composite was manufactured with 4.6% by volume of continuous polypropylene nets and 0.8% by volume of glass fibres. Specimens were tested in uniaxial tension and unloaded at three different stages i.e. about 1% strain (just before point $c$ on Figure 4.9), at 2% strain (after point $c$) and at about 5% strain. The unloading points on the stress-strain curves are shown on Figures 5.6 (a),(b),(c).

After test, each specimen was immersed in dilute hydrochloric acid for 7 days to dissolve the matrix. The matrix was washed out by flowing water and the first layer of the polypropylene was then removed with great care to reveal the glass fibre. The condition of the glass fibre was then observed under a stereoscopic microscope (Olympus SZ319111) and the results are shown in Figure 5.7 in which Figure 5.7 (a) is a representative photograph taken before loading.
Figure 5.6 Tensile stress-strain curves of the observed specimens at three different strains.
Figure 5.7 Micrographs of glass fibres in hybrid composite after acid dissolution\(^{(5.2)}\).

(a) before test  (b) at about 1% strain  (c) at about 2% strain  (d) at about 5% strain
At 1% strain Figure 5.7 (b), in spite of the presence of cracks at 2mm-7mm spacing in the matrix, no breakage of the glass fibre could be seen. At 2% strain, several substantial breaks in the strands and the failure of some filaments were observed. Although this strain was thought to be close to the failure strain of the glass fibre, every strand and filament probably had slightly different strains therefore they did not fail simultaneously. At 5% strain, there were many breaks in the strands and broken filaments were observed throughout the specimen. The effect of reinforcement by glass fibres theoretically could not exist after this point although in practice a small effect remained.

A general view of the behaviour of the composite is that all the glass fibres carry the applied load up to point c on Figure 4.9 but the strand containing filaments in which the local strain has reached the failure strain of the glass are broken at point c and consequently a sudden drop in stress is observed. However the unbroken fibres still carry some load. At strains greater than at point c, many fractures of the glass fibres occur and their role as a reinforcement will be reduced as they pull out. However further load will be still carried by the polypropylene up to the ultimate strain.
CHAPTER 6

6. LIGHT TRANSMISSION TECHNIQUE

6.1. Introduction

In Chapter 5, the microscopic observations of glass fibre failure in tensile testing gave us relatively crude information about the role of the glass fibre in hybrid composites. It was found that every glass strand and filament probably had slightly different strains and therefore they failed progressively near to the first yield point. Thus, it is important to understand the process of progressive glass failure in hybrid composites quantitatively because this affects the effective glass fibre volume at the first yield point. In order to obtain more detailed information about the fracture process of filaments and strands at increasing strain in the composite, a new technique was developed known as the light transmission method.

6.2. Literature survey on the light transmission technique

The light transmission technique has been developed on the basis of optical fibres. Optical fibres have been extensively employed in the telecommunication industry for the transmission of light signals over great distances. The first consideration of the use of optical fibres for damage sensors comes from a patent, issued in 1968 to Kinard (6-1) for monitoring of micro-meteorite damage on structures in-orbit. It was proposed to embed a grid of optical fibres under the surface of the structure. When a micro-meteorite struck the structure, a crater would be formed at the location of the impact and any optical fibre in its path would be broken. Then light would be emitted from ionized particles inside the crater and this light would be detected at the end of the fibre.

Hale et al (6-2)(6-3) reported on the use of optical fibres for crack monitoring of metallic structures. The optical fibres were adhesively bonded to the surface of the structure. When a crack was formed, any adhered fibre in its path would be broken, resulting in loss of light
transmission. In principle, the point of the break could be found by simply looking for a light bleeding spot along the fibre, assuming that intensive visible light was used at the input end and the adhesive was transparent. The ultimate tensile strain of optical fibres is of the order of 5% which is about 10 times larger than the cracking strain in brittle matrix composites. Thus, Hale used a chemical process to weaken the optical fibres. This research was aimed at developing damage-monitoring sensors for off-shore oil-drilling-rig underwater structures. Crane et al (6-4) suggested the use of optical fibres in advanced composite structures to form a damage assessment system. They verified their concept in glass fibre and carbon fibre reinforced plastic composites. However, only severe impact damage could be detected by their embedded optical fibres, because no chemical treatment was used.

Hofer (6-5) reported that the optical fibres integrated into a composite structure could serve as a reliable, automatic and remote working long-term monitoring device for structural damage. The optical fibres were broken by fractures, cracks or delaminations in a structure area and thus the light flow was interrupted. This technique could be applied in GFRP aircraft components as the damage detection system.

LeBlanc and Measures (6-6) concluded that embedded optical fibre sensors offer excellent potential as the basis of a built-in damage-assessment system for composite materials. In particular, systems based on the fracture of optical fibres indicated considerable promise for the detection and monitoring of impact damage.

Also a lot of examples using optical fibre damage sensors in concrete can be seen. The ability to monitor large areas is important when assessing the condition of large structures such as bridges (6-7). It becomes more important to monitor structures of bridges because of the increase in heavy traffic, the use of de-icing salts, air pollution, precipitation, the increased cost of insurance against loss or damage and the rising cost of conventional maintenance and repairs. The application of optical fibre sensors to bridges can be seen on the dual carriageway bridge in Oxfordshire (6-7) in U.K., the Schiessbergstrasse bridge, the Ulenbergstrasse bridge and Berlin-Marienfelde pedestrian bridge in Germany (6-8) and Notsch bridge in Austria (6-7).
Optical fibres have been successfully used in the following applications\(^{(6-7)}\):

- traditional post-tensioned bridges
- post-tensioned bridge decks employing glass fibre prestressing tendons
- dam movements
- ground and rock anchors
- soil movement
- tunnel movement

and have numerous further prospects for investigation.

In fundamental research, optical fibre sensors were also utilized for monitoring temperature, pressure, stress and strain within composite structures during curing and fabrication\(^{(6-9),(6-10),(6-11)}\).

As described above, the optical fibre has been utilized as various kinds of sensors. The technique using light transmitted down the normal AR glass fibres was studied by Hibbert\(^{(6-12)}\) to obtain quantitative information on fibre distribution in glass fibre reinforced cement (GRC) composites. Hibbert has shown that the glass fibre strands in a thickness of only about 7\,mm section of GRC composites are capable of transmitting useful quantities of light. Using such a technique, Hibbert and Grimer\(^{(6-13)}\) discussed a correlation between the variation in the number of visible strands in GRC composites and their fatigue properties.

A scanning technique using a photodiode has been developed by Rayment and Majumdar\(^{(6-14)}\), in order to estimate the glass fibre volume fractions in composites with opaque matrices. The information was derived from light transmitted along the reinforcing fibres by total internal reflection. The transmission of light through the composite was proportional to the glass fibre volume fraction up to a limiting value of about 6\%. Rayment and Majumdar have shown that the photodiode scanning technique of measuring the transmittance of GRC can successfully replace the photographic method for obtaining quantitative information on fibre volume fraction in the composite. In this section, light transmitted down the AR glass fibre
was utilized to monitor the failure of glass fibres which were assessed quantitatively using an image analyser.

6.3. Preliminary test

In this section, the light transmission technique was examined with simple equipment in order to make sure that it would be possible to pursue a more detailed study of the fracture of continuous glass fibres in the hybrid composite.

The specimen was of 75mm length, 25mm width and about 6mm thickness. When clamped in the Instron 1122 test machine there was 40mm between the grips. The illuminated surface and the observed surface were required to be flat and parallel to obtain uniform transmission of light. One end of the specimen was buried into epoxy resin vertically and polished in the same way as mentioned in section 5.4.1, after which the other end was also treated in the same manner. The specimen was set horizontally as shown in Figure 6.1.

![Schematic diagram of the light transmission test.](image)

Figure 6.1   Schematic diagram of the light transmission test.

In this test, a significant quantity of light is required in a very small area. In spite of this requirement, heat must not be allowed to affect the specimen under examination. The cold-light source KL1500 (SCHOTT Ltd.) was used as a light source, since the thermal component of the energy from the halogen lamp is filtered out and the visible light is passed through optical fibres. The maximum illumination at light outlet is approximately 10 Mlx. The light guide is a self supporting, flex-and-stay type and can illuminate a particular area.
The specimen was inserted in a slit on the support in which the opening spaces were sealed with plastic clay. Transmitted light was observed on the opposite end of the specimen by a stereoscopic microscope (Olympus SZ300300) and photographs were taken. The test was carried out in a dark room. Since the intensity of the light transmitted along the glass fibre through the composite was rather weak, high speed films, ASA 3200, were used. It was found that normal film (ASA 125) required an exposure of 5 minutes whereas the ASA 3200 film produced good results with an exposure of only 5 seconds.

Before the tensile tests, the continuity of the fibres was confirmed by checking that every glass fibre filament in a strand could transmit the light. Figure 6.2 shows a representative example with different magnifications.

Figures 6.2 (a) and (c) were observed under reflected light, and (b) and (d) were transmitted light at the same position. Glass fibre filaments can be seen as black dots in (a) and (c), and as white dots in (b) and (d). The intensity of transmitted light was altered by the angle of the supplied light. The position of the light source was adjusted to obtain the most intensive light. With one light source, the observable field was limited to 2-3mm diameter.

Although the intensity of transmitted light through individual filaments was not uniform, each filament could be clearly discerned. Since the light source, the microscope and the specimen were fixed, the observed field was limited. If they could be moved vertically and horizontally with precision, it would be possible to observe all the glass fibre on the cross section. In Hibbert's investigation, the distance of transmitted light was approximately 7mm. However the present technique was able to reliably transmit the light more than ten times as far. It was possible to transmit through fibres up to 200mm long, but the light was very weak and therefore the tests described in this section were carried out over a 75mm length.
Figure 6.2  Observation of glass fibre strands on the polished end of the specimen.

(a), (c) under reflected light    (b), (d) under transmitted light
The characteristics of strand fracture were determined in similar hybrids to those used in section 5.6. Each specimen was tested at three different tensile strains of approximately 1%, 2% and 3%, which were estimated from cross-head movement and calculated neglecting the residual strain. However since the extensometer was not used to detect the strain, the slip at the grip was included and the actual strain might be smaller than the indicated value. Photographs of the transmitted light are shown in Figures 6.3 (a)-(d). Higher magnification is in Figures 6.4 (a)-(d).

As shown in these Figures, the intensity of the transmitted light through filaments became weaker with increase in strain. No transmitted light could be seen at about 3% strain. This is because when a failure of the glass fibre occurs somewhere in the composite, the transmitted light is dispersed at the failure point and cannot go through the filament. Such an alteration of the transmitted light could not be observed uniformly in every strand and the failure of specific filaments within a strand was suspected.

The method is thought to be useful to obtain qualitative information on the glass failure, tensile behaviour of the composite and strain distribution in the strand, related to the transmitted light. Quantitative information could be obtained from improvements in the technique.
Figure 6.3  Observation of the transmitted light with strain.

(a) under reflected light  (b) transmitted light before testing
(c) about 1% strain    (d) about 2% strain
Figure 6.4  Observation of the transmitted light with strain.
(a) under reflected light
(b) transmitted light before testing
(c) about 1% strain
(d) about 2% strain
6.4. Improved light transmission test method

As a result of preliminary tests, the principle of the test was confirmed and the condition of the transmitted light could be related qualitatively to the failure of glass fibres in a composite. In this section, the experimental technique has been improved to obtain accurate information about the transmitted light.

6.4.1. Manufacture of specimens

The same mix proportions of matrix as shown on Table 2.1 in Chapter 2, were applied. The hybrid cement composites were reinforced with fibrillated polypropylene networks of 4% by volume and continuous glass fibres of 0.6% by volume. In order to arrange the glass fibre at the right position, the mould as shown in Figure 6.5 was used.

![Figure 6.5 The mould for light transmission tests.](image)

Two longer sides have slots to put the glass fibres in and the depth of the mould is 6mm which forms the thickness of the specimen. Figure 6.6 shows the arrangement of polypropylene networks and a layer of glass fibres. The glass fibre strands were fixed in the
composite about 10mm wide at the position of 2-3mm from the bottom. After the cement slurry was penetrated into fibres, the weights of about 150g were attached to the layers of glass fibres in order to keep them as straight as possible, as shown in Figure 6.7. After the composite was left in the air with the weights for 18-20 hours, it was taken out of the mould and cured under water for 28 days. All specimens were cut into strips about 85mm long by 15mm wide for light transmission tests.

![Diagram of fibre arrangement for the light transmission test.](image)

**Figure 6.6** Fibre arrangement for the light transmission test.

![Weights attached on the layers of glass fibres.](image)

**Figure 6.7** The weights attached on the layers of glass fibres.
6.4.2. Preparation and apparatus for the light transmission test

The illuminated surface and the observed surface were required to be flat and parallel to obtain uniform transmission of light. Epoxy resin was used as casting material and both surfaces were polished with a flat plate machine (Figure 6.8). The procedure and conditions are presented in Table 6.1.

![Flat plate machine](image)

**Figure 6.8** Flat plate machine.
Table 6.1 The procedure and condition of grinding and polishing specimen for light transmission test.

<table>
<thead>
<tr>
<th>Grit level</th>
<th>Pressure level</th>
<th>Rotation speed</th>
<th>Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grinding with water</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td># 500</td>
<td>2</td>
<td>150rpm</td>
<td>Till flat</td>
</tr>
<tr>
<td># 1200</td>
<td>2</td>
<td>150rpm</td>
<td>30-60 sec</td>
</tr>
<tr>
<td># 2400</td>
<td>2</td>
<td>150rpm</td>
<td>30-60 sec</td>
</tr>
<tr>
<td>Cleaning</td>
<td>After grinding, remove contamination with an ultrasonic cleaner. Then, clean with methanol and inhibisol.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Polishing with lubricant</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6μm DUR</td>
<td>2</td>
<td>150rpm</td>
<td>1-2 min.</td>
</tr>
<tr>
<td>Cleaning</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1μm DUR</td>
<td>2</td>
<td>150rpm</td>
<td>1-2 min.</td>
</tr>
<tr>
<td>Ultrasonic cleaning in methanol</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

In the preliminary test, the cold-light source KL1500 with the normal light neck with circular light area of about 5mm in diameter was used. It was difficult to supply the same intensity of the light at the same position. Then the light neck was modified to have rectangular light area of 15mm wide by 4mm thick to fit with the illuminated surface as shown in Figure 6.9. Also the specimen holder was specially made as shown in Figure 6.10. The specimen was inserted in the slot of the holder and the light neck was connected to the holder. The specimen was held vertically and touching the end of the light neck. Thus the same intensity of the light at the same position could be always obtained. The microscope was set above the specimen to observe the transmitted light and take photographs as shown in Figure 6.11. The microscope could be moved vertically and horizontally with precision to observe all the glass fibres on the cross section.
Figure 6.9  The modified light neck.

Figure 6.10  The specimen holder for the transmission of the light.
Figure 6.11  Setting view for the light transmission test.
6.4.3. Tensile tests

Uniaxial tensile tests have been carried out. The clip-on type extensometer with linear variable differential transformers (LVDT) was modified to be about 80mm long as shown in Figure 6.12. All the data of load and strain were collected by an ADU system and recorded on two X-Y recorders with different scales. The cross-head speed was constant at 2mm/min.

![Figure 6.12 The clip-on type extensometer with linear variable differential transformers.](image)

Before regular testing, it was necessary to make sure whether stress-strain curves from such a small specimen were the same as from a specimen 300mm long. A typical stress-strain curve for a small specimen about 80mm long by 15mm wide with a 40mm gauge length, is compared in Figure 6.13 with that for a normal specimen about 300mm long by 25mm wide with 100mm gauge length. The elastic region, the multiple cracking region and the post-cracking region were observed in the stress-strain curve for a small specimen, which were similar to those for a normal specimen. Thus, it was confirmed that the same behaviour could be obtained from the tensile test for small specimens.
After the initial condition of the transmitted light was observed, tensile tests were carried out. The specimen was unloaded for observation at various strains. The unloading point was decided from the stress-strain curve on the X-Y recorder, for instance, the points were supposed to be in the multiple cracking region, second elastic region and before the first yield point. The transmitted light was observed and compared with the initial condition each time. Then the specimen was re-loaded in tension. These procedures were repeated until the light disappeared.
6.5. Results and discussion

6.5.1. Stress-strain curves and transmitted light

A representative cyclic stress-strain curve for a hybrid composite reinforced with polypropylene networks and continuous glass fibres is shown in Figure 6.14. The specimen was unloaded at three different points, A, B and C in the Figure, and the transmitted light was observed with the apparatus shown in Figure 6.11. The strain at the point A is 0.9% supposed to be just after the multiple cracking region, at which many cracks have been generated in the composite. The point B at 1.3% strain is in the second elastic region where the continuous glass fibres and the polypropylene networks are stretched linearly. The point C at 1.8% strain is supposed to be just before the first yield point. After unloading at the point C, the stress-strain curve develops into the region dominated by the ductility of the polypropylene networks, where large increase in strain with little increase in stress can be seen.

![Stress-strain curve diagram](image)

**Figure 6.14** The cyclic stress-strain curve for a hybrid composite reinforced with polypropylene networks and continuous glass fibres.
At each point, the transmitted light through all the glass fibres in the composite was observed and compared with the initial condition in Figures 6.15 to 6.20. As shown in these initial conditions, the intensity of the light is not uniform due to over-exposure and under-exposure so that all the photographs of the transmitted light could not be taken in the best conditions. The alkali-resistant glass fibre contains several kinds of oxides such as $\text{Al}_2\text{O}_3$, $\text{ZrO}_2$, $\text{CaO}$ and $\text{Na}_2\text{O}$ apart from $\text{SiO}_2$ shown on Table 2.2 in Chapter 2. The transmission of the light is reduced by these oxides except for $\text{SiO}_2^{(6-15)}$. Before tensile loading, some glass filaments cannot transmit the light possibly because of an original flaw in the filaments or due to damage during polishing treatment. Therefore, the transmitted light should be always compared with the initial condition.

In Figures 6.15(A) to 6.20(A), the intensity of the light at the point A (figure 6.14) shows little difference from the initial condition. Thus, in the multiple cracking region, the matrix cracks successively without causing much failure of the glass fibres. At the point B, the intensity of the light becomes slightly weak as a whole. A part of the transmitted light disappear in some glass filaments as shown in Figure 6.17(B). After the multiple cracking region, the glass fibres and the polypropylene networks are stretched so that the stress and the strain of the hybrid composite increase and also the failure of glass filaments is initiated. At the point C just before the first yield point, a considerable amount of light has disappeared, which indicates that a lot of glass filaments are broken somewhere in the composite.

An enlarged view of the variations of the transmitted light in the glass strand is presented in Figure 6.21. Figure 6.22 shows another comparison between the initial condition and the point C. In this light transmission technique, it is difficult to observe the failure of the glass filaments in a strand continuously with increase in strain. However, from Figures 6.21 and 6.22, it seems that the failure is initiated at the edge of the strand and extended to the inside. Therefore, the characteristics of the glass filaments in contact with the cement matrix is important to the prediction of the tensile behaviour of glass/polypropylene hybrid fibre composites.
Figure 6.15 Variations of the transmitted light with increase in strain of the hybrid composite. (Observation field 1)

(A) at the point A in Figure 6.14    (B) at the point B in Figure 6.14    (C) at the point C in Figure 6.14
Figure 6.16 Variations of the transmitted light with increase in strain of the hybrid composite. (Observation field 2)

(A) at the point A in Figure 6.14  
(B) at the point B in Figure 6.14  
(C) at the point C in Figure 6.14
Figure 6.17 Variations of the transmitted light with increase in strain of the hybrid composite. (Observation field 3)
(A) at the point A in Figure 6.14  (B) at the point B in Figure 6.14  (C) at the point C in Figure 6.14
Figure 6.18 Variations of the transmitted light with increase in strain of the hybrid composite. (Observation field 4)

(A) at the point A in Figure 6.14    (B) at the point B in Figure 6.14    (C) at the point C in Figure 6.14
Figure 6.19 Variations of the transmitted light with increase in strain of the hybrid composite. (Observation field 5)
(A) at the point A in Figure 6.14  (B) at the point B in Figure 6.14  (C) at the point C in Figure 6.14
Figure 6.20 Variations of the transmitted light with increase in strain of the hybrid composite. (Observation field 6)
(A) at the point A in Figure 6.14  (B) at the point B in Figure 6.14  (C) at the point C in Figure 6.14
Figure 6.21  Typical variations of the transmitted light in the glass strand.

(A) at the point A in Figure 6.14  (B) at the point B in Figure 6.14  (C) at the point C in Figure 6.14
Figure 6.22 The comparison of the transmitted light between the initial condition and the unloaded condition at the point C.
6.5.2. Quantitative analysis of transmitted light

The variation of the transmitted light with increase in strain has been examined qualitatively in section 6.5.1. In order to predict the first yield point of the hybrid composite accurately, the failure of the glass fibres during increasing tensile stress should be understood quantitatively. Thus, the area of the transmitted light in photographs shown in Figures 6.15 to 6.20 was measured as the amount of intact glass filaments by an image analyser, Quantimet 920. These black-and-white photographs have good contrast so that the condition of the transmitted light can be taken directly from the photographs by a video camera shown in Figure 6.23 to be analysed on the screen of the Quantimet. The resolution level of the analyser was constant. The effects of over-exposure, under-exposure and focusing in the photographs were ignored and the area of the transmitted light was measured. Results of the analysis for Figures 6.15 to 6.20 are presented in Table 6.2.

Figure 6.23  The video camera connected to the image analyser, Quantimet 920.
Table 6.2 Representative results of the quantitative analysis for transmitted light in the hybrid composite reinforced with polypropylene networks and continuous glass fibres.

<table>
<thead>
<tr>
<th></th>
<th>Initial</th>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Strain (%)</td>
<td>0</td>
<td>0.90</td>
<td>1.32</td>
<td>1.79</td>
</tr>
<tr>
<td>Area of light ($\times 10^3 \mu m^2$)</td>
<td>562</td>
<td>587</td>
<td>449</td>
<td>114</td>
</tr>
<tr>
<td>Number of glass filaments</td>
<td>3652</td>
<td>3815</td>
<td>2919</td>
<td>739</td>
</tr>
<tr>
<td>Ratio of intact filaments (%)</td>
<td>100</td>
<td>100</td>
<td>79.9</td>
<td>20.2</td>
</tr>
<tr>
<td>Ratio of broken filaments (%)</td>
<td>0</td>
<td>0</td>
<td>20.1</td>
<td>79.8</td>
</tr>
</tbody>
</table>

A: just after multiple cracking region  
B: in the second elastic region  
C: just before the first yield point

The number of glass filaments was obtained from the measured area of light divided by the cross sectional area of the glass filament (diameter: 14μm). Each specimen contains 36 glass fibre strands i.e. 3600 filaments in theory. In the initial condition, the over-exposure makes the area of the light larger than theoretical area. On the other hand, flaws in the glass fibres, the damage during manufacturing and polishing and the under-exposure make the area of the light smaller. In Table 6.2, the calculated initial number of glass filaments was close to the theoretical value, however other calculated numbers were often smaller than theoretical value in this study (refer to APPENDIX 2). The ratio of intact filaments was calculated from the residual number of glass filaments divided by the initial number.

In spite of the fact that about 80% of continuous glass fibres were broken somewhere in the composite just before the first yield point, the composite still kept its rigidity. If 80% of glass fibres are locally broken at one crack face, the effective intact glass fibre volume is 20% of its actual volume, i.e. $0.6\text{vol}\% \times 0.2 = 0.12\text{vol}\%$, and therefore the hybrid composite should have failed. Thus, the broken glass filaments are probably distributed over several crack faces. The hybrid composite described in Table 6.2 had 23 cracks in the span of 40mm. Provided the failure of glass fibres is distributed uniformly over all the cracks, $79.8/23 = 3.5\%$ of glass fibres
are broken at each crack face, which means that the ratio of effective glass fibre is 96.5% of the actual fibre volume just before the first yield point. However, such a uniform distribution of glass failure is unlikely to occur and therefore the maximum ratio of broken glass filaments at one crack face is somewhere between 3.5% and 79.8%, which dominates the first yield point in the tensile behaviour of hybrid composites. The same analysis was carried out for single fibre composites reinforced with continuous glass fibres in order to obtain the maximum ratio of broken glass filaments at one crack face related to the ultimate point.

6.5.3. Comparison between hybrid and single composites

The representative cyclic stress-strain curve of single composites reinforced with 1% continuous glass fibres by volume is shown in Figure 6.24. At the point D and E, the specimen was unloaded and the transmitted light was observed under a microscope. The point F is the ultimate point in the cyclic tensile test. The strain at the point D is 0.26%, which is just after the multiple cracking region. The point E indicates 0.77% strain, which is in the second elastic region before the ultimate point.

Figure 6.24 The representative cyclic stress-strain curve for a composite reinforced with continuous glass fibres alone. ($V_f=1.0\%$)
The representative transmitted light at the point D and E compared with the initial conditions are shown in Figures 6.25 to 6.28. There are few differences between the initial condition and the point D. However at the point E, the intensity of the light has become slightly weaker throughout all the Figures.

Other specimens were unloaded at 1% and 1.5% strain before the ultimate point and observed transmitted light was analysed quantitatively as described in section 6.5.2. All the ratios of broken glass filaments at the point E were approximately 30% in Table 6.3. The ratios of broken glass filaments are plotted against strain of glass/polypropylene hybrid composites and continuous glass fibre reinforced composites in Figure 6.29.

Table 6.3 Results of quantitative analysis for transmitted light in composites reinforced with continuous glass fibres alone.

<table>
<thead>
<tr>
<th>Strain (%)</th>
<th>Initial</th>
<th>D</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of glass filaments</td>
<td>No.1</td>
<td>3866</td>
<td>3065</td>
</tr>
<tr>
<td></td>
<td>No.2</td>
<td>3302</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>No.3</td>
<td>2202</td>
<td>-</td>
</tr>
<tr>
<td>Ratio of intact filaments (%)</td>
<td>100</td>
<td>79.3</td>
<td>66.5</td>
</tr>
<tr>
<td>Ratio of broken filaments (%)</td>
<td>0</td>
<td>20.7</td>
<td>33.5</td>
</tr>
<tr>
<td>Number of cracks</td>
<td>8</td>
<td>9</td>
<td>9</td>
</tr>
</tbody>
</table>

D: just after multiple cracking region
E: in the second elastic region before the ultimate point
Figure 6.25  Variations of the transmitted light with increase in strain of the glass reinforced composite.  (Observation field 1)
(D) at the point D in Figure 6.24     (E) at the point E in Figure 6.24
Figure 6.26  Variations of the transmitted light with increase in strain of the glass reinforced composite.  (Observation field 2) (D) at the point D in Figure 6.24  (E) at the point E in Figure 6.24
Figure 6.27 Variations of the transmitted light with increase in strain of the glass reinforced composite. (Observation field 3)
(D) at the point D in Figure 6.24  (E) at the point E in Figure 6.24
Figure 6.28 Variations of the transmitted light with increase in strain of the glass reinforced composite. (Observation field 4)

(D) at the point D in Figure 6.24     (E) at the point E in Figure 6.24
Figure 6.29 The ratios of broken glass filaments with increase in tensile strain for glass/polypropylene fibre hybrid composites and glass fibre single reinforced composites.

In glass fibre single reinforced composites, the glass failure begins at strain lower than 0.26%. The ratio of broken glass filaments is almost constant at about 30% from 0.7% strain to 1.5% strain before the ultimate point. The glass fibre reinforced composites described in Table 6.3 had 8 or 9 cracks in the 40mm span. Provided that the failure of the glass fibres is distributed uniformly over all cracks, the ratio of broken filaments (%) divided by the number of cracks at 1.52% strain, i.e. 33.7/9 = 3.7% of glass fibres are broken at each crack face. Thus the maximum ratio of broken glass filaments at one crack is somewhere between 3.7% and 33.7%. The theoretical ultimate tensile stress of the continuous glass fibre reinforced composite can be calculated as $\sigma_{fgw}V_{fg} = 1500\text{MPa} \times 0.01 = 15\text{MPa}$, where $\sigma_{fgw}$ is the strength of the glass fibre and $V_{fg}$ is the glass fibre volume fraction. Experimental ultimate stresses and ratios of experiment/theory are shown in Table 6.4.
Table 6.4 The ultimate tensile stress and ratios of experiment/theory for continuous glass fibre reinforced composites.

<table>
<thead>
<tr>
<th>No. (P. 282)</th>
<th>Experimental ultimate stress (MPa)</th>
<th>Experiment / Theory</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>8.83</td>
<td>0.59</td>
</tr>
<tr>
<td>2</td>
<td>11.93</td>
<td>0.80</td>
</tr>
<tr>
<td>3</td>
<td>10.67</td>
<td>0.71</td>
</tr>
<tr>
<td>average</td>
<td>10.48</td>
<td>0.70</td>
</tr>
</tbody>
</table>

The average of experimental values is 70% of the theoretical value, and this ratio is close to the ratio of intact filaments in Table 6.3. Therefore, the effective glass fibre volume is assumed to be 70% of the actual fibre volume fraction at the ultimate stress, which implies that the maximum ratio of broken glass filaments at one crack face before final fracture is approximately 30%.

Returning to the discussion about the glass/polypropylene hybrid cement composites, the failure of about 30% of glass fibres at one crack face out of 23 cracks would dominate the stress at the first yield point. In hybrid composites, the stress is transferred uniformly due to the combination with polypropylene networks so that the failure of glass fibres is likely to be more distributed than that in the glass fibre single reinforced composite.

As described above, with respect to the glass fibre strand consist of 100 filaments used in this study, it was found that the effective fibre volume was about 70% of the actual fibre volume by the light transmission test. This effective ratio should be taken into account in any mathematical model of tensile behaviour and should enable the prediction of the stress at the first yield point precisely.
CHAPTER 7

7. MATHEMATICAL MODEL FOR TENSILE BEHAVIOUR OF GLASS/POLYPROPYLENE FIBRE HYBRID CEMENT COMPOSITES

7.1. Introduction

In the previous chapters, the results of the experiments such as tensile tests, microstructural observations and light transmission tests have been discussed. The performance of hybrid fibre reinforced composites depends on the following conditions:

- Properties of the matrix and the fibres
- Condition of the matrix: uncracked or cracked
- Type of fibre
- Volume fraction of fibres
- Geometry and surface characteristics of fibres
- Combination of the fibres

Owing to the interaction between these parameters, it is difficult to predict the mechanical behaviour of hybrid fibre reinforced composites for a given end use. The available theoretical models can deal with these parameters only to a limited extent and much progress is still needed. Most designs are based on experimental results and empirical relations.

Various models and analytical treatments have been proposed to account for the overall shape of the tensile stress-strain curve and to predict the salient points of the curve such as modulus of elasticity in the elastic region, first crack stress and strain, the strain at the end of the multiple cracking region, the ultimate stress and strain and so on. The conventional concepts include the composite materials models, fracture mechanics models and multiple cracking models. However, none of them can describe the entire response for tensile load. For example, the composite materials models may best be used to describe the elastic region and the strength of the composite, while fracture
mechanics models may be used to predict the first crack strain. At present, only a combination of these models allows the quantification of most of the characteristics of tensile behaviour.

In this Chapter, continuous glass fibres and polypropylene networks were used as reinforcing fibres. However, the principles should apply to other fibre combinations. In order to predict the tensile behaviour of the hybrid fibre reinforced composites, mathematical models have been developed to satisfy the force and strain balance in the composite\(^{(7-1)}\). Predictions have also been made of the most effective fibre combinations.

7.2. Notation

A considerable number of symbols are used in this chapter. Although the symbols are explained as they appear in the text or figures, the symbols are listed below to assist the reader.

\begin{itemize}
  \item \(A_c\): cross-sectional area of composite
  \item \(A_f\): cross-sectional area of fibre
  \item \(A_{fg}\): cross-sectional area of the glass fibre
  \item \(A_{fp}\): cross-sectional area of the polypropylene fibre
  \item \(A_m\): cross-sectional area of matrix
  \item \(\text{CTOD}\): crack tip opening displacement
  \item \(\text{CTOD}_c\): critical crack tip opening displacement
  \item \(c\): one-half of the crack length
  \item \(d\): diameter of the fibre
  \item \(E_2\): second elastic modulus of the composite in the post cracking region
  \item \(E_3\): third elastic modulus of the hybrid composite
  \item \(E_c\): modulus of elasticity of the composite
  \item \(E_f\): modulus of elasticity of the fibre
\end{itemize}
$E_{fg}$ elastic modulus of the glass fibre  
$E_{fp}$ elastic modulus of the polypropylene fibre  
$E_m$ modulus of elasticity of the matrix  
$(FYP)V_{fp(crit)}$ critical fibre volume of the polypropylene at FYP  
$G_c$ critical strain energy release rate  
$G_{db}$ fracture energy required to debond a unit surface area of fibre  
$K_c$ critical stress intensity factor  
$K'_{lc}$ modified critical stress intensity factor based on LEFM and the effective crack length  
$P_f$ perimeter of the fibre  
$P_{fg}$ perimeter of the glass fibre strand  
$P_{fp}$ perimeter of the polypropylene fibre  
$R^2$ coefficient of determination  
$r$ radius of the fibre  
$S$ fibre spacing factor  
$V_f$ volume fraction of the fibre  
$V_{f(crit)}$ critical fibre volume fraction  
$V_{fg}$ glass fibre volume fraction  
$V_{fp}$ polypropylene fibre volume fraction  
$V_m$ volume fraction of the matrix  
$x$ stress transfer length  
$x_g$ stress transfer length between glass fibre and matrix in the hybrid composite  
$x_{ig}$ intrinsic stress transfer length of the glass fibre in the single fibre composite  
$x_{ip}$ intrinsic stress transfer length of the polypropylene fibre in the single fibre composite  
$x_p$ stress transfer length between polypropylene fibre and matrix in the hybrid composite  
$\alpha$ $\frac{E_m V_m}{E_f V_f}$
\( \gamma_m \) surface energy of the matrix

\( \gamma_s \) surface energy of the material

\( \varepsilon_c \) strain in the composite

\( \varepsilon_f \) strain in the fibre

\( \varepsilon_{fgu} \) ultimate strain of the glass fibre

\( \varepsilon_{fp(FYP)} \) strain in the polypropylene fibre at the first yield point (FYP)

\( \varepsilon_{fp} \) ultimate strain of the polypropylene fibre

\( \varepsilon_{fu} \) ultimate strain of the fibre

\( \varepsilon_{g0} \) strain in glass fibre at a crack face during multiple cracking

\( \varepsilon_{g1} \) strain in glass fibre at the distance of \( x_p \) from the crack face

\( \varepsilon_{g2} \) strain in glass fibre at the distance of \( x_g/2 \) from the crack face

\( \varepsilon_m \) strain in the matrix

\( \varepsilon_{mc} \) strain at the end of the multiple cracking region

\( \varepsilon_{mu} \) ultimate matrix strain or first crack strain of the composite

\( \varepsilon_{p0} \) strain in polypropylene fibre at a crack face

\( \varepsilon_{p1} \) strain in polypropylene fibre at the distance of \( x_p \) from the crack face

\( \varepsilon_u \) ultimate tensile strain of the composite

\( \varepsilon_{u1} \) strain in the composite at FYP

\( \sigma_c \) stress in the composite

\( \sigma_{cc} \) stress at first crack in the composite

\( \sigma_f \) stress in the fibre

\( \sigma'_f \) additional stress in the fibre due to cracking of the matrix

\( \sigma_{fgu} \) ultimate strength of the glass fibre

\( \sigma_{fp(FYP)} \) stress in the polypropylene fibre at FYP

\( \sigma_{fp} \) ultimate strength of the polypropylene fibre

\( \sigma_{fu} \) ultimate strength of fibre

\( \sigma_m \) stress in the matrix

\( \sigma_{mu} \) matrix cracking strength

\( \sigma_u \) ultimate tensile strength of the composite
7.3. Background of theoretical approach

7.3.1. Composite materials models

In the elastic region, the primary variables are the moduli of elasticity of the matrix and fibres. The rule of mixtures can be used to obtain the composite response. Experimentally, the slope of the stress-strain curve up to the first cracking point can be used as the composite stiffness. When continuous parallel fibres are assumed, the strains in the matrix \( \varepsilon_m \), the fibres \( \varepsilon_f \) and the composite \( \varepsilon_c \) are equal so that there is no slip. Furthermore, the load being carried by the composite is the sum of the loads being carried by the fibre and the matrix. These equations are:

\[
\varepsilon_c = \varepsilon_m = \varepsilon_f \tag{7-1}
\]

and

\[
\sigma_c A_c = \sigma_f A_f + \sigma_m A_m \tag{7-2}
\]

where \( \sigma_c, \sigma_f \) and \( \sigma_m \) are stresses in composite, fibre and matrix, respectively. \( A_c, A_f \) and \( A_m \) stand for cross-sectional areas of composite, fibre and matrix, respectively. Then the stress and the modulus of the composite, \( \sigma_c \) and \( E_c \), can be expressed as:

\[
\sigma_c = \sigma_f V_f + \sigma_m V_m \tag{7-3}
\]

\[
E_c = E_f V_f + E_m V_m \tag{7-4}
\]

where \( E_f \) and \( E_m \) are the moduli of elasticity of the fibre and the matrix and \( V_f \) and \( V_m \) are the volume fractions of each phase. Equations (7-3) and (7-4) are valid only if the fibres are continuous and aligned in the loading direction and if there is perfect bond
between the matrix and the fibres. As the volume fraction of fibre \( V_f \) increases from 0 to 1, the composite modulus will change from \( E_m \) to \( E_f \) in proportion to \( V_f \).

Once the composite modulus \( E_c \) is known, the stress-strain relation up to the first cracking can be written as:

\[
\sigma_c = \varepsilon_c E_c = \varepsilon_m E_c = \varepsilon_f E_c \tag{7-5}
\]

Since it is assumed that there is no slip between the matrix and the fibre, the strain values of the composite \( \varepsilon_c \), matrix \( \varepsilon_m \) and fibre \( \varepsilon_f \) are all the same.

The tensile behaviour of fibre reinforced cement composites significantly depends on the fibre volume fraction. The response of a composite up to the first cracking can be described using equation (7-5). Beyond this point, assuming that the matrix does not contribute any further to the ultimate strength, the strength of the composite \( \sigma_u \) can be expressed as a function of the strength and volume fraction of the fibres, \( \sigma_{fu} \) and \( V_f \), respectively.

\[
\sigma_u = \sigma_{fu} V_f \tag{7-6}
\]

Equation (7-6) is valid for fibre volumes which exceed a critical value, \( V_f (\text{crit}) \). Below this value, the strength of the composite \( \sigma_u \) in equation (7-6) is smaller than the first crack stress \( \sigma_{cc} \) calculated by equation (7-3). Kelly \(^{(7-2)}\) has applied these equations to describe the strength of the composite as a function of fibre volume assuming continuous and aligned fibres as shown in Figure 7.1. Equation (7-3) applies for the case of \( V_f < V_f (\text{crit}) \), and equation (7-6) for the case of \( V_f > V_f (\text{crit}) \). The intersection of the two equations is at the critical fibre volume fraction, which can readily be calculated \(^{(7-3)}\) as follows.

\[
V_f (\text{crit}) = \frac{\sigma_{mu}}{\sigma_{fu} - \varepsilon_{mu} E_f + \sigma_{mu}} \tag{7-7}
\]

where \( \sigma_{mu} \) is the matrix cracking strength and \( \varepsilon_{mu} \) is the ultimate matrix strain.
Figure 7.1  Relations between composite strength $\sigma$ and fibre volume fraction $V_f$ as predicted by equation (7-3) and (7-6), for continuous and aligned fibres. Provided $E_m < E_f$. (after Kelly\textsuperscript{(7-2)})

The fibres will only contribute effectively to strength when $V_f > V_{f(crit)}$. In this case, after first cracking, the load carried by the matrix is transferred to the fibres without failure. Additional loading leads to more matrix cracking, which is still not accompanied by failure of the composite. On the other hand, when $V_f < V_{f(crit)}$, the catastrophic failure of the composite will occur because the fibre volume is insufficient to support the matrix cracking load.

7.3.2. Fracture mechanics models

Griffith\textsuperscript{(7-4)} was the first to propose the fracture mechanics model from the point of the rapid growth of pre-existing flaws or cracks in materials. It was postulated that regardless of the processing techniques, materials contain inherent flaws. The stress concentration at the tip of these flaws results in much greater stresses than those away from the crack tip. The formation of new surfaces caused by fracture requires the
consumption of energy. Furthermore, as the crack forms, the material in the vicinity of the crack tip undergoes elastic recovery, resulting in a decrease in the strain energy of the system. A crack will propagate when the decrease in the elastic strain energy is at least equal to the energy required to create new surfaces associated with the crack. Using this concept, the theoretical fracture strength for brittle, linearly elastic materials is:

$$\sigma = \left( \frac{2E\gamma_s}{\pi c} \right)^{1/2}$$  \hspace{1cm} (7-8)

where $c$ : one-half of the crack length

$\gamma_s$ : surface energy of the material

This is the basic equation of linear elastic fracture mechanics (LEFM). In LEFM, the critical strain energy release rate is defined as $G_c = 2\gamma_s$, then the criterion for catastrophic crack growth is expressed as:

$$\sigma (\pi c)^{1/2} = (EG_c)^{1/2}$$  \hspace{1cm} (7-9)

That is, fracture will occur when the crack reaches a critical size or when the stress reaches a critical value. Alternatively, when a parameter of critical stress intensity factor $K_c = \sigma \sqrt{\pi c}$ is used:

$$K_c^2 = E G_c$$  \hspace{1cm} (7-10)

However, since the LEFM parameters ($G_c$ or $K_c$) are most often insensitive to fibre content, it cannot be applied directly to composite systems such as fibre reinforced concrete. Thus LEFM was modified to explain particular phenomena such as crack arrest, stability of crack growth, crack tip opening displacement, fibre pull-out, the crack propagation resistance and so on.

The mechanics of crack arrest in concrete reinforced with small diameter steel wires was studied by Romualdi and Baston.\cite{7-5, 7-6} When cracks exist, the matrix tends to extend more than the fibres, because the stress is concentrated just ahead of the crack tip. The fibres play a roll of arresting the crack propagation. The shear bond stress distribution which causes the crack arrest is shown in Figure 7.2.
Figure 7.2 Interfacial shear stress distribution predicted by Romualdi and Baston\textsuperscript{(7-5)} in the arrest of crack propagation in a matrix between the fibres.

As a result, the stress intensity factor of the crack is effectively reduced. Romualdi and Baston introduced the concept of the fibre spacing factor, $S$, and the matrix cracking stress was found to be inversely proportional to $S$. However, there are several limitations to the application of fibre spacing factor equations for the prediction of first crack strains. In their analysis, the fibre-matrix interface is assumed to be bonded perfectly during the crack propagation process, which is simply not realistic. Swamy et al.\textsuperscript{(7-7)} have thus proposed the concept of effective fibre spacing, which takes into account modifications due to both geometrical and bond considerations.

An alternative approach to the prediction of the first crack strain was proposed by Aveston, Cooper and Kelly\textsuperscript{(7-8)}. They showed the following equation for computing the first crack strain of the matrix $\varepsilon_{mu}$. 

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where $\tau_{fu}$ is the frictional bond strength, $\gamma_m$ is the surface energy of the matrix and $r$ is the radius of the fibre. This equation was obtained by considering the changes in energy that occur when a crack is formed across a tensile specimen. It was assumed that a crack will form when the sum of the work done by the applied stresses and the elastic energy released by the matrix exceed the sum of the work of fracture of the matrix, the energy absorbed due to friction at the fibre-matrix interface, and the increase in elastic energy in fibres. Equation (7-11) indicates an effective increase in the matrix cracking strain for a high fibre volume fraction, a high interfacial frictional bond strength, and a small fibre diameter. Basically, this is another way of predicting the effect of crack arrest in accordance with the fibre spacing factor concepts proposed by Romualdi et al.

Aveston and Kelly\textsuperscript{(7-9)} have further extended the model of Aveston et al.\textsuperscript{(7-8)} to include analysis of continuous fibres which are elastically bonded to the matrix. The elastic solution of this problem is similar to solutions obtained by Cox\textsuperscript{(7-10)} and Piggot\textsuperscript{(7-11)}. However, the strains predicted by the much simpler model assuming frictional bond is not significantly different from that predicted by the modified model assuming elastic bond.

A fictitious crack model (FCM) has been proposed by Hillerborg\textsuperscript{(7-12), (713)} for fracture analysis of fibre reinforced concretes. The schematic description of FCM used to represent the crack tip behaviour in FRC is shown in Figure 7.3. The fracture zone ahead of the real crack is assumed to act as a fictitious crack which has the ability to transfer stresses. Instead of conventional fracture parameters, the tensile stress-displacement curve of the composite is introduced to describe the behaviour of the fracture zone.
Wecharatana and Shah\textsuperscript{(7-14)}, have modelled the process zone ahead of the real crack in a similar fashion, with the closing pressure function determined experimentally. They identified three distinct zones as shown in Figure 7.4.

(1) Traction free zone.
(2) Fibre bridging zone, in which stress is transferred by frictional slip of the fibres.
(3) Matrix process zone, in which there is enough continuity and aggregate interlock to transfer some stress in the matrix itself.

An interactive procedure is set up to determine the size of the process zone so that the predicted crack tip opening displacement (CTOD) at a particular load matches the experimentally observed CTOD at the load level.
Figure 7.4  Idealized description of an advancing crack and the stress field around it, in FRC. (after Wecharatana and Shah\cite{7-14})

Jenq and Shah\cite{7-15,7-16} have proposed a fracture mechanics based model to predict the crack propagation resistance of fibre reinforced cement (FRC). Fracture resistance in FRC is separated into four aspects which include:

1. Subcritical crack growth in the matrix.
2. Beginning of fibre bridging effect.
3. Post-critical crack growth in the matrix such that the stress intensity factor due to the applied load and the fibre bridging closing stresses remain constant (steady-state crack growth).
Final stage where the resistance to crack separation is provided exclusively by the fibres.

The model uses two parameters that describe the matrix fracture properties (\(K'_t\), modified critical stress intensity factor based on LEFM and the effective crack length, and CTOD\(_e\), the critical crack tip opening displacement), and a fibre pull-out stress versus slip relationship as basic input information. It has been successfully used to predict the complete load displacement behaviour of steel fibre reinforced concrete beams of several different sizes. The specification for obtaining these two parameters are proposed as a standard by the RILEM Committee on Fracture Mechanics Testing\(^{(7-17)}\).

Fracture mechanics models have also been applied to the phenomena of fibre debonding and pull-out as an alternative to the analysis\(^{(7-18),(7-19)}\) based on elastic and frictional shear stresses. In this concept, it is assumed that the debonded zone is stress free and treated as an interfacial crack. Using LEFM, the conditions leading to the propagation of this crack and debonding, can be calculated. Outwater and Murphy\(^{(7-20)}\) calculated the tensile stress in the fibre required for catastrophic debonding as:

\[
\sigma = \left( \frac{8E_fG_{db}}{d} \right)^{\frac{1}{2}}
\]  

(7-12)

where \(G_{db}\) is the energy required to debond a unit surface area of fibre and \(d\) is the diameter of fibre. Morrison et al.\(^{(7-21)}\) further extended the analysis by taking into account the frictional resistance in the debonded zone, which is the more realistic case for FRC composites. The schematic description of the model is shown in Figure 7.5.
7.3.3. Multiple cracking models

Aveston, Cooper and Kelly\textsuperscript{(7-8)} were the first to present the detailed analytical work for multiple cracking for continuous and aligned fibre reinforced composites, assuming frictional shear stress transfer. This model is well known as the ACK model and has already been described in section 5.2. The schematic description of strain distributions in fibres and matrix for the case that crack spacings are $x$ and $2x$, was shown in Figure 5.1. Multiple cracking continues at an approximately constant stress, which is equal to the first crack stress $\sigma_{ec}$, until the composite is eventually fractured into segments of length between $x$ and $2x$. Figure 7.6 shows strain distributions in fibres and matrix for the actual average crack spacing of $1.364x$\textsuperscript{(7-22)}.
Figure 7.6  Schematic description of multiple cracking.

(a) Crack spacing of 1.364x and bridging fibres.
(b) Strain distributions in fibres and matrix.

The additional stress results in an increase in fibre strain $\Delta \varepsilon_f$ at the crack surface.

$$\Delta \varepsilon_f E_f V_f = \varepsilon_{mu} E_m V_m$$  \hspace{1cm} (7-13)

Thus the total strain in fibre at the crack is:

$$\varepsilon_{mu} + \Delta \varepsilon_f = (1 + \alpha)\varepsilon_{mu}$$  \hspace{1cm} (7-14)

where $\alpha = \frac{E_m V_m}{E_f V_f}$

The matrix strain at the centre of the block is $1.364/2 \varepsilon_{mu} = 0.682 \varepsilon_{mu}$ and strain in fibre at the same position is $(1+0.318\alpha)\varepsilon_{mu}$. The increase in the composite strain during
multiple cracking can be calculated from the increase in the average strain in the fibre. Hence, the strain at the end of the multiple cracking region, $\varepsilon_{mc}$ is:

$$
\varepsilon_{mc} = \frac{1}{2}\{(1+\alpha)\varepsilon_{mu} + (1+0.318\alpha)\varepsilon_{mu}\}
$$

$$
= (1 + 0.659\alpha)\varepsilon_{mu}
$$

(B7-15)

Beyond the multiple cracking region, cracking cannot take place any more, and additional loading results in stretching of the fibres until they break. Assuming stretching continuous and aligned fibres, the elastic modulus of cracked composites in the post cracking region defined as second elastic modulus $E_2$, is $E_f V_f$ until the fibre load bearing capacity, $\varepsilon_u = \sigma_{fu} V_f$ is reached. Where, $\sigma_{fu}$ is the ultimate strength of the fibre. In Figure 7.6(b), the strain in the fibre at the crack surface increases until the ultimate strain of the fibre $\varepsilon_{fu}$ is reached. Therefore, the ultimate strain of the composite $\varepsilon_u$ is given by:

$$
\varepsilon_u = \varepsilon_{fu} - (1+\alpha)\varepsilon_{mu} + \varepsilon_{mc}
$$

(B7-16)

Equation (7-15) is substituted for $\varepsilon_{mc}$,

$$
\varepsilon_u = \varepsilon_{fu} - 0.341\alpha\varepsilon_{mu}
$$

(B7-17)

The shape of the resulting stress-strain curve is presented in Figure 7.7.

---

**Figure 7.7** Schematic description of the stress-strain curve, based on the ACK model(7-8).
The combined elastic and frictional fibre-matrix stress transfer model was later proposed by Laws\textsuperscript{(7-23)}, based on Lawrence's\textsuperscript{(7-24)} model. Laws calculated the minimum crack spacing, $x$, as a function of fibre volume fraction for a typical glass fibre reinforced cement using three different assumptions:

1. frictional stress transfer only,
2. elastic stress transfer only,
3. combined elastic and frictional stress transfer.

According to the results of this calculation, the minimum crack spacing calculated based on frictional stress transfer, is about 20\% higher than that based on the combined model. However, it appears that the simple theory adequately describes the behaviour of FRC composites, as long as the elastic shear bond strength is not much greater than the matrix tensile strength.

7.3.4. Modelling of tensile stress-strain curve

As described in the previous sections, various theoretical models have been suggested so far to understand the mechanical behaviour of FRC composites. Fracture mechanics models are extensively used to explain particular phenomena which cannot be understood by another models. However, in practice, it is difficult to define the real crack tip in the composite with complicated cracks. Thus the application of these models are limited for modelling of tensile behaviour of FRC composites. It is important that the user chooses the best suited model and is aware of the assumptions and limitations of these models used\textsuperscript{(7-25)}.

The tensile stress-strain curve is usually modelled using the ACK model\textsuperscript{(7-8),(7-9)}. The salient points on the curve shown in Figure 7.7, (initial modulus of elasticity, first crack stress and strain, strain at the end of the multiple cracking region and the ultimate stress and strain) can be determined, based on the concepts described in the previous sections, as summarized in Table 7.1\textsuperscript{(7-26)}.
Table 7.1 Relations applied to describe the salient points and slopes on the stress-strain curve of FRC composites.

<table>
<thead>
<tr>
<th>Tensile property</th>
<th>Equation</th>
<th>Concept</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial elastic modulus, $E_c$</td>
<td>(7-4)</td>
<td>Composite materials model (rule of mixtures)</td>
</tr>
<tr>
<td>First crack stress, $\sigma_{cc}$</td>
<td>(7-5)</td>
<td>Composite materials model (rule of mixtures)</td>
</tr>
<tr>
<td>First crack strain, $\varepsilon_{mc}$ (matrix crack strain)</td>
<td>(7-11)</td>
<td>Fracture mechanics model (energy balance)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>or treated as a constant</td>
</tr>
<tr>
<td>Strain at the end of multiple cracking, $\varepsilon_{mc}$</td>
<td>(7-15)</td>
<td>Multiple cracking model (ACK model)</td>
</tr>
<tr>
<td>Ultimate composite strain, $\varepsilon_u$</td>
<td>(7-17)</td>
<td>Multiple cracking model (ACK model)</td>
</tr>
<tr>
<td>Ultimate composite stress, $\sigma_u$</td>
<td>(7-6)</td>
<td>Composite materials model</td>
</tr>
</tbody>
</table>

All these concepts are available only for the composite reinforced with single type of fibre. However, when two or more types of fibres are combined to obtain effective reinforcement in a composite, these concepts cannot apply directly to describe the tensile behaviour of such hybrid fibre reinforced composites. Therefore, the ACK model should be modified for hybrid fibre reinforced systems.

7.4. Application of the ACK model for continuous glass fibre reinforced cement composites

Stress-strain curves of composites reinforced with various content of continuous glass fibres have been shown on Figure 4.4 in Chapter 4. These experimental curves will be compared with the ACK model in this section. The characteristic stress-strain curves obtained from experiments and predicted values are plotted against glass fibre volume fractions in the range of 1.5% to 4.2%, which is more than critical fibre volume.
Elastic modulus of the composite, $E_c$ (Figure 7.8)

Most of experimental plots are around the predicted line shown as a broken line, which is almost constant in this range of glass fibre volume fraction. It seems that the rule of mixtures expressed in equation (7-4) is useful in the elastic region.

![Graph showing the correlation between elastic modulus of the composite $E_c$ and glass fibre volume fraction.](image)

**Figure 7.8** Correlation between elastic modulus of the composite $E_c$ and glass fibre volume fraction.

First crack stress of the composite, $\sigma_{fc}$ (Figure 7.9)

Assuming constant matrix cracking strain, $\varepsilon_{mu}=273\times10^{-6}$, the predicted line of first crack stress calculated from equation $\sigma_{fc}=\varepsilon_{mu}E_c$ shows the same trend as that of the elastic modulus of the composite. However, experimental plots are lower than the predicted line and first crack stress apparently increases with glass fibre volume fraction. This positive trend may be caused by the crack suppression effect of the glass fibre.
Second elastic modulus in the post-multiple cracking region, $E_2$ (Figure 7.10)

In the ACK model, assuming stretching of continuous and aligned glass fibres, the modulus in the post-multiple cracking region (Figure 7.7) is expressed as $E_{fg}V_{fg}$, where the subscript of $fg$ stands for the glass fibre. The predicted line is indicated with a broken line, assuming 72GPa for the elastic modulus of the glass fibre. The experimental line was drawn, based on the simple linear regression model (the coefficient of determination $R^2 = 0.973$). The modulus of the glass fibre calculated from the slope was 60GPa which was lower than assumed value. The actual elastic modulus of the glass fibre was calculated at 73.5GPa in Chapter 2. As shown in Figure 7.10, the experimental line slightly diverged from the predicted line with increasing volume fraction. Possible reasons for the differences are that fibres which were not straight initially in the bundle were pulled into line with increased strain or fibres were not fully stressed within the bundle. Therefore, the slight divergence from the predicted line could be caused by several factors.
Ultimate stress, $\sigma_u$ (Figure 7.11)

In theory, the ultimate stress is dominated by the load bearing capacity of the glass fibre and expressed as $\sigma_{fgu}V_{fg}$, where $\sigma_{fgu}$ is the ultimate strength of the glass fibre. The predicted line is presented with a broken line, assuming 1500MPa for the ultimate stress of the glass fibre. The experimental regression line was regarded as a simple proportional line ($R^2=0.907$) and drawn below the predicted line as shown in Figure 7.11. The ultimate stress of the glass fibre was calculated to be 856MPa from the slope, which was much lower than the assumed value. In bundled filaments such as the glass fibre strand, the bonding is thought to be not uniform and the external filaments are more tightly bonded to the matrix. The bonding interface between the glass fibre and the matrix was shown on Figure 5.3 in Chapter 5. The fracture of the glass fibre filament occurs at the weakest point in the external filaments first, and the applied stress is then concentrated at the vicinity of the broken filament giving greater stresses in the remaining filaments. Thus the glass fibre strand consequently failed at lower average
stress than the predicted value for an uniformly stressed strand. Also some minor damage during manufacture or alkali attack could have affected the ultimate stress. Therefore, efficiency factors of bundled filaments and other factors may have to be taken into account to predict the actual ultimate stress. For example, according to the light transmission test, the effective ratio of the glass fibre volume was 70% of the actual fibre volume as described in Chapter 6.

![Figure 7.11 Correlation between ultimate stress $\sigma_u$ and glass fibre volume fraction.](image)

**Ultimate strain, $\varepsilon_u$ (Figure 7.12)**

The predicted ultimate strain was calculated from equation (7-17). Experimental plots are lower than the predicted values for the same reasons given for the divergence from the predicted ultimate stress as mentioned above.
Figure 7.12 Correlation between ultimate strain $\varepsilon_u$ and glass fibre volume fraction.

Theoretical stress-strain curves of composites reinforced with continuous glass fibres are presented in Figure 7.13, compared with typical experimental curves of four different glass fibre contents. Representative tensile values are shown in Table 7.2. The following values were used in the calculation for equations (7-4), (7-5), (7-15) and (7-17).

\[
\begin{align*}
E_m &= 31.6 \text{GPa} \\
E_{fg} &= 72 \text{GPa} \\
\varepsilon_{mu} &= 273 \times 10^{-6} \\
\sigma_{fgu} &= 1500 \text{MPa (Manufacturer)} \quad \quad \sigma_{fgu} = 981 \text{MPa (Measured)} \\
\varepsilon_{fgu} &= \frac{\sigma_{fgu}}{E_{fg}} = 2.1\% 
\end{align*}
\]
Figure 7.13 Comparison between theoretical stress-strain curves and typical experimental curves for continuous glass fibre reinforced cement composites.
Table 7.2 Representative tensile properties compared between theoretical values (Theory) and experimental values (Exp.).

<table>
<thead>
<tr>
<th>$V_{fg}$ (%)</th>
<th>1.68</th>
<th>2.54</th>
<th>3.25</th>
<th>4.05</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha = E_m V_m / E_{fg} V_{fg}$</td>
<td>25.7</td>
<td>-</td>
<td>16.8</td>
<td>-</td>
</tr>
<tr>
<td>$E_c$ (GPa)</td>
<td>32.7</td>
<td>30.7</td>
<td>32.6</td>
<td>33.3</td>
</tr>
<tr>
<td>$\sigma_{ct}$ (MPa)</td>
<td>8.81</td>
<td>5.52</td>
<td>8.91</td>
<td>7.00</td>
</tr>
<tr>
<td>$\varepsilon_{mc}$ (%)</td>
<td>0.49</td>
<td>-</td>
<td>0.33</td>
<td>-</td>
</tr>
<tr>
<td>$E_2 = E_{fg} V_{fg}$ (GPa)</td>
<td>1.21</td>
<td>1.22</td>
<td>1.83</td>
<td>1.52</td>
</tr>
<tr>
<td>$\sigma_u = \sigma_{gu} V_{fg}$ (MPa)</td>
<td>25.2</td>
<td>13.9</td>
<td>38.1</td>
<td>19.3</td>
</tr>
<tr>
<td>$\varepsilon_u$ (%)</td>
<td>1.86</td>
<td>1.10</td>
<td>1.94</td>
<td>1.04</td>
</tr>
</tbody>
</table>

As shown in Figure 7.13, these theoretical and experimental lines indicated fairly good agreement except for ultimate stress and strain. This inconsistency is thought to be caused by the fracture mechanism of bundled filaments as described in Chapter 6. It is confirmed that the current ACK model is sufficient to predict the tensile behaviour of the continuous glass fibre reinforced cement composite, only if an efficiency factor of bundled filaments is taken into consideration.

7.5 Mathematical model and parametric study

A typical stress-strain curve for the hybrid composite reinforced with polypropylene networks and continuous glass fibres and those of each single reinforced composite are represented in Figure 7.14
Figure 7.14  Comparison of stress-strain curves between hybrid composite and single composite reinforced with polypropylene networks and continuous glass fibres (7-1).

All four composites exhibited an initially steep slope terminating in a short nearly horizontal portion at first fracture of the matrix with multiple cracking. This is followed by a rising portion controlled by the fibre parameters. The hybrid composite was considerably enhanced in strength compared with the polypropylene composite especially up to 1-2% strain. The first maximum point of the curve was defined as the first yield point (FYP). Even after FYP, an effective enhancement was shown in the tensile behaviour up to 7% strain. The schematic tensile stress-strain curve for the hybrid composite reinforced with polypropylene networks and continuous glass fibre is shown in Figure 7.15.
Figure 7.15 Schematic stress-strain curve of composite reinforced with polypropylene networks and continuous glass fibres\(^{(7-1)}\).

The conventional ACK theory assumes that stress transfer between fibres and matrix is caused by frictional stress, and that the fibres and matrix can move freely relative to each other. This theory provides a reasonable prediction of the behaviour of cement composites reinforced with aligned continuous fibres consisting of single filament. However, when two or more kinds of fibres are used as reinforcement in the composite, the ACK theory cannot satisfactorily describe the tensile behaviour because of the synergistic effect which is produced. In order to create the theory, certain assumptions are required, and this model will be based on assumptions prepared after the ACK theory, as follows.

1. The modulus of the composite follows the simple mixture rule until the matrix cracks. The fibre and the matrix behave elastically.

2. The fibre and the matrix can move relative to each other. The stress transfer
between fibres and matrix is purely caused by frictional shear stress.

(3) Each fibre, i.e. the polypropylene network and the glass fibre, can move independently.

(4) A constant stress is transferred at the interface of fibre and matrix.

(5) Differential slip within the glass fibre bundle and in the film elements is ignored and the polypropylene network and the glass fibre are regarded as single aligned continuous fibres.

7.5.1. Elastic region from o to a (Figure 7.15)

According to the assumption (1), the elastic modulus of the hybrid composite containing polypropylene networks and glass fibres is expressed as follows.

\[ E_c = E_m V_m + E_{fp} V_{fp} + E_{fg} V_{fg} \]  

(7-18)

where the subscripts of c, m, fp and fg express the composite, the matrix, the polypropylene and the glass fibre, respectively. Assuming \( E_m = 31.6 \text{GPa} \), \( E_{fp} = 10 \text{GPa} \) and \( E_{fg} = 72 \text{GPa} \), the theoretical modulus \( E_c \) was obtained for \( V_{fp} = 4-10\% \) and \( V_{fg} = 0.2-2\% \), as shown in Figure 7.16

![Figure 7.16 Theoretical lines of the modulus of elasticity, \( E_c \).](image-url)
The scale of the ordinate is magnified to show the tendency clearly. However, the difference between these lines are relatively small so that the modulus $E_c$ is not affected much in this range of the fibre volume fraction. The comparison between a theoretical line and experimental plots is indicated in Figure 7.17.

![Figure 7.17](image)

**Figure 7.17** Comparison between a theoretical line and experimental plots for the modulus $E_c$. ($V_{tp}=4\%$)

The experimental plots are scattered below the theoretical line, and they seem to be constant regardless of the glass fibre volume fraction.

The stress at the bend over point (BOP), $\sigma_{cc}$, can be calculated from the measured first cracking strain, $\varepsilon_{mu}$.

$$\sigma_{cc} = \varepsilon_{mu} E_c$$

(7-19)

Assuming $\varepsilon_{mu}=273 \times 10^{-6}$, theoretical lines of the stress at BOP, $\sigma_{cc}$ are shown in Figure 7.18, and compared with experimental plots in Figure 7.19.
Figure 7.18  Theoretical lines of the stress at BOP, $\sigma_{cc}$.

Figure 7.19  Comparison between a theoretical line and experimental plots for the stress at BOP, $\sigma_{cc}$  \( (V_f=4\%) \)
Since $\varepsilon_{\text{mu}}$ is treated as a constant, the trend is the same as the modulus $E_c$. As shown in these Figures, the improvements of the tensile properties in the elastic region by reinforcing fibres are not very significant in the range of the fibre volume fraction, $V_{fp}=4-10\%$ and $V_{fg}=0.2-2\%$.

### 7.5.2 Multiple cracking region from a to b (Figure 7.15)

Based on the ACK theory, assuming frictional shear stress transfer (assumption (2)), the strain distribution in the vicinity of a crack face just after the first crack is shown in Figure 7.20.

![Figure 7.20 Strain distribution of fibres and matrix just after first cracking](image)

When the matrix has cracked, the load carried by the matrix at a crack face is fully thrown on to the fibres bridging a crack. The polypropylene and the glass fibre will carry different loads over different stress transfer distance, $x_p$ and $x_g$, respectively. In the ACK theory, the stress transfer length $x$ is given by (7-9).
where $\sigma'_f$ is the additional stress in the fibre due to cracking of the matrix. $\tau$ is the frictional shear stress and $A_f/P_f$ is the fibre cross-sectional area divided by its perimeter.

The stress in the fibre $\sigma'_f$ at a given strain is proportional to its elastic modulus. Since the modulus of the glass fibre is 7.10 times greater than that of the polypropylene, the transfer length $x_g$ is likely to be greater than $x_p$, even allowing for differences in $\tau$ and $A_f/P_f$ between the glass fibre and the polypropylene.

When the strain distribution in the fibres and the matrix is considered, the bond force transferred from both the polypropylene and the glass fibre to the matrix in the region between the crack face and the transfer length $x_p$ is equal to the additional load on the matrix. Thus the force balance is as follows.

From 0 to $x_p$ in Figure 7.20:

$$V_{fp} \left( \frac{V_{fp}}{A_{fp}} + P_{fp} \frac{V_{fp}}{A_{fp}} \right) x_p = E_{m} V_{m} \varepsilon_{p}$$  \hspace{1cm} (7-21)

From 0 to $x_g$, the force from the glass fibre is transferred to the matrix and also the polypropylene between the transfer length $x_p$ and $x_g$. Hence the force balance is:

$$P_{fp} \frac{V_{fp}}{A_{fp}} x_p + P_{fg} \frac{V_{fg}}{A_{fg}} x_g = E_{m} V_{m} \varepsilon_{mu} + E_{fp} V_{fp} \left( \varepsilon_{mu} - \varepsilon_{p} \right)$$  \hspace{1cm} (7-22)

where $\tau_p$ and $\tau_g$ are the frictional shear stress of the polypropylene and the glass fibre, respectively.

At crack face and at a distance $x_p$ from the crack face, the sum of forces carried by the fibres and the matrix should be equal to the force $\varepsilon_{mu} E_c$ sustained by the composite at a distance greater than $x_g$ from the crack face. Thus, the force balance at each cross-section of the composite can be expressed as follows.

At the crack face:

$$\varepsilon_{po} V_{fp} E_{fp} + \varepsilon_{go} V_{fg} E_{fg} = \varepsilon_{mu} E_c$$  \hspace{1cm} (7-23)
At $x_p$ from the crack face:

$$
\varepsilon_{pl}(V_{fp}E_{fp} + V_mE_m) + \varepsilon_{g1}V_{fg}E_{fg} = \varepsilon_{mu}E_c
$$  \hspace{1cm} (7-24)

Both the polypropylene and the glass fibre can be regarded as continuous over the whole length of the composite. Thus, the total deformation in each of them must be equal. Hence,

$$
x_p(\varepsilon_{p0} - \varepsilon_{mu}) = x_g(\varepsilon_{g0} - \varepsilon_{pl})
$$  \hspace{1cm} (7-25)

According to equation (7-20), the additional stress in the fibres alters linearly under a constant interfacial shear stress. Thus, the alteration of the strain in fibres from the crack face to a distance $x_p$ and $x_g$ is linear. The strain of the glass fibre $\varepsilon_{g1}$ at a distance $x_p$ from the crack face is derived from similar triangles between 0 and $x_g$:

$$
\varepsilon_{g1} = \varepsilon_{g0} - \left(\varepsilon_{g0} - \varepsilon_{mu}\right)\frac{x_p}{x_g}
$$  \hspace{1cm} (7-26)

In order to solve these six non-linear equations with six unknowns, $\varepsilon_{p0}$, $\varepsilon_{g0}$, $\varepsilon_{pl}$, $\varepsilon_{g1}$, $x_p$ and $x_g$, computer software called Maple V* (7-28) was used. The following values were substituted in the computation:

- $E_m = 31.6$ GPa (shown at section 2.2.2 in Chapter 2)
- $\varepsilon_{mu} = 273 \times 10^{-6}$ (shown at section 2.2.2 in Chapter 2)
- $E_{fp} = 10$ GPa (at around 1% strain on Figure 2.4 in Chapter 2)
- $\tau_p = 0.905$ MPa (measured from tensile tests for the single composite)
- $A_{fp}/P_{fp} = 17.08 \times 10^{-3}$ mm (measured from the single composite (7-29))
- $E_{fg} = 72$ GPa (shown on Table 2.3 in Chapter 2)
- $\tau_g = 2.19$ MPa (calculated for the composite reinforced with continuous glass fibres of 1.6% by volume shown on Table 5.2 in Chapter 5)

* The design and implementation of the Maple system is an on-going project of the Symbolic Computation Group at the University of Waterloo and of the Institute for Scientific Computing at ETH Zürich. Maple is distributed world-wide by Waterloo Maple Software, Waterloo, Ontario, Canada.
\[ \frac{A_{fg}}{P_{fg}} = 14.30 \times 10^{-3} \text{mm} \] (average value of measured by the image analysis shown on Table 5.1 in Chapter 5)

\[ V_{fp} = 4, 6, 8, 10\% \]

\[ V_{fg} = 0.2-2\% \text{ (every 0.2\% interval)} \]

Theoretical curves of \( \varepsilon_{p0}, \varepsilon_{g0}, \varepsilon_{p1}, \varepsilon_{g1}, x_p \) and \( x_g \) are presented in Figures 7.21 to 7.26, respectively. From each set of equations, three solutions were obtained. One of them contains negative roots which should be rejected. The other one is the case where \( \varepsilon_{p0}=\varepsilon_{g0}, \varepsilon_{p1}=\varepsilon_{g1}=\varepsilon_{mu}=273 \times 10^{-6} \) and \( x_p = x_g \), which means that the polypropylene and the glass fibre act as a single fibre. This case is one possibility, but it is unlikely to occur, because the reinforcing fibres have quite different properties and they are assumed to act independently. Thus, only one set of roots is satisfied with the condition shown in Figure 7.20, and the corresponding curves are plotted in Figures 7.21 to 7.26 as theoretical values.

![Figure 7.21 Theoretical curves of strain \( \varepsilon_{p0} \)](image-url)
Figure 7.22  Theoretical curves of strain $\varepsilon_{g0}$. 

Figure 7.23  Theoretical curves of strain $\varepsilon_{p1}$. 

169
Figure 7.24  Theoretical curves of strain $e_{gl}$.

Figure 7.25  Theoretical curves of stress transfer length $x_p$. 
Figure 7.26 Theoretical curves of stress transfer length $x_g$, minimum crack spacing for the hybrid composite.

Under the conditions described above, the matrix strain is $\varepsilon_{\text{mu}}$ at a distance greater than $x_g$ from the crack face, and hence another crack will be formed in that region. As with the ACK model, the matrix cracks successively and divides into segments of length between $x_g$ and $2x_g$ under a load equivalent to the cracking load. The strain distributions in fibres and matrix between cracks at crack spacing $2x_g$ and $x_g$ are presented in Figures 7.27 (a) and (b), respectively.
Figure 7.27 Strain distributions in fibres and matrix between cracks at crack spacing $2x_g$ (a) and $x_g$ (b).
The minimum theoretical crack spacing is expressed as $x_g$ in Figure 7.26. The value of $x_g$ was derived from equations (7-21) to (7-26) with constant frictional shear bond strength $\tau_p = 0.905 \text{MPa}$ and $\tau_g = 2.19 \text{MPa}$. With respect to the frictional shear bond strength of the polypropylene network $\tau_p$, Hughes\(^{(7-30)}\) obtained them in the range of 0.23 to 0.5MPa and 0.5 to 3.8MPa by Ohno\(^{(7-29)}\). The bond strength $\tau_g$ was obtained in the range of 1 to 3MPa from the pull-out test by Laws et al\(^{(7-31)}\). Oakley and Proctor\(^{(7-32)}\) calculated it from crack spacing for glass fibre reinforced cement sheets made by the spray-suction method. The values they deduced were of the order of 1MPa or less. Considering these values, theoretical crack spacings were calculated for the range of $0.2 \leq \tau_p \leq 3.5$ and $1 \leq \tau_g \leq 3$. Theoretical curves for the minimum combination, $\tau_p = 0.2 \text{MPa}$, $\tau_g = 1 \text{MPa}$ and for the maximum combination, $\tau_p = 3.5 \text{MPa}$, $\tau_g = 3 \text{MPa}$ are presented with the combination, $\tau_p = 0.905 \text{MPa}$, $\tau_g = 2.19 \text{MPa}$, to compare with experimental plots in Figure 7.28.

![Figure 7.28](image)

**Figure 7.28** Comparison between theoretical curves with different frictional shear bond strength and experimental plots for the crack spacing. ($t = \tau$)
experimental data

All the values are between the minimum and the maximum lines. The intrinsic stress transfer length of the polypropylene $x_{ip}$ and that of the glass fibre $x_{ig}$ are given by:

\[ x_{ip} = \frac{E_m (1-V_f) e_m}{\tau_f V_f} \frac{A_{fp}}{P_{fp}} \]  \hspace{1cm} (7-27)

\[ x_{ig} = \frac{E_m (1-V_g) e_m}{\tau_g V_g} \frac{A_{fg}}{P_{fg}} \]  \hspace{1cm} (7-28)

For the hybrid composite, $x_{ip} < x_{ip}$ and $x_{ig} < x_{ig}$. It is notable that one of the hybrid effects is to reduce the stress transfer distance of both fibres, which results in the smaller crack spacing. For instance, assuming for single fibre composites, $V_{fp} = 4.6\%$ and $V_{fg} = 0.66\%$, $x_{ip}$ and $x_{ig}$ are 3.38mm and 8.48mm, respectively. However, in the hybrid composite with $V_{fp} = 4.6\%$ and $V_{fg} = 0.66\%$, the average experimental crack spacing was measured to be 2.28mm. There appears to be an anomaly in figure 7.28 because the plots for more than 1% by volume of the glass fibre diverge from the theoretical trend. The crack spacing increases with increasing glass fibre volume fraction contrary to expectation. Assuming $V_{fg} = 1.2\%$ and $V_{fp} = 0\%$, $x_{ig}$ is calculated to be 4.64mm which is close to experimental plots for hybrid composites containing $V_{fp} = 4\%$. Therefore, for the experimental combination of $V_{fp} = 4\%$ and $V_{fg} = 1.2\%$, the crack spacing seems to be dominated by the glass fibre alone and the hybrid effect does not work efficiently. From equation (7-28), the intrinsic stress transfer length of the glass fibre $x_{ig}$ decreases with increasing the fibre volume fraction $V_{fg}$, which is likely to occur for the hybrid composite so that $x_g$ approaches $x_f$ (Figure 7.20). The strain in the fibres at a crack face is also decreased simultaneously as shown in Figures 7.21 and 7.22. For complete stress transfer between polypropylene and matrix a high polypropylene strain is required (about 1.4% strain at $V_{fp} = 4\%$). At glass fibre volumes above 1%, the strain in the polypropylene at about 0.5% may not be sufficient to develop the small crack spacings experienced with polypropylene single fibre composites. From Figure 7.28, it can be seen that at $V_{fg} < 1\%$ the hybrid effect works efficiently whereas at $V_{fg} > 1\%$ the glass fibre dominates. Although this is an approximation, it may be useful to enable a prediction...
to be made of appropriate glass and polypropylene fibre volume to achieve the best hybrid effect. Otherwise one of the fibres may dominate the tensile properties of the hybrid composite.

The strain of the composite during multiple cracking can be calculated from the mean strain in the fibre as shown in Figure 7.27. Although crack spacing is expressed as $1.364x$ in the ACK theory, experimental crack spacings are close to $x_g$ as shown in Figure 7.28. Hence the actual strain distribution is thought to be close to Figure 7.27(b). Thus, the strain at the end of the multiple cracking region $\varepsilon_{mc}$ is:

$$\varepsilon_{mc} = \frac{\varepsilon_{g0} + \varepsilon_{g2}}{2}$$

$$= \frac{1}{2} \left( \varepsilon_{g0} + \frac{\varepsilon_{g0} + \varepsilon_{mu}}{2} \right)$$

$$= \frac{3\varepsilon_{g0} + \varepsilon_{mu}}{4}$$  \hspace{1cm} (7-29)

Theoretical curves of the strain $\varepsilon_{mc}$ are shown in Figure 7.29 in which the multiple cracking region for the glass/polypropylene fibre hybrid cement composite was always less than 1% strain. In practice, the horizontal region was quite small and uneven so that the strain at the end of the multiple cracking region could not be determined easily. Thus, the comparison between theoretical and experimental values is not presented.
7.5.3. Second elastic region from b to c (Figure 7.15)

Beyond the multiple cracking region, no more cracking can take place. Further increase of the load on the composite will result in further stretching the fibres and slipping through the blocks of matrix. In the ACK theory, assuming no contribution from cracked matrix, the elastic modulus in the post multiple cracking region becomes $E_fV_f$. In Figure 7.15, both the polypropylene and the glass fibre act as reinforcement from point b to c with the modulus $E_2$. In section 4.6 in Chapter 4, the second elastic modulus $E_2$ was expressed as following equation using a multiple regression analysis. (see equation (4-8))

$$E_2 = 6.59V_{fp} + 83.1V_{fg} - 0.219$$  \hspace{1cm} (7-30)

The constant (-0.219) was derived from a statistical procedure, so that it is difficult to
rationalise. It may express the contribution from the cracked matrix or slipped fibre, but it has been ignored because the coefficients of regressor variables of $V_{fp}$ and $V_{f_g}$ are important in this analysis. The coefficients of $V_{fp}$ and $V_{f_g}$ were thought to be related to the degree of contribution of each fibre. Although the second elastic region is varied by the combination of the polypropylene and the glass fibre, $E_2$ was obtained at 1-2% strain, approximately. Using the equation (2-1) obtained in Chapter 2, the elastic modulus of the polypropylene at 1-2% strain was approximated at 7.5-9.5GPa. The average elastic modulus of the glass fibre obtained from the fibre tensile test was 73.5GPa (refer to Table 2.4). In the case of 7.5GPa for the elastic modulus of the polypropylene, the contribution ratio of the polypropylene and the glass fibre can be estimated to be $6.59/7.5 : 83.1/73.5 = 0.88 : 1.13$. When the equality of the total deformation in each fibre is considered, this ratio should approach 1:1. Therefore, the second elastic modulus $E_2$ can be expressed as follows.

$$E_2 = E_{fp}V_{fp} + E_{f_g}V_{f_g}$$

(7-31)

Theoretical lines of the modulus $E_2$ are presented in Figure 7.30 and compared with experimental plots in Figure 7.31.

![Figure 7.30 Theoretical lines of the second elastic modulus, $E_2$.](image)
In fact, the modulus of the polypropylene $E_{fp}$ is variable with strain, however, $E_{fp}$ is treated as a constant in equation (7-31). Since the glass fibre strands failed at less than 1.5% strain in tensile tests, $E_{fp}$ was calculated to be 8.3GPa at 1.5% strain. As shown in Figure 7.31, the theoretical line expresses the experimental trend reasonably.

The stress at FYP, $\sigma_{u1}$, is related to the strength of the glass fibre. Thus, $\sigma_{u1}$ is given by:

$$\sigma_{u1} = \sigma_{fp(FYP)} V_{fp} + \sigma_{fgu} V_{fg}$$

$$= \varepsilon_{fp(FYP)} E_{fp} V_{fp} + \sigma_{fgu} V_{fg}$$  \hspace{1cm} (7-32)

where $\sigma_{fp(FYP)}$ and $\varepsilon_{fp(FYP)}$ are the stress and the strain in the polypropylene at FYP, respectively. $\sigma_{fgu}$ is the ultimate stress of the glass fibre.
After the multiple cracking region, both fibres are stretched by the additional load until the strain in the glass fibre, $\varepsilon_{g0}$ in Figure 7.27 (b), reaches the ultimate strain of the glass fiber, $\varepsilon_{fgu}$. Therefore, the composite strain at FYP, $\varepsilon_{u1}$, can be expressed as follows.

$$\varepsilon_{u1} = \varepsilon_{fgu} - \varepsilon_{g0} + \varepsilon_{mc} \quad (7-33)$$

The equation (7-29) is substituted in (7-33), giving:

$$\varepsilon_{u1} = \frac{1}{4}(\varepsilon_{g0} - \varepsilon_{mu}) \quad (7-34)$$

Because the average strains in the glass fibre and the polypropylene are equal, it is assumed that after point b (Figure 7.15) the increase in strain of each fibre type at the crack is the same. Thus, the maximum strain in the polypropylene at FYP, $\varepsilon_{fp}(FYP)$, is given by:

$$\varepsilon_{fp}(FYP) = \varepsilon_{p0} + \varepsilon_{fgu} - \varepsilon_{g0} \quad (7-35)$$

The equation (7-32) is rearranged by substituting equation (7-35),

$$\sigma_{u1} = E_{fp}V_{fp}\left(\varepsilon_{p0} + \varepsilon_{fgu} - \varepsilon_{g0}\right) + \sigma_{fgu}V_{fg} \quad (7-36)$$

The ultimate strain of the glass fibre $\varepsilon_{fgu}$ was determined from the quoted ultimate stress of the glass fibre, $\sigma_{fgu}=1500\text{MPa}$, divided by the modulus $E_{fg}=72\text{GPa}$, to be 2.08%. The theoretical lines of the stress at FYP, $\sigma_{u1}$, are shown in Figure 7.32 and compared with experimental plots in Figure 7.33.
**Figure 7.32** Theoretical lines of the stress at FYP, $\sigma_{ul}$.

**Figure 7.33** Comparison between a theoretical line and experimental plots for the stress at FYP, $\sigma_{ul}$. (V$_{fp}$=4%)
As expressed in equation (7-36), the stress at FYP can be more than \( \sigma_{fpu} V_{fp} \) which is the theoretical ultimate stress for the composite reinforced with the glass fibre alone. Obviously, this is one of the important hybrid effects expressed mathematically. Provided that all the glass filaments can be stretched up to their ultimate strain and the additional load can be carried by both the polypropylene and the glass fibre, the theoretical lines can be attained. However, in fact, failure of individual glass filaments begins at a lower strain than the ultimate strain and the polypropylene is highly stretched locally so that the composite fails at lower stress than theoretical values. In order to bear the local high strain, the volume fraction of the polypropylene is required to be more than the critical fibre volume at FYP. The critical fibre volume of the polypropylene at FYP, \((FYP)V_{fpcrit}\), is given by:

\[
(FYP)V_{fpcrit} = \frac{\sigma_{u1}}{\sigma_{fpu}}
\]

(7-37)

where \(\sigma_{fpu}\) is the ultimate stress of the polypropylene. Theoretical boundaries between \(\sigma_{u1}\) and \(\sigma_{u}\) are marked by asterisks in Figure 7.32. Beyond the asterisks there is no first yield point but ultimate point. Therefore, theoretical \(\sigma_{u1}\) is shown as solid lines up to asterisks and broken lines show theoretical ultimate stress \(\sigma_{u}\) in the case of \(V_{fp}<(FYP)V_{fpcrit}\). The point \(c\) in Figure 7.15 is either first yield point or ultimate point depending upon the combination of the two fibre volume fraction.

The theoretical curves of the strain at FYP from equation (7-34), \(\varepsilon_{u1}\), are shown in Figure 7.34. The asterisks correspond to those in Figure 7.32. Broken lines present theoretical ultimate strain \(\varepsilon_{u}\) in the case of \(V_{fp}<(FYP)V_{fpcrit}\). In Figure 7.35, the comparison between a theoretical curve and experimental plots for the strain, \(\varepsilon_{u1}\), are presented. Also the effect of the frictional shear bond strength is considered for the range of \(0.2\text{MPa} \leq \tau_s \leq 0.5\text{MPa}\) and \(1\text{MPa} \leq \tau_s \leq 0.5\text{MPa}\). The experimental plots diverged from the theoretical line with increasing the glass fibre volume fraction due to the early failure of glass filaments. The strain at FYP is not affected much by the frictional shear bond strength. Such a small deviation shown in Figure 7.35, hardly affects the stress at FYP.
Figure 7.34  Theoretical curves of the strain at FYP, $\varepsilon_{ul}$.

Figure 7.35  Comparison between theoretical curves with different frictional shear bond strength and experimental plots for the strain at FYP, $\varepsilon_{ul}$.  (Vfp=4%)
7.5.4. After first yield point c (Figure 7.15)

The schematic stress-strain curve shown in Figure 7.15 shows the case where the complete failure of the composite does not occur at FYP. The tensile behaviour after FYP depends on the balance between the stress and the fibre volume fraction of the polypropylene. To carry further additional load, the volume of the polypropylene must satisfy the following inequality, otherwise the composite fails at FYP:

\[ V_{fp} > (FYP)\nu_{fp(\text{tri})} = \frac{\sigma_{\text{tri}}}{\sigma_{fp}} \]  

(7-38)

As shown in equation (7-38), the balance of the volume between the polypropylene and the glass fibre is an important factor at FYP for the design of efficient hybrid composites. Hereafter, the case satisfied with the inequality (7-38) will be discussed.

Immediately after the first failure of the glass fibre at FYP, a sudden stress reduction occurred. At that time, the additional stress was transferred to the polypropylene rapidly and the nets were extended at a faster rate than the cross-head speed resulting in a reduction of load. Therefore, the stress reduction from point c to d in Figure 7.15 is not concerned with the fracture mechanism of the composite and this phenomenon cannot be expressed by characteristic values of the composite.

Even though the glass fibre failed at FYP, the hybrid composite containing polypropylene and glass fibre still carried higher stress than that of the composite reinforced with polypropylene alone as shown in Figure 7.14. Thus, breakage of the glass fibre is thought to be localized and the polypropylene at the crack extends to a high strain to maintain the balance of stress and strain. But elsewhere the intact glass fibre still restrains the elongation of the composite. Thus at a given stress, the strain of the composite is lower than for the polypropylene only composite, due to a smaller crack opening at the other crack locations. This is thought to be a possible reason why the
polypropylene is taking, surprisingly a greater stress at a lower strain in hybrid composites.

Eventually, at high strains, all the glass fibres are broken into lengths which have debonded at every crack. The failure length of the glass fibre according to Figure 5.7 (d) seems to be much longer than the crack spacing which is determined by both the polypropylene and the glass fibres.

After all the glass fibres are broken, the tensile behaviour is dominated by the polypropylene alone from point e to f in Figure 7.15. The modulus of elasticity from e to f is defined as the third elastic modulus $E_3$ in this study, which is given by:

$$E_3 = E_{fp}V_{fp}$$  \hspace{1cm} (7-39)

Since the polypropylene film strip failed at less than 7% strain in tensile tests, $E_{fp}$ was calculated to be 4.0GPa at 7% strain from equation (2-1) presented in Chapter 2. The comparison between a theoretical line and experimental plots is shown in Figure 7.36. The modulus $E_3$ is plotted against the volume fraction of the polypropylene.
Figure 7.36 Comparison between a theoretical line and experimental plots for the third elastic modulus, $E_3$. ($V_{fg}=0.43\%$)

All the experimental plots are lower than the theoretical line. One of the possible reasons of the divergence from the theoretical line is the intermolecular shear in the polypropylene films (7-29) reducing $E_{fp}$ to less than 4.0GPa. Also the high Poisson contraction of the polypropylene and the form of the network may affect the experimental data to some extent.

The ultimate stress in the hybrid composite $\sigma_u$ can be expressed as the load bearing capacity of the polypropylene, assuming no contribution from the broken glass fibres and matrix:

$$\sigma_u = \sigma_{fp} V_{fp}$$

(7-40)

Assuming $\sigma_{fpu} = 508\text{MPa}$ (7-34), the theoretical line is presented in Figure 7.37 comparing with experimental plots. The experimental plots are lower than the
theoretical line due to the same reason as in the case of the modulus $E_3$. Also the local failure of the polypropylene causes the low ultimate stress.

Figure 7.37 Comparison between a theoretical line and experimental plots for the ultimate stress, $\sigma_u$. ($V_{fg}=0.43\%$)

With respect to the ultimate strain $\varepsilon_u$, the same solution as for the strain at FYP $\varepsilon_{ul}$, can be used. When inequality (7-38) is satisfied, the polypropylene can carry the force in the composite even after the glass fibre has failed until the strain in fibre, $\varepsilon_{p0}$ in Figure 7.27 (b), reaches the ultimate strain of the polypropylene, $\varepsilon_{fpu}$. Therefore, the ultimate strain of the composite $\varepsilon_u$ is given by:

$$\varepsilon_u = \varepsilon_{fpu} - \varepsilon_{p0} + \varepsilon_{mc}$$  \hspace{1cm} (7-41)$$

when equation (7-29) is substituted in equation (7-41),

$$\varepsilon_u = \varepsilon_{fpu} - \varepsilon_{p0} + \frac{3}{4} \varepsilon_{g0} + \frac{1}{4} \varepsilon_{mu}$$  \hspace{1cm} (7-42)$$

Assuming $\varepsilon_{fpu} = 8.1\%$ (7-34) and $\varepsilon_{mu} = 273 \times 10^{-6}$, theoretical curves are presented in Figure 7.38. The asterisks are theoretical boundaries in consideration of the critical
fibre volume of the polypropylene. When the glass fibre volume fraction is more than the volume marked by asterisks, the composite fails at the point c in Figure 7.15. Thus the theoretical ultimate strain $\varepsilon_{ul}$ is presented as broken lines in Figure 7.34. This indicates an interesting theoretical discontinuity depending on fibre volume combinations which could have significant practical consequences.

![Graph showing theoretical curves of the ultimate strain $\varepsilon_{ul}$ vs. glass fibre volume fraction](image)

**Figure 7.38 Theoretical curves of the ultimate strain, $\varepsilon_{ul}$.**

Theoretical curves of the ultimate strain are compared with experimental plots in Figure 7.39. Also, the effects of the frictional shear bond strength are represented for the range of $0.2\text{MPa} \leq \tau_p \leq 3.5\text{MPa}$ and $1\text{MPa} \leq \tau_g \leq 3\text{MPa}$. The deviation derived from different frictional shear bond strength is rather small. Although equation (7-42) contains a term of the glass fibre $\varepsilon_{g0}$, the contribution of the glass fibre to the ultimate strain of the composite is relatively small. The experimental plots for a glass fibre volume fraction of more than 0.8% are much lower than the others and cannot be expressed in equation (7-42). These plots correspond with those of strain at the point c in Figure 7.15 due to the volume of the polypropylene at FYP being less than the critical fibre volume.
This trend is presented clearly for toughness of the hybrid composites. The experimental total toughness derived from the area under the tensile stress-strain curve is plotted against the glass fibre volume fraction in Figure 7.40. The chain dotted line represents an experimental trend. The required properties of construction materials are various, which depend on a usage. Toughness is one of the important properties of composites. As shown in Figure 7.40, when high toughness is required, the glass fibre volume fraction must be lower than 0.8% for the polypropylene volume of 4-5%. Such high toughness is attributed to the polypropylene and can not be attained with the glass fibre alone. It is necessary for an efficient hybrid effect to optimize the properties of both fibres, which is shown clearly in Figure 7.40. The hybrid effect appears in various properties and the correct fibre volume combination should be chosen allowing for the type of fibres involved.
Figure 7.40 Total toughness for glass/polypropylene fibre cement composites. ($V_{fp}=4\%$)

7.5.5. Comparison between complete theoretical and experimental curves

The theoretical stress-strain curve for the hybrid composite containing glass fibres of 0.8% by volume and polypropylene networks of 4% by volume is represented in Figure 7.41, compared with the typical experimental curve. The behaviour in the region after FYP indicated by a shaded area has not been expressed theoretically so far because it is partly dependent on the testing machine used and partly dependent on the residual contribution from the broken glass fibres. Except for this region, the theoretical and experimental curves indicate a reasonable agreement. The lower boundary to the shaded region is correctly represented by the curve for the polypropylene alone.
It was found by the light transmission test that the effective glass fibre volume was about 70% of the actual fibre volume in Chapter 6. When this effective ratio was considered for the mathematical model, for example, the effective glass fibre volume fractions are about 0.6% and 0.4% for the actual fibre volume fractions of 0.8% and 0.6%, respectively. Figure 7.42 shows experimental curves and theoretical curves for which the effective ratio of the glass fibre is taken into account. As shown in this Figure, the experimental and theoretical first yield points especially indicate a better agreement than that shown in Figure 7.41. However, the reliability of the effective ratio of the glass fibre should be confirmed by further studies.

![Figure 7.41 Comparison between theoretical curve and experimental curve. (V_p=4%, V_f=0.8%) Note there is no theoretical prediction for shaded area.](Image)
7.6. Resume

A mathematical model which enables prediction of the tensile behaviour for composites reinforced with continuous polypropylene networks and glass fibre strands was obtained, and the hybrid effects were confirmed from the tensile properties. This model is also considered applicable to hybrid composites reinforced with other fibres.

It has also been shown that the hybrid effect of fibres is especially important for toughness in that the wrong volume combination of fibre types could drastically reduce the toughness. For example, an increase in glass fibre volume from 0.8% to 1% at constant polypropylene volume of 4% could reduce the total toughness by about 80% (Figure 7.40). To obtain an efficient hybrid effect, the balance of fibre volumes between the polypropylene and the glass fibre must be considered in particular, so that the polypropylene has more than the critical fibre volume at the first yield point.

Figure 7.42 Comparison between experimental curves and theoretical curves corrected by the effective glass fibre volume fraction. \( V_{f_p}=4\%, \ V_{fg}=0.8, 0.6\% \)
Otherwise, the properties of the hybrid composite may become close to those of the composite reinforced with a single fibre type. This has economic as well as technical implications.
CHAPTER 8

8. CARBON/POLYPROPYLENE FIBRE HYBRID CEMENT COMPOSITES

8.1. Introduction

Carbon fibre is a remarkable fibre material which has excellent mechanical properties and also superior thermal, chemical and electromagnetic properties. Component materials making use of these properties are widely utilized in various industrial fields.

The characteristics of carbon fibres as reinforcing fibres in cementitious composites are as follows \( (8-1), (8-2) \).

(1) An enhancement in tensile and flexural strength, toughness and impact strength can be expected due to their high strength and modulus.

(2) Carbon fibre has excellent chemical and biochemical stability. It indicates superior alkali resistance compared with the alkali resistant glass fibre. No damage is shown for long time exposure so that products with high durabilities can be obtained.

(3) The melting point and the softening point are extremely high. The melting point of the carbon fibre is higher than 3000K and its strength does not reduce rapidly at high temperature. Thus, autoclave curing is available and consequently composites with dimensional stability and fire resistance can be produced.

(4) Electrical characteristics such as the elimination of static electricity, electrical conductivity and the absorptivity of electric waves, can be added to properties of the composite. Carbon fibre indicates a volume resistivity of approximately \( 10^{-2} - 10^{-3} \Omega \text{cm} \), hence it has high electrical conductivity. Thus, for carbon fibre reinforced composites,
the conductivity can be controlled in the range from the resistivity of the concrete to about 1Ω cm. The corona discharge by the fibre to the air for charged static electricity is active because of the fine fibre diameter of <10μm. Therefore, it is a suitable material for the elimination of static electricity.

(5) Carbon fibre is safe for health. It does not show any hazard to health such as that caused by asbestos fibres. Thus, it can be used without anxiety.

As described above, carbon fibre reinforced composites can give various excellent characteristics. However, from the economic point of view, carbon fibre is too expensive to be used generally in a construction field. Carbon fibre used to be mainly PAN type made from polyacrylonitrile. Since the pitch type of carbon fibre made from the residue of petroleum and coal was developed in Japan, it has become possible to obtain cheap carbon fibres. Although pitch type of carbon fibre is inferior in strength, modulus and material stability to PAN, there is a great advantage in its cost. The cost of carbon fibre tends to be decreasing continuously, hence the use of carbon fibre reinforced composites is expected to be extended.

In this chapter, the tensile properties of hybrid composites reinforced with continuous PAN carbon fibres and polypropylene networks, are presented. The effect of the carbon fibre distribution on tensile properties is discussed and an attempt made to quantify the carbon fibre distribution in a composite. Finally, the mathematical model developed in Chapter 7, is applied to the carbon/polypropylene fibre reinforced composites (CF/PP composites) and the efficiency factor of the fibre distribution in relation to the values predicted by the theoretical model is discussed.
8.2. Tensile behaviour of carbon/polypropylene fibre hybrid cement composites

8.2.1. Stress-strain curves

Typical stress-strain curves for composites reinforced with two (C₂), four (C₄), and six layers (C₆) of continuous carbon fibres shown on Table 3.2 in Chapter 3, are presented in Figure 8.1. Also, the theoretical curve based on the ACK model is presented to compare with the experimental curves.

![Stress-strain curves](image)

**Figure 8.1** Tensile stress-strain curves for composites reinforced with 2, 4 and 6 layers of continuous carbon fibres and the theoretical curve based on the ACK model. (28 days curing)

The composites reinforced with two and four carbon fibre layers, C₂ and C₄, indicate lower stress at a given strain and lower ultimate stress than that for the composite with six carbon layers, C₆. Although the ultimate stress and strain for C₆ is about a half of the theoretical values, the stress-strain curve for C₆ shows reasonable
fit with the theoretical curve to a certain extent. In spite of containing the same
carbon fibre volume fraction, the three composites have significantly different
properties. The only variable is carbon fibre distribution which is therefore shown to
have a very significant effect although it is not included in any theoretical treatment.

Stress-strain curves of continuous and aligned carbon fibre reinforced cement
(CFRC) have been presented by Aveston et al\(^{8\text{-}3}\). These curves indicated a good
agreement between theory and experiment even for the ultimate stress and strain.
Sarker and Bailey\(^{8\text{-}4}\) also reported properties of CFRC to show effective strengthening
with continuous carbon fibres, however the ultimate tensile strengths obtained from
their experiments were on the average about 0.56 of the theoretical values. Ali,
Majumdar and Rayment\(^{8\text{-}5}\) observed less post-cracking ductility than Aveston's.
Aveston et al. and Sarker et al. used the same type of carbon fibre containing 10000
filaments in the form of tapes 75mm to 100mm wide. The former specimen size was
300×30×10mm and the latter was 250×30×6.4mm. They did not mention the
location of the carbon fibres. As Bentur and Mindess\(^{8\text{-}6}\) pointed out, such
differences in properties of CFRC may be attributed to the dispersion of the carbon
fibre during the preparation of the specimen.

Typical stress-strain curves for hybrid composites reinforced with continuous
carbon fibres of 0.6vol% and polypropylene networks of 4vol% after 7 days and 28
days curing under water are shown in Figures 8.2 and 8.3, respectively. Also, the
effect of the distribution of carbon fibres can be compared.
Figure 8.2 Tensile stress-strain curves for composites reinforced with continuous carbon fibres and polypropylene networks in the distributions of a bundle, two layers, four layers and six layers after 7 days curing under water.

Figure 8.3 Tensile stress-strain curves for CF/PP composites with different carbon fibre distributions after 28 days curing under water.

Carbon : $V_{fc}=0.6\%$, Polypropylene : $V_{fp}=4.0\%$
As compared with the composite containing polypropylene networks alone, marked enhancement in tensile strength can be seen. The effect of the carbon fibre distribution for the enhancement increases from a bundle (series PCN) to two layers (series PC2), four layers (series PC4) and six layers (series PC6) in order. However, PC6 failed at much lower strain than others as shown in these Figures.

The composites cured for 28 days indicate higher stress than that cured for 7 days especially for PC4 and PC6 due to the progress of the hydration which presumably densifies the carbon/cement interface thus increasing the bond strength. The effect of the distribution of the carbon fibre is confirmed in Figure 8.4 as well as in Figures 8.2 and 8.3 even when the fibre volume of the polypropylene is increased from 4% to 6%. Figure 8.5 shows the typical tensile stress-strain curves for the glass/polypropylene composite and the carbon/polypropylene composite compared with the polypropylene composite. In spite of a small quantity of 0.6vol%, both the glass fibre and the carbon fibre indicated a remarkable enhancement in tensile strength. Although CF/PP composites failed at lower strain, the carbon fibre exhibited the superior hybrid effect to the glass fibre up to about 1% strain. The tensile behaviour of the CF/PP composite was altered by the distribution of the carbon fibre. The correlations between the characteristic values in the tensile stress-strain curve and the distribution of the carbon fibre will be shown in the following section.
Figure 8.4  Tensile stress-strain curves for CF/PP composite reinforced with 6vol% of polypropylene networks and 0.6vol% of carbon fibres after 7 days curing.

Figure 8.5  Comparison of tensile stress-strain curve between GF/PP composite and CF/PP composite with same fibre volume fraction. (V_{fp}=4% V_{fg}=0.6% V_f=0.6% 28 days curing)
8.2.2. Effects of the carbon fibre distribution

The effects of the carbon fibre distribution on the stress at the bend over point (BOP), $\sigma_{cc}^{cp}$ and the modulus of elasticity, $E_c^{cp}$ are shown in Figure 8.6 and 8.7, respectively. The superscript $cp$ stands for the carbon/polypropylene fibre reinforced cement composite.

![Figure 8.6 Effect of the carbon fibre distribution on the stress at BOP, $\sigma_{cc}^{cp}$.](image)

The line is drawn between average values. The vertical line indicates a range of data for four or five specimens from a minimum to a maximum for each distribution. The stress at BOP increased slightly with increasing numbers of carbon fibre layers. The modulus $E_c^{cp}$ was almost constant regardless of the carbon fibre distribution. The carbon fibre distribution did not affect these properties much in the elastic region.

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Figure 8.7  Effect of the carbon fibre distribution on modulus of elasticity, $E_{cP}$. 

Figure 8.8  Effect of the carbon fibre distribution on the crack spacing.
The crack spacing was varied by the carbon fibre distribution as shown in Figure 8.8. According to the ACK theory, the crack spacing, $C$, can be expressed for the single fibre composite as follows (8-7):

$$C = 1364 \frac{V_m \sigma_{mu} A_f}{V_f' \tau P_f'}$$

where, $V_m, V_f'$: volume fraction of matrix and fibre, respectively.

$\sigma_{mu}$: strength of matrix.

$\tau$: bond strength between fibre and matrix

$A_f, P_f'$: fibre cross-sectional area and perimeter in contact with matrix, respectively.

The carbon fibre distribution as shown on Table 3.2 in Chapter 3 affects the term of $A_f/P_f$ in equation (8-1). Since the fibre volume is constant, $A_f$ is constant. However, $P_f$ is variable depending on the fibre distribution. When carbon filaments are localized such as in a bundle, the effective fibre perimeter in contact with cement is much lower than that for the distribution with six layers. Therefore, the better the fibres are dispersed, the smaller the crack spacing that can be obtained because of the larger fibre-matrix contact area. Figure 8.8 exhibits the trend as described above.

The second elastic moduli, $E_{2}^{sp}$, are plotted against the carbon fibre distribution in Figure 8.9. When the carbon fibre bundle was used, the second elastic region did not appear clearly enough to quantify. The modulus $E_{2}^{sp}$ increased noticeably with an increasing number of layers. The modulus $E_{2}^{sp}$ is dominated by fibre modulus and fibre volume in the theory as presented in equation (7-31). As far as this equation is concerned, we cannot give an explanation of this phenomenon unless some of the carbon fibres are slipping from the ends of specimen for PCN, PC2 and PC4. Thus, another dominant factor related to the carbon fibre distribution may be involved.
Figure 8.9  Effect of the carbon fibre distribution on the second elastic modulus, $E^{\text{sp}}_2$.

Figure 8.10  Effect of the carbon fibre distribution on the stress at FYP, $\sigma_{ul}^{\text{sp}}$. 
The effect on stress and strain at FYP, $\sigma_{ul}^{cp}$ and $\varepsilon_{ul}^{cp}$, are shown in Figures 8.10 and 8.11, respectively.

It is also noteworthy that the stress $\sigma_{ul}^{cp}$ and the strain $\varepsilon_{ul}^{cp}$ increased in a similar way to the modulus $E_{ul}^{cp}$ depending on the carbon fibre distribution. As for glass/polypropylene fibre composites, $\varepsilon_{ul}$ and $\sigma_{ul}$ are given by equations (7-34) and (7-36), respectively. Since the strain $\varepsilon_{g0}$ is derived from six equations (7-21) to (7-26), $\varepsilon_{ul}$ and $\sigma_{ul}$ are slightly affected by the effective fibre perimeter, $P_f$. However, the effect of $P_f$ should not be great, because the dominant factor is the fibre volume fraction and the properties of the reinforcing fibres such as the fibre strength and modulus. Therefore, the carbon fibre distribution appears to be an additional dominant factor affecting the stress and strain at FYP.

Figure 8.12 shows how the ultimate stress of the composite, $\sigma_{u}^{cp}$, depends on the carbon fibre distribution. As expressed in equation (7-40), the ultimate stress is determined by the polypropylene fibre volume fraction provided that this remains above the critical fibre volume at FYP. Thus, the carbon fibre distribution does not affect the ultimate stress as shown in Figure 8.12.

On the other hands, the ultimate strain, $\varepsilon_{u}^{cp}$, is supposed to be slightly affected by the fibre distribution because the equation (7-42) for $\varepsilon_{u}$ includes the terms of $\varepsilon_{p0}$ and $\varepsilon_{g0}$ which are related to the effective fibre perimeter $P_f$. However, as shown in Figures 8.2, 8.3 and 8.4, the composite with six carbon layers failed at lower strain because the polypropylene fibre volume was very near the critical fibre volume at FYP. The correlation between the ultimate strain and the carbon fibre distribution is not evident and hence the Figure is omitted.
Figure 8.11 Effect of the carbon fibre distribution on the strain at FYP, $\varepsilon_{ul}^{lp}$.

Figure 8.12 Effect of the carbon fibre distribution on the ultimate stress, $\sigma_u^{lp}$. 

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As described above in this section, the properties of the hybrid composites in post-cracking region such as the second elastic modulus and the first yield point were affected by the carbon fibre distribution. The number of carbon fibre layers was used as the fibre distribution parameter for the sake of convenience. However, no numerical index for the fibre distribution has been obtained so far. In order to quantify the carbon fibre distribution, a method of statistical analysis to define the carbon fibre distribution will be discussed in the next section.

8.3. Analysis of carbon fibre distribution

8.3.1. SEM observation

The microstructure of the carbon/polypropylene fibre hybrid composite was observed with a scanning electron microscope (SEM). Typical carbon fibres in the cut cross section of the hybrid composite after tensile testing are shown in Figure 8.13. The carbon fibre in the form of a bundle is presented with different magnifications in Figures 8.13 (a) and (b). Also, the features composed as the form of two layers are in (c) and (d). As shown in Figure 8.13 (a), the original carbon fibre bundle size is about 4-5mm wide by 0.6-0.7mm thick in a lens shaped form. A bundle consists of 12000 filaments. The bundle size is much larger than that of the glass fibre strand which is about 400-500μm wide by 50-80μm thick and 100 filaments per bundle. As shown in Figure 8.13 (b), since the space between filaments is too close to allow cement particles penetration, a lot of filaments are not in direct contact with matrix, which implies an unsatisfactory reinforcement of composite by the carbon fibre. On the other hand, the distribution as the form of two layers has much larger contact area with the matrix as shown in Figure 8.13 (c). Although the carbon fibres are not spread out uniformly, the number of ineffective filaments is reduced and a greater number of filament contacts with the matrix individually, as shown in Figure 8.13 (d). This trend is more evident for four and six layers’ distribution, which will be shown later in Figures 8.19 and 8.20.
In the manufacturing process of the carbon fibers, there are heat treatments for carbonization and graphitization usually under such an environment. After the treatment, only the carbon atoms exist in the portion of the fibers. Thus, the fiber surface is no longer chemically reactive and shows no chemical bonding with the matrix. In order to improve bonding, the fiber surface must be functionalized with groups like carbonyl.

Figure 8.13  Carbon fibres in the cut cross section of the hybrid composite.

(a), (b)  carbon fibre bundle
(c), (d)  composed as two layers
In the manufacturing process of the carbon fibre, there are heat treatments for carbonization and graphitization under inert gas such as nitrogen\(^{(8-8)}\). After the treatment, only the carbon atoms remain on the surface of the fibres. Thus, the fibre surface is so inactive chemically that it cannot have any chemical bonding with the matrix. In order to improve bonding, a range of active functional groups like carboxyl group -COOH, hydroxyl group -OH and carbonyl group -CO are produced on the surface by oxidative treatments such as heating in oxygen or treatment in nitric acid and sodium hypochlorite\(^{(8-9)}\). The carbon fibre is mainly designed for resin matrices and the functional groups can form chemical bonds directly with unsaturated resins. Since the groups described above are hydrophilic groups, the carbon fibre is likely to have a good affinity with hydraulic cement as well as resin. Therefore, it is important to obtain a large contact surface area between fibre and matrix for an effective chemical and physical bonding mechanism.

A polished cross section of the carbon fibre bundle is shown in Figure 8.14 (a). Each circle represents a carbon filament and the hexagonal or square packing can be seen partly. The spaces between filaments are seen to be filled with sizing materials or voids. Such filaments inside a bundle are ineffective for bonding with the matrix chemically and physically. An additional factor for bonding is the irregular surface of the carbon fibre due to the large amount of surface microroughness as shown in Figure 8.14 (b). When the carbon filaments are dispersed individually, they can have a large number of sites for chemical bonding and a large contact area with the matrix from the microstructural point of view. Thus, it is possible to increase the apparent bond strength by increasing the specific surface area in contact with cement.
Figure 8.14 Carbon fibre structure.

(a) polished cross section of the carbon bundle
(b) carbon filaments
In order to disperse the carbon fibre uniformly, various research has been carried out. Briggs\(^{(8-10)}\) introduced the filament winding process to open the filaments by a compressed air spreading device. The dispersion of the carbon fibres into individual filaments is controlled by the mixing technique when the composite is manufactured with short fibres. Effective methods of mixing with the Omni mixer\(^{(8-11)}\) or the Eirich mixer\(^{(8-12)}\) have been reported. When using an ordinary mortar mixer, 40% of the Portland cement has been replaced with silica fume in order to obtain close packing between fibre and matrix\(^{(8-13)}\). Also, Suzuki et al.\(^{(8-14)}\) have presented an unique method of mixing by using small lumps of ice instead of water. They reported effective dispersion of carbon fibres in cement even with an ordinary mixer. A different noteworthy approach was taken by Brown and Hufford\(^{(8-15)}\). Before being used to reinforce cement, the tow of carbon fibres was preimpregnated with polyvinyl acetate (PVA) resin to obtain a good fibre-matrix bond and to prevent damage suffered from the manufacturing process. The affects of polymer impregnation on the mechanical properties of carbon fibre reinforced cement have also examined by Akihama et al.\(^{(8-16)}\).

According to these quoted references, the range of the carbon fibre volume fraction is from 2% to 16%. As described in section 8.2.1, the composite was effectively strengthened in our work even by an amount of the carbon fibre as small as about 0.6vol%, but only if it is properly dispersed and used in combination with polypropylene networks. With such a small quantity of carbon fibre, the dispersion plays an important roll in determining the properties of the composite. Therefore we should have a criterion for the dispersion of the reinforcing fibre. As far as the above references are concerned, the effects of the fibre dispersion were shown by pictures or were indicated by some physical properties of the composite, such as tensile and flexural strength which depended on the fibre dispersion. However, no distribution criterion has been suggested.

In this study, one possible criterion defining fibre dispersion is contact area between fibre and matrix. In Chapter 5, a backscattered electron image was used to
determine the perimeter of the glass fibre strand. A typical backscattered electron image of the carbon fibre arranged in two layers is shown in Figure 8.15 with different magnifications. These are polished cross sections of a composite cut perpendicular to the fibres. Since the backscattered electrons are strongly dependent on the atomic number of the materials, a good image with proper contrast between the fibres and the cement matrix can be obtained. The black rectangular pieces represent polypropylene networks in Figure 8.15 (a). The black wavy band lying across the picture represents carbon fibres. The dense part of the band has a similar distribution to a fibre bundle, where the space between filaments is too small to allow penetration of cement particles. In the other part, the cement hydrate can be seen between filaments partly due to the fibres being spread out widely. However, as shown in Figure 8.15 (b), the shape of the fibre-matrix interface is so complicated that the contact area between carbon fibre and matrix cannot be determined. Therefore, a different approach has been tried to obtain another criterion for the dispersion of the carbon fibre.
Figure 8.15  Backscattered electron image of the carbon fibre layer.
8.3.2. Fibre spacing and unit cell

If the reinforcing fibres are aligned in a regular unidirectional manner, the centre to centre spacing of the fibres can be calculated from the number of fibres crossing a unit area in an arbitrary cross section of the composite. As proposed by Romualdi and Mandel\textsuperscript{(8-17)}, the centre to centre spacing of the fibres, $2R$, is given by:

$$2R = \frac{1}{\sqrt{n}} \quad (8-2)$$

where $n$ is the number of fibres passing a unit area in the cross section of the composite. However, the fibre spacing is not, in fact, determined from the number of fibres alone but also the mutual placing of the fibres in the cross section. As shown in Figure 8.16 (a), all the fibres are aligned parallel to each other in a unidirectional lamina. In an ideal situation, the fibres can be considered to be arranged in either a square or hexagonal pattern as shown in Figure 8.16 (b). Assuming that each fibre has a circular cross section and the same diameter, the volume fraction of fibres $V_f$ can be expressed for the ideal arrangements\textsuperscript{(8-9)}:

Hexagonal: $V_f = \frac{\pi}{2\sqrt{3}} \left( \frac{r}{R} \right)^2 \quad (8-3)$

Square: $V_f = \frac{\pi}{4} \left( \frac{r}{R} \right)^2 \quad (8-4)$

where $r$ is the radius of the fibre. Thus, $2R$ is given by:

Hexagonal: $2R = 2 \left( \frac{\pi}{2\sqrt{3V_f}} \right)^{\frac{1}{2}} r \quad (8-5)$

Square: $2R = 2 \left( \frac{\pi}{4V_f} \right)^{\frac{1}{2}} r \quad (8-6)$
Figure 8.16

(a) Unidirectional lamina.
(b) Hexagonal and square packing of unidirectional fibres\(^{(8,9)}\).
The maximum value of $V_f$ will be obtained when the fibres are touching, i.e. $2R=2r$. For a hexagonal array $V_{\text{fmax}}=0.907$ and for a square array $V_{\text{fmax}}=0.785$. However, as for the cement composite, the size of cement particles must be taken into account to enable penetration between the filaments. Assuming that the diameter of the carbon filament is in the same order of magnitude as the cement particle size, Waller\textsuperscript{(8-18)} pointed out that the maximum carbon fibre content is approximately $12\text{vol}\%$ with a limit of reasonable practicability of $8\text{vol}\%$. As Krenchel\textsuperscript{(8-19)} described, it would be very difficult to predict the pattern formed by the fibres in the cross section in practice and consequently impossible to carry out an exact determination of the fibre spacing, the fibre spacing nevertheless affected the mechanical properties of the composite to some extent.

In Figure 8.16 (b), the unit cell occupied by single filament is shown with dotted line. The unit cell area, $UA$, for both patterns is same and given by:

$$UA = \frac{\pi r^2}{V_f}$$  \hspace{1cm} (8-7)

The ideal fibre area fraction in a unit cell, $UA_{fi}$, corresponds with $V_f$:

$$UA_{fi} = \frac{\pi r^2}{UA} = V_f$$  \hspace{1cm} (8-8)

Simplified models of fibre distribution for square packing are shown in Figure 8.17. When the fibres are used in a form of a bundle, the fibres are so concentrated in a few unit cells (Figure 8.17 (a) ) that the local actual fibre area fraction in a unit cell, $UA_{fn}$ is much larger than the ideal uniformly distributed fibre area fraction $UA_{fi}$ (Figure 8.17 (e) ). The rest of the unit cells will be blank because the fibre volume fraction is averaged over the whole composite. When the fibres are spread out into single, double or triple layers, $UA_{fn}$ locally is reduced and becomes closer to $UA_{fi}$ as shown in Figures 8.17 (b), (c) and (d), respectively. Also the number of blank unit cells is reduced with increasing the number of fibre layers. Therefore, the fibre distribution in each cell can be quantified by the ratio of $UA_{fi}/UA_{fn}$.
Figure 8.17  Simplified models of fibre composition for square packing.

(a) fibre bundle  (b) single layer  (c) double layer  (d) triple layer
(e) uniform dispersion
The ratio of $UA_{f}/UA_{fn}$ is calculated for each pattern shown in Figure 8.17 (a) to (e). The results are presented in Table 8.1 and the ratios of total blank unit cell area $\Sigma UA_{b}$ to total area $\Sigma UA$ are also calculated.

Table 8.1  Representative values of $UA_{f}/UA_{fn}$ and ratio of the blank cell.

<table>
<thead>
<tr>
<th>Fibre distribution</th>
<th>$UA_{f}/UA_{fn}$</th>
<th>$\Sigma UA_{b}/\Sigma UA$</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>0.083</td>
<td>0.917</td>
</tr>
<tr>
<td>b</td>
<td>0.167</td>
<td>0.833</td>
</tr>
<tr>
<td>c</td>
<td>0.333</td>
<td>0.667</td>
</tr>
<tr>
<td>d</td>
<td>0.500</td>
<td>0.500</td>
</tr>
<tr>
<td>e</td>
<td>1.000</td>
<td>0.000</td>
</tr>
</tbody>
</table>

Figure 8.18  Correlation between $UA_{f}/UA_{fn}$ and $\Sigma UA_{b}/\Sigma UA$. 
As shown in Figure 8.18, there is a linear relationship between the ratios $\frac{U_{Af}}{U_{Af_n}}$ and $\frac{\Sigma U_{Af}}{\Sigma U_{A}}$. Therefore, although the ratio of $\frac{U_{Af}}{U_{Af_n}}$ is available only for the units containing fibres, the ratio of the blank unit cell is also taken into consideration by the quantification of the fibre distribution with $\frac{U_{Af}}{U_{Af_n}}$. However, such a linear relationship is satisfied only for the assumption which the fibres are arranged into layers homogeneously. In fact, the fibre arrangement in layers is irregular and the fibre location is quite unpredictable as shown in Figure 8.15.

In order to obtain experimentally a proper $\frac{U_{Af}}{U_{Af_n}}$ for each fibre distribution shown on Table 3.2 in Chapter 3, a considerable amount of measurement of the fibre area in a unit cell is required.

8.3.3. Specimen preparation for image analysis

An image analyser, Quantimet 920, was used to measure the fibre area fraction in a unit cell for each carbon fibre distribution. This equipment requires proper contrasts between carbon fibres and other materials to obtain good images. Therefore, white cement was used instead of the ordinary Portland cement as the matrix which has the greatest colour contrast to the black carbon fibre. The water/cement ratio was same as that shown on Table 2.1 in Chapter 2, however, no aggregate was mixed. The superplasticiser was added at about 0.5% by weight to the cement. The fibrillated polypropylene network used in this study contains carbon particles to prevent damage from the ultraviolet rays. Thus, it is black as well as the carbon fibre. Since the cross section of the polypropylene is rectangular and much larger than individual carbon fibre filaments as shown in Figure 8.15, we can distinguish them visually under the microscope. However, as far as an image analyser is concerned, they cannot be distinguished. In order to make the analysis easy, white polypropylene network without carbon particles was supplied by Fibronit s.p.a. for this particular use. Then the hybrid composite was manufactured into the fabrication of PCN, PC2, PC4 and PC6 as shown on Table 3.2. Specimens were cured for 28 days under water.
Micropores around fibre layers and the voids between filaments were observed in a cut cross section of the composite. The filaments in the vicinity of pores and voids are easily damaged by grinding and these flaws themselves disturb the analysis. Gypsum slurry, Aluminum oxide powder with epoxy resin and epoxy resin alone were examined as casting materials to fill pores and voids simultaneously. These casting materials were sucked up into the cross section of the composite to some extent by reduced pressure and the voids were finally filled in a mould. Although gypsum was white and gave a good contrast to carbon fibres after filling the pores, it could intrude into only relatively large pores and the strength of adhesion with matrix was so weak that it was scraped off by grinding. Aluminum oxide powder is also white and good for contrast. The particle size is approximately 1 μm and even fine pores can be filled with particles. Since aluminum oxide itself could not be hardened, it was mixed with epoxy resin. Nevertheless, in fact, it was difficult to fill voids between filaments uniformly and aluminum oxide particles were too hard to be polished. Eventually, epoxy resin alone was chosen although some contrast had to be sacrificed. Almost all the pores and voids were filled with resin and individual filaments were strengthened for grinding and polishing even though the shadow of the pores remained.

The grinding and polishing was carried out with a flat plate machine with different fineness grades of paper. The procedure and condition are the same as presented on Table 6.1 in Chapter 6.

Typical polished cross sections of the carbon fibre layers are shown in Figure 8.19. Figure 8.20 shows them with higher magnifications. Good contrasts were obtained between the carbon fibres and the white matrix. Also the white polypropylene did not disturb the analysis at all, although these fibres could be identified visually as shown in Figure 8.19 (a). The epoxy resin filled micropores and voids effectively. The cement matrix penetrated more effectively between the carbon filaments as the dispersion of the filaments improved as shown in Figures 8.19 and 8.20 from (a), (b), (c).
to (d). However, it should be remembered here that the carbon fibre dispersion for each layer was not uniform and some amount of fibre concentration area was observed even in the distribution of six layers.
Figure 8.19 Polished cross section of the carbon fibre layers.
(a) bundle (b) 2 layers (c) 4 layers (d) 6 layers
Figure 8.20 Polished cross section of the carbon fibre layers with higher magnification.

(a) bundle  (b) 2 layers  (c) 4 layers  (d) 6 layers
8.3.4 Image analysis and fibre distribution

Assuming that the radius of the carbon filament is 3.5μm and the volume fraction of the carbon fibre, \( V_{fc} \), is 0.006, the unit cell area for the composite, \( UA \), is given by equation (8-7).

\[
UA = \frac{\pi \times (3.5)^2}{0.006} = 6414 \mu m^2
\]

The ideal fibre spacing, \( 2R \), for hexagonal and square packing can be calculated from equations (8-5) and (8-6), respectively.

Hexagonal: \( 2R = 2\left(\frac{\pi}{2\sqrt{3} \times 0.006}\right)^{\frac{1}{2}} \times 3.5 = 86 \mu m \)

Square: \( 2R = 2\left(\frac{\pi}{4 \times 0.006}\right)^{\frac{1}{2}} \times 3.5 = 80 \mu m \)

In order to measure the fibre distribution, the polished cross section is set up in the microscope and a monochrome multishade image from the microscope is transmitted to the monochrome screen of an image analyser Quantimet 920. The visual display unit of the analyser has two overlapping image frames. The first is the visual frame which shows the whole of the observable field of the display. The second is the live frame which shows the particular analysing area within the visual frame. The dimension of the live frame can be adjusted and has been set as 80×80μm square for the present analysis because the ideal fibre spacing and the unit cell area are taken. In this analysis, the ideal fibre area fraction in a live frame \( UA_{fi} \) is \( V_{fc}=0.006 \) whether the packing pattern is hexagonal or square. The actual fibre area fractions in live frames \( UA_{fn} \) were obtained for each carbon fibre distribution series of PCN, PC2, PC4 and PC6 on Table 3.2. The measurement of \( UA_{fn} \) was carried out by moving a live frame along the approximate centre line of the fibre concentrated layer or area in a certain interval with minimum subjectivity. About 200 data points were obtained for each fibre distribution. As described in section 8.3.2, the fibre distribution can be quantified by the ratio of \( UA_{fi}/UA_{fn} \). Frequency distributions for the ratio of \( UA_{fi}/UA_{fn} \) are shown in Figures 8.21 (a) - (d) to compare four types of carbon fibre distribution.
Figure 8.21 Frequency distributions of the ratio of $U_{A_{fn}}/U_{A_{in}}$.

(a) bundle (PCN)  (b) 2 layers (PC2)  (c) 4 layers (PC4)  (d) 6 layers (PC6)
Statistical characteristics are presented in Table 8.2.

Table 8.2  Statistical characteristics of the ratio UAf/UAff for carbon fibre distribution.

<table>
<thead>
<tr>
<th>Distribution</th>
<th>PCN</th>
<th>PC2</th>
<th>PC4</th>
<th>PC6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of data</td>
<td>200</td>
<td>230</td>
<td>192</td>
<td>209</td>
</tr>
<tr>
<td>Mean</td>
<td>0.0133</td>
<td>0.0970</td>
<td>0.1051</td>
<td>0.1603</td>
</tr>
<tr>
<td>Maximum</td>
<td>0.0641</td>
<td>1.3793</td>
<td>0.9493</td>
<td>1.2225</td>
</tr>
<tr>
<td>Minimum</td>
<td>0.0063</td>
<td>0.0075</td>
<td>0.0093</td>
<td>0.0086</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>0.0077</td>
<td>0.1698</td>
<td>0.1274</td>
<td>0.1794</td>
</tr>
<tr>
<td>Variance (×10^2)</td>
<td>0.0062</td>
<td>2.88</td>
<td>1.62</td>
<td>3.22</td>
</tr>
<tr>
<td>Sum</td>
<td>2.6558</td>
<td>22.3061</td>
<td>20.1736</td>
<td>33.5016</td>
</tr>
<tr>
<td>Mode</td>
<td>0.0093</td>
<td>0.0478</td>
<td>0.0489</td>
<td>0.0222</td>
</tr>
<tr>
<td>Median</td>
<td>0.0105</td>
<td>0.0523</td>
<td>0.0666</td>
<td>0.1025</td>
</tr>
</tbody>
</table>

Hereafter, we will discuss how to determine a representative value of UAf/UAff for each carbon fibre distribution. A measure of central tendency in the population is most commonly used as a descriptor of the empirical distributions. The mean, mode and median are used as a general measure of central tendency.

The mean is the best-known descriptor which is computed by dividing the sum of N data value by N, where N is the total number of values in a data set.

The mode of a data set is the value that occurs most frequently. A mode can be assigned to grouped data that have a highest frequency class. The formula for computing the mode of grouped data is (8-20):

$$\text{Mode} = M_o = L + \frac{d_i}{(d_i + d_s)} c_i$$

(8-9)

where L : lower limit of the modal class

$$d_1 : \text{modal class frequency minus the frequency in the previous (smaller class}$$
The median of a set of data is a number selected to represent the middle position when the data are arrayed in order of size. The middle position in an array of N data items is the position numbered (N+1)/2. If N is odd, there is a data item at the middle position, and this item is taken as the median. If N is even, the average of the two middle data item is taken as the median. The median is thus the middle value, or the average of the two middle values, in an arrayed data sets.

The mode is rarely used as a descriptor, because a data set may have no mode or it may have two or more modes. As shown in Figure 8.21 (b), (c), there are two modal classes. Further, the mode is strongly affected by class interval whereas the mean and median are not in the present case. The mean is most commonly used measure of central tendency because it is an unbiased and consistent estimator of the mean of any population and a data set always has only one mean. However, the mean is more affected by extreme values than is the median, since each possible value directly affects the mean while only the relative magnitudes affect the median.

Schematic descriptions of the relationship between mean, mode and median are illustrated in Figure 8.22.
Figure 8.22 Schematic description of the relationship between mean, mode and median for three types of distributions. (8-20)

(a) right-skewed distribution
(b) left-skewed distribution
(c) symmetrical unimodal distribution
For skewed distributions, the median is considered a more appropriate sample descriptor for central tendency than the mean, because the mean is pulled away from the central region in the direction of skewness. According to Table 8.2, both the mean and median increase with spreading the carbon filaments and both can be representative values of $UA_{1}/UA_{2}$ for each carbon fibre distribution. Obviously, every distribution of $UA_{1}/UA_{2}$ is right-skewed as shown in Figure 8.21. Therefore, the median is considered a better measure of central tendency in the present distribution. Before using the median as a representative value of each fibre distribution, the difference of the median between two distributions, i.e. (a) and (b), (b) and (c), (c) and (d) in Figure 8.21, will have to be proved to be significant statistically.

8.3.5. The Mann-Whitney two-sample test

The Mann-Whitney two-sample test is a test of the equality or inequality of two means or medians. For a general example, random samples of sizes $n_1$ and $n_2$ are selected independently from populations 1 and 2. The test is a nonparametric test and it does not require that samples be drawn from normal populations.

The first step in the Mann-Whitney test is to assign size rank numbers with ascending or descending to the $n_1+n_2$ sample items. The sum of the ranks for a sample, $R_s$, is called its rank sum i.e. $R_{s1}$ is a sum of the $n_1$ sample ranks and $R_{s2}$ is that of $n_2$ sample ranks. Here we suppose that a pair of samples of sizes $n_1$ and $n_2$ are drawn from the same population. Then $R_{s1}$ and $R_{s2}$ for samples are computed. Next suppose we repeat the process with another pair of samples, over and over again, to generate the sampling distributions of $R_{s1}$ and $R_{s2}$ for samples of size $n_1$ and $n_2$. These sampling distributions will have means and standard deviations. By algebra, it can be expressed that $\mu_{R_{s1}}$, the mean of the distribution of $R_{s1}$ is (8-20):

$$\mu_{R_{s1}} = \frac{n_1(n_1 + n_2 + 1)}{2}$$

(8-10)

and the mean $\mu_{R_{s2}}$ of the distribution $R_{s2}$ is:
Also, both distributions have the same standard deviation $SD_{R_s}$, it is calculated with the following formula.

$$SD_{R_s} = \sqrt{\frac{n_1 n_2 (n_1 + n_2 + 1)}{12}}$$  \hspace{1cm} (8-12)

when both $n_1$ and $n_2$ are at least 10, the distributions of $R_{s1}$ and $R_{s2}$ can be approximated by normal distributions that have the mean and the standard deviation stated above.

The test can be performed on the null hypotheses $H_0: \mu_1 = \mu_2$, or $H_0: \mu_1 \leq \mu_2$, or $H_0: \mu_1 \geq \mu_2$, where $\mu_1$ and $\mu_2$ are means of populations 1 and 2, respectively. Suppose it is determined, at significance level $\delta$, whether the mean of population 2 is greater than the mean of population 1. The hypotheses are

$$H_0: \mu_1 \geq \mu_2 \quad H_a: \mu_1 < \mu_2$$  \hspace{1cm} (8-13)

where $H_a$ is the alternative hypothesis and it is true if $H_0$ is false. The decision rule is:

Reject $H_0$ if sample $z > z_\delta$  \hspace{1cm} (8-14)

Independent random samples of sizes $n_1$ and $n_2$ from the populations are selected. Then the $n_1 + n_2$ sample item are ranked in order of size, and the rank sum $R_{s2}$ is computed. Also, the test statistic is computed as follows.

$$Sample\; z = \frac{R_{s2} - \mu_{R_{s2}}}{SD_{R_s}}$$  \hspace{1cm} (8-15)

Finally, we accept or reject $H_0$ according to the decision rule (8-14).

The Mann-Whitney test has been carried out between the distributions of PCN (a) and PC2 (b), PC2 (b) and PC4 (c), and PC4 (c) and PC6 (d) in Figure 8.21 in accordance with the procedure described above. For each comparison, the rank sum $R_s$, the mean $\mu_{R_s}$ of the distribution of $R_s$, the standard deviation, $SD_{R_s}$, and the test statistic, Sample $z$ were computed as presented in Table 8.3.
Table 8.3 Results of the Mann-Whitney two-sample test.

<table>
<thead>
<tr>
<th></th>
<th>PCN and PC2</th>
<th>PC2 and PC4</th>
<th>PC4 and PC6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sample size : n</td>
<td>200</td>
<td>230</td>
<td>192</td>
</tr>
<tr>
<td>Rank sum : Rₜ</td>
<td>22943</td>
<td>69722</td>
<td>33824</td>
</tr>
<tr>
<td>Mean of dist. Rₜ : μₚₜ</td>
<td>43100</td>
<td>48645</td>
<td>38592</td>
</tr>
<tr>
<td>Standard deviation of dist. Rₜ : SDₚₜ</td>
<td>1285.37</td>
<td>1247.65</td>
<td>1159.43</td>
</tr>
<tr>
<td>Sample z</td>
<td>15.68</td>
<td>3.02</td>
<td>4.11</td>
</tr>
</tbody>
</table>

For example, the population PCN will be compared with the population PC2. The null hypothesis is the mean of population PCN, μₚₙ, is greater than the mean of population PC2, μₚ₂.

H₀ : μₚₙ ≥ μₚ₂

Thus the alternative hypothesis Hₐ is :

Hₐ : μₚₙ < μₚ₂

The test is one-tailed at significance level δ = 0.05, hence the decision rule is :

Reject H₀ if sample z > 1.64

because z₀ = z₀.05 = 1.64 from APPENDIX 4.

The sample sizes of nₚₙ and nₚ₂ are 200 and 230, respectively. After all 200+230=430 values have been ranked with ascending order from 1 for the smallest to 430 for the largest, the rank sums are computed.

Rₚₙ(ₚₙ) = 22943

Rₚ₂(ₚ₂) = 69722

From equation (8-11), the mean μₚₙ(ₚ₂) of the distribution of Rₚ₂(ₚ₂) is :

μₚₙ(ₚ₂) = nₚ₂(nₚₙ + nₚ₂ + 1) / 2

= 49565

The standard deviation SDₚₙ is calculated with equation (8-12).

SDₚₙ = \sqrt{nₚₙnₚ₂(nₚₙ + nₚ₂ + 1) / 12} = 1285.37
Then the test statistics computed:

\[
Sample: z = \frac{R_s(PC_2) - \mu_{R_s(PC_2)}}{SD_{R_s}} = 15.68
\]

It is found that the sample \( z = 15.68 \) is greater than the 1.64 in the decision rule, thus the null hypothesis \( H_0 \) can be rejected. Rejecting \( H_0 \) means that the mean of population PC2 is greater than the mean of population PCN, \( \mu_{PCN} < \mu_{PC2} \).

In the same way, following inequalities can be obtained.

\[
\begin{align*}
\mu_{PC2} &< \mu_{PC4} \\
\mu_{PC4} &< \mu_{PC6}
\end{align*}
\]

Therefore, we can conclude:

\[
\mu_{PCN} < \mu_{PC2} < \mu_{PC4} < \mu_{PC6} \tag{8-16}
\]

In the Mann-Whitney test, the median is an alternative to the mean. Inequalities (8-16) can be rewritten as:

\[
M_{d(PCN)} < M_{d(PC2)} < M_{d(PC4)} < M_{d(PC6)} \tag{8-17}
\]

where \( M_d \) is the median of each population in Figure 8.21 (a), (b), (c) and (d). Subscripts PCN, PC2, PC4 and PC6 stand for carbon fibre distribution. Consequently, inequalities (8-17) have been proved with the Mann-Whitney test and the medians \( M_{d(PCN)}, M_{d(PC2)}, M_{d(PC4)} \) and \( M_{d(PC6)} \) can be used as representative values of \( UA_f/UA_{fn} \) for each distribution of the carbon fibre.

In order to have better prediction of the tensile behaviour for the hybrid composite, further discussion about the possible usage of the ratio of \( UA_f/UA_{fn} \) will be described in section 8.4.2.
8.4. Model application and efficiency factor of fibre distributions

8.4.1. Theoretical tensile properties

The concept of the mathematical model for tensile properties is exactly the same as that described in Chapter 7. The properties of the carbon fibre have been substituted instead of the glass fibre's.

Modulus of elasticity and bend over point (BOP)

The modulus of elasticity $E_{cp}$ and the stress at BOP $\sigma_{cp}$ are given by:

$$E_{cp} = E_m V_m + E_{fp} V_{fp} + E_{fc} V_{fc}$$  (8-18)

$$\sigma_{cp} = \varepsilon_{mu} E_{cp}$$  (8-19)

The superscript $cp$ stands for the carbon/polypropylene fibre reinforced cement composite, and the subscript $fc$ stands for the carbon fibre. Theoretical lines of the modulus of elasticity, $E_{cp}$ and the stress at BOP, $\sigma_{cp}$ are shown in Figure 8.23 and 8.24, respectively. The following values were used.

- $E_m = 31.6$ GPa (shown in section 2.2.2)
- $E_{fp} = 10$ GPa (at around 1% strain on Figure 2.4 in Chapter 2)
- $E_{fc} = 228$ GPa (shown on Table 2.7 in Chapter 2)
- $\varepsilon_{mu} = 273 \times 10^{-6}$ (shown in section 2.2.2)
Figure 8.23  Theoretical lines of the modulus of elasticity, $E_{c,pp}$, for CF/PP composites.

Figure 8.24  Theoretical lines of the stress at BOP $\sigma_{c,pp}$ for CF/PP composites.
Strain in fibre and stress transfer length in the multiple cracking region

In the multiple cracking region, the following six equations are established

\[
(P_{fp} \tau_p + P_{fc} \tau_c) \frac{V_{fp}}{A_{fp}} = E \frac{V_{fc}}{A_{fc}} \epsilon_{cp} \tag{8-20}
\]

\[
P_{fp} \tau_p \frac{V_{fp}}{A_{fp}} \epsilon_{cp} + P_{fc} \tau_c \frac{V_{fc}}{A_{fc}} = E \frac{V_{m}}{A_{m}} \epsilon_{mu} + E \frac{V_{fp}}{A_{fp}} (\epsilon_{mu} - \epsilon_{cp}) \tag{8-21}
\]

\[
\epsilon_{cp} V_{fp} E_{fp} + \epsilon_{cp} V_{fc} E_{fc} = \epsilon_{mu} E_{cp} \tag{8-22}
\]

\[
\epsilon_{cp} (V_{fp} E_{fp} + V_{m} E_{m}) + \epsilon_{cp} V_{fc} E_{fc} = \epsilon_{mu} E_{cp} \tag{8-23}
\]

\[
x_p (\epsilon_{cp} - \epsilon_{mu}) = x_c (\epsilon_{cp} - \epsilon_{cp}) \tag{8-24}
\]

\[
\epsilon_{cp} = \epsilon_{cp} - (\epsilon_{cp} - \epsilon_{mu}) \frac{x_p}{x_c} \tag{8-25}
\]

Since the diameter of the carbon filament is 7\(\mu\)m (Table 2.7), the perimeter and the cross section area of the carbon, \(P_{fc}\) and \(A_{fc}\), are 7\(\pi\)\(\mu\)m and 12.25\(\pi\)\(\mu\)m\(^2\), respectively. Thus,

\[
A_{fc} / P_{fc} = 1.75 \times 10^{-3} \text{mm} (\frac{P_{fc}}{A_{fc}} = 571.4 \text{mm}^{-1})
\]

This value is obtained on the assumption that the carbon filaments are dispersed individually without touching each other, otherwise it is impossible to determine actual values as described in section 8.3.1. The frictional shear bond strength of the carbon fibre \(\tau_c\) was calculated from the crack spacing for the composite reinforced with the carbon fibre alone in the from of six layers using equation (8-26).

\[
\tau_c = 1.364 \frac{V_m V_{fc} \sigma_{mu} A_{fc}}{C P_{fc}} \tag{8-26}
\]

Substituting \(V_m = 0.9937, V_{fc} = 0.0063, \sigma_{mu} = 7.67 \text{MPa}\) (obtained from experiments), \(C = 2.4 \text{mm}\) (obtained from experiments) and \(A_{fc} / P_{fc} = 1.75 \times 10^{-3} \text{mm}\),

\[
\tau_c = 1.203 \text{MPa}
\]

On the basis of equation (8-26), Sarker and Bailey\(^{84}\) and Akihama et al\(^{82}\)
calculated the bond strength $\tau_c$ to be about 0.8MPa, with fibres of 9.2\,\mu m diameter. The bond strength $\tau_c$ of about 2.4MPa might be calculated from the data obtained by Aveston et al.\(^{8-3}\) assuming the same fibre diameter (9.2\,\mu m) as the former authors'. The obtained bond strength, $\tau_c = 1.203\text{MPa}$, seems to be reasonable, because it is in the range of 0.8 to 2.4MPa. Theoretical curves of $\varepsilon_p^\text{CP}$, $\varepsilon_c^\text{CP}$, $\varepsilon_p^\text{CP}$, $\varepsilon_c^\text{CP}$, $x_p^\text{CP}$ and $x_c^\text{CP}$ are shown in Figures 8.25 to 8.30, respectively.

**Figure 8.25** Theoretical curves of $\varepsilon_p^\text{CP}$ for the CF/PP composites.
Figure 8.26  Theoretical curves of $\varepsilon_{c0}^{\text{cp}}$ for the CF/PP composites.

Figure 8.27  Theoretical curves of $\varepsilon_{\text{p1}}^{\text{cp}}$ for the CF/PP composites.
Figure 8.28  Theoretical curves of $\varepsilon_{ci}^{sp}$ for the CF/PP composites.

Figure 8.29  Theoretical curves of $x_{p}^{sp}$ for the CF/PP composites.
Crack spacing and strain at the end of the multiple cracking region

According to the developed mathematical model described in section 7.5.2, the composite is completely separated into segments of length between $x_{cp}$ and $2x_{cp}$. When $V_{fp}=0.04$ and $V_{fc}=0.006$, $x_{cp}=1.43\text{mm}$ in Figure 8.30. The experimental crack spacing is altered by the carbon fibre distribution as shown in Figure 8.8, and the average crack spacings for the composite with four carbon fibre layers and six layers are 1.46mm and 1.37mm, respectively from the experiments. Therefore, the actual crack spacing is close to $x_{cp}$ and the theoretical minimum crack spacing of the CF/PP hybrid composite is represented as $x_{cp}$ in Figure 8.30.

The strain at the end of the multiple cracking region $\varepsilon_{mc}^{cp}$ is given by:

$$\varepsilon_{mc}^{cp} = \frac{3\varepsilon_{c0}^{cp} + \varepsilon_{mm}}{4}$$  (8-27)
Theoretical curves of $\varepsilon_{mc}^{sp}$ are shown in Figure 8.31.

**Figure 8.31** Theoretical curves of $\varepsilon_{mc}^{sp}$ for the CF/PP composites.

**Second elastic modulus and first yield point (FYP)**

The second elastic modulus $E_2^{sp}$ is given by:

$$E_2^{sp} = E_{fp} V_{fp} + E_{fc} V_{fc}$$  \hspace{1cm} (8-28)

Assuming $E_{fp}=8.3$ GPa (at 1.5% strain on Figure 2.4 in Chapter 2) and $E_{fc}=228$ GPa, theoretical lines of $E_2^{sp}$ are presented in Figure 8.32. Also, experimental values for PC2, PC4 and PC6 are plotted in the case of $V_{fp}=4\%$ and $V_{fc}=0.6\%$. The plots for PC6 are closer to the theoretical line than others, however, all the plots are below the line.
Figure 8.32 Theoretical lines of $E^p_C$ for the CF/PP composites.

The stress and strain at FYP, $\sigma^p_u$ and $\varepsilon^p_u$, are obtained by following equations.

$$
\sigma^p_u = E^p_f V^{p} (\varepsilon^p_P + \varepsilon_{fcu} - \varepsilon^{p_u}) + \sigma_{fcu} V_f
$$

$$
\varepsilon^p_u = \varepsilon_{fcu} - \frac{1}{4} (\varepsilon^{p_u} - \varepsilon_{mu})
$$

where $\sigma_{fcu}$ and $\varepsilon_{fcu}$ are the ultimate stress and strain of the carbon fibre. Assuming $\sigma_{fcu}=3790$ MPa (shown on Table 2.7 in Chapter 2) and $\varepsilon_{fcu}=\sigma_{fcu}/E_f=1.66\%$, theoretical curves of $\sigma^p_u$ and $\varepsilon^p_u$ are shown in Figures 8.33 and 8.34, respectively. Theoretical boundaries considering the critical fibre volume of the polypropylene at FYP are marked by asterisks. Beyond asterisks, theoretical ultimate stress $\sigma^p_u$ and ultimate strain $\varepsilon^p_u$ in the case of $V_f<(FYP) V_{f(p, crit)}$ is presented as broken lines in the same manner as in Figure 7.32. The experimental values are also lower than theoretical values, which implies that the inherent properties of $\sigma_{fcu}$ and $\varepsilon_{fcu}$ for the carbon fibre are not attained even in PC6 consisting of six carbon fibre layers. For a continuous unidirectional reinforcement, the orientation and fibre length efficiency factor can be ignored. Thus, the fibre distribution factor may have to be considered as described in section 8.3.
Figure 8.33  Theoretical curves of $\sigma_{ul}^{cp}$ for the CF/PP composites.

Figure 8.34  Theoretical curves of $\varepsilon_{ul}^{cp}$ for the CF/PP composites.
The third elastic modulus $E^{cp}_{3}$ and the ultimate stress $\sigma^{cp}_{u}$ for the CF/PP composite are exactly same as the GF/PP composite’s, because these are functions of the polypropylene fibre volume fraction $V_{fp}$ alone. (Refer to Figures 7.36 and 7.37)

$$E^{cp}_{3} = E_{fp}V_{fp}$$  
(8-31)

$$\sigma^{cp}_{u} = \sigma_{fsp}V_{fp}$$  
(8-32)

The ultimate strain $\varepsilon^{cp}_{u}$ is given by:

$$\varepsilon^{cp}_{u} = \varepsilon_{fsp} - \frac{3}{4}\varepsilon^{0}_{c} + \frac{1}{4}\varepsilon_{nu}$$  
(8-33)

Assuming $\varepsilon_{fsp}=8.1\%$, theoretical curves of $\varepsilon^{cp}_{u}$ are shown in Figure 8.35. The asterisks are theoretical boundaries from consideration of the critical fibre volume of the polypropylene at FYP. When the carbon fibre volume fraction is more than the asterisk point in each curve, the ultimate strain $\varepsilon^{cp}_{u}$ reduces to the strain presented as broken lines in Figure 8.34.

Figure 8.35  Theoretical curves of $\varepsilon^{cp}_{u}$ for the CF/PP composites.
8.4.2. Efficiency factor of the fibre distribution

The mechanical properties of fibre reinforced composites are mainly affected by the fibre length, the orientation of the fibres and the fibre-matrix shear bond strength when the fibre volume fraction is constant. These three factors are not independent, because the effects of both fibre length and orientation are partly dependent on the bond strength. Such efficiency factors can be determined either empirically or on the basis of analytical calculations. For the ACK model, the effect of fibre length on the multiple cracking process was treated by Laws and Proctor. They indicated that the effective rigidity of the fibre reinforcement was reduced by the greater extension of the fibres due to the additional load imposed on them during matrix cracking. In terms of the effects of fibre orientations, Aveston and Kelly extended the ACK model for random two and three dimensional fibre distribution. They showed mathematically that the minimum crack spacing was predicted to be greater than that obtained with aligned fibres by about 50% for fibres in two dimensions and 100% for a three dimensional fibre distribution.

In the present study, since the fibres are continuous and aligned to the direction of applied load, the effects of fibre length and orientation have been neglected. However, as described in section 8.2.2, the tensile properties were found to be affected by the carbon fibre distribution. It is well known in practice that the fibre distribution and dispersion is one of the important factors which affects the mechanical properties of the composite. Many investigators have made efforts to obtain better fibre distribution. Nevertheless, none of them has proposed an efficiency factor for fibre distribution. In section 8.3, the ratio of the ideal fibre area fraction to the actual fibre area fraction in a unit cell, $UA_{fi}/UA_{m}$, was suggested as a possible criterion for specifying the fibre distribution.

As shown in Figures 8.9, 10 and 11, the second elastic modulus $E^{sp}$, and stress
and strain at FYP, $\sigma_{\text{exp}}^{\text{cp}}$ and $\varepsilon_{\text{exp}}^{\text{cp}}$, are clearly affected by the carbon fibre distribution. The same parameters are therefore discussed considering the distribution factor $U_{\text{Af}}/U_{\text{Af}}$ in Figures 8.36, 8.37 and 8.38.

In many theoretical and empirical applications, efficiency factors are represented as values between 0 to 1, expressing the ratio between the actual value and the ideal value. Thus the ratios of the experimental values to the theoretical values were plotted against the fibre distribution factor $U_{\text{Af}}/U_{\text{Af}}$. The representative value of $U_{\text{Af}}/U_{\text{Af}}$ for each carbon fibre distribution was taken as the median for each distribution as discussed in section 8.3.4. The plots for the second elastic modulus $E_{\text{cp}}^{\text{exp}}$, the stress and strain at FYP, $\sigma_{\text{exp}}^{\text{cp}}$ and $\varepsilon_{\text{exp}}^{\text{cp}}$ are shown in Figures 8.36, 8.37 and 8.38, respectively. The subscripts (exp.) and (th.) stand for experimental values and theoretical values. Assuming that the theoretical value can be attained when the fibre distribution is ideal, i.e. $U_{\text{Af}}/U_{\text{Af}} = 1$, logarithmic regression curves were applied for these plots. The results of the regression analysis are presented in Table 8.4.

![Figure 8.36 Relationship between fibre distribution parameter $U_{\text{Af}}/U_{\text{Af}}$ and $E_{\text{cp}}^{\text{2(exp)}}/E_{\text{cp}}^{\text{2(th)}}$.](image)

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Figure 8.37  Relationship between fibre distribution parameter $\frac{UA_f}{UA_{th}}$ and $\sigma^{PP}_{u,l(exp)} / \sigma^{PP}_{u,l(th)}$. 

Figure 8.38  Relationship between fibre distribution parameter $\frac{UA_f}{UA_{th}}$ and $\varepsilon^{PP}_{u,l(exp)} / \varepsilon^{PP}_{u,l(th)}$. 

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Table 8.4 Logarithmic regression analysis for $E_{\text{exp}}^{\text{CP}}/E_{\text{th}}^{\text{CP}}$, $\sigma_{\text{ul,exp}}^{\text{CP}}/\sigma_{\text{ul,th}}^{\text{CP}}$ and $\varepsilon_{\text{ul,exp}}^{\text{CP}}/\varepsilon_{\text{ul,th}}^{\text{CP}}$ vs $\frac{UA_f}{UA_n}$.

$$\eta_{fd}^{\text{CP}} = a + b \ln\left(\frac{UA_f}{UA_n}\right)$$

<table>
<thead>
<tr>
<th>$\eta_{fd}^{\text{CP}}$</th>
<th>a</th>
<th>b</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{E_{\text{exp}}^{\text{CP}}}{E_{\text{th}}^{\text{CP}}}$</td>
<td>1.02</td>
<td>0.146</td>
<td>0.832</td>
</tr>
<tr>
<td>$\frac{\sigma_{\text{ul,exp}}^{\text{CP}}}{\sigma_{\text{ul,th}}^{\text{CP}}}$</td>
<td>0.97</td>
<td>0.137</td>
<td>0.944</td>
</tr>
<tr>
<td>$\frac{\varepsilon_{\text{ul,exp}}^{\text{CP}}}{\varepsilon_{\text{ul,th}}^{\text{CP}}}$</td>
<td>1.04</td>
<td>0.159</td>
<td>0.967</td>
</tr>
</tbody>
</table>

$R^2$: correlation coefficient

The efficiency of fibre reinforcement is often assessed on the basis of the enhancement in strength. Thus the efficiency factor of the fibre distribution for the CF/PP composite, $\eta_{fd}^{\text{CP}}(\sigma)$, is defined as the ratio of the actual stress to the theoretical stress, i.e. $\eta_{fd}^{\text{CP}}(\sigma_{ul}) = \frac{\sigma_{\text{ul,exp}}^{\text{CP}}}{\sigma_{\text{ul,th}}^{\text{CP}}}$. From the assumption of $\sigma_{\text{ul,exp}}^{\text{CP}} = \sigma_{\text{ul,th}}^{\text{CP}}$ for $\frac{UA_f}{UA_n}=1$, the constant of the regression equation must be 1, i.e. $a=1$ in Table 8.4. Therefore, the efficiency factor of the fibre distribution for the stress at FYP, $\eta_{fd}^{\text{CP}}(\sigma_{ul})$ is:

$$\eta_{fd}^{\text{CP}}(\sigma_{ul}) = 1 + 0.137 \ln\left(\frac{UA_f}{UA_n}\right) \quad (8-34)$$

In the same way, the efficiency factor of the fibre distribution for the strain at FYP, $\varepsilon_{ul}^{\text{CP}}$ and the second elastic modulus $E_{\text{exp}}^{\text{CP}}$ can be expressed as follows.

$$\eta_{fd}^{\text{CP}}(\varepsilon_{ul}) = 1 + 0.159 \ln\left(\frac{UA_f}{UA_n}\right) \quad (8-35)$$

$$\eta_{fd}^{\text{CP}}(E_{\text{exp}}) = 1 + 0.146 \ln\left(\frac{UA_f}{UA_n}\right) \quad (8-36)$$

These efficiency factors are available only for each particular properties, $\sigma_{\text{ul}}^{\text{CP}}$, $\varepsilon_{\text{ul}}^{\text{CP}}$ and $E_{\text{exp}}^{\text{CP}}$. The properties in the elastic region, $\sigma_{cc}^{\text{CP}}$ and $E_{c}^{\text{CP}}$, and the ultimate stress
of the CF/PP composite are not affected by the carbon fibre distribution (refer to Figures 8.6, 8.7, 8.12). The first yield point and the second elastic modulus are characteristic properties for the hybrid composite containing two types of fibre and hence the efficiency factor $\eta_{fd}$ may be dependent upon the combination of the fibre volume fractions. However, as shown in Figures 8.36, 37 and 38, it is notable that the properties of the composite improve very rapidly in the region of $0 < U_{Af}/U_{afn} < 0.2$ when more than 70% of the theoretical value is attained. In the region of $U_{Af}/U_{afn} > 0.2$, the rate of improvement becomes much slower than that in the initial region. It is impossible, in fact, to obtain perfect dispersion of fibres in a composite and composites which show 70-80% of the theoretical properties should be good enough as industrial products. Therefore, the perfect distribution, i.e. $U_{Af}/U_{afn} = 1$, is not necessary for a good composite. In order to obtain good quality of a composite, it is important to avoid poor fibre distribution, i.e. $U_{Af}/U_{afn} < 0.1$.

It should be remembered that the usage of the efficiency factor of the fibre distribution obtained here is limited and yet no theoretical foundation has been given. Nevertheless, the concept of the ratio between ideal and actual fibre area fraction in a unit cell, $U_{Af}/U_{afn}$, can give us a quantitative criterion for the fibre distribution in the composite. Also, the possible fibre distribution factors have been suggested related to the ratio of $U_{Af}/U_{afn}$. These concepts may provide guidelines for the design and development of hybrid fibre reinforced cement composites in which the strength can be optimized by modifying the reinforcing fibre distribution.
Continuous PAN type carbon fibres have been used as high stiffness fibres together with continuous polypropylene networks in cement based composites. They indicated a remarkable enhancement in tensile strength and their hybrid effect was superior to the glass fibres up to about 1% strain. However, it was found that the effect of the carbon fibre was considerably dependent upon the fibre distribution. Based on the fibre spacing theory proposed by Romualdi and Mandel[8-17], the carbon fibre distribution was quantified by using an image analyser. Although an efficiency factor for the fibre distribution is not yet applicable for general use, it is an essential parameter for the future precise prediction of composite performance.
CHAPTER 9

9. EFFECTS OF AUTOCLAVING ON CEMENT COMPOSITES CONTAINING POLYPROPYLENE, GLASS AND CARBON FIBRES

9.1. Introduction

All concrete materials are required to be cured in a suitable environment to obtain an adequate strength. Since an increase in the curing temperature of concrete increases its rate of development of strength, the gain of strength can be accelerated by curing in steam. High-pressure steam curing known as autoclaving was used in this chapter. Autoclaved concretes have several characteristics, as follows:

1. The ultimate strength can be obtained with short curing times (say within a day).

2. Efflorescence is less than general concretes because there is little water soluble Ca(OH)_2 in autoclaved concretes.

3. Shrinkage and creep are much smaller than general concretes cured in a humid condition.

In autoclaved concretes, different hydration products from those cured wet at atmospheric pressure are obtained. Thus, mix proportions have to be determined corresponding to the required properties of concretes. Autoclaving is curing with saturated steam at high temperature and pressure over 100°C and generally carried out at a range of temperature from 160°C to 180°C. Therefore, the heat-resistant properties of the reinforcing fibre have to be considered.

Since the softening point of polypropylene is in the region of 150°C approximately and the fibres melt at 160-170°C, polypropylene generally cannot be used for autoclaved composites. Also, rapid oxidation could occur at these high temperatures.
temperatures if oxygen is present adjacent to the fibre. Tests in an oven in dry air at 170°C for 8 h showed complete embrittlement of the composite with extensive chain scission of the polymer. However, there is evidence\textsuperscript{9,3} that the situation could be different in autoclaved products. The work has practical significance because if polypropylene can maintain its properties as a reinforcement in a composite even after autoclaving at around its melting temperature, the applicable field for polypropylene composites can be extended.

In this study, composites containing polypropylene alone, polypropylene and continuous glass fibres, and polypropylene and continuous carbon fibres were subjected to curing in an autoclave. The tensile stress-strain curves for the various composites were then compared.

9.2. Experimental programme

9.2.1. Materials

The mix proportions of the matrix are the same as those used in previous chapters (Table 2.1 in Chapter 2) in order to compare with the hybrid composite cured under water. This matrix was not therefore optimised to be the best matrix for autoclaving because in this work the main interest was in the integrity of the fibres at high temperatures.

Hydrated silicates in the form of tobermorite are one of the most stable crystals in hydrated products which are produced with autoclaving. The more tobermorite that is produced, the smaller the shrinkage is and the stronger the matrix that can be obtained. The product is varied by the molar ratio of CaO/SiO\textsubscript{2} and the best range of CaO/SiO\textsubscript{2} is 0.7-1.0\textsuperscript{9,4,9,5}. In this work, curing in water was a standard and the ratio of CaO/SiO\textsubscript{2} was not considered for autoclaving. Assuming that the content of CaO and SiO\textsubscript{2} in a typical ordinary Portland cement are 63% and 20%\textsuperscript{9,6} respectively, the ratio of CaO/SiO\textsubscript{2} is approximately 1.73.
With this ratio, since the lime is in surplus, $\alpha$-2CaO·SiO$_2$·H$_2$O is produced alone or coprecipitated with Ca(OH)$_2$ or tobermorite$^{(9-4)}$$^{(9-5)}$ and the strength of the matrix is expected to be reduced.

The polypropylene networks and glass fibres were the same type as described in Chapter 2. The carbon fibres were continuous and consisted of strands with 2000 filaments of 10μm diameter. The type was HM20-1091 supplied by Petoca Ltd. Japan, which was pitch-based carbon fibre. The properties are shown in Table 9.1.

Table 9.1 General properties of Petoca carbon fibre from trade literature.

<table>
<thead>
<tr>
<th>Properties</th>
<th>HM20-1091</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tensile strength (MPa)</td>
<td>2330</td>
</tr>
<tr>
<td>Tensile modulus (GPa)</td>
<td>210</td>
</tr>
<tr>
<td>Elongation (%)</td>
<td>1.1</td>
</tr>
<tr>
<td>Weight per roving (g/m)</td>
<td>0.31</td>
</tr>
<tr>
<td>Density (g/cm$^3$)</td>
<td>1.95</td>
</tr>
<tr>
<td>Filament diameter (μm)</td>
<td>10</td>
</tr>
<tr>
<td>Filament count</td>
<td>2000</td>
</tr>
</tbody>
</table>
9.2.2. Manufacture of composites and autoclaving

The composites were made by a hand lay up technique with reinforcement patterns as shown in Table 9.2.

Table 9.2 Description of the composite reinforcement.

<table>
<thead>
<tr>
<th>Fibre volume fraction</th>
<th>Fabrication</th>
</tr>
</thead>
<tbody>
<tr>
<td>PP (%)</td>
<td>GF (%)</td>
</tr>
<tr>
<td>----------------------</td>
<td>------------</td>
</tr>
<tr>
<td>3.7</td>
<td>-</td>
</tr>
<tr>
<td>4.6</td>
<td>-</td>
</tr>
<tr>
<td>3.5</td>
<td>1.2</td>
</tr>
<tr>
<td>3.7</td>
<td>-</td>
</tr>
</tbody>
</table>

PP: Polypropylene networks
GF: Glass fibres (64 loose strands/bundle)
CF: Carbon fibres

The 64 strands in each glass fibre bundle were not bonded together but were spread out during specimen manufacture to ensure good contact with the cement paste. Each fibre layer was penetrated by cement paste before the next layer was added. Thickness of the sheets was generally between 5 and 6mm, small thickness variations accounting for minor changes in fibre volume. The sheets were cut into strips 300mm long and 25mm wide after curing in water for 7 days and were sent to the Research Centre of the Asahi Glass Company Ltd. They were autoclaved with the temperature-time schedule as shown in Figure 9.1.
Figure 9.1 Temperature-time schedule for autoclaving.

Before increasing the temperature, the pressure in the vessel was reduced to -600 mmHg and maintained for 4 h. This treatment made the quantity of oxygen in the vessel low, which was thought to prevent the decomposition of the polypropylene by oxidation. Then the temperature was increased up to 170°C in 2 h and maintained for 9 h. After that, it was left to cool down below 100°C. The control specimens were cured in water at 20°C for about 60 days.

The tensile tests were carried out and the crack spacings were determined after specimen failure.
9.3. Results and discussion

The general effects at 20°C of the addition of continuous glass or carbon fibres to the polypropylene reinforcement are shown in Figure 9.2.

![Tensile stress-strain curves for composites cured in water.](image)

**Figure 9.2** Tensile stress-strain curves for composites cured in water.

(PP: polypropylene, GF: glass fibre, CF: carbon fibre.)

The stiffer reinforcements give considerably improved post-cracking performance up to 1.5% strain but have reduced strain to failure. With respect to the polypropylene and glass fibre composite, after the glass fibres have failed, the polypropylene cannot sustain the applied load due to insufficient polypropylene fibre volume. On the other hand a small reduction after peak stress can be seen in the stress-strain curve for the hybrid polypropylene/carbon fibre composite. This reduction is thought to be caused by the local fracture of the carbon fibres. After the peak stress, the applied load has been transferred to the polypropylene networks to some extent, resulting in the ultimate strain of the polypropylene and carbon fibre...
composite (about 2%) being larger than the ultimate elongation of the carbon fibre (1.11%). However, the polypropylene fibre volume was not sufficient to allow the composite to be strained up to the 5% strain of the polypropylene-only composite.

Direct comparisons of stress-strain curves for water cured and autoclaved samples from the same sheets are shown in Figures 9.3, 9.4 and 9.5.

Figure 9.3 shows the curves for samples containing 4.6% by volume of polypropylene networks only. It was rather surprising that in spite of the high curing temperature of 170°C, which is just over the melting point of unstretched polypropylene, the composite containing polypropylene still maintained its ductility on cooling even after 9 h at the elevated temperature. An explanation for this effect may be that although the polypropylene is at or above its melting temperature, the molecules in the stretched chains do not actually separate at this point but are...
effectively held in their positions in the thin channels in the cement paste. On cooling therefore a high proportion of the unheated properties can be maintained or re-established. However, since the ultimate stress and strain of the autoclaved composite were lower than those of the standard composite, the polypropylene was thought to be damaged by the high temperature to a certain extent. The stress for the autoclaved composite at the bend over point was reduced and the stress was generally lower than that of the standard composite at a given strain. The strength of the autoclaved matrix was also lower than that of the standard composite probably because the ratio of CaO/SiO₂ was too high to produce tobermorite.

Figure 9.4 Comparison between a standard and an autoclaved specimen reinforced with 3.5vol% of polypropylene and 1.2vol% of glass fibres.

Figure 9.4 shows a comparison between a standard and an autoclaved specimen reinforced with polypropylene networks and glass fibres. In the stress-strain curve for the autoclaved composite, only the horizontal region could be seen
after the bend over point and no enhancement of properties by the glass fibre was obtained. The horizontal part of the curve showed that the combined strengths of glass and polypropylene fibres had reduced to about the critical fibre volume. As it was known from Figure 9.3 that the polypropylene had only reduced in strength by about 17%, considerable weakening of the glass fibres must have occurred. Although the softening point of the glass is 500-700°C, the glass fibres were thought to be seriously damaged by the heat and alkali attack during autoclaving. The residual strain capacity was provided by the polypropylene networks.

![Stress vs Strain Graph](image)

**Figure 9.5** Comparison between a standard and an autoclaved specimen reinforced with 3.7vol% of polypropylene and 0.6vol% of carbon fibres.

Figure 9.5 shows the comparison between autoclaved and water cured samples containing a hybrid of carbon and polypropylene fibres. The autoclaved composite failed at less than 1% strain, because the volume fraction of the polypropylene was not enough to carry the additional load after carbon fibre failure.
and the polypropylene was slightly weakened by the heat during autoclaving. Apart from the damage to the polypropylene and the matrix, the carbon fibre seems to still play a role as a reinforcement. Therefore, the carbon fibre can be used in the autoclaved composite.

Representative tensile properties are shown in Table 9.3. The strain to first crack in the composite ($\varepsilon_{\text{mu}}$) was only slightly altered by autoclaving whilst the strength of the matrix ($\sigma_{\text{mu}}$) was reduced by about 20% although still being of satisfactory strength at about 7.5MPa.

The reduction of the elastic modulus of the composite $E_{\text{c}}$ was caused by both the reduced modulus of the matrix and the fibres due to autoclaving. As a result of autoclaving, the strength and stiffness of the polypropylene networks were reduced, which resulted in the reduction of the ultimate stress and strain.

The strength of the polypropylene ($\sigma_{\text{fpu}}$) in Table 9.3 was reduced from about 345 to 309 MPa showing that some changes in the molecular structure had occurred.
Table 9.3 Representative tensile properties of standard and autoclaved composites.

<table>
<thead>
<tr>
<th>Sample no.</th>
<th>$V_{fp}$ ($%$)</th>
<th>$V_{fg}$ ($%$)</th>
<th>$V_{FC}$ ($%$)</th>
<th>$\varepsilon_{mu}$ ($10^6$)</th>
<th>$\sigma_{mu}$ (MPa)</th>
<th>$\sigma_{ec}$ (MPa)</th>
<th>$E_c$ (GPa)</th>
<th>$\varepsilon_u$ (%)</th>
<th>$\sigma_u$ (MPa)</th>
<th>$\sigma_{fpu}$ (MPa)</th>
<th>$C$ (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ST</td>
<td>4.69</td>
<td>-</td>
<td>-</td>
<td>260</td>
<td>9.37</td>
<td>9.10</td>
<td>35.0</td>
<td>7.88</td>
<td>16.97</td>
<td>362</td>
<td>4.2</td>
</tr>
<tr>
<td>PP-2</td>
<td>4.65</td>
<td>-</td>
<td>-</td>
<td>280</td>
<td>9.28</td>
<td>9.03</td>
<td>32.2</td>
<td>7.19</td>
<td>17.21</td>
<td>370</td>
<td>3.6</td>
</tr>
<tr>
<td>PP-3</td>
<td>3.73</td>
<td>-</td>
<td>-</td>
<td>270</td>
<td>7.58</td>
<td>7.44</td>
<td>27.5</td>
<td>5.00</td>
<td>11.42</td>
<td>306</td>
<td>4.2</td>
</tr>
<tr>
<td>PG-1</td>
<td>3.74</td>
<td>1.23</td>
<td>-</td>
<td>320</td>
<td>9.12</td>
<td>9.12</td>
<td>28.5</td>
<td>1.44</td>
<td>19.54</td>
<td>-</td>
<td>1.8</td>
</tr>
<tr>
<td>PC-1</td>
<td>3.43</td>
<td>-</td>
<td>0.55</td>
<td>280</td>
<td>8.54</td>
<td>8.66</td>
<td>30.9</td>
<td>1.87</td>
<td>13.01</td>
<td>-</td>
<td>3.9</td>
</tr>
<tr>
<td>AC</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>PP-11</td>
<td>4.38</td>
<td>-</td>
<td>-</td>
<td>310</td>
<td>8.51</td>
<td>8.33</td>
<td>26.9</td>
<td>5.69</td>
<td>14.46</td>
<td>330</td>
<td>8.3</td>
</tr>
<tr>
<td>PP-12</td>
<td>4.59</td>
<td>-</td>
<td>-</td>
<td>310</td>
<td>7.47</td>
<td>7.33</td>
<td>23.7</td>
<td>6.72</td>
<td>14.06</td>
<td>306</td>
<td>12.5</td>
</tr>
<tr>
<td>PP-13</td>
<td>3.68</td>
<td>-</td>
<td>-</td>
<td>300</td>
<td>7.43</td>
<td>7.31</td>
<td>24.4</td>
<td>4.65</td>
<td>10.67</td>
<td>290</td>
<td>12.5</td>
</tr>
<tr>
<td>PG-11</td>
<td>3.34</td>
<td>1.14</td>
<td>-</td>
<td>300</td>
<td>7.10</td>
<td>7.17</td>
<td>23.9</td>
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<td>9.17</td>
<td>-</td>
<td>25.0</td>
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<td>1.15</td>
<td>-</td>
<td>350</td>
<td>7.72</td>
<td>7.82</td>
<td>22.3</td>
<td>2.95</td>
<td>9.67</td>
<td>-</td>
<td>16.7</td>
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<tr>
<td>PC-11</td>
<td>3.83</td>
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<td>-</td>
<td>270</td>
<td>7.06</td>
<td>7.25</td>
<td>26.8</td>
<td>0.97</td>
<td>11.99</td>
<td>-</td>
<td>12.5</td>
</tr>
<tr>
<td>PC-12</td>
<td>3.70</td>
<td>0.61</td>
<td>-</td>
<td>310</td>
<td>7.37</td>
<td>7.61</td>
<td>24.6</td>
<td>0.84</td>
<td>11.62</td>
<td>-</td>
<td>16.7</td>
</tr>
</tbody>
</table>

ST : standard curing  
AC : autoclaving  
$V_{fp}$ : volume fraction of polypropylene networks  
$V_{fg}$ : volume fraction of glass fibres  
$V_{FC}$ : volume fraction of carbon fibres  
$\varepsilon_{mu}$ : first crack strain of the composite  
$\sigma_{mu}$ : strength of the matrix  
$\sigma_{ec}$ : first crack stress of the composite  
$E_c$ : elastic modulus of the composite before the bend over point  
$\varepsilon_u$ : ultimate strain of the composite  
$\sigma_u$ : ultimate stress of the composite  
$\sigma_{fpu}$ : ultimate stress of polypropylene ($= \sigma_u / V_{fp}$)  
C : crack spacing
The crack spacing in the case of the glass fibre composites PG-1, PG-11 and PG-12 had increased from about 2 to 20mm which results from the complete loss of properties of the glass fibres resulting in the combined fibre volume not exceeding the critical fibre volume. However, for the rest of the composites the crack spacing had increased by a factor of about 3 times. According to equation (5-5) in Chapter 5, the crack spacing for fibre cements at more than the critical fibre volume is proportional to $\sigma_{mu}/\tau_{fu}$ where $\tau_{fu}$ is the fibre-matrix bond strength. The strength of the matrix ($\sigma_{mu}$) is known from Table 9.3 and therefore it can be deduced that there is a considerable weakening in bond between the polypropylene and the matrix due to autoclaving.

As described above, the polypropylene networks, which were the main reinforcement in the composite, were partly damaged by autoclaving of 9 h at 170°C. However, they maintained sufficient properties to enable the use of polypropylene to be considered as a reinforcement in autoclaved products up to 170°C even though this temperature is above the melting point of the polymer. The glass fibres used in this study were destroyed in autoclaving conditions but carbon fibres showed considerable promise as a reinforcement under high temperature steam curing conditions.

A conclusion from this pilot study is that it may be possible to extend the application of the polypropylene to autoclaved products as a reinforcement. Further work is still needed.
CHAPTER 10

10. CONCLUSIONS

10.1. Glass fibre and polypropylene film

(1) The elastic modulus of the glass fibre was almost same as the value in the trade literature. However, the ultimate stress and strain of the glass fibre was lower than the stated values. The reason is thought to be that the strength of the strand is dominated by the weakest filament in the strand. The elastic modulus of the polypropylene film varied with strain. The higher the strain was, the lower was the modulus obtained in accordance with the literature. The ultimate strength of the polypropylene film was as shown in the trade literature.

(2) The reinforcing unit of the glass fibres was not a single filament but rather a bundle of filaments. Only the external filaments had a direct contact with the matrix and therefore the measurement of the perimeter of the glass strand was required for the assessment of bond strength. This perimeter was measured by image analysis and appropriate values were obtained. Likewise, the perimeter of the polypropylene film was not a simple matter to measure, and values from previous researches were used to calculate bond strength.

10.2. Glass fibre failure and light transmission technique\(^{(10-1)}\)

(1) The sequence of glass fibre failure was confirmed in the glass/polypropylene fibre hybrid cement composite by the acid dissolution technique. The glass strand containing filaments in which the local strain had reached the failure strain of the glass were broken at around the first yield point and consequently a sudden drop in stress was observed. However the intact glass fibres still carried some load. After the first yield point, many fractures of the glass fibres occurred and their role as a reinforcement was reduced so that further load was carried by the polypropylene
alone up to the ultimate strain.

(2) In order to obtain more detailed information on the glass fibre failure, a new light transmission technique was developed. This technique was not only useful as a qualitative analysis but also as a quantitative analysis for the glass fibre failure. It was found by the light transmission test that the effective glass fibre volume was about 70% of the actual fibre volume. This effective ratio was taken into account for the mathematical model of tensile behaviour and enabled a more precise prediction to be made. However, the results were only obtained from the limited combination of glass fibres (0.6vol%) and polypropylene networks (4vol%) so that further work is still needed.

10.3 Glass/polypropylene fibre hybrid cement composites

The characteristic points in the tensile properties of the hybrid composite which are caused by the hybrid effects are as follows.

(1) In the elastic region, the properties are dominated by the rule of mixtures. The improvement in the modulus of elasticity and the stress at the bend over point is not very significant. The main effect of the fibres is in the post-cracking region.

(2) The stress carrying distance is shortened when compared with single fibre composites and consequently the crack spacing becomes smaller. Also, the multiple cracking region is shortened.

(3) After the multiple cracking region, a marked increase in stress is shown up to the first yield point (FYP) when the partial failure of the glass fibre occurs. The toughness of the composite up to FYP is greatly enhanced in comparison with that of single fibre reinforced composites.
(4) Provided that the fibre volume fraction of the polypropylene is more than the critical fibre volume at FYP, the additional load after glass fibre failure can be carried by the polypropylene up to high strains of about 7-8%.

(5) The ultimate stress after FYP is dominated by the fibre volume fraction of the polypropylene alone. The ultimate strain is affected by both fibres. A significant hybrid effect occurs in relation to total toughness which is altered drastically depending on the combination of polypropylene and glass fibres.

In order to develop the mathematical model which enabled the prediction of tensile behaviour for hybrid fibre reinforced cement composites, the ACK model was modified. Each fibre type was thought to have a different stress transfer length and the strain distribution in fibre and matrix was considered. From six equations with six unknowns, the most suitable root was derived and the reliability of the equations was assessed by the use of experimental values for the materials properties.

Theoretical curves for the tensile properties of glass/polypropylene fibre reinforced cement composites have been obtained from the parametric study. For most combinations of polypropylene and glass fibres, the theoretical curves indicated a reasonable agreement with the experimental plots. It was proved that in order to obtain a correct hybrid effect, suitable fibre volume combinations must be taken into consideration from the point of the critical fibre volume of the polypropylene at FYP.

The final comparisons between the theoretical curves and the experimental curves indicated reasonable agreement. This concept is also considered applicable to hybrid composites reinforced with other fibres.
10.4. Carbon/polypropylene fibre hybrid cement composites

When polypropylene networks are combined with continuous carbon fibres, the hybrid effect for tensile properties was greater than with the glass fibre. The mathematical model was treated in the same way as for the glass/polypropylene fibre reinforced composite described in Chapter 7. However, the hybrid effect was found to be very dependent upon the carbon fibre distribution. In order to quantify the carbon fibre distribution, the cross section of the composite was divided into a number of unit cells on a display and the actual fibre area fraction in each fibre containing cell was measured by an image analyser. The ratio between theoretical and actual fibre area fraction in each cell, \( \frac{U_{Af}}{U_{Af_{th}}}, \) was obtained to represent the fibre distribution. Further, the ratio between theoretical and actual tensile properties which was defined as the efficiency factor of the fibre distribution, \( \eta_{fd}, \) were plotted against \( \frac{U_{Af}}{U_{Af_{th}}}, \) The relative equations between \( \eta_{fd} \) and \( \frac{U_{Af}}{U_{Af_{th}}} \) were obtained by a logarithmic regression. Since the efficiency factor \( \eta_{fd} \) was deduced empirically, its use was restricted. Nevertheless, an efficiency factor for the fibre distribution is an essential parameter which needs to be considered if better predictions of composite performance are to be made.
CHAPTER 11

11. RECOMMENDATIONS FOR FURTHER WORK

In this study, fibres which are continuous and aligned parallel to the direction of tensile load were used in order to make the model simple. Even so, it was found that the effect of bundled filaments and fibre distribution need further study to be able to predict the tensile behaviour of the hybrid composite more accurately.

A light transmission technique was developed to understand the effect of the bundled glass filament fracture in the composite. However, photographs were taken only at specific strain in composites and a continuous measurement of the intensity of the transmitted light should give a significant improvement towards understanding the tensile stress-strain curve of the composite.

The investigation of the carbon fibre distribution described in this study is just a preliminary trial so that considerable further work is required. Provided a manufacturing device for fibre dispersion is developed and the degree of fibre dispersion can be controlled, the effect of the fibre distribution could be quantified more easily. A better fibre dispersion allows a larger fibre-matrix contact area and the effect of the fibre-matrix contact area could be examined with a different number of filaments per bundle. For the application of continuous carbon fibres to industrial composites, a woven cloth, mat or network made out of filaments would be expected to have a stable performance as reinforcement.

In the present study, the mix proportions and properties of the matrix have been constant. However, other applications may require not only strength but also light weight or dimensional stability. When light-weight coarse aggregate is used or when the composite is cured in an autoclave with high pressure and temperature, the properties of the composite in tension or flexure will be changed due to the alteration of the fibre-matrix bond strength. This is an important research area to study, particularly for
autoclaving, where hydration at high pressure and temperature in the matrix will alter the properties. Also the heat-resistance and the thermal effects in terms of fibre performance, will be an additional important parameter. A pilot study into the effects of autoclaving was described in Chapter 9.

Although their mechanics are rather complicated, investigations into light-weight aggregate and the autoclaving technique are essential for the development of new high value added building materials.
REFERENCES

CHAPTER 1


(1-12) Zollo, R.F., 'Collated fibrillated polypropylene fibres in FRC'. Fibre Reinforced
Concrete International Symposium, sp-81, American Concrete Institute, Detroit, Michigan, 1984, pp.397-409.


CHAPTER 2


(2-8) Nozaki, H., 'Carbon fibre reinforced concrete', Fibre reinforced concrete (FRC),


(2-12) Letter from Coutaulds Grafil Inc., 12 June 1992

(2-13) Data sheet, Grafil 33-500 6K, received from Coutaulds Grafil Inc.

CHAPTER 3


CHAPTER 4


(4-3) Kobayashi, K. and Cho, R., ' Flexural characteristics of steel fibre and

(4-4) Hasaba, S., Kawamura, M., Koizumi, T. & Takemoto, K., 'Resistibility against impact load and deformation characteristics under bending load in polymer and hybrid (polymer and steel) fibre reinforced concrete'. Fibre Reinforced Concrete ed Hoff G C., ACI, SP-81, American Concrete Institute, Detroit, 1984, pp.187-196.


CHAPTER 5


271
CHAPTER 6


(6-3) Hale, K.F., 'The application of optical fibres to structural integrity monitoring', IEEE Oceans, pp.344-348.


CHAPTER 7


(7-34) Private communication with Retiflex s.p.a.
CHAPTER 8


CHAPTER 9


CHAPTER 10


APPENDICES


APPENDIX 1. Simple linear regression

In Figures 4.10-4.13, straight lines were drawn based on the simple linear regression model. The form of a simple linear equation is:

\[ y = a - bx \]

It includes two variables, \( x \) and \( y \). The constant \( a \) is called the \( y \) intercept. The number \( b \) is the slope of the line. The straight line is determined by the least-squares method. By mathematical analysis it has been proved that \( a \) and \( b \) for a least-squares regression line can be computed with the formulas:

\[
\text{coefficient } b = \frac{n(\Sigma xy) - (\Sigma x)(\Sigma y)}{n(\Sigma x^2) - (\Sigma x)^2} \\
\text{constant } a = \frac{\Sigma y - b(\Sigma x)}{n}
\]

where \( n \): number of pairs of data (No. of observations)

In simple linear regression inferences, a consequence of using the two sample estimators \( a \) and \( b \) is that two degrees of freedom are lost. Thus, if we have \( n \) data points, the number of degrees of freedom is \( n-2 \) (Degrees of Freedom). In Figures 4.10 and 4.11, the \( y \) intercepts of regression lines were assumed to be zero for \( E_c \). In this case, one of the sample estimators is \((0, 0)\). Therefore, one degree of freedom is lost.

The sample standard error of estimate (Std Err of Y Est) is a standard deviation which measures the variations of sample points above and below the regression line. It can be computed with the formula:

\[
Se = \sqrt{\frac{\Sigma (y - \hat{y})^2}{n-2}}
\]

where \( Se \): sample standard error of estimate

\( y \): observed value
\( \hat{y} \) : estimating value on the regression line

It is also expressed as follows.

\[
Se = \sqrt{\frac{\sum y^2 - a\sum y - b\sum xy}{n-2}}
\]

If \( Se \) equals zero, all the sample points lie on the regression line. Usually, \( Se \) is not zero, and the larger \( Se \) is, the less accurate are the estimates \( \hat{y} \).

The sample standard error of the slope (Std Err of Coef) is computed with the formula

\[
Sb = \frac{Se}{\sqrt{\sum x^2 - \frac{(\sum x)^2}{n}}}
\]

where \( Sb \) : sample standard error of the slope

This is a measure of slope variation.

\( R^2 \) (R squared) is called the coefficient of determination. It is the proportion of the total variation in \( y \) that is explained by the relationship between \( y \) and \( x \). The coefficient \( R^2 \) will be 0 if none of the variation in \( y \) is explained by the relationship and \( R^2 \) will be 1 if all the variation in \( y \) is explained. Therefore \( R^2 \) is a number in the interval 0 to 1, inclusive. The ratio, explained variation divided by total variation, is

\[
R^2 = \frac{\Sigma (\hat{y} - \bar{y})^2}{\Sigma (y - \bar{y})^2}
\]

where \( \bar{y} \) : mean value of \( y \)

It is computed by the following formula

\[
R^2 = \frac{\left[n\sum xy - (\sum x)(\sum y)\right]^2}{\left[n\sum x^2 - (\sum x)^2\right]\left[n\sum y^2 - (\sum y)^2\right]}
\]
For instance, the value $R^2=0.95$ means that the total squared expected variation $\Sigma(\hat{y} - \bar{y})^2$ is 95 percent as large as the total squared variation $\Sigma(y - \bar{y})^2$. Hence, saying 95 percent of $y$ variation is explained by $x$ variation is an imprecise use of words. Nevertheless, $R^2$ is usually interpreted in this way.

When the regression line is horizontal, $R^2$ is zero because of $\hat{y} = \bar{y}$. As shown in Figures 4.10 and 4.11, since regression lines obtained after BOP are almost horizontal, $R^2$ is smaller than others. In this case, the degree of the line-fitting can be assessed by the standard error of estimate (Se : Std Err of Y Est). The smaller Se is, the better fitting are the regression lines. Both Se of horizontal lines in Figures 4.10 and 4.11 are smaller than those of lines before BOP. Therefore, although $R^2$ of these lines are small, plots from 1000 to 2500 micro-strains are regarded to be regressed straight lines well.
APPENDIX 2. Results of the light transmission test

Table A-1 Results of the light transmission test.

<table>
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<tr>
<th>Hybrid composite No</th>
<th>Initial number of glass filaments calculated from the light area</th>
<th>Calculated number of glass filaments at unloaded points</th>
<th>Strain at unloaded points (%)</th>
<th>Ratio of broken filaments at the ultimate point</th>
<th>Number of cracks</th>
</tr>
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<td>699</td>
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<td>62.6</td>
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<td>3</td>
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<td>1111</td>
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<td>52.1</td>
<td>27</td>
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<td>20.1</td>
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APPENDIX 3. Theoretical properties for glass/polypropylene fibre hybrid cement composites

Table A-2  Theoretical properties for GF/PP composites. (PP=4%)

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<th>$N_{fr}$</th>
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<th>$\sigma_{Gf}$ (MPa)</th>
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Branch number

1:  $\tau_p=3.5$MPa, $\tau_s=3$MPa
2:  $\tau_p=0.2$MPa, $\tau_s=1$MPa
### Table A-3  Theoretical properties for GF/PP composites (PP=6%)

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<th>$V_f$</th>
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<th>$E_r$ (GPa)</th>
<th>$\sigma_{fc}$(MPa)</th>
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<th>$\epsilon_{\sigma 0}$</th>
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<th>$\epsilon_{p1}$</th>
<th>$\epsilon_{p2}$</th>
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<th>$\sigma_{u1}$(MPa)</th>
<th>$\sigma_{u2}$(MPa)</th>
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### Table A-4  Theoretical properties for GF/PP composites (PP=8%)

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<th>$\epsilon_{p1}$</th>
<th>$\epsilon_{p1}$</th>
<th>$\epsilon_{p2}$</th>
<th>$\epsilon_{p2}$</th>
<th>$\sigma_{u1}$(MPa)</th>
<th>$\sigma_{u1}$(MPa)</th>
<th>$\sigma_{u2}$(MPa)</th>
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APPENDIX 4  $z$ for One-Tail area under the normal curve.

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APPENDIX 5  Theoretical properties for carbon/polypropylene fibre hybrid cement composites

Table A-6  Theoretical properties for CF/PP composites. (PP=4%)

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Table A-7  Theoretical properties for CF/PP composites. (PP=6%)

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286
### Table A-8 Theoretical properties for CF/PP composites. (PP=8%)  

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### Table A-9 Theoretical properties for CF/PP composites. (PP=10%)  

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