EXPERIMENTAL STUDIES AND MATHEMATICAL MODELLING OF THE DRAPING AND SHEAR DEFORMATION OF WOVEN FABRICS IN RESIN TRANSFER MOULDING

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ABSTRACT

This project contains experimental studies and mathematical modelling of the draping and shear deformation of woven fabrics in resin transfer moulding (RTM). A comparison between different types of woven fabrics was carried out in terms of drapeability, draping characteristics and shear deformation. The fabrics investigated included a loose plain weave, a tight plain weave, a twill and a 5 harness satin weave. A model mould was constructed for the draping experiments, comprising a hemispherical hat surrounded by a flat rim. Experimental procedures and techniques for the analysis of experimental data were developed and established regarding studies of draping and shear deformation. Detailed experimental studies were performed for the draping of each type of woven fabric and a comprehensive set of data of deformation angles was assembled. It was concluded that although similar trends were followed in the draping of all fabrics, there were differences in the amount of wrinkling and individual deformation patterns emerged with differences in deformation angles of different fabrics reaching 15°. This illustrates the need for a mechanical approach in the modelling of fabric draping if accurate predictions of deformation angles are required. Wrinkle formation during draping was investigated in draping studies and a wrinkle measure was devised. The data of deformation angles during the draping of fabrics were used to estimate the distribution of fibre volume fraction after draping. Changes in fibre orientation and fibre volume fraction after the draping of fabrics were taken into account to calculate the modulus distribution of draped fabrics, as part of resulting composite products, following the laminated plate theory.

Since in-plane shear is the major mode of deformation in the draping of woven fabrics, picture-frame shear tests were carried out for each fabric. Clear differences were observed in the shear deformation and shear locking of the various fabrics, with the tight plain weave proving the most difficult fabric to shear. A novel model for the elasticity analysis of the shear deformation of fabrics was proposed with applicability to high shear strains. It takes into account the effect of change of fibre directions during the shearing of fabric on the stresses in an orthogonal frame of reference.
The model was validated successfully with the experimental data and model parameters were determined from numerical curve fitting. The elasticity model explained part of the non-linear behaviour of fabrics in shear. The shear locking effect on shear and wrinkling of fabrics was investigated in microstructural studies and analytical models were suggested to predict the shear locking angle for each type of weave.

The solid mechanics approach was employed for preliminary computer simulations and parametric studies of the draping of fabrics by using LUSAS finite element software, supplied by FEA Ltd. A constitutive model for an elastic, orthotropic solid was incorporated for the fabric. Although the approach was promising, the assumption of an interfacial volume of low modulus between the fabric and the mould resulted in constraining the deformation of fabric.
In the name of Allah (SWT), the Benevolent, Most Merciful, I would like to express my sincere gratitude and whole-hearted appreciation to my diligent supervisor, Dr Constantina Lekakou for the devotion and untiring committed supervision she has rendered to me during this academic endeavour. I must also mention Professor Mike Bader (my co-supervisor) for the unrelenting advice and optimism he has shown to this particular research.

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NOMENCLATURE

\( a \) nodal distance along the warp direction
\( a^{(e)} \) vector of nodal displacement
\( b \) nodal distance along the weft direction

\( BS \) bending stiffness
\( (C_{11})^{1/2} \) stretch in the fibre direction
\( CR \) maximum compression ratio
\( D^{(e)} \) matrix of elastic constants

\( d_1 \) diagonal distance between warp and weft in the unstretched state
\( d_2 \) diagonal distance between warp and weft in the stretched state

\( E \) modulus matrix
\( E_f \) Young's modulus of the fibre
\( f(C_{11}) \) function defined by equation (4.13)

\( F \) deformation gradient matrix
\( F^T \) transpose of deformation gradient matrix
\( F_{12} \) in-plane compressive forces
\( F_b \) body forces
\( F_c \) concentrated loads
\( F_{\text{crit}} \) critical loads
\( F_D \) applied force on the diaphragm
\( F_s \) shear force
\( F_{\text{su}} \) surface load
\( F_{\text{v3}} \) inter-fibre compressive forces
\( F_x \) applied load from the tensile testing machine

\( g \) new dimension of sheared unit cell

\( G \) shear modulus
\( G_f \) shear modulus of the fibre
\( G_m \) shear modulus of the matrix
\( G_o \) initial shear modulus

\( h \) ply thickness
\( h_i, H_i \) lengths denoted in Figure 3.10
$h_p$ length indicated in Figure 7.23(a)

$h_w$ height of wrinkle

$I$ unit tensor

$k$ permeability tensor of the reinforcement

$\kappa g$ geodesic curvature

$K$ structure stiffness matrix

$L$ length

$L_{wp}$ warp length

$L_{wt}$ weft length

$m$ mesh size along weft direction

$M_o$ frictional stress

$n$ mesh size along warp direction

$N^{(e)}$ shape function matrix

$n_g$ geometric parameter

$N_p$ number of plies

$P$ pressure

$p$ efficiency factor

$Q$ transformed reduced stiffness matrix

$\bar{Q}$ reduced stiffness matrix

$R$ radius of the hemisphere

$\mathbf{R}$ structure force vector

$(r, \theta, \varphi)$ polar coordinates

$\mathbf{R}_b$ force vector due to element body loads

$\mathbf{R}_c$ force vector due to concentrated loads

$\mathbf{R}_o$ force vector due to initial stresses and strains

$\mathbf{R}_{su}$ force vector due to element surface tractions

$s$ length of the geodesic curve

$S$ $0.5\pi R$

$S_1$ spherical arc length along the longitudinal direction

$S_2$ spherical arc length along the equatorial direction

$\sigma$ stress tensor

$t$ time

$t_c$ width of the unit cell

$t_p$ dimension of macropore of plain weave (see Figure 7.23(a))
\( t_i \)  
material parameter for each fibre direction

\( T_i \)  
fibre stress associated with inextensibility constraint

\( \mathbf{u} \)  
displacement vector

\( V_f \)  
fibre volume fraction

\( V_{fo} \)  
fibre volume fraction of the unsheared element

\( \mathbf{v}_o \)  
superficial velocity vector

\( W \)  
yarn width

\( w_c \)  
length of the unit cell

\( w_p \)  
dimension of macropore of plain weave (see Figure 7.23(a))

\( w_w \)  
width of wrinkle

\( (x, y, z) \)  
cartesian coordinates

\( \mathbf{x} \)  
fixed, Eulerian, orthogonal frame

\( \mathbf{X} \)  
material frame

\( Y_{\text{max}} \)  
maximum vertical deformation

**Greek symbols**

\( \alpha \)  
angle between the warp and weft fibres in the deformed state

\( \Delta l \)  
difference in length

\( \varepsilon \)  
strain tensor

\( \varepsilon_o \)  
initial strain

\( \gamma \)  
shear angle

\( \mu \)  
viscosity

\( \rho \)  
density of fibre material

\( \rho_a \)  
areal density of fabric

\( \tau_s \)  
shear stress in fibre direction

\( \nu \)  
Poisson’s ratio

\( \nu_f \)  
Poisson’s ratio of the fibre

\( \nu_m \)  
Poisson’s ratio of the matrix
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1. INTRODUCTION
1.1 INTRODUCTION

Composite materials offer many exceptional properties, which are difficult to match with traditional materials such as steel, aluminium and wood. Today, composites are used by almost every dynamic, high performance structure whether on land, at sea or in the air. Composite materials allow the design and manufacture of lightweight components, which can resist corrosion, blast, fire, or impact. Weight reduction often has a direct effect on performance, leading to a compelling case for using these materials. Unfortunately, one of the major problems associated with composite materials in the field of mechanical engineering and in the automobile industry is that of cost. Considerable costs are caused by the components production process for series production and the manufacture of reinforcing semi-finished textiles such as woven fabrics or knitted multiaxial layers [http://www.spsystems.com].

In composite materials applications, woven fabrics have the potential to play a significant role due to their good reinforcing properties, the ease of handling and the well-developed weaving technologies. For instance, in composites manufacturing techniques such as the resin transfer moulding (RTM), assemblies of woven fabrics with or without binder are draped onto the mould surface. Potter (1979) described drape as follows: “Fabrics are said to possess a quality called drape. This is difficult to define, but can be described as the ability to form over three dimensional shapes without having to be cut or without having to use undue force”. The architecture of the fabric allows draping and shaping to occur over complex mould geometries, thus reducing the problem of discontinuities that sometimes occur in unidirectional fibre reinforcement.

The conformable fitting of a woven fabric to a three-dimensional surface is achieved mainly by a relative rotation of warp and weft fibre yarns, which translates into shear deformation of the fabric [Kothari et al (1989) amongst others]. The ability of woven fabrics to undergo shearing is the basis for the success of woven textiles as clothing materials.
Common features associated with the forming of woven fabrics include wrinkling, folds, cuts, bunching and many others. Due to the desire of manufacturers to minimise wastage and to achieve optimum results in the production of components, specialist knowledge and skills are needed as paramount pre-requisite for using woven fabrics in the composites industry.

With this in mind, in shaping processes, the prediction of fabric distortion during draping and the changes in fibre orientation and fibre fraction are essential for design reasons, quality control, the prediction of the permeability of preform and the estimation of mechanical properties of the composite product.

Studies of the drape forming of woven fabrics in the manufacturing of composites are generally new, mostly in the last ten years. The textiles industry has had much more experience, although fabric drape there is frequently considered over a platform in that case only part of the fabric forms onto a shape whereas the sides of the fabric hang freely. The major experimental contributions of the textile industry have focused on the development of prototype tests and apparatus for the determination of the mechanical properties (most especially shear) of woven fabrics. Early shear tests adopted by the textile industry include the Kawabata evaluation system for fabrics (KES-F) and the FAST test. Picture-frame shear tests on a tensile testing machine were developed and established later by the composite materials community.

Nevertheless, the composite materials community was concerned first with short fibre and continuous fibre mats as well as unidirectional fibre assemblies, rather than woven fabrics. Thermoplastic polymer matrices and prepregs were also first considered. As a result, viscous models were first suggested to model the shear behaviour of fibre reinforcements. However, as the present study concerns assemblies of woven fabrics to be formed dry first, and then be infiltrated by a polymer resin (as, for example, in resin transfer moulding), a solid mechanics approach was considered more appropriate. The textile industry has also adopted the solid mechanics approach regarding the mechanical shear behaviour of fabrics, in most published studies.
Concerning the mathematical modelling of fabric draping, the major contribution of the textile industry is the "fishnet" model, which has also been adopted by a large part of the composite community. This model is a type of geometric mapping approach based on a pin-joined model. The problem with such an approach is that, although it takes into account the mould geometry, it does not generally distinguish between different fabrics. As a result, a mechanical approach has been considered more suitable for this study in order to predict reliably and accurately the drapeability of woven fabrics, where the individual mechanical response of each fabric is considered together with the mould geometry.

1.2 TYPICAL TEXTILE COMPOSITES

Textile composites are produced by impregnating matrix material into dry preforms to hold the multidirectional yarns together. This is generally carried out by using liquid moulding techniques such as: autoclave processing, resin transfer moulding (RTM), structural reaction injection moulding (SRIM) and resin film infusion (RFI). This section provides an elaborate discussion on the available types of fibres and fabrics.

1.2.1 FIBRE TYPES

There are many types of fibres existing in the material world of nowadays. These are classified as natural fibres and synthetic ones. The synthetic fibres are mostly obtained from polymeric or ceramic materials. In this section a brief discussion of the composition, processing properties and applications of some of these fibres will be outlined.

1.2.1.1 GLASS FIBRE

By blending quarry products (sand, kaolin, limestone, colemanite) at 1600 °C, liquid glass is formed. The liquid is passed through micro-fine bushings and simultaneously cooled to produce glass fibre filaments of 5-24 mm diameter.
The filaments are drawn together into a strand (closely associated) or roving (loosely associated), and coated with a “size” to provide filament cohesion and protect the glass from abrasion. By variation of the “recipe”, different types of glass can be produced. The types used for structural reinforcements are as follows:

(a) E-glass (electrical) — it exists with a lower and stronger alkali content. It also possesses good tensile and compressive strength and stiffness, good electrical properties and relatively low cost, but its impact resistance is relatively poor. Depending on the type of E-glass the price ranges within £1-2/kg [http://www.spsystems.com]. E-glass is the most common form of reinforcing fibre used in polymer matrix composites.

(b) C-glass (chemical) — this is the best in terms of resistance to chemical attack. It is mainly used in the form of surface tissue in the outer layer of laminates used in chemical and water pipes and tanks.

(c) R, S, or T-glass — from manufacturers trade names for equivalent fibres having higher tensile strength and modulus than E-glass, with better-wet strength retention. Developed for aerospace and defence industries, and used in some hard ballistic armour applications. This factor, and low production volumes mean relatively high price. Depending on the type of R or S glass the price ranges within about £12-20/kg [http://www.spsystems.com].

1.2.1.2 ARAMID FIBRE

Aramid fibre is a man-made organic polymer (an aromatic polyamide) produced by spinning a solid fibre from a liquid polymer solution. The bright golden yellow filaments produced can have a range of properties, but all have high strength and low density giving very high specific strength. All grades have good resistance to impact, and lower modulus grades are used extensively in ballistic applications. Compressive strength, however, is only similar to that of E-glass.

Although most commonly known under its Dupont trade name ‘Kevlar’, there are now a number of suppliers of the fibre, most notably Akzo nobel with ‘Twaron’.
Each supplier offers several grades of aramid with various combinations of modulus and surface finish to suit various applications. As well as the high strength properties, the fibres also offer good resistance to abrasion, and chemical and thermal degradation. However, the fibres may degrade slowly when exposed to ultraviolet light. Aramid fibres are usually available in the form of rovings, with tex ranging from about 20 to 800. Typically the price of the high modulus type ranges from £15 to £25 per kg [http://www.spsystems.com].

1.2.1.3 CARBON FIBRE

The controlled oxidation, carbonisation, and graphitisation of carbon-rich organic precursors, which are already in fibre form, produce carbon fibres. The most common precursor is poly-acrylonitrile (PAN), because it gives the best carbon fibre properties, but fibres can also be made from pitch or cellulose. Variation of the graphitisation process produces either high strength fibres (at 2600 °C) or high modulus fibres (at 3000 °C) with other types in between. Once formed, the carbon fibre has a surface treatment applied to improve matrix bonding and chemical sizing which serves to protect it during handling. When carbon fibre was first produced in the late sixties the price for the basic high strength grade was about £200/kg [http://www.spsystems.com]. By 1996 the annual world-wide capacity had increased to about 7,000 tonnes and the price for the equivalent (high strength) grade was £15-40/kg [http://www.spsystems.com]. Carbon fibres are usually grouped according to the modulus band in which their properties fall. These bands are commonly referred to as high strength (HS) intermediate modulus (IM), high modulus (HM), and ultra high modulus (UHM). The filament diameter of most types is about 5-7 mm. Carbon fibre has the highest specific stiffness of any commercially available fibre, very high strength in both tension and compression, and a high resistance to corrosion, creep and fatigue. Their impact strength, however, is lower than either glass or aramid, with particularly brittle characteristics being exhibited by HM and UHM fibres.
1.2.1.4 OTHER FIBRES

There are a variety of other fibres that can be used in advanced composite structures but their use is not widespread. These include [http://www.spsystems.com]:

Polyester

This is a low density, high tenacity fibre with good impact resistance but low modulus. Its lack of stiffness usually precludes it from inclusion in a composite component, but it is useful where low weight, high impact or abrasion resistance, and low cost are required. It is mainly used as a surfacing material, as it can be very smooth, keeps weight down and works well with most resin types.

Polyethylene

In random orientation, ultra-high molecular weight polyethylene molecules give very low mechanical properties. However, if dissolved and drawn from solution into a filament by a process called gel spinning, the molecules become disentangled and aligned in the direction of the filament. The molecular alignment promotes very high tensile strength to the filament and the resulting fibre. Coupled with their low density (<1000 kg m\(^{-3}\)), these fibres have the highest specific strength of the fibres described here. However, the fibre’s tensile modulus and ultimate strength are only slightly better than E-glass and less than that of aramid or carbon. The fibre also demonstrates very low compressive strength in laminate form. These factors coupled with high price, and more importantly, the difficulty in creating a good fibre/matrix bond means that polyethylene fibres are not often used in isolation for composite components.

Quartz

A form of glass with very high silica content, much higher mechanical properties and excellent resistance to high temperatures (1000 °C+). However, the manufacturing
process and low volume production lead to a very high price (14 mm diameter-£74/kg, 9 mm diameter- £120/kg [http://www.spsystems.com]).

**Boron**

Carbon or metal fibres are coated with a layer of boron to improve the overall fibre properties. The extremely high cost of this fibre restricts its use to high temperature aerospace applications and in specialised sporting equipment. A boron/carbon hybrid, composed of carbon fibres interspersed among 80-100 mm boron fibres, in an epoxy matrix, can achieve properties greater than either fibre alone, with flexural strength and stiffness twice that of HS carbon and 14 times that of boron, and shear strength exceeding that of either fibre.

**Ceramics**

Ceramic fibres, usually in the form of very short ‘whiskers’, are mainly used in areas requiring high temperature resistance. They are more frequently associated with non-polymer matrices such as metal alloys.

**Natural**

At the other end of the scale it is possible to use fibrous plant materials such as jute and sisal as reinforcements in ‘low-tech’ applications. In these applications, the fibres low density (typically 500-600 kg m$^{-3}$) means that fairly high specific strength can be achieved.

**1.2.2 FABRIC TYPES**

Fabric reinforcements are generally classified into five basic categories according to the textile manufacturing techniques used for reinforcements [Tan et al (1997)]. These are woven fabrics, knitted fabrics, braided fabrics, multi-axial fabrics and non-crimp stitch bonded fabrics.

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In this section, processing features, major parameters affecting mechanical properties of textile composites, fabric structures and general mechanical features of these three types of textiles will be discussed.

1.2.2.1 WOVEN FABRICS

Woven fabrics are the most widely used textile reinforcements in structural applications. Woven fabrics are produced by the interlacing of warp (0°) fibres/yarn and weft (90°) fibres/yarn in a regular pattern or weave style. Each yarn is a bundle of filaments (or fibres) and its size is measured by the number of filaments in the yarn. The interlacing pattern of the warp and weft yarns is known as weave. The fabric’s integrity is maintained by the mechanical interlocking of the fibres. Drape (the ability of a fabric to conform to a complex surface), surface smoothness and stability of a fabric are controlled primarily by the weave style. The areal weight, porosity and, to a lesser degree, wet out are determined by selecting the correct combination of fibre tex and the number of fibres/cm.

Currently, most of the pure and hybrid woven fabrics used in textile composites are simple two dimensional (2D) weaves. The following is a description of some of the more commonly found weave styles (see Figure 1.1):

1.2.2.1.1 PLAIN WEAVE

Plain weave is the most common used fabric in woven composites. In a plain weaving structure each warp fibre passes alternately under and over each weft fibre. The fabric is symmetrical, with good stability and reasonable porosity. However, it is the most difficult of the weaves to drape, and the high level of fibre crimp imparts relatively low mechanical properties compared with the other weave styles. With large fibre yarns (high tex) this weave style gives excessive crimp which can reduce the modulus by up to 15% compared with a similar fraction of straight fibres [Bader & Lekakou (1997)] and therefore it tends not to be used for very heavy fabrics.

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1.2.2.1.2 TWILL WEAVE

In a twill weaving structure one or more warp fibres alternately weave over and under two or more weft fibres in a regular repeated manner. This produces the visual effect of a straight or broken diagonal 'rib' to the fabric. Twill weave has a looser interlacing and the weave is characterised by a diagonal line [Tan et al (1997)]. Superior wet out and drape is seen in the twill weave over the plain weave with only a small reduction in stability. With reduced crimp, the fabric also has a smoother surface and slightly higher mechanical properties [http://www.spsystem.com].

1.2.2.1.3 SATIN WEAVE

Satin weaves are fundamentally twill weaves modified to produce fewer intersections of warp and weft. The ‘harness’ number used in the designation (typically 4, 5 and 8) is the total number of yarns crossed and passed under, before the yarn repeats the pattern. A ‘crowsfoot’ weave is a form of satin weave with a different stagger in the repeat pattern. Satin weaves are very flat, have good wet out and a high degree of drape. The low crimp gives good mechanical properties. Satin weaves allow fibres to be woven in the closest proximity and can produce fabrics with a close ‘tight’ weave. However, the style’s low stability and asymmetry needs to be considered. The asymmetry causes one face of the fabric to have fibre running predominantly in the warp direction while the other face has fibres running predominantly in the weft direction. Care must be taken in assembling multiple layers of these fabrics to ensure that stresses are not built into the component through this asymmetric effect.

1.2.2.1.4 BASKET WEAVE

Basket weave is fundamentally the same as plain weave except that two or more warp fibres alternately interlace with two or more weft fibres. An arrangement of two warps crossing two wefts is designated 2x2 basket, but the arrangement of fibres need not be symmetrical.
Therefore it is possible to have 8x2, 5x4, etc. Basket weave is flatter, and, by having less crimp, it is stronger than a plain weave, but less stable. It may be used on heavy weight fabrics made with thick (high tex) fibres to avoid excessive crimping.

1.2.2.1.5 LENO WEAVE

Leno weave is a form of plain weave in which adjacent warp fibres are twisted around consecutive weft fibres to form a spiral pair, effectively ‘locking’ each weft in place. Leno weave improves the stability in ‘open’ fabrics, which have a low fibre count. Fabrics in leno weave are normally used in conjunction with other weave styles because if used alone their openness could not produce an effective composite component.

![Figure 1.1: Schematic of the woven fabrics, showing, (a) plain weave, (b) twill weave, (c) satin weave, (d) basket weave, (e) leno weave and (f) mock leno weave][SP Systems Composite Materials handbook]
1.2.2.1.6 MOCK LENO WEAVE

This type of weave is a version of plain weave in which occasional warp fibres, at regular intervals but usually several fibres apart, deviate from the alternate under-over interlacing and instead interlace every two or more fibres. This happens with similar frequency in the weft direction, and the overall effect is a fabric with increased thickness, rougher surface, and additional porosity.

1.2.2.2 MULTI-AXIAL FABRICS

This includes double-bias fabrics with ±45° fibre orientations (‘X’-designation), triaxial fabrics with 0°±45° and 90°±45° fibres (‘Y’ and ‘Z’-designations), and quadraxial fabrics with 0°/90°±45° fibres (‘Q’-designation) [SP systems, Composites Materials Handbook, 1999]. In conventional woven fabrics, the architecture of the weave is such that the warp and weft yarns always lie at 90° to each other and with the warp yarn always along the machine direction. In the case of the multiaxial fabrics, for instance in the triaxial woven fabric, there are two sets of warp yarns at either ±45° or ±30° and a weft yarn at 90° to the machine direction. This results in fabrics with improved planar isotropy, higher in-plane shear modulus and better handleability [Saunders (1998)].

1.2.2.3 KNITTED FABRICS

Knitted fabrics are broadly characterised into two types depending on their interlocking loops of yarns. These are the weft-knitted and the warp-knitted fabrics [Tan et al (1997)]. In the weft-knitted fabric, yarns run width-wise and, as demonstrated in Figure 1.2 (a), loops are formed by a single weft yarn. A row of loops in the longitudinal direction is called ‘wale’ or ‘warp’, and that in the width direction is called ‘course’ or ‘weft’. In the warp-knitted fabric, overlaps in alternative wale at alternate courses are produced with one thread crossing between adjacent wales in which loops incline in the course direction (Figure 1.2 (b)).
The most important factors affecting the mechanical properties of knitted fabrics are knit architecture, fibre volume fraction and yarns orientation angle [Tan et al (1997)]. Mechanical properties have contributed in making knitted fabrics unsuitable for use as reinforcements in composites compared to woven and braided fabrics. However, compared with other conventional textile fabrics, knitted fabrics possess high productivity and low cost. In addition, knitted fabrics have high extensibility, which means good formability to fit complicated mould shapes and exhibit a better resistance to impact. Hence, they are quite suitable for deep draw moulded composites.

![Diagram of knitted fabrics](image)

**Figure 1.2**: Schematic diagram of (a) Weft and (b) Warp knitted fabrics [Gommers et al (1996)]

### 1.2.2.4 BRAIDED FABRICS

Braided fabrics are constructed by intertwining or orthogonally interlacing two or more sets of yarns to form an integral structure (Figure 1.3). One set of yarns is called axial yarns while the other set is named braided yarns. So, the structure of braided fabrics consists of parallel axial yarns, interconnected with braided yarns that are placed along complex spatial orientations. Generally, braided yarns follow the +0 and -0 directions and usually interlace in either 1x1 or 2x2 patterns.

The common fabrication methods for creating braided fabrics are the traditional horn-gear method, the solid braiding method, two and four steps braiding method and track-
and-column braiding method. The major parameters affecting the mechanical properties of braided fabrics include, the braid parameters such as architecture, yarn size, yarn spacing length, fibre volume fraction and fibre orientation angles [Tan et al (1997)]. The integral structures of braided fabrics enable them to endure twisting, shearing and better impact. However, braided fabrics exhibit poor stability under axial compression in the yarn system direction.

![Diagram of 2D triaxial braid pattern](image)

*Figure 1.3: Schematic diagram of 2D triaxial braid pattern [Tan et al. (1997)].*

### 1.2.2.5 NON-CRIMP STITCH BONDED FABRICS

Presently, a large number of non-crimp stitch bonded fabrics have been developed to improve the mechanical performance of fabrics. The mechanical properties of woven laminates depend largely on the amount of crimp present in the fabrics [Bader and Lekakou (1997)]. Non-crimp stitch bonded fabrics consist of unidirectional layers of aligned rovings which have been bonded together by cross stitching with a light yarn made usually from polyester. Usually, four or more layers are bonded together in their preferred orientation such as, 0°, 90°, ±45°, to form a single layer (Figure 1.4).
Thus, depending on the mechanical performance required, a complete range of fabrics with a variety of fibre orientations could be produced from this approach. Other advantages are better mechanical properties, primarily from the fact that the fibres are always straight and non-crimped, and that more orientations of fibres are available from the increased number of layers of fabric. Also, the stitch pattern can be varied to improve the drapeability of the fabric in excess of that obtained from a unidirectional prepreg.

1.3 COMPONENTS MANUFACTURE FROM FIBRE REINFORCED COMPOSITES

The majority of parts manufactured from advanced composites are made by labour intensive hand lay-up. However, there are today several processes for the manufacture of components from fibre reinforced composites. Some of the potentially cost effective techniques are contact moulding, compression moulding of which dough moulding compound (DMC) and sheet moulding compound (SMC) are the two most common [Gutowski et al (1995)], autoclave moulding and resin transfer moulding (RTM). There are various forms in which fibre reinforcements are introduced into the polymer matrix. These could be either in the form of chopped strands as in compression moulding, or resin impregnated reinforcement (prepreg) as in autoclave.
moulding or dry reinforcement pre-placed into the mould before injection of the resin as in RTM.

The following paragraphs elaborate on some of the major techniques used in component manufacture from fibre reinforced composites.

1.3.1 PRESS MOULDING

The term “press moulding” highlighted here embraces processes where the charge is compressed between shaped dies and cured in situ. It also includes moulding processes where a plasticised charge is injected into a closed mould. This classification includes sheet and bulk moulding compounds (SMC and BMC) which although not strictly laminates, are closely related in terms of constitution and applications [Bader and Lekakou (1997)].

1.3.1.1 COLD COMPRESSION MOULDING

The term “cold” in this context means a process where no additional heat is introduced in the mouldings. In cold compression moulding, matched tooling set up in a simple-action press is used. For long production runs it may be economical to use metal tooling. While cast and aluminium alloys are used for successfully medium runs, steel or cast iron are used where maximum durability is required. Although metal tooling allows relatively higher pressure to be used, due regard must be given to the different thermal conductivity and heat capacity of the tool material compared to the corresponding thermal properties of the composite.

Dry reinforcement is placed into the open mould. An accurately metered charge of liquid resin formulated to cure rapidly at ambient temperature is then poured into the mould.
The mould is then closed so that the required temperature could be applied. This distributes the resin and causes it to percolate the reinforcement. The resin then cures, typically in about 5 minutes. On completion of the cure process, the mould is opened and the warm cured moulding is removed. It is important to regulate the quantity of resin added, as this must completely fill the interstitial spaces in the reinforcement. Too little resin will result in dry patches and too much resin is not economical and may result in excessive weight and section thickness.

1.3.1.2 HOT COMPRESSION MOULDING

"Hot compression moulding" is a process that is quite similar to the cold compression moulding. However, in this case the mould used is heated to a considerable temperature. Consequently, this has a number of important implications. First, the mould must be constructed from metallic materials such as steel, cast iron, aluminium, and zinc alloys. The heating system functions either with buried electrical resistance heaters, or by circulating oil, water or steam. In some cases, heated press platens are used and the moulds are heated only by conduction from the platens. The repercussion of the steel tooling is that higher compression pressure may be employed. Secondly, the choice of the resin must be such that it does not have to be formulated for fast cure at ambient temperature. This allows the use of systems with appropriate long pot lives and extends the choice of resin to those that can be cured at elevated temperature, such as phenolic and epoxy formulations.

1.3.1.3 SHEET MOULDING COMPOUND (SMC)

As the name implies, these are groups of materials consisting of sheet reinforcement preimpregnated with a thermosetting resin. This is mostly unsaturated polyester or vinyl ester. Often, the formulated resin is a dry paste and the reinforcement is most frequently random in-plane, but other versions using aligned continuous fibres are also available. The key modification is the addition of maturation agent, which is usually 2-5% magnesia in the polyester systems.
Mineral fillers are used in addition to glass, or other fibres to reduce the cure shrinkage and improve the surface finish.

SMCs are chiefly developed for use in high volume manufacturing, especially the automotive industries. In the basic process (see Figure 1.5), which starts with the SMC machine, a continuous sheet of material of dimensions 2 m in width and 5 mm in thickness is produced from random chopped glass strand reinforcement. The dimension of the reinforcing fibres is typically 50-75 mm length at a volume fraction of 0.3. On the continuous belt of the SMC machine, a layer of liquid resin is coated on suitable support film made from silicone- treated paper or polyethylene. As the continuous belt passes under a cutting head, chopped strand glass fibre roving cut at a predetermined length is allowed to fall freely onto the support film.

A further resin-coated film is brought in on top of the first layer to form a sandwich of random chopped glass fibre strand between the two layers of resin. The sandwich then passes through a set of kneading rollers so that consolidation of the glass and resin into a homogeneous layer would be achieved. The product is made into rolls and stored under controlled conditions for about 72 hours for the resin to mature and transform to a dry pasty compound. The mature SMC may be separated from the support film, cut
into the required size and shaped into pieces for charging into a hot compression moulding process.

1.3.1.4 BULK MOULDING COMPOUND (BMC)

BMCs are similar to SMCs both in terms of constituents and the processing possibilities. The basic difference is that a BMC is formulated only with discontinuous chopped strand glass roving, and is supplied in bulk rather than in sheet form as in the case of SMC. A crudely formed rope is the most common format of BMC. The constituents are resin, chopped glass strand, mineral filler and other additives, which include initiator and accelerator. They are mixed in a ribbon blender or similar device and then extruded into a rope, typically 20-50 mm in diameter. To produce a comparatively dry, non-sticky and easy to handle rope, the resin is formulated to mature similarly in the same way that SMC matures. In this case the fibre strands which are 10-30 mm tend to be shorter than those in SMC.

In the past, most BMCs were processed either by hot compression or transfer moulding techniques even though, presently, injection-moulding technique has dominated over hot compression technique. For the injection moulding process, the charge rope must be heated first to reduce the viscosity of the resin so that it can be injected into the hot mould and the pressure maintained until gelation has occurred. A number of special features are needed for the efficient processing of BMCs. In the initial stage a mechanical stuffer is used to charge the cold “rope” into the machine. The stuffer is based on either a reciprocating piston or a screw feeder, and carries the stiff paste of BMC into the inlet region of the plasticising screw where it will be picked up and carried down the barrel of the machine. The charge undergoes vigorous mixing action in the barrel, becomes heated up by conduction from the heated barrel and adiabatically, by the work performed on it by the screw.

During this process extra caution must be exercised in order to control the temperature of the charge inside the barrel. If the barrel is too cool, the viscosity of the charge may be too high for effective injection and mould filling. Conversely, if the charge is
overheated it may start to gel in the barrel before it is injected into the mould. The hot charge, typically at about 80-100 °C, accumulates at the front of the screw, behind the shutoff valve, and the back-pressure developed forces the screw to retract rearward. When sufficient material has become plasticized, the screw rotation is stopped, the nozzle is engaged in the mould, the shutoff is opened, and the screw is driven forward like a ram to inject the charge at high velocity into the heated mould cavity. As the cavity is filled, a high pressure is allowed to develop in the mould by continuing to apply pressure to the screw/ram. The mould is maintained at the cure temperature for the resin so that gelation occurs after a short interval and cure in 2-10 min. As soon as gelation occurs at the sprue gate the nozzle is disengaged from the mould as it is no more useful to pressure the mould and the shutoff valve is closed for the next plasticization cycle to commence. When the moulding is sufficiently cured, the mould is opened, the moulding is ejected, and the mould is closed, ready for the next moulding operation.

1.3.2 AUTOCLAVE PROCESSING

Autoclave processing is a process that is mainly designed for laminated structures from resin impregnated reinforcement sheet (prepreg). It was initially designed to meet the challenging need for composite components of the aerospace and military equipment markets of highest quality. Laminated components fabricated through this process have become a yardstick for judging the performance of other components that are manufactured through some other routes, even though autoclave processing is costly in terms of feedstock, capital investment, labour requirement and processing time. Regardless of its limitations, autoclave processing continues to be the most practicable and preferred technique for diverse applications and has exhibited to produce components that are more cost effective than those fabricated by competitive routes. There have been many reasons attributed to the dominance of this process. First, it is flexible and can be adapted to produce components in a wide range of shapes and sizes. It can be tailored to design structures that exploit the principle of parts consolidation whereby a single autoclave moulded structure may replace an assembly.

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of sometimes hundreds of individually fabricated components. Secondly, savings can be very considerable when fastening and quality assurances are considered. Finally, in the aerospace industry, product runs are sometimes very short and modifications must be incorporated frequently.

The most common basic principle of autoclave process starts with the pressure vessels. The pressure vessel is designed to accommodate pressure up to 5 bar. Most autoclave vessels used for fabricating composite components are typically 2-4 m in internal diameter and 5-10 m in length. A gas normally supplies the pressure inside the vessel, which is usually nitrogen that is circulated through heaters so that uniform temperature throughout the vessel is maintained.

The feedstock for the process is resin preimpregnated reinforcement sheet, sometimes referred to as a prepreg. Pre-impregnated materials (prepregs) are reinforcement fibres or fabrics into which a pre-catalysed resin system has been impregnated by a machine to hold the fibres in place [http//www.spsystems.com]. The prepreg is supported on a carrier film of silicone-treated paper or polymer and further protected by a cover film. A standard layer of prepreg is designed to mould to a thickness of 0.125 mm, such that 8 layers will be required to form a laminate 1 mm thick. An alternative is to use a woven cloth of standard thickness 0.25 mm per layer and impregnated in a similar fashion.

The component fabrication commences by cutting the prepreg into appropriate shapes, always ensuring that the fibres are aligned in a specified direction. These shapes are then assembled on the tool to form a laminate. A typical lay-up for a unidirectional laminate would be $0^\circ/\pm45^\circ/90^\circ/\pm45^\circ/0^\circ$. In most cases the prepreg is produced with a controlled excess of resin. This must be bled during the final production operation to produce a component with specified thickness and fibre volume fraction. In this case the raw laminate must be covered with a permeable membrane (such as a fine glass cloth treated with PTFE coating) that will allow resin to flow out of the moulding but will not stick to the surface of the component.
A layer of porous material, usually a fine woven cloth, is placed on top of the membrane to soak up the bled resin. This is termed the bleeder pack. A non-stick gas-permeable film is then applied. This is followed by a breather pack and, finally, a membrane or bag that completely seals the assembly so that the space between the outer membrane and the tool surface may be evacuated. The whole arrangement causes the outer membrane to be pressed against the laminate by atmospheric pressure. A typical schematic for the lay up can be seen in Figure 1.6.

The entire arrangement is then placed in the autoclave where the bagged moulding may be reconnected to an evacuation system to maintain the vacuum. The autoclave may then be pressurised to augment the consolidating pressure. The system is then heated according to a predetermined cure cycle. This causes the matrix to soften first, so that any excess may flow into the bleeder, and then cure and cross-link. When the resin is sufficiently cured, the temperature in the autoclave is lowered, it is depressurised, the moulding extracted, and the component separated from the mould. For an efficient and satisfactory production, the operation must be accomplished in the shortest possible time.
1.3.3 DIAPHRAGM FORMING PROCESS

The diaphragm forming technique is a potentially cost-effective technique, where the geometry of a given part is achieved in a single forming step [Gutowski et al (1995)]. It offers a significant reduction in parts cost compared to other conventional techniques when combined with automated tape lay-up and ply cutting techniques. Although, diaphragm-forming techniques were first applied to thermoplastic matrix composites in the 1970s, recently efforts have been directed towards implementation of the process utilising thermosetting polymer systems. Currently, diaphragm formed parts using thermoset composites are in production on Boeing’s 777. Thermoset systems possess several advantages over thermoplastics, including lower fabrication temperatures and pressures and, in some cases, lower material cost. The schematic of the diaphragm forming process is shown in Figure 1.7.

\[ \text{Lay up process} \]

\[ \text{Forming process} \]

\[ \text{Figure 1.7: Schematic representation of the diaphragm forming process [Gutowski et al. (1995)]} \]
1.3.4 RESIN TRANSFER MOULDING

Resin transfer moulding (RTM) is a popular composite material manufacturing process, gaining favour in the civil, aerospace, automotive and defence industries. The process possesses several advantages, including net shape production, moderately low cycle times and simple tooling requirements. RTM has showed the potential to produce low cost, high quality, geometrically complex parts. Related processes include structural reaction injection moulding (SRIM), seeman composite resin infusion moulding process (SCRIMP), and the recently developed injection compression moulding (ICM) [Lekakou and Bader (1997), Beckwith and Hyland (1998), Bickerton et al. (1997)].

The first step in the RTM process consists of placing the preform within the mould cavity. Fibre preforms are usually constructed from woven or stitched fibre mats or three-dimensional weaves, formed with fibreglass, kevlar, or carbon continuous strand fibre. The fibre preform will provide the finished piece with the majority of its structural properties. Once the mould has been sealed, a polymeric resin is injected into the mould cavity, saturating the preform and expelling any air present (see Figure 1.8).

![Figure 1.8: Schematic of the resin transfer moulding (RTM) process.](image)
The flow of the liquid resin through the porous medium is governed by the permeability of the fibrous preform [Johari (1994)]. Fibre preform permeability has a strong influence on the resin impregnation during polymer composite fabrication. Permeability together with resin viscosity determines the processability of the composite material in many applications. In most RTM studies, the flow of liquid into the fibrous medium is modelled using the Darcy’s law. The law assumes the flow rate of Newtonian fluids through saturated porous media to be proportional to the pressure drop across the medium. The porosity of the fibrous preform is assumed to be homogeneous throughout the medium. The continuity equation and Darcy’s law for the flow of the liquid through an anisotropic preform are given by:

$$\nabla \cdot \vec{v}_o = 0$$  \hspace{1cm} (1.1)

$$\vec{v}_o = -\frac{1}{\mu} \mathbf{[k]} \nabla P$$  \hspace{1cm} (1.2)

where, $\vec{v}_o$ is the superficial velocity vector, $\mu$ is the viscosity of the resin, $\mathbf{[k]}$ is the permeability tensor of the reinforcement, and $\nabla P$ is the pressure gradient. A curing reaction is initiated, most of which occurs after the complete filling of the mould cavity in RTM and the part solidifies. The finished composite product can then be removed from the mould.

The preform is of vital importance to the final properties of the finished piece. The main advantage to using a certain type of preform is that the final product structural properties can be tailored to suit the required application. In the past, ‘preforming’ has remained somewhat of an art relying to a large extent on labour intensive hand lay-up. Both braiding and draping offer the possibility of further automation in the RTM processes. Large portions of RTM preforms are manufactured using fabric style reinforcement materials. These materials include both woven and stitched fabrics and may be stacked in a variety of ways to obtain the desired properties.
‘Draping’ is a preform construction technique and, in the context of the current study, it refers to the art of bringing an initially flat fabric into contact with an arbitrary curved mould tool surface causing the deformation of the fabric [Bickerton et al (1997)]. The ability to predict the final configuration of the reinforcement material after draping is of interest to both preform manufacturers and manufacturers using the draped preforms to fabricate composite components. The final orientation of reinforcing fibres within the preform will be a major factor in determining the structural properties of the finished piece. Within an RTM moulding scenario, fibre orientation and local variations of fibre volume fraction, due to draping, can have a significant effect on the mould filling process.

Successful filling of the mould is required to produce a good finished composite product. Defects incurred through unsuccessful mould filling include both macroscopic and microscopic voids. Macroscopic voids refer to large air pockets that have been trapped within the mould cavity due to a faulty injection scheme. Microscopic voids refer to tiny air bubbles trapped within and around fibre bundles of the preform. Such voids are often very difficult to eradicate completely but attempts should be made to minimise them. Any type of void remaining in the finished product may seriously degrade the strength and quality of the part.

Presently, significant research effort has been focused on the development of numerical simulations of the RTM process including both mould filling and curing stages. Such a simulation would represent an invaluable design tool for industrial applications, allowing for full exploitation of the benefit RTM provides. These simulations will ensure that an injection scheme can be found to minimise defects and production costs before the expensive process of mould fabrication is considered. Numerical simulations of the mould filling stage also provide predictions of mould filling time and pressures experienced within the mould. This information is important for the fine-tuning of the process for larger volume production.

Computer simulations of the preforming process could of course constitute the first stage of RTM simulations. Their predictions could include values for the fibre volume

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fraction and fibre orientation, which could be used to determine the permeability of the preform, needed for the simulation of the filling stage.

At the beginning of section 1.3.4, apart from the RTM process, SRIM, SCRIMP and ICM were also mentioned. SRIM is usually related to fast reacting polymer matrices, for example of the urethane type, of a very low initial viscosity. Due to the high reactivity of the polymer, a very fast on-line mixer is required for SRIM to mix the reacting constituents. Otherwise, the set-up for SRIM is exactly the same as for RTM. In SCRIMP, the mould tool is one-sided only, the fibre preform is placed on the top and it is covered by a vacuum bag. Then, vacuum is used to inject the resin into the mould and impregnate the fibre preform.

ICM involves injecting a charge of material into a partially open mould (5-10 mm) for the full injection stroke. The mould is then fully closed for the required curing time. The completion of mould closing after the injection step causes additional material flow within the mould, which ensures optimal filling of the corners and intricate spaces. This process gives some additional control of fibre orientation as a result of the two stage moulding process. In the initial stage of the moulding procedure (injection stage), some fibre orientation in a random mat related to the cavity shape and the pressure can occur. During the compression stage the fibres of a random fibre mat reorient under a pressure that is applied equally across the whole section and high degrees of fibre orientation are generally not obtained. As a consequence, the properties are not as anisotropic as with mouldings possessing highly developed fibre orientation. This may be beneficial in some other applications where strong anisotropy in some components produces very good properties in one direction and significant inferior properties in the other.

Correct mould design is essential for the production of accurate specimen dimensions with injection/compression moulding. For ICM moulding the configurations are different from conventional tools in that the cavity is completely enclosed even at the parting line. This containment prevents material leakage and is achieved by telescoping a portion of the cavity side into a matching clearance on the core side of

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the mould. Precision fit of the telescoping areas creates the cavity pressure needed for the compression cycle.

1.4 SCOPE OF THIS STUDY

Draping of fabrics in the preforming stage of composites manufacturing techniques, such as RTM, introduces a number of complex issues that require investigation and modelling. Good drapeability of fabric reinforcement is of great importance and wrinkling is undesirable. On the other hand, draping generates local changes in the fibre volume fraction and fibre orientation of the reinforcement, which are going to cause local changes in the permeability and mechanical properties of the reinforcement. This study focuses on woven fabrics. The scope of this project is to investigate and compare the drapeability, change of fabric characteristics during draping, and shear deformation of four types of woven fabrics, as well as develop and validate mathematical techniques modelling the shear deformation and draping of fabrics in resin transfer moulding. More specifically the following specific aims are included:

(a) to study and compare the drapeability and fabric characteristics after draping of four different types of woven fabrics (a loose plain weave, a tight plain weave, a twill and a 5 harness satin weave) in resin transfer moulding;
(b) to study, compare and model the shear deformation of different types of woven fabrics, since the shear deformation is considered to be the major mode of deformation of woven fabrics in draping;
(c) to investigate and model the shear locking effect in woven fabrics, since this is a major cause for wrinkling during the preforming stage;
(d) to investigate and validate the application of the solid mechanics approach in the computer simulation of woven fabrics by using the LUSAS finite element software package supplied by FEA Ltd.

The thesis layout is as follows:
Chapter 1 (present chapter) gives a general introduction of the topic, an outline of fibre reinforcements in composite materials, a description of manufacturing techniques of fibre reinforced composites, especially RTM, and the scope and aims of this study.

Chapter 2 contains a literature survey of experimental studies and modelling techniques and simulations of the draping of fabrics, shear tests of fabrics (techniques and results), models of the shear behaviour of fibre reinforcements and studies of the effect of fabric drape on the mechanical properties and permeability.

Chapter 3 presents the studied woven fabric materials and their specifications, and the experimental techniques used in this study. Two major types of experimental studies were designed. The first part involved the draping experiments. A hemispherical mould surrounded by a flat rim was constructed that provided a model complex tool surface for the fabric to undergo significant deformation. A detailed description of the procedure of the draping experiments and the determination of the angle between crossing fibres in the deformed fabric is included in chapter 3. The second part of the experimental studies aimed at examining the shear deformation behaviour of the woven fabrics. This was achieved by conducting shear test experiments in a picture-frame type of apparatus mounted on a tensile testing machine. Chapter 3 contains the analysis methodology of the data from the shear tests. A description of hand shearing tests is also included.

Chapter 4 is on mathematical modelling. It contains the elasticity analysis of the shear deformation of fabrics which takes into account the change of fibre direction during the shearing of fabrics. It also describes the principles of the finite element continuum solid mechanics as it was applied in the computer simulation of the draping of fabrics.

Chapters 5 and 6 contain the results of the experimental studies of the draping of fabrics. Chapter 5 presents the photographs of the draped fabrics, where the different types of fabrics may be compared in terms of wrinkling and a measure of wrinkling is proposed. Chapter 6 presents the experimental data on the draping characteristics of each type of draped fabric, including maps of local angles between crossing fibres.
From these data, the distribution of fibre volume fraction and modulus in the draped fabrics, as part of composite product, is calculated.

Chapter 7 is on the results of the shear deformation of woven fabrics. The data of the picture-frame experiments are presented in different forms and curves are fitted as appropriate. The different fabrics are compared in terms of initial in-plane shear modulus, non-linear shear behaviour and locking shear angles. The elasticity theory for high shear strains, described in chapter 4, is applied to the analysis of the results and the values of certain parameters of the model are determined. With the aid of microstructural analysis, a geometrical model of the shear of a unit cell of each weave is proposed at the shear locking limit and the shear locking angles are predicted for each weave and compared with experimental data.

Chapter 8 focuses at developing a modelling approach incorporated in the LUSAS FEA computer package, to predict the drapeability of woven fabrics. The computer simulation is based on the continuum solid mechanics model, which states that the deformation behaviour of fabrics can be regarded as that of a solid continuum with certain mechanical properties. In this analysis, two types of geometry are considered for the drape simulations. These are the single curvature geometry and the double curvature geometry. The interfacial volume between the fabric sheet and the solid mould surface is modelled by flexible solid elements of a modulus of 0.1 MPa.

Chapter 9 contains conclusions and recommendations for future work.
2.1 DRAPING OF FABRICS: EXPERIMENTAL STUDIES

Potter (1979) investigated the deformation of aligned sheets and fabrics to determine the influence of stretching of the reinforcement on the production of complex structural mouldings. Three classes of fibre reinforcements were investigated: woven and similar angle-ply reinforcements; prepgs of aligned but discontinuous fibres; and two-dimensionally random fibre mats.

For a woven material he concluded that the limitations to the angular rotation of fibre yarns are due to the type of weaving. Whilst for a loose weave coherent fibre slippage interferes with the rotation of the fibre yarns, there is an inhibition to the extent to which the fibre yarns would rotate at the joints for the tight weave.

In the case of unidirectional prepgs of discontinuous fibres, Potter found that one way of determining a limit of stable deformation is to measure the load/extension curve at constant rate of displacement. According to these tests it was observed that reducing the gauge length of the test specimen reduces the amount of extension and load required, and applying the extension slowly will increase the extension of the specimen. Also, he observed that testing the material at an off-axis angle of 0°-2.5° resulted in perpendicular tearing of the fibres. Testing at 5°-15° produced tearing at an angle to the test direction and testing at an off-axis angle of 20°- 40° resulted in fibre rotation away from the test direction, leading to thinning and failure of the prepg.

For crossplied prepgs it was observed that the strength of the bond between the plies affects the amount of elongation. With well-bonded plies there is an increase in elongation when stretched at the expense of fibre rotation and the deformation mode was by both fibre rotation and slippage.

Potter concluded that the modes of deformation available for draping reinforcements depend on the number of fibre directions present. Woven fabrics can only deform by fibre rotation, and not along the fibre axis.
Unidirectional prepregs of short fibres may be stretched transversely, axially, and in shear without significant change in fibre orientation, while shaping of crossplied reinforcements is more stable and reversible when they are pre-stretched in the opposite sense prior to shaping.

Long et al (1996) investigated the effect of the depth of draw on the deformation of angle-ply reinforcements when formed on a wheel-hub mould. In order to establish this effect quantitatively, the fabrics were inscribed with a square grid and the coordinates of the deformed grid, representing the deformed fibre yarns, were obtained using a system named CAMSYS, Automated Strain Analysis and Measurement Environment (ASAME). This system used a camera to picture the deformed grid which was then monitored in a PC that allowed the three-dimensional coordinates to be measured using the CAMSYS software package.

According to the findings of this investigation, it was revealed that as the depth of deformation of a ±45° cross-ply reinforcement increases, shear deformation, which is the most dominant mode of deformation, increases. However, as the depth of draw increases it was found out that local inter-fibre slip measured from the change in grid spacing increases. These findings suggest that inter-fibre slip becomes an important mode of deformation in angle-ply reinforcements as the depth of draw increases.

Standley (1997) carried out a drape forming experiment in which a prepreg of a thick plain weave Torayca T300 carbon fibre reinforcement impregnated with Hexcel Fibredux 6376 epoxy resin was draped over a hemispherical component of radius 75mm using a single diaphragm vacuum forming process. Parametric studies conducted showed that, increasing the vacuum time from 30 s to 4 min reduced the amount of wrinkles produced. Changing the shape and size of prepregs from square to octagonal shape resulted in shifting the areas of greater wrinkling formation to the hemispherical section. Changing the forming temperature from 60 °C to 100 °C reduced the amount of wrinkles by 17% due to a decrease in viscosity of the resin. Finally, mechanical tests carried out on the draped material loaded at 0°, 15°, 30°, and 45° orientations showed that maximum deflection was produced by the 45° orientation
test. This, according to Standley, indicated that the stiffness of the material reduces with shear.

Andrews et al (1997) draped an 8-harness satin weave in 0/90° and 0/±45°-ply orientations over a rudder tip mould. Results showed that a mould inclination of 20° eliminated the wrinkles for the 0/90°-ply orientation and a mould inclination of 6° eliminated the wrinkles for the 0/±45°-ply orientation.

Wang et al (1999) conducted draping experiments of woven fabrics or woven fabric prepirgs where the materials were draped over different types of aircraft components. First of all, it was found that different starting contact points between the fabric and the mould surface might result in considerably different draped patterns. Yarn slippage was observed in carbon but not glass fabrics. For carbon fabric preforms with higher lateral bending stiffness and fewer crossover points, yarn slippage was observed when yarn lateral bending was significant.

Chu et al (1950) and Cusick (1961) studied the drapeability of fabrics which they determined by the drape coefficient, defined as the ratio of the projected area of the draped fabric specimen over the specimen's original area. Hu and Chan (1998) found that in-plane shear and bending properties of fabrics are highly correlated with the drape coefficient and sometimes internal-friction has an effect as well.

Cai et al (1994) were concerned to find a measure of formability of fabrics. For this they carried out forming experiments over a hemispherical mould geometry and they measured the load on the punch, where the punch was connected to an Instron mechanical testing machine and descended at a constant speed. From the measurements the forming energy was calculated and was used as a formability index. It was found that the plain weave was the least formable fabric but a five and an eight harness satin had a better formability index.
2.2 DRAPING OF FABRICS: MATHEMATICAL MODELLING AND COMPUTER SIMULATIONS

The problem of draping of fabrics was first studied in the textiles industry. In the field of composites, most of the published studies focus on the forming of thermoplastics. Published studies including computer simulations of the draping of fabrics in the preforming stage of processes such as resin transfer moulding are relatively few. In the past and in most of the literature cited on the drape modelling and simulation of woven fabrics, two methods of analysis have been used to study the drape behaviour of woven fabrics: (a) geometric mapping algorithms (incorporating the “fishnet” model) and (b) the continuum mechanics approach. The two main modelling approaches are covered in the literature review of sections 2.2.1 and 2.2.2 respectively.

2.2.1 MAPPING APPROACHES

Potter (1979) investigated the deformation of aligned fibre sheets in the production of complex structural mouldings. Three classes of materials, which include cloths and angle-ply reinforcements, prepps of aligned but discontinuous fibres and two-dimensionally random fibre mats were investigated. In the first part of the investigation Potter attributed the shaping of fabrics onto complex geometries to shear produced mainly by angular rotation of the warp and the weft fibre yarns. The shaping process was modelled on a hemisphere using the pin-jointed net method, otherwise known as the “fishnet” approach. In this method, the yarns are considered as a part of a net in which they can rotate freely and without any constraints at the joints.

Robertson et al (1981) pioneered research to ascertain the rearrangement of continuous fibres during the forming of a fibre cloth onto a hemispherical surface. The investigation involved both a computer simulation algorithm which was based on the “fishnet” approach, and real experimental studies. In order to develop the numerical algorithm a hemispherical surface was chosen for the shaping process. The fabric was considered to be a fisherman’s net in which the elements were interconnected at nodal
This reduced the problem to that of coordinate geometry where the nodal position followed the polar coordinates of the spherical surface. Locating the nodal coordinates involved solving three pairs of quadratic equations that described the points of coincidence of three spheres repeatedly until the whole net was covered.

From the computational predictions it was accepted that the fisherman's net could be wrapped around the hemisphere without wrinkling. However, according to the experimental studies wrinkling was observed when the vertical distance of the hemisphere reached 1.7R (R is radius of the hemisphere), depending on the fabric used. This could not be predicted by the "fishnet" algorithm.

Robertson (1984) conducted a similar simulation study on a conical surface geometry. The shape of the mould chosen consisted of a rounded cone in which the top was replaced by a spherical sector. The computer algorithm developed in the previous study was modified to cover the geometry of the spherical sector and the transitional conical surface. The placement of each nodal point started at the spherical surface with the coordinates given in polar spherical coordinates and was transformed to the conical coordinates at the point of transition.

Using the generated "fishnet" algorithms, several simulations were performed on models of cones with cone angles of 40°, 60°, and 80°. Results produced showed favourable draping without wrinkling for cone angles greater than 59°. Possible wrinkling due to the steep change of slope at the top of the cone was modelled by using a symmetry criterion. One observation from these data (simulated and real experiment) is that the deformation angles of the warp and weft fibres along the diagonal direction were found to remain constant on the entire conical surface starting from the end of spherical section.

Gutowski et al (1995) presented a paper, which compared experimental observations on the wrinkling of aligned fibre prepregs during the forming process with theoretical scaling laws based on ideal kinematics (Struik (1961), Tom and Gutowski (1990)). The kinematics approach assumed fibre inextensibility, constant interfibre spacing and
constant thickness of prepreg. According to the report, the conformance of aligned fibre prepregs to a complex geometry is achieved mainly by longitudinal in-plane shear (along the fibre direction) and inter-ply shear (see Figure 2.1). During the forming process there was the tendency for the prepregs to fail through wrinkling. This is one of the major defects associated with forming and is caused by compressive forces, which arise from significant material compression necessary to form many double curved shapes.

Figure 2.1: Schematic of the in-plane and inter-ply shear modes for aligned fibre, cross-plied composites [Gutowski et al (1995)].

Using the hemispherical geometry (Figure 2.2) as an illustration, the amount of compressive force needed by the preform to compress to the final shape was estimated. Using ideal scaling laws for composites, the critical condition that leads to laminate wrinkling was obtained by combining many factors. These included the part shape of the laminate obtained from differential geometry, shear strain mapping, the appropriate constitutive equations for the material deformation behaviour, the stiffness properties of the diaphragm in diaphragm forming and the composite's inherent resistance to buckling.

Based on this, it was found out that the following theoretical equations satisfied the critical condition required for wrinkling to appear in the forming of laminates. The critical load for wrinkling was given by:
where, $F_D$ is the applied force on the diaphragm.

\[ F_{\text{crit}} = 2F_D \]  \hspace{1cm} (2.1)

**Figure 2.2: Illustration of the compressive deformation $\Delta S$ required to form a hemisphere [Gutowski et al (1995)]**

The critical in-plane and inter-fibre compressive forces were given by:

\[ F_{12} \sim N_p h w (\tau_0 + m \Gamma_{12}^n) \]  \hspace{1cm} (2.2)

and

\[ F_{v3} \sim N_p L w (\tau_0 + m \Gamma_{v3}^n) \]  \hspace{1cm} (2.3)

where, $N_p$ is the number of plies, $h$ is the laminate thickness, $n$ is the power law shear rate exponent, $m$ is the power law coefficient and $\tau_0$ is the yield stress. The shear strain required in equations 2.2-2.3 was estimated from ideal kinematics and was given by:

\[ \Gamma_{12} = \int_{0}^{l} \kappa_g (s)ds \]  \hspace{1cm} (2.4)

where, $\kappa_g$ is the geodesic curvature and $s$ is the length of the geodesic curve.
The above procedure was used to calculate the required ideal shear strain for a variety of complex shapes that ranged from a hemisphere to a curved C-channel. There was disagreement between predictions and experimental results. From the theoretical equations and fittings with the experiment of small-scale values, critical wrinkling parameters were obtained. These parameters were used in scaling laws. Results indicated a good agreement between the model and the experiment for small extent of scaling, but not for large scaling. Deviations from the scaling laws in the latter case were attributed to deviations of the deformation process from ideal kinematics.

Heisey and Haller (1988) proposed a computational method for fitting woven fabrics to non-algebraic surfaces using numerical analysis. The method was based upon modelling the process of smoothing of fabric over surfaces. In this investigation the location of the coordinates was obtained using the differential geometry equations that describe the arc length at the point of intersection between the warp and weft fibres on the surface. These differential equations were transformed into cylindrical coordinates and formed the basic of the computer algorithm, which was solved iteratively using the Newton method.

In adopting these formulations and in order to obtain good and accurate results no assumptions were made about the surface other than that it is continuously differentiable so that the whole fabric could be mapped. Important assumptions made about the fabric were that the warp/weft intersections act as pivot points and that the distance between adjacent warp and weft intersections is small enough for the curvature of the surface to be nearly constant between intersections. From the computational analysis Heisey and Haller showed that the formulations developed produced good and reasonable results by comparing the predicted coordinates obtained from the model with those from a real experiment before the shear locking limit was reached and those obtained using a modified version of Mark and Tailor's (1956) algorithms. No comparisons were made in any area with wrinkles.

Laroche and Khanh (1994) presented a study on the prediction of the complex rearrangement that occurs during the moulding of complex components made of
woven fabrics prepregs. In this study, the deformation of the composite laminate to conform to a given geometry was modelled using the pin-jointed net model. The pin-jointed model used in this study was upgraded to take care of the yarn slippage, which was not considered in similar approaches used by other researchers. The extent of yarn slippage was assumed to increase linearly with the intra-ply shear deformation angle. The coefficient of proportionality was determined from tensile tests on ±45° fabric specimens.

They also predicted the variation in thickness of the draped fabric by assuming that the fibre volume fraction remained constant throughout the fabric during draping. There were differences between predictions of the fibre angles and experimental data and even variation between the samples of the same type of weave or between different quadrants of the cone mould geometry. The worst differences between predictions and experimental data regarded the thickness distribution results.

Trochu et al (1996) conducted a study that combined the methodology of dual kriging surface interpolation and the “fishnet” algorithm to investigate the orientations of fibres and the net shape of fabrics when draped over complex geometries. By combining these two algorithms two types of analysis were performed for the draping process. The first part of the investigation involved modelling of the parametric surface to be draped and the second part determined the coordinates of the fabric draped over the surface.

The draping algorithm was developed based on the idea of trying to cover a surface of complex shape with a fabric of aligned and continuous fibres. The model imposed that the fibres can neither stretch nor shrink. Thus the deformation of the fabric was accounted for solely by rotation of the fibres around the nodes. Another assumption was the limit of the shear angle above which wrinkling was initiated and the ability of the fabric to be formed on complex surfaces was characterised by this limit angle. So when this limit was reached the computer run was frozen and wrinkling was reported at that point.
Initially, the algorithm started by a preliminary determination of the fabric orientation, which was defined by a starting point on the surface and two curves along which the warp and weft yarns would be fitted (Figure 2.3). The next step consisted of computing the nodal coordinates of the fisherman’s net one by one. Two lines forming the boundary of each quadrant were covered by a set of nodes separated by values $a$ and $b$ which were multiples of the specified warp and weft elementary distances. The mesh size $(n, m)$ was obtained from the warp and weft lengths $L_{wp}$ and $L_{ws}$ along the principal axes by $n = L_{wp}/a$ and $m = L_{ws}/b$. The location of a given node could be calculated simply by solving the following set of non-linear equations in $(s, t)$:

$$
X(s, t) - X_{i-1,j}^2 + Y(s, t) - Y_{i-1,j}^2 + Z(s, t) - Z_{i-1,j}^2 = a^2
$$

$$
X(s, t) - X_{i,j-1}^2 + Y(s, t) - Y_{i-1,j}^2 + Z(s, t) - Z_{i-1,j}^2 = b^2
$$

where, the suffixes $(i, j-1)$ and $(i-1,j)$ refer to the positions of two previously defined nodes on the surface, and $X(s, t)$, $Y(s, t)$, $Z(s, t)$ are the positions of the next node to be determined $(i, j)$. The solution of equation (2.5) was obtained by the Newton type of iterative method.

Parameters for the kriged surface, the analytical equations of two planes ($P_1$) and ($P_2$) and lengths $a$ and $b$ of the mesh cells were fed into the computer programme.
The principal axes of the fabric were defined by the intersection of the two planes, which was also regarded as the starting point. This produced four quadrants and the computation was started by draping one of them from the starting point (0, 0). The arcs between this point and the boundaries were divided along the warp and weft directions, according to the specified lengths a, b. These elementary lengths defined the size of the grid that will be generated on the surface.

Figure 2.4: (a) grid defined by the fabric and (b) draping of patch on the parametric surface (Trochu et al (1996)).

In the next step, the non-linear equation (2.5) was solved for each new node such that given the coordinates of nodes (0, 1) and (1, 0) as shown in Figure 2.4, node (1, 1) was able to be generated. After finding (1, 1) and knowing (0, 2), node (1, 2) was determined. The process was continued row after row and column after column until
the whole patch was draped. In the final stage the four quadrants were assembled, leading to a full coverage of the surface.

According to the results obtained from the simulations, it was ascertained that in order to predict the drape process with reasonable accuracy the size of the grid should be as small as possible. Although there were no real experimental results to support this claim, comparison with other work [Robertson et al (1981)] shows that the methodology is valid. Also, when the simulation was performed for a turbine blade the resulted figures of draped reinforcement indicated that there was broad agreement between the predicted model and the experiment. By using the same algorithms, the size and the net shape of a preform that would fit a turbine blade and an irregular lamp base were simulated and compared with those for the experimental preforming of a non-crimp stitched bi-directional fabric. From the data obtained it was shown that the accuracy of the approach was maintained within the regular boundaries of the surfaces, but this tended to diminish when irregular boundaries were reached.

Another analogy to the "fishnet" model was made by Kuznetsov (1997), who implemented a numerical construction of the "Chebyshev net" model to develop a computerised geometric analysis of woven or braided reinforcements. The model idealised the fitting of a fabric onto a curved surface to be like trying to fit a net, which is characterised by the rhombic shape of its elementary cells onto the surface. In order to develop a procedure for the fitting of the net onto the surface, a cell-by-cell net construction method was adopted. To complete the construction of each cell, the coordinates of the vertices of its curvilinear rhombus were located using a simultaneous system of three non-linear equations. By building the cells into aggregates of finite size curvilinear rhombuses the whole surface was gradually covered with the net.

Although this approach has been successful in predicting the orientation of the fibres and the amount of deformation required to fit the fabric onto the surface, the accuracy of the method lies in the difficulty of computing the exact size of the rhombuses vertices used in the analysis. Secondly, the study does not provide any experimental work to verify and validate the quality of this approach.

Experimental Studies and Mathematical Modelling of the Draping and Shear Deformation of Woven Fabrics in Resin Transfer Moulding (RTM)
Bickerton et al (1997) reported developing a comprehensive computer code ("DRAPE") which was based on the "fishnet" algorithm to simulate fabric deformation during draping. It also predicted local fibre volume fraction and could be combined with the LIMS (Liquid injection moulding simulation) code to simulate mould filling processes. Using this approach the model of a fabric draped over a conical surface with an angle of 21° was generated and compared with that of an actual draped bi-directional stitched fabric.

Patterns of the shear angle along the circumference of the cone were compared with those obtained from the numerical predictions. According to observations reasonable agreement was found when comparing the deformation trends qualitatively. However, when quantitative comparison was made in the region of highest deformation, it was observed that the numerical predictions overestimated the deformation angles by about 10-15°. This behaviour was attributed to the DRAPE assumption that no slippage occurs between the weft and the warp tows. It was also found that due to the overestimation of the deformed shear angles in the region of highest deformation, the fibre volume fractions obtained from the numerical predictions were higher than those obtained from the experimental analysis.

By considering Darcy's flow into the preform and using the numerical predicted shearing angles, data of the permeability tensor for shear angles up to 36.5° of the draped fabrics were obtained. For predicted angles beyond the maximum shear angle, linear interpolation was adopted to predict the permeability tensor. These predicted permeability tensors were then inputted into a finite element computer code called LIMS to simulate the mould filling process for either constant injection pressure or constant flow rate. The predicted filling patterns were compared with those of an actual mould filling experiment in which a corn syrup liquid was used to impregnate the fabric mentioned earlier. It was observed that notable deviations from the experimental flow front shapes were found and the filling time of the predicted model was shorter than that of the actual experiment. These deviations could be attributed to the model inability to predict a decrease in the permeability values due to increase in fibre volume fraction and reorientation of fibre tows changing
the directions of principal permeability components. According to Bickerton et al these effects are very important for a correct flow front shape to be obtained.

Golden et al (1991) suggested the application of the kinematic approach for the forming of continuous fibre reinforced laminates according to which the material deforms to a given mould geometry, given the initial fibre direction in the material and taking into account the constraints of fibre inextensibility and incompressibility. The deformation was not affected by the stress level according to the kinematic approach.

Van West and Luby (1997) described the kinematics of bi-directional fabric deformation and rules for controlling wrinkling and bridging in an investigation carried out on fabric draping simulations in composite manufacturing. According to these co-workers, there are two possible mechanisms responsible for the conformation of woven fabrics onto curved surfaces. In the first mechanism it was believed that a woven fabric must undergo in-plane shear deformation (trellising) where the warp and weft yarns rotate relative to one another at crossover points. Another possible mechanism put forward, which takes place in extreme circumstances of shear deformation, is that in which the yarns spread or slip at the crossovers. The latter mechanism, however, was acknowledged to take place when the locking angle is attained which is specific to the type of weave spacing.

Simulated models of a hemisphere, truncated dome, split-ply, bead-stiffened panel and sine-wave spars were produced using CAD/CAM computer software. These predicted models were found to compare reasonably well with those from the real drape of a 5-harness satin fabric if no wrinkling occurred. From observations it was obtained that radial and tangential wrinkling (predicted by finding that the shear angle is above the shear locking limit) could even be controlled by increasing or reducing the angle between bounding constrained yarns through the direction of shaping process.

Sequel to the first publication Van West and Luby (1997) presented a simulation algorithm for the draping of fabrics in composites manufacturing based on analytical methods. The simulation algorithm for the draping process involved calculating the
coordinates of all simulated fabric node points on the surface. The method, using one warp and one weft yarn as constrained yarns as depicted in Figure 2.5, consisted of many steps. In step 1, two known points on the constrained yarn are used to calculate the location of new points. The new point and another known point on the constrained yarn are used to calculate the next new point in step 2. Using the new point and another known point, the process is repeated sequentially until all node points are calculated, row by row, quadrant by quadrant.

Figure 2.5: Sequence of calculations of fabric nodes (Van West and Luby (1997)).

The authors presented three different types of algorithms for the location of nodal points. In the first method (Figure 2.6(a)), the surface of two spheres with radii equal to a yarn segment length represent the loci of all possible positions of the ends of a warp and weft yarn segment with the other ends of the segments being the sphere's centres located at known points on the tool surface.
The intersection of the two spheres is a circle, which intersects the surface at two points one of which duplicates a known point and the other is a new point.

\[\text{circle/surface intersection} \]
\[\text{(existing node point)}\]

\[\text{sphere/sphere intersection} \]
\[\text{(circle)}\]

\[\text{yarn segment}\]
\[\text{sphere}\]
\[\text{sphere center} \]
\[\text{(existing node point)}\]

(a)

\[\text{center points} \]
\[\text{(existing node points)}\]

\[\text{circuit intersection} \]
\[\text{(existing node point)}\]

\[\text{surface} \]
\[\text{circuit}\]
\[\text{circuit}\]
\[\text{yarn segment}\]
\[\text{circuit intersection} \]
\[\text{(new node point)}\]

(b)

**Figure 2.6: (a) spherical algorithm and (b) circuit algorithm for calculating node points [Van West and Luby (1997)]**

In the second method (Figure 2.6(b)), two closed curves were generated on the surface around two known node points. The circuits represent the loci of points equidistant on
the surface from the known points. The circuits intersect at two points, one a known point and the other a new node point.

Figure 2.7 represents a different analytical approach, which requires the definition of only one "base" line on the surface, a starting point and "two sweeping" directions as the uniqueness constraints. The base line is defined by intersecting a plane with a

![Diagram of draping algorithm using one base and sweeping directions in (a) 2D flat and (b) 3D surface space [Van West and Luby (1997)].](image)

*Figure 2.7: Draping algorithm using one base and sweeping directions in (a) 2D flat and (b) 3D surface space [Van West and Luby (1997)].*
non-uniform rational B-spline (NURBS) surface. Fabric node points are calculated by intersecting the (NURBS) surface with two spheres representing yarn radii as described in the previous paragraphs.

Horsting et al (1993) performed an elaborate study on the drapeability of fabrics used for composites. In order to achieve a complete understanding of the draping process, the study was categorised into two types of analysis. These were the computer simulation and a real experimental validation analysis. The computer analysis adopted by these authors was based on a package called SIMITA G. The approach was based on the assumptions that the fibres are inextensible during deformation, fibre slip does not take place in the fabric and that the only mode of deformation is rotation of the fibre yarn at the intersections between the warp and weft tows. Certain information on parameters like the description of the body surfaces to be covered, the critical angle before jamming (or locking), the warp and weft thread spacing and a characteristic value typical of woven fabrics, which characterised their drapeability were obtained experimentally and fed into the programme.

The covering of a gear-polishing slide, a bicycle helmet, a loudspeaker membrane and a truncated cone were taken as an example of how SIMITA G can be used in drape simulations. The areas where fold deformation is likely to occur (shear angle above locking limit) were marked on the resulted drape fabric by the programme.

2.2.2 CONTINUUM MECHANICS APPROACH

The need to take into account the differences in the mechanical behaviour between different fabrics and the effects of the mechanical behaviour and friction properties on the drape forming of fabrics led to the application of mechanical approaches to be used in the modelling of fabric drape. The first approaches originated from the textiles industry where a fabric was considered either as a solid continuum sheet or as a mechanical structure or network.
Researchers from the field of composites manufacturing first considered the forming of thermoplastic sheets or prepregs reinforced with continuous or discontinuous aligned fibres or with random fibre mats. In this case, the material was considered as a continuum obeying a viscous constitutive model (see at the beginning of section 2.3). Such a model would be recommended to reinforcements in which the fibres can easily slip past each other during forming. However, since this is not usually the case for woven fabrics due to restrictions in the weave architecture, a solid mechanics approach has been applied in this study. As a result, the literature review in this section focuses on the solid mechanics modelling of the draping of fabrics.

Chen et al (1996) studied the effect of some fabric mechanical parameters on fabric drape using one of the numerical mechanical modelling approaches. The study used a mathematical modelling method that employed flexible thin shell theory to formulate the governing equations to simulate fabric drape. The governing equations were discretized using finite element formulations and then solved explicitly using the Newton-Raphson method. Using this approach, the generated model of a fabric draped on a table was simulated.

Chen et al. observed that parameters like the Young's modulus and thickness have similar effects on the fabric drape. When the modulus and the thickness were increased the drapability of the fabric decreased for an isotropic fabric, but for an orthotropic fabric the drapability was found to be less in the direction of higher stiffness than in the direction of lower stiffness. In the case of varying the shear modulus of orthotropic fabrics, observations showed that the orthotropy had an effect only in combination with high shear modulus. For the Poisson's ratio however, they found that varying the value from 0 to 0.5 had no appreciable effect on the shape or direction of the drape.

Collier et al (1991) simulated the draping behaviour of a fabric, which was considered as an orthotropic shell membrane undergoing small strain/large-displacement deformation, using a geometric non-linear finite element analysis. A non-linear finite element equilibrium equation, which incorporated both the linear and the non-linear deformation terms of the fabric, was developed.
Using the Newton-Raphson procedure, this equation was solved for a model of a circular piece of fabric draped over a pedestal. In parametric studies, parameters such as the Young's modulus and the shear modulus were found to exhibit a similar effect reported by Chen et al. However, when the effect of Poisson's ratio was considered, Collier et al observed that changing its value causes the deformation of the fabric to change dramatically. Although the values of shear modulus used in the simulations are not available for comparison, this observation slightly contradicts the findings of Chen et al.

Boisse et al. (1995) proposed a finite element formulation for the shaping of glass fabrics. The formulation was based on the potential energy consumed on deformation at the micro-level. For instance, they derived an equation for the amount of potential energy consumed by a single thread in a fabric when subjected to tensile deformation. By neglecting other possible modes of deformation during drape forming, such as inter-fibre sliding, equations were derived to obtain theoretical data for the in-plane tensile and shear behaviour of fabrics. This theoretical analysis was combined with the Green-Lagrange vector analysis to formulate a generalised constitutive equation for the deformation characteristics of the whole fabric during the shaping process. The formulation was then translated into the finite element approach and adopted to simulate the shaping process of dry fabrics (using both three and four node membrane elements) over a hemispherical dome. The predicted projected boundary of a fabric draped over a hemispherical hat indicated a very reasonable agreement with the experiment.

Using the same type of simulation tools developed and reported earlier (Boisse et al), Gelin et al (1996) conducted a study to investigate the shaping operation for the manufacture of thin composite structures by the resin transfer moulding process (RTM). In this study, drape forming of a woven fabric with fibres oriented at 0°/90°, and ±45° were carried out. The results of the drape simulations over a hemispherical geometry produced reasonable results compared with the experiment. In order to prove the capacities of the simulation tool developed, the simulation of a square cup shaping process was carried out. The results obtained from this simulation showed a
big difference with the experiment, but when the geometry of the tool used was modified the result showed a big improvement between the simulation predictions and the experiment.

In order to obtain a better understanding of the drape phenomenon of fabrics Kang and Yu (1995) developed a non-linear finite-element code to simulate the draped shapes of woven fabrics. The developed algorithm integrated both the geometric non-linearity caused by large displacements and large rotations during draping and the transverse-shear strain needed to avoid the shear locking effect common with thin plate analysis. The mechanical properties of the fabrics used in these simulations were obtained from tests using the Kawabata evaluation system (KES) (see section 2.3). In order to verify the finite element formulation two sets of analysis were performed. In the first analysis, the deflection of a rectangular strip was simulated and compared with the result of the experimental deflection of rectangular strips obtained from the warp, weft, and bias directions of wool and cotton fabrics. The second analysis compared the results of drape simulations of a fabric on both a round and a square shaped pedestal with the results of the actual experiments. In both cases there was reasonable agreement between the actual and the predicted models, although differences were present in some case studies.

Cherif and Wulfhorst (1996) reported developing a novel knitted fabric called WIMAG to study the influence of fabric architecture on drapeability. WIMAG is a high performance multiple layer fabric in which the orientations of the fibre sheets can be set as required within wide limits (0°, 90°, 30°, through 60°). Using the finite element method, both two and three-dimensional models of WIMAG were developed in which the fibre yarns were considered as beam elements interconnected at nodal points. The model incorporated the mechanical properties of each layer at nodal points, neglecting the inter-fibre sliding and the friction that exists between the layers.

The results of the drape simulations on a hemispherical surface using this model demonstrated the need for further improvements. When the model was modified using shell elements and the actual material characteristics such as the inter-fibre sliding and
the frictional coefficient between layers were taken into account the results were improved.

Blanlot (1997) conducted research to simulate the shaping process of fabrics using the finite element mechanical approach, which was based on anisotropic hypoelastic constitutive equations. The formulation incorporated Green-Nadghi's formulation in which the orthotropic axes were assumed to rotate with the rigid body frame. The evolution of the orthotropic hypoelastic law was determined by the location of the bisector of the warp and weft directions identified at each instant of the forming process and the constitutive equations of the fabric behaviour were therefore identified and up-dated at each time increment. The constitutive model was integrated in the ABAQUS finite element software.

In order to validate the above algorithm two analyses were carried out for a carbon fibre fabric with both non-updated and updated material law implemented in the ABAQUS software. When the results of the stress profiles for the two cases were compared it was shown that values from simulations with non-updated material law were insignificant compared to the values from the simulations with updated material law. Further, when a simple picture frame shear test was performed using the same carbon fibre fabric, the results of shear angle from the two types of simulations (updated and non-updated material law) differed by about 20°. However, the results of the simulations of the draping of a ±45° fabric with the updated material law showed good agreement with the real draping experiment.

Ye and Daghyani (1997) studied the forming of woven fibre fabrics by a biaxial deformation theory of woven fabrics, starting from principles of micromechanics. The fabric was considered as a structure rather than a continuum where the movement of fibre yarns in fabric shear was constrained by friction forces between crossing fibre yarns. An analytical model was described in which the shear deformation energy was calculated from the contact forces at yarn crossovers as a function of the angle of yarn rotation during the fabric shear.
2.3 SHEAR DEFORMATION OF WOVEN FABRICS: TESTING AND MODELLING

The shear behaviour of fibre reinforcements has been described by elastic, viscous and viscoelastic mechanical models. Viscous models (Roberts and Jones (1995), Wheeler and Jones (1995), Martin et al (1995)) are popular for unidirectional continuous fibre reinforcements and fibre mats in which fibres can slip easily past each other during shear. Johnson (1995) suggested a viscous model for fabric thermoplastic sheets to be thermoformed, implying that the flow of the viscous melt is dominant in the process of thermoforming and the flow behaviour is extended to the fibre yarns. Such models (also McEntee and O'Bradaigh (1996), Dykes et al (1996)), resulted in extremely high viscosities for the thermoplastic sheet to match the experimental data of stress versus shear (McGuinness and O'Bradaigh (1998)). It is suspected that a viscoelastic constitutive model would have been more appropriate for the forming of thermoplastic fabric reinforced sheets (Pickett et al (1995)).

There has been a considerable amount of work in the area of textiles to develop prototype tests and apparatus for the determination of the shear properties of fabrics. Buckling of fabrics during shear has been a major problem in designing perfect shear testing equipment. As a result most of the proposed methods are only suitable for low shear angles. An early test adopted in the textile industry is the Kawabata evaluation system for fabrics (KES-F) (Kawabata (1980), Kothari and Tandon (1989), Yu et al (1994) and Hu and Zhang (1997), amongst others) in which a sample of fabric, usually 200x50 mm, is clamped along the two opposite long edges and is sheared by moving one of the clamped edges at a constant speed (see Figure 2.8(a) and 2.9). During shear the clamped edges are kept apart by a tension, usually of 10 N/m width, applied on the fabric. This type of test is similar to a type of manual shear testing which is usually applied to determine manually the ‘locking shear angle’ of fabrics (Andrews et al (1997)).
Hu and Zhang (1997) analysed the stresses developed in a KES test and concluded that, due to the presence of corners and the presence of both tensile and shear stresses, the specimen is not subjected to pure and uniform shear. A finite element analysis yielded a distribution of shear stress along the clamping direction, where the shear stress varied from 0 at the corners to a maximum in the middle of the specimen. This was compared to the conventional shear test for a stiff material, which involves the application of torque on a cylindrical specimen, resulting in a pure shear mode of deformation.

The second type of shear test for textiles is the FAST test (Ly et al (1991), Yick et al (1996), Buckenham (1997), Boisse et al (1997), Boisse and Barr (1996)) (see Figure 2.8(b)) which employs the bias extension principle for measuring shear. The principle is also used to derive the in-plane shear response of ±45 laminates in polymer composites (ASTM D3518). However, when this test is applied to fabrics or composites under processing conditions, shearing is non-uniform throughout the specimen due to the distortion of width uniformity (McGuinness and O'Bradaigh (1997)).
Another type of proposed shear test for fabrics involves shearing of a fabric specimen, usually 200x200 mm, held within a picture hinged frame (McGuinness and O’Bradaigh (1997), (1998), Gelin et al (1996), Prodromou and Chen (1997)) (see Figure 2.8(c) and 2.10). Two diagonally opposite corners of the picture frame are pulled apart at a constant rate in a tensile testing machine. If it is assumed that the fabric is inextensible in the two fibre directions, there is only in-plane shear before wrinkling starts. It has been suggested (McGuinness and O’Bradaigh (1998)) not to clamp individual fibre bundles in the frame but let their ends rotate freely; otherwise it has been observed (McGuinness and O’Bradaigh (1998)) that fibres bend severely, slip out of the clamping frame or stretch to form an S during deformation.

Figure 2.9: Kawabata shear test apparatus [Yu et al (1994), Smith et al (1997)]
An experimental study of the shear behaviour of woven fabrics has been carried out by Kothari et al (1989). The study focussed on the effect of finishing, wetting and changes in four shear parameters of both grey and finished wool and wool blend plain and twill woven fabrics. The parameters investigated included initial shear modulus $G_0$, frictional shear stress $\tau_0$, shear rigidity $G$, and residual shear strain $\tan \theta$. A shear tester, which was based on the principle of Behre’s shear apparatus and attached to an Instron mechanical testing machine was used to obtain the shear hysteresis curve relating shear stress to shear strain for each sample. According to these authors, rectangular samples were used for shear testing throughout the study. This choice, contrary to the square samples used by most authors, was designed to produce a more uniform stress distribution and delay wrinkling during shear deformation.

On the effect of finishing, the studies observed that there was an average reduction of 85-90\% in the initial shear modulus $G_0$ and frictional shear stress $\tau_0$, 75-85\% in shear rigidity $G$, and 45-55\% in residual shear strain $\tan \theta$ because of commercial shear treatment. Accordingly, these substantial reductions in shear properties were mainly attributed to the stress relaxation of the inter-yarn pressure and frictional resistance within the fabric brought about by the finishing process. The effect of wetting on the shear properties was found to produce a volume increase effect on both the grey and the finished sample. This behaviour was related to the swelling of the yarn due to wetting, which resulted in volume change, consequently increasing the inter-fibre and inter-yarn frictions and the bending rigidity of the fabrics. Lastly, the investigation revealed that the relative humidity had the same effect as “wetting” on the shear
properties of the finished samples especially at higher relative humidity values. However, on the shear strain, the values were found to decrease initially for relative humidity values between 10-45%, and thereafter increase with it.

Smith et al (1997) performed a comprehensive analysis to evaluate the effect of shear deformation on the processing and mechanical properties of aligned reinforcements. Among the materials examined in this investigation were laminated samples of a non-crimp 0°/90° engineered fabric, a non-crimp ± 45° engineered fabric, a twill weave, a plain weave and a 4-harness satin weave. For the shear experiments, the tests were conducted using a four-bar linkage test rig similar to the Kawabata technique employed by other researchers. In the four-linkage test rig the fabric is clamped along two edges. One edge of the linkage is connected to a laboratory bench and a force applied to the opposite edge. The change in the angle as a result of the force was monitored via two perpendicular lines marked on the undeformed fabric. In this study, the authors did not provide shear force versus shear strain curves for the fabrics.

Most recently, Wang et al (1998) reported a similar investigation carried out by Smith et al (1997). The study focussed on the experimental investigation of the draping properties of reinforcement fabrics. Accordingly, four fabrics were examined, namely a plain-weave carbon fabric (PW-C), a 5-harness satin carbon fabric (5HS-C), a 4-harness satin weave glass fabric (4HS-G) and an 8-harness satin weave glass fabric (8HS-G). Initially, the effect of the specimen's aspect ratio (length/width) on the deformation patterns of the fabric was examined. It was found that a critical value of the aspect ratio must be exceeded if a meaningful and uniform deformation is to be achieved in fabrics.

Observations of the yarn slippage revealed that slippage only occurred towards the boundary of the specimen and not in the middle area where uniform deformation took place. Based on this finding the authors refuted the claim made by other authors that slippage of yarns during deformation is related to the shear angle. The yarn slippage was also found to be more pronounced for the carbon fabrics and almost absent or less pronounced in the case of the glass fabrics. In the case of the maximum shear angle
before wrinkling (also called the locking angle), the study found the values to be 31°, 26°, 28°, and 26°, for the PW-C, 5HS-C, 8HS-G, and 4HS-G respectively. When the values of the locking angles conducted using the Instron testing machine above were compared with the ones obtained from a simple shear test the values were found to be lower than the former. This anomaly was attributed to the occurrence of compressive forces emanating from lower tension force along the edges of the specimen, which play a part in producing buckling in the fabric during the simple shear test.

Andrews et al (1997) investigated the drapeability of 8-harness and 4-harness satin weaves over a rudder tip mould. In the first studies, the shear-locking angle of the woven fabrics in both wet and dry states was investigated using a simple hand shear experiment. In this experiment a piece of square shaped fabric inscribed with a square grid was laid on a table and stretched along the diagonal directions until wrinkles appeared. The maximum shear angle at which wrinkles appeared was considered as the locking angle of the fabric. The simple hand experiment was compared with two other similar experiments using simple weight pulleys and the dynamic Instron mechanical testing machine. The results indicated close fitting although, according to the investigation, the locking angle was difficult to define precisely in the Instron tests.

Prodromou et al (1997) presented the results of a study that investigated the relationship between the fabric construction (such as weave pattern and fabric parameters) and locking angle in single and multi-layer fabric set ups. The study was done with eight varieties of woven glass fabrics that included five plain weaves, two 4-harness satin weaves and an 8-harness satin weave. The shear experiments were conducted using the experimental trellis frame/Instron set up employed in many other studies. According to the observations made from these experiments the locking angle values for satin weaves were found be less than those of the plain weave fabrics. This was attributed to the size of the tow (for a given tow spacing and the inter-yarn spacing).

Vacuum forming drape experiments were conducted to show the effect of ply orientation on the wrinkling of the fabrics. In all, it was gathered that layers oriented
in the same direction produced fewer wrinkles than those with multiple orientation. Secondly, observations showed that most of the wrinkles seemed to stem from the radial lines of the mould towards the ±45° radial where the stiffness is at minimum. Using the pin-joint theory, a model was developed to predict the locking angle of the fabrics in shear. It was evidently found from the model that the locking angle depended on the ratio of the tow width/the tow spacing (spacing ratio). Some limitations were found to be imposed on the model by the coefficient of friction, compaction of the tows and variations in spacing and width of the tows, which varies from one fabric to another.

Most recently Van West et al (1997) proposed that the locking angle of woven fabrics can be predicted using the equation: \[ \theta = 2\cos^{-1}(0.7071(d_1/d_2)) \] where, \( d_1 \) and \( d_2 \) are the diagonal distance between the warp and weft yarns in the unstretched and stretched states, respectively. However, the accuracy of the above equation is not certain, as the approach was not validated experimentally.

Breuer et al (1996) studied the mechanism of wrinkle formation and ways of reducing them in a deep drawing technique of fabricating fabrics reinforced thermoplastics. This investigation was carried out for different types of reinforcing glass and carbon fabrics both dry and impregnated with a polyamide (PA-12) matrix. Shear tests were carried out in a picture-frame clamping device where the temperature was kept at room temperature for the dry fabric and at 215°C for the impregnated fabric. According to the tests, the critical shear angle before wrinkling was found to be related not only to the weave points for a twill weave fabric but also to be strongly dependent on the membrane stresses. According to the authors, membrane stresses are said to occur whenever the fabric is strained as a result of misalignment of the fibre yarns within the fabric. The magnitude of the membrane strain that leads to the formation of these stresses was given by \[ \Delta s = b \tan \delta_v \sin \gamma \] (where, \( \Delta s \) is the change in sample length due to misalignment, \( b \) is the original sample length, \( \delta_v \) is the fibre misalignment and \( \gamma \) is the shear angle).
According to observations it was found that the consequence of membrane stresses is to delay the formation of wrinkles. This effect was confirmed in the simple picture frame experiment, where the fabric was subjected to varying levels of membrane stresses by changing the magnitude of the clamping force on the diaphragm. This resulted in moving the critical shear angle ($\gamma_c$) for which wrinkles are formed from 35° to 45° and even 60°.

McBride and Chen (1997) developed a geometrical model describing the change of the unit cell geometry of a plain weave during shear and prior to buckling. The architecture of the unit cell was represented by a set of four sinusoidal curves. Trelissing and transverse yarn compaction were identified as the phenomena occurring in the shearing of fabric. An expression was derived relating the yarn width to the shear deformation.

McGuinness et al (1996) proposed a method for measuring the in-plane shear properties of unidirectional carbon reinforced PEEK laminates over a range of processing temperatures. After a series of comparisons with other approaches conducted earlier these researchers suggested that the best instrument for measuring the shear behaviour of composite components is the picture frame apparatus. Using this approach shear test experiments were carried out for shear rates ranging from 10 to 500 mm/min on laminated specimens of APC-2 which were consolidated at a pressure of 0.4 MPa and temperature of 380°C. According to the findings, the shear rate had a profound effect on the shear force required to deform the specimens. It was also found that when the specimens were heated to temperatures ranging from 350 to 380°C the shear curves showed a remarkable drop in the amount of shear force required for deformation.

A power law viscous model was developed to describe the behaviour exhibited by the unidirectional fibre PEEK specimens used in the shear tests. According to this model, the reinforced molten polymer material is idealised as a viscous material subject to the kinematic constraints of incompressibility and inextensibility in the fibre direction. Therefore the most important parameters required to characterise the behaviour of the
fabrics are the shear rate and the shear viscosity. The predicted shear force response of the material at temperatures 360-380°C was plotted on a curve where it was shown that the model works only for experiments carried out at the temperature of 360°C. For the experiments carried out at 370°C and beyond the material was thought to exhibit a shear rate dependency which is too complex to be described using a simple power law type of model.

McGuinness and O Bradaigh (1997) reported conducting work aiming at developing a model that would describe the rheological behaviour of commercially available fabric reinforced thermoplastic sheets. According to the model the material forming behaviour can be described as that of an incompressible viscous or viscoelastic material. The distinguishing feature of this set of models is a kinematic constraint of inextensibility along each set of fibre directions, together with an assumption of material incompressibility and a suitable anisotropic constitutive relationship. Kinematic quantities based on the Eulerian strain tensors and deformation tensors were adopted to describe the deformation of the material in a picture frame experiment.

Laminated samples consisting of 16 plies of G101 glass reinforced PA-12 nylon (with a 1/7 satin weave and 50% of the reinforcing fibres at each orientation) consolidated at a pressure of 0.4 MPa and a temperature of 240°C were tested in the shearing experiment. According to the observations reported by the authors, the specimen's response to shearing was rate specific and also depended on the orientation of the fibres. For positive shear, the load recorded for the specimen was observed to double that recorded for negative shear under the same conditions. However, when the test was conducted for a G201 glass fibre reinforced PA-12 nylon, (which has a 1/3 Crowfoot weave and a 90-10% bias in the number of fibres for each direction) it was found that the orientation had no effect on the shear of this material.

The force response of the specimen was fitted to the models developed earlier so that the validity of the approach as well as the experimental results could be ascertained theoretically. The force response derived from the isotropic viscous model was found to be unsuitable for both materials. The anisotropic viscous model was found to be
more successful for the G201 specimen than for the G101 specimen. The isotropic viscoelastic model interpreted the material behaviour for the G101 specimen quite well and poorly for the G201 specimen. No results were produced from the anisotropic viscoelastic model.

Asuadi et al (1994) considered woven fabrics to behave as viscoelastic materials during shear deformation processes and adopted the generalised linear viscoelasticity theory to study the shear relaxation behaviour of cotton fabrics. According to them, deformation, stress relaxation, and subsequent recovery of wool fabrics are associated with complex changes in the internal structure of wool fibres. Despite the complexity of these changes, the phenomena can be studied quantitatively using the rheological model of linear viscoelasticity. The viscoelasticity model postulated that any change in stress, $\sigma$, experienced by a material at time $t$, is related to a change in strain, $\varepsilon$, applied to the material at a time $r$ prior to time $t$ through the stress relaxation modulus of the material $E$, such that:

$$d\sigma(t) = E(t-r)d\varepsilon(r) \tag{2.6}$$

A modified form of this equation is:

$$M_o = \frac{\varepsilon(t)}{\varepsilon_o} G(0,t) \tag{2.7}$$

where, $M_o$ is the frictional stress (frictional force per unit width) at time $t$, $\varepsilon_o$ is the initial strain and $G$ is the shear stress relaxation function used to model the shear relaxation behaviour of the fabric.

The fitting for the above equation was obtained by subjecting the fabrics to shear deformation/shear relaxation experiments in a picture frame/Instron apparatus driven at 500mm/min. Relaxation stresses were measured for shear angles of 8-16° and a relaxation time of 1000 seconds. In the same manner, the residual shear strains for the fabrics were obtained. Curves of normalised shear stress versus the log of time
showed a linear behaviour indicating that the shear stress relaxation rate per decade of time for the shear angles is invariant with the increase of shear angle and shear strain. However values of the inter-fibre frictional stress versus shear strain for two wool fabrics showed the inter-fibre frictional stress to be increasing exponentially with the shear strain. Asuadi et al suggested that this observation was due mainly to variation of forces acting on the yarn cross-over point which are sufficiently large to overcome any frictional restraint that holds the yarns in place. As the shear strain increases these forces also increase until they are sufficient to cause slippage and rotation of the yarns at the yarn crossover points.

The report concluded by suggesting that the results obtained were in agreement with the facts that generally frictional restraint is a function of initial strain and absolute time. When a viscoelastic material with frictional elements is subjected to deformation and recovery cycles, the recovery proceeds until the recovery stresses are insufficient to overcome the frictional restraint.

The elastic behaviour model is often used to describe the shear deformation of fabrics before wrinkling. Many investigators used the shear tests to calculate the shear rigidity of the fabric (Yu et al (1994), Potturi et al (1995), Gelin et al (1996), Buckenham (1997)). The shear modulus of fabrics calculated from such tests has been used as input data for finite element, elastic solid mechanics simulations of draping (Boisse et al (1995), Mohammed et al (1998)). The elasticity theory has also been adopted in micromechanical analyses (Chapman and Whitcomb (1995), Vandeurzen et al (1996)).

Yu et al (1994) carried out an evaluation study in order to examine the formability of various textile fabric preforms (see Figure 2.11). In this study the Kawabata evaluation system for fabrics also called KES-F system (Kato Tech Co Ltd) was adopted for the experimental characterisation of the deformation behaviour of textile preforms in terms of in-plane tension, transverse compression, in-plane shear and out of plane bending. The test materials used by the authors in the experiment included a plain weave, a 5-harness satin weave, an 8-harness satin weave, and an angle-lock interweave (AL).
The results of these experiments showed that the fabrics were ranked in terms of the different types of rigidity as follows: (a) Tensile rigidity: angle-lock interweave<5HS<8HS/plain weave. (b) Compression rigidity: 8HS<5HS<angle-lock interweave/plain weave. (c) In-plane shear rigidity: angle-lock interweave<8HS<5HS/plain weave in the yarn direction and angle-lock interweave<5HS/plain weave<8HS in the bias direction. (d) Out-of-plane bending rigidity: angle-lock interweave<5HS<8HS/plain weave in the yarn direction and the same pattern was repeated in the bias direction.

2.4 EFFECT OF DRAPE ON THE MECHANICAL PROPERTIES OF FABRICS AND COMPOSITE PRODUCTS AND PERMEABILITY OF FABRICS

The effect of fibre orientation and fibre volume fraction on the mechanical properties of the preform or the composite product has been modelled by either devising detailed micromechanical models of the unit cell of a woven fabric (Chapman and Whitcomb (1995), Vandeurzen et al (1996)), taking into account the crimp of the fibre yarns, or by applying (Rudd et al (1993)) the Krenchel model (Krenchel (1964)) for Experimental Studies and Mathematical Modelling of the Draping and Shear Deformation of Woven Fabrics in Resin Transfer Moulding (RTM)
multidirectional continuous fibre reinforcements. In the Krenchel model, the effect of fibre volume fraction is taken into account by the rule of mixtures where the fibre modulus is multiplied by an efficiency factor, \( p \), given by

\[
p = \sum_i X_i \cos^4 \alpha_i
\]  

(2.8)

where, \( i \) refers to each fibre oriented layer, \( \alpha_i \) is the fibre orientation of layer \( i \) and \( X_i \) is the proportion of fibres oriented at \( \alpha_i \).

Rudd et al (1995) reported a study to investigate the processability and mechanical properties of bi-directional preforms for liquid composite moulding. The study set out to examine the significance of changes in fibre distribution within fabric-based preforms on the property variations, which arise from these. The investigation carried out consisted of both a theoretical as well as an experimental analysis. The theoretical part was developed using the kinematic draping analysis, which is based on the pin-jointed "fishnet" approach to predict the shape of the draped fabric. The experimental work involved a draping experiment of a non-crimpTech Textile E-LT 1134 0/90° fabric on a wheel hub.

Three types of parameters were calculated from the draped fabric. These were the fibre volume fractions, the permeability and the Young’s modulus values. In each case, the results of properties obtained showed that there was a close fitting between the experimental and the numerical data. According to the findings, the permeability values were seen to decrease from the apex of the hub along the diagonal directions to a minimum value at the rim, and the fibre volume fractions were observed to increase in this order for both the experiment and the predictions. Also, values of the mechanical properties of the real draped hub along the diagonal directions obtained from the modified rule of mixtures equation showed a gradual decrease from a nominal value at the apex to a minimum value at the rim.
Standley (1997) characterised the variables that have the greatest effect on the hot drape forming process and studied the effect of fabric shear on the mechanical properties of highly draped composite components. Two sets of analysis were carried out to investigate the mechanical properties of composite panels. In the first analysis laminates were manufactured from 5-harness satin weave tenax HTA carbon fibres reinforced with Hexcel fibredux 6376 toughened epoxy resin. The laminates were manufactured while in shear in a picture frame apparatus. Test coupons were cut from the laminates and loaded in tension to determine the elastic properties of the composites. When the values of elastic properties of the cured test coupons were compared with predicted values from the classical laminate theory and the modified rule of mixture approach, it was shown that the modified rule of mixtures produced the best results.

Hofstee et al (1998) presented an analysis report on the stiffness of thermoformed plain weave composites. In this study both in-plane shear and bending stiffnesses of the fabric sheared during the thermoforming process were modelled using two subsequent homogenising procedures. In the first procedure, the effective yarn stiffness for a repeating element of the sheared fabric lamina was determined. Secondly, undulation was accounted for, using either stiffness averaging or compliance averaging, so that the most important parameters used to define the undulation for the stiffness calculation were the maximum crimp angles, the relative volume of the arc-wise yarn segment with respect to the total yarn volume and the woven fabric thickness.

Values for these parameters were obtained from theory and were found to compare well with those obtained from experiment. The stiffness calculations using the stiffness averaging and the compliance averaging equations as a function of the shear angle for the laminate were compared with those obtained from a tensile test (ASTM D-3039) and three point bending tests (Dornier DON 128). The results indicated sufficient agreement between the geometric model and the experiment. In addition, it was found that a moderate maximum crimp angle of 7.3° leads to a significant stiffness difference of about 10GPa.
Smith et al (1997) calculated the permeability of sheared reinforcements. The values of the permeability obtained from the Carman-Kozeny equation for different shear angle values compared well with those obtained from experimental analysis. In addition, it was found that the permeability value for the sheared fabrics increases initially for ply angles up to ±40° for the engineered fabrics and ±360° for the woven fabrics. This according to the report could be attributed to the alignment of the fibres along the flow direction, but as the shear angle increases the permeability decreases due to the effect of fibre volume fraction. Furthermore, results of the mechanical properties of the sheared fabrics estimated from the Krenchel-modified rule of mixtures equation, the classical laminate theory and the experiment indicated that the former equation is valid for shear angles up to ±45° only whereas the latter equation produces reasonable results for all shear angles.

Hou (1996) reported carrying out an investigation to study the mechanical properties of a stamp formed laminated 8-harness satin woven composite made from glass (GF) and polyetherimide (PEI). The stamp forming experiment was performed by draping the inscribed-preheated laminates into a hemispherical mould. The whole arrangement consisted of an upper male mould and a female lower cavity and was heated to a very high temperature prior to the test. Strain measurements were carried out while the laminates were still being draped onto the mould. Values for the strain were obtained by the ratio of the arc length distance of the grid intersection points on the inscribed laminates and the original length of the inscribed grid intersection points along the fibre direction and 45° to the fibre direction. Tensile tests were carried out with a Zwick-148 universal testing machine by cutting specimens along the fibre direction and at 45° to the fibre direction under a heating condition similar to that encountered during the stamp forming process.

Experimental results revealed that the inelastic stretch of the material in the fibre direction under low load was found to be small, whereas in the direction of 45° to the fibre direction, the material could be stretched to large extensions at very low stress. When the effect of hold-down pressure on the part shape was investigated it was shown that parts formed with not enough hold-down pressure wrinkled at the flange.
area, parts with intermediate hold-down pressure draped well, and parts produced under excessive hold-down pressure presented buckling in the hemispherical cavity. On the effect of laminate dimensions on the part shape experiment, it was revealed that when the ratio of the area of laminate to the area of the hemisphere is large, significant wrinkling was produced around the periphery of the hemisphere cavity. Furthermore, it was observed that the more the depth of the mould cavity the more the severity of laminate wrinkling.
3. EXPERIMENTAL MATERIALS AND PROCEDURES
3.1 INTRODUCTION

Chapter 3 aims at discussing the types of woven materials and experimental procedures used in this study. The first part of the chapter deals essentially with the four types of fabrics used in both the draping and shear experiments, namely; the loose plain weave, the tight plain weave, the twill weave and the satin weave employed in both the draping and the shear experiments. Section 3.2 focuses mainly on the architectural patterns of the weaves and their specifications.

In section 3.3, the construction of the mould used in the draping experiment is fully highlighted. Section 3.4 then discusses the procedures adopted in the draping of fabrics over the hat mould surface. Subsequently, section 3.5 elaborates on the methods employed for the determination of the angles between the crossing fibres in the deformed fabric after being draped over the mould surface.

Section 3.6 discusses the shear experiments with special attention given to the construction of the picture frame, the experimental set-up and procedures for the picture-frame shear experiment. Also, this section dwells on the theoretical analysis of the picture frame deformation for the determination of the shear angle presented in chapter 7 and the hand shearing experiment. Lastly, section 3.7 explains the experimental procedure carried out in order to obtain the micrographs of the sheared fabrics employed for the shear locking-angle predictions in chapter 7.

3.2 MATERIALS

The materials used in the different stages of the experimental work of this study were fabrics, which can be typically used as reinforcements in polymer composites. Currently, most of the pure and hybrid woven fabrics used in textile composites are simple two-dimensional weaves, such as plain, twill and satin weaves, which are identified by the repeating pattern of the interlaced regions in the warp and weft directions.
In these studies, four E-glass woven fabrics were tested: a loose plain weave (LPW), a tight plain weave (TPW), a twill weave (TWILL) and a 5 harness satin weave (5HSW) respectively. The fabrics were supplied by Fothergill Engineered Fabrics Ltd. Table 3.1 presents the feature specifications of these woven fabrics [Fothergill Engineered Fabrics (1997)]. All four fabrics may be considered approximately isotropic in the two in-plane directions.

### Table 3.1: Specifications of the tested woven fabrics

<table>
<thead>
<tr>
<th>Weave type</th>
<th>LPW</th>
<th>TPW</th>
<th>TWILL</th>
<th>5HSW</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coding</td>
<td>basket</td>
<td>Y0212</td>
<td>Y0185</td>
<td>Y0227</td>
</tr>
<tr>
<td>Nom. thickness (mm)</td>
<td>0.58</td>
<td>0.48</td>
<td>0.28</td>
<td>0.23</td>
</tr>
<tr>
<td>Areal density (kg m^2)</td>
<td>0.529</td>
<td>0.546</td>
<td>0.331</td>
<td>0.297</td>
</tr>
<tr>
<td>Ends/10mm</td>
<td>1.1</td>
<td>6.7</td>
<td>11.8</td>
<td>22.4</td>
</tr>
<tr>
<td>Picks/10mm</td>
<td>1.2</td>
<td>6.3</td>
<td>11.8</td>
<td>21.3</td>
</tr>
</tbody>
</table>

LPW is also referred to as basket weave in this study.

### 3.2.1 PLAIN WEAVE

Plain weave is regarded as the most commonly used basic type of fabric for woven composites. In a plain weave structure, one warp yarn is continuously woven over and under weft yarns as shown in Figure 3.1. In this type of weave the repeating pattern in both directions is defined by the geometrical parameter \( n_g = 2 \), where \( n_g \) denotes that the weft yarn is interlaced with every warp yarn and vice versa. This makes the shifting of the yarns, or in other words the displacement of the yarns, very difficult.

The loose plain weave (LPW) fabric consisted of warp and weft yarns (or tows) containing unidirectional, continuous fibres without any twist. The tight plain weave (TPW) fabric, Y0212, had warp and weft yarns, each of which consisted of three bundles twisted together. The bundles contained continuous E-glass fibres of 9 \( \mu \)m diameter. Both warp and weft yarns had the linear density of 136 tex.
3.2.2 TWILL WEAVE

Twill weave has looser interlacing than the plain weave and the weave pattern is characterised by a diagonal line. In the twill weave, the number of warp yarns that pass over and under the weft yarns can be varied. In a 2 x 1 twill, the weft yarn floats over one and under two warp yarns, while in a 2 x 2 twill, shown in Figure 3.1, the weft yarn floats over two and under two warp yarns respectively. In a 2 x 2 twill the repeating pattern in both directions is defined by the geometric parameter \( n_g = 3 \). A twill weave fabric has generally better drapability than a plain weave. For the Y0185 twill fabric (TWILL) used in this study, each warp and weft yarn consisted of two bundles twisted together. Both types of yarns had the same linear density of 68 tex and consisted of continuous E-glass fibres of 9 \( \mu \)m diameter.

![Balanced 2x2 woven Twill](image)

![Balanced 5 Harness Satin](image)

**Figure 3.1:** Schematic of the woven fabrics used in the draping and shear experiment.
3.2.3 SATIN WEAVE

In satin weaves, the number of warp and weft yarns passing over each other before interlacing is always greater than 2 and the interlacing is always with one crossing yarn. In a five-harness satin weave, as shown in Figure 3.1 for example, one warp yarn floats over four successive weft yarns, and then under one weft yarn. The repeating pattern for a five-harness satin in both directions is defined by the geometric parameter $n_g = 5$. Satin weaves tend to have very good drapeability.

Of all the fabrics used in this study, the Y0227 five-harness satin fabric had the finest yarns and the smallest fabric thickness. Each warp and weft yarn consisted of three bundles twisted together. The yarn linear density was 22 tex and the bundles consisted of continuous E-glass fibres of 7 µm diameter.

3.3 MOULD CONSTRUCTION FOR THE DRAPING OF FABRICS

A mould was constructed for draping experiments onto a double curvature geometry, combining a curved and a flat part. The selected mould shape comprises a hemispherical dome surrounded by a flat rim. The mould was constructed in the Workshop of the Department of Materials Science and Engineering and it consists of two parts: The upper part which is regarded as the female mould was made up of transparent poly(methyl methacrylate) (PMMA - “perspex”).

A rectangular shaped solid block was used of 500 mm x 500 mm x 137 mm, in which a hemispherical cavity of 100 mm radius was machined in the central region (see Figures 3.2 and 3.3). The upper half of mould was chosen to be transparent so that the draped fabric could be viewed.

The lower mould, which is regarded as the male mould, was made up of aluminium. It consists of a square shaped flat surface of dimensions 500 mm x 500 mm with a solid aluminium dome at the centre.
The dome with a radius of 97 mm is the most important part of the lower mould, because it is here that the deformation of the fabric is effected. The whole aluminium surface sits on a movable wheel table for easy movement in the laboratory as shown in Figure 3.2.

Figure 3.2: Photograph of the hemispherical mould as used for the draping of woven fabrics.

Figure 3.3: Geometry and dimensions of the mould constructed for the draping experiments.
3.4 PROCEDURES FOR THE DRAPING EXPERIMENTS

A square piece of fabric of 360 mm x 360 mm was cut from each of the fabrics listed in Table 3.1. Then a square grid of 20x20 divisions was inscribed onto the cut sample using a thick ink pen. The grid was subdivided into four quadrants for easy identification. Four quadrants were also marked to divide the hemispherical mould surface and the surrounding flat rim on the male part. The whole idea was to fix the centre of the fabric grid on the apex of the mould dome and to fit the internal boundaries (lines of symmetry of the fabric) of the quadrants of the fabric exactly to the corresponding cross-lines of symmetry on the male mould (see Figure 3.4). The reason for this is to formalise the draping procedure of a layer of fabric onto the male mould and achieve satisfactory and accurate results.

The next stage of the drape experiment was the shaping process. The shaping process was carried out in two stages in this study. The first stage, which was the most important and difficult, was to place the fabric on the hemispherical dome and the flat rim of the male mould, so that each quadrant of the grid sits exactly over the corresponding quadrant of the mould. To facilitate this, a piece of double-sided sellotape was placed on the apex of the hemisphere and also on each of the corners on the internal boundaries of each quadrant of the mould, both on the hemispherical part and the flat surface (see Figure 3.4). Then, the central position on the grid was marked and the fabric was placed so that this point coincided with the apex. For each quadrant of the grid and the mould, the internal quadrant boundaries of the fabric were fitted manually over the corresponding lines of the mould. Using the sellotape pieces, the fitted fabric was securely constrained at this position prior to smoothing.

The second stage involved the smoothing of the fabric onto the mould surface. This was done manually at first and then using the upper half of the mould later. The fitted fabric was manually smoothed gently until each point on the deformed grid fitted on the mould surface with as little wrinkling as possible. This was achieved by lowering the upper mould slowly and gradually until the mould was closed and housed the draped fabric.
3.5 DETERMINATION OF THE ANGLES BETWEEN CROSSING FIBRES IN THE DEFORMED FABRIC

The local fibre directions in the deformed fabric are considered to be represented by the directions of the local gridlines of the grid inscribed on the fabric prior to deformation. Two methods were used to determine the local angle between the crossing gridlines at the gridpoints of the draped fabrics. The first method is simple and involves direct determination using a protractor. The second method involves numerical calculations using experimental data for the coordinates of the gridpoints.

3.5.1 DIRECT DETERMINATION OF THE ANGLE BETWEEN CROSSING GRIDLINES ON THE DEFORMED FABRIC

In order to determine the deformed angles between the fibres, a tracing paper and a protractor were used. After the mould was closed the fabric was left to stabilise and relax into the mould shape. Then the mould was opened carefully and some additional pieces of adhesive tape were added to ensure that the fabric did not change...
shape. For all fabrics that were used in the draping experiments in this study no noticeable relaxation was observed after the mould was opened. The next task was to trace the crossing gridlines at each gridpoint on the deformed fabric on a tracing paper using a pen. The tangential vectors to the gridlines at the gridpoint were then extended and the angle between these two vectors was measured using a protractor. This way, the whole grid was covered from the apex to the ends of the flat surface of the draped fabric. The local angles between the gridlines determined experimentally represent the angles between crossing fibres.

3.5.2 NUMERICAL CALCULATIONS OF THE ANGLE BETWEEN CROSSING GRIDLINES ON THE DEFORMED FABRIC, FROM COORDINATES OF GRIDPOINTS

In this method, the whole grid inscribed on the fabric is considered as consisting of quadrilateral linear elements, element nodes and element sides. The gridline parts between the grid points are represented by the element sides, which are straight-line parts connecting the element nodes. The angle $\alpha$ between two crossing gridlines at each grid point is then given as the vector angle between the corresponding element sides (see Figure 3.5) from the relation defining the inner or scalar product of two vectors:

$$\mathbf{AB} \cdot \mathbf{AC} = |\mathbf{AB}| |\mathbf{AC}| \cos \alpha$$

(3.1)

or

$$\cos \alpha = (\mathbf{AB} \cdot \mathbf{AC}) / (|\mathbf{AB}| |\mathbf{AC}|)$$

(3.2)

where, $\mathbf{AB}$ and $\mathbf{AC}$ are vectors and $|\mathbf{AB}|$ and $|\mathbf{AC}|$ are their corresponding magnitudes.
Figure 3.5: Angle between crossing gridlines as vectors angle between two adjacent element sides.

If the orthogonal rectangular coordinates of points A, B and C are \((X_A, Y_A, Z_A)\), \((X_B, Y_B, Z_B)\) and \((X_C, Y_C, Z_C)\), respectively, then the coordinates of vectors \(\mathbf{AB}\) and \(\mathbf{AC}\) are given by:

\[
\begin{align*}
X_{AB} &= X_B - X_A \\
Y_{AB} &= Y_B - Y_A \\
Z_{AB} &= Z_B - Z_A
\end{align*}
\] (3.3)

and

\[
\begin{align*}
X_{AC} &= X_C - X_A \\
Y_{AC} &= Y_C - Y_A \\
Z_{AC} &= Z_C - Z_A
\end{align*}
\] (3.4)

The scalar vector product in equation (3.2) is then given by:

\[
\mathbf{AB} \cdot \mathbf{AC} = X_{AB} X_{AC} + Y_{AB} Y_{AC} + Z_{AB} Z_{AC}
\] (3.5)

and the magnitudes are given by the relations:

\[
|\mathbf{AB}| = (X_{AB}^2 + Y_{AB}^2 + Z_{AB}^2)^{1/2}
\] (3.6)

\[
|\mathbf{AC}| = (X_{AC}^2 + Y_{AC}^2 + Z_{AC}^2)^{1/2}
\] (3.7)
Therefore, if one knows the coordinates of the points A, B and C, by using relations (3.3)-(3.7) and relation (3.2) one can determine the angle between the lines AB and AC. This procedure has been used to determine the angle between adjacent element sides of the grid inscribed on the fabric, provided that the coordinates of the element nodes or the gridpoints are determined experimentally throughout the deformed grid.

\[ \gamma = \pi/2 - \alpha \]  

(3.8)

The procedure for the determination of local angles between crossing fibres and local shear angles from the coordinates of gridpoints is applied to the deformed fabrics of both the finite element draping simulations and the experimental draping. The finite element draping simulations give in their result file the coordinates of all gridpoints in
both undeformed and deformed state. The data set can then be fed into the FORTRAN code to determine local fibre and shear angles.

In the experimental draping it is necessary to determine the coordinates of each gridpoint on the deformed grid initially inscribed on the fabric. This can be carried out following two alternative techniques. In the first technique, photographs are taken from the top (top view) and side (side view) as is shown in the results Section 5, in Figures 5.1-5.8. The top view gives the \((X, Y)\) coordinates of each gridpoint and the side view give the \(Z\) coordinate of each gridpoint. However, this technique cannot be applied well to all gridpoints, presenting particular difficulties for the points close to the edge between the hemisphere and the flat surface on the top view and the points near the edges of the hemisphere on the side view.

The other alternative technique is to measure manually arc lengths for the determination of the spherical coordinates of each gridpoint. The rectangular coordinates \((X, Y, Z)\) of each gridpoint can then be determined from the spherical coordinates \((r, \theta, \phi)\) from the following relations (Stroud (1991)), (see Figure 3.7)

\[
X = r \sin\theta \cos\phi \\
Y = r \sin\theta \sin\phi \\
Z = r \cos\theta
\]  

(3.9)

Where, \(r\) is the radius of the sphere, \(\theta\) is the angle which the node vector makes with the positive \(Z\) axis (corresponding to arc \(S_1\)) and \(\phi\) is the angle which the node vector makes with the positive \(X\) axis in Figure 3.7 (corresponding to arc \(S_2\)).

However, in order for these coordinates to be calculated the values of the arc lengths \(S_1\) and \(S_2\) must be ascertained. After the fabric shape was stabilised in the closed mould, the mould was opened carefully, as described in section 3.5.1. Two transparent measure-tapes were then used. One of the tapes was placed around the equator of one quadrant of the hemisphere (see Figure 3.7) and the other one was
positioned from the apex of the hemisphere along the longitudinal direction and finally perpendicular to the first measure-tape.

Figure 3.7: Measurement of the spherical polar coordinates of the gridpoints of the draped fabric.

The arc distance on the second tape (longitudinal) from the apex to a nodal position P represents $S_1$ (see Figure 3.7). The line then is extended along the longitudinal tape until it reaches the first tape around the equator at $P'$. The arc distance on the equator tape from a point marked as start point (X-axis) to the position $P'$ denotes $S_2$ (see Figure 3.7). To calculate $\theta$ and $\phi$ in degrees, the values of $S_1$ and $S_2$ measured were multiplied by the factor $(90/M)$ and $(90/N)$ respectively, where $M$ and $N$ are the complete boundary lengths of the quadrant along the longitudinal and equatorial directions respectively. The values of $\theta$ and $\phi$ were then substituted in relations (3.9) to calculate the rectangular coordinates $(X, Y, Z)$ of each gridpoint.
The sets \((X, Y, Z)\) were then fed into the FORTRAN code including relations (3.2)-(3.8) to calculate local angles between crossing gridlines (representing local angles between crossing fibres) and local shear angles.
3.6 SHEAR TESTING OF FABRICS

Tests were designed to study and characterise the shear behaviour of the woven fabrics. In order to examine the deformation behaviour of woven fabrics, a simple experiment was performed first, in which the fabric was stretched along the two fibre yarn directions (warp and weft directions) and along the diagonal direction. The first experiment produced almost no stretching of the fabrics along the fibre directions due to the inextensibility of the fibres. However, stretching the fabric along the diagonal direction produced shear deformational changes on the whole material. This, in combination with the results of the draping experiments (see Chapter 5), suggested that the accommodation of the fibres on a geometric mould surface during the drape process is brought about by the rotation of the fibre yarns at the warp-weft intersections, resulting in the shear deformation of the woven fabric.

The findings from the above simple tests generated the need to conduct the shear tests described below in order to have a clear understanding of the shear deformation of the fabrics. This along with the magnitude of the force involved for a certain degree of deformation will enable us to quantify the shear deformation and determine the shear-locking angle for each fabric. Picture-frame type shear tests were carried out for each fabric on a tensile testing machine.

3.6.1 THE PICTURE-FRAME APPARATUS

The picture frame, pictured in Figure 3.8, was an orthorhombic frame consisting of four freely movable arms. It was made from an aluminium light metal of dimensions 200x200 mm. Two pieces of aluminium strips hanged freely on the frame at two ends at diagonal positions. These metallic strips were purposely attached so that the whole frame could be securely mounted in the jaws of the tensile testing machine. Each arm of the picture frame had twenty pins homogeneously spaced. At the centre of each arm there was a drilled hole where bolts and nuts were inserted. These bolts and nuts were used to hold additional metallic pieces on each arm of the frame. The fabric boundaries were fixed on the pins and clamped with the clamping pieces.
3.6.2 EXPERIMENTAL SET-UP AND PROCEDURE

Each fabric material to be tested was lacerated from the bundles and made into a piece of 200 mm x 200 mm. These dimensions were chosen so that the whole picture frame was covered with the fabric piece leaving a few millimetres of fabric hanging beyond the arms. The advantage of this is to reduce tension on the fabric during the tests. On each of the fabric specimen an orthogonal grid of 10x10 was inscribed with a marker pen. The cut fabric was then placed on the frame through the pins, while the frame was in an orthogonal position. The process was carried out with care in order to avoid unnecessary damage of the fibres. Next, the metallic clamping pieces were placed on each arm of the frame through the pins and then bolted until the fabric was held securely onto the whole picture frame. The fabric was bolted onto the frame in such a way as to allow free rotation of the fibre bundles around the pins.

The whole arrangement was mounted onto a Lloyd Instruments T30K tensile testing machine attached with a 1KN-load cell. The picture frame was maintained in an orthogonal diagonal position in the Lloyd machine (see Figure 3.9). This was achieved using a protractor to measure the angles between the two arms of the frame while the
picture frame was still attached to the tensile testing machine. The Lloyd machine was set in a tensile mode at a constant crosshead speed.

The fabric's deformation was observed during the shear tests both visually and using a digital camera, which was set to take photographs at regular intervals of 3 seconds. This was meant to identify the moment when the first wrinkles would appear in order to determine the shear-locking angle. In the visual examination, the deformation of the fabrics was closely monitored and the total time from the start of the test to the time when the first wrinkle appeared was recorded. Various crosshead speeds such as 5, 10, 50 and 100 mm/min were tried in the tests.

Figure 3.9: Set-up of the picture-frame during the shear tests of the woven fabrics
3.6.3 ANALYSIS OF THE PICTURE-FRAME SHEAR TEST

Figure 3.10 illustrates the geometry of the picture frame experiment. The Lloyd Instruments machine in tensile mode measures the tensile force, $F_x$, and the extension, $2\Delta l$. The tensile force can be translated into shear components, $F_s$, by the relation

$$F_s = \frac{F_x}{2\cos(\alpha/2)}$$  \hspace{1cm} (3.10)

Assuming that the thickness, $h$, of the fabric does not change during shear before wrinkling occurs and the shear stress $\tau_s$ is homogeneous, then before the locking shear angle has been reached

$$\tau_s = \frac{F_s}{Lh}$$  \hspace{1cm} (3.11)

![Figure 3.10: Geometrical analysis of the picture-frame shear test](image)

From the geometry of Figure 3.10, it follows:
Experimental Materials and Procedures

\[ H_1 = h_1 + \Delta l \]  \hfill (3.12)

and

\[ \cos(\alpha/2) = H_1 / L \]  \hfill (3.13)

From equations (3.12) and (3.13) it follows:

\[ \cos(\alpha/2) = \frac{L \cos(\pi/4) + \Delta l}{L} \]  \hfill (3.14)

The shear angle, \( \gamma \) (see Figure 3.10), is expressed as:

\[ \gamma = \pi/2 - \alpha \]  \hfill (3.15)

The rate of change of shear angle, \( d\gamma/dt \), is given by combining equations (3.14) and (3.15) and differentiating with respect to time.

\[ \frac{d\gamma}{dt} = 2\left[ L \sin(\pi/4 - \alpha/2) \right]^{-1} \frac{d(\Delta l)}{dt} \]  \hfill (3.16)

where, \( d\gamma/dt \) is in rad/s and \( d(2\Delta l)/dt \) is the crosshead speed of the tensile testing machine.

### 3.6.4 HAND SHEARING EXPERIMENTS

The purpose of these experiments is to validate the results of shear locking angles derived from the shear tests conducted on the four woven fabrics using the picture-frame methodology. McGuinnes et al (1996) stated that the clamping process can induce a constraint on the rotation of the fabric inside the picture-frame forcing the fibres to bend severely at the point of clamping, slip out of the clamping device or
stretch to form an "S" shape during deformation. In order to confirm that the wrinkles formed during the Lloyd tensile tests were true wrinkles and not the "S" shape, a set of simple hand shearing experiments were carried out.

For these experiments, the same sample dimensions were maintained as those used in the picture-frame tests. The fabric was laid down on the laboratory bench and sheared manually and gradually while it was gently smoothed uniformly along the weft-warp directions with two hands. This was to ensure that the whole sample was covered and the shear force was distributed equally throughout the sample. The smoothing process was stopped when the first set of wrinkles appeared, or when some part of the sample was no more in contact with the surface of the laboratory bench.

3.7 MICROSTRUCTURAL ANALYSIS OF THE SHEARING OF FABRICS

A sample from each fabric was observed under the Olympus stereoscopic optical microscope, which had a camera with a zooming lens attached. The fabric sample could be either unsheared or sheared up to the maximum shear angle just at shear locking. The obtained micrographs are presented in chapter 7 and are used in the analysis of the shear locking effect.
4. MATHEMATICAL MODELLING

Experimental Studies and Mathematical Modelling of the Draping and Shear Deformation of Woven Fabrics in Resin Transfer Moulding (RTM)
4.1 INTRODUCTION

Chapter 4 focuses on the theoretical analysis of (a) the shear deformation of woven fabrics and (b) the draping of woven fabrics. Due to the fact that high shear strains are generally achieved in the draping of woven fabrics, a mathematical analysis on the basis of elasticity theory is developed and described in section 4.2 (also in Mohammed et al. (2000)). The mathematical model and relations derived from this analysis are applied in chapter 7 to analyse the experimental data of the shear testing of woven fabrics.

Section 4.3 describes the theory behind the numerical simulations of the draping of woven fabrics, which are presented in chapter 8. The computer simulations of draping in this work are based on the continuum solid mechanics analysis in which the fabric is considered as a solid sheet with mechanical properties. The finite element methodology is used to solve the resulting equations.

4.2 ELASTICITY ANALYSIS OF THE SHEAR DEFORMATION OF FABRICS

A mathematical analysis on the basis of elasticity theory is adopted to take into account the high shear strains occurring during the draping of fabrics. In this analysis, \( \mathbf{x} \) denotes position in a fixed, Eulerian, orthogonal frame of reference and \( \mathbf{X} \) denotes position in an embedded material, frame following the material deformation. The deformation gradient tensor is given by:

\[
F = \frac{\partial \mathbf{X}}{\partial \mathbf{x}} = \begin{bmatrix}
\frac{\partial X_1}{\partial x_1} & \frac{\partial X_1}{\partial x_2} \\
\frac{\partial X_2}{\partial x_1} & \frac{\partial X_2}{\partial x_2}
\end{bmatrix}
\] (4.1)

Figure 4.1 shows the picture-frame type of shear test to which the woven fabrics are subjected. The relationship between the fixed Eulerian, orthogonal, frame \( \mathbf{x} \) and the...
embedded material co-ordinate system $X$ is found by inspection of Figure 4.1. The relations are:

$$\begin{align*}
x_1 &= (\cos(\gamma / 2))X_1 + (\sin(\gamma / 2))X_2 \\
x_2 &= (\sin(\gamma / 2))X_1 + (\cos(\gamma / 2))X_2
\end{align*}$$

(4.2)

**Figure 4.1: Eulerian and material frames for the picture-frame type of shear test**

It will be convenient to have $X$ coordinates expressed in terms of the spatial Cartesian coordinates system $x$:

$$\begin{align*}
X_1 &= \left(\frac{\cos(\gamma / 2)}{\cos \gamma}\right)x_1 - \left(\frac{\sin(\gamma / 2)}{\cos \gamma}\right)x_2 \\
X_2 &= -\left(\frac{\sin(\gamma / 2)}{\cos \gamma}\right)x_1 + \left(\frac{\cos(\gamma / 2)}{\cos \gamma}\right)x_2
\end{align*}$$

(4.3)

The strain tensor $\varepsilon$ is then given by:

$$\varepsilon = \frac{1}{2} (I - F^T F)$$

(4.4)
where, \( \mathbf{I} \) is the unit tensor (whose diagonal elements are unity and non-diagonal elements are zero) and \( \mathbf{F}^T \) is the transpose of matrix \( \mathbf{F} \).

\[
\mathbf{I} = \begin{bmatrix}
1 & 0 \\
0 & 1
\end{bmatrix}
\]  
(4.5)

From equations (4.1) and (4.3)

\[
\frac{\partial X_1}{\partial x_1} = \frac{\cos(y/2)}{\cos y}
\]

\[
\frac{\partial X_2}{\partial x_2} = \frac{\cos(y/2)}{\cos y}
\]  
(4.6)

\[
\frac{\partial X_1}{\partial x_2} = \frac{\partial X_2}{\partial x_1} = \frac{-\sin(y/2)}{\cos y}
\]

Therefore:

\[
\begin{bmatrix}
\frac{\cos(y/2)}{\cos y} & \frac{-\sin(y/2)}{\cos y} \\
\frac{-\sin(y/2)}{\cos y} & \frac{\cos(y/2)}{\cos y}
\end{bmatrix}
= \mathbf{F}^T
\]  
(4.7)

and

\[
\begin{bmatrix}
\frac{\cos(y/2)}{\cos y} & \frac{-\sin(y/2)}{\cos y} \\
\frac{-\sin(y/2)}{\cos y} & \frac{\cos(y/2)}{\cos y}
\end{bmatrix}
\begin{bmatrix}
\frac{\cos(y/2)}{\cos y} & \frac{-\sin(y/2)}{\cos y} \\
\frac{-\sin(y/2)}{\cos y} & \frac{\cos(y/2)}{\cos y}
\end{bmatrix}
\]  

Experimental Studies and Mathematical Modelling of the Draping and Shear Deformation of Woven Fabrics in Resin Transfer Moulding (RTM)
Relation (4.4) can be written in two-dimensional form as follows:

\[ \varepsilon = \begin{bmatrix} \varepsilon_{11} & \varepsilon_{12} \\ \varepsilon_{12} & \varepsilon_{22} \end{bmatrix} = \frac{1}{2} \begin{bmatrix} -1 - \sec^2 \gamma & \sin \gamma \tan \gamma \\ \sin \gamma \tan \gamma & 1 - \sec^2 \gamma \end{bmatrix} \]

The Cauchy stress tensor \( \sigma \) associated with the fixed frame of reference \( \mathbf{x} \) is then given by the equation:

\[ \sigma = -P \mathbf{I} + 2\mathbf{E} \varepsilon + \sum_{i=1}^{2} T_i \mathbf{F}_i \otimes \mathbf{F}_i \]

(4.10)

where, \( \otimes \) is the kronecker product, \( P \) is an arbitrary hydrostatic pressure to satisfy the incompressibility constraint, \( \mathbf{E} \) is the modulus matrix, \( T_i \) is the fibre stress associated with the inextensibility constraint and the last term in equation (4.10) denotes the yarn extensibility limit in the fibre directions \( \mathbf{F}_i \). Qiu and Pence [1997] wrote the last term with the aid of a standard reinforcing model as:
\[ \sigma = -\Pi + 2\varepsilon \varepsilon + \sum_{i=1}^{2} \tau_i f'(C_{11}) F_i \otimes F_i \] (4.11)

where, \( \tau_i \) is a material parameter for each fibre direction (\( \tau_i = 0 \) corresponds to neo-Hookean material response and \( \tau_i \rightarrow \infty \) corresponds to totally inextensible fibre yarns) and \( (C_{11})^{1/2} \) is the stretch in the fibre direction \( F_i \) and an element of the matrix \( C \):

\[ C = F^T F \] (4.12)

\( f'(C_{11}) \) is the first derivative of \( f(C_{11}) \) where from the standard reinforcing model of Qiu and Pence (1997)

\[ f(C_{11}) = \frac{1}{2} (C_{11} - 1)^2 \] (4.13)

and

\[ f'(C_{11}) = f'(C_{22}) = (C_{11} - 1) \frac{1}{\cos^2 \gamma} - 1 \] (4.14)

Figure 4.2: Fibre directions along the material axes during the picture frame test

From Figure 4.2, the fibre vectors \( F_1 \) and \( F_2 \) along the material axes are given by:
\( \mathbf{F}_1 = (\cos \gamma / 2, \sin \gamma / 2, 0) \)
\( \mathbf{F}_2 = (\sin \gamma / 2, \cos \gamma / 2, 0) \)  

(4.15)

\[ \mathbf{F}_1 \otimes \mathbf{F}_1 = \begin{bmatrix} \cos^2(\gamma / 2) & \sin(\gamma / 2) \cos(\gamma / 2) \\ \sin(\gamma / 2) \cos(\gamma / 2) & \sin^2(\gamma / 2) \end{bmatrix} \]  

(4.16)

\[ \mathbf{F}_2 \otimes \mathbf{F}_2 = \begin{bmatrix} \sin^2(\gamma / 2) & \sin(\gamma / 2) \\ \sin(\gamma / 2) & \cos^2(\gamma / 2) \end{bmatrix} \]  

(4.17)

Therefore:

\[ \mathbf{F}_1 \otimes \mathbf{F}_1 + \mathbf{F}_2 \otimes \mathbf{F}_2 = \begin{bmatrix} \sin^2(\gamma / 2) + \cos^2(\gamma / 2) & \frac{2 \sin \gamma}{2} \\ \frac{2 \sin \gamma}{2} & \sin^2(\gamma / 2) + \cos^2(\gamma / 2) \end{bmatrix} \]  

(4.18)
Figure 4.3: Shear stress resolution along the Cartesian and material axes during the picture-frame shear test

In Figure 4.3, the measured shear stresses $\tau_s$ along the material axes are analysed into the orthogonal components of the Cauchy stress tensor as:

$$\begin{align*}
\sigma_{12} &= \sigma_{21} = \tau_s \cos(\gamma/2) \\
\sigma_{11} &= \sigma_{22} = \tau_s \sin(\gamma/2)
\end{align*}$$

\hspace{1cm} (4.19)

Where each fabric is considered symmetric in the two in-plane directions.

From equations (4.14) and (4.18) and assuming incompressible material behaviour

$$\sigma = \begin{bmatrix}
\sigma_{11} & \sigma_{12} \\
\sigma_{21} & \sigma_{22}
\end{bmatrix} = 2 \begin{bmatrix}
E_{11} & E_{12} \\
E_{21} & E_{22}
\end{bmatrix} \begin{bmatrix}
\varepsilon_{11} & \varepsilon_{12} \\
\varepsilon_{21} & \varepsilon_{22}
\end{bmatrix}$$

$$+ t \left(C_{11} - 1\right) \begin{bmatrix}
1 & \sin\gamma \\
\sin\gamma & 1
\end{bmatrix}$$

\hspace{1cm} (4.20)

From equations (4.9), (4.19) and (4.20) it follows that:

\begin{align*}
\sigma_{11} &= \sigma_{22} = \tau_s \sin(\gamma/2) = E_{11} (1 - \sec^2 \gamma) + t \left(C_{11} - 1\right) \\
\sigma_{12} &= \sigma_{21} = \tau_s \cos(\gamma/2) = E_{12} \tan \gamma \sec \gamma + t \left(C_{11} - 1\right) \sin \gamma
\end{align*}

\hspace{1cm} (4.21)
By applying relation (4.14), equation (4.21) becomes:

\[
\sigma_{11} = \sigma_{22} = E_{11} (1 - \sec^2 \gamma) + t_1 (\sec^2 \gamma - 1)
\]

\[
\sigma_{12} = \sigma_{21} = E_{12} \tan \gamma \sec \gamma + t_1 (\sec^2 \gamma - 1) \sin \gamma
\]  

(4.22)

4.3 FINITE ELEMENT CONTINUUM SOLID MECHANICS OF DRAPING

In the finite element approach of draping simulations the fabric was represented as a thin solid body with a numerical finite element mesh. The mesh should be fine enough to model accurately local features of draping such as wrinkles, folds and cuts, but may not reach the fabrics' microscale. Macro-elements can be related to the basic micro-unit of the actual fibre network regarding fibre volume fraction, porosity and fibre orientation. This could be easily demonstrated in the case of an orthogonal woven fabric such as the plain woven, satin and twill fabrics included in this study but may need more complex geometric and micro-mechanical modelling in three dimensional non-orthogonal and knitted fabrics.

Section 4.3 introduces the description of the formulation of the static analysis equilibrium equations. The equations describe the response of a structure to external loads and form the basis of the draping analysis carried out in this study based on the finite element continuum mechanics approach. In the draping analysis carried out in section 8, the load was increased in a step-wise manner and a static finite element analysis was performed at each step.

4.3.1 CONSTITUTIVE MODEL

A linear elastic, orthotropic constitutive model was employed to describe the fabric deformation at each step of static analysis, given by the following equations:
where \( \sigma_i \) are stresses, \( \varepsilon_i \) are strains and \( D \) is a matrix of orthotropic elastic properties, which is given by:

\[
D = \begin{bmatrix}
\frac{1}{E_x} & -\frac{v_{yx}}{E_y} & -\frac{v_{zx}}{E_z} & 0 & 0 & 0 \\
-\frac{v_{xy}}{E_x} & \frac{1}{E_y} & -\frac{v_{zy}}{E_z} & 0 & 0 & 0 \\
-\frac{v_{xz}}{E_x} & -\frac{v_{yz}}{E_y} & \frac{1}{E_z} & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{1}{G_{xy}} & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{1}{G_{yz}} & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{1}{G_{xz}}
\end{bmatrix}
\]

(4.24)

where, \( E_x \) is the Young’s modulus in the warp direction, \( E_y \) is the Young’s modulus in the weft direction, \( G_{xy}, G_{yz}, G_{xz} \) are the shear moduli and \( v \) is the Poison’s ratio.

The Poison’s ratios \( v_{xy}, v_{yz} \) and \( v_{zx} \) are given by:

\[
v_{yx} = \frac{v_{xy} E_y}{E_x} \quad v_{zx} = \frac{v_{xz} E_y}{E_x} \quad v_{zy} = \frac{v_{yz} E_x}{E_y}
\]

(4.25)

### 4.3.2 STATIC EQUILIBRIUM EQUATIONS

The fabric draping is described using the finite element methodology. This methodology is based on the equilibrium of a general three-dimensional body subject to the following forces.
\( F_{su} \) = surface forces
\( F_b \) = body forces
\( F_c \) = concentrated loads

The body will be displaced from its original configuration by an amount \( \mathbf{u} \), which give rise to strains \( \mathbf{\varepsilon} \) and the corresponding stresses \( \mathbf{\sigma} \).

The force equilibrium relationship may be derived in the form of the principle of virtual displacement by using strain tensors. The principle of virtual displacement states that the force equilibrium of the body requires that, for any compatible virtual displacement imposed on the body, the total internal work is equal to the total external work:

\[
\int_V \delta \varepsilon^T \sigma \, dv = \int_V \delta \mathbf{u}^T \mathbf{F}_b \, dv + \int_S \delta \mathbf{u}^T \mathbf{F}_{su} \, ds + \sum \delta \mathbf{u}^T \mathbf{F}_c
\]  

(4.26)

where, \( \delta \varepsilon \) is the strain vector corresponding to the virtual displacement vector \( \delta \mathbf{u} \).

In finite element analysis, the body is approximated as an assemblage of discrete elements interconnected at nodal points. The displacements within any element are then interpolated from the displacements at nodal points corresponding to that element.

For example for element \( e \)

\[
\mathbf{u}^{(e)} = \mathbf{N}^{(e)} \mathbf{a}^{(e)}
\]  

(4.27)

where, \( \mathbf{N}^{(e)} \) is the displacement interpolation or shape function matrix (see equation (4.28) for a four nodal quadrilateral element in figure 4.4), and \( \mathbf{a}^{(e)} \) is the vector of nodal displacements.
For linear elasticity, the stresses $\sigma$ within the finite element are related to the strains using a constitutive relationship of the form:

$$\sigma^{(e)} = D^{(e)} \varepsilon^{(e)} - \varepsilon_o^{(e)} + \sigma_o^{(e)}$$

(4.29)

where, $D^{(e)}$ is a matrix of elastic constants, and $\varepsilon_o^{(e)}$ and $\sigma_o^{(e)}$ are the initial strains and stresses respectively due to, for example, thermal effects.

Therefore using equations (4.26), (4.27), and (4.29), the virtual work equation (4.26) may be discretized to give:

$$\sum_{e=1}^{n} \delta \mathbf{a}^T \int_V \mathbf{B}^{(e)^T} D^{(e)} \mathbf{B}^{(e)} \; dV \; \mathbf{a} = \delta \mathbf{a}^T \left[ \sum_{e=1}^{n} \int_V N^{(e)^T} F^{(e)} \; dV \right]$$

$$+ \sum_{e=1}^{n} \int_s N_s^{(e)^T} F_s \; ds - \sum_{e=1}^{n} \int_V B^{(e)^T} (\sigma_o^{(e)} - D^{(e)} \varepsilon_o^{(e)}) \; dV + F_c$$

(4.30)

where, $N_s^{(e)}$ are the interpolation functions for the surface of the elements and $n$ is the number of elements in the assemblage.

By using the virtual displacement theorem, the equilibrium equations of the element assemblage becomes:

Experimental Studies and Mathematical Modelling of the Draping and Shear Deformation of Woven Fabrics in Resin Transfer Moulding (RTM)
where, $\mathbf{K}$ is the structure stiffness matrix, defined as:

$$
\mathbf{K} = \sum_{e=1}^{n} \int_{V} \mathbf{B}^{(e)T} \mathbf{D}^{(e)} \mathbf{B}^{(e)} \, dv \tag{4.32}
$$

and $\mathbf{R}$ is the structure force vector, defined as:

$$
\mathbf{R} = \mathbf{R}_b + \mathbf{R}_{su} + \mathbf{R}_o + \mathbf{R}_c \tag{4.33}
$$

and $\mathbf{R}_b$ is the force vector due to the element body loads

$$
\mathbf{R}_b = \sum_{e=1}^{n} \int_{V} \mathbf{N}^{(e)T} \mathbf{F}_b^{(e)} \, dv \tag{4.34}
$$

$\mathbf{R}_{su}$ is the force vector due to the element surface tractions

$$
\mathbf{R}_{su} = \sum_{e=1}^{n} \int_{S} \mathbf{N}^{(e)T} \mathbf{F}_{su}^{(e)} \, ds \tag{4.35}
$$

$\mathbf{R}_o$ is the force vector due to the initial stresses and strains

$$
\mathbf{R}_o = \sum_{e=1}^{n} \int_{V} \mathbf{B}^{(e)T} (\mathbf{\sigma}^{(e)} - \mathbf{D}^{(e)} \mathbf{\varepsilon}^{(e)}) \, dv \tag{4.36}
$$

$\mathbf{R}_c$ is the force vector due to concentrated loads

$$
\mathbf{R}_c = \mathbf{F}_c \tag{4.37}
$$
5. VIEWING OF THE DRAPE D FABRICS
5.1 DRAPING OF FABRICS

The ability of fabrics to form over three-dimensional shapes without having to use undue force defines the drapeability of fabrics. One method of comparing different fabrics is based on draping experiments. A hemispherical hat mould surface combined with a flat rim, described in Chapter 3, was used in this study, as it imparts biaxial deformation to the fabric. Two possible modes by which the fabrics deform to the shapes seen in Figures 5.1 to 5.4 must be considered. The first mechanism, as put forward by Potter (1979), is that the cloth acts as a pin-jointed net at each yarn crossover point and thus deforms only by angular rotation of the warp and weft fibre yarns. The other possible mode is that coherent slippage occurs at the crossover points and the yarns are drawn close to or further apart from each other.

The shapes of the draped fabrics and the deformation of the grid inscribed on each fabric prior to deformation are seen in the Figures 5.1(a-c), 5.2(a-c), 5.3(a-c) and 5.4(a-c) for the loose plain (basket) weave, the tight plain weave, the twill weave and the satin weave respectively. Each of the four draped fabrics deformed by angular rotation of the warp and weft yarns at their crossovers to produce a star shaped geometric feature. This deformation mode was observed on the grid inscribed on the fabrics prior to draping. If one considers the example of Figure 5.1 where the fabric is viewed from the top, the fibre yarns in the fabric part over the dome appear to retain much of the initial square grid with bending of the fibres in order to follow the curvilinear mould shape. At the centre of the dome, the fibre yarn angular rotation is minimal as indicated from the small change in the angles of the grid. As one moves diagonally towards the equator there is an indication of increasing fibre yarn angular rotation resulting in large variations in the grid angles towards the lower part of the hemispherical dome. The overall fibre yarn rotation in the dome part also affects the fabric part on the flat surface and creates the star shape.

Draping does not destroy the yarn formation and the weave pattern. In the draping processes shown in Figures 5.1-5.4 the initial inscribed grid remains mostly intact after draping. This is experimentally observed by following the deformations of straight lines
drawn on the fabrics prior to draping. These lines covering successive warp yarns or weft yarns become curved during draping but remain mostly continuous. However, some widening of the thickness of grid lines was observed at micro-level in the case of loose plain (basket) weave (see Figures 5.1(a-c) containing grid line discontinuities at the micro-level of yarn width, while no widening of any individual yarns was observed. After closer observation during the experiment, this indicated fibre sliding for about one yarn width, at maximum, in the case of loose plain weave. This is not so evident in the other weaves because of (a) their small yarn width and (b) the tightness of weave.
Figure 5.1: (a) Top view, (b) side view, and (c) angled view of the deformed basket weave, draped over a hemispherical mould surrounded by a flat rim.
Figure 5.2: (a) Top view, (b) side view, and (c) angled view of the deformed tight weave draped over a hemispherical mould surrounded by a flat rim
Figure 5.3: (a) Top view, (b) side view, and (c) angled view of the deformed twill weave draped over a hemispherical mould surrounded by a flat rim
Figure 5.4: (a) Top view, (b) side view, and (c) angled view of the deformed satin weave draped over a hemispherical mould surrounded by a flat rim.
5.2 WRINKLING OF FABRICS

The phenomenon of wrinkling in the draping of fabrics is caused by compressive forces induced during forming, resulting in gross buckling deformation through the entire thickness of the fabric. These compressive forces arise from significant material compression when the warp and weft fibres are sheared beyond the “locking shear angle” necessary to form the doubly curved shapes. The resulting compressive strain, which cannot be accommodated by the fabric’s shear process, results in the buckling or wrinkling of the fabric.

In Figure 5.1(a-c) the draped basket weave fabric (loose plain weave) is seen to have formed without wrinkling not only over the upper part of the hemisphere, but also the lower part of the hemisphere as well as the flat part. This is because the locking shear angle of the fabric was never reached within the mould design of the drape experiments. On the contrary when the photographs of the draped tight plain weave, twill weave and satin were examined, it can be noticed that, whereas on the hemispherical surface no wrinkles were formed, wrinkles are visibly present on the flat surface. In the dome region two simultaneous opposing processes were responsible for the absence of wrinkles. Whilst the fabric undergoes shearing to conform to the mould geometry, the upper mould, as it closes, confers a kind of smoothing forces which oppose the compressive forces, so that the effect of the latter is reduced. This is a similar effect as that observed in the diaphragm forming [Delaloye and Niedermeier (1995), O’Bradaigh et al (1993)] of fibre reinforced thermoplastics where the rubber diaphragm imparts membrane tensile stresses. As a result, the fabric experienced a smaller degree of shearing in the fibre yarn directions and the locking angle is never reached. At the transition zone, i.e. near the equator, there was reduced smoothing and the fabric was compressed deeply onto the base of the hemisphere shearing the fibres on the flat surface, in the process. Thus, because of the tightness and density of some weaves, the locking angle was reached and, when the fibres cannot accommodate the orthogonal compressive forces, they buckled and produced wrinkles.
A survey was carried out to determine the angles of the deformed elements that fell within the wrinkled areas of the draped fabrics. The whole area where wrinkles appeared for each fabric was considered for this investigation. All the element angles inside the area were traced on a transparent paper and measured with a protractor. The angle of an undeformed element without any wrinkle is 90°. The largest angle of deformed wrinkled elements, angle \( \alpha_{\text{max}} \), corresponds to the least deformed wrinkled element where there are small wrinkles; the lowest angle of deformed wrinkled elements, angle \( \alpha_{\text{min}} \), corresponds to the most deformed wrinkled element where wrinkles are more pronounced. The measurements are tabulated in Table 5.1 for the fabrics, which formed wrinkles, i.e. the tight plain weave, the twill weave and the five-harness satin weave.

<table>
<thead>
<tr>
<th>Weave</th>
<th>( \alpha_{\text{min}} ) (degrees)</th>
<th>( \alpha_{\text{max}} ) (degrees)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tight plain (TPW)</td>
<td>43-44</td>
<td>49-56</td>
</tr>
<tr>
<td>Satin (5HSW)</td>
<td>32</td>
<td>34</td>
</tr>
<tr>
<td>Twill (TWILL)</td>
<td>37</td>
<td>48</td>
</tr>
</tbody>
</table>

From the measured angles, the higher is the element angle containing the wrinkles the more susceptible the fabric may be to produce the shear locking effect. However, the values obtained do not correspond to the exact locking shear angles, which are very difficult to identify with this kind of experiment, especially as one cannot quantify the size of wrinkles. Nevertheless, these values can be used to indicate whether a certain fabric is likely to produce wrinkles when formed over a certain shear angle and to compare fabrics in terms of wrinkling during draping.
The tight plain weave has the largest angles of wrinkled elements in comparison to the other weaves. This indicates that the fibre yarns will lock earlier in shear than for the satin and twill weaves. For the same shear forces generated on the fabrics during the drape experiment, the tight plain weave will have a higher degree of wrinkling in these areas. The fabric with the next size of angles of wrinkled elements is the twill weave, which confirms that this fabric will have less wrinkling than the tight plain weave but more wrinkling than the satin weave. Also, this indicates that the fibres in the twill weave will lock at lower shear angles than in the satin weave. The satin weave is perhaps the best fabric for use in drape applications since it produces the smallest angles of wrinkled elements, which means less wrinkling when compared to the other weaves. The loose plain weave (basket weave) is of course superior to the satin weave in terms of wrinkles, since it formed no wrinkles when draped over the hemispherical dome surrounded by the flat rim in this study. However, it might have other disadvantages, for example in the area of mechanical properties, due to the large gaps between fibre yarns.

5.2.1 WRINKLE MEASURES

In order to quantify the extent of wrinkling formed on each woven fabric draped on the hemispherical hat, a wrinkle measure (WM) was devised. Since the circumferential boundary of the hemispherical dome with the flat rim is the most crucial area of the mould in this mould design in terms of fabric wrinkling, this area was selected to assess the extent of wrinkling in the complex shaped mould in this study. wrinkling generates resin pockets in the composite products and defects, which affect the properties of the product and the permeability of the draped fabric. Any wrinkle measure needs to take into account both the number of wrinkles and size of each wrinkle. The size of each wrinkle was evaluated by measuring the height and the width of each wrinkle around the circumferential boundary of the dome. The height of each wrinkle was measured by inserting a thin pin at the appropriate location and marking the point of meeting the fabric as the pin was inserted as deeply as possible in the wrinkle.
By considering the shape of each wrinkle to be a triangle (as seen in Figure 5.5) the area of each wrinkle \( A_{\text{wrinkle}} \) was estimated as:

\[
A_{\text{wrinkle}} = \frac{1}{2}(h_{w,i} \times w_{w,i})
\]  

(5.1)

Figure 5.5: Schematic of wrinkles surrounding the hemispherical dome during the draping of the woven fabrics

From Figure 5.5, the wrinkle measure (WM) was defined as:

\[
WM = \frac{\sum_{\text{all wrinkles}} \frac{1}{2}(h_{w,i} \times w_{w,i})}{2\pi Rh} = \frac{\sum A_{\text{wrinkle},i}}{2\pi Rh}
\]  

(5.2)

where, \( i \) indicates a particular wrinkle, \( h_{w,i} \) is the height of the wrinkle, \( w_{w,i} \) is the width of the wrinkle, \( h \) is the thickness of the fabric and \( R \) is the radius of the dome. The sum of all the wrinkle areas \( \sum A_{\text{wrinkle}} \), has been normalised by the cross-sectional area of a fabric draped onto the mould without wrinkling.
Table 5.2: Calculated values of total wrinkle area and wrinkle measure (WM) for tight plain weave, twill weave and satin weave

<table>
<thead>
<tr>
<th>FABRIC</th>
<th>$\Sigma A_{\text{wrinkle}, i} \ (\text{mm}^2)$</th>
<th>WM</th>
</tr>
</thead>
<tbody>
<tr>
<td>TPW</td>
<td>379</td>
<td>1.28</td>
</tr>
<tr>
<td>5HSW</td>
<td>204</td>
<td>1.44</td>
</tr>
<tr>
<td>TWILL</td>
<td>287</td>
<td>1.67</td>
</tr>
</tbody>
</table>

Table 5.2 presents the results of the total wrinkle area and the wrinkle measure for the tight plain weave, the twill weave and the satin weave after having been draped onto the hemispherical hat mould. The tight weave has the largest total wrinkle area and the 5-harness satin has the smallest wrinkle area. However if the thickness of the fabric is taken into account in the wrinkle measure (WM), it can be seen that even a smaller amount of wrinkling may have a more serious effect in a thin fabric. The conclusion is that the total wrinkle area gives an immediate and direct measure of the wrinkling that is consistent with the perception of which fabric wrinkles from viewing the draped fabrics. More work might be needed to devise an appropriate normalised wrinkle measure that will take into account the effect of wrinkles on a certain property of reinforcement or composite product.
6. DRAPING OF FABRICS:
EXPERIMENTAL RESULTS AND ANALYSIS
6.1 INTRODUCTION

Draping experiments were carried out as described in Chapter 3 for four different types of fabrics: a loose plain weave (basket weave), a tight plain weave, a satin weave and a twill weave. The aims of the draping experiments was to (a) compare the drapeability of the four types of fabrics in terms of wrinkling (see Chapter 5); (b) compare the deformed boundary profiles of the four draped fabrics; (c) measure local angles and shear angles of the deformed grid elements of the draped fabrics, assuming that these angles represent the corresponding angles between local crossing fibre yarns; (d) determine the local fibre volume fraction distribution of the draped fabrics on the basis of the experimental data of local shear angles; (e) determine the modulus distribution across the draped fabrics on the basis of experimental data, so that these values can be used for the contribution of the fabric part in the calculations of the modulus of the composite part.

6.2 DEFORMED BOUNDARY PROFILES OF THE DRAPED FABRICS

Figure 6.1 represents a flat view of the deformed boundary profiles of the woven fabrics draped over the hemispherical dome surrounded by a flat rim. Examination of the deformed boundaries reveals that the fabrics deformed to form a star shaped feature and most of the deformation takes place at the central position of the four edges. At these positions, the fabrics deform deeply inside a distance of about 52 mm to produce curved edges, generating an overall star shaped deformation feature. Considering the boundaries of the four weaves, it can be seen that with the exception of the satin weave all the other weaves produce a similar deformation pattern and the boundaries of the twill, tight plain and loose plain weaves are almost identically superimposed onto one another. The slight deviation of the satin weave could be attributed to the thickness value of 0.23 mm, which is the lowest in comparison to the other weaves. Also, another possibility could be the slippery nature of the satin weave, which makes it very difficult to handle during drape experiments.
Initially the inscribed grid occupies a total area of 102400mm², which represents the total area available for deformation. However it was evaluated from the experimental profiles of Figure 6.1 that after draping the area of the grid shrank to about 93500 mm², which represents 91% of the initial undeformed area. The area difference represents either shrinkage occurring locally in areas of high shear, as in the case of loose plain weave (basket weave), or the extra fabric in folds and wrinkles, as in the case of tight plain weave, the satin and twill weaves.

Figure 6.1: Experimental deformed boundary profiles of the twill weave, tight plain weave, loose plain weave (basket weave) and satin weave fabrics, draped over a hemispherical dome surrounded by a flat rim

6.3 DISTRIBUTION OF FIBRE ORIENTATION ANGLES AND SHEAR ANGLES

Local fibre orientation is assumed to be represented by the orientation of the deformed gridlines of the grid inscribed on the fabric prior to draping. So, the first task was to measure the angles \( \alpha \) of the grid elements (see Figure 3.6) of the fabric after draping.
Figures 6.2-6.5 show the angle $\alpha$ along the diagonal direction of the grid as a function of the normalised arc length away from the apex of the hemisphere for two different draped samples of fabric for each of the four types of draped fabric. For each fabric, the angle was measured either by direct determination (see section 3.5.1) or numerical evaluation from experimental data of the coordinates of the gridpoints from photographs (see section 3.5.2).

The technique of measuring the coordinates of the gridpoints from photographs had some difficulties regarding the edges of the draped object. Hence, the first technique of the direct determination of the angle of the grid elements was preferred and the results in chapter 6 are from this technique. Figures 6.2-6.5 shows that the measured angles of grid elements had a maximum difference of 8-10° between sample 1 and sample 2 demonstrating the variation between different draped samples of the same fabric, also including the experimental error.
Figure 6.2: Plots of the numerical and direct values of the angle of the grid elements along the diagonal fabric direction as a function of the normalised arc distance L/S from the apex for the basket weave fabric draped over a hemispherical mould surrounded by a flat rim.

Figure 6.3: Plots of the numerical and direct values of the angle of the grid elements along the diagonal fabric direction as a function of the normalised arc distance L/S from the apex for the tight plain weave fabric draped over a hemispherical mould surrounded by a flat rim.
Figure 6.4: Plots of the numerical and direct values of the angle of the grid elements along the diagonal fabric direction as a function of the normalised arc distance \( L/S \) from the apex for the twill weave fabric draped over a hemispherical mould surrounded by a flat rim.

Figure 6.5: Plots of the numerical and direct values of the angle of the grid elements along the diagonal fabric direction as a function of the normalised arc distance \( L/S \) from the apex for the satin weave fabric draped over a hemispherical mould surrounded by a flat rim.
Figures 6.6-6.9 present shaded contours interpolated from the map of measurements of the deformed grid element angle as a function of location on the draped fabric for the loose plain weave (basket weave), tight plain weave twill weave and satin weave respectively. The detailed experimental data are presented in appendix 2. These contours provide us with the zones of deformation patterns on the entire double curvature hemispherical surface and the surrounding flat surface. Starting from the apex of the dome, one notices that no fabric shear deformation was recorded and the original 90° grid element angle, α, was retained. Hence, this area can be regarded as a zero deformation zone. The zero deformation zone extends around the midlines of the fabric, forming a diamond or a cross of elaborate shape, depending on the fabric (see Figures 6.6-6.9). Next to the zero shear deformation zone is the transition zone where within each of the four quarters of the hemisphere, the fibres bend and shear at the yarn crossovers to form onto the hemisphere and the recorded angles of deformed elements are less than 90°. As we move away from the apex toward the edge of the hemispherical dome, there are zones of increasing shear, where the fabric deformation continues to rise. Toward the end of the hemisphere, (equator), the fabric deformation reaches its peak, which extends to the region where the hemisphere ends and the flat section begins. In this region, the highest attainable fabric deformation was obtained around the diagonals of the fabric.

When one considers the grid elements angles contours for the loose plain weave (basket weave) first (see Figure 6.6), it can be seen that the pattern follows the general trends as described above. A dark diamond of 90° element angles is observed around the apex of the hemispherical dome, which corresponds to the zero-shear zone. After that, there are about nine other zones where the grid elements angle ranges between 84-88 to 52-56° within the hemispherical part. At the equator, where the hemispherical dome changes to the flat surface the fabric deformation is maximum and a grid element angle of 34° was recorded. However, this maximum shear is concentrated only in a small area along the diagonal direction of the fabric quadrant. Outside this zone of maximum deformation, shear starts to decrease at four successive zones on the flat section and the grid element angle ranges between 64° and 68° at the tail end.
A similar trend can be observed for the tight weave fabric in Figure 6.7 although again the contour pattern of the grid element angles is not identical to the previous patterns of the loose plain weave. Starting from the apex, again a central zero-deformation zone is observed of a diamond/elaborate cross shape. Thereafter, the transition zone follows where grid element angles in the range of 80-84° were recorded in each fabric quadrant. After this zone, the rest of the zones experience an increase in fabric deformation until shear reaches a maximum value and grid element angles in the range of 52-56° were recorded close to the end of hemispherical dome. At the transition zone to the flat section, grid element angle of 42° was observed.

When the grid element angle contours are considered for the twill weave (see Figure 6.8), a similar pattern is observed but by no means identical to those of the loose and tight plain weave. The zero-deformation zone around the apex is of cross shape the sides of which extend along the midlines of the fabric indicating minimum shear deformation due to symmetry. Then moving to the transition zone, where the deformation begins, the grid element angles range between 81-86°. From there onwards there is continuous increase in the deformation of the fabric in each quadrant and Grid element angles in the range of 76-81° and 45-50° near the end of the hemisphere were observed. In the transition region where the hemispherical surface ends and the flat section begins, a minimum grid element angle of 34° was recorded. After this zone, the rest of the flat section consists of several zones of deformation. In these zones deformation falls gradually and the deformed grid element angles lie in the range 45-50° until they reach 72-76° at the fabric tail end.

Figure 6.9 presents the interpolated shaded contours for the satin weave. The general trend of shear deformation is the same as for the other weaves. The zero deformation zones is of an elaborate cross-shape. Some lack of rotational symmetry is observed in the central cross-pattern, which is also observed in the deformed boundary profile of the satin weave in figure 6.1. A maximum grid element angle of 38° is recorded on the diagonal line on the flat part just after the dome edge. After that the grid element angle increases until it reaches 64-69° at the fabric’s tail end.
Figure 6.6: Shaded contours of the deformation angle (α) of grid elements after the draping of the loose plain weave (basket weave) onto a hemispherical mould surrounded by a flat rim.
Figure 6.7: Shaded contours of the deformation angle ($\alpha$) of grid elements after the draping of the tight weave onto a hemispherical mould surrounded by a flat rim.
Figure 6.8: Shaded contours of the deformation angle ($\alpha$) of grid elements after the draping of the twill weave onto a hemispherical mould surrounded by a flat rim.
Figure 6.9: Shaded contours of the deformation angle (α) of grid elements after the draping of the satin weave onto a hemispherical mould surrounded by a flat rim.
Figure 6.10 presents the grid element angle, \( \alpha \), curves of the loose plain (basket) weave, tight plain weave, satin weave and twill weave along the diagonal direction as a function of the normalised distance (L/S) from the apex. The normalising factor is \( 0.5 \pi R \) where \( R \) is the radius of the dome mould.

Hence, at \( L/S = 0 \) is the apex, at \( L/S = 1 \) is the dome edge, \( L/S < 1 \) is within the dome and \( L/S > 1 \) is on the flat surface surrounding the dome. The four experimental curves of Figure 6.10 display the same general features for the four fabrics, but are not identical. The grid element angle starts from values close to 90° in the zero-deformation zone at the apex of the dome (\( L/S = 0 \)) and decreases to a minimum angle around \( L/S = 1 \) at the edge of the dome where the maximum shear occurs. Thereafter, deformation starts declining progressively and this continues up to the end of the fabric.

From the value of the angles of the grid elements (Figure 6.10), the local shear angles, \( \gamma \), can be calculated as \( \gamma = \pi/2 - \alpha \). Figure 6.11 presents the corresponding curves of shear angle for the loose plain weave (basket weave), tight plain weave, twill and satin
weaves against the normalised arc distance (L/S) from the apex along the diagonal direction of each fabric. The shear angle rises from 0 at the apex of the dome to a maximum around L/S = 1 and then it falls again on the flat surface along the tails of the fabric. The loose plain weave (basket weave) and the twill weave exhibit the highest shear angle of 56°, followed by the satin weave, which has a maximum shear angle of 52° and lastly the tight plain weave with a maximum shear deformation angle of 48°. At points where L/S>1 the flat section begins and the fabrics experience less deformation resulting in low shear angle values.

Figure 6.10: Plots of the values of the shear angle along the diagonal fabric direction as a function of the normalised arc distance L/S from the apex for the four woven fabrics draped over a hemispherical mould surrounded by a flat rim

6.4 DETERMINATION OF THE FIBRE VOLUME FRACTION DISTRIBUTION OF THE DRAPEFABRICS

The next step is to calculate local changes in the fibre volume fraction due to the inhomogeneous shear distribution on the draped fabric. Let us consider an

Experimental Studies and Mathematical Modelling of the Draping and Shear Deformation of Woven Fabrics in Resin Transfer Moulding (RTM)
undecormed, rectangular fabric element of sides a and b (see Figure 6.12). During draping it is considered that shearing is the dominant deformation mode, according to which the original fabric element changes as shown in Figure 6.12.

![Figure 6.12: Pure shear of a fabric element](image)

It is assumed that the fabric element deforms following pure shear, the lengths of its sides a and b remains unchanged during shear, there is no compression and the element thickness, h, remains unchanged. In this case, taking into account that the element mass remains the same before and after shear, it can be written that:

\[ V_{fo} \cdot hab = V_f \cdot abh \sin \alpha \]  

(6.1)

where, \( V_{fo} \) is the fibre volume fraction of the unsheared element, \( V_f \) is the fibre volume fraction after shearing. Rearranging equation 6.1 gives:

\[ \frac{V_f}{V_{fo}} = \frac{hab}{habsin \alpha} = \frac{1}{\sin \alpha} \]  

(6.2)

The values of \( V_{fo} \) can be obtained from the expression below

\[ V_{fo} = \frac{N_p \rho_a}{h \rho} \]  

(6.3)
where $\rho_s$ is the areal density of the weave (kg/m³), $N_p$ is the number of plies which in our case is taken as 1, $h$ is the ply thickness (m), and $\rho$ is the density of fibre glass (kg/m³). In this study, the value for $h$ for the single draped fabric layer was taken as the nominal thickness of the uncompressed fabric, as given in Table 3.1 for the different fabrics.

By using equations 6.2, 6.3, and data from Table 3.1, the distributions of the fibre volume fractions of the sheared grid elements of the draped fabrics were calculated. The initial values of the fibre volume fractions, $V_{fo}$, in the unsheared grid were determined to be 0.36, 0.44, 0.46 and 0.50 for the loose plain weave (basket weave), tight plain weave, twill and satin weave, respectively.

Figure 6.13 shows the graphs of the fibre volume fraction of the sheared grid elements along the diagonal direction of the four weaves used in the experiments, as functions of the normalised arc distance L/S from the apex of the dome. The curves start from $V_{fo}$ at the apex of the dome, where is the zero-deformation zone, and rise to a maximum

![Graph of fibre volume fraction](image-url)
around \( L/S = 1 \), where the maximum shear occurs in the fibre yarn directions. Then the fibre volume fraction falls along the diagonal line of the flat part of the fabric.

**Table 6.1: Initial and maximum fibre volume fractions for each draped weave.**

<table>
<thead>
<tr>
<th>Fabric</th>
<th>( V_{fo} )</th>
<th>( V_{f\text{max}} )</th>
<th>( V_{f\text{max}} / V_{fo} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>twill weave</td>
<td>0.46</td>
<td>0.83</td>
<td>1.80</td>
</tr>
<tr>
<td>tight plain weave</td>
<td>0.44</td>
<td>0.66</td>
<td>1.50</td>
</tr>
<tr>
<td>loose plain weave</td>
<td>0.36</td>
<td>0.64</td>
<td>1.77</td>
</tr>
<tr>
<td>satin weave</td>
<td>0.50</td>
<td>0.82</td>
<td>1.64</td>
</tr>
</tbody>
</table>

Table 6.1 summarises the initial fibre volume fractions of the weaves used in the analysis and the maximum fibre volume fractions calculated from the shear data of the drape experiments. Compression of weaves results to a maximum fibre volume fraction around 0.62-0.65 [Saunders et al (1998, 1999)] corresponding to the packing limit. Beyond this limit, it is reasonable to expect that in-plane compression wrinkling or ply thickening will occur. The loose plain weave reaches a maximum calculated \( V_f \) close to this limit after draping. However, the calculated maximum \( V_f \) for the tight plain weave, twill weave and satin weave clearly exceed the maximum acceptable \( V_f \) and, hence, they exhibit wrinkling after been draped in the hat mould geometry. A thick line around \( V_f = 0.65 \) in Figure 6.13 represents the plateau of approximately acceptable maximum \( V_f \). The corresponding cells of unacceptable \( V_{f\text{max}} \) after draping (as calculated by relation 6.2) have been shaded in Table 6.1. It must be pointed out that after wrinkling the assumptions associated with relation (6.2) are unlikely to hold and also the angles between the gridlines might not represent the local angles between crossing fibre yarns.

### 6.5 DETERMINATION OF THE MODULUS DISTRIBUTION IN DRAPED FABRICS

Changes in the fibre orientation and the fibre volume fraction in the draping of fabrics result in an inhomogeneous distribution of mechanical properties of the corresponding composite product. In this section, the modulus distribution of fabrics draped over a

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hemispherical hat will be evaluated on the basis of laminate plate theory [Powell (1983), Daniel and Ishai (1994), Mathews and Rawlings (1995) for example]. Each ply of woven fabric is considered as a combination of two layers (see Figure 6.14) at 0 and 90° for an unsheared fabric, and at 0 and α degrees for a fabric sheared by an angle γ. The effect of crimp and the type of weave are not incorporated in this type of theoretical analysis.

Each layer of unidirectional fibres in Figure 6.14 has mechanical properties depending on the principal directions 1 and 2 (where 1 is the fibre direction and 2 is perpendicular to the fibre direction) and on the fibre volume fraction. Table 6.2 presents the elastic properties of E-glass [Naik (1994)], the material from which the fibres are made for the fabrics studied in the experimental programme of this work, where $E_r$ is the fibre Young’s modulus, $\nu_r$ is the fibre major Poisson’s ratio and $G_r$ is the fibre shear modulus. These properties were used in the modulus calculations of this study.

<table>
<thead>
<tr>
<th>$E_r$ (GPa)</th>
<th>$\nu_r$</th>
<th>$G_r$ (GPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>72</td>
<td>0.3</td>
<td>27.7</td>
</tr>
</tbody>
</table>

Assuming that the fibre volume fraction in each equivalent uni-directional fibre ply changes during shear by the same amount as the overall fibre volume fraction of the fabric, the effect of the change of $V_f$ during shear on the elastic properties of each unidirectional fibre ply is given by equations [Powell (1983)], where subscript f and n refer to the fibre and matrix respectively.

$$E_i = E_f V_f + E_m (1 - V_f)$$  \hspace{1cm} (6.4)
(Rule of mixture)
\[
\frac{1}{E_2} = \frac{V_f}{E_f} + \frac{1-V_f}{E_m} \quad (6.5)
\]

\[
v_{12} = v_f V_f + v_m (1-V_f) \quad (6.6)
\]

\[
\frac{1}{G_{12}} = \frac{V_f}{G_f} + \frac{1-V_f}{G_m} \quad (6.7)
\]

The influence of the change in fibre orientation during the draping of a fabric on the modulus of the draped fabric, as part of composite product, can be ascertained by considering the rectilinear system of coordinates \((x, y)\). This is related to the system of principal directions \((1, 2)\) for each ply in unsheared and sheared state as is illustrated in Figure 6.14.

**Figure 6.14: Representation of a woven fabric by a combination of two plies of unidirectional fibres in unsheared and sheared state**
According to Figure 6.14 the system of coordinates \((x, y)\) is aligned with the principal directions \((1, 2)\) of the \(0^\circ\) ply both in unsheared and sheared state, whereas the \(90^\circ\) ply changes direction with respect to the \((x, y)\) system of coordinates after shear. The relationship between stress and strain in each ply is expressed by:

\[
\begin{bmatrix}
\sigma_x \\
\sigma_y \\
\sigma_{xy}
\end{bmatrix} =
\begin{bmatrix}
Q_{11} & Q_{12} & Q_{16} \\
Q_{21} & Q_{22} & Q_{26} \\
Q_{16} & Q_{26} & Q_{66}
\end{bmatrix}
\begin{bmatrix}
\varepsilon_x \\
\varepsilon_y \\
\varepsilon_{xy}
\end{bmatrix}
\]

where, \(\sigma\) is the stress tensor, \(\varepsilon\) is the strain tensor and \(\bar{Q}\) is the transformed reduced stiffness matrix.

The stiffness matrix of a uni-directional fibre ply is defined as:

\[
\begin{bmatrix}
Q_{ij}
\end{bmatrix} =
\begin{bmatrix}
Q_{11} & Q_{12} & Q_{16} \\
Q_{21} & Q_{22} & Q_{26} \\
Q_{16} & Q_{26} & Q_{66}
\end{bmatrix}
\]

where \(Q_{11}, Q_{12}, Q_{22}\) and \(Q_{66}\) are called the reduced stiffnesses and can be expressed in terms of the elastic properties of a uni-directional fibre ply [Powell (1983)] as:

\[
\begin{align*}
Q_{11} &= E_1/ (1-\nu_{12}\nu_{21}) \\
Q_{22} &= E_2/ (1-\nu_{12}\nu_{21}) \\
Q_{12} &= \nu_{21}E_1/ (1-\nu_{12}\nu_{21}) \\
Q_{66} &= G_{12}
\end{align*}
\]

Table 6.3 presents the transformed reduced stiffness matrix components \(Q_{11}\) and \(Q_{22}\) for each ply of a woven fabric in the \((x, y)\) system of coordinates in the unsheared and sheared state. The stiffness of components of the \(0^\circ\) ply remain the same in the unsheared and sheared state since the \(0^\circ\) ply is aligned with the \((x, y)\) system and does...
not change orientation after shear. The $\alpha$ degree ply is at 90° with respect to the (x, y) system in the sheared state.

Table 6.3: transformed reduced stiffness matrix components $Q_{11}$ and $Q_{22}$ of each uni-directional fibre ply of a woven fabric in the (x, y) system of coordinates, in unsheared and sheared fabric state

<table>
<thead>
<tr>
<th></th>
<th>0° ply</th>
<th>90° degree ply</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Unsheared state</strong></td>
<td>$Q_{11}$</td>
<td>$Q_{22}$</td>
</tr>
<tr>
<td>$Q_{11}$</td>
<td>$Q_{22}$</td>
<td>$Q_{22}$</td>
</tr>
<tr>
<td><strong>Sheared state</strong></td>
<td>$Q_{11}$</td>
<td>$Q_{22}$</td>
</tr>
<tr>
<td></td>
<td>$Q_{22}$</td>
<td>$Q_{11}$</td>
</tr>
</tbody>
</table>

The transformed reduced stiffness matrix components of the $\alpha$ degree ply in an off-axis loading test in the x and y directions are given by the following relations [Powell (1983)].

\[
\begin{align*}
Q_{11} &= Q_{11} m^4 + 2(Q_{21} + 2Q_{66})n^2 m^2 + Q_{22} n^4 \\
Q_{22} &= Q_{22} m^4 + 2(Q_{12} + 2Q_{66})n^2 m^2 + Q_{22} n^4 \\
Q_{12} &= (Q_{11} + Q_{22} - 4Q_{66})n^2 m^2 + Q_{12}(n^4 + m^4) \\
Q_{66} &= (Q_{11} + Q_{22} - 2Q_{12} - 2Q_{66})n^2 m^2 + Q_{66}(n^4 + m^4) \\
Q_{16} &= (Q_{11} - Q_{12} - 2Q_{66})n m^3 - (Q_{22} - Q_{12} - 2Q_{66})n^3 m \\
Q_{26} &= (Q_{11} - Q_{12} - 2Q_{66})n^3 m - (Q_{22} - Q_{12} - 2Q_{66})n m^3
\end{align*}
\]

where, $m = \cos \alpha$ and $n = \sin \alpha$

Once the components of the transformed reduced stiffness matrix are calculated for each equivalent uni-directional fibre ply of a woven fabric, the transformed reduced stiffness matrix of the “fabric laminate” under loading in the (x, y) directions is given by the relation.

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\[
\left[ Q_{ij} \right]_{\text{trans}} = 0.5 \left( \left[ Q_{ij} \right]_{\text{trans}} + \left[ Q_{ij} \right]_{\text{trans}} \right)
\] (6.12)

which originates from the relation giving the laminate force matrix, \( N \)

\[
\left[ N \right]_{\text{laminate}} = \sum_{n=1}^{N} \int_{h_{n-1}}^{h_{n}} \left[ \sigma_{h} \right]_{n} dz = \sum_{n=1}^{N} \int_{h_{n-1}}^{h_{n}} \left[ Q_{h} \right]_{n} \varepsilon^{0} dz + \sum_{n=1}^{N} \int_{h_{n-1}}^{h_{n}} \left[ Q_{h} \right]_{n} \left[ k \right]_{n} dz
\] (6.13)

where the laminate thickness consists of \( N \) uni-directional fibre plies, \((h_{n}-h_{n-1})\) is the thickness of each ply, \( \left[ Q_{h} \right]_{n} \) is the transformed reduced stiffness matrix of the ply \( n \) ply, \( \varepsilon^{0} \) is the mid-plane strain and \( \left[ k \right] \) is the mid-plane plate curvature. By assuming that the fabric thickness is very small, the second term of the sum in equation (6.13) is not very significant. Also, by assuming that each of the warp and weft uni-directional fibre plies in a woven fabric has the same thickness and fibre volume fraction, the definition of the transformed reduced stiffness matrix of the laminate is established and relation (6.12) derived.

The above analysis and calculation procedure of the major components of the transformed reduced stiffness matrix of a sheared fabric laminate loaded in the \((x, y)\) directions is applied to the loose plain weave (basket weave) draped over the hemispherical hat mould. This fabric was chosen because it draped without wrinkling and the evaluated values of fibre volume fraction according to relation (6.2) did not exceed the maximum packing (see figure 6.1). The weave can be considered balanced (see table 3.1) and the fabric can be considered as in-plane symmetric material.

First the effect of the change of \( V_f \) due to shear during draping was incorporated in the elastic properties in the principal directions \((1, 2)\) of each uni-directional fibre ply, according to equations (6.4)-(6.7). (At this stage, 0 values were assumed for the mechanical properties of the matrix, so that the matrix effect is not incorporated in the results). Figure 6.15 presents the effect of the \( V_f \) change on \( E_1 \) and \( E_2 \). It is evident that \( E_1 \) is the most important modulus value and it increases as one moves away from
the apex along the diagonal direction of the fabric. It reaches a maximum value at the edge of the hemisphere where the value of $V_f$ and shear are maximum. The value then falls just after the edge of the mould and this continues gradually up to the flat surface. $E_2$ is negligible for the uni-directional E-glass fibre ply, which is considered dry that is without any polymer matrix.

After the effect of $V_f$ during shear has been taken into account, the major reduced stiffness for a uni-directional E-glass fibre ply can be calculated from relations (6.10). (Notice that $\nu_{21} = \nu_{12} E_2/E_1$ [Powell (1983)]. These stiffness values are directly applicable to the $0^\circ$ ply and are presented in Figure 6.16. Again, it can be seen that $Q_{11}$ is the most important stiffness component, which corresponds to $E_1$ and follows its trend due to the effect of changing $V_f$ across the draped ply.

The transformed reduced stiffnesses of the $\alpha$ degree uni-directional fibre ply were evaluated from equation (6.11), which expressed the effect of the change in fibre orientation. The effect of change in the fibre volume fraction after drape was also taken into account in the values of the reduced stiffness $[Q_{ij}]$ in the principal directions. Hence, Figure 6.17 presents the transformed reduced stiffness of the $\alpha$ degree uni-directional fibre ply of the loose plain weave fabric (basket weave) draped over the hemispherical hat for loading in the $(x, y)$ directions (see Figure 6.14), incorporating the effect of changing $V_f$ and changing fibre direction during shear. It can be seen that at the apex ($L/S = 0$) $Q_{22}$ is maximum and is the important reduced stiffness, whereas the other reduced stiffnesses are negligible. At this location $\alpha = 90^\circ$ and $Q_{22}$ corresponds to the modulus along the orthogonal fibre direction of the fabric. As one moves away from the apex along the diagonal direction of the fabric, the fibre orientation changes due to shear and as a result $Q_{22}$ decreases whereas $Q_{11}$ increases reaching a maximum at $L/S = 1$.

By using equation (6.12) the transformed reduced stiffnesses of the "fabric laminate" were calculated as a combination of the corresponding stiffnesses of the $0$ and $\alpha$ degree uni-directional fibre plies, when loading is applied in the $(x, y)$ directions.
Figure 6.14. Figure 6.18 presents the reduced stiffnesses of the loose plain weave (basket weave) fabric along the normalised diagonal distance (L/S) outwards from the apex when the fabric is draped over the hemispherical hat. These stiffnesses are with respect to the loading in the (x, y) directions (see Figure 6.14).

A recent paper [Mohammed et al, “Experimental studies and analysis of the draping of woven fabrics” to be published in Composites A] includes a similar case-study for the determination of the reduced stiffnesses of a laminate formed over the same hemispherical hat geometry, as in this study, where the laminate consists of a layer of the basket weave fabric and the epoxy matrix.
Draping of Fabrics: Experimental Results and Analysis

Figure 6.15: The effect of V_f change on the elastic moduli (E_1 and E_2), in the principal directions of a uni-directional E-glass fibre ply in the draping of the loose plain weave (basket weave) over a hemispherical dome surrounded by a flat rim.

Figure 6.16: The reduced stiffness of the θ° uni-directional fibre ply in the draping of the loose plain weave (basket weave) over a hemispherical dome surrounded by a flat rim.

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Figure 6.17: The transformed reduced stiffnesses of the α degree uni-directional fibre ply in the draping of the loose plain weave (basket weave) over a hemispherical dome surrounded by a flat rim. Loading in the (x, y) directions (see Figure 6.14)

Figure 6.18: The reduced stiffnesses of the loose plain weave (basket weave) draped over a hemispherical dome surrounded by a flat rim where the loading is in the (x, y) directions in Figure 6.14
7. SHEAR DEFORMATION AND MICROMECHANICS OF WOVEN FABRICS: RESULTS

Experimental Studies and Mathematical Modelling of the Draping and Shear Deformation of Woven Fabrics in Resin Transfer Moulding (RTM)
7.1 SHEAR TEST EXPERIMENTS

This chapter presents the results of the shear test experiments, the experimental procedure of which was described in section 3.6. First, photographs will be presented illustrating the wrinkling of the woven fabrics during the picture-frame and manual shear tests. The results of the picture-frame shear tests will be presented in the form of shear curves of the Lloyd applied load versus shear angle and of the Lloyd applied load versus shear rate. Further analysis will follow via polynomial fitting and application of the elasticity theory to determine the mechanical properties and parameters associated with the shear deformation of woven fabrics. Finally, a microstructural analysis of the shear deformation of woven fabrics will be presented to model and predict the geometric shear locking of woven fabrics.

7.2 VIEWING OF THE SHEAR DEFORMED FABRICS

Figure 7.1 presents the pictures of the fabrics that have been deformed in the picture-frame experiment described in Chapter 3. In each deformed frame, the appearance of wrinkles can be seen, formed during the deformation process.

For the loose plain (basket) weave (Figure 7.1(a)) observations have shown that the wrinkles (though hardly noticed) appear close to the jaws of the Lloyd machine, along the diagonal direction. It can be seen that the stretching of the fabric inside the frame is not evenly distributed. This is indicated by the deformation of fibres close to the jaws of the Lloyd which appear to be stretched more than the fibres toward the middle of the fabrics. The trend in wrinkle formation could be attributed to the two forces responsible for the deformation of the fabric inside the frame. While shear forces are responsible for shearing the fabric along the directions of the warp and weft fibre yarns, there are also compressive forces acting inside the fabric emanating from the movement of the frame during the shear test. When a certain critical maximum shear angle has been reached the shear forces from the applied Instron load are hindered by the resistance of the fabric to shear as a result of the closure of the gaps between the fibre yarns.
Shear Deformation and Micromechanics of Woven Fabrics: Results

Figure 7.1: Deformation and wrinkling of (a) loose plain weave, (b) tight plain weave (c) twill weave and (d) satin weave during the picture-frame Lloyd shear tests.
As a result of this development the compressive forces, which are more prevalent at the upper and lower pivot points of the picture frame arms predominate, thereby producing wrinkling at these two positions.

In the case of the tight plain weave (Figure 7.1(b)), it can be seen that the wrinkles appear extended from the upper to the lower pivot point along the diagonal direction of the frame. This behaviour can be attributed to the tight weave pattern which resulted in high fabric resistance to both shear forces and out-of-plane expansion of the fibre yarns at low shear angles. So in this case, wrinkling appeared early and was homogeneous along the whole length of fabric between the upper and lower pivot point of the picture frame.

In the case of the twill weave (Figure 7.1(c)) the same observations found in the tight plain weave can be seen here. Wrinkles are formed along the diagonal direction where the fabric is being stretched. This observation could also be explained in the same manner as that for the tight plain weave. However, what is noticeable in this case is that the deformation of the frame is more than that for the tight plain weave. This means that the twill weave can be sheared to a larger shear angle before it wrinkles. The reason for this could be traced to the weaving pattern of the twill weave, which provides more freedom for the warp and weft fibre yarns to rotate. Furthermore, the thickness of the fabric (0.28 mm) makes the fabric lighter and easier for the fibre yarns to rotate to larger shear angles than the tight plain weave. However, it must be mentioned that thinner fabrics bend and buckle easier in in-plane compression due to reduced fabric rigidity. This means that a reduction in thickness for the thin, twill and satin weave fabrics was unfavourable because it must have brought wrinkling a little earlier in these fabrics.

For the satin weave (Figure 7.1(d)) the appearance of wrinkles is not along the diagonal direction, but instead wrinkles are formed erratically within the fabric. There are two reasons that could be put forward to explain this behaviour. The first reason is attributed to the lack of symmetry of satin fabrics as reported by Spencer (1999). Secondly, the satin weave fabric was very slippery and, therefore, it was very difficult to handle and secure it onto the picture-frame.
Figure 7.2: Manual shear tests for (a) loose plain weave and (b) twill weave showing the appearance of wrinkles.

Figure 7.2 shows the pictures of the deformed basket and twill weaves, which have been sheared manually with bare hands as explained in section 3.6.4. It can be seen that for the basket weave (Figure 7.2(a)) the same deformation pattern exhibited in the picture frame was also observed here. The wrinkles are formed towards the upper and lower corners of the fabric and along the diagonal direction. From the picture of the sheared fabric, the shear deformation angle of the fabric appears to be close to that of the picture-frame deformed fabric.

Similarly, in Figure 7.2(b) the deformation pattern and the appearance of wrinkles for the twill weave are analogous to that of the twill weave in the picture-frame test. The magnitude of the deformation angle is comparable to that of the picture-frame test. However, this assertion will be verified in the following sections.

7.3 SHEAR TEST CURVES: APPLIED LOAD VERSUS SHEAR ANGLE

In this section, the results of the picture-frame weft-warp shear deformation tests (see Figure 7.3(a)) for the four woven fabrics will be presented in the form of shear deformation curves. Two types of shear deformation curves will be presented in all.
These are for applied Lloyd load versus shear angle and also for the applied Lloyd load versus shear rate. The curves presented will be for crosshead speeds of 5, 10, 50, and 100 mm/min for each of the woven fabrics.

The tensile machine in tensile mode measures the tensile force, $F_x$, and the extension of the fabric $2\Delta l$. The extension of the fabric was translated into shear angle using equations (3.14) and (3.15). In this case, the values of the tensile load were plotted as a function of shear angle. Figures 7.4(a)-(d) present such shear deformation curves from three picture-frame shear test experiments for each type of fabric at a crosshead speed of 5 mm/min. The purpose of this is to study the variation between different shear test experiments for the same weave under the same nominal test conditions. Variations incorporate experimental error, variations in the fixing of fabric onto the frame and variations within the roll of fabric material.

![Diagram](image)

Figure 7.3: (a) weft-warp and (b) warp-weft configuration of the fabric in the picture-frame shear tests
Experimental Studies and Mathematical Modelling of the Draping and Shear Deformation of Woven Fabrics in Resin Transfer Moulding (RTM)
Figure 7.4: Applied tensile load versus deformation curves for (a) loose plain weave, (b) tight plain weave, (c) twill weave and (d) satin weave obtained during the picture-frame experiment (weft-warp configuration)
Figures 7.5-7.8 present the results of the applied tensile Lloyd load versus shear angle at different crosshead speeds in weft-warp type of shear deformation tests (see Figure 7.3(a)). The data points in each curve have been taken as the average of three tests with a new fabric sample in each test. Figure 7.5 presents the Lloyd applied load versus shear angle curves for the basket weave. It can be observed that, initially, the deformation of the fabric was linear for shear angles up to about 40°. Beyond this angle, the deformation of the fabric changes to become non-linear. This observation is visible for all the crosshead speeds used in the tests.

Furthermore, the effect of the crosshead speed on the deformation of the basket weave may be considered to be small, within the experimental error of testing. This error could emanate either from the variation in the amount of tension introduced during the clamping procedure, or from the distortion of the yarn spacing in the fabric during the clamping and mounting process, which could affect the slipping of the yarns during the shear tests. In this, it has to be borne in mind that this a very loose type of weave. This type of inconsistent mismatch in the load-deformation curves at different crosshead speeds of the tensile testing machine was also reported in some of the data reported by McGuinness and O'Braidaigh (1997). Wrinkles appeared at a high shear angle of about 60°.

Figure 7.6 presents the Lloyd applied load versus shear angle for the tight plain weave. It can be seen that there is a similar trend in the deformation curve exhibited by the fabric for the various crosshead speeds. In this case too as in the basket weave, no clear trend in the dependence of the shear deformation on the crosshead speed is observed, although in this case the differences are not so pronounced as in the case of the basket weave. The small differences could be traced back to the architecture of the tight plain weave, which is more orderly and regular than the loose plain weave. Also, the fact that the yarn spacing between the warp and weft yarns is small makes the possibility of yarn slippage minimal during shear deformation. It can be seen in Figure 7.6 that wrinkles appeared early at a shear angle of 10°. This observation was similarly made for all crosshead speeds.
Figure 7.7 presents the curves of the applied Lloyd load versus shear angle for the twill weave at the various crosshead speeds. The effect of the crosshead speed is more pronounced for the twill weave than that for the loose plain weave and tight plain weave. Although no consistent trend of the load with the crosshead speed is obvious for the range of crosshead speeds used, it must be mentioned that the twill weave is difficult to mount due to its small thickness and the architecture of the weave. It can be seen that in the case of the twill weave the wrinkles appeared at a shear angle of about 25° and within the linear region of the deformation curve.

Figure 7.8 presents the results of the shear tests in the form of applied Lloyd load versus shear angle for the satin weave. As can be observed, there is a general inconsistent trend in the load-deformation curves for the different crosshead speeds of the tensile testing machine used. This inconsistency is more pronounced than in the case of the loose and tight plain weaves. This type of behaviour could be attributed to the difficulties in the handling of the fabric as a result of its small thickness and its slippery nature. Another issue of interest in the case of a satin weave is the degree of symmetry since satin weaves are generally non-symmetric as shown by Spencer (1999).

A deformation curve in a warp-weft configuration (see Figure 7.3(b)) is presented in Figure 7.8 at a crosshead speed of 5 mm/min. The data points on this curve are again the average of three picture-frame shear tests. Some difference is observed when compared to the corresponding curve in weft-warp configuration, but this difference is within the expected experimental error. In the case of the satin weave, the shear-locking angle, which was determined by the appearance of wrinkles was observed at a shear angle of 26°. This is quite close to the shear-locking angle of the twill weave. It must also be mentioned that the thickness of the two weaves does not vary significantly (see Table 3.1).
Figure 7.5: Total applied tensile load versus shear angle curves obtained during the deformation of a loose plain weave fabric in a picture-frame experiment (weft-warp configuration).

Figure 7.6: Total applied tensile load versus shear angle curves obtained during the deformation of a tight plain weave fabric in a picture-frame experiment (weft-warp configuration).
Figure 7.7: Total applied tensile load versus shear angle curves obtained during the deformation of a twill weave fabric in a picture-frame experiment (weft-warp configuration).

Figure 7.8: Total applied tensile load versus shear angle curves obtained during the deformation of a satin weave in a picture-frame experiment.
7.4 SHEAR LOCKING ANGLES

The shear tests were also used to detect wrinkling during the shearing of the fabrics. Table 7.1 presents the maximum shear angles or the shear locking angles for the four woven fabrics. These are also indicated in the shear curves of Figures 7.5-7.8 as the "wrinkling point".

Table 7.1: Values of the maximum shear locking angles above which wrinkling appears.

<table>
<thead>
<tr>
<th>FABRIC</th>
<th>PICTURE-FRAME TEST</th>
<th>MANUAL TEST</th>
<th>MANUAL TEST + SMOOTHING</th>
</tr>
</thead>
<tbody>
<tr>
<td>Basket weave (LPW)</td>
<td>61°</td>
<td>54°</td>
<td>70°</td>
</tr>
<tr>
<td>Tight plain weave (TPW)</td>
<td>10°</td>
<td>9°</td>
<td>38°</td>
</tr>
<tr>
<td>Twill weave (Twill)</td>
<td>25°</td>
<td>24°</td>
<td>40°</td>
</tr>
<tr>
<td>Satin weave (5HSW)</td>
<td>26°</td>
<td>25°</td>
<td>45°</td>
</tr>
</tbody>
</table>

As can be seen from the table, the loose plain weave can be sheared to very high shear angles without wrinkling. The next fabrics in terms of low wrinkling are the twill and the satin weaves. The tight plain weave wrinkles at very low shear angles of around 10°. What is apparent from these results is that the Lloyd test and the manual test values are very close to each other. An important observation about the maximum shear locking angle for the four fabrics is that smoothing during shearing can delay wrinkling to much higher shear angles. This observation is useful, most especially to draping procedures.
7.5 SHEAR TEST RESULTS: APPLIED LOAD VERSUS SHEAR RATE

Figures 7.9-7.12 present the results of the picture-frame shear tests (weft-warp configuration, see Figure 3(a)) in the form of applied total tensile load, $F_x$, versus shear rate, determined by equation (3.16). In all four weaves, there is a clear trend in the movement of the load-shear rate curves for the different crosshead speeds of the tensile testing machine. As the crosshead speed increases from 5 mm/min to 100 mm/min, the shear curve moves to the right that is to positions of higher shear rate.

From the shear curves it can be seen that for crosshead speeds of 5-10 mm/min the shear rate is not affected much by the load change. For crosshead speeds of 50 mm/min and 100 mm/min, the deformation curves assumed a non-linear behaviour, meaning that the rate of shear deformation of the fabrics does not increase linearly with the applied load.
Figure 7.9: Total applied tensile load versus shear rate curves obtained during the deformation of a basket weave in a picture-frame experiment (weft-warp configuration)

Figure 7.10: Total applied tensile load versus shear rate curves obtained during the deformation of a tight plain weave in a picture-frame experiment (weft-warp configuration)
Figure 7.11: Total applied tensile load versus shear rate curves obtained during the deformation of a twill weave in a picture-frame experiment (weft-warp configuration).

Figure 7.12: Total applied tensile load versus shear rate curves obtained during the deformation of a satin weave in a picture-frame experiment (weft-warp configuration).
7.6 POLYNOMIAL CURVE FITTING OF SHEAR TEST DATA

In this section, the shear stress along the fibre yarn directions will be presented as a function of shear angle in the form of polynomial curves for the four woven fabrics. These curves will be used to determine the shear modulus of the woven fabrics for small shear strains of 1%.

The total applied tensile LLoyd load, $F_s$, in Figures 7.5-7.8 was first converted to shear force components, $F_s$, by using equation (3.10) and subsequently to mean shear stresses, $\tau_s$, along the warp and weft fibre yarn directions (which change during fabric shear) by using equation (3.11). Figure 7.13 represents the resulted curves of the shear stress, $\tau_s$, as a function of the shear angle for the four woven fabrics used in the picture-frame shear tests.

![Figure 7.13: Comparison of the shear behaviour of the four woven fabrics during the picture-frame shear tests at a crosshead speed of 5 mm/min.](image)

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It can be seen in Figure 7.13 that the satin weave seems to have the lowest initial shear modulus, but after a shear angle of about 38-40° the shear curve rises higher than that of the loose plain weave. This behaviour is also exhibited by the twill weave, although it has initially a higher shear modulus and rises above the curve of the loose plain weave at lower shear angles. The shear curve of the tight plain weave for the whole test is higher than that of the other weaves. This behaviour suggests that the tight plain weave has the highest shear modulus of all the woven fabrics tested.

If the response of the fabrics in shear deformation, illustrated in Figure 7.13, is considered to be represented by non-linear elastic mechanical behaviour, polynomial curve fitting can be used in the regression analysis of the experimental points for each fabric, namely:

Loose plain weave (LPW) - basket weave:

\[ \tau_s = 0.1241\gamma^3 + 622.71\gamma \]  \hspace{1cm} (7.1)

Tight plain weave (TPW):

\[ \tau_s = 0.0028\gamma^5 - 2.7694\gamma^3 + 2588.1\gamma \]  \hspace{1cm} (7.2)

Twill (TWILL):

\[ \tau_s = 0.587\gamma^3 + 569.23\gamma \]  \hspace{1cm} (7.3)

5-Harness satin weave (5HSW):

\[ \tau_s = 0.4718\gamma^3 + 115.43\gamma \]  \hspace{1cm} (7.4)

All polynomial shear curves described by relations (7.1)-(7.4) are symmetric about the origin. In the equations \( \tau_s \) represents a measure of shear stress along the directions of the fibres in the fabric, which are non-orthogonal during shear. However, it is
possible to assume that the fibre directions are approximately orthogonal at small shear strains, $\epsilon_{12}$, and determine an initial, Secant, in-plane, shear modulus, $G_{12}$, for each fabric at $\epsilon_{12} = 1\%$.

Table 7.2: Values of the initial shear modulus obtained from the polynomial regression analysis of the data of shear stress for the woven fabrics.

<table>
<thead>
<tr>
<th>FABRICS</th>
<th>$G_{12}$ (kPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Basket weave (LPW)</td>
<td>35</td>
</tr>
<tr>
<td>Tight plain weave (TLW)</td>
<td>148</td>
</tr>
<tr>
<td>Twill weave (TWILL)</td>
<td>32</td>
</tr>
<tr>
<td>Satin weave (5HSW)</td>
<td>6.6</td>
</tr>
</tbody>
</table>

Table 7.2 displays the values of the initial shear modulus $G_{12}$ for the four woven fabrics, which were obtained from the polynomial regression curves of Figures (7.14)-(7.17) for small shear strains of 1%. As can be seen from the table, the tight plain weave has the highest shear modulus compared to the other fabrics. This development is expected because the tight plain weave is the most difficult fabric to shear and it was also found to produce wrinkles at a very low shear angle. The next fabric in terms of shear modulus is the basket weave, followed closely by the twill weave. The reason for the lower value of the shear modulus for the loose plain weave could be attributed to the architecture of the weave as elaborated in section 7.3. The satin weave has the lowest shear modulus of all the four weaves. The ease of slipping between fibre yarns and the small thickness of satin weave must have contributed to the ease with which it can be sheared initially during the deformation process.

Figures 7.14-7.17 present the polynomial shear curves generated from the data in Figure 7.13. These curves are computed for both the positive and negative cycles of shear deformation. For the negative cycle to be computed and to correspond to the curve of positive cycle, the odd power polynomials are considered. From the relation ($\epsilon_{12} = \tan \gamma / \cos \gamma$) it was possible to calculate the value of the shear angle $\gamma$ at a small shear strain of 1%. The value of the shear angle was computed to be about $0.8^\circ$ for
this small shear strain. It was then substituted into the equation \( G_{12} = \frac{\tau_s}{\tan \gamma \cos \gamma} \), with \( \tau_s \) determined from Figures 7.14-7.17, to obtain the shear modulus for the woven fabrics. The generated values were presented in Table 7.2.
Figure 7.14: Polynomial curve fitting of shear test data for the basket weave (LPW).

Figure 7.15: Polynomial curve fitting of shear test data for the tight plain weave (TPW).
Figure 7.16: Polynomial curve fitting of shear test data for the twill (TWILL) weave.

Figure 7.17: Polynomial curve fitting of shear test data for the satin (5HSW) weave.
7.7 RESULTS OF THE ELASTICITY ANALYSIS OF THE SHEAR DEFORMATION OF FABRICS

The shear stress, $\tau_s$, presented in Figure 7.13 was along the warp and weft fibre directions which change during the fabric shear, especially at high shear deformations. Therefore, there is a need to apply an elasticity analysis to analyse the above shear stress in an orthogonal frame of reference and determine the material properties needed for the picture-frame shear test and the draping of fabrics at all levels of shear deformation. In this section equations (4.1)-(4.22) were used for the analysis of the experimental data of Figure 7.13. The analysis has been applied before the limit of shear locking and wrinkling of fabric. According to the data in Table 7.1, the loose plain weave - LPW- did not wrinkle within the range of experimental data presented in Figure 7.13 whereas the satin - 5HSW- and twill - TWILL- weaves wrinkled around a shear angle, $\gamma$, of 25°. Therefore, the theoretical analysis was applied to almost the whole data range for the loose plain weave - LPW- (basket weave), and for the satin - 5HSW- and twill - TWILL- weaves up to $\gamma = 25^\circ$. The analysis was not applied to the tight plain weave - TPW- because it wrinkled very early at $\gamma = 10^\circ$.

The shear stress data were analysed into a shear stress component and a normal stress component in an orthogonal frame of reference, according to equations (4.19). $f_{12}(\gamma)$ and $f_{11}(\gamma)$ were calculated as defined by relations (7.5) (see equations (4.22)), from the experimental data of the shear angle corresponding to the shear stress data, and by using values for $E_{11}$, $E_{12}$ and $t_i$ determined from a linear regression fitting of the data of $\sigma_{11}$ versus $f_{11}(\gamma)$ and $\sigma_{12}$ versus $f_{12}(\gamma)$.

$$\sigma_{11} = \sigma_{22} = E_{11}(1 - \sec^2 \gamma) + t_i (\sec^2 \gamma - 1) = f_{11}(\gamma)$$

$$\sigma_{12} = \sigma_{21} = E_{12} \tan \gamma \sec \gamma + t_i (\sec^2 \gamma - 1) \sin \gamma = f_{12}(\gamma)$$

(7.5)

Figure 7.18 demonstrates the linear regression fitting of the applied data when $\sigma_{12}$ is plotted against the function $f_{12}(\gamma)$ for the loose plain weave - LPW- (basket weave),

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twill - TWILL- and satin - 5HSW - weave. It can be seen in Figure 7.18 that the fitting between the experimental data and the lines is very good.

![Graph showing fitting of experimental shear data](image)

**Figure 7.18:** Fitting of the experimental shear data according to equations (4.19) and (7.5) for the loose plain weave, the twill weave and the satin weave.

In this manner, $E_{11}$, $E_{12}$ and $t_i$ were determined. Table 7.3 presents the determined values for the moduli and the material parameter $t_i$ from the linear regression analysis of the experimental data for the three investigated fabrics. Figures 7.19-7.21 illustrate the agreement between the experimental data (equation (4.19)) and the predictions (according to equation (4.22) and the determined values in Table 7.3), when the shear stress $\sigma_{12}$ is plotted against the shear angle.

The values of the determined parameters in Table 7.3 cover a wide range of shear data up to fabric wrinkling and not just initial small shear strains covered by the Secant shear modulus at a strain of 1%. As can be seen the values of $E_{12}$ can be compared favourably with the values obtained in Table 7.2, with the closest agreement exhibited by the twill weave. These constant values of Table 7.3 can then be employed in the
Shear Deformation and Micromechanics of Woven Fabrics. Results

Elasticity analysis for each fabric for large strains before wrinkling. Figures 7.19-7.21 demonstrates non-linear behaviour of the shear stress in an orthogonal frame as a function of shear angle even before fabric wrinkling. The non-linear nature of the experimental data and the predicted curves of the shear stress, $\sigma_{12}$, versus shear angle, $\gamma$, in Figures 7.19-7.21 is due mainly to the change of the angle between the warp and weft fibre yarn directions during shear.

Table 7.3: Values for $E_{11}$, $E_{12}$ and $t_i$ determined from the regression analysis of the data of shear deformation of a loose plain weave (LPW), a twill (TWILL) and a satin (5HSW) weave.

<table>
<thead>
<tr>
<th>FABRIC</th>
<th>$E_{11}$ (kPa)</th>
<th>$E_{12}$ (kPa)</th>
<th>$t_i$ (kPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>LPW</td>
<td>13</td>
<td>20</td>
<td>0.050</td>
</tr>
<tr>
<td>TWILL</td>
<td>40</td>
<td>35</td>
<td>18</td>
</tr>
<tr>
<td>5HSW</td>
<td>27</td>
<td>16</td>
<td>16</td>
</tr>
</tbody>
</table>

Figure 7.19: Experimental data and predictions of the shear stress (in-plane, orthogonal directions) versus shear angle for the loose plain weave.
Figure 7.20: Experimental data and predictions of the shear stress (in-plane, orthogonal directions) versus shear angle for the twill weave.

Figure 7.21: Experimental data and predictions of the shear stress (in-plane, orthogonal directions) versus shear angle for the satin weave.

Experimental Studies and Mathematical Modelling of the Draping and Shear Deformation of Woven Fabrics in Resin Transfer Moulding (RTM)
7.8 MICROSTRUCTURAL ANALYSIS OF THE SHEAR LOCKING EFFECT

The next step after investigating shear deformation of fabrics in terms of mechanical testing is to study the wrinkling phenomenon. Wrinkling is associated with the level of in-plane compressive stresses, which develop during the shearing of fabrics, the thickness of fabric, its stiffness and its microstructure. External tensile stresses during draping, as a result of manual smoothing of fabric for example, counteract the compressive stresses and reduce wrinkling.

At microstructural level, wrinkling of a fabric during shear is associated with the maximum locking shear. Once shear locking is reached, the rise of shear stress with shear angle is expected to be sharp and highly non-linear in comparison with Figures 7.19-7.21. Figure 7.22 illustrates the microstructural changes during shearing of the tight plain weave, the twill weave and the 5-harness satin weave when each of these fabrics is undergoing shear from the unsheared state to a maximum shear angle. The aim of this section is to devise a procedure for the prediction of the locking shear angle. At lock, it is considered that there are no macro-pores between bundles and the structure is compressed (in-plane) to the maximum fibre fraction, sometimes also involving compression of the bundles.

In plain weaves, it is assumed that the maximum shear angle at shear locking is associated with the disappearance of the macropores between fibre yarns. This is consistent with the analysis of the shear locking effect in plain weaves by Prodromou and Chen (1997) although they defined and considered slightly different geometric parameters from the parameters of this study.
Figure 7.22: Microstructural changes in the woven fabrics during shear for the (a) unsheared (b) sheared states.
Figure 7.23(a) represents the basic cell in a plain weave. Parameters to be considered in this study include the length \( h_p \), the dimensions of the original macropore, \( w_p \) and \( t_p \), and the yarn width, \( W \). Considering that there is no in-plane compression of fibre yarns during the shear deformation, the shear angle for a given fabric can be expressed as:

\[
\gamma = \frac{\pi}{2} - \alpha
\]  

(7.6)

where, according to Figure 7.23(a)

\[
\alpha = \alpha_1 + \alpha_2
\]  

(7.7)

\[
\sin \alpha_1 = \frac{h_p}{t_p}
\]  

(7.7)

\[
\sin \alpha_2 = \frac{h_p}{w_p}
\]  

such that:

\[
\alpha_1 = \arcsin \left( \frac{h_p}{w_p} \right)
\]  

(7.8)

\[
\alpha_2 = \arcsin \left( \frac{h_p}{t_p} \right)
\]  

From equations (7.6)-(7.8),

\[
\gamma = \frac{\pi}{2} - \arcsin \left( \frac{h_p}{w_p} \right) - \arcsin \left( \frac{h_p}{t_p} \right)
\]  

(7.9)

The aim is to minimise \( h_p \) at the maximum shear-locking angle.
Figure 7.23: (a) basic cell in a plain weave as considered in the microstructural analysis of shear deformation and (b) unit cell in the unsheared and sheared state as considered in the analysis of the shear locking effect for the satin and twill weaves.
Regarding the maximum shear angle at wrinkling for the plain weaves in Table 7.2, it was observed that wrinkling in the picture-frame test appeared before the macropores closed something that was particularly obvious in the case of tight plain weave. This fabric was particularly stiff to in-plane shear due to strong “anchoring” of fibre yarns at the crossover points. For the plain weaves shear locking at microstructural level occurred in the manual shear test in which the fabric was also stretched during manual smoothing. Since only the loose plain weave and the tight plain weave contain macropore spaces between the fibre yarns, equation (7.9) was applied in the shear locking angle predictions only for these two weaves.

### Table 7.4: Microstructural parameters used for the prediction of shear locking angle for the LPW and the TPW.

<table>
<thead>
<tr>
<th>FABRIC</th>
<th>h_p (mm)</th>
<th>W (mm)</th>
<th>t_p (mm)</th>
<th>w_p (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>LPW</td>
<td>0.404</td>
<td>5.05</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>TPW</td>
<td>0.100</td>
<td>1.25</td>
<td>0.2</td>
<td>0.28</td>
</tr>
</tbody>
</table>

Microstructural data were obtained from Figure 7.22 for the tight plain weave and from direct visual observation and measurements for the loose plain weave and tabulated in Table 7.4. These parameters were then substituted into equation (7.9) so that the maximum shear-locking angle for both tested plain weaves can be predicted as:

(a) Loose plain weave: \( \gamma = \frac{\pi}{2} - \arcsin(0.404/3) - \arcsin(0.404/2) \)

\[ \begin{align*}
&= 90 - 7.74 - 11.65 \\
&= 70.6^\circ
\end{align*} \]

(b) Tight plain weave: \( \gamma = \frac{\pi}{2} - \arcsin(0.1/0.28) - \arcsin(0.1/0.2) \)

\[ \begin{align*}
&= 90 - 20.9 - 30 \\
&= 39.1^\circ
\end{align*} \]

The \( h_p \) values in Table 7.4 and the above calculations correspond to the shear locking point, where it was found that \( h_p/W = 0.08 \) for both LPW and TPW for the predicted shear locking angles to agree with the experimental data.
These two values are consistent with the values obtained during the manual shear test with smoothing for the loose and tight plain weaves presented in Table 7.1.

The 5-harness satin and the twill weaves seemed to shear easily and it was considered that wrinkling in the picture-frame test was associated with shear locking at microstructural level. If \( w_c \) and \( t_c \) are the dimensions of the unsheared unit cell of the satin weave, then \( w_c \) and \( g \) are the new dimensions of unit cell due mainly to the compression of the fibre yarns during shear (see Figure 7.23(b)). The relation between the shear angle and the change in dimensions of the sheared unit cell was found to be:

\[
\tan \gamma = \frac{f}{g} \tag{7.10}
\]

where,

\[
g = t_c \cdot CR \tag{7.11}
\]

and CR is the in-plane compression ratio.

Finally, it was found empirically that

\[
f = \frac{V_{f_0} \cdot w_c}{V_{f_{\text{max}}} \cdot CR} - w_c \tag{7.12}
\]

where, \( V_{f_0} \) and \( V_{f_{\text{max}}} \) are the initial (unsheared state) and maximum packing volume fractions, respectively, and CR is the maximum compression ratio of the fibre yarn.
Table 7.5: Microstructural parameters for the prediction of shear locking angle for the twill weave and the satin-5HSW.

<table>
<thead>
<tr>
<th>FABRIC</th>
<th>( w_c \text{ (mm)} )</th>
<th>( t_c \text{ (mm)} )</th>
<th>( g \text{ (mm)} )</th>
<th>CR</th>
<th>( V_{f_0} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>TWILL</td>
<td>2.6</td>
<td>2.6</td>
<td>1.5</td>
<td>0.58</td>
<td>0.46</td>
</tr>
<tr>
<td>5HSW</td>
<td>4.4</td>
<td>2.85</td>
<td>1.9</td>
<td>0.64</td>
<td>0.50</td>
</tr>
</tbody>
</table>

In Table 7.5, microstructural parameters obtained from Figure 7.22 are tabulated for the twill and the satin weave. These are the data that are to be used in the prediction of the shear locking angles for these fabrics. By assuming that the maximum packing fibre volume fraction \( V_{f_{\text{max}}} \) in the locked shear state would be 0.65 for the satin weave and 0.63 for the twill weave and substituting the data from Table 7.5 into equations (7.10)-(7.12), the following predicted shear locking angles can be obtained:

(a) 5HSW: \( \gamma = 25^\circ \)

This agrees with the experimental data of Table 7.1 for 5HSW.

If the same relations were applied to the twill weave, the locking shear angle would be predicted as:

(b) TWILL: \( \gamma = 24^\circ \)

Again, this agrees with the experimental data of Table 7.1 for the twill weave.

At this point, it must be mentioned that the CR values in Table 7.5 and the above predictions correspond to the shear locking point so that the predicted shear locking angles fit the experimental data.
8. FINITE ELEMENT SIMULATIONS OF THE DRAPING OF FABRICS
8.1 INTRODUCTION

Computer simulations of the draping/forming of fabrics in a mould were carried out by using the solid mechanics approach and the finite element analysis (FEA), as is described in chapter 4. In this approach the fabric was represented by a thin solid body with a numerical finite element mesh. The FEA simulations of the draping of fabrics were performed by using LUSAS 12 software package supplied by FEA Ltd. A quasi-static finite element analysis was applied in which the load was increased in a step-wise manner. The simulations covered two mould geometries: (a) a single curvature geometry which was a half-cylindrical shell and (b) a double curvature geometry which was a hemispherical mould surrounded by a flat rim, as in the draping experiments of Chapters 5 and 6. Parametric studies included the effects of the thickness of the fabric, mechanical properties and mesh size.

8.2 FINITE ELEMENT MODEL MESH AND INPUT DATA: SINGLE CURVATURE DRAPING

For the single curvature type of drape analysis, a simple semi-cylindrical component was chosen. Figure 8.1 shows the finite element model constructed using the MYSTRO part of the FEA package. The semi-cylindrical solid mould surface has a length of 30 mm and a diameter of 80 mm. It was meshed using 8-noded quadrilateral (QSM8) shell elements. The numerical mesh of the mould surface consisted of a regular grid of 10x4 elements with a total of 149 nodes.

The fabric, with the dimensions of 80 mm length and 30 mm width, was modelled as a thin, flat, solid sheet. The thickness of the sheet was chosen to represent the thickness of a particular fabric. A thickness of 0.28 mm was used as input datum for the simulation results presented in Figure 8.2. The fabric sheet was meshed numerically using a regular grid of 10x4 hexahedral (HX20), orthotropic solid elements, which produce a total of 965 nodes. The interfacial interactions between the fabric and the solid mould were modelled by considering that a very soft material occupied the volume between the fabric and the mould.

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Figure 8.1: (a) FEA model for the draping of fabrics over a single curvature cylindrical geometry and (b) model of the fabric which was meshed with 10x4 hexahedral elements
This volume was meshed using 10x4 isotropic, hexahedral (HX20), solid elements. Fixed node boundary conditions were applied to all the nodes of the solid mould surface (hemi-cylindrical surface), so that the mould surface could not be translated nor rotated during the FEA simulation of fabric draping. A homogeneously distributed vertical load was applied on the top surface of the fabric to cause fabric deformation, so that the fabric is formed onto the mould surface. The purpose of this type of loading condition was to simulate a matched die type of forming operation where the fabric deforms under the weight of a top mould part, matching in shape the mould bottom part. As a first approximation, orthotropic, elastic mechanical properties were given to the fabric with the following values:

**Table 8.1: Data of the mechanical properties used for the single curvature drape model [Hull (1992) pp. 91 and Yu et al. (1994)]**

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young’s modulus</td>
<td></td>
</tr>
<tr>
<td>$E_{xx}(\text{N/mm}^2)$</td>
<td>$76 \times 10^3$</td>
</tr>
<tr>
<td>$E_{yy}(\text{N/mm}^2)$</td>
<td>$76 \times 10^3$</td>
</tr>
<tr>
<td>$E_{zz}(\text{N/mm}^2)$</td>
<td>$76 \times 10^3$</td>
</tr>
<tr>
<td>Shear modulus</td>
<td></td>
</tr>
<tr>
<td>$G_{xy}(\text{N/mm}^2)$</td>
<td>$2.1 \times 10^4$</td>
</tr>
<tr>
<td>$G_{yz}(\text{N/mm}^2)$</td>
<td>$2.1 \times 10^4$</td>
</tr>
<tr>
<td>$G_{zx}(\text{N/mm}^2)$</td>
<td>$1.04 \times 10^2$</td>
</tr>
</tbody>
</table>

It must be noted that the $(x, y, z)$ system of coordinates in Chapter 8 corresponds to the $(x, z, y)$ system of coordinates in Chapter 4.

The interfacial interactions between the fabric and the semi-cylindrical mould surface were modelled by considering the volume between the fabric and the mould surface as a soft isotropic solid of modulus 0.1 N mm$^{-2}$.

### 8.2.1 FINITE ELEMENT SIMULATIONS

In this section the results of three sequential quasi-static FEA simulations are presented where the uniformly distributed vertical load on the fabric is increased in three steps. Figure 8.2(a–d) illustrates the deformed fabric sheet at successive steps. Step-wise increase of load is a useful methodology employed in finite element quasi-static simulations of large deformations. This methodology is analogous to the numerical procedure adopted in geometrically non-linear deformation analyses. An obvious benefit arising from the display of Figure 8.2 is that the draping process can
Figure 8.2: Step-wise load increase FEA simulations of the draping of a fabric over a semi-cylindrical mould surface.
be monitored at different stages on the computer and identify the onset of potential problems.

Bending of the fabric sheet seems to be the main form of deformation in the draping of a fabric sheet over single curvature mould geometry. A total (sum of all load steps), uniformly distributed vertical load of 0.127 N per node was used to achieve the final draped fabric shape presented in Figure 8.2.

8.3 FINITE ELEMENT MODEL MESH AND INPUT DATA: DOUBLE CURVATURE MOULD GEOMETRY

In this analysis, a hemispherical mould surface was chosen as an ideal shape to simulate the drapeability of woven fabrics on a double curved geometry. The simulation was performed using the continuum solid mechanics approach used in the draping simulations regarding the single curvature mould geometry. Also, in these simulations the FEA computer code-LUSAS version 12.1 supplied by FEA Ltd- was used for the numerical analysis.

Figure 8.3 presents the finite element model including a female mould and the fabric at the top. The mould, which is of a hat shape, consists of a hemispherical part at the centre, surrounded by a flat rim (or flange). The spherical portion of the mould has a radius of 98 mm and a curvature radius of 3 mm at the dome/flat rim transition. The fabric is a square piece of 320 x 320 mm and a thickness of 0.23 mm (same thickness as for the five-harness satin fabric, 5HSW, see Table 3.1).

The mould surface was meshed using a regular grid of 8-noded quadrilateral shell elements (QSM8). The fabric was meshed numerically using a 20x20 regular grid of hexahedral (HX20) solid elements. The interfacial space between the fabric and the mould was meshed also using the hexahedral (HX20) solid elements. This resulted in a grid fabric in a standard FEA simulation of draping with 3003 nodes.
Finite Element Simulations of the Draping of Fabrics

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Fixed node boundary conditions were applied to all nodes of the solid mould surface, so that the mould could not be translated nor rotated during the FEA simulation of fabric draping. The forming process was carried out by applying a homogeneously distributed vertical load on the top surface of the fabric within a central circular region covering the mould. This load was meant to simulate the action of the punch in a matched die type of forming operation. In this analysis, the fabric was considered as an elastic, orthotropic, solid sheet with the values of mechanical properties as given in Table 8.1.

Shear deformation of the fabric sheet is the dominant mechanism of deformation during the draping of orthogonal woven fabrics over double curvature mould geometry. Hence, the in-plane shear modulus is the most important property to be considered. This study aims at investigating the validity of LUSAS FEA package for draping simulations. As a first approximation, linear elastic properties were assigned to the fabric sheet. The assigned in-plane shear modulus value in the standard simulations was very close to that of the 5 harness satin weave and quite similar to that of the loose plain weave (basket weave) verified from the initial part of the experimental load deformation curves in Chapter 7. Parametric studies were also carried out to study the effect of the change in the in-plane shear modulus on the drapeability of the fabric (see section 8.3.2.2).

The interfacial interactions between the fabric and the mould were modelled by considering the interfacial volume to be a soft isotropic solid with a modulus of 0.1N mm$^{-2}$.

### 8.3.1 FINITE ELEMENT SIMULATIONS AND PARAMETRIC STUDIES

The draping was simulated using a quasi-static type of FEA analysis in which the uniformly distributed vertical load on the fabric was increased in three steps. In each step, a static linear finite element analysis was carried based on the continuum mechanics approach using theoretical equations of section 4.3. The simulation time obviously depends on the computer power and capacity. Such that with a 16MB
Experimental Studies and Mathematical Modelling of the Draping and Shear Deformation of Woven Fabrics in Resin Transfer Moulding (RTM)
(RAM) Intel Pentium microprocessor computer a static linear finite element analysis was completed in about 4500 CPU. (Three steps were taken for each load increment). Figure 8.4 presents the fabric deformation at the different steps of load increment. A total (sum of all load-steps) homogeneously distributed, vertical total vertical load of 0.0125 N per node was applied to achieve the draped shape in figure 8.4(d). It can be observed in figure 8.4 that the draping deformation takes place by rotation of gridlines which account for the in-plane shearing of the draping fabric sheet. High shear occurs along the diagonal line of the fabric as one moves away from the central region of the hemispherical cavity. The sides of the fabric sheet curve in the middle during the draping in the hemispherical mould. No wrinkling has occurred in this draping simulation.

Stepwise FEA simulations are useful because changes in the shape of the fabric, as it passes through the various stages of the deformation process, can also be observed as they develop and their progress can be studied. From this, the origin of certain possible undesirable features, such as wrinkles, folds and some other abnormalities in the draped shape, could be easily traced.

The second stage of FEA simulations of draping over a double curvature mould geometry involved a series of parametric studies which investigated the effects of changes in the values of fabric thickness (geometrical parameter), mechanical properties of fabric and size of the numerical mesh (numerical parameter). This is essential to determine first of all the extent to which the numerical model is capable of predicting fabric drape for a wide range of values of mechanical properties. Secondly, the influence of material properties can be examined on the draping of different types of weaves. Change of numerical parameters brings numerical optimisation of the FEA simulations. In the process of parametric studies it was also possible to compare some predictions with experimental data.
8.3.2.1 EFFECT OF FABRIC THICKNESS ON DRAPE

In this study, the thickness of the fabric was varied from 0.28 mm to 0.58 mm and the applied uniformly distributed, vertical load was kept the same. The aim of this is to investigate the effect of the change in thickness on the shape of the deformed fabric and also on the amount of deformation achieved by draped fabric. According to observed results it was revealed that the fabric with the smaller thickness deformed more than the fabric with the larger thickness. However, the shape of the deformed fabric was found not to be affected by the change in the thickness if the load was increased so that the thick fabric would conform into the mould. The reason for this could be related to the influence of the thickness on the bending stiffness of the fabric, where according to Daniel and Ishai (1994), the bending stiffness for plates or shells is given by

\[ B_{ij} = \frac{h^3}{12} Q_{ij} \]  

(8.1)

where, \( Q_{ij} \) are components of the reduced stiffness matrix expressed in terms of the elastic properties (see equations (6.10)), \( i, j = 1, 2 \) and 6, and \( h \) is the thickness of the fabric sheet.

Equation (8.1) shows that the bending stiffness for plates or shells is proportional to the third power of thickness. This means that as the thickness of the fabric is increased the bending stiffness also increases and the fabric will find it difficult to drape.

8.3.2.2 EFFECT OF YOUNG’S MODULUS ON DRAPE

In this analysis the values of other parameters such as thickness, in-plane shear modulus and Poisson's ratio were kept constant. The values of the Young's moduli in Table 8.1 (\( E_{xx} = E_{yy} = E_{zz} \)), were varied from 76x10^3 to 1 N mm^-2. The result of this leads to a reduction in the value of the bending stiffness seen in equation (8.1).
Based on data from the drape simulations the fabric with the lower stiffness was found to deform more readily than the one with the higher stiffness. This shows that the Young’s moduli of the fabric have similar effects on the drapeability of the fabric as that of the thickness.

### 8.3.2.3 EFFECT OF SHEAR MODULUS ON DRAPE

The shear moduli investigated in these simulations are $G_{xy}$, $G_{yz}$, and $G_{zx}$ in the three orthogonal directions. The simulations were conducted by keeping all the other numerical parameters constant and varying the shear modulus $G_{xy}$, $G_{yz}$, or $G_{zx}$. In the first simulation, the shear modulus had the same value of $21 \times 10^3$ N mm$^{-2}$ in all three directions. Figure 8.5(a) reveals that the fabric deformed in the usual manner to produce the hemispherical shape at the central region of the fabric, but beyond the rim of the hemisphere, the flat tail end observed in Figure 8.4(d) was not formed.

The values of shear moduli $G_{xy}$ and $G_{yz}$ were then kept at $21 \times 10^3$ N mm$^{-2}$ and the value of shear modulus $G_{zx}$ at $1 \times 10^2$ N mm$^{-2}$. This is the standard FEA draping simulation presented in Figure 8.4. When the draped fabric is examined, it can be noticed that the region beyond the rim deforms to produce the tail end and the flatness of the fabric in this section has improved.

Similar simulations were carried out subsequently by reducing the value of the transverse shear modulus $G_{xy}$ and $G_{yz}$ in the range of $21 \times 10^3 - 1$ N mm$^{-2}$ while keeping the in-plane shear modulus $G_{zx}$ at $1 \times 10^2$ N mm$^{-2}$. Each time the modulus was reduced it was noticed that the flatness of the fabric around the mould rim increased (see Figure 8.5(b)).
Finite Element Simulations of the Draping of Fabrics

Figure 8.5: Predicted shape of the draped fabric by FEA when (a) the shear moduli \( G_{xy}, G_{yz} \) and \( G_{xz} \) are \( 21 \times 10^4 \) and (b) \( G_{xy} = G_{yz} = 1 \text{N/mm}^2 \) and \( G_{xz} = 1.04 \times 10^2 \text{N/mm}^2 \)
Figure 8.6 presents the values of shear angles along the diagonal direction of the draped fabric, as obtained from the FEA simulations, as a function of the normalised arc length $L/S$. For the values of $G_{zx}$ of $1.04 \times 10^{-2}$, 0.1 and 1000 N mm$^{-2}$ the values of the shear angles were almost the same as shown by the coinciding curves in Figure 8.6. However, when the value of the shear modulus $G_{zx}$ was changed to $21 \times 10^3$ N mm$^{-2}$ there was a sharp drop in the shear angle curve especially near the rim where $L/S = 0.95$. These results show that the drapeability of woven fabrics is not sensitive to the in-plane shear modulus for relatively low values of the in-plane shear modulus, but high values of the in-plane shear modulus lower significantly the drapeability of woven fabrics.

It is also noticed in Figure 8.6 that the predicted values of shear angles are lower than the experimental data, although the general shape of the curve is similar. One of the reasons could be that the modulus of the interfacial volume between the fabric and the mould restrained the deformation of the fabric. It was attempted to lower the modulus of the interfacial volume to 0.01 N mm$^{-2}$ but, unfortunately, the FEA simulation with
the LUSAS software package crashed. The value of 0.1 N mm\(^{-2}\) was the lowest value of the modulus of the interfacial volume for which LUSAS was able to run.

### 8.3.2.4 EFFECT OF MESH SIZE ON DRAPE

In Figure 8.7 the effect of mesh size on the FEA predictions of the draping of woven fabrics is examined. Two mesh sizes of 20x20 and 40x40 elements were chosen for this investigation. According to the curves of the values of shear angles along the diagonal direction of the draped fabric against the normalised arc length (L/S), there is no significant difference between the mesh sizes of 20x20 and 40x40 elements in affecting the amount of predicted fabric deformation.

When we attempted to increase the size of the mesh to 50x50 and 100x100 elements, the simulations crashed due to insufficient memory of the computer.

Figure 8.7: predicted shear angle curves along the diagonal direction of the fabric against the normalised arc length for two mesh sizes.
9. CONCLUSIONS AND RECOMMENDATIONS FOR FURTHER WORK
Conclusions and Recommendations for Further Work

9.1 CONCLUSIONS

This study focused on experimental investigations and mathematical modelling of the drapeability, draping and shear deformation of woven fabrics in resin transfer moulding (RTM). The investigation was channelled toward three main areas, namely:

(a) Experimental studies of the drapeability and fabric characteristics after draping of four woven fabrics, i.e. a loose plain weave (basket weave), a tight plain weave (TPW), a twill weave (TWILL) and a satin (5HSW) weave.

(b) Shear test experiments and mathematical modelling of the shear deformation behaviour of the four woven fabrics mentioned in (a), including the shear locking angle which indicates wrinkle formation in these weaves.

(c) Computer simulation of the draping of woven fabrics using continuum solid mechanics approach which was incorporated in the LUSAS finite element analysis (FEA) software.

On the experimental part of the investigations, a double curvature hemispherical hat geometry was chosen to test the drapeability of a loose plain weave, a tight plain weave, a twill weave and a 5-harness satin weave. The experiments were carried out in the dry state i.e. without the fabric being wetted with resin. This was designed to simulate the preforming operation in resin transfer moulding (RTM).

In all, it was found out that the four weaves could be draped over the mould surface without any major inherent difficulties. However, in terms of wrinkle appearance it was found out that the tight plain weave produced the highest extent of wrinkling, while the loose plain weave could be draped without the appearance of wrinkles. The twill and the satin weaves produced wrinkles but not as prominent as the tight plain weave. A wrinkle measure was proposed to estimate the extent of wrinkling formed on each woven fabric draped on the hemispherical hat. It was found that the tight plain weave has the largest total wrinkle area, which was followed by the satin and the twill weave. However, by considering the thickness of the fabric it was seen that even a small amount of wrinkling might have a significant effect on a thin fabric. Shear deformation of fabric by rotation of the warp and weft yarns was prominent in the draping of all woven fabrics. From observations of the grid initially inscribed on each
it was concluded that some fibre sliding, at maximum for one yarn width (which was a considerable width size in the case of basket weave), was present in the draping of the loose plain weave (basket weave).

Variations in the extent of wrinkle formation were associated with differences in the shear behaviour of the fabrics during draping which was thought to depend on the structural characteristics of the fabric, sizes of macropores between yarns and the yarn sliding properties. It was observed that a smoothing process during the drape forming of fabrics (for example by closing the mould or manual smoothing of fabrics) resulted in reduced wrinkling and higher shear locking angles. This was thought to be due to the fact that the “smoothing forces” contribute to overcome the “anchoring forces” in the rotation of fibre yarns at the yarn crossover points and help the weave to achieve maximum shear and in-plane shrinkage of its macropores.

After draping a square piece from each fabric on the hemispherical hat mould, the draped fabric boundaries were compared between fabrics and the local deformation angles of the inscribed grid were measured for each draped fabric. In general, similar trends were followed in the draping of all fabrics including a star shaped draped fabric boundary and a maximum shear angle at a location where the hemispherical dome meets the flat rim surface, along the diagonal line of the fabric piece. These general trends would have been also predicted by algorithms based on the “fishnet” approach (see section 2.2.1). However, it was also concluded that differences in the draping of the various woven fabrics existed regarding the extent of wrinkle formation and the maps of the local deformation angles between fibre yarns or the individual patterns of contours of the local deformation angles. These differences in the deformation angles between different fabrics could reach 15°. Such differences in local fibre orientation and corresponding local fibre volume fraction would result in considerable differences in the permeability (Heardman et al (1999)) and mechanical properties of fibre reinforcement. So, the conclusion was that a mechanical approach would be preferable in the modelling and simulation of fabric draping, if accurate predictions of deformation angles and other characteristics of the draped fabric are required.
The values of local fibre volume fractions of the draped fabrics were calculated from the data of the shear angles along the diagonal line of the draped fabrics. Fibre volume fraction and fibre orientation distributions become important when considering local changes in the permeability of the draped fabric during the RTM process and also changes in the mechanical properties of the fabricated component. It was ascertained from the results that changes in the local fibre volume fractions of the draped fabric occurred from the apex of the dome to the rim of the dome. At the apex of the mould dome, where the fabric was not deformed, the volume fraction of the fibres remained unchanged, but after the apex of the dome the fibre volume fraction increased and reached a peak at the rim of the dome where shear deformation was maximum.

Likewise, calculations of the modulus distribution of draped fabrics, as part of the resulting composite product on the basis of changes in fibre orientation and fibre volume fraction (where the calculations were carried out for the basket weave which did not exhibit any wrinkling), showed local changes in the values of the components of the reduced stiffness matrix \( Q \) in the principal directions of the \( 0^\circ \) ply. More specifically, the \( Q_{11} \) component increased from 15 GPa at the apex of the dome to a maximum of about 35 GPa at the boundary between the dome and the flat rim, along the diagonal direction of the draped fabric.

Picture-frame shear tests were carried out in a tensile mechanical testing machine for each fabric. From the profiles of the curves of shear load versus shear angle for varying crosshead speeds it was shown that the weave structure has the major influence on the shear deformation behaviour of woven fabrics in comparison with the crosshead speed. Based on a polynomial curve fitting of the experimental data and regression analysis, the values of the Secant shear modulus of the fabrics at a strain value of 1% were different for the various fabrics: the 5 harness satin weave achieved the lowest shear modulus value of 6.6 kPa and the tight plain weave had the highest shear modulus value of 148 kPa. This is consistent with the weave architecture, which is very tight and rigid for the tight plain weave and loose and flexible for the satin weave.
By using the elasticity model for high shear strains, described in section 4.2, to analyse the shear deformation of the loose plain weave (basket weave), the twill (TWILL) and satin (5HSW) weave for the data range of the shear deformation curves before the wrinkling point, non-linear elastic shear deformation was predicted before wrinkling, due to the rotation of fibre yarns during shear, so that the angle between the warp and weft yarns becomes increasingly less than 90° whereas the components of the stress tensor have to be in orthogonal directions. Very good agreement was observed between the model predictions and the experimental data whereas model parameters were fitted from curve fitting. The model parameters included a shear modulus the value of which was calculated as 16 kPa for the satin weave, 20 kPa for the basket weave and 35 kPa for the twill weave.

In the mechanical shear test experiments for the woven fabrics it was shown that wrinkles appeared after a maximum shear angle, otherwise called the “shear locking angle”. It was found that the tight plain weave produced the lowest shear-locking angle of 10°. The twill weave and the satin weave produced the next lower shear locking angles around 25°. The highest shear locking-angle of 61° was produced by the basket weave. Similar values were also obtained when the experiment was performed manually. One important observation made was that manual shearing with smoothing delayed the appearance of wrinkles to higher shear locking angles. This observation is very important for the draping process in the fabric preforming stage of RTM.

After microstructural analysis of the shear locking effect, two microstructural models were developed to device a procedure for predicting the shear locking angles of woven fabrics. The loose plain weave and the tight plain weave presented shear locking in shearing with smoothing when the macropores between the fibre yarns closed. The model could estimate with a very good accuracy the values of shear locking-angles values obtained from the experiments. Another model was developed for the twill and satin weaves, which incorporates parameters such as the maximum fibre volume fraction and the in-plane compression ratio of the fabric when comparing the sheared and unsheared states. Fibre yarn compression was considered for the twill and satin weaves. The values of the shear locking-angles calculated using
the two models showed very close fitting between predictions and experiments for all four weaves, provided that appropriate values were given to selected parameters ($h_p$ for plain weaves and CR for the twill and satin weaves).

Finite element analysis of the draping of fabrics was based on the continuum solid mechanics approach. In order to validate the computer simulations, preliminary studies were carried out where the fabric was considered as a linear elastic orthotropic solid. So, obviously the non-linear shear deformation curves derived in chapter 7 were not incorporated in the constitutive model of the finite element analysis of draping and, hence, no wrinkling predictions were expected at this stage. Low values of the shear modulus, comparable to the experimental values of the shear modulus of the satin weave, were used as input data for the simulations. A similar fabric deformation trend was found in the draping predictions as in the experimental results of draping over the hemispherical hat mould, with the maximum shear angle predicted at the boundary between the hemispherical dome and the flat rim, along the diagonal direction of the fabric. However, the shear angles achieved after draping were under predicted in comparison with the experimental data. This was thought to be due to the assumption of an interfacial volume between the fabric and the mould of low modulus of 0.1 MPa. This looked as if it restricted the deformation of low shear modulus fabrics. It was not possible to run the LUSAS software package for lower input values of the modulus of the interfacial volume, because the run crushed. It was not possible to incorporate the sliding surface boundary condition (friction properties) in the LUSAS software package with the solid mesh elements used in this finite element simulation.

However, it was thought that the continuum solid mechanics approach has still a good potential in simulations of fabric draping if an appropriate software package is used to incorporate suitable friction properties between the fabric and the mould. Results of the parametric studies of draping of the LUSAS finite element analysis showed that the fabric thickness has a considerable influence on the amount of deformation that fabrics can attain when draped over the hemispherical hat mould under a certain load. This finding was also confirmed by a theoretical analysis using the plate bending stiffness. Furthermore, the out of plane shear stiffness of the fabric has been found to change the shape of the draped fabric for a certain load by changing the flatness of draped fabric on the flat section of the mould. The shear angle distribution of the
draped fabric was found to be not so sensitive to the in-plane shear modulus for relatively low values of the in-plane shear modulus, but high values of the in-plane shear modulus lowered significantly the drapeability of woven fabric. This is consistent with the findings of Dong et al (1999) who carried out parametric finite element studies of the draping of woven fabrics following the continuum solid mechanics approach and taking into account the friction properties between the fabric and the mould.

9.2 RECOMMENDATIONS FOR FUTURE WORK

Based on the findings of this study, recommendations for future work focus on the following areas:

Experimental drape analysis should be carried out on fabricated composite components of the four woven fabrics to determine the effect of the resin on the shear angle, the formation of wrinkles and on the mechanical properties of the final composite product.

Secondly, drape experimental analysis, carried out in this study on the four woven fabrics, can be similarly performed on other geometric architectures of fibre reinforcements.

Finally, many layers of each of the woven fabrics can be employed in the drape experiment instead of the one layer square shaped fabric used in this study.

In the case of shear tests it is recommended that the same experiments should be conducted on resin impregnated woven fabrics. This is to provide data to allow modelling of the forming of prepregs. Following this, there is also the desirability to develop microstructural models for predicting the shear locking angle, which incorporate the effect of the impregnating resin.
On the computer simulations of the draping of fabrics using the continuum mechanics approach, a different finite element computer package could be used where it will be feasible to incorporate friction properties between the fabric and the mould. Lastly, the mechanical model for the dry fabrics used in the simulation analysis of this study can be changed to a different model or different values of mechanical properties more suitable for a resin impregnated fabric to simulate forming of prepregs.
10. REFERENCES

Experimental Studies and Mathematical Modelling of the Draping and Shear Deformation of Woven Fabrics in Resin transfer Moulding (RTM)


References


Experimental Studies and Mathematical Modelling of the Draping and Shear Deformation of Woven Fabrics in Resin transfer Moulding (RTM)


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R. E. Robertson, "Continuous fibre rearrangement during the moulding of fibre composites. 11: flat cloth to a rounded cone", Polymer Composites 5(3), 1984, pp. 191-197.


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D. Standley, "Characterising the hot drape forming process and the effect of fibre shearing on the mechanical properties of highly draped composites components", Proceedings of ICCM-11, July 14th-18th, 1997, pp. 559-568, Gold Coast Australia, vol IV.


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APPENDICES

Experimental Studies and Mathematical Modelling of the Draping and Shear Deformation of Woven Fabrics in Resin transfer Moulding (RTM)
Appendix 1: Fortran computer programme used to generate the shear angle data of draped fabrics

PROGRAM ANGLE
C .... THIS IS A FORTRAN PROGRAM
C .... WRITTEN BY U. Mohammed
C .... FOR THE DETERMINATION OF ANGLE BETWEEN
C .... SETS OF THREE POINTS ON A MESH
DIMENSION X(4000), Y(4000), Z(4000), ANGLE(4000)
integer j, m, n, nd
WRITE(*, *) 'GIVE NUMBER OF REQUIRED ANGLES'
READ(*, *) M
C .... M=NO OF REQUIRED ANGLES
WRITE(*, *) 'GIVE NUMBER OF POINTS OF UNDEFORMED MESH'
READ(*, *) N
C .... N=NO OF POINTS OF UNDERDEFORMED MESH
WRITE(*, *) 'GIVE NUMBER OF POINTS OF DEFORMED MESH'
READ(*, *) ND
C .... ND=NO OF POINTS OF DEFORMED MESH
C
C .... UNIT 9=UNDEFORMED COORDINATES
C .... UNIT 8=DEFORMATIONS
C .... UNIT 7=POINTS ASSOCIATED WITH EACH ANGLE
C .... UNIT 6=RESULTS OF ANGLES
OPEN(UNIT =9, FILE= 'c:\umar\dat1.txt')
OPEN(UNIT= 8, FILE= 'c:\umar\umar2Ob.txt')
OPEN(UNIT= 7, FILE= 'c:\umar\dat3.txt')
OPEN(UNIT= 6, FILE= 'c:\umar\resin1.txt')
C
C
C .... CALCULATE COORDINATES OF DEFORMED MESH
C write(*,*) 'i j x(j) y(j) z(j)'
DO 10 I=1,N
READ(9,*) J, x(j), y(j), z(j)
10 CONTINUE
DO 20 I=1,ND
READ(8,*) J,DX,DY,DZ

C write(*, *) J,DX,DY,DZ
X(J)=X(J)+DX
Y(J)=Y(J)+DY
Z(J)=Z(J)+DZ
C write(*, *)x(j), y(j), z(j)
20 CONTINUE
C
C
C......CALCULATE ANGLES
C......J=POINT OF ANGLE, J1=MIDPOINT OF FIRST SIDE OF ANGLE
C......J2=MIDPOINT OF SECOND SIDE OF ANGLE
C......WRITE(5,*) J J1 J2 ANGLE(J) DEFANGLE(J)
DO 50 I=1,M
READ(7,*) J,J1,J2
XA=(X(J1)-X(J))
XB=(X(J2)-X(J))
YA=(Y(J1)-Y(J))
YB=(Y(J2)-Y(J))
ZA=(Z(J1)-Z(J))
ZB=(Z(J2)-Z(J))
A=(XA*XA+YA*YA+ZA*ZA)
A=SQRT(A)
B=(XB*XB+YB*YB+ZB*ZB)
B=SQRT(B)
C......write(*,*)m, j, j1, j2,x(j1), x(j), x(j2)
AA=(XA*XB+YA*YB+ZA*ZB)/(A*B)
ANGLE(J)=ACOS(AA)
ANGLE(J)=90*ANGLE(J)/1.57
DEFANGLE=90-ANGLE(J)
WRITE(*, *) J,J1,J2,ANGLE(J), DEFANGLE

Experimental Studies and Mathematical Modelling of the Draping and Shear Deformation of Woven Fabrics in Resin transfer Moulding (RTM)
50 CONTINUE
CLOSE(7)
CLOSE(8)
CLOSE(9)
CLOSE(5)
STOP
END
Appendix 2: Table of experimental shear-angles data obtained from the method described in section 3.5 for the basket weave, tight plain weave, twill weave and satin weave

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<th>Satin weave</th>
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Experimental Studies and Mathematical Modelling of the Draping and Shear Deformation of Woven Fabrics in Resin transfer Moulding (RTM)
### Experimental Studies and Mathematical Modelling of the Draping and Shear Deformation of Woven Fabrics in Resin transfer Moulding (RTM)

|   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| 73 | 17 | 83 | 7 | 75 | 15 | 73 | 17 | 68 | 22 | 71 | 19 | 63 | 27 | 70 | 20 | 57 | 33 | 59 | 31 | 62 | 28 | 61 | 29 |
| 49 | 41 | 53 | 37 | 55 | 35 | 54 | 36 | 54 | 36 | 47 | 43 | 45 | 45 | 38 | 52 | 54 | 42 | 57 | 33 | 60 | 30 | 67 | 23 | 56 | 34 |
| 86 | 4 | 89 | 1 | 88 | 2 | 83 | 7 | 90 | 0 | 78 | 12 | 80 | 10 | 78 | 12 | 73 | 17 | 74 | 16 | 69 | 21 | 70 | 20 |
| 63 | 27 | 59 | 31 | 84 | 6 | 66 | 24 | 49 | 41 | 57 | 33 | 48 | 42 | 54 | 36 | 50 | 40 | 46 | 44 | 37 | 53 | 52 | 38 |
| 48 | 42 | 44 | 46 | 12 | 78 | 47 | 43 | 48 | 42 | 36 | 54 | 42 | 48 | 41 | 49 | 56 | 34 | 53 | 37 | 54 | 36 | 52 | 38 |
| 62 | 28 | 59 | 31 | 61 | 29 | 59 | 31 | 90 | 0 | 87 | 3 | 86 | 4 | 81 | 9 | 78 | 12 | 79 | 11 | 79 | 11 | 81 | 9 |
| 65 | 25 | 74 | 16 | 36 | 54 | 80 | 10 | 60 | 30 | 65 | 25 | 44 | 46 | 43 | 47 | 39 | 51 | 52 | 38 | 50 | 40 | 48 | 42 |
| 48 | 42 | 45 | 45 | 40 | 50 | 55 | 35 | 34 | 56 | 42 | 48 | 34 | 56 | 38 | 52 | 44 | 46 | 47 | 43 | 48 | 41 | 49 | 47 |
| 51 | 39 | 61 | 29 | 45 | 45 | 58 | 32 | 63 | 27 | 65 | 25 | 59 | 31 | 62 | 28 | 82 | 8 | 86 | 4 | 84 | 6 | 78 | 12 |
| 75 | 15 | 74 | 16 | 86 | 4 | 64 | 26 | 59 | 31 | 66 | 24 | 59 | 31 | 60 | 30 | 71 | 19 | 56 | 34 | 66 | 24 | 48 | 42 |
| 57 | 33 | 54 | 36 | 50 | 40 | 45 | 45 | 41 | 49 | 53 | 37 | 53 | 37 | 52 | 38 | 37 | 53 | 54 | 36 | 57 | 33 | 48 | 42 |
| 49 | 41 | 59 | 31 | 55 | 35 | 56 | 34 | 55 | 35 | 67 | 23 | 56 | 34 | 63 | 27 | 76 | 14 | 75 | 15 | 67 | 23 | 65 | 25 |
| 84 | 6 | 90 | 0 | 89 | 1 | 85 | 5 | 77 | 13 | 90 | 0 | 81 | 9 | 69 | 21 | | | | | | | |

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Table showing data for experimental studies and mathematical modelling of draping and shear deformation of woven fabrics in resin transfer moulding (RTM).
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**Bold:** Shear-angles in the diagonal direction used to generate the shear angle profiles in chapter 6