Performance of Multicell Joint Processing Planar Cellular Uplink in the Presence of Relay Nodes

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Abstract: In this paper we compare the uplink performance of various deployment scenarios in the planar (hexagonal) cellular system where the received signals at multiple cells are jointly processed by a central processor. A mathematical model of the planar cellular system is presented and its performance metric is based on information theoretic sum rate, and group rate share. The system deployment is as follows; the density of the base station is decreased, relay node replaces alternate base stations and in the last scenario, alternate bases stations are removed while the left over base stations are equipped with an additional antenna. It is observed that in reduction in the density of base stations resulted in reduction in the achievable sum rate and makes the group rate unfair. The introduction of additional antenna resulted in an improvement in the rate (compared with the reduction in density scenario) without any improvement in group rate fairness to cell user at the cell edge. The replacement of base stations with relay node and the implementation of orthogonal amplify and forward scheme resulted in a reduction in the achievable sum rate however it produced a significant improvement in group rate fairness. The result suggest that relay deployment in multicell joint processing system does not improve the sum rate of the system rather it improves the rate share for cell edge users making the user rate distribution fairer.

Keywords: Uplink, Group rate distribution, Sum rate, Relay

1. Introduction

The surge in high data rate services has evoked a need for performance improvement in wireless communications. Multicell joint processing, as originally proposed by Wyner [9] and Hanly-Whiting [3], is a promising approach to achieve higher spectrum efficiency in the cellular systems. Multicell joint processing eliminate the problem of inter-cellular interference thus leading to an improvement on the spectrum efficiency.

Wyner in [9], studies the benefits of multicell joint processing assuming an additive white Gaussian noise (AWGN) channel with no Rayleigh fading. Somekh and Shamai [7] extend Wyner's model by incorporating the narrowband single-tap Rayleigh fading. Extended models for progressively more practical considerations are studied in [5, 2]. They all show the potential improvements in spectrum efficiency when multicell joint processing is deployed.

Recently there has been an interest in the deployment of Relay nodes to improve the performance of the cellular systems and some studies were performed for the use of relay nodes in the context of multicell joint processing [6] where both Amplify and Forward [6] and Compress and Forward [8] schemes have been studied. Simeone et al [6] consider a TDMA cellular system and for keeping the model mathematically tractable they assume that the users in each cell have the same relative position with respect to the closes base station. Distance dependent path loss is modeled with a single system-wide parameter. The main finding is that the benefit of Amplify and Forward relaying is limited to the regime of low to moderate transmission rates.

In [4] we investigated sum-rate and user rate distribution of multicell joint processing in linear cellular system.In this paper we extend the linear cellular architecture in [4] to the planar architecture. The rest of the paper is organized as follows. The objective and methodology of this paper is presented in section 2 and 3 respectively. In section 4 the technology description is presented. Section 5 shows the calculation of the information theoretic sum-rate and group rates. In section 6 we discuss results for the four selected cases and 7 concludes the work.

2. Objective

The objective is to show that the deployment of relay node in planar multicell joint processing system does not improve the sum rate, rather makes the rate distribution for cell edge users fairer

3. Methodology

In this paper we extend our work on multicell joint processing linear cellular sysytem to a planar cellular system. Each of the cell is sub divided into groups based on their distance to the base station. All the base station are assumed to be connected to a central processor via a high capacity and delay-less back-haul.The relays are positioned at the vertices of the hexagonal shaped planar cellular model separated from the base station with a distance R equivalent to the radius of the cell. We consider a narrowband single tap wireless channel, the information theoretic sum-rate and group rate are estimated for the four cases to compare their performance

4. Technology Description

A hexagonal cellular array with N cells is considered. We assume that each of the cells has K users which are randomly distributed within the cell coverage and that all the K users are served simultaneously .We also assume that each of the Base Station (BS) and User Terminal (UT) are equipped with only one antenna. In our model, each cell is sub divided into Z groups based on the distance to the base station. We also introduced scanning order which depict the decoding order that is to be implemented. Figure 1 illustrates the scanning order implemented. Scanning starts from cell 1 and runs in an anti clockwise direction through the first group represented by b_i with z = 0, followed by the next layered group, z = 1 up to when z = Z-1

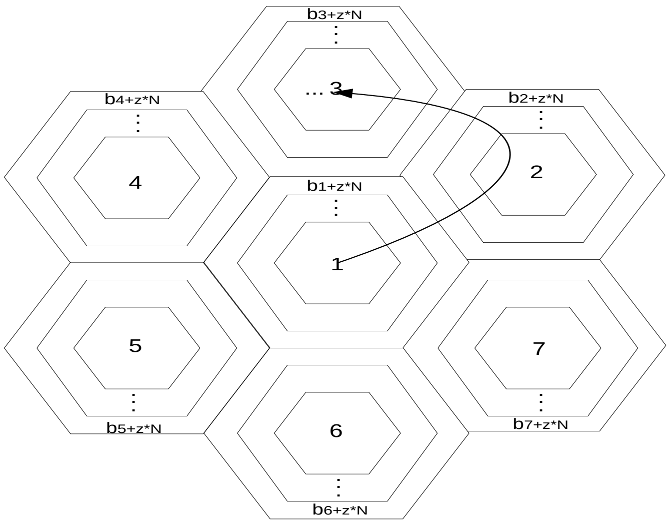


Figure 1: Scanning order for joint processing of planar cellular uplink

The channel model is based on the group classification. The signal transmitted from the user terminal must fulfill the transmit power constraint given by $\mathrm{E}[xx^h] \leq P$ where E the expectation of random variables and P is the power constraint at each source. The transmitted signal from all source nodes are stacked into a single vector $\mathbf{x}$ where $\mathbf{x} = [x_1 x_2 x_3 ... x_{NK}]^T$ $\mathbf{x}\ \varepsilon\ C^{NK}$

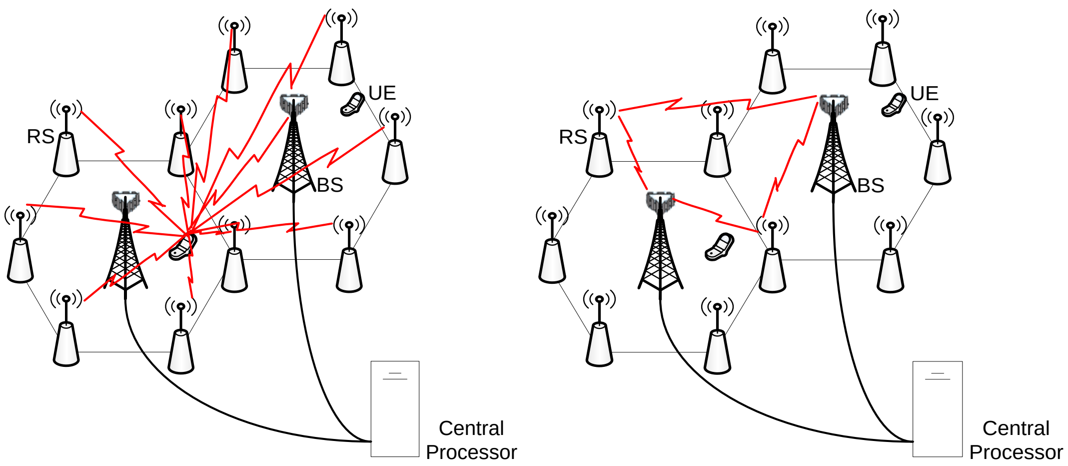
The channel from each source node to the destination node is given by h_{izd} .The channel matrix for each group is given as $\mathbf{H}_f$ .where $f = b_i + zN$ With the implementation of the scanning order in figure 1 the overall channel matrix is expressed as;

$\mathbf{H} = [\mathbf{H}_1\ \mathbf{H}_2 \ldots \mathbf{H}_{ZN-1}\, \mathbf{H}_{ZN}]$ $, \mathbf{H}\ \varepsilon\ C^{NK}$ The overall received vector at the destination node can be expressed as:

$$\mathbf{y} = \mathbf{Hx} + \mathbf{w} \qquad , \mathbf{y}\ \varepsilon\ C^N \tag{1}$$

Where $\mathbf{w}\ \varepsilon\ C^N$ is the Additive White Gaussian Noise (AWGN) vector at the destination nodes with zero mean and variance σ_0^2.The covariance matrix of the noise vector $\mathbf{R_w} = E[\mathbf{ww}^h] = \sigma_0^2 \mathbf{I}_N$ where $\mathbf{I}_N$ is an N x N identity matrix.

In modeling the channel gain h_{izd} two factors are considered: one factor models the path loss experienced by the signal over the transmission distance while the other factor models the Rayleigh fading. In this work we assume a narrow band, single-tap flat Rayleigh fading channel. Thus h_{izd} . Is expressed as: $h_{izd} = \psi_{izd} * g_{izd}$ where g_{izd} is the flat fading Rayleigh coefficient, normalized to unit power and circularly symmetric i.i.d. Gaussian. ψ_{izd} is the path loss factor obtained from the inverse power law $\psi_{izd} = L_0 (1 + D_{d,s})^{-\eta/2}$.; Where L_0 is the loss at the reference distance, $D_{d,s}$ is the distance between the destination and the reference distance at the source and η is the path loss exponent.The log-normal shadowing effect is not considered in our system model



UT transmit, BS and relay receives	UT silent, relay forward data

Figure 2: System scenario and two phases of uplink transmission

The direction of transmission from the source node is omnidirectional, and it is received by several base stations at the absence of relay nodes. The joint decoding is performed at the central processor. The mathematical representation for this scenario with N base stations and NK user terminals is given in (1). The sum capacity with joint receiver at the receiver can be expressed as:

$$C_{sum} = \mathrm{E}(\log_2 \det(\mathbf{I}_N + \mathbf{HQ}_x\mathbf{H}^h\mathbf{R}_{\mathbf{w}}^{-1}) \qquad (2)$$

$\mathbf{Q}_{\mathbf{x}}$ and $\mathbf{R}_{\mathbf{w}}$ are the covariance of the input vector $\mathbf{x}$ and the noise vector $\mathbf{w}$ respectively. Fig. 2 is an illustration of the system model.

At the introduction of relay nodes, we implement the orthogonal amplify and forward scheme which results in the use of two channel resources for the transmission from the source node to the destination node. Consequently, the user terminal increase their transmit power to compensate for not transmitting on the second channel resource. Users terminals transmits vector $\mathbf{x}_u$ on the first channel resource which is received by all nearby relay nodes and base stations. The received vector at the base station in this phase is given by:

$$\mathbf{y}_1 = \mathbf{H}_{BU}\mathbf{x}_u + \mathbf{w}_{B1} \qquad (3)$$

Where $\mathbf{w}_{B1}$ is a N x 1 vector of the Gaussian noise at the base station array in this phase. $\mathbf{H}_{BU}$ is the matrix that represents the direct channel from the user to the base station and is formed from the group channel matrix following the earlier introduced scanning order

$$\mathbf{H}_{BU} = [\mathbf{H}_{BU1}\ \mathbf{H}_{BU2} \ldots \mathbf{H}_{BU,ZN-1}\ \mathbf{H}_{BU.ZN}] \qquad ,\mathbf{H}_{BU}\ \varepsilon\, C^{N*NK}$$

The received signal vector at the relay node is given as

$$\mathbf{y}_1^R = \mathbf{H}_{RU}\mathbf{x}_u + \mathbf{w}_R \qquad (4)$$

Where $\mathbf{w}_R$ is a L x 1 vector of the Gaussian noise at the relay array (L being the number of relay nodes). $\mathbf{H}_{RU}$ matrix represents the channel between the user terminals and the relay nodes .and it's also formed based on the scanning order.

$$\mathbf{H}_{RU} = [\mathbf{H}_{RU1}\ \mathbf{H}_{RU2} \ldots \mathbf{H}_{RU.ZN-1}\ \mathbf{H}_{RU.ZN}] \qquad ,\mathbf{H}_{RU}\ \varepsilon\, C^{L*NK}$$

The signal received at the relay node is then retransmitted to the base station.

$$\mathbf{y}_2 = \mathbf{H}_{BR}[\mathrm{M}(\mathbf{H}_{RU}\mathbf{x}_u + \mathbf{w}_R)] + \mathbf{w}_{B2} \qquad (5)$$

Where $\mathbf{w}_{B2}$ is a N x 1 vector of the Gaussian noise at the base station array in this phase. $\mathbf{H}_{BR}$ is the channel matrix between the relay node and the base stations. $\mathbf{H}_{BR}\ \varepsilon\, C^{N*L}$. M is a diagonal matrix of size LxL containing individual relay amplification factor μ_j.

$$\mathrm{M} = \begin{bmatrix} \mu_1 & \cdots & 0 \\ \vdots & \ddots & 0 \\ 0 & 0 & \mu_l \end{bmatrix}$$

Each relay amplification factor must meet the transmit power constraint of the relay node given by: $\mathrm{E}[\mathbf{x}_{Ri}\mathbf{x}_{Rj}^h] \le P_{Rj}$ where $\mathbf{x}_{Rj} = \mu_j \mathbf{y}_{1j}^R$, $\mathbf{y}_{1j}^R$ is the received vector at the j^th relay, P_{Rj} is the power constraint that must not be exceeded at the j^th relay station. The amplification factor the j^th relay based on the power constraint is given by:

$$\mu_j^2 \le \frac{P_{Rj}}{tr(P\sum_{i=1}^{NK}(h_{ru}h_{ru}^h) + N_0)} \tag{7}$$

5. Performance Evaluation

The sum capacity expression given in (2) applies for cases without relay node. For cases with implementation of relay node, the overall received signal at the base station is obtained by stacking the relayed signal vector and the direct signal vector from user terminals which are then jointly decoded. Thus (3) and (6) are stacked together resulting in:

$$\begin{bmatrix} \mathbf{y}_1 \\ \mathbf{y}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{H}_{BU} \\ \mathbf{H}_{BU}\mathrm{M}\mathbf{H}_{RU} \end{bmatrix} \mathbf{x}_u + \begin{bmatrix} \mathbf{w}_{B1} \\ \mathbf{H}_{BR}\mathrm{M}\mathbf{w}_{BR} + \mathbf{w}_{B2} \end{bmatrix} \tag{8}$$

Which can also be expressed as

$$\overline{\mathbf{y}} = \overline{\mathbf{H}}\mathbf{x}_u + \overline{\mathbf{w}} \tag{9}$$

The maximum achievable sum rate is given as:

$$R_{OAF} = \frac{1}{2}\log\det(\mathbf{I}_{2N} + 2P\overline{\mathbf{H}}\,\overline{\mathbf{H}}^h\mathbf{R}_{\overline{\mathbf{w}}}^{-1}) \tag{10}$$

The factor of half comes as a result of the use of two channel resources while the $2P$ compensates for the non transmission of the user on the second channel resource. The overall noise covariance matrix $\mathbf{R}_{\overline{\mathbf{w}}}$ is given as

$$\mathbf{R}_{\overline{\mathbf{w}}} = \sigma^2 \begin{bmatrix} \mathbf{I}_N & \mathbf{O}_N \\ \mathbf{O}_N & \mathbf{I}_N + \mathbf{H}_{BR}\mathrm{M}^2\mathbf{H}_{BR}^h \end{bmatrix} \tag{11}$$

Where $\mathbf{I}_N$ and $\mathbf{O}_N$ are (N x N) identity matrix and null matrix respectively.

In this model we also examine the group sum rate and the group user rate. The group sum rate defines the sum of users rate in a group, while is the group user rate is the average achievable rate of a user in one of the earlier defined groups. We obtain the group sum rate by following the decoding order in illustrated in figure 1.This involves decoding groups that are at the edge of the cells first while groups closer to the base station are decoded last. Users in group f = 1 are decoded last and they experience no interference since users in the other group have been decoded and removed from the received signal. The sum group rate for users in group f:

$$r_f = \log\det\left(\frac{\mathbf{I}_{2N} + 2P\sum_{j=1}^{f}\mathbf{h}_j\mathbf{h}_j^h\mathbf{R}_{\overline{\mathbf{w}}}^{-1}}{\mathbf{I}_{2N} + 2P\sum_{j=1}^{f-1}\mathbf{h}_j\mathbf{h}_j^h\mathbf{R}_{\overline{\mathbf{w}}}^{-1}}\right) \tag{12}$$

The group user rate is given by $r_g = \dfrac{r_f}{no\ of\ users\ in\ group}$

6. Results

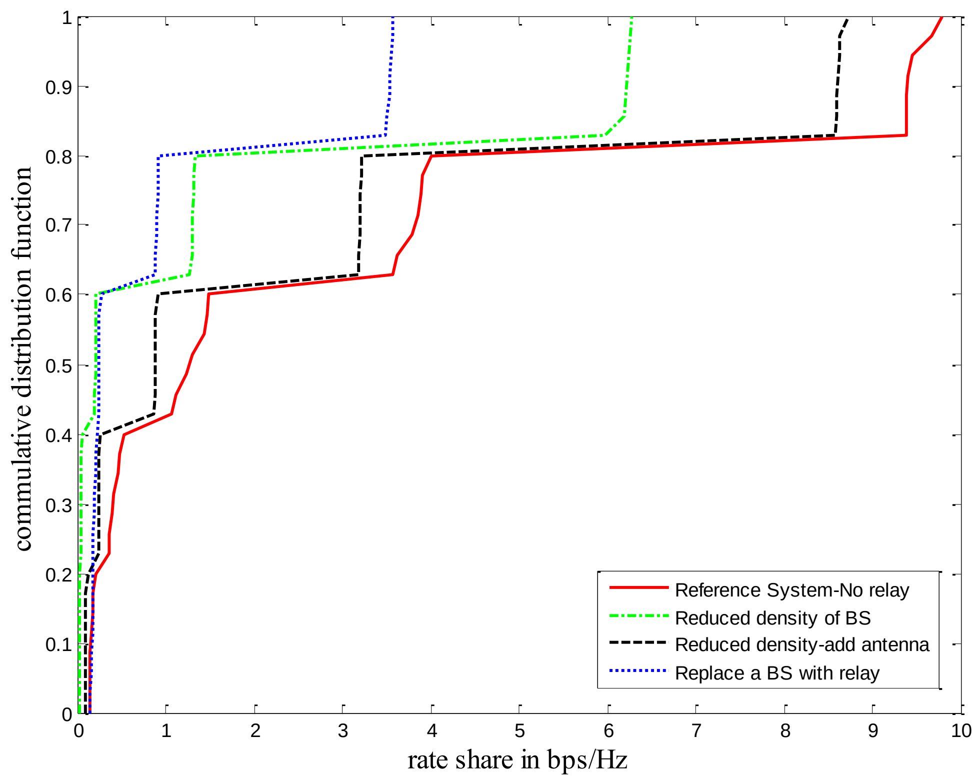


Figure 3: Comparison of rate share CDF for various planar cellular deployments.

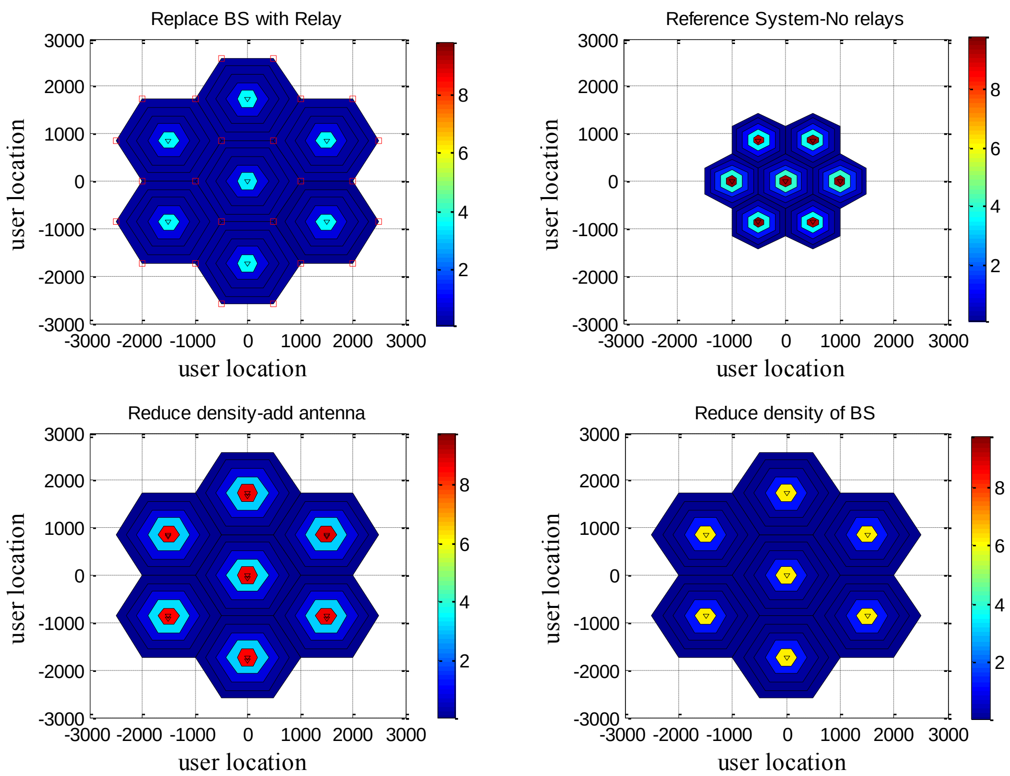


Figure 4: Group user rate as a function of user location for various cellular deployments.

We simulate a hexagonal cellular system with N = 7 base stations each with a coverage radius R = 1Km. We sub-divide each of the cells into 5 groups based on their distance to the base station [0-R/5,R/5-2R/5, 2R/5-3R/5, 3R/5-4R/5,4 R/5-R].Users are randomly distributed over the entire cellular coverage with a density of 9/q users/Km^2 (area of hexagon is qR^2). The transmit power constraint of the user terminals and the relay nodes are 100mW and 1W respectively. A path loss exponent of η = 4 and a path gain $L_0 = -38dB$ at a reference distance of 1m is considered to correspond to empirical models. Noise spectral density $N_0 = -169dBm$ is assumed and a subcarrier of narrow band spectrum (W=50 KHz) is considered where the noise power is calculated as $\sigma^2 = N_0 W$. Four different cases are considered in the simulation which are: Case A represents the reference system with a radius $R/\sqrt{3}$ (No relay), and its achievable sum rate calculated in (2). Case B represent a scenario with reduced density of BS (radius = R) and the achievable sum rate expression remains same. In Case C (radius = R) the base station is equipped with two additional antennas to compensate for increase in coverage area of the base station. Finally, in Case D the removed base station is replaced with a relay node and its achievable sum rate is given in (10).

In Fig. 3 we compare the rate share CDF of the four cases of the multicell joint processing of planar cellular system considered. It could be observed that 60% of the users in case D had higher rate than the users in Case B, 20% of users in case D experience higher rate than users in case C and about 20% of users in case D rates fairly equal to that of case A.

In Fig. 4 the group user rate is plotted as a function of user location for the four cases considered. It could be observed that for cases A, B and C the group user decreases from its peak at the centre of the cell to its minima at cell edges. Reducing the density of base station resulted in a reduction in the group rate while the addition of antenna tries to compensate for the reduced rate. Replacing of base station with relay produced a fairer distribution in group rate compared to the other cases by providing higher rates to users closer to the cell edges.

Comparing the results in Fig 3 and 4 showed that replacing of base station with relay resulted (case D) in higher rate at the cell edges than the cell edges of case B, C and almost a similar performance with the cell edges in A. At regions very close to the base station, the group user rate for case A, B and C is about twice that of case D

7. Conclusion

In this paper we have investigated the performance of joint processing cellular uplink for the planar cellular array by analyzing various deployment conditions. The information theoretic sum rate, group sum rate and group user rate share are derived and calculated for the various cases. It was observed that decreasing the density of the base station resulted in the loss in achievable sum rate. Equipping the base station with additional antenna tries to compensate for the rate lost as a reduction in base station density. It was observed that the sum rate when the relay was introduced was much lower than that without relay. However it was observed that introduction of relay resulted in a fairer distribution of the user rate. The work suggests that the relay nodes can play an important role to reshape the user rate shares in the cellular environment even in the case where the base stations are fully cooperating for the decoding of the uplink signals

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