Downlink Sum-rate Performance for Cooperation and Coordination between Three Interfering Sectors

Oluwakayode Onireti, Muhammad Ali Imran, Reza Hoshyar
Centre for Communications Systems Research, University of Surrey
Guildford, United Kingdom
email: {o.onireti, m.imran, r.hoshyar}@surrey.ac.uk

***Abstract*— In this paper we investigate the performance of various transmission schemes for the downlink cellular system. These schemes are either biased towards interference minimisation or the efficient utilisation of the available resources. A mathematical model for the various schemes is presented and the performance measure is based on the information theoretic sum rate and the user rate share. The identified schemes are: avoid intra-cell and tolerate inter-cell interference, full orthogonality, single cell cooperation, cooperation for critical users only and the full cooperation scheme. Two categories of users are identified in this paper: non critical users which are close to the base station and the critical user which are at the cell boundary. It is observed that the full cooperation provides an upper bound on the achievable sum rate and has a user rate distribution whose fairness can be improved by allocating more power to the critical users. The full orthogonality scheme has the fairest user rate distribution and much lower achievable sum rate compared with the full cooperation. The performance of the cooperation of critical user scheme can approach the upper bound of full cooperation scheme with lower complexity.**

***Index Terms*— Cellular, Sum rate, User rate share, Cooperation, Downlink.**

I. INTRODUCTION

Multi-user downlink communication has received a lot of interest in recent time. One key area of interest is on intra-cell and inter-cell interference management through cooperation and coordination. A scheme to achieve an interference free system is via exclusive use of channel resource by partitioning of the resource to create orthogonality which however degrades the spectral efficiency. Transmission schemes that do not perform exclusive use of channel resources share the same degree of freedom and thus experience interference [1]. The downlink is modeled as a broadcast channel with one transmitter sending independent signals to multiple receivers.

The capacity region of a degraded broadcast channel (single antenna at the transmitter) can be achieved by interference subtraction at the receiver [2]. Costa [3] showed that the capacity of a channel with additive Gaussian noise remains unchanged in the presence of an interference signal as long as full knowledge of the interfering signal is available at the transmitter. This concept is referred to dirty paper coding (DPC). The DPC concept as been extended to broadcast channel [4], [5], [6], [7].

In [7] the authors showed that capacity region of broadcast channel is also achievable for both degraded and non degraded broadcast channel using DPC. In [6] the authors showed that DPC strategy achieve the capacity for Multiple-input multiple output broadcast channels (MIMO-BC). The duality concept between the MIMO multiple access channel (MIMO-MAC) and the MIMO-BC [5], [8] leads to solving the non convexity of MIMO-BC capacity region by transforming it to convex dual MIMO dual-MAC.

Shamai et al [9] work on linear cellular architecture shows that, central encoding system with dirty paper coding produced a better performance than local processing (no cooperation) at each base station. In this work we introduce the limited cooperation concept which involves the cooperation of only the critical users and compare performance with other schemes.

We consider the interfering sectors of three adjacent cells in a cellular system. Assume a scenario of uniformly distributed users in each cell where user terminals are served by their closest base station which is named as a “serving BS”. Focusing on the downlink channel and assuming aggressive reuse of frequency (same frequency used in these three cells),inter-cell interference result when signals transmitted to users in a cell interferes with the signal transmitted to other adjacent cell users by their serving BS. In addition to the inter-cell interference, the user within a single sector of a cell can also interfere if they are forced to use the same resources resulting in intra-cell interference. To tilt the trade-off between “higher efficiency due to non-exclusive use” and “lower rate due to interference” require an efficient management of interference.

Consider a general scenario as shown in Fig. 1. The downlink signal power required to achieve a certain signal to noise ratio at the receiver end is much smaller for a user close to the serving BS. These users are termed non-critical users. In contrast, the users located towards the boundaries of the coverage area require a larger transmit power and are also more susceptible to receive strong interference from the neighbouring BS. These users are labelled critical users. Since

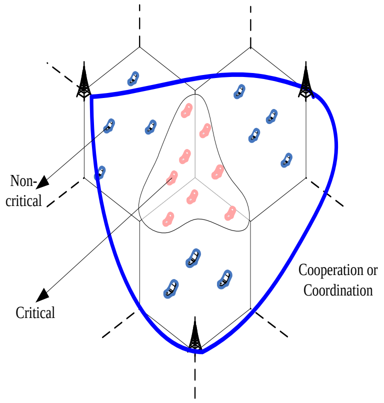

Fig. 1: Critical and Non-critical users

interference management will incur some overhead (as it requires control messages) the system may focus on managing it only where extremely necessary. In certain scenarios, it may suffice to confine ourselves to manage the service to the critical users only, in order to control the main source of interference in the system.

We impose a constraint on non-exclusivity that no more than two users within a sector share same resource. For a given user snapshot this constraint can be met by scheduling. Hence from the given set of users we can select a pair of users to be served possibly on a frequency resource unit. In order to keep the interference for at least one member of the pair weak as possible, we pair a critical user with a non critical one depicted in Fig. 2.

The inter-site distance is represented by D and the distance of each site from the common boundary of the three sectors is given by d . Assuming that the non-critical user is at a distance βd from the serving base station and the critical user is at the same distance from the common boundary of the three sectors. The relation between the parameter d and D follow from the simple geometric arguments and is given as $D/2 = d\cos(\pi/6)$.

II. CHANNEL MODEL

We index the base station (BS) by variable n such that $n\,\varepsilon\{1,2,N\}$, with $N=3$ for the considered scenario. Users are indexed by k . Consider that there are two users in each cell in the above scenario we have $k\,\varepsilon\{1,2,\ldots K\}$, with $K=6$. We assume single antenna at the user terminal and the base stations. The wireless channel from each BS transmitter to each user receiver is given by $h_{k,n}$ and consists of two factors as expressed as $h_{k,n} = \varsigma_{k,n} g_{k,n}$. Where $\varsigma_{k,n}$ represents the distance dependent path gain (loss since it is less than 1) and $g_{k,n}$ represents the random complex channel gain for the narrow band Rayleigh fading i.e. $g_{k,n} \sim CN(0,1)$. Rayleigh fading is assumed to be i.i.d. for each link.

The received signal at each user indexed by k is given as:

$$y_k = \sum_{n=1}^{N} h_{k,n} x_n + w_{k,n}$$

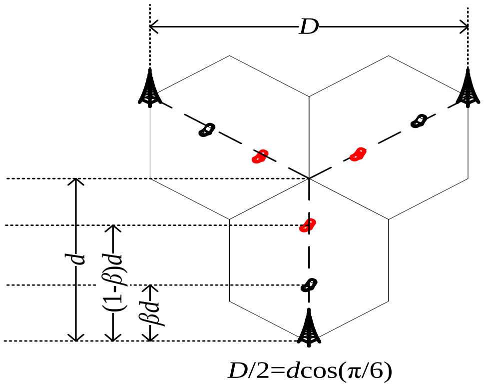

Fig.2: Scenario Considered

where $w_{k,n} \sim CN(0,\sigma^2)$ represents the complex Gaussian noise at the receiver of each user with zero mean and noise variance σ^2. x_n is the aggregated message transmitted by BS n .

The objective for each user to decode its intended message in the presence of noise and the unwanted interfering signals transmitted to the other user in the system.

In light of information theoretic arguments, the maximum rate at which error free communication can be maintained depends on the ratio of signal power to the unwanted interference and noise power, referred to as SNIR. If the transmitted signal power for each user is given as $P_{k,n}$ in cell n then the signal to noise plus interference ratio (SNIR) for the k^{th} user's receiver is given as:

$$\gamma_{k,n} = \frac{\left|h_{k,n}\right|^2 P_{k,n}}{\left(\sum_{m\neq n}\left|h_{k,m}\right|^2 P_m + \sum_{j\neq k}\left|h_{k,n}\right|^2 P_{j,n}\right)+\sigma^2} \qquad (1.1)$$

With a power constraint such that $P_n \le P$ .The information theoretic user rate share (bit/sec/Hz) for the user k will be given as: $r_{k,n} = \log_2(1+\gamma_{k,n})$ (1.2)

And the sum rate for the system is given by

$$R_{sum} = \sum_{n=1}^{N}\sum_{k=1}^{K} r_{k,n}$$

III. TRANSMISSION STRATEGIES

Several transmission strategies are possible in order to meet two competing objectives: keeping the interference low and efficient resource utilisation. Some strategies are biased towards the first objective and others are biased towards the second. Several such schemes are compared against a scheme which is known to be optimal (but most complex) scheme. This optimal scheme provides upper-bound for bench marking the performance of the other schemes. However, with current hardware capabilities and encoding and decoding time constraints, this scheme is not practically feasible.

A. *Avoid intra-cell and tolerate inter-cell interference*

In this scenario the resource are split into two partitions to ensure the exclusive use within each cell while the same resource is reused in all three neighbouring cells. Intra-cell interference is eliminated in this scheme. Assuming two orthogonal resources unit are used and the power allocated to the exclusive user of each resource unit in each cell is given by the max power constraint of that cell i.e. P_n. In this scenario $P_n = P_{k,n}$. The sum rate is given as:

$$R_{sum} = \frac{1}{2}\sum_{n=1}^{N}\sum_{k=1}^{K}\log_2\left(1+\frac{\left|h_{k,n}\right|^2 P_{k,n}}{\sum_{m\neq n}^{N}\left|h_{k,m}\right|^2 P_m + \sigma^2}\right) \tag{1.3}$$

B. *Full orthogonality*

In this case the available degrees of freedom (time and frequency) are split into the total number of users that we are attempting to serve simultaneously in the whole cellular system i.e. K. Both inter-cell interference and intra-cell interfe rrence do not exist in this scenario. We assume that the total power is allocated to the exclusive user of each resource unit which implies that $P_n = P_{k,n}$. And the sum rate is given as:

$$R_{sum} = \frac{1}{K}\sum_{n=1}^{N}\sum_{k=1}^{K}\log_2\left(1+\frac{\left|h_{k,n}\right|^2 P_{k,n}}{\sigma^2}\right)$$

(1.4)

C. *Single Cell Cooperation (user pairing)*

In this scenario users tolerate the inter-cell interference and the intra-cell interference. Same frequency resource is used in all the three cells resulting in inter-cell interference. Even within a given cell we allocate the same frequency resource (e.g. a subcarrier) to two users with diverse channels (one close to the BS and the other at the edge of the cell) and therefore split the transmit power between them. One user is encoded first and has to tolerate the interference from the other user but the user that is encoded second can pre cancel the known interference caused by the user within the same cell that was encoded first. For any given cell n the individual user SINR is given as:

$$\gamma_{k,n\,\varepsilon\,\kappa_1} = \frac{\left|h_{k,n}\right|^2 P_{k,n}}{\left(\sum_{m\neq n}^{N}\left|h_{k,m}\right|^2 P_m + \left|h_{k,n}\right|^2 P_{j,n}\right) + \sigma^2} \tag{1.5}$$

$$\gamma_{k,n\,\varepsilon\,\kappa_{n2}} = \frac{\left|h_{k,n}\right|^2 P_{k,n}}{\sum_{m\neq n}^{N}\left|h_{k,m}\right|^2 P_m + \sigma^2} \tag{1.6}$$

where $\gamma_{k,n\varepsilon\kappa_1}$ is the SINR of the first encoded user while $\gamma_{k,n\varepsilon\kappa_2}$ represents the SINR of the second encoded user after interference pre-cancelling. $P_{k,n}$ and $P_{j,n}$ are the transmit power split for users in the cell. The sum rate is given as:

$$R_{sum} = \sum_{n=1}^{N}\sum_{k=1}^{K}\log_2\left(1+\gamma_{k,n}\right)$$

D. *Cooperation for critical users only*

We assume that encoding for critical users can be jointly performed with the knowledge of channel state information of all critical users. The codebooks for the non critical users are designed to pre-subtract the joint codeword of the critical users. Let κ_c represent the set of critical users and κ_{nc} represents the set of non-critical users. π_c be the permutation of set κ_c. Let the encoding order be such that the critical user's signal are encoded first followed by the signal for the non-critical users. The SINR for each critical user is given by:

$$\gamma_{\pi_c(k)} = \frac{\left|h_{\pi_c(k),n_k}\right|^2 P_{\pi_c(k)}}{\left(\sum_{j\varepsilon\kappa_{nc}}\left|h_{\pi_c(k),n_j}\right|^2 P_j + \sum_{j=k+1}^{\left|\kappa_c\right|}\left|h_{\pi_c(j),n_j}\right|^2 P_j\right) + \sigma^2}$$

And for each non-critical user is given as

$$\gamma_{(k)} = \frac{\left|h_{k,n}\right|^2 P_{(k)}}{\sum_{j\varepsilon\kappa_{nc\backslash k}}\left|h_{k,n_j}\right|^2 P_j + \sigma^2} \tag{1.7}$$

Notation n_j identifies the "serving BS" for user j.

In this scenario each non-critical user tolerates the interference of all other non-critical users simultaneously using the same resource in the neighbouring sectors. The sum rate is given as:

$$R_{sum} = \sum_{k\varepsilon\kappa_c\cup\kappa_{nc}} r_{k,n} \tag{1.8}$$

E. *Full Cooperation*

For this benchmark scheme all base station jointly transmit signal for each user terminal. User signal are encoded in a specific order. The user which is encoded first in the order does not have any prior information for the user signal to be encoded for the others following in the sequence of encoding. However each subsequently encoded user can take care of the prior encoded signals (or in other words pre-cancels the known interference at the encoding end). In this manner the user which is encoded last has effectively gotten rid of all other user's interference signal and its receiver enjoys the best signal to noise ratio and highest rate share.

Since the sum rate of the system is given by the dual multiple access channel (dual MAC) as detailed in [5,8]. The channel matrix for the dual-MAC is given by $\mathbf{H}$ where entry at n^{th} row and k^{th} column, is given as $h_{k,n}$. The sum rate (for actual downlink broadcast channel and uplink dual-MAC) is given as $R_{sum} = \log\det\left[\sigma^2\mathbf{I} + \sum_{k=1}^{K} q_k \mathbf{h}_k \mathbf{h}_k^h\right]$ where $\mathbf{I}$ is an identity matrix of dimension NxK and q_k is the power transmitted by the user k in the dual uplink. The channel vector $h_{k,n}$ is the k^{th} column of dual-uplink channel matrix $\mathbf{H}$.

For a specified user decoding order π where $\pi = [\pi(1), \pi(2), \ldots \pi(K)]$ is a permutation of $\{1,2,\ldots K\}$, the user rate share for each user at position k in the decoding order is given as:

$$r_{\pi(k)} = \log_2 \frac{\det\left[\sigma^2 \mathbf{I} + \sum_{j=1}^{k} P_{\pi(j)} h_{\pi(j)} h_{\pi(j)}^{h}\right]}{\det\left[\sigma^2 + \sum_{j=1}^{k-1} P_{\pi(j)} h_{\pi(j)} h_{\pi(j)}^{h}\right]} \qquad (1.9)$$

According to the duality principle [4] there exist a set of downlink power $\{p_k\}$ that can achieve the same rate K-tuple as the dual-MAC while satisfying the same power in the two systems.

The maximum sum power provides us the information theoretic capacity of such a system however following inherent assumptions are worth noting. The channel state information is assumed to be known at the base station transmitters and noise statistics at the BS receiver and user terminal receiver are assumed to be identical. Finally since this is a theoretic upper bound it does not suggest any practical modulation and coding scheme that will achieve this rate but only provides the maximum value that can be theoretically achieved with zero error probability.

TABLE 1: TRANSMISSION SCHEMES CONSIDERED

Scheme label	Scheme Description
1	Avoid Intra-cell and tolerate inter-cell interference
2	Full orthogonality
3	Single cell cooperation (user pairing)
4	Cooperation for Critical Users Only
5	Full Cooperation

IV. RESULTS AND INSIGHTS

We assume that the total power within each cell is constrained by $P_n = P$ and it is split between the critical and non-critical users in each cell by a factor $0 \leq \delta \leq 1$, such that the share of the critical user is $P_{k,n\varepsilon\kappa_c} = \delta P_n$ and the share for the non critical user is $P_{k,n\varepsilon\kappa_{nc}} = (1-\delta)P_n$ .As a starting point we take the power share factor of 0.5. We distribute the user as shown in Fig. 2 with $\beta = 0.2$. The total power constraint for each base station antenna is 50W. Single tap Rayleigh fading channel is used with the following parameters for path loss: $\eta = 3.5$, $L_0 = -34.5dB$, $W = 100KHz$, $N_0 = -169dBm$ with $\sigma^2 = N_0 W$. The inter-site distance is varied from 100m to 5km.In our first simulation we set $\delta = 0.5$ sharing the power equally between the critical and non critical users. The sum rate of the system in bits/sec/Hz is plotted against the inter-site distance, see Fig. 3. Following observation can be made:

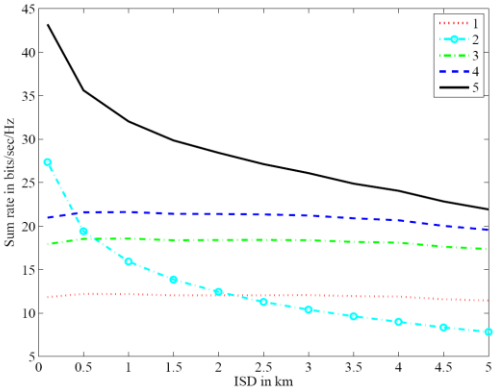

Fig 3: sum rate plotted against the inter-site distance. Power equally shared between the critical and non-critical users. (Refer to Table 1)

- The full cooperation scheme provides an upper bound on the achievable sum-rate. However, this scheme requires a huge amount of control signalling data for joint encoding of the signals.

- The sum rate of full cooperation scheme and full orthogonality scheme decreases for the increasing inter-site distance. This is because the cell size increases and due to larger path loss (smaller path gains) the received power is reduced while the noise power stays the same.

- For very high inter-site density (less than 500m), the full orthogonality scheme perform better than any other reuse scheme except for the full cooperation.

- For a wide range of inter-site distances, the scheme where we cooperate only for the critical user performs better than all other schemes except the global cooperation. The gap between the global cooperation and the cooperation for the critical users only, closes down for larger inter-site distances.

- Pairing scheme, where cooperation is performed on the single cell level, perform close to the scheme where cooperation for the critical user performed. This type of cooperation is easier to perform in the sense that the need for exchanging message between the physically dispersed base stations is reduced.

- The schemes 1,3,4 show an interference limited behaviour where the inter-site distance does not significantly affect the sum rate of the system since the positive impact due to the reduced inter-site interference is approximately balanced by the negative impact of larger path loss in the channel.

- The tolerance of inter-cell interference becomes feasible when the inter-site distance is relatively large

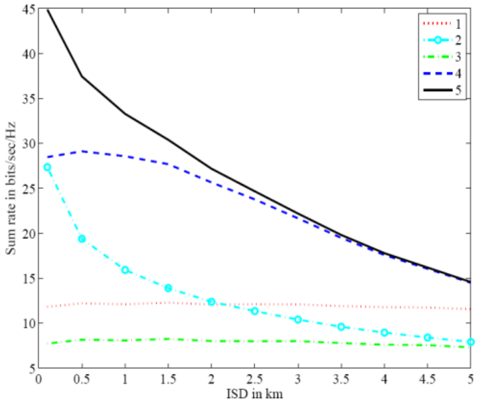

Fig.4: Sum rate plotted against the inter-site distance. Only 1% power allocated to the non critical users and the rest (99%) allocated to the critical ones. (Refer to Table 1)

In the second simulation we set $\delta = 0.99$ giving the critical user of each cell a power share of 99% and the rest of the power (1%) to the non-critical users. The sum rate of the system in bit/sec/Hz is plotted against the inter-site distance, see Fig. 4. Similar to the previous case following additional observation can be made:

- Cooperation for critical users is better scheme and approaches the upper bound of full cooperation scheme when the inter-site distance is large.

- The full orthogonal scheme and the intra-cell orthogonal schemes are not affected by power splitting

We also calculate the user share for all 6 users in the system equal power sharing approach as presented in Fig. 5 (rate shares of user stacked in each bar and the total height of the bar represents the sum rate. From top to bottom we have the rate share for a critical user and non critical user of one cell and then the next cell in the same order and so on) that:

- The scheme with full orthogonality has the fairest distribution of rate share among all six users

- Cooperation schemes can significantly increase the sum rate of the system at a small cost of reducing some rate for the critical users but this approach results in a relatively unfair distribution of user rates.

For unequal power share in Fig. 6 (99% power allocated to the critical users and 1% for non critical users), the following additional observation are made.

- For the case of cooperation, the rate share for the critical users improved at the cost of the share of the non-critical users when compared to the distribution of rates in case of equal power case. Even in the case of full cooperation a reasonably fair distribution is observed

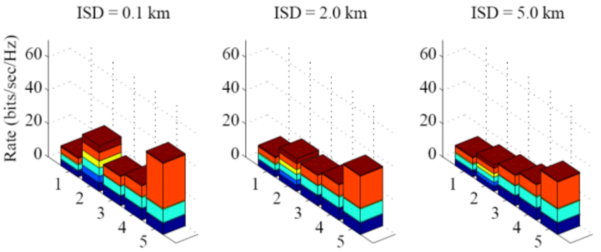

Fig.5: user rate share with equal power share for critical and non critical users

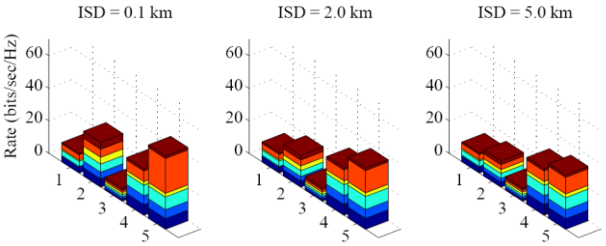

Fig. 6: User rate share with 1% of power allocated to the non critical users and the rest (99%) allocated to the critical ones.

V. CONCLUSION

The main conclusion of this study is that cooperation between the downlink transmitters of interfering cells is an effective measure to manage interference. Since cooperation requires a high volume of control and other signal to be exchanged between the base stations, limited cooperation may provide a more practical suboptimal alternative. In this regard it is observed that the cooperation between the cells only for the encoding of the critical users can harness a large portion of the cooperation gain. Cooperation within the cell is useful especially if we intend to increase the share of the critical users at the cost of non-critical users for making the rate sharing more fair.

VI. REFERENCES

[1] R. H. Etkin, D. N. C. Tse, H. Wang, "Gaussian interference channel capacity to within one bit," *IEEE Trans. Inform. Theory*, vol. 54, no. 12, pp. 5534-5562, Dec. 2008.

[2] T. Cover, "Broadcast channels," *IEEE Trans. Inform. Theory*, vol. 18, no 1, pp. 2-14, Jan. 1972

[3] M. Costa, "Writing on dirty paper," *IEEE Trans. Inform Theory* vol.29, no.3, pp. 439-441, 1983.

[4] N. Jindal, S. Vishwanath and A. Goldsmith "On the duality of Gaussian multiple-access and broadcast channels," *IEEE Trans. Inform. Theory*,vol. 50, pp. 768-783, May 2004.

[5] S. Vishwanath, N. Jindal and A. Goldsmith "Duality, achieving rates and sum rate capacity of Gaussian MIMO broadcast channels," *IEEE Trans. Inform Theory* Vol. 49, no. 10, pp. 2658-2668, Oct. 2003.

[6] H.Weingarten, Y. Steinberg, and S. Shamai (Shitz), "The capacity region of the Gaussian multiple-input multiple –output broadcast channel*,"* *IEEE Trans. Inform. Theory*, vol. 52, no. 9, pp 3936-3964, Sep. 2006.

[7] W. Yu, J. M. Cioffi, "Trellis precoding for broadcast channel," *in Proc. Vehicular Technology Conf*., May 2000, pp. 2888-2892

[8] D. Tse and P. Viswanath. "Sum capacity of the vector Gaussian broadcast channel and uplink downlink capacity duality," *IEEE Trans. Inform Theory*, *Vol*. 49, no.8, pp. 1912-1921, Aug. 2003.

[9] S. Shamai, O.Simeone, O.Somekh, and H.V. Poor "Joint multicell processing for downlink channels with limited backhaul," *IEEE Trans. Inform Theory and Applic. Workshop*, pp. 345-349, 2008.